Correct Solutions for Rotating Black Holes

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This paper introduces correct solutions for rotating black holes and electrically charged rotating black holes. The solutions are based on the space metric of a rotating spherical body approximated by a mass-point, which is a new metric to General Relativity introduced and proved using Einstein's equations in the previous paper (Progr. Phys., 2014, v. 20, 79–99) as an extension of Schwarzschild's mass-point metric. According to the solutions, rotating black holes have the shape of an oblate spheroid, flattened at the poles, where its radius is equal to the gravitational radius of the body, and thickened at the equator. The introduced black hole solutions are mathematically and physically correct, because they have no limitations, unlike Kerr's solution and the Kerr-Newman solution, which, since they are obtained using the tetrad formalism, are valid only in an infinitely small vicinity of the surface of a rotating black hole.

1 Non-rotating black holes

Definition 1: A black hole is a type of cosmic body, the gravitational field of which is so strong that light cannot escape from its surface.

This is the original definition of black holes according to the founders of the black hole problem — the Reverend John Michell, who in 1783 wrote his article in which he first outlined his idea of such cosmic objects [1], and also Pierre-Simon Laplace, who in 1796, independently of Michell, in Chapter 6 of his *Exposition du Système du Monde* gave a definition of black holes [2, p. 305], and then in 1799 provided a mathematical justification for such objects in the framework of Classical Mechanics [3].

See the 2009 study of the history of the black hole problem [4] and the papers [5–7] referred therein.

In the General Theory of Relativity, the geometric basis is not a three-dimensional Euclidean space, as in Classical Mechanics, but a four-dimensional pseudo-Riemannian space (space-time), black holes are defined from the general formula of the Riemannian space (space-time) metric*

$$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta = \\ = g_{00}\, dx^0 dx^0 + 2\, g_{0i}\, dx^0 dx^i + g_{ik}\, dx^i dx^k, \qquad (1)$$

in which specific formulae for the components of the fundamental metric tensor $g_{\alpha\beta}$ determine the geometry and distribution of matter of the particular Riemannian space (space-time) that we are considering.

Usually, the definition of black holes in General Relativity is given in terms of the zero (time) component

$$g_{00} = \left(1 - \frac{\mathrm{w}}{c^2}\right)^2 \qquad (2)$$

of the fundamental metric tensor $g_{\alpha\beta}$, based on the assertion that the difference of g_{00} from 1 indicates the deviation of real time intervals $d\tau$ from ideal (unperturbed and homogeneous) time intervals dt, which is determined *only* by the potential of the acting gravitational field $\mathrm{w} = c^2\left(1 - \sqrt{g_{00}}\right)$.

This definition of black holes says:

Definition 2: A black hole is a type of cosmic body, on the surface of which $g_{00} = 0$ and, hence its physical radius is equal to the gravitational radius $r_g = 2GM/c^2$, calculated for its mass M. The entire mass of such a body is under its gravitational radius, which means that this body is in the state of gravitational collapse, i.e., the body is a *gravitational collapsar*.

This definition of black holes originates from the mass-point space metric introduced in 1916 by Karl Schwarzschild [8], which is known as the *Schwarzschild mass-point metric*. This metric†

$$ds^2 = \left(1 - \frac{r_g}{r}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{r_g}{r}} - r^2 \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \qquad (3)$$

describes a spherically symmetric space filled with a gravitational field created by a massive spherical island of substance, which is approximated by a material point, where r is the radial distance from the barycentre of the massive island (which is the coordinate origin), and $r_g = 2GM/c^2$ is the gravitational radius of the island, calculated for its mass M. According to this metric, the non-zero components of the fundamental metric tensor $g_{\alpha\beta}$ of such a space are

$$\left.\begin{array}{ll} g_{00} = 1 - \dfrac{r_g}{r}, & g_{11} = -\dfrac{1}{1 - \frac{r_g}{r}} \\ g_{22} = -r^2, & g_{33} = -r^2 \sin^2\theta \end{array}\right\}. \qquad (4)$$

According to the views commonly accepted in the 1920–1930s among the scientists working in the field of General

*Here $\alpha, \beta = 0, 1, 2, 3$ are four-dimensional (space-time) indices, and $i, k = 1, 2, 3$ are three-dimensional spatial indices.

