Galaxy Clusters: Quantum Celestial Mechanics (QCM) Rescues MOND?

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Although the MOND radial acceleration $g = \sqrt{g_N a_0}$ for the acceleration of objects in a low acceleration environment less than $a_0 = -1.2 \times 10^{-10}$ m/s^2 has been extremely successful for single galaxies, the much higher mass clusters of galaxies do not have enough baryonic mass to comply. We consider the possibility that the MOND a_0 value, instead of being a universal constant, actually depends upon both the total baryonic mass of the gravitationally bound system and its total angular momentum, as derived by Quantum Celestial Mechanics (QCM) from the general relativistic Hamilton-Jacobi equation. If the total angular momentum of the galaxy cluster is less than expected, then the MOND radial acceleration expression can remain valid.

1 Introduction

Galaxy rotation velocities do not match Newton's Law of Universal Gravitation, $g = -GM/r^2$, for stars experiencing low gravitational radial accelerations [1]. The stars at all large orbital radii where the radial acceleration is less than about 10^{-10} m/s^2 are moving at nearly identical velocities instead of decreasing to the lower velocity values predicted by Newton's Law.

Initial attempts to alter Newton's Law failed, so the dark matter hypothesis became the alternative explanation with the consequence that Newton's Law could apply once again [2]. However, two important challenges to dark matter continue to exist: (1) no predicted dark matter particle has ever been detected in at least 50 years of experimental searches [3, 4], and (2) a modification of gravitation called MOND (MOdified Newtonian Dynamics) exists and agrees extremely well with single galaxy rotation curves [5] and has predicted many other physical properties that have been found to hold true for single galaxies and other gravitationally bound systems [6,7].

Even though fitting the rotation curves of single galaxies is remarkably successful, MOND does not fit the radial acceleration values for clusters of galaxies [8,9]. There is a significant disagreement with the MOND gravitational acceleration expression

$$g = \sqrt{g_N a_0}\,, \tag{1}$$

where the MOND acceleration constant $a_0 = -1.2 \times 10^{-10}$ m/s^2 and g_N = $-GM(< r)/r^2$ is the Newtonian acceleration for enclosed baryonic mass M. This gravitational expression using the a_0 value has been shown to hold true for all single galaxies and is assumed to be true for clusters of galaxies.

However, the measurements for galaxy clusters reveal that the observed acceleration g_{obs} is greater than the acceleration value g predicted by this MOND expression at the low radial acceleration environments where the expression should be true. Fig. 1 shows the discrepancy between the dynamic mass and all the observed mass of the baryons in the gas and the stars within the cluster, with the data from [8]. Some clusters need as much as a factor of 5 more baryonic mass for the MOND expression to hold true, i.e., be at the straight line where the dynamic mass and the observed mass values agree.

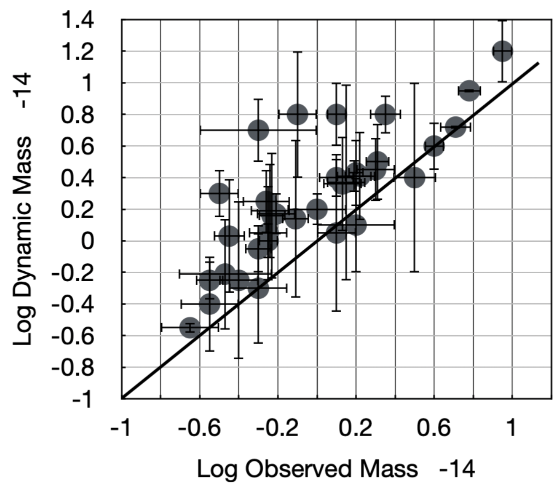

Fig. 1: Log scales around 10^{14} solar masses for galaxy clusters. If MOND is to prevail with its constant a_0 value, all the clusters should be at the straight line. Data is a representative selection from Sanders (2003).

This discrepancy between g_{obs} and the MOND predicted g has been attributed to missing baryonic mass M in the cluster, but several searches have not found any more mass than already determined. Consequently, to fit the actual observed radial accelerations for all galaxy clusters, a distribution of dark matter has been proposed so that Newton's Law applies to galaxy clusters.

