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SPECIAL REPORT

PLANCK, the Satellite: a New Experimental Test of General Relativity

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If the origin of a microwave background (EMB) is the Earth, what would be its density and associated dipole anisotropy measured at different altitudes from the surface of the Earth? The mathematical methods of the General Theory of Relativity are applied herein to answer these questions. The density of the EMB is answered by means of Einstein's equations for the electromagnetic field of the Earth. The dipole anisotropy, which is due to the rapid motion of the source (the Earth) in the weak intergalactic field, is analysed by using the geodesic equations for light-like particles (photons), which are mediators for electromagnetic radiation. It is shown that the EMB decreases with altitude so that the density of its energy at the altitude of the COBE orbit (900km) is 0.68 times less than that at the altitude of a U2 aeroplane (25 km). Furthermore, the density at the 2nd Lagrange point (1.5 million km, the position of the WMAP and PLANCK satellites) should be only $\sim 10^{-7}$ of the value detected by a U2 aeroplane or at the COBE orbit. The dipole anisotropy of the EMB doesn't depend on altitude from the surface of the Earth, it should be the same irrespective of the altitude at which measurements are taken. This result is in support to the experimental and observational analysis conducted by P.-M. Robitaille, according to which the 2.7 K microwave background, first observed by Penzias and Wilson, is not of cosmic origin, but of the Earth, and is generated by the oceans. WMAP indicated the same anisotropy of the microwave background at the 2nd Lagrange point that near the Earth. Therefore when PLANCK, which is planned on July, 2008, will manifest the 2.7 K monopole microwave signal deceased at the 2nd Langrange point, it will be a new experimental verification of Einstein's theory.

1 Introduction

Our recent publication [1] was focused on the mathematical proof in support to the claim made by P.-M. Robitaille: according to the experimental and observational analysis conducted by him [3–10], the 2.7 K monopole microwave background, first detected by Penzias and Wilson [2], is not of cosmic origin, but of the Earth, and is generated by oceanic water*. As shown in the framework of Robitaille's concept, the anisotropy of the background, observed on the 3.35 mK dipole component of it†, is due to the rapid motion of the whole field in common with its source, the Earth, in a weak intergalactic field so that the anisotropy of the observed microwave background has a purely relativistic origin [21].‡

If the microwave background is of the earthy origin, the density of the field should obviously decrease with altitude from the surface of the Earth. The ground-bound measurements and those made on board of the COBE satellite, at the altitude 900 km, were processed very near the oceans which aren't point-like sources, so the observations were unable to manifest the change of the field density with altitude. Another case — the 2nd Lagrange point, which is located as far as 1.5 mln km from the Earth, the position of the WMAP satellite and the planned PLANCK satellite.

A problem is that WMAP has only differential instruments on board: such an instrument, having a few channels for incoming photons, registers only the difference between the number of photons in the channels. WMAP therefore targeted measurements of the anisotropy of the field, but was unable to measure the field density. PLANCK, which is planned on July, 2008, is equipped by absolute instruments (with just one channel for incoming photons, an absolute instrument gets the integral density of the monopole and all the multipole components of the field). Hence PLANCK will be able to measure the field density at the 2nd Lagrange point.

We therefore were looking for a theory which would be able to represent the density and anisotropy of the Earth's microwave background as the functions of altitude from the Earth's surface.

In our recent publication [1], we created such a theory with use of the mathematical methods of the General The-

*Robitaille reported the result first in 1999 and 2001 in the short communications [1, 2], then detailed explanation of the problem was given by him in the journal publications [3–6] and also in the reports [9, 10].

†The 3.35 mK dipole component of the background was first observed in 1969 by Conklin [11] in a ground-based observation. Then it was studied by Henry [12], Corey [13], and also Smoot, Gorenstein, and Muller (the latest team organized a stratosphere observation on board of a U2 aeroplane [14]). The history of the discovery and all the observations is given in detail in Lineweaver's paper [15]. The anisotropy of the dipole component was found later, in the COBE space mission then verified by the WMAP space mission [16–20].

‡This conclusion is based on that fact that, according to the General Theory of Relativity, photons exceeded from a source at radial directions should be carried out with the space wherein this source moves so that the spherical distribution of the signals should experience an anisotropy in the direction of the motion of this source in the space [22, 23].

ory of Relativity where the physical characteristics of fields are expressed through the geometrical characteristics of the space itself. We have split our tasks into two particular problems: if a microwave background originates from the Earth, what would be the dependency of its density and relativistic anisotropy with altitude? The first problem was solved via Einstein's equations for the electromagnetic field of the Earth. The second problem was solved using the geodesic equations for light-like particles (photons) which are mediators for electromagnetic radiation.

We have determined, according to our solutions [1], that a microwave background that originates at the Earth decreases with altitude so that the density of the energy of such a background in the COBE orbit (the altitude 900 km) is 0.68 times less than that at the altitude of a U2 aeroplane. The density of the energy of the background at the L2 point is only $\sim 10^{-7}$ of the value detected by a U2 aeroplane or at the COBE orbit. The dipole anisotropy of such an earthy microwave background, due to the rapid motion of the Earth relative to the source of a weak intergalactic field which is located in depths of the cosmos, doesn't depend on altitute from the surface of the Earth. Such a dipole will be the same irrespective of the position at which measurements are taken.

In principle, the first problem — how the density of an earthy-origin microwave background decreases with altitude — may be resolved by the methods of classical physics. But this is possible only in a particular case where the space is free of rotation. In real, the Earth experiences daily rotation. We therefore should take into account that fact that the rotation makes the observer's local space non-holonomic: in such a space the time lines are non-orthogonal to the spatial section, so the Riemannian curvature of the space is non-zero. A satellite's motion around the Earth should be also taken into account for the local space of an observer which is located on board of the satellite. Therefore in concern of a real experiment, in both cases of ground-based and satellite-based observations, the first problem can be resolved only in the framework of the General Theory of Relativity.

The second problem can never been resolved in the framework of classical physics due to the purely relativistic origin of the field anisotropy we are considering.

WMAP registered the same parameters of the microwave background anisotropy that the registered by COBE near the Earth. This is according to our theory.

Therefore when PLANCK will manifest the 2.7 K monopole microwave signal deceased at the 2nd Langrange point, with the same anisotropy of the background that the measured near the Earth (according to WMAP which is as well located at the 2nd Langange point), this will be a new experimental verification of the General Theory of Relativity.

A drawback of our theory was only that complicate way in which it was initially constructed. As a result, our recently published calculation [1] is hard to reproduce by the others who have no mathematical skills in the very specific areas of General Relativity, which are known to only a close circle of the specialists who are no many in the world. We therefore were requested for many additional explanations by those readers who tried to repeat the calculation.

Due to that discussion, we found another way to give representation of our result with much unused stuff removed. We also gave an additional explanation to those parts of our calculation, which were asked by the readers. As a result a new representation of our calculation, with the same result, became as simple as easy to peroduce by everyone who is free in tensor algebra. This representation is given here.

2 The local space metric of a satellite-bound observer

A result of real measurement processed by an observer depends on the properties of his local space. These properties are completely determined by the metric of this space. We therefore are looking for the metric of the local space of an observer, who is located on board of a statellite moved in the Earth's gravitational field.

As one regularly does in construction for a metric, we take a simplest metric which is close to the case we are considering, then modify the metric by introduction of those additional factors which are working in our particular case.

Here is how we do it.

As was proven in the 1940's by Abraham Zelmanov, on the basis of the theory of hon-holonomic manifolds [24] constructed in the 1930's by Schouten then applied by Zelmanov to the four-dimensional pseudo-Riemannian space of General Relativity, the non-holonomity of such a space (i.e. the non-orthogonality of the time lines to the spatial section, that is expressed as $g_{0i} \neq 0$ in the fundamental metric tensor $g_{\alpha\beta}$) is manifest as the three-dimensional rotation of this space. Moreover, Zelmanov proven that any non-holonomic space has nonzero Riemannian curvature (nonzero Riemann-Christoffel tensor) due to $g_{0i} \neq 0$. All these was first reported in 1944 by him in his dissertation thesis [25], then also in the latter publications [26–28].

In practice this means that the physical space of the Earth, the planet, is non-holonomic and curved due to the daily rotation of it. This is in addition to that fact that the Earth's space is curved due to the gravitational field of the Earth, described in an approximation by Schwarzschild metric of a centrally symmetric gravitational field, created by a spherical mass in emptiness. The space metric of a satellite-bound observer should also take into account that fact that the satellite moves along its orbit in the Earth's space around the terrestrial globe (the central mass that produces the field). In addition to it the Earth, in common with the satellite and the observer located in it, rapidly moves in the physical space of the Universe associated to the weak intergalactic microwave field. This fact should also be taken into account in the metric.

First, we consider a simplest non-holonomic space — a space wherein all $g_{0i} \neq 0$, and they have the same numerical

$$ds^2 = \left(1 - \frac{2GM}{c^2 r} - \frac{\omega^2 r^2}{c^2}\right) c^2 dt^2 + \frac{2v\,(\cos\varphi + \sin\varphi)}{c}\, cdt\, dr + \frac{2r\,[v\,(\cos\varphi - \sin\varphi) - \omega r]}{c}\, cdt\, d\varphi + \frac{2v}{c}\, cdt\, dz - \\ - \left(1 + \frac{2GM}{c^2 r}\right) dr^2 - r^2 d\varphi^2 - dz^2 \qquad (7)$$

$$ds^2 = \left(1 - \frac{2GM}{c^2 r} - \frac{\omega^2 r^2}{c^2} + \frac{2v\mathrm{v}}{c^2}\right) c^2 dt^2 + \frac{2v\,(\cos\varphi + \sin\varphi)}{c}\, cdt\, dr + \frac{2r\,[v\,(\cos\varphi - \sin\varphi) - \omega r]}{c}\, cdt\, d\varphi + \frac{2v}{c}\, cdt\, dz - \\ - \left(1 + \frac{2GM}{c^2 r}\right) dr^2 + \frac{2v\mathrm{v}\,(\cos\varphi + \sin\varphi)}{c^2}\, dr\, dz - r^2 d\varphi^2 + \frac{2r\mathrm{v}\,[v\,(\cos\varphi - \sin\varphi) - \omega r]}{c^2}\, d\varphi\, dz - \left(1 - \frac{2v\mathrm{v}}{c^2}\right) dz^2 \qquad (8)$$

values. According to Zelmanov [25–28], such a space experiences rotation around all three axes with the same linear velocity $v = v_1 = v_2 = v_3$, where $v_i = -\frac{c\, g_{0i}}{\sqrt{g_{00}}}$. As obvious, the metric of such a non-holonomic space is

$$ds^2 = c^2 dt^2 + \frac{2v}{c}\, cdt\,(dx + dy + dz) - dx^2 - dy^2 - dz^2. \quad (1)$$

For easy taking the Earth's field into account, we change to the cylindrical coordinates r, φ, z, where the r-axis is directed from the centre of gravity of the Earth along its radius. The corresponding transformations of the coordinates are $x = r\cos\varphi$, $y = r\sin\varphi$, $z = z$ so that the metric (1) represented in the new coordinates is

$$ds^2 = c^2 dt^2 + \frac{2v}{c}\,(\cos\varphi + \sin\varphi)\, cdt\, dr + \\ + \frac{2vr}{c}\,(\cos\varphi - \sin\varphi)\, cdt\, d\varphi + \frac{2v}{c}\, cdt\, dz - \\ - dr^2 - r^2 d\varphi^2 - dz^2. \quad (2)$$

Next we introduce the factor of the Earth's gravitational field in the same way as it is made in Schwarzschild metric (see §100 in *The Classical Theory of Fields* [29]) — the metric of a spherically symmetric gravitational field, produced by a sperical mass M in emptiness, which in the cylindrical coordinates is

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{2GM}{c^2 r}} - r^2 d\varphi^2 - dz^2, \quad (3)$$

where we should take into account that fact that $\frac{2GM}{c^2 r}$ is small value, so we have

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \left(1 + \frac{2GM}{c^2 r}\right) dr^2 - \\ - r^2 d\varphi^2 - dz^2. \quad (4)$$

Besides, we should take into account the factor of rotational motion of the observer, in common with the satellite, along its orbit around the Earth. We see how to do it in the example of a plane metric in the cylindrical coordinates

$$ds^2 = c^2 dt^2 - dr^2 - r^2 d\varphi^2 - dz^2, \quad (5)$$

where we change the reference frame to another one, which rotates relative to the initially reference frame with a constant angular velocity ω. By the applying the transformation of the coordinates $r' = r$, $\varphi' = \varphi + \omega t$, $x' = z$, we obtain ds^2 in the rotating reference frame*

$$ds^2 = \left(1 - \frac{\omega^2 r^2}{c^2}\right) c^2 dt^2 - \frac{2\omega r^2}{c}\, cdt\, d\varphi - dr^2 - \\ - r^2 d\varphi^2 - dz^2. \quad (6)$$

Following with the aforementioned steps†, we obtain the metric of the *local physical space* of a satellite-bound observer which takes all properties of such a space into account. This resulting metric is represented in formula (7).

This metric will be used by us in calculation for the density of the Earth microwave background, measured by an observer on board of a satellite of the Earth.

This metric is definitely curved due to two factors: non-zero gravitational potential $\mathrm{w} = c^2\,(1 - \sqrt{g_{00}}) \neq 0$ and the space non-holonomity $g_{0i} \neq 0$. Hence we are able to consider Einstein equations in such a space.

On the other hand this metric doesn't take into account that fact that the Earth microwave background, in common with the Earth, moves in a weak intergalactic field with a velocity of $\mathrm{v} = 365 \pm 18$ km/sec (as observational analysis indicates it). To calculate the accociated dipole anisotropy of the Earth microwave background, which is due to the motion, we should use such a space metric which takes this motion into account. To do it we take the metric (7) then apply Lorentz' transformations to the z-coordinate (we direct the z-axis with the motion of the Earth in the weak intergalactic field) and time with an obvious approximation of $\mathrm{v} \ll c$ and high order terms omitted: $z' = z + \mathrm{v}t$, $t' = t + \frac{\mathrm{v}z}{c^2}$. In other word, we "move" the whole local physical space of an earthy satellite-bound observer relative to the source of the weak intergalactic field. As a result the local physical space of such an observer and all physical fields connected to the Earth should experience a drift in the z-direction and a corresponding change the

*See §10.3 in [27], or §3.6 in [28] for detail.

†As known in Riemannian geometry, which is particular to metric geometries, a common metric can be deduced as a superposition of all the particular metrics each of whom takes a particular property of the common space into account.

local physically observed time that should has a sequel on the observed charachteristics of the Earth's microwave field.

The resulting metric we have obtained after the transformation is (8). We will use this metric in calculation for the anisotropy of the Earth microwave background measured by a satellite-bound observer.

3 The density of the Earth's microwave background at the 2nd Lagrange point

To calculate the density of a field (distributed matter) dependent from the properties of the space wherein this field is situated we should operate with Einstein's equations

$$R_{\alpha\beta} - \frac{1}{2}\, g_{\alpha\beta}\, R = -\,\kappa\, T_{\alpha\beta} + \lambda\, g_{\alpha\beta}\,, \tag{9}$$

the left side of which is for the space geometry, while the right side describes distributed matter (it is with the energy-momentum tensor of distributed matter and the λ-term which describes the distribution of physical vacuum).

Projection of the energy-momentum tensor $T_{\alpha\beta}$ onto the time line and spatial section of an observer's local physical space gives the properties of distributed matter observed by him [25–28]: the density of the energy of distributed matter $\rho = \frac{T_{00}}{g_{00}}$, the density of the momentum $J^i = \frac{c\,T_0^i}{\sqrt{g_{00}}}$, and the stress-tensor $U^{ik} = c^2 T^{ik}$. To express the first of these observable quantities through the observable properties of the local physical space is a task in our calculation.

To reach this task we should project the whole Einstein equations onto the ime line and spatial section of the metric space (7) with taking into account that fact that the energy-momentum tensor is of an electromagnetic field. The left side of the projected equations will be containing the observable properties of the local space of such an observer, while the right side will be containing the aforementioned observable properties of distributed matter (the Earth microwave background, in our case). Then we can express the density of the Earth microwave background ρ as a function of the observable properties of the local space.

Einstein's equations projected onto the time line and spatial section of a common case were obtained in the 1940's by Zelmanov [25–28], and are quite complicate in the left side (the observable properties of the local space). We therefore first should obtain the observable properties of the given space (7), then decide what propetries can be omitted from consideration in the framework of our problem.

According to the theory of physical observable quantities of the General Theory of Relativity [25–28], the observable properties of a space are the three-dimensional quantities which are invariant within the fixed spatial section of an observer (so-called *chronometrically invariant quantities*). Those are the three-dimensional metric tensor h_{ik}, the gravitational inertial force F_i, the angular velocity of the space rotation A_{ik} (known as the non-holonomity tensor), the space deformation tensor D_{ik}, the three-dimensional Christoffel symbols Δ^i_{kn}, and the three-dimensional curvature C_{iklj}, expressed through the gravitational potential $\mathrm{w} = c^2\,(1-\sqrt{g_{00}})$ and the linear velocity of the space rotation $v_i = -\,\frac{c\,g_{0i}}{\sqrt{g_{00}}}$ (whose components are $v^i = -\,c\,g^{0i}\sqrt{g_{00}}$ and $v_i = h_{ik}\,v^k$):

$$h_{ik} = -\,g_{ik} + \frac{g_{0i}\,g_{0k}}{g_{00}} = -\,g_{ik} + \frac{1}{c^2}\,v_i v_k\,, \tag{10}$$

$$h^{ik} = -\,g^{ik}, \qquad h^i_k = -\,g^i_k = \delta^i_k\,, \tag{11}$$

$$F_i = \frac{1}{\sqrt{g_{00}}}\left(\frac{\partial \mathrm{w}}{\partial x^i} - \frac{\partial v_i}{\partial t}\right), \tag{12}$$

$$A_{ik} = \frac{1}{2}\left(\frac{\partial v_k}{\partial x^i} - \frac{\partial v_i}{\partial x^k}\right) + \frac{1}{2c^2}\left(F_i v_k - F_k v_i\right), \tag{13}$$

$$D_{ik} = \frac{1}{2}\frac{{}^*\partial h_{ik}}{\partial t}\,, \qquad D^{ik} = -\,\frac{1}{2}\frac{{}^*\partial h^{ik}}{\partial t}\,, \tag{14}$$

$$D = h^{ik} D_{ik} = D^k_k = \frac{{}^*\partial \ln\sqrt{h}}{\partial t}\,, \quad h = \det\|\,h_{ik}\|\,, \tag{15}$$

$$\Delta^i_{jk} = \frac{1}{2}\,h^{im}\left(\frac{{}^*\partial h_{jm}}{\partial x^k} + \frac{{}^*\partial h_{km}}{\partial x^j} - \frac{{}^*\partial h_{jk}}{\partial x^m}\right), \tag{16}$$

$$\Delta^j_{jk} = \frac{{}^*\partial \ln\sqrt{h}}{\partial x^k}\,, \tag{17}$$

$$C_{lkij} = \frac{1}{4}\left(H_{lkij} - H_{jkil} + H_{klji} - H_{iljk}\right), \tag{18}$$

$$C_{kj} = C_{kij\cdot}^{\cdots i} = h^{im} C_{kimj}\,, \quad C = C^j_j = h^{lj} C_{lj}\,. \tag{19}$$

Here $\frac{{}^*\partial}{\partial t} = \frac{1}{\sqrt{g_{00}}}\frac{\partial}{\partial t}$ and $\frac{{}^*\partial}{\partial x^i} = \frac{\partial}{\partial x^i} + \frac{1}{c^2}\,v_i\,\frac{{}^*\partial}{\partial t}$ are the chronometrically invariant differential operators, while

$$H_{lki\cdot}^{\cdots j} = \frac{{}^*\partial \Delta^j_{il}}{\partial x^k} - \frac{{}^*\partial \Delta^j_{kl}}{\partial x^i} + \Delta^m_{il}\Delta^j_{km} - \Delta^m_{kl}\Delta^j_{im} \tag{20}$$

is Zelmanov's tensor constructed by him on the basis of the non-commutativity of the second chronometrically invariant derivatives of an arbitrary spatial vector taken in a given three-dimensional spatial section

$${}^*\nabla_i{}^*\nabla_k\, Q_l - {}^*\nabla_k{}^*\nabla_i\, Q_l = \frac{2A_{ik}}{c^2}\frac{{}^*\partial Q_l}{\partial t} + H_{lki\cdot}^{\cdots j}\, Q_j\,, \tag{21}$$

where ${}^*\nabla_i\, Q_l = \frac{{}^*\partial Q_l}{\partial x^i} - \Delta^j_{ji}\, Q_l$ is the chronometrically invariant derivative of the vector (${}^*\nabla_i\, Q^l = \frac{{}^*\partial Q^l}{\partial x^i} + \Delta^j_{ji}\, Q^l$ respectively). The tensor $H_{lki\cdot}^{\cdots j}$ was introduced by Zelmanov similarly to Schouten's tensor of the theory of non-holonomic manifolds [24] so that the three-dimensional curvature tensor C_{lkij} possesses all the algebraic properties of the Riemann-Christoffel curvature tensor in the spatial section.

We take the components of the fundamental metric tensor $g_{\alpha\beta}$ from the metric of the local physical space of a satellite-bound observer (7), then calculate the aforementioned observable quantities. In this calculation we take into account that fact that $\frac{2GM}{c^2 r}$ and $\frac{\omega^2 r^2}{c^2}$ are in order of 10^{-9} near the surface of the Earth, and the values decrease with altitude. We therefore operate these terms according to the rules of small values. We also neglect all high order terms. We however cannot neglect $\frac{2GM}{c^2 r}$ and $\frac{\omega^2 r^2}{c^2}$ in $g_{00} = 1 - \frac{2GM}{c^2 r} - \frac{\omega^2 r^2}{c^2}$ when calculating the gravitational potential $\mathrm{w} = c^2 (1 - \sqrt{g_{00}})$ according to the rule of small values

$$\begin{aligned} \mathrm{w} &= c^2 \left(1 - \sqrt{1 - \frac{2GM}{c^2 r} - \frac{\omega^2 r^2}{c^2}}\right) = \\ &= c^2 \left\{1 - \left(1 + \frac{GM}{c^2 r} + \frac{\omega^2 r^2}{2c^2}\right)\right\} \end{aligned} \tag{22}$$

because these terms are multiplied by c^2. We also assume the linear velocity of the space rotation v to be small to the velocity of light c. We assume that v doesn't depend from the z-coordinate. This assumption is due to the fact that the Earth, in common with its space, moves relative to a weak intergalactic microwave background that causes the anisotropy of the Earth's microwave field.

As a result we obtain the substantially non-zero components of the characteristics of the space

$$\mathrm{w} = \frac{GM}{r} + \frac{\omega^2 r^2}{2}\,, \tag{23}$$

$$\left.\begin{aligned} v_1 &= -v\,(\cos\varphi + \sin\varphi) \\ v_2 &= -r\left[v\,(\cos\varphi - \sin\varphi) - \omega r\right] \\ v_3 &= -v \end{aligned}\right\}, \tag{24}$$

$$\left.\begin{aligned} &F_1 = (\cos\varphi + \sin\varphi)\, v_t + \omega^2 r - \frac{GM}{r^2} \\ &F_2 = r\,(\cos\varphi - \sin\varphi)\, v_t\,, \qquad F_3 = v_t \end{aligned}\right\}, \tag{25}$$

$$\left.\begin{aligned} A_{12} &= \omega r + \frac{1}{2}\big[(\cos\varphi + \sin\varphi)\, v_\varphi - \\ &\qquad\qquad - r\,(\cos\varphi - \sin\varphi)\, v_r\big] \\ A_{23} &= -\frac{v_\varphi}{2}\,, \qquad A_{13} = -\frac{v_r}{2} \end{aligned}\right\}, \tag{26}$$

$$\left.\begin{aligned} &h_{11} = h_{33} = 1\,, \quad h_{22} = r^2, \quad h^{11} = h^{33} = 1 \\ &h^{22} = \frac{1}{r^2}\,, \qquad h = r^2, \qquad \frac{\partial \ln\sqrt{h}}{\partial r} = \frac{1}{r} \end{aligned}\right\}, \tag{27}$$

$$\Delta^1_{22} = -r\,, \qquad \Delta^2_{12} = \frac{1}{r} \tag{28}$$

while all components of the tensor of the space deformation D_{ik} and the three-dimensional curvature C_{iklj} are negligible in the framework of the first order approximation (the four-dimensional Riemannian curvature isn't negligible).

The quantities v_r, v_φ, and v_t denote the partial derivatives of the linear velocity of the space rotation v by the respective coordinate and time. (Here $v_z = 0$ according to the initially assumptions in the framework of our problem.)

We consider the projected Einstein equations in complete form, published in [25–28]

$$\left.\begin{aligned} &\frac{{}^*\partial D}{\partial t} + D_{jl}D^{jl} + A_{jl}A^{lj} + \left({}^*\nabla_j - \frac{1}{c^2}F_j\right)F^j = \\ &= -\frac{\varkappa}{2}\left(\rho c^2 + U\right) + \lambda c^2 \\ &{}^*\nabla_j\left(h^{ij}D - D^{ij} - A^{ij}\right) + \frac{2}{c^2}F_j A^{ij} = \varkappa J^i \\ &\frac{{}^*\partial D_{ik}}{\partial t} - \left(D_{ij} + A_{ij}\right)\left(D^j_k + A^{\cdot j}_{k\cdot}\right) + D D_{ik} + \\ &+ 3A_{ij}A^{\cdot j}_{k\cdot} + \frac{1}{2}\left({}^*\nabla_i F_k + {}^*\nabla_k F_i\right) - \frac{1}{c^2}F_i F_k - \\ &- c^2 C_{ik} = \frac{\varkappa}{2}\left(\rho c^2 h_{ik} + 2U_{ik} - U h_{ik}\right) + \lambda c^2 h_{ik} \end{aligned}\right\}. \tag{29}$$

We withdraw the λ-term, the space deformation D_{ik}, and the three-dmensional curvature C_{iklj} from consideration. We also use the aforemonetioned assumptions on small values and high order terms that reduce the chronometrically invariant differential operators to the regular differential operators: $\frac{{}^*\partial}{\partial t} = \frac{\partial}{\partial t}$, $\frac{{}^*\partial}{\partial x^i} = \frac{\partial}{\partial x^i}$. As a result of all these, the projected Einstein equations take the simplified form

$$\left.\begin{aligned} &\frac{\partial F^i}{\partial x^i} + \frac{\partial \ln\sqrt{h}}{\partial x^i}F^i - A_{ik}A^{ik} = -\frac{\varkappa}{2}\left(\rho c^2 + U\right) \\ &\frac{\partial A^{ik}}{\partial x^k} + \frac{\partial \ln\sqrt{h}}{\partial x^k}\, A^{ik} = -\varkappa J^i \\ &2A_{ij}A^{\cdot j}_{k\cdot} + \frac{1}{2}\left(\frac{\partial F_i}{\partial x^k} + \frac{\partial F_k}{\partial x^i} - 2\Delta^m_{ik}F_m\right) = \\ &\qquad = \frac{\varkappa}{2}\left(\rho c^2 h_{ik} + 2U_{ik} - U h_{ik}\right) \end{aligned}\right\}. \tag{30}$$

We substitute hereto the obtained observable characheristics of the local physical space of a satellite-bound observer. Because the value v is assumed to be small, we neglect not only the square of it, but also the square of its derivative and the products of the derivatives.

The Einstein equations (30) have been written for a space filled with an arbitrary matter, which is described by the energy-momentum tensor written in the common form $T_{\alpha\beta}$. In other word, the distributed matter can be the superposition of an electromagnetic field, dust, liquid or other matter. Concerning our problem, we consider only an electromagnetic field. As known [29], the energy-momentum tensor $T_{\alpha\beta}$ of any electromagnetic field should satisfy the condition $T = \rho c^2 - U$. We therefore assume that the right side of the Einstein equations contains the energy-momentum tensor of only an electromagnetic field (no dust, liquid, or other matter distributed near the Earth). In other word we should mean, in the right side,

$$\rho c^2 = U\,. \tag{32}$$

Besides, because all measurement in the framework of our problem are processed by an observer on board of a satel-

$$\left.\begin{aligned}
&-2\omega^2-2\omega\left(\cos\varphi+\sin\varphi\right)\frac{v_\varphi}{r}+2\omega\left(\cos\varphi-\sin\varphi\right)v_r+\left(\cos\varphi+\sin\varphi\right)v_{tr}+\left(\cos\varphi-\sin\varphi\right)\frac{v_{t\varphi}}{r}=-\kappa\rho c^2\\
&\frac{1}{2}\left[\left(\cos\varphi+\sin\varphi\right)\left(\frac{v_r}{r}+\frac{v_{\varphi\varphi}}{r^2}\right)+\left(\cos\varphi-\sin\varphi\right)\left(\frac{v_\varphi}{r^2}-\frac{v_{r\varphi}}{r}\right)\right]=-\kappa J^1\\
&\frac{1}{2}\left[\left(\cos\varphi+\sin\varphi\right)\left(\frac{v_\varphi}{r^3}-\frac{v_{r\varphi}}{r^2}\right)+\left(\cos\varphi-\sin\varphi\right)\frac{v_{rr}}{r}\right]=-\kappa J^2\\
&\frac{1}{2}\left(v_{rr}+\frac{v_r}{r}+\frac{v_{\varphi\varphi}}{r^2}\right)=-\kappa J^3\\
&2\omega^2+2\omega\left(\cos\varphi+\sin\varphi\right)\frac{v_\varphi}{r}-2\omega\left(\cos\varphi-\sin\varphi\right)v_r+\left(\cos\varphi+\sin\varphi\right)v_{tr}=\kappa U_{11}\\
&\frac{r^2}{2}\left[\left(\cos\varphi+\sin\varphi\right)\frac{v_{t\varphi}}{r^2}+\left(\cos\varphi-\sin\varphi\right)\frac{v_{tr}}{r}\right]=\kappa U_{12}\\
&\omega\frac{v_\varphi}{r}+\frac{1}{2}v_{tr}=\kappa U_{13}\\
&2\omega^2+2\omega\left(\cos\varphi+\sin\varphi\right)\frac{v_\varphi}{r}-2\omega\left(\cos\varphi-\sin\varphi\right)v_r+\left(\cos\varphi-\sin\varphi\right)\frac{v_{t\varphi}}{r}=\kappa\frac{U_{22}}{r^2}\\
&\frac{r^2}{2}\left(\frac{v_{t\varphi}}{r^2}-2\omega\frac{v_r}{r}\right)=\kappa U_{23}\\
&\kappa U_{33}=0
\end{aligned}\right\}\tag{31}$$

lite, we should also take into account the weightlessness condition

$$\frac{GM}{r^2}=\omega^2 r\,.\tag{33}$$

As a result, we obtain the system of the projected Einstein equations (30) in the form (31) which is specific to the real physical space of a satellite-bound observer.

In other word, that fact that we used the conditions (32) and (33) means that our theoretical calculation targets measurement of an electromagnetic field in the weightlessness state in an orbit of the Earth.

We are looking for the quantity ρ as a function of the properties of the space from the first (scalar) equation of the Einstein equantions (31). This isn't a trivial task, because the aforementioned scalar Einstein equation

$$\begin{aligned}\kappa\rho c^2&=2\omega^2+2\omega\left(\cos\varphi+\sin\varphi\right)\frac{v_\varphi}{r}-\\&-2\omega\left(\cos\varphi-\sin\varphi\right)v_r-\left(\cos\varphi+\sin\varphi\right)v_{tr}-\\&-\left(\cos\varphi-\sin\varphi\right)\frac{v_{t\varphi}}{r}\end{aligned}\tag{34}$$

contains the distribution functions of the linear velocity of the space rotation (the functions v_r, v_φ, and v_t), which are unknown. We therefore should first find the functions.

According to our asumption, $\rho c^2=U$. Therefore $\kappa\rho c^2$ and κU are the same in the framework of our problem. We calculate the quantity

$$\kappa U=\kappa h^{ik}U_{ik}=\kappa\left(U_{11}+\frac{U_{22}}{r^2}+U_{33}\right)\tag{35}$$

as the sum of the 5th and the 8th equations of the system of the Einstein equations (31) with taking into account that fact that, in our case, $U_{33}=0$ (as seen from the 10th equation, with $\rho c^2=U$). We obtain

$$\begin{aligned}\kappa U&=4\omega^2+4\omega\left(\cos\varphi+\sin\varphi\right)\frac{v_\varphi}{r}-\\&-4\omega\left(\cos\varphi-\sin\varphi\right)v_r+\left(\cos\varphi+\sin\varphi\right)v_{tr}+\\&+\left(\cos\varphi-\sin\varphi\right)\frac{v_{t\varphi}}{r}\,.\end{aligned}\tag{36}$$

Subtracting $\kappa\rho c^2$ (34) from κU (36) then equalizing the result to zero, according to the electromagnetic field condition $\rho c^2=U$, we obtain the geometrization condition for the electromagnetic field

$$\begin{aligned}&\omega^2+\omega\left(\cos\varphi+\sin\varphi\right)\frac{v_\varphi}{r}-\omega\left(\cos\varphi-\sin\varphi\right)v_r+\\&+\left(\cos\varphi+\sin\varphi\right)v_{tr}+\left(\cos\varphi-\sin\varphi\right)\frac{v_{t\varphi}}{r}=0\,.\end{aligned}\tag{37}$$

With this condition, all the components of the energy-momentum tensor of the field $T_{\alpha\beta}$ (the right side of the Einstein equations) are expressed in only the properties of the space (the left side of the Einstein equations). Hence we have geometrized the electromagnetic field. This is an important result: earlier only isotropic electromagnetic fields (they are satisfying Rainich's condition and Nordtvedt-Pagels condition) were geometrized.

To find the distribution functions of v, we consider the conservation law $\nabla_\sigma T^{\alpha\sigma}=0$, expressed in terms of the phys-

$$\left.\begin{aligned}&(\cos\varphi-\sin\varphi)\left(\frac{v_{tr\varphi}}{r}-\frac{v_{t\varphi}}{r^2}\right)+\omega\,(\cos\varphi+\sin\varphi)\left(\frac{v_{r\varphi}}{r}-\frac{v_{\varphi}}{r^2}\right)-\\&-\omega\,(\cos\varphi-\sin\varphi)\,v_{rr}+(\cos\varphi+\sin\varphi)\,v_{trr}=0\\&(\cos\varphi+\sin\varphi)\left(\frac{v_{tr\varphi}}{r^2}-\frac{v_{t\varphi}}{r^3}\right)+(\cos\varphi-\sin\varphi)\left(\frac{v_{t\varphi\varphi}}{r^3}+\frac{v_{tr}}{r^2}\right)+\\&+\omega\,(\cos\varphi+\sin\varphi)\left(\frac{v_{\varphi\varphi}}{r^3}+\frac{v_r}{r^2}\right)+\omega\,(\cos\varphi-\sin\varphi)\left(\frac{v_{\varphi}}{r^3}-\frac{v_{r\varphi}}{r^2}\right)=0\end{aligned}\right\}\tag{41}$$

ical observed quantities [25–28]

$$\left.\begin{aligned}&\frac{{}^*\partial\rho}{\partial t}+D\rho+\frac{1}{c^2}\,D_{ij}U^{ij}\;+\\&\qquad+\left({}^*\nabla_i-\frac{1}{c^2}\,F_i\right)J^i-\frac{1}{c^2}\,F_iJ^i=0\\&\frac{{}^*\partial J^k}{\partial t}+2\left(D^k_i+A_{i\cdot}^{\cdot k}\right)J^i\;+\\&\qquad+\left({}^*\nabla_i-\frac{1}{c^2}\,F_i\right)U^{ik}-\rho F^k=0\end{aligned}\right\}\tag{38}$$

which, under the assumptions specific in our problem, is

$$\left.\begin{aligned}&\frac{\partial J^i}{\partial x^i}+\frac{\partial\ln\sqrt{h}}{\partial x^i}\,J^i=0\\&\frac{\partial J^k}{\partial t}+2A_{i\cdot}^{\cdot k}J^i+\frac{\partial U^{ik}}{\partial x^i}+\Delta^k_{im}U^{im}+\\&\qquad+\frac{\partial\ln\sqrt{h}}{\partial x^i}\,U^{ik}-\rho F^k=0\end{aligned}\right\}.\tag{39}$$

The first (scalar) equation of the system of the conservation equations (39) means actually that the chronometrically invariant derivative of the vector J^i is zero

$${}^*\nabla_iJ^i=\frac{\partial J^i}{\partial x^i}+\frac{\partial\ln\sqrt{h}}{\partial x^i}\,J^i=0\,,\tag{40}$$

i.e. the flow of the vector J^i (the flow of the density of the field momentum) is constant. So, the first equation of (39) satisfies identically as ${}^*\nabla_iJ^i=0$.

The rest three (vectorial) equations of the system (39), with the properties of the local space of a satellite-bound observer and the components of the energy-momentum tensor substituted (the latest should be taken from the Einstein equations), take the form (41). As seen, only first two equations still remaining meaningful, while the third of the vectorial equations of conservation vanishes becoming the identity zero equals zero.

In other word, we have obtained the equations of the conservation law specific to the real physical space of a satellite-bound observer.

Let's suppose that the function v has the form

$$v=T(t)\,r\,e^{i\varphi},\tag{42}$$

hence the partial derivatives of this function are

$$\left.\begin{aligned}&v_r=T\,e^{i\varphi} && v_{\varphi}=i\,T\,r\,e^{i\varphi}\\&v_{tr}=\dot{T}\,e^{i\varphi} && v_{t\varphi}=i\,\dot{T}\,r\,e^{i\varphi}\\&v_{rr}=0 && v_{trr}=0\\&v_{tr\varphi}=i\,\dot{T}\,e^{i\varphi} && v_{t\varphi\varphi}=-\dot{T}\,r\,e^{i\varphi}\\&v_{\varphi\varphi}=-T\,r\,e^{i\varphi} && v_{r\varphi}=i\,T\,e^{i\varphi}\end{aligned}\right\}.\tag{43}$$

After the functions substituted into the equations of the conservation law (41), we see that the equations satisfy identically. Hence $v=T(t)\,r\,e^{i\varphi}$ is exact solution of the conservation equations with respect to v.

Now we need to find only the unknown function $T(t)$. This function can be found from the electromagnetic field condition $\rho c^2=U$ expressed by us through the properties of the space itself as the formula (37).

We assume that the satellite, on board of which the observer is located, displaces at small angle along its orbit during the process of his observation. This is obvious assumption, because the very fast registration process for a single photon. Therefore φ is small value in the framework of our problem. Hence in concern of the formula (37), we should mean $\cos\varphi\simeq 1+\varphi$ and $\sin\varphi\simeq\varphi$. We also take into account only real parts of the function v and its derivatives. (This is due to that fact that a real instrument processes measurement with only real quantities.) Concerning those functions which are contained in the formula (37), all these means that

$$\left.\begin{aligned}&v=T\,r\,(1+\varphi)\\&v_r=T\,(1+\varphi) && \frac{v_{\varphi}}{r}=-T\,\varphi\\&v_{tr}=\dot{T}\,(1+\varphi) && \frac{v_{t\varphi}}{r}=-\dot{T}\,\varphi\end{aligned}\right\}.\tag{44}$$

Substituting these into (37), we obtain

$$(1+2\varphi)\,\dot{T}-(1+2\varphi)\,\omega T+\omega^2=0\,,\tag{45}$$

or, because $\varphi=\omega t$ and ω is small value (we also neglect the terms which order is higher than ω^2),

$$\dot{T}-\omega T=-\frac{\omega^2}{1+2\omega t}=-\omega^2\,(1-2\omega t)=-\omega^2.\tag{46}$$

This is a linear differential equation of the first order

$$\dot{y}+f(t)\,y=g(t)\tag{47}$$

whose exact solution is (see Part I, Chapter I, §4.3 in Erich Kamke's reference book [30])

$$y = e^{-F}\left(y_0 + \int_{t_0=0}^{t} g(t)\, e^F dt\right), \tag{48}$$

where

$$F(t) = \int f(t)\, dt\,. \tag{49}$$

We substitute $f = -\omega$ and $g = -\omega^2$. So we obtain, for small values of ω,

$$e^F = e^{\int -\omega dt} = e^{-\omega t}, \qquad e^{-F} = e^{\omega t}, \tag{50}$$

hence the function y is

$$y = e^{\omega t}\left(y_0 - \omega^2 \int_{t_0=0}^{t} e^{-\omega t} dt\right) = \\ = e^{\omega t}\left[y_0 + \omega\left(e^{-\omega t} - 1\right)\right]. \tag{51}$$

We assume the numerical value of the function $y = T(t)$ to be zero at the initial moment of observation: $y_0 = T_0 = 0$. As a result we obtain the solution for the function $T(t)$:

$$T = \omega\left(1 - e^{\omega t}\right). \tag{52}$$

Applying this solution, we can find a final formula for the density of the energy of the Earth's microwave background $W = \rho c^2$ observed by a satellite-bound observer.

First, we substitute the distribution functions of v (44) into the initially formula for ρc^2 (34) which is originated from the scalar Einstein equation. Assuming $\cos\varphi \simeq 1 + \varphi$ and $\sin\varphi \simeq \varphi$, we obtain

$$\kappa\rho c^2 = -2\omega^2 - 2\omega T(1 + 2\varphi) - (1 + 2\varphi)\,\dot{T}\,. \tag{53}$$

Then we do the same substitution into the geometrization condition (37) which is originated from the Einstein equations, and is necessary to be applied to our case due to that fact that we have only an electromagnetic field distributed in the space ($\rho c^2 = U$ in the right side of the Einstein equations, as for any electromagnetic field). After algebra the geometrization condition (37) takes the form

$$\omega^2 - \omega T\,(1 + 2\varphi) + (1 + 2\varphi)\,\dot{T} = 0\,. \tag{54}$$

We express $(1 + 2\varphi)\,\dot{T} = \omega T\,(1 + 2\varphi) - 4\omega^2$ from this formula, then substitute it into the previous expression (53) with taking into accout that fact that the angle φ is a small value. As a result, we obtain

$$\rho c^2 = \frac{3\omega}{\kappa}\,(\omega - T) = \frac{3\omega^2}{\kappa}\left[1 - \left(1 - e^{\omega t}\right)\right]. \tag{55}$$

Expanding the exponent into the series $e^{\omega t} = 1 + \omega t + \frac{1}{2}\,\omega^2 t^2 + \ldots \simeq 1 + \omega t$ and taking into account that fact that ω is small value*, we arrive to the final formula for calculation the density of the energy of the Earth's microwave background observed on board of a satellite

$$\rho c^2 = \frac{3\omega^2}{\kappa}\,, \tag{56}$$

which is obviously dependent on altitude from the surface of the Earth due to that fact that $\omega = \sqrt{GM_{\oplus}/R^3}$.

With this final formula (55), we calculate the ratio between the density of the Earth's microwave background expected to be measured at different altitudes from the surface of the Earth. According to this formula, the ratio between the density at the altitude of the COBE orbit ($R_{\mathrm{COBE}} = 6{,}370 + 900 = 7{,}270$ km) and that at the altitude of a U2 aeroplane ($R_{\mathrm{U2}} = 6{,}370 + 25 = 6{,}395$ km) should be

$$\frac{\rho_{\mathrm{COBE}}}{\rho_{\mathrm{U2}}} = \frac{R^3_{\mathrm{U2}}}{R^3_{\mathrm{COBE}}} \simeq 0.68\,, \tag{57}$$

the ratio between the density at the 2nd Lagrange point ($R_{\mathrm{L2}} = 1.5$ million km) and that at the COBE orbit should be

$$\frac{\rho_{\mathrm{L2}}}{\rho_{\mathrm{COBE}}} = \frac{R^3_{\mathrm{COBE}}}{R^3_{\mathrm{L2}}} \simeq 1.1\times10^{-7}, \tag{58}$$

and the ratio between the density at the 2nd Lagrange point and that at the altitude of a U2 aeroplane should be

$$\frac{\rho_{\mathrm{L2}}}{\rho_{\mathrm{U2}}} = \frac{R^3_{\mathrm{U2}}}{R^3_{\mathrm{L2}}} \simeq 7.8\times10^{-8}. \tag{59}$$

As a result of our calculation, processed on the basis of the General Theory of Relativity, we see that a microwave background field which originates in the Earth (the Earth microwave background) should have almost the same density at the position of a U2 aeroplane and the COBE satellite. However, at the 2nd Lagrange point (1.5 million km from the Earth, the point of location of the WMAP satellite and the planned PLANCK satellite), the density of the background should be only $\sim10^{-7}$ of that registered either by the U2 aeroplane or by the COBE satellite.

4 The anisotropy of the Earth's microwave background at the 2nd Lagrange point

We consider the anisotropy of the Earth's microwave background which is due to the rapid motion of the source of this field (the Earth) in a weak intergalactic microwave field†. From views of physics this means that photons, the mediators for electromagnetic radiation, being radiated by the source of the field (the Earth) should experience a carrying in the direc-

*The quantity $\omega = \sqrt{GM_{\oplus}/R^3}$, the frequency of the rotation of the Earth space for an observer existing in the weightless state, takes its maximum numerical value at the equator of the Earth's surface, where $\omega = 1.24\times10^{-3}$ sec^{-1}, and decreases with altitude above the surface.

†As observatonal analysis indicates it, the Earth moves in the weak intergalactic field with a velocity of $\mathrm{v} = 365\pm18$ km/sec in the direction of the anisotropy.

tion whereto the Earth flies in the weak intergalactic field. From mathematical viewpoint this problem can be formulated as a shift of the trajectories experienced by photons of the Earth's microwave field in the direction of this motion.