†The commonly accepted mathematical form of this metric given above was derived not by Schwarzschild himself, but immediately after his death in 1916 independently by Johannes Droste and David Hilbert [9, 10].

Relativity, real time intervals $d\tau$ are expressed through ideal (unperturbed, homogeneous) time intervals dt as

$$d\tau = \sqrt{g_{00}}\, dt\,, \tag{5}$$

see §84 on distances and time intervals in *The Classical Theory of Fields* by Landau and Lifshitz [11], the 1st edition of which was published in 1939.

Since $g_{00} = 1 - \frac{r_g}{r} = 0$ on the surface of a gravitational collapsar ($r = r_g$), physically observable time stops ($d\tau = 0$) on its surface from the point of view of an external observer and, hence, no signal can escape from this body. In other words, this is a cosmic object called a *black hole*.

Some criticism to the black hole concept is based on the fact that r in the formula of the Schwarzschild mass-point metric is a radial coordinate (as in any space metric written in spherical coordinates r, θ, φ), and not the physical radius of the massive spherical island of substance creating the gravitational field. For the details of this criticism, see [12, 13] and references therein. This is true, but this fact does not cancel the existence of a space breaking on a spherical surface of the radius $r_g = 2GM/c^2$ from the barycentre of the massive island (due to the breaking $g_{00} = 0$ in the space metric on this surface). Of course, a spherical massive island that creates a gravitational field described by the Schwarzschild mass-point metric can have any radius R. But if its radius is equal to the gravitational radius $R = r_g$ calculated for its mass M, then this body is definitely a gravitational collapsar (black hole). See my remarks [14] on the above criticism.

Gravitational collapsars are conceivable not only in the form of a collapsed spherical body, i.e., they are associated not only with the Schwarzschild mass-point metric. It can be any cosmic body, the physical radius of which is equal to its gravitational radius (and, therefore, $g_{00} = 0$ on its surface). So, in 2010 Larissa Borissova introduced a new cosmological model, according to which the entire observable Universe is a de Sitter collapsar — a de Sitter space (this is a constant curvature spherical space filled with the physical vacuum) in the state of gravitational collapse: its radius (which is the same as the curvature radius of space) is equal to its gravitational radius. She called this model the *de Sitter bubble* [15]. Also, in our common monograph on the internal constitution of stars, *Inside Stars* [16], we considered liquid black holes; the space inside such a collapsar is determined by Schwarzschild's metric of the space inside a liquid sphere.

2 The complete formula for real time

As was mentioned above, the key point of the black hole solution in General Relativity is the stopping of real time ($d\tau = 0$) on the surface of a black hole, which is usually determined from the formula for real time intervals $d\tau = \sqrt{g_{00}}\, dt$, commonly accepted in the 1920–1930s [11, §84].

At the same time, the problem of determining real time intervals is not trivial, and is a particular case of the general problem of determining physically observable quantities in the space-time of General Relativity.

Initially, only heuristic considerations were used for determining physical observable quantities in General Relativity. For example, physically observable (real) time intervals $d\tau$ were assumed to be the square root of the first (time) term $g_{00}\, dx^0 dx^0 = g_{00}\, c^2 dt^2$ of the square of the four-dimensional (space-time) interval ds^2, i.e., $d\tau = \sqrt{g_{00}}\, dt$. It was heuristically assumed that three-dimensional components of a four-dimensional vector form a three-dimensional observable vector, and its time component is the observable potential of the vector field. And so forth and so on, which generally does not prove that these quantities can be actually observed.

Only in 1944 Abraham Zelmanov developed a versatile mathematical method that unambiguously determined physically observable quantities in the space-time of General Relativity as the projections of four-dimensional quantities onto the time line and the three-dimensional spatial section, associated with an observer. Such projections are invariant throughout the spatial section of the observer (his observable three-dimensional reference space), i.e., they are "chrono-metric invariants" in his reference frame and depend on the properties of his reference space, such as the gravitational potential, rotation, deformation, curvature, etc. For this reason, Zelmanov called his mathematical method the *theory of chronometric invariants* or the *chronometrically invariant formalism*.

Although Zelmanov presented his work in 1944 in his lengthy doctoral dissertation and later in two short papers, one of which was published in English in 1956 [17], his chronometrically invariant formalism remained outside attention of the scientific community over decades. His main works were published in English only in the 2000s [18, 19]. See the comprehensive survey of the Zelmanov formalism [20], where I and Larissa Borissova collected almost everything that we know on this subject personally from Zelmanov and based on our own research studies.