We propose a different explanation for the acceleration discrepancy, one that allows the MOND acceleration expression $g = \sqrt{g_N a_0}$ to be correct for galaxy clusters as well as for single galaxies. In 2003, H. G. Preston and I investigated [10] an approach to gravitation that we called quantum celestial mechanics (QCM) in which the general relativistic Hamilton-Jacobi equation is converted into a new scalar gravitational wave equation (GWE). In different metrics the GWE allows us to propose some new explanations for specific types of gravitational behavior.

In the Schwarzschild metric the GWE leads to the quantization of angular momentum *per unit mass* because both Newtonian gravitational attraction and a QCM gravitational repulsion exists for orbiting bodies. All confirmed planetary systems, including the Solar System, have been shown to agree with this QCM prediction [11], i.e. the orbital planetary equilibrium radii of the multi-planetary systems are only at the QCM predicted subset of all possible equilibrium radii that are allowed by Newton's Law.

We also derived the above MOND gravitational expression, which revealed that the MOND acceleration a_0 will have slightly different values in different single galaxies depending on the total baryonic mass M_T of the gravitationally bound system and its total angular momentum L_T,

$$a_0 = \frac{G^3 M_T^7}{n^4 L_T^4}\,, \tag{2}$$

with n an integer.

This dependency of a_0 upon both M_T and L_T will allow us to re-interpret the MOND expression for g so that clusters of galaxies, as well as single galaxies, satisfy $g_{obs} = g$ without the need for dark matter.

2 Derivation of QCM and MOND, a brief review

From the general relativistic Hamilton-Jacobi equation,

$$g^{\alpha\beta}\,\frac{\partial S}{\partial x^\alpha}\,\frac{\partial S}{\partial x^\beta} - \mu^2 c^2 = 0 \tag{3}$$

the transformation

$$\Psi = e^{iS'/\mu c H} \tag{4}$$

introduces a wave function Ψ, with S the action, μ the mass of the orbiting object, and $S' = S/\mu c$ so that the equivalence principle is obeyed. For a detailed derivation, see [10]. Here we have defined a system scale length H by

$$H = \frac{L_T}{M_T c} \tag{5}$$

for the total gravitationally bound system mass M_T having total angular momentum L_T and c being the speed of light in vacuum.

Following through with the mathematical steps produces a scalar gravitational wave equation (GWE)

$$g^{\alpha\beta}\,\frac{\partial^2 \Psi}{\partial x^\alpha \partial x^\beta} + \frac{\Psi}{H^2} = 0\,. \quad \text{(GWE)} \tag{6}$$

Expressing the GWE in the Schwarzschild metric, a separation of variables leads to differential equations in coordinates (t, r, θ, ϕ) that produce quantization conditions. The angular parts dictate the quantization of angular momentum *per unit mass* for orbital angular momentum L as

$$\frac{L}{\mu} = mcH \tag{7}$$

for integer m. The radial equation leads to the quantization of energy per unit mass

$$E_n = -\mu c^2\,\frac{r_g^2}{8n^2 H^2} \tag{8}$$

for integer n.

Using the virial theorem and E_n, we obtain the velocity v in terms of the Schwarzschild radius r_g and H,

$$v = \frac{r_g c}{2nH}\,. \tag{9}$$

Whence, with the radial acceleration $g = v^2/r$, we derive the MOND acceleration expression from

$$g = \frac{r_g^2 c^2}{4n^2 H^2 r} = \sqrt{\frac{GM}{r^2}\left(\frac{G^3 M_T^7}{n^4 L_T^4}\right)}. \tag{10}$$

Therefore, the MOND acceleration a_0 is not a universal constant but is determined to be

$$a_0 = \frac{G^3 M_T^7}{n^4 L_T^4}\,, \tag{11}$$

explicitly expressing its dependency upon both the total mass M_T of the system and its total angular momentum L_T.

3 Discussion

We have begun with the successful MOND espression $g = \sqrt{g_N a_0}$ using $a_0 = -1.2{\times}10^{-10}$ m/s^2, with a_0 having this value for all single galaxies. But if we assume that a_0 has this same value for galaxy clusters, then the baryonic mass discrepancy shown in Fig. 1 arises.