A light-like free particle, e.g. a free photon, moves along isotropic geodesic trajectories whose four-dimensional (general covariant) equations are [25–28]

$$\frac{dK^{\alpha}}{d\sigma} + \Gamma^{\alpha}_{\mu\nu} K^{\mu} \frac{dx^{\nu}}{d\sigma} = 0\,, \tag{60}$$

where $K^{\alpha} = \frac{\Omega}{c}\frac{dx^{\alpha}}{d\sigma}$ is the four-dimensional wave vector of the photon (the vector satisfies the condition $K_{\alpha}K^{\alpha} = 0$ which is specific to any isotropic vector), Ω is the proper cyclic frequency of the photon, while $d\sigma$ is the three-dimensional chronometrically invariant (observable) spatial interval determined as $d\sigma^2 = \left(-g_{ik} + \frac{g_{0i}\,g_{0k}}{g_{00}}\right) dx^i dx^k = h_{ik}\, dx^i dx^k$. The quantity $d\sigma$ is chosen as a parameter of differentiation along isotropic geodesics, because along them the four-dimensional interval is zero $ds^2 = c^2 d\tau^2 - d\sigma^2 = 0$ while $d\sigma = c d\tau \neq 0$ (where $d\tau$ is the interval of the physical observable time determined as $d\tau = \sqrt{g_{00}}\, dt + \frac{g_{0i}}{c\sqrt{g_{00}}}\, dx^i$).

In terms of the physical observables, the isotropic geodesic equations are represented by their projections on the time line and spatial section of an observer [25–28]

$$\left.\begin{aligned} &\frac{d\Omega}{d\tau} - \frac{\Omega}{c^2} F_i c^i + \frac{\Omega}{c^2} D_{ik} c^i c^k = 0 \\ &\frac{d}{d\tau}\left(\Omega c^i\right) + 2\Omega\left(D^i_k + A^{\cdot i}_{k\cdot}\right) c^k - \\ &\qquad - \Omega F^i + \Omega \Delta^i_{kn} c^k c^n = 0 \end{aligned}\right\} \tag{61}$$

where $c^i = \frac{dx^i}{d\tau}$ is the three-dimensional vector of the observable velocity of light (the square of the vector satisfies $c_k c^k = h_{ik} c^i c^k = c^2$ in the spatial section of the observer). The first of the equations (the scalar equation) represents the law of energy for the particle, while the vectorial equation is the three-dimensional equation of its motion.

The terms $\frac{F_i}{c^2}$ and $\frac{D_{ik}}{c^2}$ are negligible in the framework of our assumption. We obtain, from the scalar equation of (61), that the proper frequency of the photons, registered by the observer, is constant. In such a case the vectorial equations of isotropic geodesics (61), written in component notation, are

$$\left.\begin{aligned} &\frac{dc^1}{d\tau} + 2\left(D^1_k + A^{\cdot 1}_{k\cdot}\right) c^k - F^1 + \Delta^1_{22}\, c^2 c^2 + \\ &\qquad + 2\Delta^1_{23}\, c^2 c^3 + \Delta^1_{33}\, c^3 c^3 = 0 \\ &\frac{dc^2}{d\tau} + 2\left(D^2_k + A^{\cdot 2}_{k\cdot}\right) c^k - F^2 + 2\Delta^2_{12}\, c^1 c^2 + \\ &\qquad + 2\Delta^2_{13}\, c^1 c^3 + \Delta^2_{33}\, c^3 c^3 = 0 \\ &\frac{dc^3}{d\tau} + 2\left(D^3_k + A^{\cdot 3}_{k\cdot}\right) c^k - F^3 + \Delta^3_{11}\, c^1 c^1 + \\ &\qquad + 2\Delta^3_{12}\, c^1 c^2 + 2\Delta^3_{13}\, c^1 c^3 + \\ &\qquad + \Delta^3_{22}\, c^2 c^2 + 2\Delta^3_{23}\, c^2 c^3 = 0 \end{aligned}\right\}, \tag{62}$$

where $c^1 = \frac{dr}{d\tau}$, $c^2 = \frac{d\varphi}{d\tau}$, and $c^3 = \frac{dz}{d\tau}$, while $\frac{d}{d\tau} = \frac{^*\partial}{\partial t} + \mathrm{v}^i \frac{^*\partial}{\partial x^i}$.

We direct the z-axis of our cylindrical coordinates along the motion of the Earth in the weak intergalactic field. In such a case the local physical space of a satellite-bound observer is described by the metric (8). We therefore will solve the isotropic geodesic equations in the metric (8).

The metric (7) we used in the first part of the problem is a particular to the metric (8) in a case, where $\mathrm{v} = 0$. Therefore the solution $v = T(t)\, r\, e^{i\varphi}$ (42) we have obtained for the metric (7) is also lawful for the generatized metric (8). We therefore calculate the observable characteristics of the space with taking this function into account. As earlier, we take into account only real part of the function $e^{i\varphi} = \cos\varphi + i\sin\varphi \simeq \simeq (1+\varphi) + i\varphi$. We also take into account the derivatives of this function (43) and the function $T = \omega\,(1 - e^{\omega t})$ we have found earlier (52).

As well as in the first part of the problem, we assume φ to be small value: $\cos\varphi \simeq 1 + \varphi$ and $\sin\varphi \simeq \varphi$. Because ω is small value too, we neglect $\omega^2\varphi$ terms. We also take the weightlessness condition $\frac{GM}{r^2} = \omega^2 r$ into account in calculation for the gravitational inertial force. It should be noted that the weightlessness condition is derived from the derivative of the gravitational potential $\mathrm{w} = c^2\,(1 - \sqrt{g_{00}})$. We therefore cannot mere substitute the weightlessness condition into $g_{00} = 1 - \frac{2GM}{c^2 r} - \frac{\omega^2 r^2}{c^2} + \frac{2v\mathrm{v}}{c^2}$ taken from the metric (8). We first calculate $\mathrm{w} = c^2\,(1 - \sqrt{g_{00}})$, then take derivative of it by the respective coordinate that is required in the formula for the gravitational inertial force F_i (12). Only then the weightlessness condition $\frac{GM}{r^2} = \omega^2 r$ is lawful to be substituted.

Besides these, we should take into account that fact that the anisotropy of a field is a second order effect. We therefore cannot neglect the terms divided by c^2. This is in contrast to the first part of the problem, where we concerned only a first order effect. As a result the space deformation and the three-dimensional curvature, neglected in the first part, now cannot be neglected. We however take into account only the space deformation D_{ik}. The three-dimensional curvature C_{iklj} isn't considered here due to the fact that this quantity isn't contained in the equations of motion.

In the same time, in the framework of our assumption for a weak gravitational field and a low speed of the space rotation, $\frac{^*\partial}{\partial t} = \frac{1}{\sqrt{g_{00}}}\frac{\partial}{\partial t} \simeq \frac{\partial}{\partial t}$ and $\frac{^*\partial}{\partial x^i} = \frac{\partial}{\partial x^i} + \frac{1}{c^2} v_i \frac{^*\partial}{\partial t} \simeq \frac{\partial}{\partial x^i}$.

Applying all these conditions to the definitions of v_i, h_{ik}, F_i, A_{ik}, D_{ik}, and Δ^i_{km}, given in Page 7, we obtain substantially non-zero components of the characteristics of the space whose metric is (8):

$$\mathrm{w} = \frac{GM}{r} + \frac{\omega^2 r^2}{2} - v\,\mathrm{v}\,, \tag{63}$$

$$\left.\begin{aligned} v_1 &= \omega^2 t r \\ v_2 &= \omega r^2\,(\omega t + 1) \\ v_3 &= \omega^2 t r \end{aligned}\right\}, \tag{64}$$

$$\left.\begin{aligned} &\ddot{r}-\omega^{2}\left(t-\frac{r\,\mathrm{v}}{c^{2}}\right)\dot{z}+\omega^{2}\left(r-\mathrm{v}t\right)+\frac{\omega^{2}\mathrm{v}\,t}{c^{2}}\,\dot{z}^{2}=0\\ &\ddot{\varphi}+2\omega\left(1+\frac{\omega t}{2}\right)\frac{\dot{r}}{r}+\frac{\omega^{2}\mathrm{v}}{c^{2}}\,\dot{z}+\omega^{2}+\frac{2\omega\,\mathrm{v}\left(1+\frac{\omega t}{2}\right)}{c^{2}r}\,\dot{r}\dot{z}=0\\ &\ddot{z}+\omega^{2}\left(t+\frac{r\,\mathrm{v}}{c^{2}}\right)\dot{r}+\frac{2\omega^{2}\mathrm{v}\,r}{c^{2}}\,\dot{z}+\omega^{2}r+\frac{\omega^{2}\mathrm{v}\,t}{c^{2}}\,\dot{r}^{2}+\frac{2\omega^{2}\mathrm{v}\,t}{c^{2}}\,\dot{r}\dot{z}=0 \end{aligned}\right\} \tag{70}$$

$$\dot{r}^{2}+\frac{2\omega^{2}r\,\mathrm{v}\,t}{c^{2}}\,\dot{r}\dot{z}+\left(1-\frac{2\omega^{2}r\,\mathrm{v}\,t}{c^{2}}\right)\dot{z}^{2}=c^{2} \tag{71}$$

$$\left.\begin{aligned} &F_{1}=-\omega^{2}\left(r-\mathrm{v}t\right)\\ &F_{2}=-\omega^{2}r^{2}\\ &F_{3}=-\omega^{2}r \end{aligned}\right\}, \tag{65}$$

$$\left.\begin{aligned} &A_{12}=\omega\,r\left(1+\frac{\omega t}{2}\right)\\ &A_{23}=0\\ &A_{13}=\frac{\omega^{2}t}{2} \end{aligned}\right\}, \tag{66}$$

$$\left.\begin{aligned} &h_{11}=1\,, && h_{13}=\frac{\omega^{2}\mathrm{v}\,t\,r}{c^{2}}\\ &h_{22}=r^{2}, && h_{23}=\frac{\omega\,r^{2}\mathrm{v}\left(1+\omega t\right)}{c^{2}}\\ &h_{33}=1-\frac{2\omega^{2}\mathrm{v}\,t\,r}{c^{2}}\\ &h^{11}=1\,, && h^{13}=-\frac{\omega^{2}\mathrm{v}\,t\,r}{c^{2}}\\ &h^{22}=\frac{1}{r^{2}}\,, && h^{23}=-\frac{\omega\,\mathrm{v}\left(1+\omega t\right)}{c^{2}}\\ &h^{33}=1+\frac{2\omega^{2}\mathrm{v}\,t\,r}{c^{2}}\\ &h=r^{2}\left(1+\frac{2\omega^{2}\mathrm{v}\,t\,r}{c^{2}}\right) \end{aligned}\right\}, \tag{67}$$

$$\left.\begin{aligned} &D_{13}=\frac{\omega^{2}r\,\mathrm{v}}{2\,c^{2}}\,, && D_{23}=\frac{\omega^{2}r^{2}\mathrm{v}}{2\,c^{2}}\\ &D_{33}=\frac{\omega^{2}r\,\mathrm{v}}{c^{2}}\,, && D=\frac{\omega^{2}r\,\mathrm{v}}{c^{2}} \end{aligned}\right\}, \tag{68}$$

$$\left.\begin{aligned} &\Delta^{1}_{22}=-r\,, && \Delta^{1}_{23}=-\frac{\omega\,r\,\mathrm{v}}{c^{2}}\left(1+\frac{\omega t}{2}\right)\\ &\Delta^{1}_{33}=\frac{\omega^{2}\mathrm{v}\,t}{c^{2}}\,, && \Delta^{2}_{12}=\frac{1}{r}\\ &\Delta^{2}_{13}=\frac{\omega\,\mathrm{v}}{c^{2}r}\left(1+\frac{\omega t}{2}\right), && \Delta^{3}_{11}=\frac{\omega^{2}\mathrm{v}\,t}{c^{2}}\\ &\Delta^{3}_{12}=\frac{\omega^{2}r\,\mathrm{v}\,t}{2c^{2}}\,, && \Delta^{3}_{13}=-\frac{\omega^{2}\mathrm{v}\,t}{c^{2}} \end{aligned}\right\}, \tag{69}$$

where we present only those components of Christoffel's symbols which will be used in the geodesic equations (equations of of motion).

After substitution of the components, the vectorial equations of isotropic geodesic (62) take the form (70). The condition $h_{ik}\,c^{i}c^{k}=c^{2}$ — a chronometrically invariant expression of the condition $ds^{2}=c^{2}d\tau^{2}-d\sigma^{2}=0$, which is specific to isotropic trajectories — takes the form (71).

We consider a light beam (a couple of photons) travelling from the Earth along the radial direction r. Therefore, looking for anisotropy in the distribution of the photons' trajectories in the field, we are interested to solve only the third isotropic geodesic equation of (70), which is the equation of motion of a photon along the z-axis orthogonal to the light beam's direction r.

Before to solve the equation, a few notes on our assumptions should be made.

First, because the Earth moves relative to the weak microwave background with a velocity v^{i} along the z-direction, only $\mathrm{v}^{3}=\dot{z}$ of the components v^{i} is non-zero. Besides that, as easy to see from our previous considerations, we should mean $\frac{^*\partial}{\partial t}=\frac{1}{\sqrt{g_{00}}}\frac{\partial}{\partial t}\simeq\frac{\partial}{\partial t}$ and $\frac{^*\partial}{\partial z}=\frac{\partial}{\partial x^{i}}+\frac{1}{c^{2}}v_{3}\frac{^*\partial}{\partial t}\simeq\frac{\partial}{\partial z}=0$. Hence, we apply $\frac{d}{d\tau}=\frac{^*\partial}{\partial t}+\mathrm{v}^{3}\frac{^*\partial}{\partial z}=\frac{d}{\partial t}$ to our calculation.

Second, the orbital velocity of a satellite of the Earth, $\sim$8 km/sec, is much lesser than the velocity of light. We therefore assume that a light beam doesn't sense the orbital motion of such a satellite. The coordinate φ in the equations of isotropic geodesics is related to the light beam (a couple of single photons), not the rotation of the reference space of a satellite bound observer. Hence, we assume $\varphi=\mathrm{const}$ in our calculation, i.e. $c^{2}=\frac{d\varphi}{dt}=\dot{\varphi}=0$.

Third, we are talking about the counting for signle photons in a detector which is located on board of a satellite. The process of the measurement is actually instant. In other word, the measurement is processed very close the moment $t_{0}=0$. Hence we assume $\dot{z}=0$ in our calculation, while the acceleration $\ddot{z}$ can be non-zero in the z-direction orthogonally to the initially r-direction of such a photon.

Fourth, we apply the relations $\dot{r}=c$ and $r=ct$ which are obvious for such a photon. If such a photon, travelling initially in the r-direction, experiences a shift to the z-direction (the direction of the motion of the Earth relative to the weak intergalactic field), the distribution of photons of the Earth's microwave field has an anisotropy to the z-direction.

After taking all the factors into account, the third equation of the system (70), which is the equation of motion of a single photon in the z-direction, takes the simple form

$$\ddot{z} + \omega^2 \left(ct + \frac{r\,\mathrm{v}}{c} \right) + \omega^2 \left(r + \mathrm{v}t \right) = 0 \qquad (72)$$

which, due to the weightlessness condition $\frac{GM}{r^2} = \omega^2 r$ and the condition $r = ct$, is

$$\ddot{z} + \frac{2GM_{\oplus}}{c^2 t^2} \left(1 + \frac{\mathrm{v}}{c} \right) = 0\,. \qquad (73)$$

Integration of this equation gives

$$\dot{z} = \frac{2GM_{\oplus}}{cr} \left(1 + \frac{\mathrm{v}}{c} \right) = \dot{z}' + \Delta z'. \qquad (74)$$

The first term of the solution (74) manifests that fact that such a photon, initially launched in the r-direction (radial direction) in the gravitational field of the Earth, is carried into the z-direction by the rotation of the space of the Earth. The second term, $\Delta z'$, manifests the carriage of the photon into the z-direction due to the motion of the Earth in this direction through the weak intergalactic field.

As a result we obtain the carriage of the three-dimensional vector of the observable velocity of light from the initially r-direction to the z-direction, due to the common motion of the space of the Earth in the point of observation:

$$\frac{\Delta \dot{z}'}{\dot{z}'} = \frac{\mathrm{v}}{c}\,. \qquad (75)$$

Such a carriage of a photon radiated from the Earth's surface, being applied to a microwave background generated by oceanic water, reveals the anisotropy associated with the dipole component of the microwave background.

As seen from the obtained formula (75), such a carriage of a photon into the z-direction, doesn't depend on the path travelled by such a photon in the radial direction r from the Earth. In other word, the anisotropy associated with the dipole component of the Earth microwave background shouldn't be dependent on altitude from the surface of the Earth: the anisotropy of the Earth microwave background should be the same if measured on board a U2 aeroplane (25 km), at the orbit of the COBE satellite (900 km), and at the 2nd Langrange point (the WMAP satellite and PLANCK satellite, 1.5 million km from the Earth).

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SPECIAL REPORT

New Approach to Quantum Electrodynamics

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It is shown that a photon with a specific frequency can be identified with the Dirac magnetic monopole. When a Dirac-Wilson line forms a Dirac-Wilson loop, it is a photon. This loop model of photon is exactly solvable. From the winding numbers of this loop-form of photon, we derive the quantization properties of energy and electric charge. A new QED theory is presented that is free of ultraviolet divergences. The Dirac-Wilson line is as the quantum photon propagator of the new QED theory from which we can derive known QED effects such as the anomalous magnetic moment and the Lamb shift. The one-loop computation of these effects is simpler and is more accurate than that in the conventional QED theory. Furthermore, from the new QED theory, we have derived a new QED effect. A new formulation of the Bethe-Salpeter (BS) equation solves the difficulties of the BS equation and gives a modified ground state of the positronium. By the mentioned new QED effect and by the new formulation of the BS equation, a term in the orthopositronium decay rate that is missing in the conventional QED is found, resolving the orthopositronium lifetime puzzle completely. It is also shown that the graviton can be constructed from the photon, yielding a theory of quantum gravity that unifies gravitation and electromagnetism.

1 Introduction

It is well known that the quantum era of physics began with the quantization of energy of electromagnetic field, from which Planck derived the radiation formula. Einstein then introduced the light-quantum to explain the photoelectric effects. This light-quantum was regarded as a particle called photon [1–3]. Quantum mechanics was then developed, ushering in the modern quantum physics era. Subsequently, the quantization of the electromagnetic field and the theory of Quantum Electrodynamics (QED) were established.

In this development of quantum theory of physics, the photon plays a special role. While it is the beginning of quantum physics, it is not easy to understand as is the quantum mechanics of other particles described by the Schrödinger equation. In fact, Einstein was careful in regarding the light-quantum as a particle, and the acceptance of the light-quantum as a particle called photon did not come about until much later [1]. The quantum field theory of electromagnetic field was developed for the photon. However, such difficulties of the quantum field theory as the ultraviolet divergences are well known. Because of the difficulty of understanding the photon, Einstein once asked: "What is the photon?" [1].

On the other hand, based on the symmetry of the electric and magnetic field described by the Maxwell equation and on the complex wave function of quantum mechanics, Dirac derived the concept of the magnetic monopole, which is hypothetically considered as a particle with magnetic charge, in analogy to the electron with electric charge. An important feature of this magnetic monopole is that it gives the quantization of electric charge. Thus it is interesting and important to find such particles. However, in spite of much effort, no such particles have been found [4, 5].

In this paper we shall establish a mathematical model of photon to show that the magnetic monopole can be identified as a photon. Before giving the detailed model, let us discuss some thoughts for this identification in the following.

First, if the photon and the magnetic monopole are different types of elementary quantum particles in the electromagnetic field, it is odd that one type can be derived from the other. A natural resolution of this oddity is the identification of the magnetic monopole as a photon.

The quantum field theory of the free Maxwell equation is the basic quantum theory of photon [6]. This free field theory is a linear theory and the models of the quantum particles obtained from this theory are linear. However, a stable particle should be a soliton, which is of the nonlinear nature. Secondly, the quantum particles of the quantum theory of Maxwell equation are collective quantum effects in the same way the phonons which are elementary excitations in a statistical model. These phonons are usually considered as quasi-particles and are not regarded as real particles. Regarding the Maxwell equation as a statistical wave equation of electromagnetic field, we have that the quantum particles in the quantum theory of Maxwell equation are analogous to the phonons. Thus they should be regarded as quasi-photons and have properties of photons but not a complete description of photons.

In this paper, a nonlinear model of photon is established. In the model, we show that the Dirac magnetic monopole can be identified with the photon with some frequencies. We provide a $U(1)$ gauge theory of Quantum Electrodynamics (QED), from which we derive photon as a quantum Dirac-Wilson loop $W(z,z)$ of this model. This nonlinear loop model of the photon is exactly solvable and thus may be regarded as a quantum soliton. From the winding numbers of this loop model of the photon, we derive the quantization property of energy in Planck's formula of radiation and the quantization property of charge. We show that the quantization property of charge is derived from the quantization property of energy (in Planck's formula of radiation), when the magnetic monopole is identified with photon with certain frequencies. This explains why we cannot physically find a magnetic monopole. It is simply a photon with a specific frequency.

From this nonlinear model of the photon, we also construct a model of the electron which has a mass mechanism for generating mass of the electron. This mechanism of generating mass supersedes the conventional mechanism of generating mass (through the Higgs particles) and makes hypothesizing the existence of the Higgs particles unnecessary. This explains why we cannot physically find such Higgs particles.

The new quantum gauge theory is similar to the conventional QED theory except that the former is not based on the four dimensional space-time $(t,\mathbf{x})$ but is based on the proper time s in the theory of relativity. Only in a later stage in the new quantum gauge theory, the space-time variable $(t,\mathbf{x})$ is derived from the proper time s through the Lorentz metric $ds^2 = dt^2 - d\mathbf{x}^2$ to obtain space-time statistics and explain the observable QED effects.

The derived space variable $\mathbf{x}$ is a random variable in this quantum gauge theory. Recall that the conventional quantum mechanics is based on the space-time. Since the space variable $\mathbf{x}$ is actually a random variable as shown in the new quantum gauge theory, the conventional quantum mechanics needs probabilistic interpretation and thus has a most mysterious measurement problem, on which Albert Einstein once remarked: "God does not play dice with the universe." In contrast, the new quantum gauge theory does not involve the mentioned measurement problem because it is not based on the space-time and is deterministic. Thus this quantum gauge theory resolves the mysterious measurement problem of quantum mechanics.

Using the space-time statistics, we employ Feynman diagrams and Feynman rules to compute the basic QED effects such as the vertex correction, the photon self-energy and the electron self-energy. In this computation of the Feynman integrals, the dimensional regularization method in the conventional QED theory is also used. Nevertheless, while the conventional QED theory uses it to reduce the dimension 4 of space-time to a (fractional) number n to avoid the ultraviolet divergences in the Feynman integrals, the new QED theory uses it to increase the dimension 1 of the proper time to a number n less than 4, which is the dimension of the space-time, to derive the space-time statistics. In the new QED theory, there are no ultraviolet divergences, and the dimensional regularization method is not used for *regularization*.

After this increase of dimension, the renormalization method is used to derive the well-known QED effects. Unlike the conventional QED theory, the renormalization method is used in the new QED theory to compute the space-time statistics, but not to remove the ultraviolet divergences, since the ultraviolet divergences do not occur in the new QED theory. From these QED effects, we compute the anomalous magnetic moment and the Lamb shift [6]. The computation does not involve numerical approximation as does that of the conventional QED and is simpler and more accurate.

For getting these QED effects, the quantum photon propagator $W(z,z')$, which is like a line segment connecting two electrons, is used to derive the electrodynamic interaction. (When the quantum photon propagator $W(z,z')$ forms a closed circle with $z=z'$, it then becomes a photon $W(z,z)$.) From this quantum photon propagator, a photon propagator is derived that is similar to the Feynman photon propagator in the conventional QED theory.

The photon-loop $W(z,z)$ leads to the renormalized electric charge e and the mass m of electron. In the conventional QED theory, the bare charge e_0 is of less importance than the renormalized charge e, in the sense that it is unobservable. In contrast, in this new theory of QED, the bare charge e_0 and the renormalized charge e are of equal importance. While the renormalized charge e leads to the physical results of QED, the bare charge e_0 leads to the universal gravitation constant G. It is shown that $e = n_e e_0$, where n_e is a very large winding number and thus e_0 is a very small number. It is further shown that the gravitational constant $G = 2e_0^2$ which is thus an extremely small number. This agrees with the fact that the experimental gravitational constant G is a very small number. The relationships, $e = n_e e_0$ and $G = 2e_0^2$, are a part of a theory unifying gravitation and electromagnetism. In this unified theory, the graviton propagator and the graviton are constructed from the quantum photon propagator. This construction leads to a theory of quantum gravity. In short, a new theory of quantum gravity is developed from the new QED theory in this paper, and unification of gravitation and electromagnetism is achieved.

In this paper, we also derive a new QED effect from the seagull vertex of the new QED theory. The conventional Bethe-Salpeter (BS) equation is reformulated to resolve its difficulties (such as the existence of abnormal solutions [7–32]) and to give a modified ground state wave function of the positronium. By the new QED effect and the reformulated BS equation, another new QED effect, a term in the orthopositronium decay rate that is missing in the conventional QED is discovered. Including the discovered term, the computed orthopositronium decay rate now agrees with the experimental rate, resolving the *orthopositronium lifetime puzzle* completely [33–52]. We note that the recent resolution of this orthopositronium lifetime puzzle resolves the puzzle only partially due to a special statistical nature of this new term in the orthopositronium decay rate.

This paper is organized as follows. In Section 2 we give a brief description of a new QED theory. With this theory, we introduce the classical Dirac-Wilson loop in Section 3. We show that the quantum version of this loop is a nonlinear exactly solvable model and thus can be regarded as a soliton. We identify this quantum Dirac-Wilson loop as a photon with the $U(1)$ group as the gauge group. To investigate the properties of this Dirac-Wilson loop, we derive a chiral symmetry from the gauge symmetry of this quantum model. From this chiral symmetry, we derive, in Section 4, a conformal field theory, which includes an affine Kac-Moody algebra and a quantum Knizhnik-Zamolodchikov (KZ) equation. A main point of our model on the quantum KZ equation is that we can derive two KZ equations which are dual to each other. This duality is the main point for the Dirac-Wilson loop to be exactly solvable and to have a winding property which explains properties of photon. This quantum KZ equation can be regarded as a quantum Yang-Mills equation.

In Sections 5 to 8, we solve the Dirac-Wilson loop in a form with a winding property, starting with the KZ equations. From the winding property of the Dirac-Wilson loop, we derive, in Section 9 and Section 10, the quantization of energy and the quantization of electric charge which are properties of photon and magnetic monopole. We then show that the quantization property of charge is derived from the quantization property of energy of Planck's formula of radiation, when we identify photon with the magnetic monopole for some frequencies. From this nonlinear model of photon, we also derive a model of the electron in Section 11. In this model of electron, we provide a mass mechanism for generating mass to electron. In Section 12, we show that the photon with a specific frequency can carry electric charge and magnetic charge, since an electron is formed from a photon with a specific frequency for giving the electric charge and magnetic charge. In Section 13, we derive the statistics of photons and electrons from the loop models of photons and electrons.

In Sections 14 to 22, we derive a new theory of QED, wherein we perform the computation of the known basic QED effects such as the photon self-energy, the electron self-energy and the vertex correction. In particular, we provide simpler and more accurate computation of the anomalous magnetic moment and the Lamb shift. Then in Section 23, we compute a new QED effect. Then from Section 24 to Section 25, we reformulate the Bethe-Salpeter (BS) equation. With this new version of the BS equation and the new QED effect, a modified ground state wave function of the positronium is derived. Then by this modified ground state of the positronium, we derive in Section 26 another new QED effect, a term missing in the theoretic orthopositronium decay rate of the conventional QED theory, and show that this new theoretical orthopositronium decay rate agrees with the experimental decay rate, completely resolving the orthopositronium life time puzzle [33–52].

In Section 27, the graviton is derived from the photon. This leads to a new theory of quantum gravity and a new unification of gravitation and electromagnetism. Then in Section 28, we show that the quantized energies of gravitons can be identified as dark energy. Then in a way similar to the construction of electrons by photons, we use gravitons to construct particles which can be regarded as dark matter. We show that the force among gravitons can be repulsive. This gives the diffusion phenomenon of dark energy and the accelerating expansion of the universe [53–57].

2 New gauge model of QED

Let us construct a quantum gauge model, as follows. In probability theory we have the Wiener measure ν which is a measure on the space $C[t_0, t_1]$ of continuous functions [58]. This measure is a well defined mathematical theory for the Brownian motion and it may be symbolically written in the following form:

$$d\nu = e^{-L_0} dx\,, \tag{1}$$

where $L_0 := \frac{1}{2}\int_{t_0}^{t_1} \left(\frac{dx}{dt}\right)^2 dt$ is the energy integral of the Brownian particle and $dx = \frac{1}{N}\prod_t dx(t)$ is symbolically a product of Lebesgue measures $dx(t)$ and N is a normalized constant.

Once the Wiener measure is defined we may then define other measures on $C[t_0, t_1]$, as follows [58]. Let a potential term $\frac{1}{2}\int_{t_0}^{t_1} V dt$ be added to L_0. Then we have a measure ν_1 on $C[t_0, t_1]$ defined by:

$$d\nu_1 = e^{-\frac{1}{2}\int_{t_0}^{t_1} V dt} d\nu\,. \tag{2}$$

Under some condition on V we have that ν_1 is well defined on $C[t_0, t_1]$. Let us call (2) as the Feynman-Kac formula [58].

Let us then follow this formula to construct a quantum model of electrodynamics, as follows. Then similar to the formula (2) we construct a quantum model of electrodynamics from the following energy integral:

$$\begin{aligned} -\int_{s_0}^{s_1} D\,ds := -\int_{s_0}^{s_1} \Big[&\tfrac{1}{2}\left(\tfrac{\partial A_1}{\partial x^2} - \tfrac{\partial A_2}{\partial x^1}\right)^* \left(\tfrac{\partial A_1}{\partial x^2} - \tfrac{\partial A_2}{\partial x^1}\right) + \\ &+ \left(\tfrac{dZ^*}{ds} + ie_0 (\textstyle\sum_{j=1}^2 A_j \tfrac{dx^j}{ds}) Z^*\right) \times \\ &\times \left(\tfrac{dZ}{ds} - ie_0 (\textstyle\sum_{j=1}^2 A_j \tfrac{dx^j}{ds}) Z\right)\Big] ds\,, \end{aligned} \tag{3}$$

where the complex variable $Z = Z(z(s))$ and the real variables $A_1 = A_1(z(s))$ and $A_2 = A_2(z(s))$ are continuous functions in a form that they are in terms of a (continuously differentiable) curve $z(s) = C(s) = (x^1(s), x^2(s))$, $s_0 \leqslant s \leqslant s_1$, $z(s_0) = z(s_1)$ in the complex plane where s is a parameter representing the proper time in relativity. (We shall also write

$z(s)$ in the complex variable form $C(s) = z(s) = x^1(s) + + ix^2(s)$, $s_0 \leqslant s \leqslant s_1$.) The complex variable $Z = Z(z(s))$ represents a field of matter, such as the electron (Z^* denotes its complex conjugate), and the real variables $A_1 = A_1(z(s))$ and $A_2 = A_2(z(s))$ represent a connection (or the gauge field of the photon) and e_0 denotes the (bare) electric charge.

The integral (1.1) has the following gauge symmetry:

$$\begin{aligned} Z'(z(s)) &:= Z(z(s))\, e^{ie_0 a(z(s))} \\ A'_j(z(s)) &:= A_j(z(s)) + \tfrac{\partial a}{\partial x^j}\,, \quad j = 1, 2 \end{aligned} \tag{4}$$

where $a = a(z)$ is a continuously differentiable real-valued function of z.

We remark that this QED theory is similar to the conventional Yang-Mills gauge theories. A feature of (1.1) is that it is not formulated with the four-dimensional space-time but is formulated with the one dimensional proper time. This one dimensional nature let this QED theory avoid the usual ultraviolet divergence difficulty of quantum fields. As most of the theories in physics are formulated with the space-time let us give reasons of this formulation. We know that with the concept of space-time we have a convenient way to understand physical phenomena and to formulate theories such as the Newton equation, the Schrödinger equation, e.t.c. to describe these physical phenomena. However we also know that there are fundamental difficulties related to space-time such as the ultraviolet divergence difficulty of quantum field theory. To resolve these difficulties let us reexamine the concept of space-time. We propose that the space-time is a statistical concept which is not as basic as the proper time in relativity. Because a statistical theory is usually a convenient but incomplete description of a more basic theory this means that some difficulties may appear if we formulate a physical theory with the space-time. This also means that a way to formulate a basic theory of physics is to formulate it not with the space-time but with the proper time only as the parameter for evolution. This is a reason that we use (1.1) to formulate a QED theory. In this formulation we regard the proper time as an independent parameter for evolution. From (1.1) we may obtain the conventional results in terms of space-time by introducing the space-time as a statistical method.

Let us explain in more detail how the space-time comes out as a statistics. For statistical purpose when many electrons (or many photons) present we introduce space-time (t, x) as a statistical method to write ds^2 in the form

$$ds^2 = dt^2 - d\mathrm{x}^2. \tag{5}$$

We notice that for a given ds there may have many dt and $d\mathrm{x}$ which correspond to many electrons (or photons) such that (5) holds. In this way the space-time is introduced as a statistics. By (5) we shall derive statistical formulas for many electrons (or photons) from formulas obtained from (1.1). In this way we obtain the Dirac equation as a statistical equation for electrons and the Maxwell equation as a statistical equation for photons. In this way we may regard the conventional QED theory as a statistical theory extended from the proper-time formulation of this QED theory (From the proper-time formulation of this QED theory we also have a theory of space-time statistics which give the results of the conventional QED theory). This statistical interpretation of the conventional QED theory is thus an explanation of the mystery that the conventional QED theory is successful in the computation of quantum effects of electromagnetic interaction while it has the difficulty of ultraviolet divergence.

We notice that the relation (5) is the famous Lorentz metric. (We may generalize it to other metric in General Relativity.) Here our understanding of the Lorentz metric is that it is a statistical formula where the proper time s is more fundamental than the space-time (t, x) in the sense that we first have the proper time and the space-time is introduced via the Lorentz metric only for the purpose of statistics. This reverses the order of appearance of the proper time and the space-time in the history of relativity in which we first have the concept of space-time and then we have the concept of proper time which is introduced via the Lorentz metric. Once we understand that the space-time is a statistical concept from (1.1) we can give a solution to the quantum measurement problem in the debate about quantum mechanics between Bohr and Einstein. In this debate Bohr insisted that with the probability interpretation quantum mechanics is very successful. On the other hand Einstein insisted that quantum mechanics is incomplete because of probability interpretation. Here we resolve this debate by constructing the above QED theory which is a quantum theory as the quantum mechanics and unlike quantum mechanics which needs probability interpretation we have that this QED theory is deterministic since it is not formulated with the space-time.

Similar to the usual Yang-Mills gauge theory we can generalize this gauge theory with $U(1)$ gauge symmetry to non-abelian gauge theories. As an illustration let us consider $SU(2) \otimes U(1)$ gauge symmetry where $SU(2) \otimes U(1)$ denotes the direct product of the groups $SU(2)$ and $U(1)$.

Similar to (1.1) we consider the following energy integral:

$$\begin{aligned} L := \int_{s_0}^{s_1} \Big[\tfrac{1}{2}\, Tr\,(D_1 A_2 - D_2 A_1)^*(D_1 A_2 - D_2 A_1) + \\ + (D_0^* Z^*)(D_0 Z) \Big]\, ds\,, \end{aligned} \tag{6}$$

where $Z = (z_1, z_2)^T$ is a two dimensional complex vector; $A_j = \sum_{k=0}^{3} A_j^k t^k$ $(j = 1, 2)$ where A_j^k denotes a component of a gauge field A^k; $t^k = i\,T^k$ denotes a generator of $SU(2) \otimes U(1)$ where T^k denotes a self-adjoint generator of $SU(2) \otimes U(1)$ (here for simplicity we choose a convention that the complex i is absorbed by t^k and t^k is absorbed by A_j; and the notation A_j is with a little confusion with the notation A_j in the above formulation of (1.1) where $A_j, j = 1, 2$ are real valued); and $D_l = \frac{\partial}{\partial x_l} - e_0 (\sum_{j=1}^{2} A_j \frac{dx^j}{ds})$ for $l = 1, 2$; and $D_0 = \frac{d}{ds} - e_0 (\sum_{j=1}^{2} A_j \frac{dx^j}{ds})$ where e_0 is the bare electric charge for general interactions including the strong and weak interactions.

From (6) we can develop a nonabelian gauge theory as similar to that for the above abelian gauge theory. We have that (6) is invariant under the following gauge transformation:

$$\begin{aligned} Z'(z(s)) &:= U(a(z(s)))\, Z(z(s)) \\ A'_j(z(s)) &:= U(a(z(s)))\, A_j(z(s))\, U^{-1}(a(z(s))) + \\ &+ U(a(z(s)))\tfrac{\partial U^{-1}}{\partial x^j}(a(z(s))), \quad j=1,2 \end{aligned} \tag{7}$$

where $U(a(z(s))) = e^{a(z(s))}$; $a(z(s)) = \sum_k e_0 a^k(z(s))t^k$ for some functions a^k. We shall mainly consider the case that a is a function of the form $a(z(s)) = \sum_k \mathrm{Re}\,\omega^k(z(s))t^k$ where ω^k are analytic functions of z. (We let the function $\omega(z(s)) := \sum_k \omega^k(z(s))t^k$ and we write $a(z) = \mathrm{Re}\,\omega(z)$.)

The above gauge theory is based on the Banach space X of continuous functions $Z(z(s))$, $A_j(z(s))$, $j=1,2$, $s_0 \leqslant s \leqslant s_1$ on the one dimensional interval $[s_0, s_1]$.

Since L is positive and the theory is one dimensional (and thus is simpler than the usual two dimensional Yang-Mills gauge theory) we have that this gauge theory is similar to the Wiener measure except that this gauge theory has a gauge symmetry. This gauge symmetry gives a degenerate degree of freedom. In the physics literature the usual way to treat the degenerate degree of freedom of gauge symmetry is to introduce a gauge fixing condition to eliminate the degenerate degree of freedom where each gauge fixing will give equivalent physical results [59]. There are various gauge fixing conditions such as the Lorentz gauge condition, the Feynman gauge condition, etc. We shall later in the Section on the Kac-Moody algebra adopt a gauge fixing condition for the above gauge theory. This gauge fixing condition will also be used to derive the quantum KZ equation in dual form which will be regarded as a quantum Yang-Mill equation since its role will be similar to the classical Yang-Mill equation derived from the classical Yang-Mill gauge theory.

3 Classical Dirac-Wilson loop

Similar to the Wilson loop in quantum field theory [60] from our quantum theory we introduce an analogue of Wilson loop, as follows. (We shall also call a Wilson loop as a Dirac-Wilson loop.)

Definition A classical Wilson loop $W_R(C)$ is defined by:

$$W_R(C) := W(z_0, z_1) := Pe^{e_0 \int_C A_j dx^j}, \tag{8}$$

where R denotes a representation of $SU(2)$; $C(\cdot) = z(\cdot)$ is a fixed closed curve where the quantum gauge theories are based on it as specific in the above Section. As usual the notation P in the definition of $W_R(C)$ denotes a path-ordered product [60–62].

Let us give some remarks on the above definition of Wilson loop, as follows.

1. We use the notation $W(z_0, z_1)$ to mean the Wilson loop $W_R(C)$ which is based on the whole closed curve $z(\cdot)$. Here for convenience we use only the end points z_0 and z_1 of the curve $z(\cdot)$ to denote this Wilson loop (We keep in mind that the definition of $W(z_0, z_1)$ depends on the whole curve $z(\cdot)$ connecting z_0 and z_1).

Then we extend the definition of $W_R(C)$ to the case that $z(\cdot)$ is not a closed curve with $z_0 \neq z_1$. When $z(\cdot)$ is not a closed curve we shall call $W(z_0, z_1)$ as a Wilson line.

2. In constructing the Wilson loop we need to choose a representation R of the $SU(2)$ group. We shall see that because a Wilson line $W(z_0, z_1)$ is with two variables z_0 and z_1 a natural representation of a Wilson line or a Wilson loop is the tensor product of the usual two dimensional representation of the $SU(2)$ for constructing the Wilson loop. ⋄

A basic property of a Wilson line $W(z_0, z_1)$ is that for a given continuous path A_j, $j = 1, 2$ on $[s_0, s_1]$ the Wilson line $W(z_0, z_1)$ exists on this path and has the following transition property:

$$W(z_0, z_1) = W(z_0, z)W(z, z_1) \tag{9}$$

where $W(z_0, z_1)$ denotes the Wilson line of a curve $z(\cdot)$ which is with z_0 as the starting point and z_1 as the ending point and z is a point on $z(\cdot)$ between z_0 and z_1.

This property can be proved as follows. We have that $W(z_0, z_1)$ is a limit (whenever exists) of ordered product of $e^{A_j \triangle x^j}$ and thus can be written in the following form:

$$\begin{aligned} W(z_0, z_1) &= I + \int_{s'}^{s''} e_0 A_j(z(s))\tfrac{dx^j(s)}{ds}\,ds + \\ &+ \int_{s'}^{s''} e_0 A_j(z(s_2))\tfrac{dx^j(s_2)}{ds} \times \\ &\times \left[\int_{s'}^{s_2} e_0 A_j(z(s_3))\tfrac{dx^j(s_3)}{ds} ds_3\right] ds_2 + \cdots \end{aligned} \tag{10}$$

where $z(s') = z_0$ and $z(s'') = z_1$. Then since A_i are continuous on $[s', s'']$ and $x^i(z(\cdot))$ are continuously differentiable on $[s', s'']$ we have that the series in (10) is absolutely convergent. Thus the Wilson line $W(z_0, z_1)$ exists. Then since $W(z_0, z_1)$ is the limit of ordered product we can write $W(z_0, z_1)$ in the form $W(z_0, z)W(z, z_1)$ by dividing $z(\cdot)$ into two parts at z. This proves the basic property of Wilson line. ⋄

Remark (classical and quantum versions of Wilson loop) From the above property we have that the Wilson line $W(z_0, z_1)$ exists in the classical pathwise sense where A_i are as classical paths on $[s_0, s_1]$. This pathwise version of the Wilson line $W(z_0, z_1)$; from the Feynman path integral point of view; is as a partial description of the quantum version of the Wilson line $W(z_0, z_1)$ which is as an operator when A_i are as operators. We shall in the next Section derive and define a quantum generator J of $W(z_0, z_1)$ from the quantum gauge theory. Then by using this generator J we shall compute the quantum version of the Wilson line $W(z_0, z_1)$.

We shall denote both the classical version and quantum version of Wilson line by the same notation $W(z_0, z_1)$ when there is no confusion. ⋄

By following the usual approach of deriving a chiral symmetry from a gauge transformation of a gauge field we have the following chiral symmetry which is derived by applying an analytic gauge transformation with an analytic function ω for the transformation:

$$\begin{aligned} W(z_0, z_1) \mapsto \quad & W'(z_0, z_1) = \\ & = U(\omega(z_1)) W(z_0, z_1) U^{-1}(\omega(z_0)) , \end{aligned} \tag{11}$$

where $W'(z_0, z_1)$ is a Wilson line with gauge field:

$$A'_\mu = \frac{\partial U(z)}{\partial x^\mu} U^{-1}(z) + U(z)\, A_\mu\, U^{-1}(z) . \tag{12}$$

This chiral symmetry is analogous to the chiral symmetry of the usual guage theory where U denotes an element of the gauge group [61]. Let us derive (11) as follows. Let $U(z) := := U(\omega(z(s)))$ and $U(z + dz) \approx U(z) + \frac{\partial U(z)}{\partial x^\mu} dx^\mu$ where $dz = (dx^1, dx^2)$. Following [61] we have

$$\begin{aligned} & U(z + dz)(1 + e_0 dx^\mu A_\mu) U^{-1}(z) = \\ & = U(z + dz) U^{-1}(z) + e_0 dx^\mu U(z+dz) A_\mu U^{-1}(s) \\ & \approx 1 + \tfrac{\partial U(z)}{\partial x^\mu} U^{-1}(z) dx^\mu + e_0 dx^\mu U(z+dz) A_\mu U^{-1}(s) \\ & \approx 1 + \tfrac{\partial U(z)}{\partial x^\mu} U^{-1}(z) dx^\mu + e_0 dx^\mu U(z) A_\mu U^{-1}(z) \\ & =: 1 + \tfrac{\partial U(z)}{\partial x^\mu} U^{-1}(z) dx^\mu + e_0 dx^\mu U(z) A_\mu U^{-1}(z) \\ & =: 1 + e_0 dx^\mu A'_\mu . \end{aligned} \tag{13}$$

From (13) we have that (11) holds.

As analogous to the WZW model in conformal field theory [65, 66] from the above symmetry we have the following formulas for the variations $\delta_\omega W$ and $\delta_{\omega'} W$ with respect to this symmetry (see [65] p.621):

$$\delta_\omega W(z, z') = W(z, z')\omega(z) \tag{14}$$

and

$$\delta_{\omega'} W(z, z') = -\omega'(z') W(z, z') , \tag{15}$$

where z and z' are independent variables and $\omega'(z') = \omega(z)$ when $z' = z$. In (14) the variation is with respect to the z variable while in (15) the variation is with respect to the z' variable. This two-side-variations when $z \neq z'$ can be derived as follows. For the left variation we may let ω be analytic in a neighborhood of z and extended as a continuously differentiable function to a neighborhood of z' such that $\omega(z') = 0$ in this neighborhood of z'. Then from (11) we have that (14) holds. Similarly we may let ω' be analytic in a neighborhood of z' and extended as a continuously differentiable function to a neighborhood of z such that $\omega'(z) = 0$ in this neighborhood of z. Then we have that (15) holds.

4 Gauge fixing and affine Kac-Moody algebra

This Section has two related purposes. One purpose is to find a gauge fixing condition for eliminating the degenerate degree of freedom from the gauge invariance of the above quantum gauge theory in Section 2. Then another purpose is to find an equation for defining a generator J of the Wilson line $W(z, z')$. This defining equation of J can then be used as a gauge fixing condition. Thus with this defining equation of J the construction of the quantum gauge theory in Section 2 is then completed.