In short, the chronometrically invariant projections of any four-dimensional quantity are calculated using operators of projection, which take the physical properties and geometric structure of the observer's physical space into account. Thus, the four-dimensional displacement vector dx^α ($\alpha = 0, 1, 2, 3$), projected onto the time line of an observer, represents the physically observable (real) chr.inv.-time interval

$$d\tau = \sqrt{g_{00}}\, dt - \frac{1}{c^2}\, v_i\, dx^i, \quad i = 1, 2, 3, \tag{6}$$

and the projection of dx^α onto the three-dimensional spatial section associated with the observer is the physically observable three-dimensional chr.inv.-displacement vector dx^i.

Here g_{00} is expressed through the physically observable chr.inv.-potential w of the gravitational field that fills the observer's space

$$\mathrm{w} = c^2\left(1 - \sqrt{g_{00}}\right), \qquad \sqrt{g_{00}} = 1 - \frac{\mathrm{w}}{c^2}\,, \tag{7}$$

and v_i is the three-dimensional vector of the linear velocity of rotation of the observer's space

$$v_i = -\frac{c\,g_{0i}}{\sqrt{g_{00}}}\,, \quad v^i = -c\,g^{0i}\sqrt{g_{00}}\,, \quad v_i = h_{ik}\,v^k. \tag{8}$$

The square of the three-dimensional physically observable chr.inv.-interval is determined as

$$d\sigma^2 = h_{ik}\,dx^i dx^k \tag{9}$$

using the three-dimensional chr.inv.-metric tensor h_{ik}

$$h_{ik} = -g_{ik} + \frac{1}{c^2}\,v_i v_k\,, \quad h^{ik} = -g^{ik}, \quad h^i_k = \delta^i_k\,, \tag{10}$$

which is the chr.inv.-projection of the fundamental metric tensor $g_{\alpha\beta}$ onto the spatial section associated with the observer and possesses all properties of $g_{\alpha\beta}$ throughout the spatial section (the observer's three-dimensional space).

Thus, the square of the four-dimensional (space-time) interval $ds^2 = g_{\alpha\beta}\,dx^\alpha dx^\beta$ is expressed in terms of chronometrically invariant (physically observable) quantities as

$$ds^2 = c^2 d\tau^2 - d\sigma^2. \tag{11}$$

The above has aftermaths for the black hole solution. The complete formula for physically observable (real) time intervals $d\tau = \sqrt{g_{00}}\,dt - \frac{1}{c^2}\,v_i dx^i$ (6) differs from $d\tau = \sqrt{g_{00}}\,dt$ (5) given in §84 of *The Classical Theory of Fields* by the second term, determined by the rotation of space. They coincide only if space does not rotate. Therefore, since the stopping of observable time ($d\tau = 0$) defines black holes in General Relativity ($d\tau = 0$ on the surface of a body means that no signal can leave this body), the condition

$$d\tau = \sqrt{g_{00}}\,dt - \frac{1}{c^2}\,v_i dx^i = 0 \tag{12}$$

which follows from the chronometrically invariant formalism should give a black hole solution for rotating black holes.

3 The correct solution for a rotating black hole

It is obvious that a correct solution for a rotating black hole should follow from the space metric of a rotating spherical body, approximated by a mass-point. Such a metric was introduced and proved using Einstein's field equations in the previous paper [21].

This metric was derived on the basis of the Schwarzschild mass-point metric (3) by assuming that the space rotates together with the body itself along the equatorial coordinate axis φ, i.e., along the geographical longitudes of the body, with the linear velocity $v_3 = \omega r^2 \sin^2\theta$. In addition, it was assumed that the rotation of space is stationary, i.e., the angular velocity ω of this rotation is constant ($\omega = const$). Since by definition of v_i (8) we have

$$v_3 = \omega r^2 \sin^2\theta = -\frac{c\,g_{03}}{\sqrt{g_{00}}}\,, \tag{13}$$

then the metric of a rotating spherical body approximated by a mass-point has the form

$$ds^2 = \left(1 - \frac{r_g}{r}\right) c^2 dt^2 - 2\,\omega r^2 \sin^2\theta \sqrt{1 - \frac{r_g}{r}}\, dt\, d\varphi - \frac{dr^2}{1 - \frac{r_g}{r}} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{14}$$