According to QCM, there can be two possible causes for the discrepancy between the observed radial acceleration g_{obs} and the predicted MOND value g in the galaxy clusters, the values of total baryonic mass value M_T and the total angular momentum L_T. All the baryonic mass M_T in the gas and stars, etc., has been identified. However, we do not know the total angular momentum L_T of any galaxy cluster.

QCM predicts that the a_0 value depends upon the ratio M_T^7/L_T^4. We already know the baryonic M_T for the clusters, but there are no published values of L_T for any cluster. Therefore, we must estimate the L_T values for different galaxy clusters if the MOND $g = g_{obs}$ is to hold true.

In some clusters of galaxies the dynamical mass M_{dyn} has been determined to be as much as a factor of 5 larger than the actual observed mass M_{obs} of the hot gas and the stellar content. Therefore, the ratio M_T^7/L_T^4 for a_0 in these galaxy clusters must be up to 5 times larger than for single galaxies in order to have $g_{obs} = \sqrt{g_N a_0}$.

We assume that the expected L_T value is the one that makes the MOND $a_0 = -1.2 \times 10^{-10}$ m/s^2. Then in the general case, if there is a factor f reduction in the baryonic mass

M_T, QCM requires

$$f a_0 = \frac{G^3 M_T^7}{n^4 L_T^4} \tag{12}$$

which, for n = 1 and $M_{obs} = M_T/f$, means

$$L_T^4 = \frac{G^3 M_{obs}^7}{f^8 a_0}\,. \tag{13}$$

For example, if $f = 2$, the L_T value would be 4 times smaller than expected for the M_{obs}. This L_T value then makes the product $g_N a_0$ guarantee $g = g_{obs}$. Thus. each galaxy cluster could have a unique a_0 value.

There exists several possible sources of a lower angular momentum total than expected:

1. the intracluster (IC) gas that comprises about 90% of the baryonic mass of the cluster could, in part or as a whole, have a slower rotation speed than gas in single galaxies,
2. the IC stars are known to rotate slower than many stars,
3. the angular momentum vectors of the galaxies in the cluster may have a greater variety of directions than expected, thereby decreasing their vector sum.

Whether any or all of these possible sources of lower angular momentum are the cause of the different a_0 values for the galaxy clusters has yet to be determined.

4 Conclusion

Recent measurements have verified that there is not enough baryonic mass for the successful MOND gravitational acceleration expression for single galaxies

$$g = \sqrt{g_N a_0} \tag{14}$$

to be true for clusters of galaxies, where g_N is the Newtonian gravitational radial acceleration and the MOND $a_0 = -1.2 \times 10^{-10}$ m/s^2 is assumed to be a universal constant.

However, a_0 may not be a universal constant as originally proposed. We briefly reviewed the quantum celestial mechanics (QCM) derivation of a_0 from the general relativistic Hamilton-Jacobi equation to obtain the acceleration

$$g = \frac{r_g^2 c^2}{4n^2 H^2 r} = \sqrt{\frac{GM}{r^2}\left(\frac{G^3 M_T^7}{n^4 L_T^4}\right)}. \tag{15}$$

Therefore, QCM dictates

$$a_0 = \frac{G^3 M_T^7}{n^4 L_T^4}\,, \tag{16}$$

showing that a_0 depends upon both the total baryonic mass M_T and its total angular momentum L_T of any gravitationally bound system obeying the Schwarzschild metric. For single galaxies, this QCM expression for a_0 varies less than a few percent and therefore a_0 can be assumed universal.

However, in more massive gravitationally bound systems such as clusters of galaxies, a_0 could have different values in order to satisfy the MOND expression $g = \sqrt{g_N a_0}$. If galaxy clusters possess significantly less angular momentum than is expected for the measured total baryonic mass, this MOND expression can be satisfied still. Several possible reasons for the lesser total angular momentum values were suggested.

We await total angular momentum estimates for galaxy clusters in the near future to establish whether the MOND acceleration a_0 has a different value for galaxy clusters and whether the MOND expression $g = \sqrt{g_N a_0}$ continues to hold true.

Acknowledgements

We thank Sciencegems for continued support and encouragement to investigate various challenges in gravitation.

Received on September 26, 2024

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