We shall derive a quantum loop algebra (or the affine Kac-Moody algebra) structure from the Wilson line $W(z, z')$ for the generator J of $W(z, z')$. To this end let us first consider the classical case. Since $W(z, z')$ is constructed from $SU(2)$ we have that the mapping $z \to W(z, z')$ (We consider $W(z, z')$ as a function of z with z' being fixed) has a loop group structure [63, 64]. For a loop group we have the following generators:

$$J^a_n = t^a z^n \qquad n = 0, \pm 1, \pm 2, \dots \tag{16}$$

These generators satisfy the following algebra:

$$[J^a_m, J^b_n] = i f_{abc} J^c_{m+n} . \tag{17}$$

This is the so called loop algebra [63, 64]. Let us then introduce the following generating function J:

$$J(w) = \sum_a J^a(w) = \sum_a j^a(w)\, t^a, \tag{18}$$

where we define

$$J^a(w) = j^a(w) t^a := \sum_{n=-\infty}^{\infty} J^a_n(z)(w - z)^{-n-1}. \tag{19}$$

From J we have

$$J^a_n = \frac{1}{2\pi i} \oint_z dw\, (w - z)^n J^a(w) , \tag{20}$$

where $\oint_z$ denotes a closed contour integral with center z. This formula can be interpreted as that J is the generator of the loop group and that J^a_n is the directional generator in the direction $\omega^a(w) = (w - z)^n$. We may generalize (20) to the following directional generator:

$$\frac{1}{2\pi i} \oint_z dw\, \omega(w)\, J(w) , \tag{21}$$

where the analytic function $\omega(w) = \sum_a \omega^a(w) t^a$ is regarded as a direction and we define

$$\omega(w) J(w) := \sum_a \omega^a(w) J^a. \tag{22}$$

Then since $W(z, z') \in SU(2)$, from the variational formula (21) for the loop algebra of the loop group of $SU(2)$ we have that the variation of $W(z,z')$ in the direction $\omega(w)$ is given by

$$W(z,z')\frac{1}{2\pi i}\oint_z dw\,\omega(w)\,J(w)\,. \tag{23}$$

Now let us consider the quantum case which is based on the quantum gauge theory in Section 2. For this quantum case we shall define a quantum generator J which is analogous to the J in (18). We shall choose the equations (34) and (35) as the equations for defining the quantum generator J. Let us first give a formal derivation of the equation (34), as follows. Let us consider the following formal functional integration:

$$\begin{aligned}&\langle W(z,z')A(z)\rangle := \\ &\quad := \int dA_1 dA_2 dZ^* dZ e^{-L} W(z,z')A(z),\end{aligned} \tag{24}$$

where $A(z)$ denotes a field from the quantum gauge theory. (We first let z' be fixed as a parameter.)

Let us do a calculus of variation on this integral to derive a variational equation by applying a gauge transformation on (24) as follows. (We remark that such variational equations are usually called the Ward identity in the physics literature.)

Let (A_1, A_2, Z) be regarded as a coordinate system of the integral (24). Under a gauge transformation (regarded as a change of coordinate) with gauge function $a(z(s))$ this coordinate is changed to another coordinate (A_1', A_2', Z'). As similar to the usual change of variable for integration we have that the integral (24) is unchanged under a change of variable and we have the following equality:

$$\begin{aligned}&\int dA_1' dA_2' dZ'^* dZ' e^{-L'} W'(z,z')A'(z) = \\ &\quad = \int dA_1 dA_2 dZ^* dZ e^{-L} W(z,z')A(z)\,,\end{aligned} \tag{25}$$

where $W'(z,z')$ denotes the Wilson line based on A_1' and A_2' and similarly $A'(z)$ denotes the field obtained from $A(z)$ with (A_1, A_2, Z) replaced by (A_1', A_2', Z').

Then it can be shown that the differential is unchanged under a gauge transformation [59]:

$$dA_1' dA_2' dZ'^* dZ' = dA_1 dA_2 dZ^* dZ\,. \tag{26}$$

Also by the gauge invariance property the factor e^{-L} is unchanged under a gauge transformation. Thus from (25) we have

$$0 = \langle W'(z,z')A'(z)\rangle - \langle W(z,z')A(z)\rangle\,, \tag{27}$$

where the correlation notation $\langle\cdot\rangle$ denotes the integral with respect to the differential

$$e^{-L} dA_1 dA_2 dZ^* dZ \tag{28}$$

We can now carry out the calculus of variation. From the gauge transformation we have the formula:

$$W'(z,z') = U(a(z))W(z,z')U^{-1}(a(z'))\,, \tag{29}$$

where $a(z) = \mathrm{Re}\,\omega(z)$. This gauge transformation gives a variation of $W(z,z')$ with the gauge function $a(z)$ as the variational direction a in the variational formulas (21) and (23). Thus analogous to the variational formula (23) we have that the variation of $W(z,z')$ under this gauge transformation is given by

$$W(z,z')\frac{1}{2\pi i}\oint_z dw\,a(w)\,J(w)\,, \tag{30}$$

where the generator J for this variation is to be specific. This J will be a quantum generator which generalizes the classical generator J in (23).

Thus under a gauge transformation with gauge function $a(z)$ from (27) we have the following variational equation:

$$\begin{aligned}0 = \Big\langle W(z,z')\Big[&\delta_a A(z) + \\ &+ \frac{1}{2\pi i}\oint_z dw\,a(w)J(w)A(z)\Big]\Big\rangle,\end{aligned} \tag{31}$$

where $\delta_a A(z)$ denotes the variation of the field $A(z)$ in the direction $a(z)$. From this equation an ansatz of J is that J satisfies the following equation:

$$W(z,z')\Big[\delta_a A(z) + \frac{1}{2\pi i}\oint_z dw\,a(w)J(w)A(z)\Big] = 0\,. \tag{32}$$

From this equation we have the following variational equation:

$$\delta_a A(z) = \frac{-1}{2\pi i}\oint_z dw\,a(w)J(w)A(z)\,. \tag{33}$$

This completes the formal calculus of variation. Now (with the above derivation as a guide) we choose the following equation (34) as one of the equation for defining the generator J:

$$\delta_\omega A(z) = \frac{-1}{2\pi i}\oint_z dw\,\omega(w)J(w)A(z)\,, \tag{34}$$

where we generalize the direction $a(z) = \mathrm{Re}\,\omega(z)$ to the analytic direction $\omega(z)$. (This generalization has the effect of extending the real measure of the pure gauge part of the gauge theory to include the complex Feynman path integral since it gives the transformation $ds \to -i\,ds$ for the integral of the Wilson line $W(z,z')$.)

Let us now choose one more equation for determine the generator J in (34). This choice will be as a gauge fixing condition. As analogous to the WZW model in conformal field theory [65–67] let us consider a J given by

$$J(z) := -k_0 W^{-1}(z,z')\,\partial_z W(z,z')\,, \tag{35}$$

where we define $\partial_z = \partial_{x^1} + i\partial_{x^2}$ and we set $z' = z$ after the differentiation with respect to z; $k_0 > 0$ is a constant which is fixed when the J is determined to be of the form (35) and the minus sign is chosen by convention. In the WZW model [65, 67] the J of the form (35) is the generator of the chiral symmetry of the WZW model. We can write the J in (35) in the following form:

$$J(w) = \sum_a J^a(w) = \sum_a j^a(w)\, t^a. \tag{36}$$

We see that the generators t^a of $SU(2)$ appear in this form of J and this form is analogous to the classical J in (18). This shows that this J is a possible candidate for the generator J in (34).

Since $W(z, z')$ is constructed by gauge field we need to have a gauge fixing for the computations related to $W(z, z')$. Then since the J in (34) and (35) is constructed by $W(z, z')$ we have that in defining this J as the generator J of $W(z, z')$ we have chosen a condition for the gauge fixing. In this paper we shall always choose this defining equations (34) and (35) for J as the gauge fixing condition.

In summary we introduce the following definition.

Definition The generator J of the quantum Wilson line $W(z, z')$ whose classical version is defined by (8), is an operator defined by the two conditions (34) and (35). $\diamond$

Remark We remark that the condition (35) first defines J classically. Then the condition (34) raises this classical J to the quantum generator J. $\diamond$

Now we want to show that this generator J in (34) and (35) can be uniquely solved. (This means that the gauge fixing condition has already fixed the gauge that the degenerate degree of freedom of gauge invariance has been eliminated so that we can carry out computation.)

Let us now solve J. From (11) and (35) the variation $\delta_\omega J$ of the generator J in (35) is given by [65, p. 622] and [67]:

$$\delta_\omega J = [J, \omega] - k_0 \partial_z \omega\,. \tag{37}$$

From (34) and (37) we have that J satisfies the following relation of current algebra [65–67]:

$$J^a(w) J^b(z) = \frac{k_0 \delta_{ab}}{(w-z)^2} + \sum_c i f_{abc} \frac{J^c(z)}{(w-z)}\,, \tag{38}$$

where as a convention the regular term of $J^a(w)J^b(z)$ is omitted. Then by following [65–67] from (38) and (36) we can show that the J^a_n in (18) for the corresponding Laurent series of the quantum generator J satisfy the following Kac-Moody algebra:

$$[J^a_m, J^b_n] = i f_{abc} J^c_{m+n} + k_0\, m\, \delta_{ab} \delta_{m+n,0}\,, \tag{39}$$

where k_0 is usually called the central extension or the level of the Kac-Moody algebra.

Remark Let us also consider the other side of the chiral symmetry. Similar to the J in (35) we define a generator J' by:

$$J'(z') = k_0 \partial_{z'} W(z, z') W^{-1}(z, z')\,, \tag{40}$$

where after differentiation with respect to z' we set $z = z'$. Let us then consider the following formal correlation:

$$\begin{aligned} &\langle A(z') W(z, z') \rangle := \\ &:= \int dA_1 dA_2 dZ^* dZ A(z')\, W(z, z')\, e^{-L}, \end{aligned} \tag{41}$$

where z is fixed. By an approach similar to the above derivation of (34) we have the following variational equation:

$$\delta_{\omega'} A(z') = \frac{-1}{2\pi i} \oint_{z'} dw A(z') J'(w) \omega'(w)\,, \tag{42}$$

where as a gauge fixing we choose the J' in (42) be the J' in (40). Then similar to (37) we also have

$$\delta_{\omega'} J' = [J', \omega'] - k_0 \partial_{z'} \omega'. \tag{43}$$

Then from (42) and (43) we can derive the current algebra and the Kac-Moody algebra for J' which are of the same form of (38) and (39). From this we have $J' = J$. $\diamond$

Now with the above current algebra J and the formula (34) we can follow the usual approach in conformal field theory to derive a quantum Knizhnik-Zamolodchikov (KZ) equation for the product of primary fields in a conformal field theory [65–67]. We derive the KZ equation for the product of n Wilson lines $W(z, z')$. Here an important point is that from the two sides of $W(z, z')$ we can derive two quantum KZ equations which are dual to each other. These two quantum KZ equations are different from the usual KZ equation in that they are equations for the quantum operators $W(z, z')$ while the usual KZ equation is for the correlations of quantum operators. With this difference we can follow the usual approach in conformal field theory to derive the following quantum Knizhnik-Zamolodchikov equation [65, 66, 68]:

$$\begin{aligned} &\partial_{z_i} W(z_1, z'_1) \cdots W(z_n, z'_n) = \\ &= \tfrac{-e_0^2}{k_0 + g_0} \textstyle\sum_{j \neq i}^n \frac{\sum_a t^a_i \otimes t^a_j}{z_i - z_j} W(z_1, z'_1) \cdots W(z_n, z'_n)\,, \end{aligned} \tag{44}$$

for $i = 1, \ldots, n$ where g_0 denotes the dual Coxeter number of a group multiplying with e_0^2 and we have $g_0 = 2e_0^2$ for the group $SU(2)$ (When the gauge group is $U(1)$ we have $g_0 = 0$). We remark that in (44) we have defined $t^a_i := t^a$ and:

$$\begin{aligned} &t^a_i \otimes t^a_j W(z_1, z'_1) \cdots W(z_n, z'_n) := W(z_1, z'_1) \cdots \\ &\cdots [t^a W(z_i, z'_i)] \cdots [t^a W(z_j, z'_j)] \cdots W(z_n, z'_n)\,. \end{aligned} \tag{45}$$

It is interesting and important that we also have the following quantum Knizhnik-Zamolodchikov equation with respect to the z'_i variables which is dual to (44):

$$\begin{aligned} &\partial_{z'_i} W(z_1, z'_1) \cdots W(z_n, z'_n) = \\ &= \tfrac{-e_0^2}{k_0 + g_0} \textstyle\sum_{j \neq i}^n W(z_1, z'_1) \cdots W(z_n, z'_n) \frac{\sum_a t^a_i \otimes t^a_j}{z'_j - z'_i} \end{aligned} \tag{46}$$

for $i=1,\dots,n$ where we have defined:

$$W(z_1,z_1')\cdots W(z_n,z_n')t_i^a\otimes t_j^a := W(z_1,z_1')\cdots \\ \cdots[W(z_i,z_i')t^a]\cdots[W(z_j,z_j')t^a]\cdots W(z_n,z_n')\,. \tag{47}$$

Remark From the quantum gauge theory we derive the above quantum KZ equation in dual form by calculus of variation. This quantum KZ equation in dual form may be considered as a quantum Euler-Lagrange equation or as a quantum Yang-Mills equation since it is analogous to the classical Yang-Mills equation which is derived from the classical Yang-Mills gauge theory by calculus of variation. ⋄

5 Solving quantum KZ equation in dual form

Let us consider the following product of two quantum Wilson lines:

$$G(z_1,z_2,z_3,z_4) := W(z_1,z_2)W(z_3,z_4)\,, \tag{48}$$

where the quantum Wilson lines $W(z_1,z_2)$ and $W(z_3,z_4)$ represent two pieces of curves starting at z_1 and z_3 and ending at z_2 and z_4 respectively.

We have that this product $G(z_1,z_2,z_3,z_4)$ satisfies the KZ equation for the variables z_1, z_3 and satisfies the dual KZ equation for the variables z_2 and z_4. Then by solving the two-variables-KZ equation in (44) we have that a form of $G(z_1,z_2,z_3,z_4)$ is given by [69–71]:

$$e^{-\hat{t}\log[\pm(z_1-z_3)]}C_1\,, \tag{49}$$

where $\hat{t}:=\frac{e_0^2}{k_0+g_0}\sum_a t^a\otimes t^a$ and C_1 denotes a constant matrix which is independent of the variable z_1-z_3.

We see that $G(z_1,z_2,z_3,z_4)$ is a multi-valued analytic function where the determination of the $\pm$ sign depended on the choice of the branch.

Similarly by solving the dual two-variable-KZ equation in (46) we have that G is of the form

$$C_2\,e^{\hat{t}\log[\pm(z_4-z_2)]}\,, \tag{50}$$

where C_2 denotes a constant matrix which is independent of the variable z_4-z_2.

From (49), (50) and letting:

$$C_1 = Ae^{\hat{t}\log[\pm(z_4-z_2)]}\,, \quad C_2 = e^{-\hat{t}\log[\pm(z_1-z_3)]}A\,, \tag{51}$$

where A is a constant matrix we have that $G(z_1,z_2,z_3,z_4)$ is given by:

$$G(z_1,z_2,z_3,z_4) = e^{-\hat{t}\log[\pm(z_1-z_3)]}Ae^{\hat{t}\log[\pm(z_4-z_2)]}\,, \tag{52}$$

where at the singular case that $z_1=z_3$ we define $\log[\pm(z_1-z_3)]=0$. Similarly for $z_2=z_4$.

Let us find a form of the initial operator A. We notice that there are two operators $\Phi_\pm(z_1-z_3):=e^{-\hat{t}\log[\pm(z_1-z_3)]}$ and $\Psi_\pm(z_4-z_2)=e^{\hat{t}\log[\pm(z_4-z_2)]}$ acting on the two sides of A respectively where the two independent variables z_1,z_3 of $\Phi_\pm$ are mixed from the two quantum Wilson lines $W(z_1,z_2)$ and $W(z_3,z_4)$ respectively and the the two independent variables z_2,z_4 of $\Psi_\pm$ are mixed from the two quantum Wilson lines $W(z_1,z_2)$ and $W(z_3,z_4)$ respectively. From this we determine the form of A as follows.

Let D denote a representation of $SU(2)$. Let $D(g)$ represent an element g of $SU(2)$ and let $D(g)\otimes D(g)$ denote the tensor product representation of $SU(2)$. Then in the KZ equation we define

$$[t^a\otimes t^a][D(g_1)\otimes D(g_1)]\otimes[D(g_2)\otimes D(g_2)] := \\ := [t^aD(g_1)\otimes D(g_1)]\otimes[t^aD(g_2)\otimes D(g_2)] \tag{53}$$

and

$$[D(g_1)\otimes D(g_1)]\otimes[D(g_2)\otimes D(g_2)][t^a\otimes t^a] := \\ := [D(g_1)\otimes D(g_1)t^a]\otimes[D(g_2)\otimes D(g_2)t^a]\,. \tag{54}$$

Then we let $U(\mathbf{a})$ denote the universal enveloping algebra where $\mathbf{a}$ denotes an algebra which is formed by the Lie algebra $su(2)$ and the identity matrix.

Now let the initial operator A be of the form $A_1\otimes A_2\otimes A_3\otimes A_4$ with $A_i, i=1,\dots,4$ taking values in $U(\mathbf{a})$. In this case we have that in (52) the operator $\Phi_\pm(z_1-z_3)$ acts on A from the left via the following formula:

$$t^a\otimes t^aA = [t^aA_1]\otimes A_2\otimes[t^aA_3]\otimes A_4\,. \tag{55}$$

Similarly the operator $\Psi_\pm(z_4-z_2)$ in (52) acts on A from the right via the following formula:

$$At^a\otimes t^a = A_1\otimes[A_2t^a]\otimes A_3\otimes[A_4t^a]\,. \tag{56}$$

We may generalize the above tensor product of two quantum Wilson lines as follows. Let us consider a tensor product of n quantum Wilson lines: $W(z_1,z_1')\cdots W(z_n,z_n')$ where the variables z_i, z_i' are all independent. By solving the two KZ equations we have that this tensor product is given by:

$$W(z_1,z_1')\cdots W(z_n,z_n') = \\ = \prod_{ij}\Phi_\pm(z_i-z_j)A\prod_{ij}\Psi_\pm(z_i'-z_j')\,, \tag{57}$$

where $\prod_{ij}$ denotes a product of $\Phi_\pm(z_i-z_j)$ or $\Psi_\pm(z_i'-z_j')$ for $i,j=1,\dots,n$ where $i\neq j$. In (57) the initial operator A is represented as a tensor product of operators $A_{iji'j'}$, $i,j,i',j'=1,\dots,n$ where each $A_{iji'j'}$ is of the form of the initial operator A in the above tensor product of two-Wilson-lines case and is acted by $\Phi_\pm(z_i-z_j)$ or $\Psi_\pm(z_i'-z_j')$ on its two sides respectively.

6 Computation of quantum Wilson lines

Let us consider the following product of two quantum Wilson lines:

$$G(z_1,z_2,z_3,z_4) := W(z_1,z_2)W(z_3,z_4)\,, \tag{58}$$

where the quantum Wilson lines $W(z_1, z_2)$ and $W(z_3, z_4)$ represent two pieces of curves starting at z_1 and z_3 and ending at z_2 and z_4 respectively. As shown in the above Section we have that $G(z_1, z_2, z_3, z_4)$ is given by the following formula:

$$G(z_1, z_2, z_3, z_4) = e^{-\hat{t}\log[\pm(z_1-z_3)]} A e^{\hat{t}\log[\pm(z_4-z_2)]}, \quad (59)$$

where the product is a 4-tensor.

Let us set $z_2 = z_3$. Then the 4-tensor $W(z_1, z_2)W(z_3, z_4)$ is reduced to the 2-tensor $W(z_1, z_2)W(z_2, z_4)$. By using (59) the 2-tensor $W(z_1, z_2)W(z_2, z_4)$ is given by:

$$\begin{aligned} &W(z_1, z_2)W(z_2, z_4) = \\ &\quad = e^{-\hat{t}\log[\pm(z_1-z_2)]} A_{14} e^{\hat{t}\log[\pm(z_4-z_2)]}, \end{aligned} \quad (60)$$

where $A_{14} = A_1 \otimes A_4$ is a 2-tensor reduced from the 4-tensor $A = A_1 \otimes A_2 \otimes A_3 \otimes A_4$ in (59). In this reduction the $\hat{t}$ operator of $\Phi = e^{-\hat{t}\log[\pm(z_1-z_2)]}$ acting on the left side of A_1 and A_3 in A is reduced to acting on the left side of A_1 and A_4 in A_{14}. Similarly the $\hat{t}$ operator of $\Psi = e^{\hat{t}\log[\pm(z_4-z_2)]}$ acting on the right side of A_2 and A_4 in A is reduced to acting on the right side of A_1 and A_4 in A_{14}.

Then since $\hat{t}$ is a 2-tensor operator we have that $\hat{t}$ is as a matrix acting on the two sides of the 2-tensor A_{14} which is also as a matrix with the same dimension as $\hat{t}$. Thus Φ and Ψ are as matrices of the same dimension as the matrix A_{14} acting on A_{14} by the usual matrix operation. Then since $\hat{t}$ is a Casimir operator for the 2-tensor group representation of $SU(2)$ we have that Φ and Ψ commute with A_{14} since Φ and Ψ are exponentials of $\hat{t}$. (We remark that Φ and Ψ are in general not commute with the 4-tensor initial operator A.) Thus we have

$$\begin{aligned} &e^{-\hat{t}\log[\pm(z_1-z_2)]} A_{14} e^{\hat{t}\log[\pm(z_4-z_2)]} = \\ &\quad = e^{-\hat{t}\log[\pm(z_1-z_2)]} e^{\hat{t}\log[\pm(z_4-z_2)]} A_{14}. \end{aligned} \quad (61)$$

We let $W(z_1, z_2)W(z_2, z_4)$ be as a representation of the quantum Wilson line $W(z_1, z_4)$:

$$\begin{aligned} W(z_1, z_4) &:= W(z_1, w_1)W(w_1, z_4) = \\ &= e^{-\hat{t}\log[\pm(z_1-w_1)]} e^{\hat{t}\log[\pm(z_4-w_1)]} A_{14}. \end{aligned} \quad (62)$$

This representation of the quantum Wilson line $W(z_1, z_4)$ means that the line (or path) with end points z_1 and z_4 is specific that it passes the intermediate point $w_1 = z_2$. This representation shows the quantum nature that the path is not specific at other intermediate points except the intermediate point $w_1 = z_2$. This unspecification of the path is of the same quantum nature of the Feynman path description of quantum mechanics.

Then let us consider another representation of the quantum Wilson line $W(z_1, z_4)$. We consider the three-product $W(z_1, w_1)W(w_1, w_2)W(w_2, z_4)$ which is obtained from the three-tensor $W(z_1, w_1)W(u_1, w_2)W(u_2, z_4)$ by two reductions where $z_j,\ w_j,\ u_j,\ j = 1, 2$ are independent variables. For this representation we have:

$$\begin{aligned} &W(z_1, w_1)W(w_1, w_2)W(w_2, z_4) = e^{-\hat{t}\log[\pm(z_1-w_1)]} \times \\ &\times e^{-\hat{t}\log[\pm(z_1-w_2)]} e^{\hat{t}\log[\pm(z_4-w_1)]} e^{\hat{t}\log[\pm(z_4-w_2)]} A_{14}. \end{aligned} \quad (63)$$

This representation of the quantum Wilson line $W(z_1, z_4)$ means that the line (or path) with end points z_1 and z_4 is specific that it passes the intermediate points w_1 and w_2. This representation shows the quantum nature that the path is not specific at other intermediate points except the intermediate points w_1 and w_2. This unspecification of the path is of the same quantum nature of the Feynman path description of quantum mechanics.

Similarly we may represent $W(z_1, z_4)$ by path with end points z_1 and z_4 and is specific only to pass at finitely many intermediate points. Then we let the quantum Wilson line $W(z_1, z_4)$ as an equivalent class of all these representations. Thus we may write:

$$\begin{aligned} W(z_1, z_4) &= W(z_1, w_1)W(w_1, z_4) = \\ &= W(z_1, w_1)W(w_1, w_2)W(w_2, z_4) = \cdots \end{aligned} \quad (64)$$

Remark Since A_{14} is a 2-tensor we have that a natural group representation for the Wilson line $W(z_1, z_4)$ is the 2-tensor group representation of the group $SU(2)$.

7 Representing braiding of curves by quantum Wilson lines

Consider again the $G(z_1, z_2, z_3, z_4)$ in (58). We have that $G(z_1, z_2, z_3, z_4)$ is a multi-valued analytic function where the determination of the $\pm$ sign depended on the choice of the branch.

Let the two pieces of curves represented by $W(z_1, z_2)$ and $W(z_3, z_4)$ be crossing at w. In this case we write $W(z_i, z_j)$ as $W(z_i, z_j) = W(z_i, w)W(w, z_j)$ where $i = 1, 3,\ j = 2, 4$. Thus we have

$$\begin{aligned} &W(z_1, z_2)W(z_3, z_4) = \\ &\quad = W(z_1, w)W(w, z_2)W(z_3, w)W(w, z_4). \end{aligned} \quad (65)$$

If we interchange z_1 and z_3, then from (65) we have the following ordering:

$$W(z_3, w)W(w, z_2)W(z_1, w)W(w, z_4). \quad (66)$$

Now let us choose a branch. Suppose these two curves are cut from a knot and that following the orientation of a knot the curve represented by $W(z_1, z_2)$ is before the curve represented by $W(z_3, z_4)$. Then we fix a branch such that the product in (59) is with two positive signs:

$$W(z_1, z_2)W(z_3, z_4) = e^{-\hat{t}\log(z_1-z_3)} A e^{\hat{t}\log(z_4-z_2)}. \quad (67)$$

Then if we interchange z_1 and z_3 we have

$$W(z_3, w)W(w, z_2)W(z_1, w)W(w, z_4) = \\ = e^{-\hat{t}\log[-(z_1-z_3)]}Ae^{\hat{t}\log(z_4-z_2)}. \tag{68}$$

From (67) and (68) as a choice of branch we have

$$W(z_3, w)W(w, z_2)W(z_1, w)W(w, z_4) = \\ = RW(z_1, w)W(w, z_2)W(z_3, w)W(w, z_4)\,, \tag{69}$$

where $R = e^{-i\pi\hat{t}}$ is the monodromy of the KZ equation. In (69) z_1 and z_3 denote two points on a closed curve such that along the direction of the curve the point z_1 is before the point z_3 and in this case we choose a branch such that the angle of $z_3 - z_1$ minus the angle of $z_1 - z_3$ is equal to π.

Remark We may use other representations of the product $W(z_1, z_2)W(z_3, z_4)$. For example we may use the following representation:

$$W(z_1, w)W(w, z_2)W(z_3, w)W(w, z_4) = \\ = e^{-\hat{t}\log(z_1-z_3)}e^{-2\hat{t}\log(z_1-w)}e^{-2\hat{t}\log(z_3-w)} \times \\ \times Ae^{\hat{t}\log(z_4-z_2)}e^{2\hat{t}\log(z_4-w)}e^{2\hat{t}\log(z_2-w)}. \tag{70}$$

Then the interchange of z_1 and z_3 changes only $z_1 - z_3$ to $z_3 - z_1$. Thus the formula (69) holds. Similarly all other representations of $W(z_1, z_2)W(z_3, z_4)$ will give the same result. $\diamond$

Now from (69) we can take a convention that the ordering (66) represents that the curve represented by $W(z_1, z_2)$ is up-crossing the curve represented by $W(z_3, z_4)$ while (65) represents zero crossing of these two curves.

Similarly from the dual KZ equation as a choice of branch which is consistent with the above formula we have

$$W(z_1, w)W(w, z_4)W(z_3, w)W(w, z_2) = \\ = W(z_1, w)W(w, z_2)W(z_3, w)W(w, z_4)R^{-1}, \tag{71}$$

where z_2 is before z_4. We take a convention that the ordering in (71) represents that the curve represented by $W(z_1, z_2)$ is under-crossing the curve represented by $W(z_3, z_4)$. Here along the orientation of a closed curve the piece of curve represented by $W(z_1, z_2)$ is before the piece of curve represented by $W(z_3, z_4)$. In this case since the angle of $z_3 - z_1$ minus the angle of $z_1 - z_3$ is equal to π we have that the angle of $z_4 - z_2$ minus the angle of $z_2 - z_4$ is also equal to π and this gives the R^{-1} in this formula (71).

From (69) and (71) we have

$$W(z_3, z_4)W(z_1, z_2) = RW(z_1, z_2)W(z_3, z_4)R^{-1}, \tag{72}$$

where z_1 and z_2 denote the end points of a curve which is before a curve with end points z_3 and z_4. From (72) we see that the algebraic structure of these quantum Wilson lines $W(z, z')$ is analogous to the quasi-triangular quantum group [66, 69].

8 Computation of quantum Dirac-Wilson loop

Consider again the quantum Wilson line $W(z_1, z_4)$ given by $W(z_1, z_4) = W(z_1, z_2)W(z_2, z_4)$. Let us set $z_1 = z_4$. In this case the quantum Wilson line forms a closed loop. Now in (61) with $z_1 = z_4$ we have that the quantities $e^{-\hat{t}\log\pm(z_1-z_2)}$ and $e^{\hat{t}\log\pm(z_1-z_2)}$ which come from the two-side KZ equations cancel each other and from the multi-valued property of the log function we have:

$$W(z_1, z_1) = R^N A_{14}\,, \qquad N = 0, \pm1, \pm2, \ldots \tag{73}$$

where $R = e^{-i\pi\hat{t}}$ is the monodromy of the KZ equation [69].

Remark It is clear that if we use other representation of the quantum Wilson loop $W(z_1, z_1)$ (such as the representation $W(z_1, z_1) = W(z_1, w_1)W(w_1, w_2)W(w_2, z_1)$) then we will get the same result as (73).

Remark For simplicity we shall drop the subscript of A_{14} in (73) and simply write $A_{14} = A$.

9 Winding number of Dirac-Wilson loop as quantization

We have the equation (73) where the integer N is as a winding number. Then when the gauge group is $U(1)$ we have

$$W(z_1, z_1) = R^N_{U(1)}A\,, \tag{74}$$

where $R_{U(1)}$ denotes the monodromy of the KZ equation for $U(1)$. We have

$$R^N_{U(1)} = e^{iN\frac{\pi e_0^2}{k_0+g_0}}\,, \qquad N = 0, \pm1, \pm2, \ldots \tag{75}$$

where the constant e_0 denotes the bare electric charge (and $g_0 = 0$ for $U(1)$ group). The winding number N is as the quantization property of photon. We show in the following Section that the Dirac-Wilson loop $W(z_1, z_1)$ with the abelian $U(1)$ group is a model of the photon.

10 Magnetic monopole is a photon with a specific frequency

We see that the Dirac-Wilson loop is an exactly solvable nonlinear observable. Thus we may regard it as a quantum soliton of the above gauge theory. In particular for the abelian gauge theory with $U(1)$ as gauge group we regard the Dirac-Wilson loop as a quantum soliton of the electromagnetic field. We now want to show that this soliton has all the properties of photon and thus we may identify it with the photon.

First we see that from (75) it has discrete energy levels of light-quantum given by

$$h\nu := N\frac{\pi e_0^2}{k_0}\,, \qquad N = 0, 1, 2, 3, \ldots \tag{76}$$

where h is the Planck's constant; ν denotes a frequency and the constant $k_0 > 0$ is determined from this formula. This formula is from the monodromy $R_{U(1)}$ for the abelian gauge theory. We see that the Planck's constant h comes out from this winding property of the Dirac-Wilson loop. Then since this Dirac-Wilson loop is a loop we have that it has the polarization property of light by the right hand rule along the loop and this polarization can also be regarded as the spin of photon. Now since this loop is a quantum soliton which behaves as a particle we have that this loop is a basic particle of the above abelian gauge theory where the abelian gauge property is considered as the fundamental property of electromagnetic field. This shows that the Dirac-Wilson loop has properties of photon. We shall later show that from this loop model of photon we can describe the absorption and emission of photon by an electron. This property of absorption and emission is considered as a basic principle of the light-quantum hypothesis of Einstein [1]. From these properties of the Dirac-Wilson loop we may identify it with the photon.

On the other hand from Dirac's analysis of the magnetic monopole we have that the property of magnetic monopole comes from a closed line integral of vector potential of the electromagnetic field which is similar to the Dirac-Wilson loop [4]. Now from this Dirac-Wilson loop we can define the magnetic charge q and the minimal magnetic charge q_{min} which are given by:

$$eq := enq_{min} := n_e e_0 n \frac{n_m e_0 \pi}{k_0}, \quad n = 0, 1, 2, 3, \ldots \tag{77}$$

where $e := n_e e_0$ is as the observed electric charge for some positive integer n_e; and $q_{min} := \frac{n_m e_0 \pi}{k_0}$ for some positive integer n_m and we write $N = nn_e n_m, n = 0, 1, 2, 3, \ldots$ (by absorbing the constant k_0 to e_0^2 we may let $k_0 = 1$).

This shows that the Dirac-Wilson loop gives the property of magnetic monopole for some frequencies. Since this loop is a quantum soliton which behaves as a particle we have that this Dirac-Wilson loop may be identified with the magnetic monopole for some frequencies. Thus we have that photon may be identified with the magnetic monopole for some frequencies. With this identification we have the following interesting conclusion: Both the energy quantization of electromagnetic field and the charge quantization property come from the same property of photon. Indeed we have:

$$nh\nu_1 := n \frac{n_e n_m e_0^2 \pi}{k_0} = eq, \qquad n = 0, 1, 2, 3, \ldots \tag{78}$$

where ν_1 denotes a frequency. This formula shows that the energy quantization gives the charge quantization and thus these two quantizations are from the same property of the photon when photon is modelled by the Dirac-Wilson loop and identified with the magnetic monopole for some frequencies. We notice that between two energy levels neq_{min} and $(n+1)eq_{min}$ there are other energy levels which may be regarded as the excited states of a particle with charge ne.

11 Nonlinear loop model of electron

In this Section let us also give a loop model to the electron. This loop model of electron is based on the above loop model of the photon. From the loop model of photon we also construct an observable which gives mass to the electron and is thus a mass mechanism for the electron.

Let $W(z, z)$ denote a Dirac-Wilson loop which represents a photon. Let Z denotes the complex variable for electron in (1.1). We then consider the following observable:

$$W(z, z)Z\,. \tag{79}$$

Since $W(z, z)$ is solvable we have that this observable is also solvable where in solving $W(z, z)$ the variable Z is fixed. We let this observable be identified with the electron. Then we consider the following observable:

$$Z^* W(z, z)Z\,. \tag{80}$$

This observable is with a scalar factor Z^*Z where Z^* denotes the complex conjugate of Z and we regard it as the mass mechanism of the electron (79). For this observable we model the energy levels with specific frequencies of $W(z, z)$ as the mass levels of electron and the mass m of electron is the lowest energy level $h\nu_1$ with specific frequency ν_1 of $W(z, z)$ and is given by:

$$mc^2 = h\nu_1\,, \tag{81}$$

where c denotes the constant of the speed of light and the frequency ν_1 is given by (78). From this model of the mass mechanism of electron we have that electron is with mass m while photon is with zero mass because there does not have such a mass mechanism $Z^* W(z, z)Z$ for the photon. From this definition of mass we have the following formula relating the observed electric charge e of electron, the magnetic charge q_{min} of magnetic monopole and the mass m of electron:

$$mc^2 = eq_{min} = h\nu_1\,. \tag{82}$$

By using the nonlinear model $W(z, z)Z$ to represent an electron we can then describe the absorption and emission of a photon by an electron where photon is as a parcel of energy described by the loop $W(z, z)$, as follows. Let $W(z, z)Z$ represents an electron and let $W_1(z_1, z_1)$ represents a photon. Then the observable $W_1(z_1, z_1)W(z, z)Z$ represents an electron having absorbed the photon $W_1(z_1, z_1)$. This property of absorption and emission is as a basic principle of the hypothesis of light-quantum stated by Einstein [1]. Let us quote the following paragraph from [1]:

> …First, the light-quantum was conceived of as a parcel of energy as far as the properties of pure radiation (no coupling to matter) are concerned. Second, Einstein made the assumption — he call it the heuristic principle — that also in its coupling to matter (that is, in emission and absorption), light is created or annihilated in similar discrete parcels of energy. That, I

believe, was Einstein's one revolutionary contribution to physics. It upset all existing ideas about the interaction between light and matter...

12 Photon with specific frequency carries electric and magnetic charges

In this loop model of photon we have that the observed electric charge $e := n_e e_0$ and the magnetic charge q_{min} are carried by the photon with some specific frequencies. Let us here describe the physical effects from this property of photon that photon with some specific frequency carries the electric and magnetic charge. From the nonlinear model of electron we have that an electron $W(z,z)Z$ also carries the electric charge when a photon $W(z,z)$ carrying the electric and magnetic charge is absorbed to form the electron $W(z,z)Z$. This means that the electric charge of an electron is from the electric charge carried by a photon. Then an interaction (as the electric force) is formed between two electrons (with the electric charges).

On the other hand since photon carries the constant e_0^2 of the bare electric charge e_0 we have that between two photons there is an interaction which is similar to the electric force between two electrons (with the electric charges). This interaction however may not be of the same magnitude as the electric force with the magnitude e^2 since the photons may not carry the frequency for giving the electric and magnetic charge. Then for stability such interaction between two photons tends to give repulsive effect to give the diffusion phenomenon among photons.

Similarly an electron $W(z,z)Z$ also carries the magnetic charge when a photon $W(z,z)$ carrying the electric and magnetic charge is absorbed to form the electron $W(z,z)Z$. This means that the magnetic charge of an electron is from the magnetic charge carried by a photon. Then a closed-loop interaction (as the magnetic force) may be formed between two electrons (with the magnetic charges).

On the other hand since photon carries the constant e_0^2 of the bare electric charge e_0 we have that between two photons there is an interaction which is similar to the magnetic force between two electrons (with the magnetic charges). This interaction however may not be of the same magnitude as the magnetic force with the magnetic charge q_{min} since the photons may not carry the frequency for giving the electric and magnetic charge. Then for stability such interaction between two photons tends to give repulsive effect to give the diffusion phenomenon among photons.

13 Statistics of photons and electrons

The nonlinear model $W(z,z)Z$ of an electron gives a relation between photon and electron where the photon is modelled by $W(z,z)$ which is with a specific frequency for $W(z,z)Z$ to be an electron, as described in the above Sections. We want to show that from this nonlinear model we may also derive the required statistics of photons and electrons that photons obey the Bose-Einstein statistics and electrons obey the Fermi-Dirac statistics. We have that $W(z,z)$ is as an operator acting on Z. Let $W_1(z,z)$ be a photon. Then we have that the nonlinear model $W_1(z,z)W(z,z)Z$ represents that the photon $W_1(z,z)$ is absorbed by the electron $W(z,z)Z$ to form an electron $W_1(z,z)W(z,z)Z$. Let $W_2(z,z)$ be another photon. The we have that the model $W_1(z,z)W_2(z,z)W(z,z)Z$ again represents an electron where we have:

$$\begin{aligned} W_1(z,z)W_2(z,z)W(z,z)Z &= \\ &= W_2(z,z)W_1(z,z)W(z,z)Z\,. \end{aligned} \tag{83}$$

More generally the model $\prod_{n=1}^{N} W_n(z,z)W(z,z)Z$ represents that the photons $W_n(z,z), n=1,2,\ldots,N$ are absorbed by the electron $W(z,z)Z$. This model shows that identical (but different) photons can appear identically and it shows that photons obey the Bose-Einstein statistics. From the polarization of the Dirac-Wilson loop $W(z,z)$ we may assign spin 1 to a photon represented by $W(z,z)$.

Let us then consider statistics of electrons. The observable $Z^*W(z,z)Z$ gives mass to the electron $W(z,z)Z$ and thus this observable is as a scalar and thus is assigned with spin 0. As the observable $W(z,z)Z$ is between $W(z,z)$ and $Z^*W(z,z)Z$ which are with spin 1 and 0 respectively we thus assign spin $\frac{1}{2}$ to the observable $W(z,z)Z$ and thus electron represented by this observable $W(z,z)Z$ is with spin $\frac{1}{2}$.

Then let Z_1 and Z_2 be two independent complex variables for two electrons and let $W_1(z,z)Z_1$ and $W_2(z,z)Z_2$ represent two electrons. Let $W_3(z,z)$ represents a photon. Then the model $W_3(z,z)(W_1(z,z)Z_1 + W_2(z,z)Z_2)$ means that two electrons are in the same state that the operator $W_3(z,z)$ is acted on the two electrons. However this model means that a photon $W(z,z)$ is absorbed by two distinct electrons and this is impossible. Thus the models $W_3(z,z)W_1(z,z)Z_1$ and $W_3(z,z)W_2(z,z)Z_2$ cannot both exist and this means that electrons obey Fermi-Dirac statistics.

Thus this nonlinear loop model of photon and electron gives the required statistics of photons and electrons.

14 Photon propagator and quantum photon propagator

Let us then investigate the quantum Wilson line $W(z_0,z)$ with $U(1)$ group where z_0 is fixed for the photon field. We want to show that this quantum Wilson line $W(z_0,z)$ may be regarded as the quantum photon propagator for a photon propagating from z_0 to z.

As we have shown in the above Section on computation of quantum Wilson line; to compute $W(z_0,z)$ we need to write $W(z_0,z)$ in the form of two (connected) Wilson lines: $W(z_0,z) = W(z_0,z_1)W(z_1,z)$ for some z_1 point. Then we have:

$$W(z_0, z_1)W(z_1, z) = \\ = e^{-\hat{t}\log[\pm(z_1-z_0)]}Ae^{\hat{t}\log[\pm(z-z_1)]}, \tag{84}$$

where $\hat{t} = -\frac{e_0^2}{k_0}$ for the $U(1)$ group (k_0 is a constant and we may for simplicity let $k_0 = 1$) where the term $e^{-\hat{t}\log[\pm(z-z_0)]}$ is obtained by solving the first form of the dual form of the KZ equation and the term $e^{\hat{t}\log[\pm(z_0-z)]}$ is obtained by solving the second form of the dual form of the KZ equation.

Then we may write $W(z_0, z)$ in the following form:

$$W(z_0, z) = W(z_0, z_1)W(z_1, z) = e^{-\hat{t}\log\frac{(z_1-z_0)}{(z-z_1)}}A. \tag{85}$$

Let us fix z_1 with z such that:

$$\frac{|z_1 - z_0|}{|z - z_1|} = \frac{r_1}{n_e^2} \tag{86}$$

for some positive integer n_e such that $r_1 \leqslant n_e^2$; and we let z be a point on a path of connecting z_0 and z_1 and then a closed loop is formed with z as the starting and ending point. (This loop can just be the photon-loop of the electron in this electromagnetic interaction by this photon propagator (85).) Then (85) has a factor $e_0^2 \log\frac{r_1}{n_e^2}$ which is the fundamental solution of the two dimensional Laplace equation and is analogous to the fundamental solution $\frac{e^2}{r}$ (where $e := e_0 n_e$ denotes the observed (renormalized) electric charge and r denotes the three dimensional distance) of the three dimensional Laplace equation for the Coulomb's law. Thus the operator $W(z_0, z) = W(z_0, z_1)W(z_1, z)$ in (85) can be regarded as the quantum photon propagator propagating from z_0 to z.

We remark that when there are many photons we may introduce the space variable $\mathbf{x}$ as a statistical variable via the Lorentz metric $ds^2 = dt^2 - d\mathbf{x}^2$ to obtain the Coulomb's law $\frac{e^2}{r}$ from the fundamental solution $e_0^2 \log\frac{r_1}{n_e^2}$ as a statistical law for electricity (We shall give such a space-time statistics later).

The quantum photon propagator (85) gives a repulsive effect since it is analogous to the Coulomb's law $\frac{e^2}{r}$. On the other hand we can reverse the sign of $\hat{t}$ such that this photon operator can also give an attractive effect:

$$W(z_0, z) = W(z_0, z_1)W(z_1, z) = e^{\hat{t}\log\frac{(z-z_1)}{(z_1-z_0)}}A, \tag{87}$$

where we fix z_1 with z_0 such that:

$$\frac{|z - z_1|}{|z_1 - z_0|} = \frac{r_1}{n_e^2} \tag{88}$$

for some positive integer n_e such that $r_1 \geqslant n_e^2$; and we again let z be a point on a path of connecting z_0 and z_1 and then a closed loop is formed with z as the starting and ending point. (This loop again can just be the photon-loop of the electron in this electromagnetic interaction by this photon propagator (85).) Then (87) has a factor $-e_0^2 \log\frac{r_1}{n_e^2}$ which is the fundamental solution of the two dimensional Laplace equation and is analogous to the attractive fundamental solution $-\frac{e^2}{r}$ of the three dimensional Laplace equation for the Coulomb's law.

Thus the quantum photon propagator in (85), and in (87), can give repulsive or attractive effect between two points z_0 and z for all z in the complex plane. These repulsive or attractive effects of the quantum photon propagator correspond to two charges of the same sign and of different sign respectively.

On the other hand when $z = z_0$ the quantum Wilson line $W(z_0, z_0)$ in (85) which is the quantum photon propagator becomes a quantum Wilson loop $W(z_0, z_0)$ which is identified as a photon, as shown in the above Sections.