where, as is seen from the above formula,

$$\left.\begin{aligned} g_{00} &= 1 - \frac{r_g}{r}\,, \qquad g_{03} = -\frac{\omega r^2 \sin^2\theta}{c}\sqrt{1 - \frac{r_g}{r}} \\ g_{11} &= -\frac{1}{1 - \frac{r_g}{r}}\,, \qquad g_{22} = -r^2, \quad g_{33} = -r^2 \sin^2\theta \end{aligned}\right\}, \tag{15}$$

and, hence, non-zero lower-index components of the chr.inv.-metric tensor h_{ik} (10) are

$$\left.\begin{aligned} h_{11} &= \frac{1}{1 - \frac{r_g}{r}}\,, \qquad h_{22} = r^2 \\ h_{33} &= r^2 \sin^2\theta \left(1 + \frac{\omega^2 r^2 \sin^2\theta}{c^2}\right) \end{aligned}\right\}, \tag{16}$$

while its upper-index components h^{ik}, since the matrix h_{ik} is strict diagonal, i.e., all of its non-diagonal components (for which $i \neq k$) are zero, are $h^{ik} = (h_{ik})^{-1}$ just like the invertible matrix components to any diagonal matrix.

To check the above rotating metric, we calculate $v^2 = v_i v^i = h_{ik} v^i v^k$. Since $v^i = h^{ik} v_k$, we obtain the following

$$v^2 = v_i v^i = \frac{\omega^2 r^2 \sin^2\theta}{1 + \frac{\omega^2 r^2 \sin^2\theta}{c^2}}\,, \qquad v = \frac{\omega r \sin\theta}{\sqrt{1 + \frac{\omega^2 r^2 \sin^2\theta}{c^2}}}\,, \tag{17}$$

therefore, the dimension of v is [cm/sec] as it should be. If the space rotates slowly, then the above formula transforms to $v = \omega r \sin\theta$ [cm/sec] as in Classical Mechanics.

In fact, the space metric (14) describes a spherically symmetric space, which is filled with the gravitational field created by a rotating spherical island of substance (approximated by a mass-point) and rotates together with this body.

The introduced and proved metric (14) is a new space metric to General Relativity, which is a modern extension and replacement of the Schwarzschild mass-point metric (3), because in the space of the Schwarzschild metric a massive body creating gravitational field does not rotate. Moreover, this metric is the *basic space metric in the Universe*, characterizing the physically observable field of any real cosmic body, be it a planet, star, galaxy or something else (since all real cosmic bodies rotate).

Consider the black hole condition $d\tau = 0$ in the space of a rotating spherical body approximated by a mass-point, i.e., according to the space metric (14).

Independently of the specific metric of space, the black hole condition $d\tau = 0$ (12) can be transformed to the form

$$d\tau = \left(\sqrt{g_{00}} - \frac{1}{c^2}\, v_i u^i\right) dt = 0\,, \qquad u^i = \frac{dx^i}{dt}\,, \tag{18}$$

where ideal (unperturbed, homogeneous) time intervals are $dt \neq 0$ and, therefore,

$$\sqrt{g_{00}} - \frac{1}{c^2}\, v_i u^i = 0\,, \tag{19}$$

while u^i is the coordinate velocity of a source of signals (in this case — along the surface, on which physically observable time stops). Thus, the black hole condition $d\tau = 0$ (12) took its detailed form (19).

In the space of a rotating spherical body, approximated by a mass-point, i.e., in the space of the metric (14), the obtained detailed formula (19) of the black hole condition $d\tau = 0$ takes the particular form characteristic of this space metric

$$\sqrt{1 - \frac{r_g}{r}} - \frac{1}{c^2}\, v_3 u^3 = 0\,. \tag{20}$$

From this form of the black hole condition $d\tau = 0$, since $r_g = 2GM/c^2$ is the gravitational radius of the body, calculated for its mass M,* we then derive the *black hole solution*, which is a distance $r = r_c$ from the barycentre of the body at which physically observable time for signals stops, i.e., the signals disappear for an external observer.