Let us then derive a form of photon propagator from the quantum photon propagator $W(z_0, z)$. Let us choose a path connecting z_0 and $z = z(s)$. We consider the following path:

$$z(s) = z_1 + a_0\big[\theta(s_1 - s)e^{-i\beta_1(s_1-s)} + \\ + \theta(s - s_1)e^{i\beta_1(s_1-s)}\big], \tag{89}$$

where $\beta_1 > 0$ is a parameter and $z(s_0) = z_0$ for some proper time s_0; and a_0 is some complex constant; and θ is a step function given by $\theta(s) = 0$ for $s < 0$, $\theta(s) = 1$ for $s \geqslant 0$. Then on this path we have:

$$W(z_0, z) = \\ = W(z_0, z_1)W(z_1, z) = e^{\hat{t}\log\frac{(z-z_1)}{(z_1-z_0)}}A = \\ = e^{\hat{t}\log\frac{a_0[\theta(s-s_1)e^{-i\beta_1(s_1-s)}+\theta(s_1-s)e^{i\beta_1(s_1-s)}]}{(z_1-z_0)}}A = \\ = e^{\hat{t}\log b[\theta(s-s_1)e^{-i\beta_1(s_1-s)}+\theta(s_1-s)e^{i\beta_1(s_1-s)}]}A = \\ = b_0\big[\theta(s - s_1)e^{-i\hat{t}\beta_1(s_1-s)} + \theta(s_1 - s)e^{i\hat{t}\beta_1(s_1-s)}\big]A \tag{90}$$

for some complex constants b and b_0. From this chosen of the path (89) we have that the quantum photon propagator is proportional to the following expression:

$$\frac{1}{2\lambda_1}\big[\theta(s - s_1)e^{-i\lambda_1(s-s_1)} + \theta(s_1 - s)e^{i\lambda_1(s-s_1)}\big] \tag{91}$$

where we define $\lambda_1 = -\hat{t}\beta_1 = e_0^2\beta_1 > 0$. We see that this is the usual propagator of a particle $x(s)$ of harmonic oscillator with mass-energy parameter $\lambda_1 > 0$ where $x(s)$ satisfies the following harmonic oscillator equation:

$$\frac{d^2x}{ds^2} = -\lambda_1^2 x(s). \tag{92}$$

We regard (91) as the propagator of a photon with mass-energy parameter λ_1. Fourier transforming (91) we have the following form of photon propagator:

$$\frac{i}{k_E^2 - \lambda_1}, \tag{93}$$

where we use the notation k_E (instead of the notation k) to denote the proper energy of photon. We shall show in the next Section that from this photon propagator by space-time statistics we can get a propagator with the k_E replaced by the energy-momemtum four-vector k which is similar to the Feynman propagator (with a mass-energy parameter $\lambda_1 > 0$). We thus see that the quantum photon propagator $W(z_0, z)$ gives a classical form of photon propagator in the conventional QED theory.

Then we notice that while $\lambda_1 > 0$ which may be think of as the mass-energy parameter of a photon the original quantum photon propagator $W(z_0, z)$ can give the Coulomb potential and thus give the effect that the photon is massless. Thus the photon mass-energy parameter $\lambda_1 > 0$ is consistent with the property that the photon is massless. Thus in the following Sections when we compute the vertex correction and the Lamb shift we shall then be able to let $\lambda_1 > 0$ without contradicting the property that the photon is massless. This then can solve the infrared-divergence problem of QED.

We remark that if we choose other form of paths for connecting z_0 and z we can get other forms of photon propagator corresponding to a choice of gauge. From the property of gauge invariance the final result should not depend on the form of propagators. We shall see that this is achieved by renormalization. This property of renormalizable is as a property related to the gauge invariance. Indeed we notice that the quantum photon propagator with a photon-loop $W(z, z)$ attached to an electron represented by Z has already given the renormalized charge e (and the renormalized mass m of the electron) for the electromagnetic interaction.

It is clear that this renormalization by the quantum photon propagator with a photon-loop $W(z, z)$ is independent of the chosen photon propagator (because it does not need to choose a photon propagator). Thus the renormalization method as that in the conventional QED theory for the chosen of a photon propagator (corresponding to a choice of gauge) should give the observable result which does not depend on the form of the photon propagators since these two forms of renormalization must give the same effect of renormalization.

In the following Section and the Sections from Section 16 to Section 23 on Quantum Electrodynamics (QED) we shall investigate the renormalization method which is analogous to that of the conventional QED theory and the computation of QED effects by using this renormalization method.

15 Renormalization

In this Section and the following Sections from Section 16 to Section 23 on Quantum Electrodynamics (QED) we shall use the density (1.1) and the notations from this density where A_j, $j = 1, 2$ are real components of the photon field. Following the conventional QED theory let us consider the following renormalization:

$$A_j = z_A^{\frac{1}{2}} A_{jR}\,, \;\; j = 1, 2; \quad Z = z_Z^{\frac{1}{2}} Z_R\,; \qquad e_0 = \frac{z_e}{z_Z z_A^{\frac{1}{2}}}\, e = \frac{1}{n_e}\, e\,; \tag{94}$$

where z_A, z_Z, and z_e are renormalization constants to be determined and A_{Rj}, $j = 1, 2$, Z_R are renormalized fields. From this renormalization the density D of QED in (1.1) can be written in the following form:

$$\begin{aligned}
D &= \tfrac{1}{2}\, z_A \left(\tfrac{\partial A_{1R}}{\partial x^2} - \tfrac{\partial A_{2R}}{\partial x^1}\right)^* \left(\tfrac{\partial A_{1R}}{\partial x^2} - \tfrac{\partial A_{2R}}{\partial x^1}\right) + \\
&+ z_Z \left(\tfrac{dZ_R^*}{ds} + ie\,(\textstyle\sum_{j=1}^2 A_{jR}\tfrac{dx^j}{ds})Z_R^*\right) \times \\
&\times \left(\tfrac{dZ_R}{ds} - ie\,(\textstyle\sum_{j=1}^2 A_{jR}\tfrac{dx^j}{ds})Z_R\right) = \\
&= \Big\{ \tfrac{1}{2} \left(\tfrac{\partial A_{1R}}{\partial x^2} - \tfrac{\partial A_{2R}}{\partial x^1}\right)^* \left(\tfrac{\partial A_{1R}}{\partial x^2} - \tfrac{\partial A_{2R}}{\partial x^1}\right) + \\
&+ \tfrac{dZ_R^*}{ds}\tfrac{dZ_R}{ds} + \mu^2 Z_R^* Z_R - \mu^2 Z_R^* Z_R + \\
&+ i\,e\,(\textstyle\sum_{j=1}^2 A_{jR}\tfrac{dx^j}{ds})Z_R^*\tfrac{dZ_R}{ds} - \\
&- i\,e\,(\textstyle\sum_{j=1}^2 A_{jR}\tfrac{dx^j}{ds})\tfrac{dZ_R^*}{ds}Z_R + \\
&+ e^2(\textstyle\sum_{j=1}^2 A_{Rj}\tfrac{dx^j}{ds})^2 Z_R^* Z_R \Big\} + \\
&+ \Big\{ (z_A - 1)\big[\tfrac{1}{2}\left(\tfrac{\partial A_{1R}}{\partial x^2} - \tfrac{\partial A_{2R}}{\partial x^1}\right)^* \left(\tfrac{\partial A_{1R}}{\partial x^2} - \tfrac{\partial A_{2R}}{\partial x^1}\right)\big] + \\
&+ (z_Z - 1)\tfrac{dZ_R^*}{ds}\tfrac{dZ_R}{ds} + \\
&+ (z_e - 1)\big[+ie\,(\textstyle\sum_{j=1}^2 A_{jR}\tfrac{dx^j}{ds})Z_R^*\tfrac{dZ_R}{ds} - \\
&- i\,e(\textstyle\sum_{j=1}^2 A_{jR}\tfrac{dx^j}{ds})\tfrac{dZ_R^*}{ds}Z_R\big] + \\
&+ (\tfrac{z_e^2}{z_Z} - 1)e^2(\textstyle\sum_{j=1}^2 A_{jR}\tfrac{dx^j}{ds})^2 Z_R^* Z_R \Big\} := \\
&:= D_{phy} + D_{cnt}\,,
\end{aligned} \tag{95}$$

where D_{phy} is as the physical term and the D_{cnt} is as the counter term; and in D_{phy} the positive parameter μ is introduced for perturbation expansion and for renormalization.

Similar to that the Ward-Takahashi identities in the conventional QED theory are derived by the gauge invariance of the conventional QED theory; by using the gauge invariance of this QED theory we shall also derive the corresponding Ward-Takahashi identities for this QED theory in the Section on electron self-energy. From these Ward-Takahashi identities we then show that there exists a renormalization procedure such that $z_e = z_Z$; as similar to that in the conventional QED theory. From this relation $z_e = z_Z$ we then have:

$$e_0 = \frac{e}{z_A^{\frac{1}{2}}} = \frac{1}{n_e}\, e \tag{96}$$

and that in (95) we have $\frac{z_e^2}{z_Z} - 1 = z_e - 1$.

16 Feynman diagrams and Feynman rules for QED

Let us then transform ds in (1.1) to $\frac{1}{(\beta+ih)}ds$ where $\beta, h > 0$ are parameters and h is as the Planck constant. The parameter h will give the dynamical effects of QED (as similar to the conventional QED). Here for simplicity we only consider the limiting case that $\beta \to 0$ and we let $h = 1$. From this transformation we get the Lagrangian $\mathcal{L}$ from $-\int_{s_0}^{s_1} D ds$ changing to $\int_{s_0}^{s_1} \mathcal{L} ds$. Then we write $\mathcal{L} = \mathcal{L}_{phy} + \mathcal{L}_{cnt}$ where $\mathcal{L}_{phy}$ corresponds to D_{phy} and $\mathcal{L}_{cnt}$ corresponds to D_{cnt}. Then from the following term in $\mathcal{L}_{phy}$:

$$-i\left[\left(\frac{dZ_R}{ds}\right)^* \frac{dZ_R}{ds} - \mu^2 Z_R^* Z_R\right] \tag{97}$$

and by the perturbation expansion of $e^{\int_{s_0}^{s_1} \mathcal{L} ds}$ we have the following propagator:

$$\frac{i}{p_E^2 - \mu^2} \tag{98}$$

which is as the (primitive) electron propagator where p_E denotes the proper energy variable of electron.

Then from the pure gauge part of $\mathcal{L}_{phy}$ we get the photon propagator (93), as done in the above Sections and the Section on photon propagator.

Then from $\mathcal{L}_{phy}$ we have the following seagull vertex term:

$$ie^2\left(\sum_{j=1}^{2} A_{jR}\frac{dx^j}{ds}\right)^2 Z_R^* Z_R\,. \tag{99}$$

This seagull vertex term gives the vertex factor ie^2. (We remark that the ds of the paths $\frac{dx^j}{ds}$ are not transformed to $-ids$ since these paths are given paths and thus are independent of the transformation $ds \to -ids$.)

From this vertex by using the photon propagator (93) in the above Section we get the following term:

$$\frac{ie^2}{2\pi}\int \frac{i\,dk_E}{k_E^2 - \lambda_1^2} = -\frac{ie^2}{2\lambda_1} =: -i\,\omega^2. \tag{100}$$

The parameter ω is regarded as the mass-energy parameter of electron. Then from the perturbation expansion of $e^{\int_{s_0}^{s_1} \mathcal{L} ds}$ we have the following geometric series (which is similar to the Dyson series in the conventional QED):

$$\begin{aligned} &\tfrac{i}{p_E^2-\mu^2} + \tfrac{i}{p_E^2-\mu^2}(-i\omega^2 + i\mu^2)\tfrac{i}{p_E^2-\mu^2} + \cdots = \\ &= \tfrac{i}{p_E^2-\mu^2-\omega^2+\mu^2} = \tfrac{i}{p_E^2-\omega^2}\,, \end{aligned} \tag{101}$$

where the term $i\mu$ of $-i\,\omega^2 + i\mu^2$ is the $i\mu$ term in $\mathcal{L}_{phy}$. (The other term $-i\mu$ in $\mathcal{L}_{phy}$ has been used in deriving (98).) Thus we have the following electron propagator:

$$\frac{i}{p_E^2 - \omega^2}\,. \tag{102}$$

This is as the electron propagator with mass-energy parameter ω. From ω we shall get the mass m of electron. (We shall later introduce a space-time statistics to get the usual electron propagator of the Dirac equation. This usual electron propagator is as the statistical electron propagator.) As the Feynman diagrams in the conventional QED we represent this electron propagator by a straight line.

In the above Sections and the Section on the photon propagator we see that the photon-loop $W(z, z)$ gives the renormalized charge $e = n_e e_0$ and the renormalized mass m of electron from the bare charge e_0 by the winding numbers of the photon loop such that m is with the winding number factor n_e. Then we see that the above one-loop energy integral of the photon gives the mass-energy parameter ω of electron which gives the mass m of electron. Thus these two types of photon-loops are closely related (from the relation of photon propagator and quantum photon propagator) such that the mass m obtained by the winding numbers of the photon loop $W(z, z)$ reappears in the one-loop energy integral (100) of the photon.

Thus we see that even there is no mass term in the Lagrangian of this gauge theory the mass m of the electron can come out from the gauge theory. This actually resolves the mass problem of particle physics that particle can be with mass even without the mass term. Thus we do not need the Higgs mechanism for generating masses to particles.

On the other hand from the one-loop-electron form of the seagull vertex we have the following term:

$$\frac{ie^2}{2\pi}\int \frac{idp_E}{p_E^2 - \mu^2} = -\frac{ie^2}{2\mu} =: -i\lambda_2^2\,. \tag{103}$$

So for photon from the perturbation expansion of $e^{\int_{s_0}^{s_1} \mathcal{L} ds}$ we have the following geometric series:

$$\begin{aligned} &\tfrac{i}{k_E^2-\lambda_1^2} + \tfrac{i}{k_E^2-\lambda_1^2}(-i\lambda_2^2)\tfrac{i}{k_E^2-\lambda_1^2} + \cdots = \\ &= \tfrac{i}{k_E^2-\lambda_1^2-\lambda_2^2} =: \tfrac{i}{k_E^2-\lambda_0^2}\,, \end{aligned} \tag{104}$$

where we define $\lambda_0^2 = \lambda_1^2 + \lambda_2^2$. Thus we have the following photon propagator:

$$\frac{i}{k_E^2 - \lambda_0^2}\,, \tag{105}$$

which is of the same form as (93) where we replace λ_1 with λ_0. As the Feynman diagrams in the conventional QED we represent this photon propagator by a wave line.

Then the following interaction term in $\mathcal{L}_{phy}$:

$$\begin{aligned} &-ie\tfrac{dZ_R^*}{ds}\left(\textstyle\sum_{j=1}^{2} A_{jR}\tfrac{dx^j}{ds}\right)Z_R + \\ &\qquad + ie\tfrac{dZ_R}{ds}\left(\textstyle\sum_{j=1}^{2} A_{jR}\tfrac{dx^j}{ds}\right)Z_R^* \end{aligned} \tag{106}$$

gives the vertex factor $(-ie)(p_E + q_E)$ which corresponds to the usual vertex of Feynman diagram with two electron straight lines (with energies p_E and q_E) and one photon wave line in the conventional QED.

Then as the Feynman rules in the conventional QED a sign factor $(-1)^n$, where n is the number of the electron loops in a Feynman diagram, is to be included for the Feynman diagram.

17 Statistics with space-time

Let us introduce space-time as a statistical method for a large amount of basic variables Z_R and A_{1R}, A_{2R}. As an illustration let us consider the electron propagator $\frac{i}{p_E^2-\omega^2}$ and the following Green's function corresponding to it:

$$\frac{i}{2\pi}\int\frac{e^{-ip_E(s-s')}dp_E}{p_E^2-\omega^2}\,, \tag{107}$$

where s is the proper time.

We imagine each electron (and photon) occupies a space region (This is the creation of the concept of space which is associated to the electron. Without the electron this space region does not exist). Then we write

$$p_E(s-s') = p_E(t-t') - \mathbf{p}(\mathbf{x}-\mathbf{x}')\,, \tag{108}$$

where $\mathbf{p}(\mathbf{x}-\mathbf{x}')$ denotes the inner product of the three dimensional vectors $\mathbf{p}$ and $\mathbf{x}-\mathbf{x}'$ and $(t,\mathbf{x})$ is the time-space coordinate where $\mathbf{x}$ is in the space region occupied by $Z_R(s)$ and that

$$\omega^2-\mathbf{p}^2 = m^2 > 0\,, \tag{109}$$

where m is the mass of electron. This mass m is greater than 0 since each Z_R occupies a space region which implies that when $t-t'$ tends to 0 we can have that $|\mathbf{x}-\mathbf{x}'|$ does not tend to 0 ($\mathbf{x}$ and $\mathbf{x}'$ denote two coordinate points in the regions occupied by $Z_R(s)$ and $Z_R(s')$ respectively) and thus (109) holds. Then by linear summing the effects of a large amount of basic variables Z_R and letting ω varies from m to ∞ from (107), (108) and (109) we get the following statistical expression:

$$\frac{i}{(2\pi)^4}\int\frac{e^{-ip(x-x')}dp}{p^2-m^2}\,, \tag{110}$$

which is the usual Green's function of a free field with mass m where p is a four vector and $x=(t,\mathbf{x})$.

The result of the above statistics is that (110) is induced from (107) with the scalar product p_E^2 of a scalar p_E changed to an indefinite inner product p^2 of a four vector p and the parameter ω is reduced to m.

Let us then introduce Fermi-Dirac statistics for electrons. As done by Dirac for deriving the Dirac equation we factorize p^2-m^2 into the following form:

$$\begin{aligned} p^2-m^2 &= (p_E-\omega)(p_E+\omega) = \\ &= (\gamma_\mu p^\mu - m)(\gamma_\mu p^\mu + m)\,, \end{aligned} \tag{111}$$

where γ_μ are the Dirac matrices. Then from (110) we get the following Green's function:

$$\begin{aligned} &\tfrac{i}{(2\pi)^4}\int e^{-ip(x-x')}\tfrac{\gamma_\mu p^\mu+m}{p^2-m^2}dp = \\ &= \tfrac{i}{(2\pi)^4}\int\tfrac{e^{-ip(x-x')}dp}{\gamma_\mu p^\mu - m}\,. \end{aligned} \tag{112}$$

Thus we have the Fermi-Dirac statistics that the statistical electron propagator is of the form $\frac{i}{\gamma_\mu p^\mu-m}$ which is the propagator of the Dirac equation and is the electron propagator of the conventional QED.

Let us then consider statistics of photons. Since the above quantum gauge theory of photons is a gauge theory which is gauge invariant we have that the space-time statistical equation for photons should be gauge invariant. Then since the Maxwell equation is the only gauge invariant equation for electromagnetism which is based on the space-time we have that the Maxwell equation must be a statistical equation for photons.

Then let us consider the vertexes. The tree vertex (106) with three lines (one for photon and two for electron) gives the factor $-ie(p_E+q_E)$ where p_E and q_E are from the factor $\frac{dZ_R}{ds}$ for electron.

We notice that this vertex is with two electron lines (or electron propagator) and one photon line (or photon propagator). In doing a statistics on this photon line when it is as an external electromagnetic field on the electron this photon line is of the statistical form $\gamma_\mu A^\mu$ where A^μ denotes the four electromagnetic potential fields of the Maxwell equation. Thus the vertex $-ie(p_E+q_E)$ after statistics is changed to the form $-ie(p_E+q_E)\frac{\gamma^\mu}{2}$ where for each γ^μ a factor $\frac{1}{2}$ is introduced for statistics.

Let us then introduce the on-mass-shell condition as in the conventional QED theory (see [6]). As similar to the on-mass-shell condition in the conventional QED theory our on-mass-shell condition is that $p_E = m$ where m is the observable mass of the electron. In this case $-ie(p_E+q_E)\frac{\gamma^\mu}{2}$ is changed to $-iem\gamma^\mu$. Then the m is absorbed to the two external spinors $\frac{1}{\sqrt{E}}u$ (where E denotes the energy of the electron satisfied the Dirac equation while the E of p_E is only as a notation) of the two electrons lines attached to this vertex such that the factor $\frac{1}{\sqrt{E}}$ of spin 0 of the Klein-Gordon equation is changed to the factor $\sqrt{\frac{m}{E}}$ of spinors of the Dirac equation. In this case we have the magnitude of p_E and q_E reappears in the two external electron lines with the factor $\sqrt{m}$. The statistical vertex then becomes $-ie\gamma^\mu$. This is exactly the usual vertex in the conventional QED. Thus after a space-time statistics on the original vertex $-ie(p_E+q_E)$ we get the statistical vertex $-ie\gamma^\mu$ of the conventional QED.

18 Basic effects of Quantum Electrodynamics

To illustrate this new theory of QED let us compute the three basic effects of QED: the one-loop photon and electron self-energies and the one-loop vertex correction.

As similar to the conventional QED we have the Feynman rules such that the one-loop photon self-energy is given by the following Feynman integral:

$$i\Pi_0(k_E) := i^2(-i)^2\frac{e^2}{2\pi}\times \\ \times\int\frac{(2p_E+k_E)(2p_E+k_E)dp_E}{(p_E^2-\omega^2)((k_E+p_E)^2-\omega^2)}, \tag{113}$$

where e is the renormalized electric charge.

Then as the Feynman rules in the conventional QED for the space-time statistics a statistical sign factor $(-1)^j$, where j is the number of the electron loops in a Feynman diagram, will be included for the Feynman diagram. Thus for the one-loop photon self-energy (113) a statistical factor $(-1)^j$ will be introduced to this one-loop photon self-energy integral.

Then similarly we have the Feynman rules such that the one-loop electron self-energy is given by the following Feynman integral:

$$-i\Sigma_0(p_E) := i^2(-i)^2\frac{e^2}{2\pi}\times \\ \times\int\frac{(2p_E-k_E)(2p_E-k_E)dk_E}{(k_E^2-\lambda_0^2)((p_E-k_E)^2-\omega^2)}. \tag{114}$$

Similarly we have the Feynman rules that the one-loop vertex correction is given by the following Feynman integral:

$$(-ie)\Lambda_0(p_E,q_E) := \\ := (i)^3(-i)^3\frac{e^3}{2\pi}\int\frac{(2p_E-k_E)(2q_E-k_E)(p_E+q_E-2k_E)dk_E}{((p_E-k_E)^2-\omega^2)((q_E-k_E)^2-\omega^2)(k_E^2-\lambda_0^2)}. \tag{115}$$

Let us first compute the one-loop vertex correction and then compute the photon self-energy and the electron self-energy.

As a statistics we extend the one dimensional integral $\int dk_E$ to the n-dimensional integral $\int d^nk$ $(n\to 4)$ where $k=(k_E,\mathbf{k})$. This is similar to the dimensional regularization in the conventional quantum field theories (However here our aim is to increase the dimension for statistics which is different from the dimensional regularization which is to reduce the dimension from 4 to n to avoid the ultraviolet divergence). With this statistics the factor 2π is replaced by the statistical factor $(2\pi)^n$. From this statistics on (115) we have the following statistical one loop vertex correction:

$$\frac{e^3}{(2\pi)^n}\int_0^1 dx\int_0^1 2ydy\int d^nk\times \\ \times\frac{4p_Eq_E(p_E+q_E)-2k_E((p_E+q_E)^2+4p_Eq_E)+5k_E^2(p_E+q_E)-2k_E^3}{[k^2-2k(pxy+q(1-x)y)-p_E^2xy-q_E^2(1-x)y+m^2y+\lambda^2(1-y)]^3}, \tag{116}$$

where $k^2=k_E^2-\mathbf{k}^2$, and $\mathbf{k}^2$ is from the free parameters ω, λ_0 where we let $\omega^2=m^2+\mathbf{k}^2$, $\lambda_0^2=\lambda^2+\mathbf{k}^2$ for the electron mass m and a mass-energy parameter λ for photon; and:

$$\left.\begin{aligned} &k(pxy+q(1-x)y) := k_E(p_Exy+q_E(1-x)y)\\ &-\mathbf{k}\cdot\mathbf{0} = k_E(p_Exy+q_E(1-x)y)\end{aligned}\right\}. \tag{117}$$

By using the formulae for computing Feynman integrals we have that (116) is equal to (see [6, 72]):

$$\begin{aligned}&\frac{ie^3}{(2\pi)^n}\int_0^1 dx\int_0^1 2ydy\times\\ &\times\left[\frac{4p_Eq_E(p_E+q_E)\pi^{\frac{n}{2}}\Gamma(3-\frac{n}{2})}{\Gamma(3)(\Delta-r^2)^{3-2}}\frac{1}{(-\Delta+r^2)^{2-\frac{n}{2}}}-\right.\\ &-\frac{2((p_E+q_E)^2+4p_Eq_E)\pi^{\frac{n}{2}}\Gamma(3-\frac{n}{2})r}{\Gamma(3)(\Delta-r^2)^{3-2}}\frac{1}{(-\Delta+r^2)^{2-\frac{n}{2}}}+\\ &+\frac{5(p_E+q_E)\pi^{\frac{n}{2}}\Gamma(3-1-\frac{n}{2})\frac{n}{2}}{\Gamma(3)(\Delta-r^2)^{3-2-1}}\frac{1}{(-\Delta+r^2)^{2-\frac{n}{2}}}+\\ &+\frac{5(p_E+q_E)\pi^{\frac{n}{2}}\Gamma(3-\frac{n}{2})r^2}{\Gamma(3)(\Delta-r^2)^{3-2}}\frac{1}{(-\Delta+r^2)^{2-\frac{n}{2}}}-\\ &-\frac{\frac{(n+2)}{2}2\pi^{\frac{n}{2}}\Gamma(3-1-\frac{n}{2})r}{\Gamma(3)(\Delta-r^2)^{3-2-1}}\frac{1}{(-\Delta+r^2)^{2-\frac{n}{2}}}-\\ &\left.-\frac{2\pi^{\frac{n}{2}}\Gamma(3-\frac{n}{2})r^3}{\Gamma(3)(\Delta-r^2)^{3-2}}\frac{1}{(-\Delta+r^2)^{2-\frac{n}{2}}}\right]=:\\ &=: (-ie)\Lambda(p_1,p_2),\end{aligned} \tag{118}$$

where we define:

$$\left.\begin{aligned} &r := p_Exy+q_E(1-x)y\\ &\Delta := p_E^2xy+q_E^2(1-x)y-m^2y-\lambda^2(1-y)\end{aligned}\right\}. \tag{119}$$

We remark that in this statistics the p_E and q_E variables are remained as the proper variables which are derived from the proper time s.

Let us then introduce the Fermi-Dirac statistics on the electron and we consider the on-mass-shell case as in the conventional QED. We shall see this will lead to the theoretical results of the conventional QED on the anomalous magnetic moment and the Lamb shift.

As a Fermi-Dirac statistics we have shown in the above Section that the vertex term $-ie(p_E+q_E)$ is replaced with the vertex term $-ie(p_E+q_E)\frac{\gamma^\mu}{2}$. Then as a Fermi-Dirac statistics in the above Section we have shown that the statistical vertex is $-ie\gamma^\mu$ under the on-mass-shell condition. We notice that this vertex agrees with the vertex term in the conventional QED theory.

Let us then consider the Fermi-Dirac statistics on the one-loop vertex correction (118). Let us first consider the following term in (118):

$$\frac{ie^3}{(2\pi)^n}\int_0^1 dx\int_0^1 2ydy\times \\ \times\frac{\pi^2(p_E+q_E)4p_Eq_E}{\Gamma(3)(\Delta-r^2)^{3-2}}\frac{1}{(-\Delta+r^2)^{2-\frac{n}{2}}}, \tag{120}$$

where we can (as an approximation) let $n=4$. From Fermi-Dirac statistics we have that this term gives the following statistics:

$$\frac{ie^3}{(2\pi)^4}\int_0^1 dx\int_0^1 2ydy\,\frac{\pi^2(p_E+q_E)\frac{1}{2}\gamma^\mu 4p_Eq_E}{\Gamma(3)(\Delta-r^2)^{3-2}}. \tag{121}$$

Then we consider the case of on-mass-shell. In this case we have $p_E=m$ and $q_E=m$. Thus from (121) we have the following term:

$$\frac{ie^3}{(2\pi)^4}\int_0^1 dx\int_0^1 2ydy\,\frac{\pi^2\gamma^\mu 4p_E\,q_E}{\Gamma(3)(\Delta-r^2)^{3-2}}\,, \qquad (122)$$

where a mass factor $m=\frac{1}{2}(p_E+q_E)$ has been omitted and put to the external spinor of the external electron as explained in the above Section on space-time statistics. In (122) we still keep the expression $p_E\,q_E$ even though in this case of on-mass-shell because this factor will be important for giving the observable Lamb shift, as we shall see. In (122) because of on-mass-shell we have (as an approximation we let $n=4$):

$$(\Delta-r^2)^{3-2}=-\lambda^2(1-y)-r^2= \\ -\lambda^2(1-y)-m^2y^2\,. \qquad (123)$$

Thus in the on-mass-shell case (122) is of the following form:

$$ie\gamma^\mu\frac{\alpha}{\pi}\int_0^1 dx\int_0^1 ydy\,\frac{\pi^2 p_E\,q_E}{-\lambda^2(1-y)-m^2y^2}\,, \qquad (124)$$

where $\alpha=\frac{e^2}{4\pi}$ is the fine structure constant. Carrying out the integrations on y and on x we have that as $\lambda\to 0$ (124) is equal to:

$$(-ie)\,\gamma^\mu\frac{\alpha}{\pi}\frac{p_E\,q_E}{m^2}\log\frac{m}{\lambda}\,, \qquad (125)$$

where the proper factor $p_E\,q_E$ will be for a linear space-time statistics of summation. We remark that (125) corresponds to a term in the vertex correction in the conventional QED theory with the infra-divergence when $\lambda=0$ (see [6]). Here since the parameter λ has not been determined we shall later find other way to determine the effect of (125) and to solve the infrared-divergence problem.

Let us first rewrite the form of the proper value $p_E\,q_E$. We write $p_E\,q_E$ in the following space-time statistical form:

$$p_E\,q_E=-2p'\cdot p\,, \qquad (126)$$

where p and p' denote two space-time four-vectors of electron such that $p^2=m^2$ and $p'^2=m^2$. Then we have

$$\begin{aligned} p_E\,q_E &= \\ &=\tfrac{1}{3}\,(p_E\,q_E+p_E\,q_E+p_E\,q_E)=\tfrac{1}{3}\,(m^2-2p'\cdot p+m^2) \\ &=\tfrac{1}{3}\,(m^2-2p'\cdot p+m^2)=\tfrac{1}{3}\,(p'^2-2p'\cdot p+p^2) \\ &=\tfrac{1}{3}\,(p'-p)^2 \\ &=:\ \tfrac{1}{3}\,q^2, \end{aligned} \qquad (127)$$

where following the convention of QED we define $q=p'-p$. Thus from (125) we have the following term:

$$(-ie)\gamma^\mu\frac{\alpha}{3\pi}\frac{q^2}{m^2}\log\frac{m}{\lambda}\,, \qquad (128)$$

where the parameter λ are to be determined. Again this term (128) corresponds to a term in the vertex correction in the conventional QED theory with the infrared-divergence when $\lambda=0$ (see [6]).

Let us then consider the following term in (118):

$$\frac{-ie^3}{(2\pi)^n}\int_0^1 dx\,\times \\ \times\int_0^1\frac{2((p_E+q_E)^2+4p_E\,q_E)\pi^{\frac{n}{2}}r2ydy}{\Gamma(3)(\Delta-r^2)^{3-2}(-\Delta+r^2)^{2-\frac{n}{2}}}\,. \qquad (129)$$

For this term we can (as an approximation) also let $n=4$ and we have let $\Gamma\,(3-\frac{n}{2})=1$. As similar to the conventional QED theory we want to show that this term gives the anomalous magnetic moment and thus corresponds to a similar term in the vertex correction of the conventional QED theory (see [6]).

By Fermi-Dirac statistics the factor (p_E+q_E) in (129) of $(p_E+q_E)^2$ gives the statistical term $(p_E+q_E)\frac{1}{2}\gamma^\mu$. Thus with the on-mass-shell condition the factor (p_E+q_E) gives the statistical term $m\gamma^\mu$. Thus with the on-mass-shell condition the term $(p_E+q_E)^2$ gives the term $m\gamma^\mu(p_E+q_E)$. Then the factor (p_E+q_E) in this statistical term also give $2m$ by the on-mass-shell condition. Thus by Fermi-Dirac statistics and the on-mass-shell condition the factor $(p_E+q_E)^2$ in (129) gives the statistical term $\gamma^\mu 2m^2$. Then since this is a (finite) constant term it can be cancelled by the corresponding counter term of the vertex giving the factor $-ie\gamma^\mu$ and having the factor z_e-1 in (95). From this cancellation the renormalization constant z_e is determined. Since the constant term is depended on the $\delta>0$ which is introduced for space-time statistics we have that the renormalization constant z_e is also depended on the $\delta>0$. Thus the renormalization constant z_e (and the concept of renormalization) is related to the space-time statistics.

At this point let us give a summary of this renormalization method, as follows.

Renormalization

1. The renormalization method of the conventional QED theory is used to obtain the renormalized physical results. Here unlike the conventional QED theory the renormalization method is not for the removing of ultraviolet divergences since the QED theory in this paper is free of ultraviolet divergences.

2. We have mentioned in the above Section on photon propagator that the property of renormalizable is a property of gauge invariance that it gives the physical results independent of the chosen photon propagator.

3. The procedure of renormalization is as a part of the space-time statistics to get the statistical results which is independent of the chosen photon propagator. ⋄

Let us then consider again the above computation of the one-loop vertex correction. We now have that the (finite) constant term of the one-loop vertex correction is cancelled by the corresponding counter term with the factor z_e-1 in (95).

Thus the nonconstant term (128) is renormalized to be the following renormalized form:

$$(-ie)\,\gamma^{\mu}\,\frac{\alpha}{3\pi}\,\frac{q^2}{m^2}\,\log\frac{m}{\lambda}\,. \tag{130}$$

Let us then consider the following term in (129):

$$\frac{-ie^3}{(2\pi)^n}\int_0^1 dx\int_0^1 2y\,dy\,\frac{8p_E\,q_E\,\pi^{\frac{n}{2}}\,\Gamma(3-\frac{n}{2})r}{\Gamma(3)(\Delta-r^2)^{3-\frac{n}{2}}}\,, \tag{131}$$

where we can (as an approximation) let $n=4$. With the on-mass-shell condition we have that $\Delta - r^2$ is again given by (123). Then letting $\lambda = 0$ we have that (131) is given by:

$$\frac{-ie\alpha}{4\pi}\int_0^1 dx\int_0^1 y\,dy\,.\,\frac{8p_E\,q_E}{-r} \tag{132}$$

With the on-mass-shell condition we have $r = my$. Thus this term (132) is equal to:

$$(-ie)\frac{-\alpha}{4\pi m}\,8p_E\,q_E\,. \tag{133}$$

Again the factor $p_E\,q_E$ is for the exchange of energies for two electrons with proper energies p_E and q_E respectively and thus it is the vital factor. This factor is then for the space-time statistics and later it will be for a linear statistics of summation for the on-mass-shell condition. Let us introduce a space-time statistics on the factor $p_E\,q_E$, as follows. With the on-mass-shell condition we write $p_E\,q_E$ in the following form:

$$p_E\,q_E = \frac{1}{2}\,(mp_E + q_E m) = \frac{1}{2}\,m\,(p_E + q_E)\,. \tag{134}$$

Then we introduce a space-time statistics on the proper energies p_E and q_E respectively that p_E gives a statistics βp and q_E gives a statistics $\beta p'$ where p and p' are space-time four vectors such that $p^2 = m^2$; $p'^2 = m^2$; and β is a statistical factor to be determined.

Then we have the following Gordan relation on the space-time four vectors p and p' respectively (see [6] [72]):

$$\left.\begin{aligned} p^{\mu} &= \gamma^{\mu}(p\cdot\gamma) + i\sigma^{\mu\nu}p_{\nu} \\ p^{\mu\prime} &= (p'\cdot\gamma)\gamma^{\mu} - i\sigma^{\mu\nu}p'_{\nu} \end{aligned}\right\}, \tag{135}$$

where p^{μ} and $p^{\mu\prime}$ denote the four components of p and p' respectively. Thus from (134) and the Gordan relation (135) we have the following space-time statistics:

$$\begin{aligned} &\tfrac{1}{2}\,(mp_E + q_E m) = \\ &= \tfrac{1}{2}\,m\,\beta(\gamma^{\mu}(p\cdot\gamma) + (p'\cdot\gamma)\gamma^{\mu} - i\sigma^{\mu\nu}q_{\nu})\,, \end{aligned} \tag{136}$$

where following the convention of QED we define $q = p' - p$.

From (136) we see that the space-time statistics on p_E for giving the four vector p needs the product of two Dirac γ-matrices. Then since the introducing of a Dirac γ-matrix for space-time statistics requires a statistical factor $\frac{1}{2}$ we have that the statistical factor $\beta = \frac{1}{4}$.

Then as in the literature on QED when evaluated between polarization spinors, the $p'\cdot\gamma$ and $\gamma\cdot p$ terms are deduced to the mass m respectively. Thus the term $\frac{1}{2}m\beta(\gamma^{\mu}p\cdot\gamma + p'\cdot\gamma\gamma^{\mu})$ as a constant term can be cancelled by the corresponding counter term with the factor $z_e - 1$ in (95).

Thus by space-time statistics on $p_E\,q_E$ from (133) we get the following vertex correction:

$$(-ie)\,\frac{i\alpha}{4\pi m}\,\sigma^{\mu\nu}q_{\nu} \tag{137}$$

where $q = p - p'$ and the factor 8 in (133) is cancelled by the statistical factor $\frac{1}{2}\beta = \frac{1}{8}$. We remark that in the way of getting (137) a factor m has been absorbed by the two polarization spinors u to get the form $\sqrt{\frac{m}{E}u}$ of the spinors of external electrons.

Then from (137) we get the following exact second order magnetic moment:

$$\frac{\alpha}{2\pi}\,\mu_0\,, \tag{138}$$

where $\mu_0 = \frac{1}{2m}$ is the Dirac magnetic moment as in the literature on QED (see [6]).

We see that this result is just the second order anomalous magnetic moment obtained from the conventional QED (see [6] [72]- [78]). Here we can obtain this anomalous magnetic moment exactly while in the conventional QED this anomalous magnetic moment is obtained only by approximation under the condition that $|q^2| \ll m^2$. The point is that we do not need to carry out a complicate integration as in the literature in QED when the on-mass-shell condition is applied to the proper energies p_E and q_E, and with the on-mass-shell condition applied to the proper energies p_E and q_E the computation is simple and the computed result is the exact result of the anomalous magnetic moment.

Let us then consider the following terms in the one-loop vertex correction (118):

$$\begin{aligned} &\tfrac{ie^3}{(2\pi)^n}\int_0^1 dx\int_0^1 2y\,dy\;\times \\ &\times\Big[\tfrac{5(p_E+q_E)\pi^{\frac{n}{2}}\Gamma(3-1-\frac{n}{2})\frac{n}{2}}{\Gamma(3)(\Delta-r^2)^{3-2-1}}\,\tfrac{1}{(-\Delta+r^2)^{2-\frac{n}{2}}} + \\ &+\tfrac{5(p_E+q_E)\pi^{\frac{n}{2}}\Gamma(3-\frac{n}{2})r^2}{\Gamma(3)(\Delta-r^2)^{3-2}}\,\tfrac{1}{(-\Delta+r^2)^{2-\frac{n}{2}}} - \\ &-\tfrac{\frac{(n+2)}{2}2\pi^{\frac{n}{2}}\Gamma(3-1-\frac{n}{2})r}{\Gamma(3)(\Delta-r^2)^{3-2-1}}\,\tfrac{1}{(-\Delta+r^2)^{2-\frac{n}{2}}} - \\ &-\tfrac{2\pi^{\frac{n}{2}}\Gamma(3-\frac{n}{2})r^3}{\Gamma(3)(\Delta-r^2)^{3-2}}\,\tfrac{1}{(-\Delta+r^2)^{2-\frac{n}{2}}}\Big]. \end{aligned} \tag{139}$$

From the on-mass-shell condition we have $\Delta - r^2 = -r^2$ where we have set $\lambda = 0$. The first and the second term are with the factor $(p_E + q_E)$ which by Fermi-Dirac statistics gives the statistics $(p_E + q_E)\frac{1}{2}\gamma^\mu$. Then from the following integration:

$$\int_0^1 dx \int_0^1 2yr dy = \\ = \int_0^1 dx \int_0^1 2y^2(p_E x + (1-x)q_E)dy \tag{140}$$

we get a factor $(p_E + q_E)$ for the third and fourth terms. Thus all these four terms by Fermi-Dirac statistics are with the statistics $(p_E + q_E)\frac{1}{2}\gamma^\mu$. Then by the on-mass-shell condition we have that the statistics $(p_E + q_E)\frac{1}{2}\gamma^\mu$ gives the statistics $m\gamma^\mu$. Thus (139) gives a statistics which is of the form $(\gamma^\mu \cdot \text{constant})$. Thus this constant term can be cancelled by the corresponding counter term with the factor $z_e - 1$ in (95).

Thus under the on-mass-shell condition the renormalized vertex correction $(-ie)\Lambda_R(p', p)$ from the one-loop vertex correction is given by the sum of (128) and (137):

$$(-ie)\Lambda_R(p', p) = \\ = (-ie)\left[\gamma^\mu \frac{\alpha}{3\pi}\frac{q^2}{m^2}\log\frac{m}{\lambda} + \frac{i\alpha}{4\pi m}\sigma^{\mu\nu}q_\nu\right]. \tag{141}$$

19 Computation of the Lamb shift: Part I

The above computation of the vertex correction has not been completed since the parameter λ has not been determined. This appearance of the nonzero λ is due to the on-mass-shell condition. Let us in this Section complete the above computation of the vertex correction by finding another way to get the on-mass-shell condition. By this completion of the above computation of the vertex correction we are then able to compute the Lamb shift.

As in the literature of QED we let $\omega_{\min}$ denote the minimum of the (virtual) photon energy in the scatting of electron. Then as in the literature of QED we have the following relation between $\omega_{\min}$ and λ when $\frac{v}{c} \ll 1$ where v denotes the velocity of electron and c denotes the speed of light (see [6, 68–74]):

$$\log 2\omega_{\min} = \log\lambda + \frac{5}{6}. \tag{142}$$

Thus from (141) we have the following form of the vertex correction:

$$(-ie)\gamma^\mu \frac{\alpha}{3\pi}\frac{q^2}{m^2}\left[\log\frac{m}{2\omega_{\min}} + \frac{5}{6}\right] + \\ + (-ie)\gamma^\mu \frac{ie\alpha i\sigma^{\mu\nu}q_\nu}{4\pi m}. \tag{143}$$

Let us then find a way to compute the following term in the vertex correction (143):

$$(-ie)\gamma^\mu \frac{\alpha}{3\pi}\frac{q^2}{m^2}\log\frac{m}{2\omega_{\min}}. \tag{144}$$

The parameter $2\omega_{\min}$ is for the exchanging (or shifting) of the proper energies p_E and q_E of electrons. Thus the magnitudes of p_E and q_E correspond to the magnitude of $\omega_{\min}$. When the $\omega_{\min}$ is chosen the corresponding p_E and q_E are also chosen and vise versa.

Since $\omega_{\min}$ is chosen to be very small we have that the corresponding proper energies p_E and q_E are very small that they are no longer equal to the mass m for the on-mass-shell condition and they are for the virtual electrons. Then to get the on-mass-shell condition we use a linear statistics of summation on the vital factor $p_E q_E$. This means that the large amount of the effects $p_E q_E$ of the exchange of the virtual electrons are to be summed up to statistically getting the on-mass-shell condition.

Thus let us consider again the one-loop vertex correction (118) where we choose p_E and q_E such that $p_E \ll m$ and $q_E \ll m$. This chosen corresponds to the chosen of $\omega_{\min}$. We can choose p_E and q_E as small as we want such that $p_E \ll m$ and $q_E \ll m$. Thus we can let $\lambda = 0$ and set $p_E = q_E = 0$ for the p_E and q_E in the denominators $(\Delta - r^2)^{3-2}$ in (118). Thus (118) is approximately equal to:

$$\frac{ie^3}{(2\pi)^n}\int_0^1 dx\int_0^1 dy\left[\frac{4p_E q_E(p_E+q_E)\pi^{\frac{n}{2}}\Gamma(3-2)}{-m^2} - \right. \\ - \frac{2((p_E+q_E)^2+4p_E q_E)\pi^{\frac{n}{2}}\Gamma(3-2)r}{-m^2} + \\ + \frac{5(p_E+q_E)\pi^{\frac{n}{2}}\Gamma(2-\frac{n}{2})\frac{n}{2}}{(-\Delta+r^2)^{2-\frac{n}{2}}} + \frac{5(p_E+q_E)\pi^{\frac{n}{2}}\Gamma(3-2)r^2}{-m^2} - \\ \left. - \frac{\frac{(n+2)}{2}2\pi^{\frac{n}{2}}\Gamma(2-\frac{n}{2})r}{(\Delta-r^2)^{2-\frac{n}{2}}} + \frac{2\pi^{\frac{n}{2}}\Gamma(3-2)r^3}{-m^2}\right]. \tag{145}$$

Let us then first consider the four terms in (145) without the factor $\Gamma(2-\frac{n}{2})$. For these four terms we can (as an approximation) let $n=4$. Carry out the integrations $\int_0^1 dx\int_0^1 ydy$ of these four terms we have that the sum of these four terms is given by:

$$(ie)\frac{\alpha\pi^2}{4\pi^3 m^2}\left[4p_E q_E(p_E + q_E) - \right. \\ - \frac{1}{2}((p_E + q_E)^2 + 4p_E q_E)(p_E + q_E) + \\ + \frac{5}{9}(p_E + q_E)(p_E^2 + q_E^2 + p_E q_E) - \\ \left. - \frac{1}{8}(p_E^3 + q_E^3 + p_E^2 q_E + p_E q_E^2)\right] = \\ = (ie)\frac{\alpha\pi^2}{4\pi^3 m^2}(p_E + q_E)\left[\frac{5}{72}p_E^2 + \frac{5}{72}q_E^2 - \frac{14}{9}p_E q_E\right], \tag{146}$$

where the four terms of the sum are from the corresponding four terms of (145) respectively.

Then we consider the two terms in (145) with the factor $\Gamma(2-\frac{n}{2})$. Let $\delta := 2 - \frac{n}{2} > 0$. We have:

$$\Gamma(\delta)\cdot(-\Delta + r^2)^{-\delta} = \tag{147}$$

$$= \left(\frac{1}{\delta} + \text{a finite limit term as } \delta \to 0\right)\cdot e^{-\delta\log(-\Delta+r^2)}.$$

We have:

$$\frac{1}{\delta}\cdot e^{-\delta\log(-\Delta+r^2)} = \\ = \frac{1}{\delta}\cdot\left[1 - \delta\log(-\Delta + r^2) + 0(\delta^2)\right]. \tag{148}$$

Then we have:

$$-\tfrac{1}{\delta}\cdot\delta\log(-\Delta+r^2)=$$
$$=-\log m^2y-\log\tfrac{1}{m^2}\times$$
$$\times\big[m^2-p_E^2x-q_E^2(1-x)+(p_Ex+q_E(1-x))^2y\big]=$$
$$=-\log m^2y-\log\Big[1-\tfrac{p_E^2x(1-xy)+q_E^2(1-x)(1-(1-x)y)}{m^2}+$$
$$+\tfrac{2p_Eq_Ex(1-x)y}{m^2}+0\Big(\tfrac{p_E^2+q_E^2}{m^2}\Big)\Big]. \tag{149}$$

Then the constant term $-\log m^2y$ in (149) can be cancelled by the corresponding counter term with the factor z_e-1 in (95) and thus can be ignored. When $p_E^2\ll m^2$ and $q_E^2\ll m^2$ the second term in (149) is approximately equal to:

$$f(x,y):=\tfrac{p_E^2x(1-xy)+q_E^2(1-x)(1-(1-x)y)}{m^2}-\\ -\tfrac{2p_Eq_Ex(1-x)y}{m^2}. \tag{150}$$

Thus by (150) the sum of the two terms in (145) having the factor $\Gamma(2-\frac{n}{2})$ is approximately equal to:

$$\tfrac{ie^3}{(2\pi)^n}\int_0^1dx\int_0^1ydy\,f(x,y)\times\\ \times\big[5(p_E+q_E)\pi^{\frac{n}{2}}\tfrac{n}{2}-2\pi^{\frac{n}{2}}\tfrac{(n+2)}{2}r\big], \tag{151}$$

where we can (as an approximation) let $n=4$. Carrying out the integration $\int_0^1dx\int_0^1ydy$ of the two terms in (151) we have that (151) is equal to the following result:

$$(ie)\tfrac{\alpha\pi^2}{4\pi^3m^2}(p_E+q_E)\times\\ \times\big[(-5\cdot\tfrac{1}{9}\cdot 2p_Eq_E)+(-\tfrac{7}{24}p_E^2-\tfrac{7}{24}q_E^2+\tfrac{3}{9}p_Eq_E)\big], \tag{152}$$

where the first term and the second term in the $[\cdot]$ are from the first term and the second term in (151) respectively.

Combining (146) and (151) we have the following result which approximately equal to (145) when $p_E^2\ll m^2$ and $q_E^2\ll m^2$:

$$(-ie)\frac{\alpha\pi^2}{4\pi^3m^2}(p_E+q_E)\left[\frac{2}{9}p_E^2+\frac{2}{9}q_E^2+\frac{7}{3}p_Eq_E\right], \tag{153}$$

where the exchanging term $\frac{7}{3}p_Eq_E$ is of vital importance.

Now to have the on-mass-shell condition let us consider a linear statistics of summation on (153). Let there be a large amount of virtual electrons z_j, $j\in J$ indexed by a set J with the proper energies $p_{Ej}^2\ll m^2$ and $q_{Ej}^2\ll m^2$, $j\in J$. Then from (153) we have the following linear statistics of summation on (153):

$$\tfrac{(-ie)\alpha\pi^2(p_{Ej_0}+q_{Ej_0})}{4\pi^3m^2}\times\\ \times\Big[\tfrac{2}{9}\textstyle\sum_j(p_{Ej}^2+q_{Ej}^2)+\tfrac{7}{3}\sum_jp_{Ej}q_{Ej}\Big], \tag{154}$$

where for simplicity we let:

$$p_{Ej}+q_{Ej}=p_{Ej'}+q_{Ej'}=p_{Ej_0}+q_{Ej_0}=2m_0 \tag{155}$$

for all $j,j'\in J$ and for some (bare) mass $m_0\ll m$ and for some $j_0\in J$. Then by applying Fermi-Dirac statistics on the factor $p_{Ej_0}+q_{Ej_0}$ in (154) we have the following Fermi-Dirac statistics for (154):

$$(-ie)\tfrac{\alpha\pi^2}{4\pi^3m^2}\tfrac{1}{2}\gamma^\mu(p_{Ej_0}+q_{Ej_0})\times\\ \times\big[\tfrac{2}{9}\textstyle\sum_j(p_{Ej}^2+q_{Ej}^2)+\tfrac{7}{3}\sum_jp_{Ej}q_{Ej}\big]= \\ =(-ie)\tfrac{\alpha\pi^2\gamma^\mu m_0}{4\pi^3m^2}\big[\tfrac{2}{9}\textstyle\sum_j(p_{Ej}^2+q_{Ej}^2)+\tfrac{7}{3}\sum_jp_{Ej}q_{Ej}\big]. \tag{156}$$

Then for the on-mass-shell condition we require that the linear statistical sum $m_0\frac{7}{3}\sum_jp_{Ej}q_{Ej}$ in (156) is of the following form:

$$m_0\frac{7}{3}\sum_jp_{Ej}q_{Ej}=\beta_0m\,\frac{7}{3}\,q^2, \tag{157}$$

where $q^2=(p'-p^2)$ and the form $mq^2=m(p'-p^2)$ is the on-mass-shell condition which gives the electron mass m; and that β_0 is a statistical factor (to be determined) for this linear statistics of summation and is similar to the statistical factor $(2\pi)^n$ for the space-time statistics.

Then we notice that (156) is for computing (144) and thus its exchanging term corresponding to $\sum_jp_{Ej}q_{Ej}$ must be equal to (144). From (156) we see that there is a statistical factor 4 which does not appear in (144). Since this exchanging term in (156) must be equal to (144) we conclude that the statistical factor β_0 must be equal to 4 so as to cancel the statistical factor 4 in (156). (We also notice that there is a statistical factor π^2 in the numerator of (156) and thus it requires a statistical factor 4 to form the statistical factor $(2\pi)^2$ and thus $\beta_0=4$.) Thus we have that for the on-mass-condition we have that (156) is of the following statistical form:

$$(-ie)\frac{\alpha\pi^2}{\pi^3m^2}m\gamma^\mu\left[\beta_2\,\frac{2}{9}\,m^2+\beta_2'\,\frac{2}{9}\,m^2+\frac{7}{3}\,q^2\right]. \tag{158}$$

Then from (158) we have the following statistical form:

$$(-ie)\frac{\alpha\pi^2}{\pi^3m^2}\gamma^\mu\left[\beta_2\,\frac{2}{9}\,m^2+\beta_2'\,\frac{2}{9}\,m^2+\frac{7}{3}\,q^2\right], \tag{159}$$

where the factor m of $m\gamma^\mu$ has been absorbed to the two external spinors of electron. Then we notice that the term corresponding to $\beta_2\frac{2}{9}m^2+\beta_2'\frac{2}{9}m^2$ in (159) is as a constant term and thus can be cancelled by the corresponding counter term with the factor z_e-1 in (95). Thus from (159) we have the following statistical form of effect which corresponds to (144):

$$(-ie)\gamma^\mu\,\frac{\alpha}{\pi m^2}\,\frac{7}{3}\,q^2. \tag{160}$$

This effect (160) is as the total effect of q^2 computed from the one-loop vertex with the minimal energy $\omega_{\min}$ and thus includes the effect of q^2 from the anomalous magnetic moment. Thus we have that (144) is computed and is given by the following statistical form:

$$
\begin{aligned}
&(-ie)\gamma^\mu \tfrac{\alpha}{3\pi}\tfrac{q^2}{m^2}\log\tfrac{m}{2\omega_{\min}} = \\
&\qquad = (-ie)\gamma^\mu \tfrac{\alpha}{3\pi}\tfrac{q^2}{m^2}\left[7-\tfrac{3}{8}\right],
\end{aligned} \tag{161}
$$

where the term corresponding to the factor $\frac{3}{8}$ is from the anomalous magnetic moment (137) as computed in the literature of QED (see [6]). This completes our computation of (144). Thus under the on-mass-shell condition the renormalized one-loop vertex $(-ie)\Lambda_R(p',p)$ is given by:

$$
\begin{aligned}
&(-ie)\Lambda_R(p',p) = \\
&= (-ie)\left[\gamma^\mu \tfrac{\alpha q^2}{3\pi m^2}\left(7+\tfrac{5}{6}-\tfrac{3}{8}\right) + \tfrac{i\alpha}{4\pi m}\,\sigma^{\mu\nu}q_\nu\right].
\end{aligned} \tag{162}
$$

This completes our computation of the one-loop vertex correction.

20 Computation of photon self-energy

To compute the Lamb shift let us then consider the one-loop photon self energy (113). As a statistics we extend the one dimensional integral $\int dp_E$ to the n-dimensional integral $\int d^np$ ($n \to 4$) where $p = (p_E, \mathbf{p})$. This is similar to the dimensional regularization in the existing quantum field theories (However here our aim is to increase the dimension for statistics which is different from the dimensional regularization which is to reduce the dimension from 4 to n to avoid the ultraviolet divergence). With this statistics the factor 2π is replaced by the statistical factor $(2\pi)^n$. From this statistics on (113) we have that the following statistical one-loop photon self-energy:

$$
\begin{aligned}
&(-1)i^2(-i)^2\tfrac{e^2}{(2\pi)^n}\times \\
&\qquad \times \int_0^1 dx \int \tfrac{(4p_E^2+4p_Ek_E+k_E^2)d^np}{(p^2+2pkx+k_E^2x-m^2)^2},
\end{aligned} \tag{163}
$$

where $p^2 = p_E^2 - \mathbf{p}^2$, and $\mathbf{p}^2$ is from $\omega^2 = m^2 + \mathbf{p}^2$; and:

$$
pk := p_Ek_E - \mathbf{p}\cdot\mathbf{0} = p_Ek_E\,. \tag{164}
$$

As a Feynman rule for space-time statistics a statistical factor (-1) has been introduced for this photon self-energy since it has a loop of electron particles.

By using the formulae for computing Feynman integrals we have that (163) is equal to:

$$
\begin{aligned}
&\tfrac{(-1)ie^2}{(2\pi)^n}\int_0^1 dx\,\times \\
&\times\left[\tfrac{k_E^2(4x^2-4x+1)\pi^{\frac{n}{2}}\Gamma(2-\frac{n}{2})}{\Gamma(2)(m^2-k_E^2x(1-x))^{2-\frac{n}{2}}} + \tfrac{\pi^{\frac{n}{2}}\Gamma(2-1-\frac{n}{2})\frac{n}{2}}{\Gamma(2)(m^2-k_E^2x(1-x))^{2-1-\frac{n}{2}}}\right].
\end{aligned} \tag{165}
$$

Let us first consider the first term in the $[\cdot]$ in (165). Let $\delta := 2-\frac{n}{2} > 0$. As for the one-loop vertex we have

$$
\begin{aligned}
&\Gamma(\delta)\cdot(m^2-k_E^2\,x(1-x))^{-\delta} = \\
&= \left(\tfrac{1}{\delta} + \text{a finite term as } \delta\to 0\right)\cdot e^{-\delta\log(m^2-k_E^2x(1-x))}.
\end{aligned} \tag{166}
$$

We have

$$
\begin{aligned}
&\tfrac{1}{\delta}\cdot e^{-\delta\log(m^2-k_E^2x(1-x))} = \\
&= \tfrac{1}{\delta}\cdot\left[1-\delta\log(m^2-k_E^2x(1-x)) + 0(\delta^2)\right].
\end{aligned} \tag{167}
$$

Then we have

$$
\begin{aligned}
&-\tfrac{1}{\delta}\cdot\delta\log(m^2-k_E^2x(1-x)) = \\
&\qquad = -\log m^2 - \log\left[1-\tfrac{k_E^2x(1-x)}{m^2}\right].
\end{aligned} \tag{168}
$$

Then the constant term $-\log m^2$ in (168) can be cancelled by the corresponding counter term with the factor $z_A - 1$ in (95) and thus can be ignored. When $k_E^2 \ll m^2$ the second term in (168) is approximately equal to:

$$
\frac{k_E^2x(1-x)}{m^2}\,. \tag{169}
$$

Carrying out the integration $\int_0^1 dx$ in (163) with $-\log\left[1-\frac{k_E^2x(1-x)}{m^2}\right]$ replaced by (169), we have the following result:

$$
\int_0^1 dx(4x^2-4x+1)\frac{k_E^2x(1-x)}{m^2} = \frac{k_E^2}{30m^2}\,. \tag{170}
$$

Thus as in the literature in QED from the photon self-energy we have the following term which gives contribution to the Lamb shift:

$$
\frac{k_E^2}{30m^2} = \frac{(p_E-q_E)^2}{30m^2}\,, \tag{171}
$$

where $k_E = p_E - q_E$ and p_E, q_E denote the proper energies of virtual electrons. Let us then consider statistics of a large amount of photon self-energy (168). When there is a large amount of photon self-energies we have the following linear statistics of summation:

$$
\frac{\sum_i k_{Ei}^2}{30m^2}\,, \tag{172}
$$

where each i represent a photon. Let us write:

$$
k_{Ei}^2 = (p_{Ei}-q_{Ei})^2 = p_{Ei}^2 - 2p_{Ei}q_{Ei} + q_{Ei}^2\,. \tag{173}
$$

Thus we have:

$$
\begin{aligned}
&\textstyle\sum_i k_{Ei}^2 = \sum_i(p_{Ei}-q_{Ei})^2 = \\
&\qquad = \textstyle\sum_i(p_{Ei}^2+q_{Ei}^2) - 2\sum_i p_{Ei}q_{Ei}\,.
\end{aligned} \tag{174}
$$

Now as the statistics of the vertex correction we have the following statistics:

$$
\sum_i p_{Ei}q_{Ei} = 4(p'-p)^2 = 4q^2, \tag{175}
$$

where 4 is a statistical factor which is the same statistical factor of case of the vertex correction and p, p' are on-mass-shell four vectors of electrons. As the the statistics of the vertex correction this statistical factor cancels another statistical factor 4. On the other hand as the statistics of the vertex correction we have the following statistics:

$$\sum_i p_{Ei}^2 = \beta_3 m^2, \qquad \sum_i q_{Ei}^2 = \beta_4 m^2, \tag{176}$$

where β_3 and β_4 are two statistical factors. As the case of the vertex correction these two sums give constant terms and thus can be cancelled by the corresponding counter term with the factor $z_A - 1$ in (95). Thus from (174) we have that the linear statistics of summation $\sum_i k_{Ei}^2$ gives the following statistical renormalized photon self-energies Π_R and Π_M (where we follow the notations in the literature of QED for photon self-energies Π_M):

$$\begin{aligned} i\Pi_R(k_E) &= ik_E^2\Pi_M(k_E) = \\ &= ik_E^2\tfrac{\alpha}{4\pi}\tfrac{8q^2}{30m^2} = ik_E^2\tfrac{\alpha}{3\pi}\tfrac{q^2}{5m^2}\,, \end{aligned} \tag{177}$$

where we let $k_{Ei}^2 = k_E^2$ for all i.

Let us then consider the second term in the $[\cdot]$ in (165). This term can be written in the following form:

$$\begin{aligned} &\tfrac{\pi^{\frac{n}{2}}\Gamma(2-\frac{n}{2})\frac{n}{2}}{(1-\frac{n}{2})\Gamma(2)(m^2-k_E^2x(1-x))^{2-1-\frac{n}{2}}} = \\ &= \tfrac{\pi^{\frac{n}{2}}\Gamma(2-\frac{n}{2})\frac{n}{2}}{(1-\frac{n}{2})\Gamma(2)}\left[(m^2-k_E^2x(1-x))+0(\delta)\right] = \\ &= k_E^2\left[\tfrac{1}{\delta}\cdot\tfrac{(-1)\pi^{\frac{n}{2}}\Gamma(2-\frac{n}{2})\frac{n}{2}}{(1-\frac{n}{2})\Gamma(2)}\,x(1-x)\right] + \\ &+ \left[\tfrac{1}{\delta}\cdot\tfrac{\pi^{\frac{n}{2}}\Gamma(2-\frac{n}{2})\frac{n}{2}}{(1-\frac{n}{2})\Gamma(2)}\,m^2+0(\delta))\right] \end{aligned} \tag{178}$$

Then the first term in (178) under the integration $\int_0^1 dx$ is of the form $(k_E^2 \cdot \text{constant})$. Thus this term can also be cancelled by the counter-term with the factor $z_A - 1$ in (95). In summary the renormalization constant z_A is given by the following equation:

$$\begin{aligned} (-1)^3 i(z_A-1) &= (-i)\Big\{\tfrac{1}{\delta}\cdot\tfrac{e^2\pi^{\frac{n}{2}}}{(2\pi)^n}\int_0^1 dx\, \times \\ &\times\left[(4x^2-4x+1)-\tfrac{nx(1-x)}{2-n}\right]+c_A\Big\}, \end{aligned} \tag{179}$$

where c_A is a finite constant when $\delta \to 0$. From this equation we have that z_A is a very large number when $\delta > 0$ is very small. Thus $e_0 = z_e\left(z_Z z_A^{1/2}\right)^{-1} e = \frac{1}{n_e}e$ is a very small constant when $\delta > 0$ is very small (and since $\frac{e^2}{4\pi} = \alpha = \frac{1}{137}$ is small) where shall show that we can let $z_e = z_Z$.

Then the second term in (178) under the integration $\int_0^1 dx$ gives a parameter $\lambda_3 > 0$ for the photon self-energy since $\delta > 0$ is as a parameter.

Combing the effects of the two terms in the $[\cdot]$ in (165) we have the following renormalized one-loop photon self-energy:

$$i\left(\Pi_R(k_E)+\lambda_3\right). \tag{180}$$

Then we have the following Dyson series for photon propagator:

$$\begin{aligned} &\tfrac{i}{k_E^2-\lambda_0}+\tfrac{i}{k_E^2-\lambda_0}(i\Pi_R(k_E)+i\lambda_3)\tfrac{i}{k_E^2-\lambda_0}+\cdots = \\ &= \tfrac{i}{k_E^2(1+\Pi_M)-(\lambda_0-\lambda_3)} =: \\ &=: \tfrac{i}{k_E^2(1+\Pi_M)-\lambda_R}\,, \end{aligned} \tag{181}$$

where λ_R is as a renormalized mass-energy parameter. This is as the renormalized photon propagator. We have the following approximation of this renormalized photon propagator:

$$\frac{i}{k_E^2(1+\Pi_M)-\lambda_R} \approx \frac{i}{k_E^2-\lambda_R}(1-\Pi_M)\,. \tag{182}$$

21 Computation of the Lamb shift: Part II

Combining the effect of vertex correction and photon self-energy we can now compute the Lamb shift. Combining the effect of photon self-energy $(-ie\gamma^\mu)[-\Pi_M]$ and vertex correction we have:

$$\begin{aligned} &(-ie)\Lambda_R(p',p)+(-ie\gamma^\mu)[-\Pi_M] = \\ &= (-ie)\left[\gamma^\mu\tfrac{\alpha q^2}{3\pi m^2}\left(7+\tfrac{5}{6}-\tfrac{3}{8}-\tfrac{1}{5}\right)+\tfrac{i\alpha}{4\pi m}\sigma^{\mu\nu}q_\nu\right]. \end{aligned} \tag{183}$$

As in the literature of QED let us consider the states $2S_{\frac{1}{2}}$ and the $2P_{\frac{1}{2}}$ in the hydrogen atom [6, 72–78]. Following the literature of QED for the state $2S_{\frac{1}{2}}$ an effect of $\frac{\alpha q^2}{3\pi m^2}\left(\frac{3}{8}\right)$ comes from the anomalous magnetic moment which cancels the same term with negative sign in (183). Thus by using the method in the computation of the Lamb shift in the literature of QED we have the following second order shift for the state $2S_{\frac{1}{2}}$:

$$\Delta E_{2S_{\frac{1}{2}}} = \frac{m\alpha^5}{6\pi}\left(7+\frac{5}{6}-\frac{1}{5}\right). \tag{184}$$

Similarly by the method of computing the Lamb shift in the literature of QED from the anomalous magnetic moment we have the following second order shift for the state $2P_{\frac{1}{2}}$:

$$\Delta E_{2P_{\frac{1}{2}}} = \frac{m\alpha^5}{6\pi}\left(-\frac{1}{8}\right). \tag{185}$$

Thus the second order Lamb shift for the states $2S_{\frac{1}{2}}$ and $2P_{\frac{1}{2}}$ is given by:

$$\Delta E = \Delta E_{2S_{\frac{1}{2}}} - \Delta E_{2P_{\frac{1}{2}}} = \frac{m\alpha^5}{6\pi}\left(7+\frac{5}{6}-\frac{1}{5}+\frac{1}{8}\right) \tag{186}$$

or in terms of frequencies for each of the terms in (186) we have:

$$\begin{aligned} \Delta\nu &= 952 + 113.03 - 27.13 + 16.96 = \\ &= 1054.86 \text{ Mc/sec.} \end{aligned} \tag{187}$$

This agrees with the experimental results [6, 72–78]:

$$\begin{aligned} \Delta\nu^{\text{exp}} &= 1057.86 \pm 0.06 \text{ Mc/sec} \\ \text{and } &= 1057.90 \pm 0.06 \text{ Mc/sec.} \end{aligned} \tag{188}$$

22 Computation of the electron self-energy

Let us then consider the one-loop electron self-energy (113). As a statistics we extend the one dimensional integral $\int dk_E$ to the n-dimensional integral $\int d^n k$ $(n \to 4)$ where $k = (k_E, \mathbf{k})$. This is similar to the dimensional regularization in the existing quantum field theories (However here our aim is to increase the dimension for statistics which is different from the dimensional regularization which is to reduce the dimension from 4 to n to avoid the ultraviolet divergence). With this statistics the factor 2π is replaced by the statistical factor $(2\pi)^n$. From this statistics on (114) we have that the following statistical one-loop electron self-energy $-i\Sigma(p_E)$:

$$-i\Sigma(p_E) := i^2(-i)^2 \frac{e^2}{(2\pi)^n}\int_0^1 dx \int d^n k \times \times \frac{(k_E^2 - 4p_E k_E + 4p_E^2)d^n k}{(k^2 - 2kpx + p_E^2 x - xm^2 - (1-x)\lambda^2)^2}, \tag{189}$$

where $k^2 = k_E^2 - \mathbf{k}^2$, and $\mathbf{k}^2$ is from $\omega^2 = m^2 + \mathbf{k}^2$ and $\lambda_0^2 = \lambda^2 + \mathbf{k}^2$; and $kp := k_E p_E - \mathbf{k}\cdot\mathbf{0} = k_E p_E$. By using the formulae for computing Feynman integrals we have that (189) is equal to:

$$\frac{ie^2}{(2\pi)^n}\int_0^1 dx \Big[\frac{p_E^2(x^2-4x+4)\pi^{\frac{n}{2}}\Gamma(2-\frac{n}{2})}{\Gamma(2)(xm^2+(1-x)\lambda^2-p_E^2x(1-x))^{2-\frac{n}{2}}} + $$
$$+ \frac{\pi^{\frac{n}{2}}\Gamma(2-1-\frac{n}{2})\frac{n}{2}}{\Gamma(2)(xm^2+(1-x)\lambda^2-p_E^2x(1-x))^{2-1-\frac{n}{2}}}\Big] =$$
$$= \frac{ie^2}{(2\pi)^n}\int_0^1 dx \Big\{p_E^2(x^2-4x+4)\pi^{\frac{n}{2}} \times$$
$$\times \left[\left(\tfrac{1}{\delta} + O(\delta)\right)\cdot e^{-\delta\log(xm^2+(1-x)\lambda^2-p_E^2x(1-x))}\right] -$$
$$- \pi^{\frac{n}{2}}\frac{n}{2}\frac{1}{\delta}\left[xm^2 + (1-x)\lambda^2 - p_E^2x(1-x) + 0(\delta)\right]\Big\} =$$
$$= \frac{ie^2}{(2\pi)^n}\int_0^1 dx \Big\{p_E^2(x^2-4x+4)\pi^{\frac{n}{2}}\big[\tfrac{1}{\delta} - \tfrac{1}{\delta}\times$$
$$\times\,\delta\log(xm^2 + (1-x)\lambda^2 - p_E^2x(1-x)) + 0(\delta)\big] -$$
$$- \pi^{\frac{n}{2}}\frac{n}{2}\frac{1}{\delta}\left[xm^2 + (1-x)\lambda^2 - p_E^2x(1-x) + 0(\delta)\right]\Big\} =$$
$$= \frac{ie^2}{(2\pi)^n}\int_0^1 dx \Big\{p_E^2(x^2-4x+4)\pi^{\frac{n}{2}} \times$$
$$\times\left[\tfrac{1}{\delta} - \log(xm^2 + (1-x)\lambda^2 - p_E^2x(1-x)) + 0(\delta)\right] -$$
$$- \pi^{\frac{n}{2}}\frac{n}{2}\frac{1}{\delta}\left[xm^2 + (1-x)\lambda^2 - p_E^2x(1-x) + 0(\delta)\right]\} =$$
$$= \frac{ie^2}{(2\pi)^n}\int_0^1 dx \Big\{p_E^2(x^2-4x+4)\pi^{\frac{n}{2}} \times$$
$$\times\big[\tfrac{1}{\delta} - \log(xm^2 + (1-x)\lambda^2) -$$
$$- \log\left(1 - \tfrac{p_E^2x(1-x)}{xm^2+(1-x)\lambda^2}\right) + 0(\delta)\big] -$$
$$- \pi^{\frac{n}{2}}\frac{n}{2}\frac{1}{\delta}\left[xm^2 + (1-x)\lambda^2 - p_E^2x(1-x) + 0(\delta)\right]\Big\} =:$$

$$=: \frac{ie^2}{(2\pi)^n}\int_0^1 dx \Big\{p_E^2(x^2-4x+4)\pi^{\frac{n}{2}}\big[\tfrac{1}{\delta} - - \log(xm^2 + (1-x)\lambda^2) - \log\left(1 - \tfrac{p_E^2x(1-x)}{xm^2+(1-x)\lambda^2}\right) + + 0(\delta)\big] + p_E^2\cdot\tfrac{1}{\delta}\pi^{\frac{n}{2}}\tfrac{n}{2}x(1-x)\Big\} + i\omega_3\,, \tag{190}$$

where $\omega_3 > 0$ is as a mass-energy parameter.

Then we notice that from the expressions for $\Sigma_0(p_E)$ and $\Lambda_0(p_E, q_E)$ in (114) and (115) we have the following identity:

$$\frac{\partial}{\partial p_E}\Sigma_0(p_E) = -\Lambda_0(p_E, p_E) + + \frac{i4e^2}{2\pi}\int dk \frac{-k_E + 2p_E}{(k_E^2-\lambda_0^2)((p_E-k_E)^2-\omega^2)}\,. \tag{191}$$

This is as a Ward-Takahashi identity which is analogous to the corresponding Ward-Takahashi identity in the conventional QED theory [6].

From (114) and (115) we get their statistical forms by changing $\int dk$ to $\int d^n k$. From this summation form of statistics and the identity (191) we then get the following statistical Ward-Takahashi identity:

$$\frac{\partial}{\partial p_E}\Sigma(p_E) = -\Lambda(p_E, p_E) + + \frac{i4e^2}{(2\pi)^n}\int_0^1 dx\int d^n k \frac{-k_E + 2p_E}{(k^2 - 2kpx + p_E^2x - xm^2 - (1-x)\lambda^2)^2}\,, \tag{192}$$

where $\Sigma(p_E)$ denotes the statistical form of $\Sigma_0(p_E)$ and is given by (189) and $\Lambda(p_E, q_E)$ denotes the statistical form of $\Lambda_0(p_E, q_E)$ as in the above Sections.

After the differentiation of (190) with respect to p_E the remaining factor p_E of the factor p_E^2 of (190) is absorbed to the external spinors as the mass m and a factor $\frac{\gamma^\mu}{2}$ is introduced by space-time statistics, as the case of the statistics of the vertex correction $\Lambda_0(p_E, q_E)$ in the above Sections. From the absorbing of a factor p_E to the external spinors for both sides of this statistical Ward-Takahashi identity we then get a statistical Ward-Takahashi identity where the Taylor expansion (of the variable p_E) of both sides of this statistical Ward-Takahashi identity are with constant term as the beginning term. From this Ward-Takahashi identity we have that these two constant terms must be the same constant. Then the constant term, denoted by $C(\delta)$, of the vertex correction of this Ward-Takahashi identity is cancelled by the counterterm with the factor $z_e - 1$ in (95), as done in the above computation of the renormalized vertex correction $A_R(p', p)$. (At this point we notice that in computing the constant term of the vertex correction some terms with the factor p_E has been changed to constant terms under the on-mass-shell condition $p_E = m$. This then modifies the definition of $C(\delta)$).

On the other hand let us denote the constant term for the electron self-energy by $B(\delta)$. Then from the above statistical Ward-Takahashi identity we have the following equality:

$$B(\delta) + a_1\cdot\frac{1}{\delta} + b_1 = C(\delta)\,, \tag{193}$$

where a_1, b_1 are finite constants when $\delta \to 0$ and the term $a_1 \cdot \frac{1}{\delta}$ is from the second term in the right hand side of (192).

Let us then compute the constant term $B(\delta)$ for the electron self-energy, as follows. As explained in the above the constant term for the electron self-energy can be obtained by differentiation of (190) with respect to p_E and the removing of the remaining factor p_E of p_E^2. We have:

$$\begin{aligned}
&\frac{\partial}{\partial p_E}\Big\{ \frac{ie^2}{(2\pi)^n} \int_0^1 dx p_E^2 (x^2-4x+4)\pi^{\frac{n}{2}} \times \\
&\times \left[\tfrac{1}{\delta} - \log(xm^2 + (1-x)\lambda^2 - p_E^2 x(1-x))\right] + \\
&+ p_E^2 \cdot \tfrac{1}{\delta}\pi^{\frac{n}{2}} \tfrac{n}{2} \tfrac{ie^2}{(2\pi)^n} \int_0^1 x(1-x)dx + i\omega_3 \Big\} = \\
&= \frac{ie^2}{(2\pi)^n} \int_0^1 dx 2p_E (x^2-4x+4)\pi^{\frac{n}{2}} \times \\
&\times \left[\tfrac{1}{\delta} - \log(xm^2 + (1-x)\lambda^2 - p_E^2 x(1-x))\right] + \\
&+ \frac{ie^2}{(2\pi)^n} \int_0^1 dx \frac{p_E^2(x^2-4x+4)\pi^{\frac{n}{2}} \cdot 2p_E x(1-x)}{xm^2+(1-x)\lambda^2 - p_E^2 x(1-x)} + \\
&+ 2p_E \cdot \tfrac{1}{\delta}\pi^{\frac{n}{2}} \tfrac{n}{2} \tfrac{ie^2}{(2\pi)^n} \int_0^1 x(1-x)dx\,.
\end{aligned} \tag{194}$$

Then by Taylor expansion of (194) and by removing a factor $2p_E$ from (194) the constant term for the electron self-energy is given by:

$$\begin{aligned}
B(\delta) &:= \frac{-e^2}{(2\pi)^n} \int_0^1 dx (x^2-4x+4)\pi^{\frac{n}{2}} \times \\
&\times \left[\tfrac{1}{\delta} - - \log(xm^2 + (1-x)\lambda^2)\right] - \\
&- \tfrac{1}{\delta}\pi^{\frac{n}{2}} \tfrac{n}{2} \frac{e^2}{(2\pi)^n} \int_0^1 x(1-x)dx\,.
\end{aligned} \tag{195}$$

Then as a renormalization procedure for the electron self-energy we choose a $\delta_1 > 0$ which is related to the δ for the renormalization of the vertex correction such that:

$$B(\delta_1) = B(\delta) + a_1 \cdot \frac{1}{\delta} + b_1\,. \tag{196}$$

This is possible since $B(\delta)$ has a term proportional to $\frac{1}{\delta}$. From this renormalization procedure for the electron self-energy we have:

$$B(\delta_1) = C(\delta)\,. \tag{197}$$

This constant term $B(\delta_1)$ for the electron self-energy is to be cancelled by the counter-term with the factor $z_Z - 1$ in (95). We have the following equation to determine the renormalization constant z_Z for this cancellation:

$$(-1)^3 i(z_Z - 1) = (-i)B(\delta_1)\,. \tag{198}$$

Then from the equality (197) we have $z_e = z_Z$ where z_e is determined by the following equation:

$$(-1)^3 i(z_e - 1) = (-i)C(\delta)\,. \tag{199}$$

Cancelling $B(\delta_1)$ from the electron self-energy (190) we get the following renormalized one-loop electron self-energy:

$$\begin{aligned}
&-ip_E^2\Sigma_R(p_E) + i\omega_3^2 := -ip_E^2 \tfrac{\alpha}{4\pi} \times \\
&\times \int_0^1 dx(x^2-4x+4)\log\left[1 - \frac{p_E^2 x(1-x)}{xm^2+(1-x)\lambda^2}\right] + i\omega_3^2\,.
\end{aligned} \tag{200}$$

We notice that in (200) we can let $\lambda = 0$ since there is no infrared divergence when $\lambda = 0$. This is better than the computed electron self-energy in the conventional QED theory where the computed one-loop electron self-energy is with infrared divergence when $\lambda = 0$ [6].

From this renormalized electron self-energy we then have the renormalized electron propagator obtained by the following Dyson series:

$$\begin{aligned}
&\frac{i}{p_E^2-\omega^2} + \frac{i}{p_E^2-\omega^2}(-ip_E^2\Sigma_R(p_E) + i\omega_3^2)\frac{i}{p_E^2-\omega^2} + \cdots = \\
&= \frac{i}{p_E^2(1-\Sigma_R(p_E)) - (\omega^2-\omega_3^2)} =: \\
&=: \frac{i}{p_E^2(1-\Sigma_R(p_E)) - \omega_R^2}\,,
\end{aligned} \tag{201}$$

where $\omega_R^2 := \omega^2 - \omega_3^2$ is as a renormalized electron mass-energy parameter. Then by space-time statistics from the renormalized electron propagator (201) we can get the renormalized electron propagator in the spin-$\frac{1}{2}$ form, as that the electron propagator $\frac{i}{\gamma_\mu p^\mu - m}$ in the spin-$\frac{1}{2}$ form can be obtained from the electron propagator $\frac{i}{p_E^2-\omega^2}$.

23 New effect of QED

Let us consider a new effect for electron scattering which is formed by two seagull vertexes with one photon loop and four electron lines. This is a new effect of QED because the conventional spin $\frac{1}{2}$ theory of QED does not have this seagull vertex. The Feynman integral corresponding to the photon loop is given by

$$\begin{aligned}
&\frac{i^2(i)^2e^4}{2\pi} \int \frac{dk_E}{(k_E^2-\lambda_0^2)((p_E-q_E-k_E)^2-\lambda_0^2)} = \\
&= \frac{e^4}{2\pi}\int_0^1 \int \frac{dk_E}{(k_E^2 - 2k_E(p_E-q_E)x + (p_E-q_E)^2 x - \lambda_0^2)^2} = \\
&= \frac{e^4}{2\pi}\int_0^1 \int \frac{dk_E}{(k_E^2 - 2k_E(p_E-q_E)x + (p_E-q_E)^2 x - \lambda_0^2)^2}\,.
\end{aligned} \tag{202}$$

Let us then introduce a space-time statistics. Since the photon propagator of the (two joined) seagull vertex interactions is of the form of a circle on a plane we have that the appropriate space-time statistics of the photons is with the two dimensional space for the circle of the photon propagator. From this two dimensional space statistics we then get a three dimensional space statistics by multiplying the statistical factor $\frac{1}{(2\pi)^3}$ of the three dimensional space statistics and by concentrating in a two dimensional subspace of the three dimensional space statistics.

Thus as similar to the four dimensional space-time statistics with the three dimensional space statistics in the above

Sections from (202) we have the following space-time statistics with the two dimensional subspace:

$$\frac{e^4}{(2\pi)^4}\int_0^1\int\frac{d^3k}{(k_E^2-2k_E(p_E-q_E)x+(p_E-q_E)^2x-\mathbf{k}^2-\lambda_4^2)^2} = \\ = \frac{e^4}{(2\pi)^4}\int_0^1 dx\int\frac{d^3k}{(k^2-2k\cdot(p_E-q_E,0)x+(p_E-q_E)^2x-\lambda_4^2)^2}\,, \tag{203}$$

where the statistical factor $\frac{1}{(2\pi)^3}$ of three dimensional space has been introduced to give the factor $\frac{1}{(2\pi)^4}$ of the four dimensional space-time statistics; and we let $k=(k_E,\mathbf{k})$, $k^2 = k_E^2-\mathbf{k}^2$ and since the photon energy parameter λ_0 is a free parameter we can write $\lambda_0^2=\mathbf{k}^2+\lambda_4^2$ for some λ_4.

Then a delta function concentrating at 0 of a one dimensional momentum variable is multiplied to the integrand in (203) and the three dimensional energy-momentum integral in (203) is changed to a four dimensional energy-momentum integral by taking the corresponding one more momentum integral.

From this we get a four dimensional space-time statistics with the usual four dimensional momentum integral and with the statistical factor $\frac{1}{(2\pi)^4}$. After this additional momentum integral we then get (203) as a four dimensional space-time statistics with the two dimensional momentum variable.

Then to get a four dimensional space-time statistics with the three dimensional momentum variable a delta function concentrating at 0 of another one dimensional momentum variable is multiplied to (203) and the two dimensional momentum variable of (203) is extended to the corresponding three dimensional momentum variable. From this we then get a four dimensional space-time statistics with the three dimensional momentum variable.

Then we have that (203) is equal to:

$$\frac{e^4}{(2\pi)^4}\frac{i\pi^{\frac{3}{2}}\Gamma(2-\frac{3}{2})}{\Gamma(2)}\int_0^1\frac{dx}{((p_E-q_E)^2x(1-x)-\lambda_4^2)^{\frac{1}{2}}}\,. \tag{204}$$

Then since the photon mass-energy parameter λ_4 is a free parameter for space-time statistics we can write λ_4 in the following form:

$$\lambda_4^2 = (\mathbf{p}-\mathbf{q})^2x(1-x)\,, \tag{205}$$

where $\mathbf{p}-\mathbf{q}$ denotes a two dimensional momentum vector.

Then we let $p-q=(p_E-q_E,\mathbf{p}-\mathbf{q})$. Then we have:

$$(p_E-q_E)^2\,x(1-x)-\lambda_4^2 = \\ = (p_E-q_E)^2\,x(1-x)-(\mathbf{p}-\mathbf{q})^2\,x(1-x) = \\ = (p-q)^2\,x(1-x)\,. \tag{206}$$

Then we have that (204) is equal to:

$$\frac{e^4}{(2\pi)^4}\frac{i\pi^{\frac{3}{2}}\Gamma(2-\frac{3}{2})}{\Gamma(2)}\int_0^1\frac{dx}{((p-q)^2x(1-x))^{\frac{1}{2}}} \\ = \frac{e^4}{(2\pi)^4}\frac{i\pi\pi^{\frac{3}{2}}\Gamma(2-\frac{3}{2})}{\Gamma(2)}\frac{1}{((p-q)^2)^{\frac{1}{2}}} = \\ = \frac{e^4 i}{16\pi((p-q)^2)^{\frac{1}{2}}} = \frac{e^2\alpha i}{4((p-q)^2)^{\frac{1}{2}}}\,. \tag{207}$$

Thus we have the following potential:

$$V_{seagull}(p-q) = \frac{e^2\alpha i}{4((p-q)^2)^{\frac{1}{2}}}\,. \tag{208}$$

This potential (208) is as the seagull vertex potential.

We notice that (208) is a new effect for electron-electron or electron-positron scattering. Recent experiments on the decay of positronium show that the experimental orthopositronium decay rate is significantly larger than that computed from the conventional QED theory [33–52]. In the following Section 24 to Section 26 we show that this discrepancy can be remedied with this new effect (208).

24 Reformulating the Bethe-Salpeter equation

To compute the orthopositronium decay rate let us first find out the ground state wave function of the positronium. To this end we shall use the Bethe-Salpeter equation. It is well known that the conventional Bethe-Salpeter equation is with difficulties such as the relative time and relative energy problem which leads to the existence of nonphysical solutions in the conventional Bethe-Salpeter equation [7–32]. From the above QED theory let us reformulate the Bethe-Salpeter equation to get a new form of the Bethe-Salpeter equation. We shall see that this new form of the Bethe-Salpeter equation resolves the basic difficulties of the Bethe-Salpeter equation such as the relative time and relative energy problem.

Let us first consider the propagator of electron. Since electron is a spin-$\frac{1}{2}$ particle its statistical propagator is of the form $\frac{i}{\gamma_\mu p^\mu - m}$. Thus before the space-time statistics the spin-$\frac{1}{2}$ form of electron propagator is of the form $\frac{i}{p_E-\omega}$ which can be obtained from the electron propagator $\frac{i}{p_E^2-\omega^2}$ by the factorization: $p_E^2-\omega^2=(p_E-\omega)(p_E+\omega)$. Then we consider the following product which is from two propagators of two spin-$\frac{1}{2}$ particles:

$$[p_{E1}-\omega_1][p_{E2}-\omega_2] = \\ = p_{E1}p_{E2}-\omega_1 p_{E2}-\omega_2 p_{E1}+\omega_1\omega_2 =: \\ =: p_E^2-\omega_b^2, \tag{209}$$

where we define $p_E^2=p_{E1}p_{E2}$ and $\omega_b^2 := \omega_1 p_{E2}+\omega_2 p_{E1} - \omega_1\omega_2$. Then since ω_1 and ω_2 are free mass-energy parameters we have that ω_b is also a free mass-energy parameter with the requirement that it is to be a positive parameter.

Then we introduce the following reformulated relativistic equation of Bethe-Salpeter type for two particles with spin-$\frac{1}{2}$:

$$\phi_0(p_E,\omega_b) = \frac{i^2\lambda'}{[p_{E1}-\omega_1][p_{E2}-\omega_2]}\times \\ \times\int\frac{ie^2\phi_0(q_E,\omega_b)dq_E}{((p_E-q_E)^2-\lambda_0^2)}\,, \tag{210}$$

where we use the photon propagator $\frac{i}{k_E^2-\lambda_0^2}$ (which is of the effect of Coulomb potential) for the interaction of these two particles and we write the proper energy k_E^2 of this potential in the form $k_E^2 = (p_E - q_E)^2$; and λ' is as the coupling parameter. We shall later also introduce the seagull vertex term for the potential of binding.

Let us then introduce the space-time statistics. Since we have the seagull vertex term for the potential of binding which is of the form of a circle in a two dimensional space from the above Section on the seagull vertex potential we see that the appropriate space-time statistics is with the two dimensional space. Thus with this space-time statistics from (210) we have the following reformulated relativistic Bethe-Salpeter equation:

$$\phi_0(p) = \frac{-\lambda'}{p^2-\gamma_0^2}\int \frac{i d^3q}{(p-q)^2}\,\phi_0(q)\,, \tag{211}$$

where we let the free parameters ω_b and λ_0 be such that $p^2 = p_E^2 - \mathbf{p}^2$ with $\omega_b^2 = \mathbf{p}^2 + \gamma_0^2$ for some constant $\gamma_0^2 = \frac{1}{a^2} > 0$ where a is as the radius of the binding system; and $(p-q)^2 = (p_E - q_E)^2 - (\mathbf{p}-\mathbf{q})^2$ with $\lambda_0^2 = (\mathbf{p}-\mathbf{q})^2$. We notice that the potential $\frac{i\alpha}{(p-q)^2}$ of binding is now of the usual (relativistic) Coulomb potential type. In (211) the constant e^2 in (210) has been absorbed into the parameter λ' in (211).

We see that in this reformulated Bethe-Salpeter equation the relative time and relative energy problem of the conventional Bethe-Salpeter equations is resolved [7–32]. Thus this reformulated Bethe-Salpeter equation will be free of abnormal solutions.

Let us then solve (211) for the relativistic bound states of particles. We show that the ground state solution $\phi_0(p)$ can be exactly solved and is of the following form:

$$\phi_0(p) = \frac{1}{(p^2-\gamma_0^2)^2}\,. \tag{212}$$

We have:

$$\begin{aligned}
&\frac{1}{((p-q)^2)}\frac{1}{(q^2-\gamma_0^2)^2} = \\
&= \frac{(2+1-1)!}{(2-1)!(1-1)!}\int_0^1 \frac{(1-x)dx}{[x(p-q)^2+(1-x)(q^2-\gamma_0^2)^2]^3} = \\
&= \frac{(2+1-1)!}{(2-1)!(1-1)!}\int_0^1 \frac{(1-x)dx}{[q^2+2xpq+xp^2-(1-x)\gamma_0^2]^3} = \\
&= 2\int_0^1 \frac{(1-x)dx}{[q^2+2xpq+xp^2-(1-x)\gamma_0^2]^3}\,.
\end{aligned} \tag{213}$$

Thus we have:

$$\begin{aligned}
&i\int \frac{d^3q}{((p-q)^2)(q^2-\gamma_0^2)^2} = \\
&= i2\int_0^1 (1-x)dx \int \frac{d^3q}{[q^2+2xpq+xp^2-(1-x)\gamma_0^2]^3} = \\
&= i^2\,\frac{2\pi^{\frac{3}{2}}\Gamma(3-\frac{3}{2})}{\Gamma(3)}\int_0^1 \frac{(1-x)dx}{[+x(1-x)p^2-(1-x)\gamma_0^2]^{\frac{3}{2}}} = \\
&= -\frac{2\pi^{\frac{3}{2}}\Gamma(3-\frac{3}{2})}{\Gamma(3)}\int_0^1 \frac{dx}{[+xp^2-\gamma_0^2][(1-x)(xp^2-\gamma_0^2)]^{\frac{3}{2}}} = \\
&= -\frac{2\pi^{\frac{3}{2}}\Gamma(3-\frac{3}{2})}{\Gamma(3)}\frac{\partial^2}{\partial(\gamma_0^2)^2}\int_0^1 dx\left[\frac{xp^2-\gamma_0^2}{1-x}\right]^{\frac{3}{2}} = \\
&= -\frac{2\pi^{\frac{3}{2}}\Gamma(3-\frac{3}{2})}{\Gamma(3)}\frac{\partial^2}{\partial(\gamma_0^2)^2}\int_0^1 dx\left[\frac{p^2-\gamma_0^2}{1-x}-p^2\right]^{\frac{3}{2}} = \\
&= -\frac{2\pi^{\frac{3}{2}}\Gamma(3-\frac{3}{2})}{\Gamma(3)}\frac{\partial^2}{\partial(\gamma_0^2)^2}\int_1^\infty \frac{dt}{t^2}\left[(p^2-\gamma_0^2)t-p^2\right]^{\frac{3}{2}} = \\
&= -\frac{2\pi^{\frac{3}{2}}\Gamma(3-\frac{3}{2})}{\Gamma(3)}\int_1^\infty \frac{dt}{t^2}\left[(p^2-\gamma_0^2)t-p^2\right]^{\frac{-3}{2}} = \\
&= -\frac{2\pi^{\frac{3}{2}}\Gamma(3-\frac{3}{2})}{\Gamma(3)}\frac{1}{(p^2-\gamma_0^2)}\int_{\gamma_0^2}^\infty x^{\frac{-3}{2}}dx = \\
&= -\frac{\pi^2}{2}\frac{1}{\gamma_0(p^2-\gamma_0^2)}\,.
\end{aligned} \tag{214}$$

Then let us choose λ' such that $\lambda' = \frac{2\gamma_0}{\pi^2}$. From this value of λ' we see that the BS equation (211) holds. Thus the ground state solution is of the form (212). We see that when $p_E = 0$ and $\omega_b^2 = \mathbf{p}^2 + \gamma_0^2$ then this ground state gives the well known nonrelativistic ground state of the form $\frac{1}{(\mathbf{p}^2+\gamma_0^2)^2}$ of binding system such as the hydrogen atom.