So forth, assuming that the linear velocity with which the space rotates together with the body itself $v_3 = \omega r^2 \sin^2\theta$ is much less than the speed of light, and the source of signals rests on the body's surface, i.e., its coordinate velocity along the equatorial axis φ is $u^3 = \frac{d\varphi}{dt} = \omega$, we obtain the *black hole solution for a rotating black hole*

$$r_c = \frac{r_g}{1 - \frac{1}{c^4}\,\omega^4 r^4 \sin^4\theta} \simeq r_g \left(1 + \frac{1}{c^4}\,\omega^4 r^4 \sin^4\theta\right) \geqslant r_g\,. \tag{21}$$

According to the obtained black hole solution (21), since $\sin\theta = 0$ at the poles of a rotating black hole (as in the previous paper [21] we assume that the θ coordinate is the polar angle measured from the North Pole), the second term in the brackets vanishes at the poles and has no effect. As a result, the radius r_c of a rotating black hole coincides with its gravitational radius ($r_c = r_g$) at the North Pole and South Pole.

In contrast, $\sin\theta = 1$ at the equator and, hence, the equatorial radius of a rotating black hole is greater than its gravitational radius r_g by a length

$$\Delta r = r_g \frac{\omega^4 r^4}{c^4}\,, \tag{22}$$

which is greater the faster the black hole rotates.

*At the distance $r_g = 2GM/c^2$ from its barycentre the space has a breaking, which manifests itself in the form of the condition $g_{00} = 0$.

Therefore, according to the black hole solution (21) that we have obtained, we arrive at the conclusion:

Conclusion: Rotating black holes are not spheres, but have the *shape of an oblate spheroid*, flattened at the poles, where its radius is equal to the gravitational radius of the body, and thickened at the equator, where its radius exceeds the gravitational radius (due to rotation). The faster a black hole rotates, the thicker its body is at the equator compared to the poles.

This means that, according to the black hole solution obtained above, signals arriving at the poles of a rotating gravitational collapsar disappear for an external observer when they arrive at its gravitational radius (as in the case of a non-rotating collapsar). However, if signals arrive at a rotating gravitational collapsar at latitudes other than the poles, then they disappear at a distance greater than its gravitational radius (this distance exceeding the gravitational radius is maximum at the equator).

For this reason, it is reasonable to reconsider the initial definition of black holes in General Relativity, which is based on the gravitational collapse condition $g_{00} = 0$ (see Definition 2 in the beginning of this article). Since, according to the solution obtained above for rotating black holes, the equatorial radius of a rotating black hole exceeds its gravitational radius, we must replace the initial definition of black holes with a more general one, according to which black holes are defined as objects, on the surface of which physically observable time stops:

Definition 3: A black hole is a type of cosmic body, on the surface of which time stops from the point of view of an external observer (the interval of physically observable time is zero $d\tau = 0$ on its surface) and, hence, no one signal can escape the surface of the body.

4 The correct solution for an electrically charged rotating black hole

Consider another case, where the considered spherical island of substance (approximated by a mass-point) possesses an electric charge q. In this case, the space of the mass-point is filled with not only the gravitational field created by it, but also a spherically symmetric electric (electromagnetic) field, i.e., is filled with distributed matter. The space of an electrically charged mass-point is described by the Reissner-Nordström metric

$$ds^2 = \left(1 - \frac{r_g}{r} + \frac{r_q^2}{r^2}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{r_g}{r} + \frac{r_q^2}{r^2}} - r^2 \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{23}$$

which is an extension of the Schwarzschild mass-point metric, first considered in 1916 by Hans Reissner [22], and then, in 1918, by Gunnar Nordström [23]. The Reissner-Nordström metric uses the same denotations as the Schwarzschild mass-point metric, with an addition of

$$r_q^2 = \frac{Gq^2}{4\pi\varepsilon_0 c^4}\,, \tag{24}$$

where q is the electric charge of the massive island (source of the gravitational field and the electromagnetic field), G is the gravitational constant, and $\frac{1}{4\pi\varepsilon_0}$ is Coulomb's force constant.