25 Bethe-Salpeter equation with seagull vertex potential

Let us then introduce the following reformulated relativistic Bethe-Salpeter equation which is also with the seagull vertex potential of binding:

$$\begin{aligned}
\phi(p) = &\frac{-\lambda'}{p^2-\gamma_0^2}\times \\
&\times\int\left[\frac{i}{(p-q)^2}+\frac{i\alpha}{4((p-q)^2)^{\frac{1}{2}}}\right]\phi(q)d^3q\,,
\end{aligned} \tag{215}$$

where a factor e^2 of both the Coulomb-type potential and the seagull vertex potential is absorbed to the coupling constant λ'.

Let us solve (215) for the relativistic bound states of particles. We write the ground state solution in the following form:

$$\phi(p) = \phi_0(p) + \alpha\phi_1(p)\,, \tag{216}$$

where $\phi_0(p)$ is the ground state of the BS equation when the interaction potential only consists of the Coulomb-type potential. Let us then determine the $\phi_1(p)$.

From (215) by comparing the coefficients of the $\alpha^j, j = 0, 1$ on both sides of BS equation we have the following equation for $\phi_1(p)$:

$$\begin{aligned}
\phi_1(p) = &\frac{-\lambda'}{p^2-\gamma_0^2}\int\left[\frac{i}{4((p-q)^2)^{\frac{1}{2}}}\right]\phi_0(q)d^3q\,+ \\
&+\frac{-\lambda'}{p^2-\gamma_0^2}\int\left[\frac{i}{((p-q)^2)}+\frac{i\alpha}{4((p-q)^2)^{\frac{1}{2}}}\right]\phi_1(q)d^3q\,.
\end{aligned} \tag{217}$$

This is a nonhomogeneous linear Fredholm integral equation. We can find its solution by perturbation. As a first order approximation we have the following approximation of $\phi_1(p)$:

$$
\begin{aligned}
\phi_1(p) &\approx \tfrac{-\lambda'}{p^2-\gamma_0^2}\int\tfrac{i}{4((p-q)^2)^{\frac12}}\,\phi_0(q)d^3q \;= \\
&= \tfrac{-\lambda'}{p^2-\gamma_0^2}\int\tfrac{i}{4((p-q)^2)^{\frac12}}\,\tfrac{1}{(q^2-\gamma_0^2)^2}\,d^3q \;= \\
&= \tfrac{-\lambda'}{p^2-\gamma_0^2}\,\tfrac{i\Gamma(1+\frac12+2-1)}{4\Gamma(1+\frac12-1)\Gamma(1+2-1)}\int_0^1 y^{\frac12}(1-y)dy\;\times \\
&\times\int\tfrac{d^3q}{[q^2-2qpy+p^2y-(1-y)\gamma_0^2]^{2+\frac12}} \;= \\
&= \tfrac{-\lambda'}{p^2-\gamma_0^2}\,\tfrac{i\Gamma(\frac12+2)}{4\Gamma(\frac12)\Gamma(2)}\int_0^1\tfrac{i\pi^{\frac32}\Gamma(\frac52-\frac32)y^{\frac12}(1-y)dy}{\Gamma(\frac52)(p^2y(1-y)-(1-y)\gamma_0^2)} \;= \\
&= \tfrac{\lambda'}{p^2-\gamma_0^2}\,\tfrac{\pi^{\frac32}}{4\Gamma(\frac12)}\int_0^1 y^{\frac12}dy\,\tfrac{1}{(p^2y-\gamma_0^2)} \;= \\
&= \tfrac{\lambda'\pi}{p^2-\gamma_0^2}\,\tfrac{1}{4|p|\gamma_0}\log\left|\tfrac{|p|-\gamma_0}{|p|+\gamma_0}\right| \;= \\
&= \tfrac{\pi}{p^2-\gamma_0^2}\,\tfrac{2\gamma_0}{\pi^2}\,\tfrac{1}{4|p|\gamma_0}\log\left|\tfrac{|p|-\gamma_0}{|p|+\gamma_0}\right| \;= \\
&= \tfrac{1}{2\pi(p^2-\gamma_0^2)|p|}\log\left|\tfrac{|p|-\gamma_0}{|p|+\gamma_0}\right|,
\end{aligned}
\tag{218}
$$

where $|p| = \sqrt{p^2}$.

Thus we have the ground state $\phi(p) = \phi_0(p) + \alpha\phi_1(p)$ where p denotes an energy-momentum vector with a two dimensional momentum. Thus this ground state is for a two dimensional (momentum) subspace. We may extend it to the ground state of the form $\phi(p) = \phi_0(p) + \alpha\bar{\phi}_1(p)$ where p denotes a four dimensional energy-momentum vector with a three dimensional momentum; and due to the special nature that $\phi_1(p)$ is obtained by a two dimensional space statistics the extension $\bar{\phi}_1(p)$ of $\phi_1(p)$ to with a three dimensional momentum is a wave function obtained by multiplying $\phi_1(p)$ with a delta function concentrating at 0 of a one dimensional momentum variable and the variable p of $\phi_1(p)$ is extended to be a four dimensional energy-momentum vector with a three dimensional momentum.

Let us use this form of the ground state $\phi(p) = \phi_0(p) + \alpha\bar{\phi}_1(p)$ to compute new QED effects in the orthopositronium decay rate where there is a discrepancy between theoretical result and the experimental result [33–52].

26 New QED effect of orthopositronium decay rate

From the seagull vertex let us find new QED effect to the orthopositronium decay rate where there is a discrepancy between theory and experimental result [33–52]. Let us compute the new one-loop effect of orthopositronium decay rate which is from the seagull vertex potential.

From the seagull vertex potential the positronium ground state is modified from $\phi(p)=\phi_0(p)$ to $\phi(p)=\phi_0(p)+\alpha\bar{\phi}_1(p)$. Let us apply this form of the ground state of positronium to the computation of the orthopositronium decay rate.

Let us consider the nonrelativistic case. In this case we have $\phi_0(\mathbf{p}) = \frac{1}{(\mathbf{p}^2+\gamma_0^2)^2}$ and:

$$\phi_1(\mathbf{p}) = \frac{-1}{2\pi(\mathbf{p}^2+\gamma_0^2)|\mathbf{p}|}\log\left|\frac{|\mathbf{p}|-\gamma_0}{|\mathbf{p}|+\gamma_0}\right|. \tag{219}$$

Let M denotes the decay amplitude. Let M_0 denotes the zero-loop decay amplitude. Then following the approach in the computation of the positronium decay rate [33–52] the first order decay rate Γ is given by:

$$
\begin{aligned}
&\int 8\pi^{\frac12}\gamma_0^{\frac52}\left[\phi_0(\mathbf{p})+\alpha\bar{\phi}_1(\mathbf{p})\right]M_0(\mathbf{p})d^3\mathbf{p} =: \\
&\qquad =: \Gamma_0 + \alpha\Gamma_{seagull}\,,
\end{aligned}
\tag{220}
$$

where $8\pi^{\frac12}\gamma_0^{\frac52}$ is the normalized constant for the usual unnormalized ground state wave function ϕ_0 [33–52].

We have that the first order decay rate Γ_0 is given by [33–52]:

$$
\begin{aligned}
\Gamma_0 &:= \tfrac{1}{(2\pi)^3}\int 8\pi^{\frac12}\gamma_0^{\frac52}\phi_0(\mathbf{p})M_0(\mathbf{p})d^3\mathbf{p} \;= \\
&= \tfrac{1}{(2\pi)^3}\int\tfrac{8\pi^{\frac12}\gamma_0^{\frac52}}{(\mathbf{p}^2+\gamma_0^2)^2}M_0(\mathbf{p})d^3\mathbf{p} \;\approx \\
&\approx \psi_0(\mathbf{r}=0)M_0(0) \;= \\
&= \tfrac{8\pi^{\frac12}\gamma_0^{\frac52}}{(2\pi)^3}\int\tfrac{d^3\mathbf{p}}{(\mathbf{p}^2+\gamma_0^2)^2}M_0(0) \;= \\
&= \tfrac{8\pi^{\frac12}\gamma_0^{\frac52}}{(2\pi)^3}\,\tfrac{\pi^2}{\gamma_0}M_0(0) \;= \\
&= \tfrac{1}{(\pi a^3)^{\frac12}}\,M_0(0)\,,
\end{aligned}
\tag{221}
$$

where $\psi_0(\mathbf{r})$ denotes the usual nonrelativistic ground state wave function of positronium; and $a = \frac{1}{\gamma_0}$ is as the radius of the positronium. In the above equation the step $\approx$ holds since $\phi_0(\mathbf{p}) \to 0$ rapidly as $\mathbf{p} \to \infty$ such that the effect of $M_0(\mathbf{p})$ is small for $\mathbf{p} \neq 0$; as explained in [33]- [52].

Then let us consider the new QED effect of decay rate from $\bar{\phi}_1(\mathbf{p})$. As the three dimensional space statistics in the Section on the seagull vertex potential we have the following statistics of the decay rate from $\bar{\phi}_1(\mathbf{p})$:

$$
\begin{aligned}
\Gamma_{seagull} &= \tfrac{1}{(2\pi)^3}\int 8\pi^{\frac12}\gamma_0^{\frac52}\,\bar{\phi}_1(\mathbf{p})M_0(\mathbf{p})d^3\mathbf{p} \;= \\
&= \tfrac{1}{(2\pi)^3}\int 8\pi^{\frac12}\gamma_0^{\frac52}\,\phi_1(\mathbf{p})M_0(\mathbf{p})d^2\mathbf{p} \;\approx \\
&\approx \tfrac{8\pi^{\frac12}\gamma_0^{\frac52}}{(2\pi)^3}\int\phi_1(\mathbf{p})M_0(0)d^2\mathbf{p} \;= \\
&= \tfrac{-8\pi^{\frac12}\gamma_0^{\frac52}}{(2\pi)^3}\int\tfrac{\log\left|\frac{|\mathbf{p}|-\gamma_0}{|\mathbf{p}|+\gamma_0}\right|}{2\pi(\mathbf{p}^2+\gamma_0^2)|\mathbf{p}|}d^2\mathbf{p}M_0(0) \;= \\
&= \tfrac{8\pi^{\frac12}\gamma_0^{\frac52}}{(2\pi)^3 2\pi}\,\tfrac{\pi^3}{2\gamma_0}M_0(0) \;= \\
&= \tfrac{1}{4(\pi a^3)^{\frac12}}M_0(0)\,,
\end{aligned}
\tag{222}
$$

where the step $\approx$ holds as similar the equation (221) since in the two dimensional integral of $\phi_1(\mathbf{p})$ we have that $\phi_1(\mathbf{p}) \to 0$ as $\mathbf{p} \to \infty$ such that it tends to zero as rapidly as the three dimensional case of $\phi_0(\mathbf{p}) \to 0$.

Thus we have:

$$\alpha\Gamma_{seagull} = \frac{\alpha}{4}\,\Gamma_0\,. \tag{223}$$

From the literature of computation of the orthopositronium decay rate we have that the computed orthopositronium decay rate (up to the order α^2) is given by [33–52]:

$$\Gamma_{\text{o-Ps}} = \Gamma_0\big[1 + A\tfrac{\alpha}{\pi} + \tfrac{\alpha^2}{3}\log\alpha + B(\tfrac{\alpha}{\pi})^2 - \tfrac{\alpha^3}{2\pi}\log^2\alpha\big] = \\ = 7.039934(10)\,\mu\text{s}^{-1}, \tag{224}$$

where $A = -10.286\,606(10)$, $B = 44.52(26)$ and $\Gamma_0 = \frac{9}{2}(\pi^2 - 9)m\alpha^6 = 7.211\,169\,\mu\text{s}^{-1}$.

Then with the additional decay rate from the seagull vertex potential (or from the modified ground state of positronium) we have the following computed orthopositronium decay rate (up to the order α^2):

$$\Gamma_{\text{o-Ps}} + \alpha\Gamma_{seagull} = \\ = \Gamma_0\big[1 + (A + \tfrac{\pi}{4})\tfrac{\alpha}{\pi} + \tfrac{\alpha^2}{3}\log\alpha + B(\tfrac{\alpha}{\pi})^2 - \tfrac{\alpha^3}{2\pi}\log^2\alpha\big] = \\ = 7.039934(10) + 0.01315874\ \mu\text{s}^{-1} = \\ = 7.052092(84)\,\mu\text{s}^{-1}. \tag{225}$$

This agrees with the two Ann Arbor experimental values where the two Ann Arbor experimental values are given by: $\Gamma_{\text{o-Ps}}(\text{Gas}) = 7.0514(14)\,\mu\text{s}^{-1}$ and $\Gamma_{\text{o-Ps}}(\text{Vacuum}) = 7.0482(16)\,\mu\text{s}^{-1}$ [33, 34].

We remark that for the decay rate $\alpha\Gamma_{seagull}$ we have only computed it up to the order α. If we consider the decay rate $\alpha\Gamma_{seagull}$ up to the order α^2 then the decay rate (225) will be reduced since the order α of $\Gamma_{seagull}$ is of negative value.

If we consider only the computed orthopositronium decay rate up to the order α with the term $B(\frac{\alpha}{\pi})^2$ omitted, then $\Gamma_{\text{o-Ps}} = 7.038202\ \mu\text{s}^{-1}$ (see [33–52]) and we have the following computed orthopositronium decay rate:

$$\Gamma_{\text{o-Ps}} + \alpha\Gamma_{seagull} = 7.05136074\,\mu\text{s}^{-1}. \tag{226}$$

This also agrees with the above two Ann Arbor experimental values and is closer to these two experimental values.

On the other hand the Tokyo experimental value given by $\Gamma_{\text{o-Ps}}(\text{Powder}) = 7.0398(29)\,\mu\text{s}^{-1}$ [35] may be interpreted by that in this experiment the QED effect $\Gamma_{seagull}$ of the seagull vertex potential is suppressed due to the special two dimensional statistical form of $\Gamma_{seagull}$ (Thus the additional effect of the modified ground state ϕ of the positronium is suppressed). Thus the value of this experiment agrees with the computational result $\Gamma_{\text{o-Ps}}$. Similarly the experimental result of another Ann Arbor experiment given by $7.0404(8)\,\mu\text{s}^{-1}$ [36] may also be interpreted by that in this experiment the QED effect $\Gamma_{seagull}$ of the seagull vertex potential is suppressed due to the special two dimensional statistical form of $\Gamma_{seagull}$.

27 Graviton constructed from photon

It is well known that Einstein tried to find a theory to unify gravitation and electromagnetism [1, 79, 80]. The search for such a theory has been one of the major research topics in physics [80–88]. Another major research topic in physics is the search for a theory of quantum gravity [89–120]. In fact, these two topics are closely related. In this Section, we propose a theory of quantum gravity that unifies gravitation and electromagnetism.

In the above Sections the photon is as the quantum Wilson loop with the $U(1)$ gauge group for electrodynamics. In the above Sections we have also shown that the corresponding quantum Wilson line can be regarded as the photon propagator in analogy to the usual concept of propagator. In this section from this quantum photon propagator, the quantum graviton propagator and the graviton are constructed. This construction forms the foundation of a theory of quantum gravity that unifies gravitation and electromagnetism.

It is well known that Weyl introduced the gauge concept to unify gravitation and electromagnetism [80]. However this gauge concept of unifying gravitation and electromagnetism was abandoned because of the criticism of the path dependence of the gauge (it is well known that this gauge concept later is important for quantum physics as phase invariance) [1]. In this paper we shall use again Weyl's gauge concept to develop a theory of quantum gravity which unifies gravitation and electromagnetism. We shall show that the difficulty of path dependence of the gauge can be solved in this quantum theory of unifying gravitation and electromagnetism.

Let us consider a differential of the form $g(s)ds$ where $g(s)$ is a field variable to be determined. Let us consider a symmetry of the following form:

$$g(s)ds = g'(s')ds', \tag{227}$$

where s is transformed to s' and $g'(s)$ is a field variable such that (227) holds. From (227) we have a symmetry of the following form:

$$g(s)^* g(s) ds^2 = g'^*(s') g'(s') ds'^2, \tag{228}$$

where $g^*(s)$ and $g'^{*(s)}$ denote the complex conjugate of $g(s)$ and $g'(s)$ respectively. This symmetry can be considered as the symmetry for deriving the gravity since we can write $g(s)^* g(s) ds^2$ into the following metric form for the four dimensional space-time in General Relativity:

$$g(s)^* g(s) ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \tag{229}$$

where we write $ds^2 = a_{\mu\nu}dx^\mu dx^\nu$ for some functions $a_{\mu\nu}$ by introducing the space-time variable x^μ, $\mu = 0, 1, 2, 3$ with x^0 as the time variable; and $g_{\mu\nu} = g(s)^* g(s) a_{\mu\nu}$. Thus from the symmetry (227) we can derive General Relativity.

Let us now determine the variable $g(s)$. Let us consider $g(s) = W(z_0, z(s))$, a quantum Wilson line with $U(1)$ group where z_0 is fixed. When $W(z_0, z(s))$ is the classical Wilson line then it is of path dependence and thus there is a difficulty to use it to define $g(s) = W(z_0, z(s))$. This is also the difficulty of Weyl's gauge theory of unifying gravitation and electromagnetism. Then when $W(z_0, z(s))$ is the quantum Wilson line because of the quantum nature of unspecification of paths we have that $g(s) = W(z_0, z(s))$ is well defined where the whole path of connecting z_0 and $z(s)$ is unspecified (except the two end points z_0 and $z(s)$).

Thus for a given transformation $s' \to s$ and for any (continuous and piecewise smooth) path connecting z_0 and $z(s)$ the resulting quantum Wilson line $W'(z_0, z(s(s')))$ is again of the form $W(z_0, z(s)) = W(z_0, z(s(s')))$. Let $g'(s') = W'(z_0, z(s(s')))\frac{ds}{ds'}$. Then we have:

$$\begin{aligned} & g'^*(s')g'(s')ds'^2 = \\ &= W'^*(z_0, z(s(s')))W'(z_0, z(s(s')))(\tfrac{ds}{ds'})^2 ds'^2 = \\ &= W^*(z_0, z(s))W(z_0, z(s))(\tfrac{ds}{ds'})^2 ds'^2 = \\ &= g(s)^* g(s) ds^2. \end{aligned} \tag{230}$$

This shows that the quantum Wilson line $W(z_0, z(s))$ can be the field variable for the gravity and thus can be the field variable for quantum gravity since $W(z_0, z(s))$ is a quantum field variable.

Then we consider the operator $W(z_0, z)W(z_0, z)$. From this operator $W(z_0, z)W(z_0, z)$ we can compute the operator $W^*(z_0, z)W(z_0, z)$ which is as the absolute value of this operator. Thus this operator $W(z_0, z)W(z_0, z)$ can be regarded as the quantum graviton propagator while the quantum Wilson line $W(z_0, z)$ is regarded as the quantum photon propagator for the photon field propagating from z_0 to z. Let us then compute this quantum graviton propagator $W(z_0, z)W(z_0, z)$. We have the following formula:

$$\begin{aligned} & W(z, z_0)W(z_0, z) = \\ &\quad = e^{-\hat{t}\log[\pm(z-z_0)]} A\, e^{\hat{t}\log[\pm(z_0-z)]}, \end{aligned} \tag{231}$$

where $\hat{t} = -\frac{e_0^2}{k_0}$ for the $U(1)$ group ($k_0 > 0$ is a constant and we may let $k_0 = 1$) where the term $e^{-\hat{t}\log[\pm(z-z_0)]}$ is obtained by solving the first form of the dual form of the KZ equation and the term $e^{\hat{t}\log[\pm(z_0-z)]}$ is obtained by solving the second form of the dual form of the KZ equation.

Then we change the $W(z, z_0)$ of $W(z, z_0)W(z_0, z)$ in (231) to the second factor $W(z_0, z)$ of $W(z, z_0)W(z_0, z)$ by reversing the proper time direction of the path of connecting z and z_0 for $W(z, z_0)$. This gives the graviton propagator $W(z_0, z)W(z_0, z)$. Then the reversing of the proper time direction of the path of connecting z and z_0 for $W(z, z_0)$ also gives the reversing of the first form of the dual form of the KZ equation to the second form of the dual form of the KZ equation. Thus by solving the second form of dual form of the KZ equation we have that $W(z_0, z)W(z_0, z)$ is given by:

$$\begin{aligned} W(z_0, z)W(z_0, z) &= e^{\hat{t}\log[\pm(z-z_0)]} A e^{\hat{t}\log[\pm(z-z_0)]} = \\ &= e^{2\hat{t}\log[\pm(z-z_0)]} A\,. \end{aligned} \tag{232}$$

In (232) let us define the following constant G:

$$G := -2\hat{t} = 2\,\frac{e_0^2}{k_0}\,. \tag{233}$$

We regard this constant G as the gravitational constant of the law of Newton's gravitation and General Relativity. We notice that from the relation $e_0 = \left(z_A^{\frac{1}{2}}\right)^{-1} e = \frac{1}{n_e}\, e$ where the renormalization number $n_e = z_A^{\frac{1}{2}}$ is a very large number we have that the bare electric charge e_0 is a very small number. Thus the gravitational constant G given by (233) agrees with the fact that the gravitational constant is a very small constant. This then gives a closed relationship between electromagnetism and gravitation.

We remark that since in (232) the factor $-G\log r_1 = G\log\frac{1}{r_1} < 0$ (where we define $r_1 = |z - z_0|$ and r_1 is restricted such that $r_1 > 1$) is the fundamental solution of the two dimensional Laplace equation we have that this factor (together with the factor $e^{-G\log r_1} = e^{G\log\frac{1}{r_1}}$) is analogous to the fundamental solution $-G\frac{1}{r}$ of the three dimensional Laplace equation for the law of Newton's gravitation. Thus the operator $W(z_0, z)W(z_0, z)$ in (232) can be regarded as the graviton propagator which gives attractive effect when $r_1 > 1$. Thus the graviton propagator (232) gives the same attractive effect of $-G\frac{1}{r}$ for the law of Newton's gravitation.

On the other hand when $r_1 \leqslant 1$ we have that the factor $-G\log r_1 = G\log\frac{1}{r_1} \geqslant 0$. In this case we may consider that this graviton propagator gives repulsive effect. This means that when two particles are very close to each other then the gravitational force can be from attractive to become repulsive. This repulsive effect is a modification of $-G\frac{1}{r}$ for the law of Newton's gravitation for which the attractive force between two particles tends to ∞ when the distance between the two particles tends to 0.

Then by multiplying two masses m_1 and m_2 (obtained from the winding numbers of Wilson loops in (73) of two particles to the graviton propagator (232) we have the following formula:

$$Gm_1 m_2 \log\frac{1}{r_1}\,. \tag{234}$$

From this formula (234) by introducing the space variable $\mathbf{x}$ as a statistical variable via the Lorentz metric: $ds^2 =$

$= dt^2 - d\mathbf{x}^2$ we have the following statistical formula which is the potential law of Newton's gravitation:

$$-GM_1M_2\,\frac{1}{r}\,, \tag{235}$$

where M_1 and M_2 denotes the masses of two objects.

We remark that the graviton propagator (232) is for matters. We may by symmetry find a propagator $f(z_0, z)$ of the following form:

$$f(z_0, z) := e^{-2\hat{t}\log[\pm(z-z_0)]}A\,. \tag{236}$$

When $|z - z_0| > 1$ this propagator $f(z_0, z)$ gives repulsive effect between two particles and thus is for anti-matter particles where by the term anti-matter we mean particles with the repulsive effect (236). Then since $|f(z_0, z)| \to \infty$ as $|z - z_0| \to \infty$ we have that two such anti-matter particles can not physically exist. However in the following Section on dark energy and dark matter we shall show the possibility of another repulsive effect among gravitons.

As similar to that the quantum Wilson loop $W(z_0, z_0)$ is as the photon we have that the following double quantum Wilson loop can be regarded as the graviton:

$$W(z_0, z)W(z_0, z)W(z, z_0)W(z, z_0)\,. \tag{237}$$

28 Dark energy and dark matter

By the method of computation of solutions of KZ equations and the computation of the graviton propagator (232) we have that (237) is given by:

$$\begin{aligned} &W(z_0, z)W(z_0, z)W(z, z_0)W(z, z_0) = \\ &= e^{2\hat{t}\log[\pm(z-z_0)]}A_g e^{-2\hat{t}\log[\pm(z-z_0)]} = \\ &= R^{2n}A_g,\; n = 0, \pm1, \pm2, \pm3, \ldots \end{aligned} \tag{238}$$

where A_g denotes the initial operator for the graviton. Thus as similar to the quantization of energy of photons we have the following quantization of energy of gravitons:

$$h\nu = 2\pi e_0^2 n, \quad n = 0, \pm1, \pm2, \pm3, \ldots \tag{239}$$

As similar to that a photon with a specific frequency can be as a magnetic monopole because of its loop nature we have that the graviton (237) with a specific frequency can also be regarded as a magnetic monopole (which is similar to but different from the magnetic monopole of the photon kind) because of its loop nature. (This means that the loop nature gives magnetic property.)

Since we still can not directly observe the graviton in experiments the quantized energies (239) of gravitons can be identified as dark energy. Then as similar to the construction of electrons from photons we construct matter from gravitons by the following formula:

$$W(z_0, z)W(z_0, z)W(z, z_0)W(z, z_0)Z\,, \tag{240}$$

where Z is a complex number as a state acted by the graviton.

Similar to the mechanism of generating mass of electron we have that the mechanism of generating the mass m_d of these particles is given by the following formula:

$$m_d c^2 = 2\pi e_0^2 n_d = \pi G n_d = h\nu_d \tag{241}$$

for some integer n_d and some frequency ν_d.

Since the graviton is not directly observable it is consistent to identify the quantized energies of gravitons as dark energy and to identify the matters (240) constructed by gravitons as dark matter.

It is interesting to consider the quantum gravity effect between two gravitons. When a graviton propagator is connected to a graviton we have that this graviton propagator is extended to contain a closed loop since the graviton is a closed loop. In this case as similar to the quantum photon propagator this extended quantum graviton propagator can give attractive or repulsive effect. Then for stability the extended quantum graviton propagator tends to give the repulsive effect between the two gravitons. Thus the quantum gravity effect among gravitons can be repulsive which gives the diffusion of gravitons and thus gives a diffusion phenomenon of dark energy. Furthermore for stability more and more open-loop graviton propagators in the space form closed loops. Thus more and more gravitons are forming and the repulsive effect of gravitons gives the accelerating expansion of the universe [53–57].

Let us then consider the quantum gravity effect between two particles of dark matter. When a graviton propagator is connected to two particles of dark matter not by connecting to the gravitons acting on the two particles of dark matter we have that the graviton propagator gives only attractive effect between the two particles of dark matter. Thus as similar to the gravitational force among the usual non-dark matters the gravitational force among dark matters are mainly attractive. Then when the graviton propagator is connected to two particles of dark matter by connecting to the gravitons acting on the two particles of dark matter then as the above case of two gravitons we have that the graviton propagator can give attractive or repulsive effect between the two particles of dark matter.

29 Conclusion

In this paper a quantum loop model of photon is established. We show that this loop model is exactly solvable and thus may be considered as a quantum soliton. We show that this nonlinear model of photon has properties of photon and magnetic monopole and thus photon with some specific frequency may be identified with the magnetic monopole. From the discrete winding numbers of this loop model we can derive the quantization property of energy for the Planck's formula of radiation and the quantization property of electric charge. We show that the charge quantization is derived from the energy quantization. On the other hand from the nonlinear model of photon a nonlinear loop model of electron is established. This model of electron has a mass mechanism which generates mass to the electron where the mass of the electron is from the photon-loop. With this mass mechanism for generating mass the Higgs mechanism of the conventional QED theory for generating mass is not necessary.

We derive a QED theory which is not based on the four dimensional space-time but is based on the one dimensional proper time. This QED theory is free of ultraviolet divergences. From this QED theory the quantum loop model of photon is established. In this QED theory the four dimensional space-time is derived for statistics. Using the space-time statistics, we employ Feynman diagrams and Feynman rules to compute the basic QED effects such as the vertex correction, the photon self-energy and the electron self-energy. From these QED effects we compute the anomalous magnetic moment and the Lamb shift. The computation is of simplicity and accuracy and the computational result is better than that of the conventional QED theory in that the computation is simpler and it does not involve numerical approximation as that in the conventional QED theory where the Lamb shift is approximated by numerical means.

From the QED theory in this paper we can also derive a new QED effect which is from the seagull vertex of this QED theory. By this new QED effect and by a reformulated Bethe-Salpeter (BS) equation which resolves the difficulties of the BS equation (such as the existence of abnormal solutions) and gives a modified ground state wave function of the positronium. Then from this modified ground state wave function of the positronium a new QED effect of the orthopositronium decay rate is derived such that the computed orthopositronium decay rate agrees with the experimental decay rate. Thus the *orthopositronium lifetime puzzle* is completely resolved where we also show that the recent resolution of this orthopositronium lifetime puzzle only partially resolves this puzzle due to the special nature of two dimensional space statistics of this new QED effect.

By this quantum loop model of photon a theory of quantum gravity is also established where the graviton is constructed from the photon. Thus this theory of quantum gravity unifies gravitation and electromagnetism. In this unification of gravitation and electromagnetism we show that the universal gravitation constant G is proportional to e_0^2 where e_0 is the bare electric charge which is a very small constant and is related to the renormalized charge e by the formula $e_0 = \frac{1}{n_e} e$ where the renormalized number n_e is a very large winding number of the photon-loop. This relation of G with e_0 (and thus with e) gives a closed relationship between gravitation and electromagnetism. Then since gravitons are not directly observable the quantized energies of gravitons are as dark energy and the particles constructed by gravitons are as dark matter. We show that the quantum gravity effect among particles of dark matter is mainly attractive (and it is possible to be repulsive when a graviton loop is formed in the graviton propagator) while the quantum gravity effect among gravitons can be repulsive which gives the diffusion of gravitons and thus gives the diffusion phenomenon of dark energy and the accelerating expansion of the universe.

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SPECIAL REPORT

Reconsideration of the Uncertainty Relations and Quantum Measurements

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Discussions on uncertainty relations (UR) and quantum measurements (QMS) persisted until nowadays in publications about quantum mechanics (QM). They originate mainly from the conventional interpretation of UR (CIUR). In the most of the QM literarure, it is underestimated the fact that, over the years, a lot of deficiencies regarding CIUR were signaled. As a rule the alluded deficiencies were remarked disparately and discussed as punctual and non-essential questions. Here we approach an investigation of the mentioned deficiencies collected in a conclusive ensemble. Subsequently we expose a reconsideration of the major problems referring to UR and QMS. We reveal that all the basic presumption of CIUR are troubled by insurmountable deficiencies which require the indubitable failure of CIUR and its necessary abandonment. Therefore the UR must be deprived of their statute of crucial pieces for physics. So, the aboriginal versions of UR appear as being in postures of either (i) thought-experimental fictions or (ii) simple QM formulae and, any other versions of them, have no connection with the QMS. Then the QMS must be viewed as an additional subject comparatively with the usual questions of QM. For a theoretical description of QMS we propose an information-transmission model, in which the quantum observables are considered as random variables. Our approach directs to natural solutions and simplifications for many problems regarding UR and QMS.

1 Introduction

The uncertainty relations (UR) and quantum measurements (QMS) constitute a couple of considerable popularity, frequently regarded as a crucial pieces of quantum mechanics (QM). The respective crucial character is often glorified by assertions like:

(i) UR are expression of "*the most important principle of the twentieth century physics*" [1];

(ii) the description of QMS is "*probably the most important part of the theory (QM)*" [2].

The alluded couple constitute the basis for the so-called Conventional Interpretation of UR (CIUR). Discussions about CIUR are present in a large number of early as well as recent publications (see [1–11] and references therein). Less mentioned is the fact that CIUR ideas are troubled by a number of still unsolved deficiencies. As a rule, in the main stream of CIUR partisan publications, the alluded deficiencies are underestimated (through unnatural solutions or even by omission).

Nevertheless, during the years, in scientific literature were recorded remarks such as:

(i) UR "*are probably the most controverted formulae in the whole of the theoretical physics*" [12];

(ii) "*the word ("measurement") has had such a damaging efect on the discussions that… it should be banned altogether in quantum mechanics*" [13];

(iii) "*the idea that there are defects in the foundations of orthodox quantum theory is unquestionable present in the conscience of many physicists*" [14];

(iv) "*Many scientists have considered the conceptual framework of quantum theory to be unsatisfactory. The very foundations of Quantum Mechanics is a matter that needs to be resolved in order to achieve and gain a deep physical understanding of the underlying physical procedures that constitute our world*" [15].

The above mentioned status of things require further studies and probably new views. We believe that a promising strategy to satisfy such requirements is to develop an investigation guided by the following objectives (obj.):

(obj.1) to identify the basic presumptions of CIUR;

(obj.2) to reunite together all the significant deficiencies of CIUR;

(obj.3) to examine the verity and importance of the respective deficiencies;

(obj.4) to see if such an examination defends or incriminate CIUR;

(obj.5) in the latter case to admit the failure of CIUR and its abanonment;

(obj.6) to search for a genuine reinterpretation of UR;

(obj.7) to evaluate the consequences of the UR reinterpretation for QMS;

(obj.8) to promote new views about QMS;

(obj.9) to note a number of remarks on some adjacent questions.

A such guided investigation we are approaching in the next sections of this paper. The present approach try to complete and to improve somewhat less elaborated ideas from few of our previous writings. But, due to a lot of unfortunate chances, and contrary to my desire, the respective writings were edited in modest publications [16–18] or remained as preprints registred in data bases of LANL and CERN libraries (see [19]).

2 Shortly on CIUR history and its basic presumptions

The story of CIUR began with the Heisenberg's seminal work [20] and it starts [21] from the search of general answers to the primary questions (q.):

(q.1) Are all measurements affected by measuring uncertainties?

(q.2) How can the respective uncertainties be described quantitatively?

In connection with the respective questions, in its subsequent extension, CIUR promoted the suppositions (s.):

(s.1) The measuring uncertainties are due to the perturbations of the measured microparticle (system) by its interactions with the measuring instrument;

(s.2) In the case of macroscopic systems the mentioned perturbations can be made arbitrarily small and, consequently, always the corresponding uncertainties can be considered as negligible;

(s.3) On the other hand, in the case of quantum microparticles (of atomic size) the alluded perturbations are essentially unavoidable and consequently for certain measurements (see below) the corresponding uncertainties are non-negligible.

Then CIUR limited its attention only to the quantum cases, for which restored to an amalgamation of the following motivations (m.):

(m.1) Analysis of some thought (gedanken) measuring experiments;

(m.2) Appeal to the theoretical version of UR from the existing QM.

Notification: In the present paper we will use the term "observable" (introduced by CIUR literature) for denoting a physical quantity referring to a considered microparticle (system).

Now let us return to the begining of CIUR history. Firstly [20, 22], for argumentation of the above noted motivation (m.1) were imagined some thought experiments on a quantum microparticle, destined to simultaneous measurements of two (canonically) conjugated observables A and B (such are coordinate q and momentum p or time t and energy E). The corresponding "*thought experimental*" (*te*) uncertainties were noted with $\Delta_{te}A$ and $\Delta_{te}B$. They were found as being interconnected trough the following *te*-UR

$$\Delta_{te}A \cdot \Delta_{te}B \geqslant \hbar\,, \tag{1}$$

where $\hbar$ denotes the reduced Planck constant.

As regard the usage of motivation (m.2) in order to promote CIUR few time later was introduced [23, 24] the so-called Robertson Schrödinger UR (RSUR):

$$\Delta_{\Psi}A \cdot \Delta_{\Psi}B \geqslant \frac{1}{2}\left|\left\langle [\hat{A}, \hat{B}] \right\rangle_{\Psi}\right|. \tag{2}$$

In this relation one finds usual QM notations i.e.: (i) $\hat{A}$ and $\hat{B}$ denote the quantum operators associated with the observables A and B of the same microparticle, (ii) $\Delta_{\Psi}A$ and $\Delta_{\Psi}B$ signify the standard deviation of the respective observables, (iii) $\langle(\ldots)\rangle_{\Psi}$ represents the mean value of $(\ldots)$ in the state described by the wave function Ψ, (iv) $[\hat{A}, \hat{B}]$ depict the commutator of the operators $\hat{A}$ and $\hat{B}$ (for some other details about the QM notations and validity of RSUR (2) see the next section).

CIUR was built by regarding the relations (1) and (2), as standard (reference) elements. It started through the writings (and public lectures) of the so-called Copenhagen School partisans. Later CIUR was adopted, more or less explicitely, in a large number of publications.

An attentive examination of the alluded publications show that in the main CIUR is builded onthe following five basic presumptions (***P***):

$\boldsymbol{P_1}$: Quantities $\Delta_{te}A$ and $\Delta_{\Psi}A$ from relations (1) and (2) denoted by a unique symbol ΔA, have similar significance of measuring uncertainty for the observable A refering to the same microparticle. Consequently the respective relations have the same generic interpretation as UR regarding the simultaneous measurements of observables A and B of the alluded microparticle;

$\boldsymbol{P_2}$: In case of a solitary observable A, for a microparticle, the quantity ΔA can have always an unbounded small value. Therefore such an obvservable can be measured without uncertainty in all cases of microparticles (systems) and states;

$\boldsymbol{P_3}$: When two observables A and B are commutable (i.e $[\hat{A}, \hat{B}] = 0$) relation (2) allows for the quantities ΔA and ΔB, regarding the same microparticle, to be unlimitedly small at the same time. That is why such observables can be measured simultaneously and without uncertainties for any microparticle (system) or state. Therefore they are considered as compatible;

$\boldsymbol{P_4}$: If two observables A and B are non-commutable (i.e. $[\hat{A}, \hat{B}] \neq 0$) relation (2) shows that, for a given microparticle, the quantities ΔA and ΔB can be never reduced concomitantly to null values. For that reason such observables can be measured simultaneously only with non-null and interconnected uncertainties, irrespective of the microparticle (system) or state. Hence such observables are considered as incompatible;

$\boldsymbol{P_5}$: Relations (1) and (2), Planck's constant $\hbar$ as well as the measuring peculiarities noted in $\boldsymbol{P_4}$ are typically QM things which have not analogies in classical (non-quantum) macroscopic physics.

Here it must recorded the fact that, in individual publications from the literature which promote CIUR, the above noted presumptions $\boldsymbol{P_1}$–$\boldsymbol{P_5}$ often appear in non-explicit forms and are mentioned separately or only few of them. Also in the same publications the deficiencies of CIUR are omited or underestimated. On the other hand in writings which tackle the deficiencies of CIUR the respective deficiencies are always discussed as separate pieces not reunited in some elucidative ensembles. So, tacitly, in our days CIUR seems to remain a largely adopted doctrine which dominates the questions regarding the foundation and interpretation of QM.

3 Examination of CIUR deficiencies regarded in an elucidative collection

In oder to evaluate the true significance of deficiences regarding CIUR we think that it must discussed together many such deficiences reunited, for a good examination, in an elucidative collection. Such a kind of discussion we try to present below in this section.

Firstly let us examine the deficiences regarding the relation (1). For such a purpose we note the following remark (***R***):

$\boldsymbol{R_1}$: On the relation (1)
In reality the respective relation is an improper piece for a reference/standard element of a supposed solid doctrine such as CIUR. This fact is due to the to the circumstance that such a relation has a transitory/temporary character because it was founded on old resolution criteria (introduced by Abe and Rayleigh — see [22,25]). But the respective criteria were improved in the so-called super-resolution techniques worked out in modern experimental physics (see [26–31] and references). Then it is possible to imagine some super-resolution-thought-experiments (*srte*). So, for the corresponding *srte*-uncertainties $\Delta_{srte}A$ and $\Delta_{srte}B$ of two observables A and B the following relation can be promoted

$$\Delta_{srte}A \cdot \Delta_{srte}B \leqslant \hbar\,. \tag{3}$$

Such a relation is possibly to replace the CIUR basic formula (1). But the alluded possibility invalidate the presumtion $\boldsymbol{P_1}$ and incriminate CIUR in connection with one of its main points.
End of $\boldsymbol{R_1}$

For an argued examination of CIUR deficiences regarding the relation (2) it is of main importance the following remark:

$\boldsymbol{R_2}$: On the aboriginal QM elements
Let us remind briefly some significant elements, selected from the aboriginal framework of usual QM. So we consider a QM microparticle whose state (of orbital nature) is described by the wave function Ψ. Two observables A_j $(j = 1, 2)$ of the respective particle will be described by the operators $\hat{A}_j$. The notation (f, g) will be used for the scalar product of the functions f and g. Correspondingly, the quantities $\langle A_j\rangle_\Psi = (\Psi\,, \hat{A}_j\Psi)$ and $\delta_\Psi\hat{A}_j = \hat{A}_j - \langle\hat{A}_j\rangle_\Psi$ will depict the mean (expected) value respectively the deviation-operator of the observable A_j regarded as a random variable. Then, by denoting the two observable with $A_1 = A$ and $A_2 = B$, we can be write the following Cauchy-Schwarz relation:

$$\begin{aligned}\left(\delta_\Psi\hat{A}\Psi,\, \delta_\Psi\hat{A}\Psi\right)\left(\delta_\Psi\hat{B}\Psi,\, \delta_\Psi\hat{B}\Psi\right) \geqslant \\ \geqslant \left|\left(\delta_\Psi\hat{A}\Psi,\, \delta_\Psi B\Psi\right)\right|^2.\end{aligned} \tag{4}$$

For an observable A_j considered as a random variable the quantity $\Delta_\Psi A_j = \left(\delta_\Psi\hat{A}_j\Psi,\, \delta_\Psi\hat{A}_j\Psi\right)^{\frac{1}{2}}$ signifies its standard deviation. From (4) it results directly that the standard deviations $\Delta_\Psi A$ and $\Delta_\Psi B$ of the mentioned observables satisfy the relation

$$\Delta_\Psi A \cdot \Delta_\Psi B \geqslant \left|\left(\delta_\Psi\hat{A}\Psi,\, \delta_\Psi B\Psi\right)\right|, \tag{5}$$

which can be called *Cauchy-Schwarz formula* (CSF). Note that CSF (5) (as well as the relation (4)) is always valid, i.e. for all observables, paricles and states. Here it is important to specify the fact that the CSF (5) is an aboriginal piece which implies the subsequent and restricted RSUR (1) only in the cases when the operators $\hat{A} = \hat{A}_1$ and $\hat{B} = \hat{A}_2$ satisfy the conditions

$$\left(\hat{A}_j\Psi, \hat{A}_k\Psi\right) = \left(\Psi, \hat{A}_j\hat{A}_k\Psi\right), \qquad (j, k = 1, 2)\,. \tag{6}$$

Indeed in such cases one can write the relation

$$\begin{aligned}&\left(\delta_\Psi\hat{A}\Psi,\, \delta_\Psi\hat{B}\Psi\right) = \\ &= \tfrac{1}{2}\left(\Psi,\, \left(\delta_\Psi\hat{A}\cdot\delta_\Psi\hat{B}\Psi + \delta_\Psi\hat{B}\cdot\delta_\Psi\hat{A}\right)\Psi\right) - \\ &- \tfrac{i}{2}\left(\Psi,\, i\,[\hat{A},\, \hat{B}]\Psi\right),\end{aligned} \tag{7}$$

where the two terms from the right hand side are purely real and imaginary quantities respectively. Therefore in the mentioned cases from (5) one finds

$$\Delta_\Psi A \cdot \Delta_\Psi B \geqslant \frac{1}{2}\left|\left\langle[\hat{A},\, \hat{B}]\right\rangle_\Psi\right| \tag{8}$$

i.e. the well known RSUR (2).

The above reminded aboriginal QM elements prove the following fact. In reality for a role of standard (reference) piece regarding the interpretation of QM aspects must be considered the CSF (5) but not the RSUR (2). But such a reality

incriminate in an indubitable manner all the basic presumptions $\boldsymbol{P_1}$–$\boldsymbol{P_5}$ of CIUR.
End of $\boldsymbol{R_2}$

The same QM elenments reminded in $\boldsymbol{R_2}$, motivate the next remark:

$\boldsymbol{R_3}$: On a denomination used by CIUR
The denomination "*uncertainty*" used by CIUR for quantities like $\Delta_\Psi A$ from (2) is groundless because of the following considerations. As it was noted previously in the aboriginal QM framework, $\Delta_\Psi A$ signifies the standard deviation of the observable A regarded as a random variable. The mentioned framework deals with theoretical concepts and models about the intrinsic (inner) properties of the considered particle but not with aspects of the measurements performed on the respective particle. Consequently, for a quantum microparticle, the quantity $\Delta_\Psi A$ refers to the intrinsic characteristics (reflected in fluctuations) of the observable A. Moreover it must noted the following realities:

(i) For a particle in a given state the quantity $\Delta_\Psi A$ has a well defined value connected with the corresponding wave function Ψ;

(ii) The value of $\Delta_\Psi A$ is not related with the possible modifications of the accuracy regarding the measurement of the observable A.

The alluded realities are attested by the fact that for the same state of the measured particle (i.e. for the same value of $\Delta_\Psi A$) the measuring uncertainties regarding the observable A can be changed through the improving or worsening of experimental devices/procedures. Note that the above mentioned realities imply and justify the observation [32] that, for two variables x and p of the same particle, the usual CIUR statement "*as Δx approaches zero, Δp becomes infinite and vice versa*" is a doubtful speculation. Finally we can conclude that the ensemble of the things revealed in the present remark contradict the presumptions $\boldsymbol{P_2}$–$\boldsymbol{P_4}$ of CIUR. But such a conclusion must be reported as a serious deficience of CIUR.
End of $\boldsymbol{R_3}$

A class of CIUR conceptual deficiences regards the following pairs of canonically conjugated observables: L_z-φ, N-ϕ and E-t ($L_z = z$ component of angular momentum, $\varphi =$ azimuthal angle, $N =$ number, $\phi =$ phase, $E =$ energy, $t =$ time). The respective pairs were and still are considered as being unconformable with the accepted mathematical rules of QM. Such a fact roused many debates and motivated various approaches planned to elucidate in an acceptable manner the missing conformity (for significant references see below within the remarks $\boldsymbol{R_4}$–$\boldsymbol{R_6}$). But so far such an elucidation was not ratified (or admited unanimously) in the scientific literature. In reality one can prove that, for all the three mentioned pairs of observables, the alluded unconformity refers not to conflicts with aboriginal QM rules but to serious disagreements with RSUR (2). Such proofs and their consequences for CIUR we will discuss below in the following remarks:

$\boldsymbol{R_4}$: On the pair L_z-φ
The parts of above alluded problems regarding of the pair L_z-φ were examined in all of their details in our recent paper [33]. There we have revealed the following indubitable facts:

(i) In reality the pair L_z-φ is unconformable only in respect with the secondary and limited piece which is RSUR (2);

(ii) In a deep analysis, the same pair proves to be in a natural conformity with the true QM rules presented in $\boldsymbol{R_2}$;

(iii) The mentioned conformity regards mainly the CSF (5) which can degenerate in the trivial equality $0 = 0$ in some cases rgarding the pair L_z-φ.

But such facts points out an indubitable deficience of CIUR's basic presumption $\boldsymbol{P_4}$.
End of $\boldsymbol{R_4}$

$\boldsymbol{R_5}$: On the pair N- ϕ
The involvement of pair N-ϕ in debates regarding CIUR started [35] subsequently of the Dirac's idea [36] to transcribe the ladder (lowering and raising) operators $\hat{a}$ and $\hat{a}^+$ in the forms

$$\hat{a} = e^{i\hat{\phi}}\sqrt{\hat{N}}\,, \qquad \hat{a}^+ = \sqrt{\hat{N}}e^{-i\hat{\phi}}. \tag{9}$$

By adopting the relation $[\hat{a}, \hat{a}^+] = \hat{a}\hat{a}^+ - \hat{a}^+\hat{a} = 1$ from (9) it follows that the operators $\hat{N}$ and $\hat{\phi}$ satisfy the commutation formula

$$[\hat{N}, \hat{\phi}] = i\,. \tag{10}$$

This relation was associated directly with the RSUR (2) respectively with the presumption $\boldsymbol{P_4}$ of CIUR. The mentioned association guided to the rash impression that the N-ϕ pair satisfy the relation

$$\Delta_\Psi N \cdot \Delta_\Psi \phi \geqslant \frac{1}{2}\,. \tag{11}$$

But, lately, it was found that relation (11) is false — at least in some well-specified situations. Such a situation appears in the case of a quantum oscillator (QO). The mentioned falsity can be pointed out as follows. The Schrödinger equation for a QO stationary state has the form:

$$E\Psi = \frac{1}{2m_0}\,\hat{p}^2\Psi + \frac{1}{2}\,m_0\,\omega^2\hat{x}^2\Psi\,, \tag{12}$$

where m_0 and ω represent the mass and (angular) frequency of QO while $\hat{p} = -i\hbar\frac{\partial}{\partial x}$ and $x = x\cdot$ denote the operators of the Cartesian moment p and coordinate x. Then the operators $\hat{a}$, $\hat{a}^+$ and $\hat{N}$ have [34] the expressions

$$\hat{a} = \frac{m_0\omega\hat{x} + i\hat{p}}{\sqrt{2m_0\omega\hbar}}\,, \quad \hat{a}^+ = \frac{m_0\omega\hat{x} - i\hat{p}}{\sqrt{2m_0\omega\hbar}}\,, \quad \hat{N} = \hat{a}^+\hat{a}\,. \tag{13}$$

The solution of the equation (12) is an eigenstate wave function of the form

$$\Psi_n(x) = \Psi_n(\xi) \propto \exp\left(-\frac{\xi^2}{2}\right)\mathcal{H}_n(\xi)\,, \qquad (14)$$

where $\xi = x\sqrt{\frac{m_0\omega}{\hbar}}$, while $n = 0, 1, 2, 3, \ldots$ signifies the oscillation quantum number and $\mathcal{H}_n(\xi)$ stand for Hermite polinomials of ξ. The noted solution correspond to the energy eigenvalue $E = E_n = \hbar\omega(n + \frac{1}{2})$ and satisfy the relation $\hat{N}\Psi_n(x) = n \cdot \Psi_n(x)$.

It is easy to see that in a state described by a wave function like (14) one find the results

$$\Delta_\Psi N = 0\,, \quad \Delta_\Psi \phi \leqslant 2\pi\,. \qquad (15)$$

The here noted restriction $\Delta\phi \leqslant 2\pi$ (more exactly $\Delta\phi = \pi/\sqrt{3}$ — see below in (19)) is due to the natural fact that the definition range for ϕ is the interval $[0, 2\pi)$. Through the results (15) one finds a true falsity of the presumed relation (11). Then the harmonization of N-ϕ pair with the CIUR doctrine reaches to a deadlock. For avoiding the mentioned deadlock in many publications were promoted various adjustements regarding the pair N-ϕ (see [35, 37–43] and references therein). But it is easy to observe that all the alluded adjustements are subsequent (and dependent) in respect with the RSUR (2) in the following sense. The respective adjustements consider the alluded RSUR as an absolutely valid formula and try to adjust accordingly the description of the pair N-ϕ for QO. So the operators $\hat{N}$ and $\hat{\phi}$, defined in (9) were replaced by some substitute (*sbs*) operators $\hat{N}_{sbs} = f(\hat{N})$ and $\hat{\phi}_{sbs} = g(\hat{\phi})$, where the functions f and g are introduced through various ad hoc procedures. The so introduced substitute operators $\hat{N}_{sbs}$ and $\hat{\phi}_{sbs}$ pursue to be associated with corresponding standard deviations $\Delta_\Psi N_{sbs}$ and $\Delta_\Psi \phi_{sbs}$ able to satisfy relations resembling more or less with RSUR (2) or with (11). But we appreciate as very doubtful the fact that the afferent "*substitute observables*" N_{sbs} and ϕ_{sbs} can have natural (or even useful) physical significances. Probably that this fact as well as the ad hoc character of the functions f and g constitute the reasons for which until now, in scientific publications, it does not exist a unanimous agreement able to guarantee a genuine elucidation of true status of the N-ϕ pair comparatively with CIUR concepts.

Our opinion is that an elucidation of the mentioned kind can be obtained only through a discussion founded on the aboriginal QM elements presented above in the remark $\boldsymbol{R_2}$. For approaching such a discussion here we add the following supplementary details. For the alluded QO the Schrödinger equation (12) as well as its solution (14) are depicted in a "*coordinate x-representation*". But the same equation and solution can be described in a "*phase ϕ-representation*". By taking into account the relation (10) it results directly that tn the ϕ-representation the operators $\hat{N}$ and $\hat{\phi}$ have the expressions $\hat{N} = i\left(\frac{\partial}{\partial\phi}\right)$ and $\hat{\phi} = \phi\cdot$. In the same representation the Schrödinger equation (12) takes the form

$$E\Psi_n(\phi) = \hbar\omega\left(i\frac{\partial}{\partial\phi} + \frac{1}{2}\right)\Psi_n(\phi) \qquad (16)$$

where $\phi \in [0, 2\pi)$. Then the solution of the above equation is given by the relation

$$\Psi_n(\phi) = \frac{1}{\sqrt{2\pi}}\exp(in\phi) \qquad (17)$$

with $n = \frac{E}{\hbar\omega} - \frac{1}{2}$. If, similarly with te case of a classical oscillator, for a QO the energy E is considered to have non-negative values one finds $n = 0, 1, 2, 3, \ldots$.

Now, for the case of a QO, by taking into account the wave function (17), the operators $\hat{N}$ and $\hat{\phi}$ in the ϕ-representation, as well as the aboriginal QM elements presented in $\boldsymbol{R_2}$, we can note the following things. In the respective case it is verified the relation

$$(\hat{N}\Psi_n, \hat{\Phi}\Psi_n) = (\Psi_n, \hat{N}\hat{\Phi}\Psi_n) + i\,. \qquad (18)$$

This relation shows directly the circumstance that in the mentioned case the conditions (6) are not fulfiled by the operators $\hat{N}$ and $\hat{\phi}$ in connection with the wave function (17). But such a circumstance point out the observation that in the case under discussion the RSUR (2)/(8) is not valid. On the other hand one can see that CSF (5) remains true. In fact it take the form of the trivial equality $0 = 0$ because in the due case one obtains

$$\Delta_\Psi N = 0\,, \ \Delta_\Psi\phi = \frac{\pi}{\sqrt{3}}\,, \ \left(\delta\hat{N}\Psi_n, \delta\,\hat{\phi}\,\Psi_n\right) = 0\,. \qquad (19)$$

The above revealed facts allow us to note the following conclusions. In case of QO states (described by the wave functions (14) or (17)) the N-ϕ pair is in a complete disagreement with the RSUR (2)/(8) and with the associated basic presumption $\boldsymbol{P_4}$ of CIUR. But, in the alluded case, the same pair is in a full concordance with the aboriginal QM element by the CSF (5). Then it is completely clear that the here noted concclusions reveal an authentic deficience of CIUR.

Observation: Often in CIUR literature the N-ϕ pair is discussed in connection with the situations regarding ensembles of particles (e.g. fuxes of photons). But, in our opinion, such situations are completely different comparatively with the above presented problem about the N-ϕ pair and QO wave functions (states). In the alluded situations the Dirac's notations/formulas (9) can be also used but they must be utilized strictly in connection with the wave functions describing the respective ensembles. Such utilization can offer examples in which the N-ϕ pair satisfy relations which are semblable with RSUR (2) or with the relation (11). But it is less probable that the alluded examples are able to consolidate the CIUR concepts. This because in its primary form CIUR regards on the first place the individual quantum particles but not ensembles of such particles.

End of $\boldsymbol{R_5}$

R_6: On the E-t pair

Another pair of (canonically) conjugated observables which are unconformable in relation with the CIUR ideas is given by energy E and time t. That is why the respective pair was the subject of a large number of (old as well as recent) controversial discussions (see [2, 44–48] and references therein). The alluded discussions were generated by the following observations. On one hand, in conformity with the CIUR tradition, in terms of QM, E and t regarded as conjugated observables, ought to be described by the operators

$$\hat{E} = i\hbar \frac{\partial}{\partial t}, \qquad \hat{t} = t\cdot \tag{20}$$

respectively by the commutation relation

$$\left[\hat{E}, \hat{t}\right] = i\hbar . \tag{21}$$

In accordance with the RSUR (2) such a description require the formula

$$\Delta_\Psi E \cdot \Delta_\Psi t \geqslant \frac{\hbar}{2} . \tag{22}$$

On the other hand because in usual QM the time t is a deterministic but not a random variable for any quantum situation (particle/system and state) one finds the expressions

$$\Delta_\Psi E = \text{a finite quantity}, \qquad \Delta_\Psi t \equiv 0 . \tag{23}$$

But these expressions invalidate the relation (22) and consequently show an anomaly in respect with the CIUR ideas (especially with the presumption P_4). For avoiding the alluded anomaly CIUR partisans invented a lot of adjusted $\Delta_\Psi E - \Delta_\Psi t$ formulae destined to substitute the questionable relation (22) (see [2, 44–48] and references). The mentioned formulae can be written in the generic form

$$\Delta_v E \cdot \Delta_v t \geqslant \frac{\hbar}{2} . \tag{24}$$

Here $\Delta_v E$ and $\Delta_v t$ have various (v) significances such as:

(i) $\Delta_1 E$ = line-breadth of the spectrum characterizing the decay of an excited state and $\Delta_1 t$ = half-life of the respective state;

(ii) $\Delta_2 E = \hbar \Delta\omega$ = spectral width (in terms of frequency ω) of a wave packet and $\Delta_2 t$ = temporal width of the respective packet;

(iii) $\Delta_3 E = \Delta_\Psi E$ and $\Delta_3 t = \Delta_\Psi A \cdot \left(d \langle A \rangle_\Psi / dt\right)^{-1}$, with A = an arbitrary observable.

Note that in spite of the efforts and imagination implied in the disputes connected with the formulae (24) the following observations remain of topical interest.

(i) The diverse formulae from the family (24) are not mutually equivalent from a mathematical viewpoint. Moreover they have no natural justification in the framework of usual QM (that however give a huge number of good results in applications);

(ii) In the specific literature (see [2, 44–48] and references therein) none of the formulas (24) is agreed unanimously as a correct substitute for relation (22).

Here it must be added also another observation regarding the E-t pair. Even if the respective pair is considered to be described by the operators (20), in the true QM terms, one finds the relation

$$\left(\hat{E}\Psi, \, \hat{t}\Psi\right) = \left(\Psi, \, \hat{E}\,\hat{t}\Psi\right) - i\hbar . \tag{25}$$

This relation shows clearly that for the E-t pair the condition (6) is never satisfied. That is why for the respective pair the RSUR (2)/(8) is not applicable at all. Nevertheless for the same pair, described by the operators (20), the CSF (5) is always true. But because in QM the time t is a deterministic (i.e. non-random) variable in all cases the mentioned CSF degenerates into the trivial equality $0 = 0$.

Due to the above noted observations we can conclude that the applicability of the CIUR ideas to the E-t pair persists in our days as a still unsolved question. Moreover it seems to be most probabble the fact that the respective question can not be solved naturally in accordance with the authentic and aboriginal QM procedures. But such a fact must be reported as a true and serious deficience of CIUR.

End of R_6

In the above remarks R_1–R_6 we have approached few facts which through detailed examinations reveal indubitable deficiences of CIUR.The respective facts are somewhat known due to their relative presence in the published debates. But there are a number of other less known things which poit out also deficiences of CIUR. As a rule, in publications, the respective things are either ignored or mentioned with very rare occasions. Now we attempt to re-examine the mentioned things in a spirit similar with the one promoted in the remarks R_1–R_6 from the upper part of this section. The announced re-examination is given below in the next remarks.

R_7: On the commutable observables

For commutable observables CIUR adopt the presumtion P_3 because the right hand side term from RSUR (2) is a null quantity. But as we have shown in remark R_2 the respective RSUR is only a limited by-product of the general relation which is the CSF (5). However by means of the alluded CSF one can find examples where two commutable observable A and B can have simultaneously non-null values for their standard deviations ΔA and ΔB.

An example of the mentioned kind is given by the cartesian momenta p_x and p_y for a particle in a 2D potential well. The observables p_x and p_y are commutable because $[\hat{p_x}, \hat{p_y}] = 0$. The well is delimited as follows: the potential energy V is null for $0 < x_1 < a$ and $0 < y_1 < b$ respectively $V = \infty$ otherwise, where $0 < a < b$, $x_1 = \frac{(x+y)}{\sqrt{2}}$ and $y_1 = \frac{(y-x)}{\sqrt{2}}$. Then for the particle in the lowest energetic state one finds

$$\Delta_\Psi p_x = \Delta_\Psi p_y = \hbar\,\frac{\pi}{ab}\sqrt{\frac{a^2+b^2}{2}}\,, \tag{26}$$

$$|\langle(\delta_\Psi \hat{p}_x\Psi,\, \delta_\Psi \hat{p}_y\Psi)\rangle| = \left(\frac{\hbar\pi}{ab}\right)^2 \cdot \left(\frac{b^2-a^2}{2}\right). \tag{27}$$

With these expressions it results directly that for the considered example the momenta p_x and p_y satisfy the CSF (5) in a non-trivial form (i.e. as an inequality with a non-null value for the right hand side term).

The above noted observations about commutable observables constitute a fact that conflicts with the basic presumption $\boldsymbol{P_3}$ of CIUR. Consequently such a fact must be reported as an element which incriminates the CIUR doctrine.
End of $\boldsymbol{R_7}$

$\boldsymbol{R_8}$: On the eigenstates
The RSUR (2) fails in the case when the wave function Ψ describes an eigenstate of one of the operators $\hat{A}$ or $\hat{B}$. The fact was mentioned in [49] but it seems to remain unremarked in the subsequent publications. In terms of the here developed investigations the alluded failure can be discussed as follows. For two non-commutable observables A and B in an eigenstate of A one obtains the set of values: $\Delta_\Psi A = 0$, $0 < \Delta_\Psi B < \infty$ and $\langle[\hat{A},\, \hat{B}]\rangle_\Psi \neq 0$. But, evidently, the respective values infringe the RSUR(2). Such situations one finds particularly with the pairs L_z-φ in some cases detailed in [33] and N-ϕ in situations presented above in $\boldsymbol{R_5}$.

Now one can see that the question of eigenstates does not engender any problem if the quantities $\Delta_\Psi A$ and $\Delta_\Psi B$ are regarded as QM standard deviations (i.e.characteristics of quantum fluctuations) (see the next Section). Then the mentioned set of values show that in the respective eigenstate A has not fluctuations (i.e. A behaves as a deterministic variable) while B is endowed with fluctuations (i.e. B appears as a random variable). Note also that in the cases of specified eigenstates the RSUR (2) are not valid. This happens because of the fact that in such cases the conditions (6) are not satisfied. The respective fact is proved by the observation that its opposite imply the absurd result

$$a\cdot\langle B\rangle_\Psi = \left\langle[\hat{A},\, \hat{B}]\right\rangle_\Psi + a\cdot\langle B\rangle_\Psi \tag{28}$$

with $\langle[\hat{A},\, \hat{B}]\rangle_\Psi \neq 0$ and $a =$ eigenvalue of $\hat{A}$ (i.e. $\hat{A}\Psi = a\Psi$). But in the cases of the alluded eigenststes the CSF (5) remain valid. It degenerates into the trivial equality $0 = 0$ (because $\delta_\Psi \hat{A}\,\Psi = 0$).

So one finds a contradiction with the basic presumption $\boldsymbol{P_4}$ — i.e. an additional and distinct deficiency of CIUR.
End of $\boldsymbol{R_8}$

$\boldsymbol{R_9}$: On the multi-temporal relations
Now let us note the fact RSUR (2)/(8) as well as its precursor CSF (5) are one-temporal formulas. This because all the quantities implied in the respective formulas refer to the same instant of time. But the mentioned formulas can be generalized into multi-temporal versions, in which the corresponding quantities refer to different instants of time. So CSF (5) is generalizable in the form

$$\Delta_{\Psi_1} A\cdot\Delta_{\Psi_2} B \geqslant \left|\left(\delta_{\Psi_1}\hat{A}\,\Psi_1,\, \delta_{\Psi_2}\hat{B}\Psi_2\,,\right)\right| \tag{29}$$

where Ψ_1 and Ψ_2 represent the wave function for two different instants of time t_1 and t_2. If in (29) one takes $|t_2 - t_1| \to \infty$ in the CIUR vision the quantities $\Delta_{\Psi_1} A$ and $\Delta_{\Psi_2} B$ have to refer to A and B regarded as independent solitary observables. But in such a regard if $\left(\delta_{\Psi_1}\hat{A}\,\Psi_1,\, \delta_{\Psi_2}\hat{B}\,\Psi_2\right) \neq 0$ the relation (29) refute the presumption $\boldsymbol{P_2}$ and so it reveals another additional deficience of CIUR. Note here our opinion that the various attempts [50, 51], of extrapolating the CIUR vision onto the relations of type (29) are nothing but artifacts without any real (physical) justification. We think that the relation (29) does not engender any problem if it is regarded as fluctuations formula (in the sense which will be discussed in the next Section). In such a regard the cases when $\left(\delta_{\Psi_1}\hat{A}\,\Psi_1,\, \delta_{\Psi_2}\hat{B}\,\Psi_2\right) \neq 0$ refer to the situations in which, for the time moments t_1 and t_2, the corresponding fluctuations of A and B are correlated (i.e. statistically dependent).

Now we can say that, the previuosly presented discussion on the multi-temporal relations, disclose in fact a new deficiency of CIUR.
End of $\boldsymbol{R_9}$

$\boldsymbol{R_{10}}$: On the many-observable relations
Mathematically the RSUR (2)/(8) is only a restricted by-product of CSF (5) which follows directly from the two-observable true relation (4). But further one the alluded relation (4) appear to be merely a simple two-observable version of a more general many-observable formula. Such a genaral formula has the the form

$$\det\left[\left(\delta_\Psi \hat{A}_j\Psi,\, \delta_\Psi \hat{A}_k\Psi\right)\right] \geqslant 0\,. \tag{30}$$

Here $\det[\alpha_{jk}]$ denotes the determinant with elements α_{jk} and $j = 1, 2, \ldots, r$; $k = 1, 2, \ldots, r$ with $r \geqslant 2$. The formula (30) results from the mathematical fact that the quantities $\left(\delta_\Psi \hat{A}_j\Psi,\, \delta_\Psi \hat{A}_k\Psi\right)$ constitute the elements of a Hermitian and non-negatively defined matrix (an abstract presentation of the mentioned fact can be found in [52]).

Then, within a consistent judgment of the things, for the many-observable relations (30), CIUR must to give an interpretation concordant with its own doctrine (summarized in its basic presumptions $\boldsymbol{P_1}$–$\boldsymbol{P_5}$). Such an interpretation was proposed in [53] but it remained as an unconvincing thing (because of the lack of real physical justifications). Other discussions about the relations of type (30) as in [38] elude any interpretation of the mentioned kind. A recent attempt [54] meant to promote an interpretation of relations like (30), for three or more observables. But the respective attempt has not a helping value for CIUR doctrine. This is because instead of consolidating the CIUR basic presumptions $\boldsymbol{P_1}$–$\boldsymbol{P_5}$) it seems rather to support the idea that the considered relations are fluctuations formulas (in the sense which will be discussed bellow in the next Section). We opine that to find a CIUR-concordant interpretation for the many-observable relations (30) is a difficult (even impossible) task on natural ways (i.e. without esoteric and/or non-physical considerations). An exemplification of the respective difficulty can be appreciated by investigating the case of observables $A_1 = p$, $A_2 = x$ and $A_3 = H =$ energy in the situations described by the wave functions (14) of a QO.

Based on the above noted appreciations we conclude that the impossibility of a natural extension of CIUR doctrine to a interpretation regarding the many-observable relations (30) rveal another deficience of the respective doctrine.

End of $\boldsymbol{R_{10}}$

$\boldsymbol{R_{11}}$: On the quantum-classical probabilistic similarity

Now let us call attention on a quantum-classical similarity which directly contradicts the presumption $\boldsymbol{P_5}$ of CIUR. The respective similarity is of probabilistic essence and regards directly the RSUR (2)/(8) as descendant from the CSF (5). Indeed the mentioned CSF is completely analogous with certain two-observable formula from classical (phenomenolgical) theory of fluctuations for thermodynamic quantities. The alluded classical formula can be written [55, 56] as follows

$$\Delta_w \mathbb{A} \cdot \Delta_w \mathbb{B} \geqslant \left| \langle \delta_w \mathbb{A} \cdot \delta_w \mathbb{B} \rangle_w \right| . \tag{31}$$

In this formula $\mathbb{A}$ and $\mathbb{B}$ signify two classical global observables which characterize a thermodynamic system in its wholeness. In the same formula w denotes the phenomenological probability distribution, $\langle(\ldots)\rangle_w$ represents the mean (expected value) of the quantity $(\ldots)$ evaluated by means of w while $\Delta_w \mathbb{A}$, $\Delta_w \mathbb{B}$ and $\langle \delta_w \mathbb{A} \cdot \delta_w \mathbb{B} \rangle_w$ stand for characteristics (standard deviations respectively correlation) regarding the fluctuations of the mentioned observables. We remind the appreciation that in classical physics the alluded characteristics and, consequently, the relations (31) describe the intrinsic (own) properties of thermodynamic systems but not the aspects of measurements performed on the respective systems. Such an appreciation is legitimated for example by the research regarding the fluctuation spectroscopy [57] where the properties of macroscopic (thermodynamic) systems are evaluated through the (spectral components of) characteristics like $\Delta_w \mathbb{A}$ and $\langle \delta_w \mathbb{A} \cdot \delta_w \mathbb{B} \rangle_w$.

The above discussions disclose the groundlessness of idea [58–60] that the relations like (31) have to be regarded as a sign of a macroscopic/classical complementarity (similar with the quantum complementarity motivated by CIUR presumption $\boldsymbol{P_4}$). According to the respective idea the quantities $\Delta_w \mathbb{A}$ and $\Delta_w \mathbb{B}$ appear as macroscopic uncertainties. Note that the mentioned idea was criticized partially in [61, 62] but without any explicit specification that the quantities $\Delta_w \mathbb{A}$ and $\Delta_w \mathbb{B}$ are quatities which characterise the macroscopic fluctuations.

The previously notified quantum-classical similarity together with the reminded significance of the quantities implied in (31) suggests and consolidates the following regard (argued also in $\boldsymbol{R_3}$). The quantities $\Delta_\Psi A$ and $\Delta_\Psi B$ from RSUR (2)/(8) as well as from CSF (5) must be regarded as describing intrinsic properties (fluctuations) of quantum observables A and B but not as uncertainties of such observables.

Now, in conclusion, one can say that the existence of classical relations (31) contravenes to both presumptions $\boldsymbol{P_1}$ and $\boldsymbol{P_5}$ of CIUR. Of course that such a conclusion must be announced as a clear deficience of CIUR.

End of $\boldsymbol{R_{11}}$

$\boldsymbol{R_{12}}$: On the higher order fluctuations moments

In classical physics the fluctuations of thermodynamic observables $\mathbb{A}$ and $\mathbb{B}$ implied in (31) are described not only by the second order probabilistic moments like $\Delta_w \mathbb{A}$, $\Delta_w \mathbb{B}$ or $\langle \delta_w \mathbb{A}\, \delta_w \mathbb{B} \rangle_w$. For a better evaluation the respective fluctuations are characterized additionally [63] by higher order moments like $\left\langle (\delta_w \mathbb{A})^r \, (\delta_w \mathbb{B})^s \right\rangle_w$ with $r + s \geqslant 3$. This fact suggests the observation that, in the context considered by CIUR, we also have to use the quantum higher order probabilistic moments like $\left(\left(\delta_\Psi \hat{A}_j\right)^r \Psi, \left(\delta_\Psi \hat{A}_k\right)^s \Psi \right)$, $r + s \geqslant 3$. Then for the respective quantum higher order moments CIUR is obliged to offer an interpretation compatible with its own doctrine. But it seems to be improbable that such an interpretation can be promoted through credible (and natural) arguments resulting from the CIUR own presumptions.

That improbability reveal one more deficience of CIUR.

End of $\boldsymbol{R_{12}}$

$\boldsymbol{R_{13}}$: On the so-called "macroscopic operators"

Another obscure aspect of CIUR was pointed out in connection with the question of the so called "macroscopic operators". The question was debated many years ago (see [64, 65] and references) and it seems to be ignored in the lsat decades, although until now it was not elucidated. The question appeared due to a forced transfer of RSUR (2) for the cases of quantum statistical systems. Through such a transfer CIUR partisans promoted the formula

$$\Delta_\rho \mathbb{A} \cdot \Delta_\rho \mathbb{B} \geqslant \frac{1}{2} \left| \left\langle [\hat{\mathbb{A}}, \hat{\mathbb{B}}] \right\rangle_\rho \right| . \tag{32}$$

This formula refers to a quantum statistical system in a state described by the statistical operator (density matrix) $\hat{\rho}$.

With $\mathbb{A}$ and $\mathbb{B}$ are denoted two macroscopic (global) observables associated with the operators $\hat{\mathbb{A}}$ and $\hat{\mathbb{B}}$. The quantity

$$\Delta_\rho \mathbb{A} = \left\{ \mathrm{Tr}\left[\left(\hat{\mathbb{A}} - \langle \mathbb{A} \rangle_\rho \right)^2 \right] \right\}^{\frac{1}{2}}$$

denotes the standard deviation of the macroscopic observable $\mathbb{A}$ regarded as a (generalised) random variable. In its expression the respective quantity imply the notation $\langle \mathbb{A} \rangle_\rho = \mathrm{Tr}\left(\hat{\mathbb{A}} \hat{\rho}\right)$

for the mean (expected) value of the macroscopic observable $\mathbb{A}$.

Relation (32) entailed discussions because of the conflict between the following two findings:

(i) On the one hand (32) is introduced by analogy with RSUR (2) on which CIUR is founded. Then, by extrapolating CIUR, the quantities $\Delta_\rho \mathbb{A}$ and $\Delta_\rho \mathbb{B}$ from (32) should be interpreted as (global) uncertainties subjected to stipulations as the ones indicated in the basic presumption $\boldsymbol{P_1}$;

(ii) On the other hand, in the spirit of the presumption $\boldsymbol{P_5}$, CIUR agrees the posibility that macroscopic observables can be measured without any uncertainty (i.e. with unbounded accuracy). For an observable the mentioned possibility should be independent of the fact that it is measured solitarily or simultaneously with other observables. Thus, for two macroscopic (thermodynamic) observables, it is senselessly to accept CIUR basic presumptions $\boldsymbol{P_3}$ and $\boldsymbol{P_4}$.

In order to elude the mentioned conflict it was promoted the idea to abrogate the formula (32) and to replace it with an adjusted macroscopic relation concordant with CIUR vision. For such a purpose the global operators $\hat{\mathbb{A}}$ and $\hat{\mathbb{B}}$ from (32) were substituted [64, 65] by the so-called "macroscopic operators" $\hat{\mathcal{A}}$ and $\hat{\mathcal{B}}$. The respective "macroscopic operators" are considered to be representable as quasi-diagonal matrices (i.e. as matrices with non-null elements only in a "microscopic neighbourhood" of principal diagonal). Then one supposes that $\left[\hat{\mathcal{A}}, \hat{\mathcal{B}}\right] = 0$ for any pairs of "macroscopic observables" $\mathbb{A}$ and $\mathbb{B}$. Consequently instead of (32) it was introduced the formula

$$\Delta_\rho \mathcal{A} \cdot \Delta_\rho \mathcal{B} \geqslant 0\,. \tag{33}$$

In this formula CIUR partisans see the fact that the uncertainties $\Delta_\rho \mathcal{A}$ and $\Delta_\rho \mathcal{B}$ can be unboundedly small at the same time moment, for any pair of observables $\mathbb{A}$ and $\mathbb{B}$ and for any system. Such a fact constitute the CIUR vision about macroscopic observables. Today it seems to be accepted the belief that mentioned vision solves all the troubles of CIUR caused by the formula (32).

A first disapproval of the mentioned belief results from the following observations:

(i) Relation (32) cannot be abrogated if the entire mathematical apparatus of quantum statistical physics is not abrogated too. More exactly, the substitution of operators from the usual global version $\hat{\mathbb{A}}$ into a "macroscopic" variant $\hat{\mathcal{A}}$ is a senseless invention as long as in practical procedures of quantum statistical physics [66, 67] for lucrative operators one uses $\hat{\mathbb{A}}$ but not $\hat{\mathcal{A}}$;

(ii) The substitution $\hat{\mathbb{A}} \to \hat{\mathcal{A}}$ does not metamorphose automatically (32) into (33), because if two operators are quasi-diagonal, in sense required by the partisans of CIUR, it is not surely that they commute.

For an ilustration of the last observatiom we quote [68] the Cartesian components of the global magnetization $\vec{\mathbb{M}}$ of a paramagnetic system formed of N independent $\frac{1}{2}$-spins. The alluded components are described by the global operators

$$\hat{\mathbb{M}}_\alpha = \frac{\gamma\hbar}{2}\,\hat{\sigma}_\alpha^{(1)} \oplus \frac{\gamma\hbar}{2}\,\hat{\sigma}_\alpha^{(2)} \oplus \cdots \oplus \frac{\gamma\hbar}{2}\,\hat{\sigma}_\alpha^{(N)}\,, \tag{34}$$

where $\alpha = x, y, z$; γ = magneto-mechanical factor and $\hat{\sigma}_\alpha^{(i)}$ = Pauli matrices associated to the i-th spin (particle). Note that the operators (34) are quasi-diagonal in the sense required by CIUR partisans, i.e. $\hat{\mathbb{M}}_\alpha \equiv \hat{\mathcal{M}}_\alpha$. But, for all that, they do not commute because $\left[\hat{\mathcal{M}}_\alpha, \hat{\mathcal{M}}_\beta\right] = i\hbar\gamma\cdot\varepsilon_{\alpha\beta\mu}\cdot\hat{\mathcal{M}}_\mu$ ($\varepsilon_{\alpha\beta\mu}$ denote the Levi-Civita tensor).

A second disproval of the belief induced by the substitution $\hat{\mathbb{A}} \to \hat{\mathcal{A}}$ is evidenced if the relation (32) is regarded in an ab original QM approach like the one presented in $\boldsymbol{R_2}$. In such regard it is easy to see that in fact the formula (32) is only a restrictive descendant from the generally valid relation

$$\Delta_\rho \mathbb{A} \cdot \Delta_\rho \mathbb{B} \geqslant \left|\left\langle \delta_\rho \mathbb{A} \cdot \delta_\rho \mathbb{B} \right\rangle_\rho\right|, \tag{35}$$

where $\delta_\rho \hat{\mathbb{A}} = \hat{\mathbb{A}} - \langle \mathbb{A} \rangle_\rho$. In the same regard for the "macroscopic operators" $\mathcal{A}$ and $\mathcal{B}$ instead of the restricted relation (33) it must considered the more general formula

$$\Delta_\rho \mathcal{A} \cdot \Delta_\rho \mathcal{B} \geqslant \left|\left\langle \delta_\rho \mathcal{A} \cdot \delta_\rho \mathcal{B} \right\rangle_\rho\right|. \tag{36}$$

The above last two relations justify the following affirmations:

(i) Even in the situations when $\left[\hat{\mathcal{A}}, \hat{\mathcal{B}}\right] = 0$ the product $\Delta_\rho \mathcal{A} \cdot \Delta_\rho \mathcal{B}$ can be lower bounded by a non-null quantity. This happens because it is possible to find cases in which the term from the right hand side of (36) has a non-null value;

(ii) In fact the substitution $\hat{\mathbb{A}} \to \hat{\mathcal{A}}$ replace (35) with (36). But for all that the alluded replacement does not guaranttee the validity of the relation (33) and of the corresponding speculations.

The just presented facts warrant the conclusion that the relation (32) reveal a real deficiency of CIUR. The respective deficiency cannot be avoided by resorting to the so-called "macroscopic operators". But note that the same relation does not rise any problem if it is considered together with (35) as formulas which refer to the fluctuations of macroscopic (global) observables regarding thermodynamic systems.

End of $\boldsymbol{R_{13}}$

$\boldsymbol{R_{14}}$: On the similarities between calassical Boltzmann's and quantum Planck's constants k_B and $\hbar$

The quantum-classical similarity revealed in $\boldsymbol{R_{11}}$ entails also a proof against the CIUR presumption $\boldsymbol{P_5}$. According to the respective presumptions the Planck constant $\hbar$ has no analog in classical (non-quantum) physics. The announced proof can be pointed out as follows.

The here discussed similarity regards the groups of classical respectively quantum relations (31) and (5) (the last ones including their restricted descendant RSUR (2)/(8)). The respective relations imply the standard deviations $\Delta_w \mathbb{A}$ or $\Delta_\Psi A$ associated with the fluctuations of the corresponding classical and quantum observables. But mathematically the standard deviation indicate the randomness of an observable. This in the sense that the alluded deviation has a positive or null value as the corresponding observable is a random or, alternatively, a deterministic (non-random) variable. Therefore the quantities $\Delta_w \mathbb{A}$ and $\Delta_\Psi A$ can be regarded as similar indicators of randomness for the classical respectively quantum observables.

For diverse cases (of observables, systems and states) the classical standard deviations $\Delta_w \mathbb{A}$ have various expressions in which, apparently, no common element seems to be implied. Nevertheless such an element can be found out [69] as being materialized by the Boltzmann constant k_B. So, in the framework of phenomenological theory of fluctuations (in Gaussian approximation) one obtains [69]

$$(\Delta_w \mathbb{A})^2 = k_\beta \cdot \sum_\alpha \sum_\beta \frac{\partial \bar{\mathbb{A}}}{\partial \bar{\mathbb{X}}_\alpha} \cdot \frac{\partial \bar{\mathbb{A}}}{\partial \bar{\mathbb{X}}_\beta} \cdot \left(\frac{\partial^2 \bar{\mathbb{S}}}{\partial \bar{\mathbb{X}}_\alpha \partial \bar{\mathbb{X}}_\beta} \right)^{-1} . \quad (37)$$

In this relation $\bar{\mathbb{A}} = \langle \mathbb{A} \rangle_w$, $\bar{\mathbb{S}} = \bar{\mathbb{S}}(\bar{\mathbb{X}}_\alpha)$ denotes the entropy of the system written as a function of independent thermodynamic variables $\bar{\mathbb{X}}_\alpha$, $(\alpha = 1, 2, \ldots, r)$ and $(a_{\alpha\beta})^{-1}$ represent the elements for the inverse of matrix $(a_{\alpha\beta})$. Then from (37) it result that the expressions for $(\Delta_w \mathbb{A})^2$ consist of products of k_B with factors which are independent of k_B. The respective independence is evidenced by the fact that the alluded factors must coincide with deterministic (non-random) quantities from usual thermodynamics (where the fluctuations are neglected). Or it is known that such quantities do not imply k_B at all. See [69] for concrete exemplifications of the relations (37) with the above noted properties.

Then, as a first aspect, from (37) it results that the fluctuations characteristics $(\Delta_w \mathbb{A})^2$ (i.e. dispersions = squares of the standard deviations) are directly proportional to k_B and, consequently, they are non-null respectively null quantities as $k_B \neq 0$ or $k_B \to 0$. (Note that because k_B is a physical constant the limit $k_B \to 0$ means that the quantities directly proportional with k_B are negligible comparatively with other quantities of same dimensionality but independent of k_B.) On the other hand, the second aspect (mentioned also above) is the fact that $\Delta_w \mathbb{A}$ are particular indicators of classical randomness. Conjointly the two mentioned aspects show that k_B has the qualities of an authentic generic indicator of thermal randomness which is specific for classical macroscopic (thermodynamic) systems. (Add here the observation that the same quality of k_B can be revealed also [69] if the thermal randomness is studied in the framework of classical statistical mechanics).

Now let us discuss about the quantum randomness whose indicators are the standard deviations $\Delta_\Psi A$. Based on the relations (26) one can say that in many situations the expressions for $(\Delta_\Psi A)^2$ consist in products of Planck constant $\hbar$ with factors which are independent of $\hbar$. (Note that a similar situation can be discovered [33] for the standard deviations of the observables L_z and φ in the case of quantum torsion pendulum.) Then, by analogy with the above discussed classical situations, $\hbar$ places itself in the posture of generic indicator for quantum randomness.

In the mentioned roles as generic indicators k_B and $\hbar$, in direct connections with the quantities $\Delta_w \mathbb{A}$ and $\Delta_\Psi A$, regard the onefold (simple) randomness, of classical and quantum nature respectively. But in physics is also known a twofold (double) randomness, of a combined thermal and quantum nature. Such a kind of randomness one encounters in cases of quantum statistical systems and it is evaluated through the standard deviations $\Delta_\rho \mathbb{A}$ implied in relations (32) and (35). The expressions of the mentioned deviations can be obtained by means of the fluctuation-dissipation theorem [70] and have the form

$$(\Delta_\rho \mathbb{A})^2 = \frac{\hbar}{2\pi} \int_{-\infty}^{\infty} \coth\left(\frac{\hbar\omega}{2k_B T} \right) \chi''(\omega)\, d\omega\, . \quad (38)$$

Here $\chi''(\omega)$ denote the imaginary parts of the susceptibility associated with the observable $\mathbb{A}$ and T represents the temperature of the considered system. Note that $\chi''(\omega)$ is a deterministic quantity which appear also in non-stochastic framework of macroscopic physics [71]. That is why $\chi''(\omega)$ is independent of both k_B and $\hbar$. Then from (38) it results that k_B and $\hbar$ considered together appear as a couple of generic indicators for the twofold (double) randomness of thermal and quantum nature. The respective randomness is negligible when $k_B \to 0$ and $\hbar \to 0$ and significant when $k_B \neq 0$ and $\hbar \neq 0$ respectively.

The above discussions about the classical and quantum randomness respectively the limits $k_B \to 0$ and $\hbar \to 0$ must be supplemented with the following specifications.

(i) In the case of the classical randomness it must considered the following fact. In the respective case one associates the limits $k_B \to 0$ respectively "*(classical) microscopic approach*" $\to$ "*(classical) macroscopic approach*". But in this context $k_B \to 0$ is concomitant with the condition $N \to 0$ (N = number of microscopic constituents (molecules) of the considered system). The respective concomitance assures the transformation $k_B N \to \nu R$, i.e. transition of physical quantities from "*microscopic version*" into a "*macroscopic version*" (because R sidnify the macoscopic gass constant and ν denotes the macroscopic amount of substance;

(ii) On the other hand in connection with the quantum case it must taken into account the following aspect. The corresopnding randomness regards the cases of observables of orbital and spin types respectively;

(iii) In the orbital cases the limit $\hbar \to 0$ is usually associated with the quantum $\to$ classical limit. The respective limit implies an unbounded growth of the values of some quantum numbers so as to ensure a correct limit for the associated observables regarding orbital movements. Then one finds [72, 73] that, when $\hbar \to 0$, the orbital-type randomness is in one of the following two situations:

(a) it converts oneself in a classical-type randomness of the corresponding observables (e.g. in the cases of φ and L_z of a torsional pendulum or of x and p of a rectilinear oscillator), or

(b) it disappears, the corresponding observables becoming deterministic classical variables (e.g. in the case of the distance r of the electron in respect with the nucleus in a hydrogen atom);

(iv) The quantum randomness of spin-type regards the spin observables. In the limit $\hbar \to 0$ such observables disappear completely (i.e. they lose both their mean values and the affined fluctuations).

In the alluded posture the Planck constant $\hbar$ has an authentic classical analog represented by the Boltzmann constant k_B. But such an analogy contradicts strongly the presumption $\boldsymbol{P_5}$ and so it reveals a new deficience of CIUR.
End of $\boldsymbol{R_{14}}$

Within this section, throgh the remarks $\boldsymbol{R_1}$–$\boldsymbol{R_{14}}$, we examined a collection of things whose ensemble point out deficiencies which incriminate all the basic presumptions $\boldsymbol{P_1}$–$\boldsymbol{R_5}$ of CIUR, considered as single or grouped pieces. In regard to the truth qualities of the respective deficiences here is the place to note the following completion remark:

$\boldsymbol{R_{15}}$: On the validity of the above signallized CIUR deficiences
The mentioned deficiencies are indubitable and valid facts which can not be surmounted (avoided or rejected) by solid and verisimilar arguments taken from the inner framework of CIUR doctrine.
End of $\boldsymbol{R_{15}}$

4 Consequences of the previous examination

The discussions belonging to the examination from the previous section impose as direct consequences the following remarks:

$\boldsymbol{R_{16}}$: On the indubitable failure of CIUR
In the mentioned circumstances CIUR proves oneself to be indubitably in a failure situation which deprives it of necessary qualities of a valid scientific construction. That is why CIUR must be abandoned as a wrong doctrine which, in fact, has no real value.
End of $\boldsymbol{R_{16}}$

$\boldsymbol{R_{17}}$: On the true significance of the relations (1) **and** (2)
The alluded abandonment has to be completed by a natural re-interpretation of the basic CIUR's relations (1) and (2). We opine that the respective re-interpretation have to be done and argued by taking into account the discussions from the previous Section, mainly those from the remarks $\boldsymbol{R_1}$, $\boldsymbol{R_2}$ and $\boldsymbol{R_3}$. We appreciate that in the alluded re-interpretation must be included the following viewpoints:

(i) On the one hand the relations (1) remain as provisional fictions destitute of durable physical significance;

(ii) On the other hand the relations (2) are simple fluctuations formulae, from the same family with the microscopic and macroscopic relations from the groups (4), (5), (29), (30) respectively (31), (32), (35);

(iii) None of the relations (1) and (2) or their adjustments have not any connection with the description of QMS.