We introduce the space metric of an electrically charged rotating body (approximated by a mass-point) analogous to the mass-point space metric of a rotating body (14), which together with its space stationary rotates with a constant angular velocity $\omega = const$ and the linear velocity $v_3 = \omega r^2 \sin^2\theta$ along the equatorial coordinate axis φ, i.e., along the geographical longitudes of the body. The resulting space metric of an electrically charged rotating soherical body approximated by a mass-point has the form

$$ds^2 = \left(1 - \frac{r_g}{r} + \frac{r_q^2}{r^2}\right) c^2 dt^2 - 2\omega r^2 \sin^2\theta \sqrt{1 - \frac{r_g}{r} + \frac{r_q^2}{r^2}}\, dt\, d\varphi - \frac{dr^2}{1 - \frac{r_g}{r} + \frac{r_q^2}{r^2}} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{25}$$

where, as follows from the above formula,

$$\left.\begin{aligned} g_{00} &= 1 - \frac{r_g}{r} + \frac{r_q^2}{r^2} \\ g_{03} &= -\frac{\omega r^2 \sin^2\theta}{c}\sqrt{1 - \frac{r_g}{r} + \frac{r_q^2}{r^2}} \\ g_{11} &= -\frac{1}{1 - \frac{r_g}{r} + \frac{r_q^2}{r^2}} \\ g_{22} &= -r^2, \quad g_{33} = -r^2 \sin^2\theta \end{aligned}\right\}, \tag{26}$$

and, hence, non-zero lower-index components of the chr.inv.-metric tensor h_{ik} (10) are

$$\left.\begin{aligned} h_{11} &= \frac{1}{1 - \frac{r_g}{r} + \frac{r_q^2}{r^2}}\,, \qquad h_{22} = r^2 \\ h_{33} &= r^2 \sin^2\theta \left(1 + \frac{\omega^2 r^2 \sin^2\theta}{c^2}\right) \end{aligned}\right\}, \tag{27}$$

while its upper-index components are $h^{ik} = (h_{ik})^{-1}$ just like the invertible matrix components to any diagonal matrix.

The introduced space metric (25) describes a spherically symmetric space, which is filled with the gravitational field and the electromagnetic field, which are created by a rotating electrically charged spherical island of substance (approximated by a mass-point) and rotates together with this body.

The introduced metric (25) is a new space metric to General Relativity, which is a modern extension of the Schwarzschild mass-point metric (3), the recently introduced metric of a rotating spherical body approximated by a mass-point (14) and the Reissner-Nordström metric (23).

This metric is proved using Einstein's field equations absolutely analogous to the metric of a rotating spherical body approximated by a mass-point, which was introduced and proved in the recent paper [21], because it differs only by one additional term in g_{00}, which takes the electric charge q into account. The only difference in the proof is that the right-hand side of the Einstein equations in this case is non-zero and contains physically observable components of the energy-momentum tensor of the electromagnetic field, and the Riemannian conditions for the metric take the electromagnetic field into account. This proof is easy to repeat by anyone, following with the recent paper [21]. We therefore omit this proof in the present paper, since the main task here is to obtain solutions for black holes.

In the space of the rotating Reissner-Nordström metric (25) that we just introduced, the detailed general formula (19) of the black hole condition $d\tau = 0$ takes the form

$$\sqrt{1 - \frac{r_g}{r} + \frac{r_q^2}{r^2}} - \frac{1}{c^2}\, v_3 u^3 = 0\,. \tag{28}$$

From here, assuming that the effect created by the electromagnetic field of an electrically charged black hole (the third term under the square root) is much weaker than the effect of its gravitational field (the second term), which is a natural assumption for a gravitational collapsar due to its super-strong gravitational field, we then derive the black hole solution for an electrically charged black hole, which is a distance from its barycentre at which physically observable time for signals stops (they disappear for an external observer).

As above in the case of a regular rotating black hole, we assume that the linear velocity $v_3 = \omega r^2 \sin^2\theta$ with which the space rotates together with the electrically charged body itself is much less than the speed of light, and the source of signals rests on the body's surface ($u^3 = \frac{d\varphi}{dt} = \omega$). Thus, we obtain the formula of the *black hole solution for an electrically charged rotating black hole*

$$r_c = \frac{r_g}{1 - \frac{1}{c^4}\,\omega^4 r^4 \sin^4\theta + \frac{r_q^2}{r^2}} \simeq r_g \left(1 + \frac{1}{c^4}\,\omega^4 r^4 \sin^4\theta - \frac{r_q^2}{r^2}\right). \tag{29}$$

According to the obtained black hole solution (29), the radius of an electrically charged rotating black hole is shorter at its poles than the gravitational radius $r_g = 2GM/c^2$ calculated for its mass M, and is less thick at the equator than the equatorial radius of a regular rotating black hole. In other words, the electric charge (and the electromagnetic field) of an electrically charged rotating black hole makes its shape more flattened from the poles and less thick at the equator than a regular rotating black hole

$$r_{c\,(\text{poles})} = \frac{r_g}{1 + \frac{r_q^2}{r^2}} \simeq r_g \left(1 - \frac{r_q^2}{r^2}\right) < r_g\,, \tag{30}$$

$$r_{c\,(\text{equator})} = \frac{r_g}{1 - \frac{\omega^4 r^4}{c^4} + \frac{r_q^2}{r^2}} \simeq r_g \left(1 + \frac{\omega^4 r^4}{c^4} - \frac{r_q^2}{r^2}\right). \tag{31}$$

An electrically charged rotating black hole has a radius equal to its gravitational radius r_g at geographic latitudes in the northern and southern hemispheres, where the sine of the polar angle θ is

$$\sin\theta = \frac{c\sqrt{r_q}}{\omega\, r^{3/2}}\,, \qquad r_q^2 = \frac{Gq^2}{4\pi\varepsilon_0 c^4}\,. \tag{32}$$

Finally, the obtained black hole solution for electrically charged black holes (29) leads us at the conclusion:

Conclusion: Electrically charged rotating black holes are not spheres, but have the *shape of an oblate spheroid*, flattened at the poles, where its radius is shorter that the gravitational radius of the body, and thickened at the equator, where its radius exceeds the gravitational radius (due to rotation). The faster a black hole rotates, the thicker its body is at the equator compared to the poles. The greater its electric charge, the shorter its radius is at the poles compared to its gravitational radius and the less thick its equatorial radius.

That is, according to the obtained black hole solution, signals arriving at the poles of an electrically charged rotating gravitational collapsar disappear for an external observer at an altitude less than its gravitational radius. But, if signals arrive at such a collapsar at the equator, then they disappear at a distance greater than its gravitational radius. Signals disappear at a distance of the gravitational radius from the barycentre of an electrically charged rotating gravitational collapsar at geographic latitudes in the northern and southern hemispheres, where the effect of the collapsar's rotation is completely compensated by the effect of its electric charge.

P.S. It should be noted that the significance of the black hole solutions obtained in the present paper contrasts with Kerr's solution and the Kerr-Newman solution.

Kerr's solution for a rotating black hole [24] and the Kerr-Newman solution for an electrically charged rotating black hole [25] were introduced in the early 1960s, based on the respective space (space-time) metrics that they derived using a special version of the tetrad formalism called the Newman-Penrose formalism [26]. In the tetrad formalism, all quantities given in the four-dimensional pseudo-Riemannian space (space-time of General Relativity, which is generally curved, inhomogeneous and anisotropic) are projected onto a tangential space, which is a four-dimensional flat, homogeneous and isotropic space (space-time) tangential to the given Riemannian space at the point where you are looking for a solution. Thus, the tetrad formalism solves all problems of General Relativity in this tangential flat space.

The advantage of this mathematical formalism is that in the tangential flat homogeneous and isotropic space there are no singularities (space discontinuities), and complicated problems of General Relativity are expressed in a simple mathematical form. On the other hand, the tetrad formalism has a serious drawback that has prevented it from becoming the main mathematical tool of the researchers working in the field of General Relativity. Quantities associated with objects and the geometric structure of the Riemannian space can be projected onto a tangential flat space only in an infinitely small vicinity of the projection point (because they are absent in the tangential flat space). Therefore, all calculation results obtained using the tetrad formalism or any modification of it are valid only in an infinitely small vicinity of the projection point in the tangential flat space (and not in the Riemannian space itself), and these results are not integrable to another point of the Riemannian space.

Kerr's solution and the Kerr-Newman solution were derived using the Newman-Penrose formalism (a modification of the tetrad formalism) in the tangential flat space, in which objects of General Relativity do not actually exist. Therefore, the physical reality of their theoretical results is questionable. In addition, Kerr's solution and the Kerr-Newman solution have a serious limitation. Namely — these solutions are valid only in an infinitely small vicinity of the surface of a rotating black hole in the tangential flat space, and not in the Riemannian space itself (in which all objects of General Relativity exist, including gravitational collapsars).

In contrast, the solutions obtained here for rotating black holes and electrically charged rotating black holes are mathematically and physically correct, since they were derived in the Riemannian space itself, have no limitations, and are integrable over the entire space.

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