Consequently in fact the relations (1) and (2) must be regarded as pieces of fiction respectively of mathematics without special or extraordinary status/significance for physics.
End of $\boldsymbol{R_{17}}$

$\boldsymbol{R_{18}}$: On the non-influences towards the usual QM
The above noted reconsideration of CIUR does not disturb in some way the framework of usual QM as it is applied concretely in the investigations of quantum microparticles. (Few elements from the respective framework are reminded above in the remark $\boldsymbol{R_2}$).
End of $\boldsymbol{R_{18}}$

5 Some considerations on the quantum and classical measurements

The question regarding the QMS description is one of the most debated subject associated with the CIUR history. It generated a large diversity of viewpoints relatively to its importance and/or approach (see [1–9] and references). The respective diversity inserts even some extreme opinions such are the ones noted in the Section 1 of the present paper. As a notable aspect many of the existing approaches regarding the alluded question are grounded on some views which presume and even try to extend the CIUR doctrine. Such views (v.) are:

(v.1) The descriptions of QMS must be developed as confirmations and extensions of CIUR concepts;

(v.2) The peculiarities of QMS incorporated in CIUR presumptions $\boldsymbol{P_2}$–$\boldsymbol{P_4}$ are connected with the corresponding features of the measuring perturbations. So in the cases of observables refered in $\boldsymbol{P_2}$–$\boldsymbol{P_3}$ respectively in $\boldsymbol{P_4}$ the alluded perturbations are supposed to have an avoidable respectively an unavoidable character;

(v.3) In the case of QMS the mentioned perturbations cause specific jumps in states of the measured quantum microparticles (systems). In many modern texts the respective jumps are suggested to be described as follows. For a quantum observable A of a microparticle in the state Ψ a QMS is assumed to give as result a single value say a_n which is one of the eigenvalues of the associated operator $\hat{A}$. Therefore the description of the respective QMS must include as essential piece a "collapse" (sudden reduction) of the wave function i.e a relation of the form:

$$\Psi\begin{pmatrix}\text{before}\\ \text{measurement}\end{pmatrix} \to \Psi_n\begin{pmatrix}\text{after}\\ \text{measurement}\end{pmatrix}, \qquad (39)$$

where $\Psi_n\,(\mathtt{after\ measurement})$ denotes the eigenfunction of the operator $\hat{A}$ corresponding to the eigenvalue a_n;

(v.4) With regard to the observables of quantum and classical type respectively the measuring inconveniences (perturbations and uncertainties) show an essential difference. Namely they are unavoidable respectively avoidable characteristics of measurements. The mentioned difference must be taken into account as a main point in the descriptions of the measurements regarding the two types of observables;

(v.5) The description of QMS ought to be incorporated as an inseparable part in the framework of QM. Adequately QM must be considered as a unitary theory both of intrinsic properties of quantum microparticles and of measurements regarding the respective properties.

Here is the place to insert piece-by-piece the next remark:

R_{19}: Counter-arguments to the above views

The above mentioned views about QMS must be appreciated in conformity with the discussions detailed in the previous sections. For such an appreciation we think that it must taken into account the following counter-arguments (c-a):

(c-a.1) According to the remark R_{16}, in fact CIUR is nothing but a wrong doctrine which must be abandoned. Consequently CIUR has to be omitted but not extended in any lucrative scientific question, particularly in the description of QMS. That is why the above view (v.1) is totally groundless;

(c-a.2) The view (v.2) is inspired and argued by the ideas of CIUR about the relations (1) and (2). But, according to the discussions from the previous sections, the respective ideas are completely unfounded. Therefore the alluded view (v.2) is deprived of any necessary and well-grounded justification;

(c-a.3) The view (v.3) is inferred mainly from the belief that the mentioned jumps have an essential importance for QMS.

But the respective belief appears as entirely unjustified if one takes into account the following natural and indubitable observation [74]: "*it seems essential to the notion of measurement that it answers a question about the given situation existing before the measurement. Whether the measurement leaves the measured system unchanged or brings about a new and different state of that system is a second and independent question*".

Also the same belief apperars as a fictitious thing if we take into account the quantum-classical probabilistic similarity presented in the remark R_{11}. According to the respective similarity, a quantum observable must be regarded mathematically as a random variable.Then a measurement of such a observable must consist not in a single trial (which give a unique value) but in a statistical selection/sampling (which yields a spectrum of values). For more details regarding the measurements of random observables see below in this and in the next sections.

So we can conclude that the view (v.3) is completely unjustified;

(c-a.4) The essence of the difference between classical and quantum observables supposed in view (v.4) is questionable at least because of the following two reasons:

(a) In the classical case the mentioned avoidance of the measuring inconveniences have not a significance of principle but only a relative and limited value (depending on the performances of measuring devices and procedures). Such a fact seems to be well known by experimenters.

(b) In the quantum case until now the alluded unavoidableness cannot be justified by valid arguments of experimental nature (see the above remark R_{16} and the comments regarding the relation (3));

(c-a.5) The viev (v.5) proves to be totally unjustified if the usual conventions of physics are considered. According to the respective conventions, in all the basic chapters of physics, each observable of a system is regarded as a concept "*per se*" (in its essence) which is denuded of measuring aspects. Or QM is nothing but such a basic chapter, like classical mechanics, thermodynamics, electrodynamics or statistical physics. On the other hand in physics the measurements appear as main pourposes for experiments. But note that the study of the experiments has its own problems [75] and is done in frameworks which are additional and distinct in respect with the basic chapters of physics. The above note is consolidated by the observation that [76]: "*the procedures of measurement (comparison with standards) has a part which cannot be described inside the branch of physics where it is used*".

Then, in contrast with the view (v.5), it is natural to accept the idea that QM and the description of QMS have to remain distinct scientific branches. However the two branches have to use some common concepts and symbols. This happens because, in fact, both of them also imply elements regarding the same quantum microparticles (systems).

The here presented counter-arguments contradict all the above oresented views (v.1)–(v.5) promoted in many of the existing approaches regarding the QMS description.

End of $\boldsymbol{R_{19}}$

On the basis of discussions presented in $\boldsymbol{R_{11}}$ and reminded in (c-a.3) from $\boldsymbol{R_{19}}$ a quantum observables must be considered as random variables having similar characteristics which corespond to the classical random observables. Then it results that, on principle, the description of QMS can be approached in a manner similar with the one regarding the corresponding classical measurements. That is why below we try to resume a model promoted by us in [77, 78] and destined to describe the measurement of classical random observables.

For the announced resume we consider a classical random observable from the family discussed in $\boldsymbol{R_{11}}$. Such an observable and its associated probability distribution will be depicted with the symbols $\widetilde{\mathbb{A}}$ respectively $w = w(a)$. The individual values a of $\widetilde{\mathbb{A}}$ belong to the spectrum $a \in (-\infty, \infty)$). For the considered situation a measurement preserve the spectrum of $\widetilde{\mathbb{A}}$ but change the dustribution $w(a)$ from a "*in*" (input) version $w_{in}(a)$ into an "*out*" (output) reading $w_{out}(a)$. Note that $w_{in}(a)$ describes the intrinsic properties of the measured system while $w_{out}(a)$ incorporates the information about the same system, but obtained on the recorder of measuring device. Add here the fact that, from a general perspective, the distributions $w_{in}(a)$ and $w_{out}(a)$ incorporate informations referring to the measured system. That is why a measurement appears as an "*informational input $\to$ output transmision process*". Such a process is symbolized by a transformation of the form $w_{in}(a) \to w_{out}(a)$. When the measurement is done by means of a device with stationary and linear characteristics, the the mentioned transformation can described as follows:

$$w_{out}(a) = \int_{-\infty}^{\infty} G(a, a')\, w_{in}(a')\, da'. \tag{40}$$

Here the kernel$G(a, a')$ represents a transfer probability with the significances:

(i) $G(a, a')\, da$ enotes the (infnitesimal) probability that by measurement the *in*-value a' of $\widetilde{\mathbb{A}}$ to be recorded in the *out*-interval $(a; a + da)$;

(ii) $G(a, a')\, da'$ stands for the probability that the *out*-value a to result from the *in*-values which belong to the interval $(a'; a' + da')$.

Due to the mentioned significances the kernel $G(a, a')$ satisfies the conditions

$$\int_{-\infty}^{\infty} G(a,\, a')\, da = \int_{-\infty}^{\infty} G(a,\, a')\, da' = 1\,. \tag{41}$$

Add here the fact that, from a physical perspective, the kernel $G(a, a')$ incorporates the theoretical description of all the characteristics of the measuring device. For an ideal device which ensure $w_{out}(a) = w_{in}(a)$ it must be of the form $G(a, a') = \delta(a - a')$ (with $\delta(a - a')$ denoting the Dirac's function of argument $a - a'$).

By means of $w_\eta(a)$ ($\eta = in, out$) the corresponding global (or numerical) characteristics of $\widetilde{\mathbb{A}}$ regarded as random variable can be introduced. In the spirit of usual practice of physics we refer here only to the two lowest order such characteristics. They are the η — mean (expected) value $\langle \mathbb{A} \rangle_\eta$ and η — standard deviations $\Delta_\eta \mathbb{A}$ defined as follows

$$\left.\begin{aligned} \langle \mathbb{A} \rangle_\eta &= \int_{-\infty}^{\infty} a\, w_\eta(a)\, da \\ (\Delta_\eta \mathbb{A})^2 &= \left\langle \left(\mathbb{A} - \langle \mathbb{A} \rangle_\eta \right)^2 \right\rangle_\eta \end{aligned}\right\}. \tag{42}$$

Now, from the general perspective of the present paper, it is of interest to note some observations about the measuring uncertainties (errors). Firstly it is important to remark that for the discussed observable $\mathbb{A}$, the standard deviations $\Delta_{in}\mathbb{A}$ and $\Delta_{out}\mathbb{A}$ are not estimators of the mentioned uncertainties. Of course that the above remark contradicts some loyalities induced by CIUR doctrine. Here it must be pointed out that:

(i) On the one hand $\Delta_{in}\mathbb{A}$ together with $\langle \mathbb{A} \rangle_{in}$ describe only the intrinsic properties of the measured system;

(ii) On the other hand $\Delta_{out}\mathbb{A}$ and $\langle \mathbb{A} \rangle_{out}$ incorporate composite information about the respective system and the measuring device.

Then, in terms of the above considerations, the measuring uncertainties of $\mathbb{A}$ are described by the following error indicators (characteristics)

$$\left.\begin{aligned} \varepsilon\{\langle \mathbb{A} \rangle\} &= |\langle \mathbb{A} \rangle_{out} - \langle \mathbb{A} \rangle_{in}| \\ \varepsilon\{\Delta\, \mathbb{A}\} &= |\Delta_{out}\mathbb{A} - \Delta_{in}\mathbb{A}| \end{aligned}\right\}. \tag{43}$$

Note that because $\mathbb{A}$ is a random variable for an acceptable evaluation of its measuring uncertainties it is completely insufficient the single indicator $\varepsilon\{\langle \mathbb{A} \rangle\}$. Such an evaluation requires at least the couple $\varepsilon\{\langle \mathbb{A} \rangle\}$and $\varepsilon\{\Delta\, \mathbb{A}\}$ or even the differences of the higher order moments like

$$\varepsilon\left\{\left\langle (\delta\mathbb{A})^n \right\rangle\right\} = \left| \left\langle (\delta_{out}\mathbb{A})^n \right\rangle_{out} - \left\langle (\delta_{in}\mathbb{A})^n \right\rangle_{in} \right|, \tag{44}$$

where $\delta_\eta \mathbb{A} = \widetilde{\mathbb{A}} - \langle \mathbb{A} \rangle_\eta$; $\eta = in,\ out$; $n \geqslant 3$).

Now we wish to specify the fact that the errors (uncertainties) indicators (43) and (44) are *theoretical (predicted) quantities*. This because all the above considerations consist in a theoretical (mathematical) modelling of the discussed measuring process. Or within such a modelling we operate only with theoretical (mathematical) elements presumed to reflect in a plausible manner all the main characteristics of the respective process. On the other hand, comparatively, in experimental physics, the indicators regarding the measuring errors (uncertainties) are *factual entities* because they are estimated on the basis of factual experimental data. But such entities are discussed in the framework of observational error studies.

6 An informational model for theoretical description of QMS

In the above, (c-a.5) from $\boldsymbol{R_{19}}$, we argued forthe idea that QM and the description of QMS have to remain distinct scientific branches which nevertheless have to use some common concepts and symbols. Here we wish to put in a concrete form the respective idea by recommending a reconsidered model for description of QMS. The announced model will assimilate some elements discussed in the previous section in connecton with the measuremens of classical random observables.

We restrict our considerations only to the measurements of quantum observables of orbital nature (i.e. coordinates, momenta, angles, angular momenta and energy). The respective observables are described by the following operators $\hat{A}_j$ $(j = 1, 2, \ldots, n)$ regarded as generalized random variables. As a measured system we consider a spinless microparticle whose state is described by the wave function $\Psi = \Psi(\vec{r})$, taken in the coordinate representation ($\vec{r}$ stand for microparticle position). Add here the fact that, because we consider only a non-relativistic context, the explicit mention of time as an explicit argument in the expression of Ψ is unimportant.

Now note the observation that the wave function $\Psi(\vec{r})$ incorporate information (of probabilistic nature) about the measured system. That is why a QMS can be regarded as a *process of information transmission*: from the measured microparticle (system) to the recorder of the measuring device. Then, on the one hand, the input (in) information described by $\Psi_{in}(\vec{r})$ refers to the intrinsic (own) properties of the respective micropraticle (regarded as information source). The expression of $\Psi_{in}(\vec{r})$ is deducible within the framework of usual QM (e.g. by solving the adequate Schrödinger equation). On the other hand, the output (out) information, described by the wave function $\Psi_{out}(\vec{r})$, refers to the data obtained on the device recorder (regarded as information receiver). So the measuring device plays the role of the transmission channel for the alluded information. Accordingly the measurement appears as a processing information operation. By regarding the things as above the description of the QMS must be associated with the transformation

$$\Psi_{in}(\vec{r}) \to \Psi_{out}(\vec{r})\,. \tag{45}$$

As in the classical model (see the previous section), without any loss of generality, here we suppose that the quantum observables have identical spectra of values in both in- and out-situations. In terms of QM the mentioned supposition means that the operators $\hat{A}_j$ have the same mathematical expressions in both in- and out-readings. The respective expressions are the known ones from the usual QM.

In the framework delimited by the above notifications the description of QMS requires to put the transformation (45) in concrete forms by using some of the known QM rules. Additionally the same description have to assume suggestions from the discussions given in the previous section about measurements of classical random obsevables. That is why, in our opinion, the transformation (45) must be detailed in terms of quantum probabilities carriers. Such carriers are the probabilistic densities ρ_η and currents $\vec{J}_\eta$ defined by

$$\rho_\eta = |\Psi_\eta|^2\,, \qquad \vec{J}_\eta = \frac{\hbar}{m_0}\,|\Psi_\eta|^2 \cdot \nabla\Phi_\eta\,. \tag{46}$$

Here $|\Psi_\eta|$ and Φ_η represents the modulus and the argument of Ψ_η respectively (i.e. $\Psi_\eta = |\Psi_\eta|\,exp(i\Phi_\eta)$) and m_0 denotes the mass of microparticle.

The alluded formulation is connected with the observations [79] that the couple ρ–$\vec{J}$ "*encodes the probability distributions of quantum mechanics*" and it "*is in principle measurable by virtue of its effects on other systems*". To be added here the possibility [80] of taking in QM as primary entity the couple ρ_{in}–$\vec{J}_{in}$ but not the wave function Ψ_{in} (i.e. to start the construction of QM from the continuity equation for the mentioned couple and subsequently to derive the Schrödinger equation for Ψ_{in}).

According to the above observations the transformations (45) have to be formulated in terms of ρ_η and $\vec{J}_\eta$. But ρ_η and $\vec{J}_\eta$ refer to the position and the motion kinds of probabilities respectively. Experimentally the two kinds can be regarded as measurable by distinct devices and procedures. Consequently the mentioned formulation has to combine the following two distinct transformations

$$\rho_{in} \to \rho_{out}\,, \qquad \vec{J}_{in} \to \vec{J}_{out}\,. \tag{47}$$

The considerations about the classical relation (40) suggest that, by completely similar arguments, the transformations (47) admit the following formulations

$$\rho_{out}(\vec{r}) = \iiint \Gamma(\vec{r}, \vec{r}\,')\,\rho_{in}\left(\vec{r}\,'\right)\,d^3\vec{r}\,' \tag{48}$$

$$J_{out;\,\alpha} = \sum_{\beta=1}^{3} \iiint \Lambda_{\alpha\beta}(\vec{r}, \vec{r}\,')\,J_{in;\,\beta}(\vec{r}\,')\,d^3\vec{r}\,'\,. \tag{49}$$

In (49) $J_{\eta;\alpha}$ with $\eta = in, out$ and $\alpha = 1, 2, 3 = x, y, z$ denote Cartesian components of $\vec{J}_\eta$.

Note the fact that the kernels Γ and $\Lambda_{\alpha\beta}$ from (48) and (49) have significance of transfer probabilities, completely analogous with the meaning of the classical kernel $G(a, a')$ from (40). This fact entails the following relations

$$\iiint \Gamma(\vec{r}, \vec{r}\,')\,d^3\vec{r} = \iiint \Gamma(\vec{r}, \vec{r}\,')\,d^3\vec{r}\,' = 1\,, \tag{50}$$

$$\sum_{\alpha=1}^{3} \iiint \Lambda_{\alpha\beta}\left(\vec{r}, \vec{r}'\right) d^3\vec{r} = \\ = \sum_{\beta=1}^{3} \iiint \Lambda_{\alpha\beta}\left(\vec{r}, \vec{r}'\right) d^3\vec{r}' = 1 \,. \tag{51}$$

The kernels Γ and $\Lambda_{\alpha\beta}$ describe the transformations induced by QMS in the data (information) about the measured microparticle. Therefore they incorporate some extra-QM elements regarding the characteristics of measuring devices and procedures. The respective elements do not belong to the usual QM framework which refers to the intrinsic (own) characteristics of the measured microparticle (system).

The above considerations facilitate an evaluation of the effects induced by QMS on the probabilistic estimators of here considered orbital observables A_j. Such observables are described by the operators $\hat{A}_j$ whose expressions depend on $\vec{r}$ and ∇. According to the previous discussions the mentioned operators are supposed to remain invariant under the transformations which describe QMS. So one can say that in the situations associated with the wave functions Ψ_η ($\eta = in, out$) the mentioned observables are described by the following probabilistic estimators/characteristics (of lower order): mean values $\langle A_j \rangle_\eta$, correlations $C_\eta\left(A_j,\, A_k\right)$ and standard deviations $\Delta_\eta A_j$. With the usual notation $(f, g) = \int f^* g\, d^3\vec{r}$ for the scalar product of functions f and g, the mentioned estimators are defined by the relations

$$\left.\begin{aligned} &\langle A_j \rangle_\eta = \left(\Psi_\eta,\, \hat{A}_j \Psi_\eta\right) \\ &\delta_\eta \hat{A}_j = \hat{A}_j - \langle A_j \rangle_\eta \\ &C_\eta\left(A_j,\, A_k\right) = \left(\delta_\eta \hat{A}_j\, \Psi_\eta,\, \delta_\eta \hat{A}_k\, \Psi_\eta\right) \\ &\Delta_\eta A_j = \sqrt{C_\eta\left(A_j,\, A_j\right)} \end{aligned}\right\} . \tag{52}$$

Add here the fact that the in-version of the estimators (52) are calculated by means of the wave function Ψ_{in}, known from the considerations about the inner properties of the investigated system (e.g. by solving the corresponding Schrödinger equation).

On the other hand the out-version of the respective estimators can be evaluated by using the probability density and current ρ_{out} and $\vec{J}_{out}$. So if $\hat{A}_j$ does not depend on ∇ (i.e. $\hat{A}_j = A_j(\vec{r})$) in evaluating the scalar products from (52) one can use the evident equality $\Psi_\eta \hat{A}_j\, \Psi_\eta = \hat{A}_j\, \rho_\eta$. When $\hat{A}_j$ depends on ∇ (i.e. $\hat{A}_j = A_j(\nabla)$) in the same products can be appealed to the substitution

$$\Psi_\eta^* \,\nabla \Psi_\eta = \frac{1}{2}\,\nabla \rho_\eta + \frac{im}{\hbar}\,\vec{J}_\eta \,, \tag{53}$$

$$\Psi_\eta^* \,\nabla^2 \Psi_\eta = \rho_\eta^{\frac{1}{2}}\,\nabla^2\,\rho_\eta^{\frac{1}{2}} + \frac{im}{\hbar}\,\nabla \vec{J}_\eta - \frac{m^2}{\hbar^2}\,\frac{\vec{J}_\eta^2}{\rho_\eta} \,. \tag{54}$$

The mentioned usage seems to allow the avoidance of the implications regarding [79] "*a possible nonuniqueness of current*" (i.e. of the couple ρ_η–$\vec{J}_\eta$).

Within the above presented model of QMS the errors (uncertainties) associated with the measurements of observables A_j can be evaluated through the following indicators

$$\left.\begin{aligned} &\varepsilon\left\{\langle A_j \rangle\right\} = \left|\langle A_j \rangle_{out} - \langle A_j \rangle_{in}\right| \\ &\varepsilon\left\{C\left(A_j, A_k\right)\right\} = \left|C_{out}\left(A_j, A_k\right) - C_{in}\left(A_j, A_k\right)\right| \\ &\varepsilon\left\{\Delta\, A_j\right\} = \left|\Delta_{out}\, A_j - \Delta_{in}\, A_j\right| \end{aligned}\right\} . \tag{55}$$

These quantum error indicators are entirely similar with the classical ones (43). Of course that, mathematically, they can be completed with error indicators like $\varepsilon\left\{\left(\left(\delta_\Psi \hat{A}_j\right)^r \Psi,\, \left(\delta_\Psi \hat{A}_k\right)^s \Psi\right)\right\}$, $r + s \geqslant 3$, which regard the higher order probabilistic moments mentioned in $\boldsymbol{R_{12}}$.

The above presented model regarding the description of QMS is exemplified in the end of this paper in Annex.

Now is the place to note that the out-version of the estimators (52), as well as the error indicators (55), have a theoretical significance.

In practice the verisimilitude of such estimators and indicators must be tested by comparing them with their *experimental (factual) correspondents* (obtained by sampling and processing of the data collected from the recorder of the measuring device). If the test is confirmative both theoretical descriptions, of QM intrinsic properties of system (microparticle) and of QMS, can be considered as adequate. But if the test gives an invalidation of the results, at least one of the mentioned descriptions must be regarded as inadequate.

In the end of this section we wish to add the following two observations:

(i) The here proposed description of QMS does not imply some interconnection of principle between the measuring uncertainties of two distinct observables. This means that from the perspective of the respective description there are no reasons to discuss about a measuring compatibility or incompatibility of two observables;

(ii) The above considerations from the present section refer to the QMS of orbital observables. Similar considerations can be also done in the case of QMS regarding the spin observables. In such a case besides the probabilities of spin-states (well known in QM publications) it is important to take into account the spin current density (e.g. in the version proposed recently [81]).

7 Some conclusions

We starred the present paper from the ascertained fact that in reality CIUR is troubled by a number of still unsolved deficiencies. For a primary purpose of our text, we resumed the CIUR history and identified its basic presumptions.Then, we attempt to examine in details the main aspects as well as the validity of CIUR deficiencies regarded in an elucidative collection.

The mentioned examination, performed in Section 3 reveal the following aspects:

(i) A group of the CIUR deficiencies appear from the application of usual RSUR (2) in situations where, mathematically, they are incorrect;

(ii) The rest of the deficiencies result from unnatural associations with things of other nature (e.g. with the thought experimental relations or with the presence/ absence of $\hbar$ in some formulas);

(iii) Moreover one finds that, if the mentioned applications and associations are handled correctly, the alluded deficiencies prove themselves as being veridic and unavoidable facts. The ensemble of the respective facts invalidate all the basic presumptions of CIUR.

In consensus with the above noted findings, in Section 4, we promoted the opinion that CIUR must be abandoned as an incorrect and useless (or even misleading) doctrine. Conjointly with the respective opinion we think that the primitive UR (the so called Heisenberg's relations) must be regarded as:

(i) fluctuation formulas — in their theoretical RSUR version (2);

(ii) fictitious things, without any physical significance — in their thought-experimental version (1).

Abandonment of CIUR requires a re-examination of the question regarding QMS theoretical description. To such a requirement we tried to answer in Sections 5 and 6. So, by a detailed investigation, we have shown that the CIUR-connected approaches of QMS are grounded on dubitable (or even incorrect) views.

That is why we consider that the alluded question must be reconsidered by promoting new and more natural models for theoretical description of QMS. Such a model, of somewhat informational concept, is developed in Section 6 and it is exemplified in Annex.

Of course that, as regards the QMS theoretical description, our proposal from Section 6, can be appreciated as only one among other possible models. For example, similarly with the discussions regarding classical errors [77, 78], the QMS errors can be evaluated through the informational (Shannon) entopies.

It is to be expected that, in connection with QMS, other models will be also promoted in the next moths/years. But as a general rule all such models have to take into account the indubitable fact that the usual QM and QMS theoretical description must be refered to distinct scientific questions (objectives).

Annex: A simple exemplification for the model presented in Section 6

For the announced exemplification let us refer to a microparticle in a one-dimensional motion along the x-axis. We take $\Psi_{in}(x) = |\Psi_{in}(x)| \cdot \exp\{i\Phi_{in}(x)\}$ with

$$|\Psi_{in}(x)| \propto \exp\left\{-\frac{(x-x_0)^2}{4\sigma^2}\right\}, \quad \Phi_{in}(x) = kx\,. \quad (56)$$

Note that here as well as in other relations from this Annex we omit an explicit notation of the normalisation constants which can be added easy by the interesed readers.

Correspondingly to the Ψ and Φ from (56) we have

$$\rho_{in}(x) = |\Psi_{in}(x)|^2\,, \quad J_{in}(x) = \frac{\hbar k}{m_0}\,|\Psi_{in}(x)|^2\,. \quad (57)$$

So the intrinsic properties of the microparticle are described by the parameters x_0, σ and k.

If the errors induced by QMS are small the kernels Γ and Λ in (48)–(49) can be considered of Gaussian forms like

$$\Gamma(x,x') \propto \exp\left\{-\frac{(x-x')^2}{2\gamma^2}\right\}, \quad (58)$$

$$\Lambda(x,x') \propto \exp\left\{-\frac{(x-x')^2}{2\lambda^2}\right\}, \quad (59)$$

where γ and λ describe the characteristics of the measuring devices. Then for ρ_{out} and J_{out} one finds

$$\rho_{out}(x) \propto \exp\left\{-\frac{(x-x')^2}{2(\sigma^2+\gamma^2)}\right\}, \quad (60)$$

$$J_{out}(x) \propto \hbar k \cdot \exp\left\{-\frac{(x-x')^2}{2(\sigma^2+\lambda^2)}\right\}. \quad (61)$$

It can been seen that in the case when both $\gamma \to 0$ and $\lambda \to 0$ the kernels $\Gamma(x,x')$ and $\Lambda(x,x')$ degenerate into the Dirac's function $\delta(x-x')$. Then $\rho_{out} \to \rho_{in}$ and $J_{out} \to J_{in}$. Such a case corresponds to an ideal measurement. Alternatively the cases when $\gamma \neq 0$ and/or $\lambda \neq 0$ are associated with non-ideal measurements.

As observables of interest we consider coordinate x and momentum p described by the operators $\hat{x} = x\cdot$ and $\hat{p} = -i\hbar\frac{\partial}{\partial x}$. Then, in the measurement modeled by the expressions (56),(58) and (59), for the errors (uncertainties) of the considered observables one finds

$$\varepsilon\{\langle x\rangle\} = 0\,, \quad \varepsilon\{\langle p\rangle\} = 0\,, \quad \varepsilon\{C(x,p)\} = 0\,, \quad (62)$$

$$\varepsilon\{\Delta x\} = \sqrt{\sigma^2+\gamma^2} - \sigma\,, \quad (63)$$

$$\varepsilon\{\Delta p\} = \hbar \left| \left[\frac{k^2(\sigma^2+\gamma^2)}{\sqrt{(\sigma^2+\lambda^2)(\sigma^2+2\gamma^2-\lambda^2)}} - \right.\right.$$
$$\left.\left. - \; k^2 + \frac{1}{4(\sigma^2+\gamma^2)} \right]^{\frac{1}{2}} - k \right| . \tag{64}$$

If in (56) we restrict to the values $x_0 = 0$, $k = 0$ and $\sigma = \sqrt{\frac{\hbar}{2m_0\omega}}$ our system is just a linear oscillator in its ground state (m_0 = mass and ω = angular frequency). This means that the "*in*"-wave function (56) has the same expression with the one from (14) for $n = 0$. As observable of interest we consider the energy described by the Hamiltonian

$$\hat{H} = -\frac{\hbar^2}{2m_0}\frac{d^2}{dx^2} + \frac{m_0\,\omega^2}{2}\,x^2. \tag{65}$$

Then for the respective observable one finds

$$\langle H\rangle_{in} = \frac{\hbar\,\omega}{2}\,, \qquad \Delta_{in}H = 0\,, \tag{66}$$

$$\langle H\rangle_{out} = \frac{\omega\left[\hbar^2 + \left(\hbar + 2m\,\omega\,\gamma^2\right)^2\right]}{4\left(\hbar + 2m_0\,\omega\,\gamma^2\right)}\,, \tag{67}$$

$$\Delta_{out}\,H = \frac{\sqrt{2}\,m\,\omega^2\,\gamma^2\,\left(\hbar + m\,\omega\,\gamma^2\right)}{\left(\hbar + 2m\,\omega\,\gamma^2\right)}\,. \tag{68}$$

The corresponding errors of mean value resoectively of standard deviation of oscillator energy have the expressions

$$\varepsilon\{\langle H\rangle\} = |\langle H\rangle_{out} - \langle H\rangle_{in}| \neq 0\,, \tag{69}$$

$$\varepsilon\{\Delta H\} = |\Delta_{out}\,H - \Delta_{in}H| \neq 0\,. \tag{70}$$

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A Model of Discrete-Continuum Time for a Simple Physical System

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Proceeding from the assumption that the time flow of an individual object is a real physical value, in the framework of a physical kinetics approach we propose an analogy between time and temperature. The use of such an analogy makes it possible to work out a discrete-continuum model of time for a simple physical system. The possible physical properties of time for the single object and time for the whole system are discussed.

Commonly, time is considered to be a fundamental property of the Universe, and the origin which is not yet clear enough for natural sciences, although it is widely used in scientific and practical activity. Different hypotheses of temporal influence on physical reality and familiar topics have been discussed in modern scientific literature (see, e. g., [1–3] and references therein). In particular, the conception of discrete time-space was proposed in order to explain a number of physical effects (e.g., the problem of asymmetry of some physical phenomena and divergences in field theory) [2–4]. Following this theme, in the present paper we shall consider some aspects of the pattern of discrete-continuum time for a single object and for the whole system. We will focus on the difference between time taken as a property of a single object and a property of the system. We would also touch upon the question of why the discreteness of time is not obvious in ordinary conditions.

As a "given" property of existence time is assumed to be an absolutely passive physical factor and the flow of time is always uniform in ordinary conditions (here we consider the non-relativistic case) for all objects of our world. Therefore classical mechanics proceeds from the assumption that the properties of space and time do not depend on the properties of moving material objects. However even mechanics suggests that other approaches are possible.

From the point of view of classical mechanics a reference frame is in fact a geometrical reference frame of each material object with an in-built "clock" registering time for each particular object. So in fixing the reference frame we deal with the time of each unique object only and subsequently this time model is extended to all other objects of concrete reality. Thus, we always relate time to some concrete object (see, e. g., [5]). Here we seem to neglect the fact that such an assumption extends the time scale as well as the time flow of only one object onto reality in general. This approach is undoubtedly valid for mechanical systems. In the framework of such an approach there is no difference between the time of an unique object and the time of the system containing a lot of objects.

But is it really so? Will the time scale of the system taken as a whole be the same as the time scale of each of the elements constituting the system? It is of interest to consider the opposite case, i.e. when time for a single object and time for the system of objects do not coincide. So we set out to try to develop a time model for a physical system characterized by continuum and discrete time properties which arise from the assumption that the time flow of an individual object is a real physical value as, for example, the mass or the charge of the electron. In other words, following Mach, we are going to proceed from the assumption that if there is no matter, there is no time.

In order to show the plausibility of such an approach we shall consider a set of material N objects, for example, structureless particles without any force-field interaction between them. Each object is assumed to have some individual physical characteristics and each object is the carrier of its own local time, i.e. for each i-object we shall define its own time flow with some temporal scale θ_i as

$$dt_i = \theta_i dt, \tag{1}$$

where t is the ordinary Newtonian time. Generally speaking, one can expect dependence of θ_i on the physical characteristics of the object, for example, both kinetic and potential energy of the object. However, here we shall restrict ourselves to consideration of the simple case when $\theta_i = \text{const}$.

Since we associate objects with particles we shall also assume that there are collisions between particles and the value of θ_i remains constant until the object comes into contact with other objects, as θ_i may be changed only during the impact, division or merger of objects. This means that the dynamics of a single object without interaction with other objects is determined only by its own time t_i. If, however, we consider the dynamics of an i-object with another j-object we have to take into account some common time of i- and j-objects which we are to determine.

This consideration suggests that in order to describe the whole system (here we shall use the term "system" to denote a set of N objects which act as a single object) one should use something close or similar to a physical kinetics pattern where macroscopic parameters like density, temperature etc., are defined by averaging the statistically significant ensemble of objects. In particular, for the system containing N particles the temperature may be written as

$$T = \frac{1}{N}\sum_{i=1}^{N} v_i^2 - \left(\frac{1}{N}\sum_{i=1}^{N} v_i\right)^2, \tag{2}$$

where v_i is the velocity of the i-object. For the whole system we introduce the general time τ to replace the local time of the i-object (1) as

$$d\tau = (1 + D)\, dt, \tag{3}$$

where D is determined by the differential relation

$$D(\tau) = \left[\frac{1}{N}\sum_{i=1}^{N} (\theta_i)^2 - \left(\frac{1}{N}\sum_{i=1}^{N} \theta_i\right)^2\right]^{1/2}. \tag{4}$$

By such a definition the general time of the system is the sum total of its Newtonian times and some nonlinear time $D(t_i)$ which is a function that depends on the dispersion of the individual times dt_i. It is noteworthy that this simplest possible statistical approach is similar to that of [6, 7].

It is quite evident that we have Newtonian-like time even if $D = \text{const} \neq 0$. Indeed, from (3) it follows that

$$\tau = (1 + D)\, t. \tag{5}$$

The pure Newtonian case in relation (3) is realized when all objects have the same temporal scales $\theta_i = \theta_0$.

At the same time there exist a number of cases in which the violation of the pure Newtonian case may occur. For example, let us assume that we have got a system where some number of objects would perish, disappear, whilst another set of the objects might come into existence. In this case the number N is variable and we have to consider D as an explosive step-like function with respect to N, which we ought to integrate (3) only in some interval $t_0 \leqslant t \leqslant t_x$ where D remains constant. Here it is obvious that the value of such an interval $t_x - t_0$ is initially unknown. Instead of the Newtonian continuum time (5) we now get a piecewise linear continuous time which is determined by the following recurrence relation

$$\tau = (1 + D)(t - t_0) + \tau_0\,, \qquad t_0 \leqslant t \leqslant t_x\,, \tag{6}$$

where $\tau_0 = \tau(\tau_0)$. This relation remains true whilst D is not changed. At the moment of local time $t = t_x$ the value D becomes $D + \Delta D$, so we have to redefine τ_0 and other parameters as

$$\left.\begin{array}{l} \tau_0 := \tau(t_x) = (1 + D)(t_x - t_0) + \tau_0 \\ t_0 := t_x\,, \quad D := D + \Delta D \end{array}\right\}. \tag{7}$$

Thus, instead of the linear Newtonian time for a single object we get the broken linear dependence for the time of the whole system if the number of objects forming this system is continuously changing.

Since in reality the majority of objects, as a rule, form some systems consisting of elementary units, it can be concluded that the number of constituent elements might change, as was shown above in the example considered. In this case D becomes variable and one deals with the manifestation of a piecewise linear dependence of time.

However, it is clear that the effects of this non-uniform time can be revealed to best advantage in a system with a rather small N, since in the limit $N \to \infty$ the parameter D becomes little sensitive to the changes in N. That is the basic reason why, in ordinary conditions, we may satisfy ourselves with the Newtonian time conception alone.

In the present paper we have tried to draw an analogy between time and temperature for the simplest possible physical system without collective interaction of the objects constituting the system, in order to show the difference in the definition of time for unique objects and for whole systems. One should consider this case as a basic simplified example of the system where the discrete-continuum properties of time may be observed. Thus one should consider it as a rather artificial case since there are no physical objects without field-like interactions between them.

However, despite the simplified case considered above, the piecewise linear properties of time may in fact be observed in reality (in ordinary, non-simplified conditions), though they are by no means obvious. In order to reveal the of dispersion time $D(\tau)$ it is necessary to create some specific experimental conditions. Temporal effects, in our opinion, are best observed in systems characterized by numerous time scales and a relatively small number of constituent elements.

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Models for Quarks and Elementary Particles — Part I: What is a Quark?

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A quark is not a tiny sphere. The formal model idea is based on a vector group which is constructed like an outer vector product. The vectors perform dynamic movements. Two vectors (vector pair) which rotate in opposite directions in a plane have an increasing and diminishing result vector as consequence. At the same time the vector group rotates about the bisectrix of the vector pair. The two movements matched to each other result in that the tip of the resultant vector draws so-called geometrical locus loops in a plane. The u- and the d-quarks have characteristic loops. Each vector group has its own orthogonal, hyperbolic space. By joining three such spaces each, two groups of spaces, one group with a quasi-Euclidian and one group with a complex space are obtained. Based on the u- and d-quarks characterized with their movements and spaces a first elementary particle order is compiled.

1 Introduction

The models are presented in a comprehensive work* and comprise a large number of aspects. Not all of these can be reflected in the present publication. For this reason, only the prominent aspects are presented in four short Parts I to IV.

It is clear that the answer to the question of the heading cannot originate from experiments. A quark is a part of the confinements, of the interior of the elementary particles, which are not accessible for experiments. For this reason the answer in the present case is based on a model, (*lexically = draft, hypothetical presentation to illustrate certain statements; hypothesis = initially unconfirmed assumption of legitimacy with the objective of making them a guaranteed part of our knowledge through confirmation later on*) which on the one hand is based on secured, e.g. QED, physical theories (*lexically = scientific presentation, system of scientific principles*). The answer is not based on one or several axioms (*lexically = immediately obvious tenet which in itself cannot be justified*).

The model or the models were developed during a journey of thinking taking decades from the galaxies to the quarks, to the elementary particles, back to the stars and again to the confinement, the universe as a puzzle.

2 The vector principle

The photon contains electric and magnetic fields and is described with appropriate vectors. This formal description possibility is utilised. Why does the photon have the electric and magnetic vectors positioned vertically to the direction of flight and vertically with regard to each other, the understanding of this will be developed during the course of the model development. For this reason it is obvious that a long distance over highly formal stretches was covered which is not re-enacted here in detail.

It is highly productive to start from the idea of the outer vector product known from mathematics: two vectors of identical size start in a coordinate origin and open up a plane. The resultant vector (EV) stands vertically on this plane and likewise starts in the coordinate origin. In the next step the three vectors of the outer vector product are given a dynamic characteristic. Two movements are introduced:

Firstly, the two identically sized vectors perform a movement in opposite directions. Since the angle between the two vectors **V1** and **V2** is called 2φ, this is described as φ-rotation or φ-swivelling; see Fig. 1.

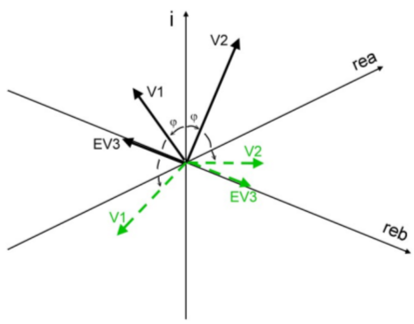

Fig. 1

If the two vectors according to Fig. 1 perform smaller and opposing φ-swivel movements, the resultant vector **EV3** becomes greater and smaller in its orientation.

Secondly, the entire vector arrangement measured by Fig. 1 performs a rotation about the bisectrix between the vectors **V1** and **V2**. Since this angle of rotation is referred to as ρ, this rotation is a ρ-rotation or a ρ-swivelling. If the vectors **V1** and **V2** during the ρ-rotation enclose a fixed angle, the vector tip of the **EV** draws an arc of a circle. However,

*There is a homepage under the Internet address www.universum-un.de where a book with the title “Models for Quarks and Elementary Particles” will be displayed, having a volume of approximately 250 DIN A4 pages.

should the angle 2φ between **V1** and **V2** change during the ρ-rotation, the tip of the **EV** deviates from the arc of the circle; see Fig. 2.

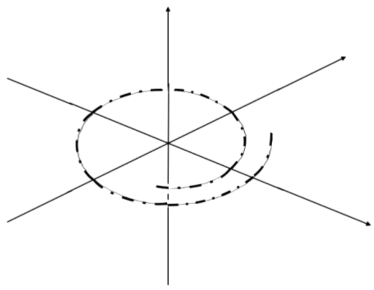

Fig. 2

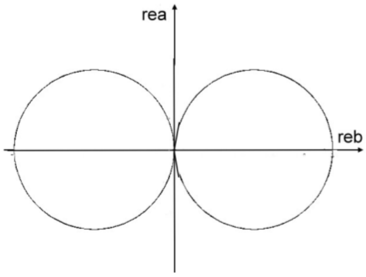

Fig. 3

It is immediately obvious that there are a huge number of possible combinations of the two φ- and ρ-movements in a coordinate system. In developing the models attention was paid to ensure that only φ- and ρ-movements that were matched to one another were considered. If for example each vector **V1** and **V2** starting from the vertical axis covers an angle $\varphi = 90°$ and the **EV** at the same time covers an angle of $\rho = 180°$ in the horizontal plane, the tip of the **EV** draws a loop in a plane. Assuming two vector pairs (one drawn black and one green) with the arrangement as in Fig. 1, two loops, see for instance Fig. 3 are obtained. Loops of this type are called geometric loci or geometric locus loops.

3 The three types of space

The limitation to a defined few coordinated φ- and ρ-movements is not yet sufficient to understand quarks. It is necessary to go beyond the Euclidian space with three orthogonal axes. At the same time, the principle of the vectors, especially that of the outer vector product should be maintained. The transition is made from the Euclidian space to the hyperbolic space with right angles between the axes. Here, it must be decided if the hyperbolic space should have one or two imaginary axes. Just as in the case of the vectors only very few models with matched φ- and ρ-movements were found to be carrying further, only few variants carry further with the space as well. (It has not been possible to find a similarly selective way from the amount of the approximately 10^{500} string theories and, in my opinion, will never be found either.)

Just as an outer vector product is productive as idea, it is also productive for the outer vector product, (for the first quark generation) to assume an orthogonal, hyperbolic space with two real axes and one imaginary axis; Fig. 4.

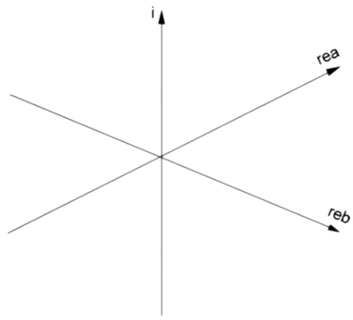

Fig. 4

So as not to create any misunderstanding at this point: it is not that several vector groups (one vector pair, **VP**, and one **EV** each) are placed in a hyperbolic orthogonal space with two real axes and one imaginary axis but each vector group has its own hyperbolic space. Here, the **VP** can be positioned in the real plane or in a Gaussian plane.

Various combinations of the vector groups are possible, as a result of which individual spaces can also be combined differently. As with the φ- and ρ-movements and as with the hyperbolic space, a selection has to be performed also with the combination of individual spaces. Fig. 5 to Fig. 9 show such a selection. The choice of words of the captions to the Figures becomes clear only as this text progresses.

Taking into account quantum chromodynamics, which prescribes three-quark particles, the result of the selected combinations of such individual spaces is the following: only two groups of combined spaces of three vector groups each are obtained: either spaces which in each of the three orientations have at least one real axis (if applicable, superimposed by an imaginary axis) and are therefore called "quasi-Euclidian" (see Fig. 7 and Fig. 8), or spaces which only have imaginary axes in one of the three orientations and are therefore called "complex" (see Fig. 6).

Particles of three quarks have either a quasi-Euclidian or a complex overall space. The Euclidian space from the view of this model is fiction.

Note for Fig. 6 to Fig. 9: Variants of three hyperbolic spaces linked in the coordinate origin consisting of the hyperbolic spaces of a dual-coordination and the hyperbolic space of a singular quark in various arrangements.

4 The four quarks (of the first generation)

Taking into account the construction of a vector group, the matched φ- and ρ-movements, the orientation of **VP** and **EV** in the hyperbolic space and the electric charge a geometrical locus loop according to Fig. 10 is obtained for the d-quark

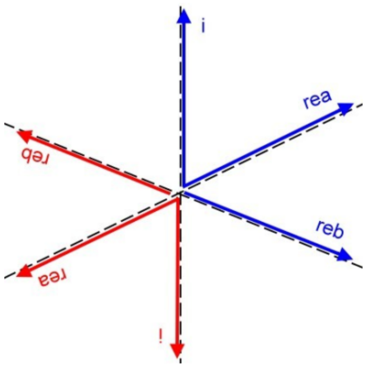

Fig. 5: The two ideal-typically arranged hyperbolic spaces of a dual-coordination as $\underline{dd}$, $\underline{uu}$, $\underline{\overline{dd}}$, $\underline{\overline{uu}}$, linked in the coordinate origin.

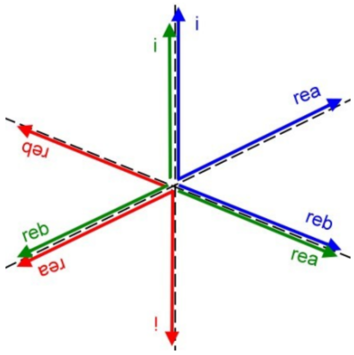

Fig. 6

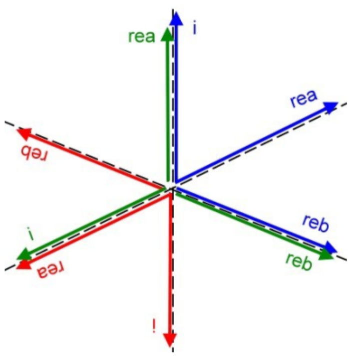

Fig. 7

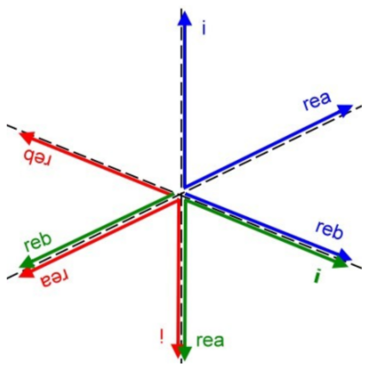

Fig. 8

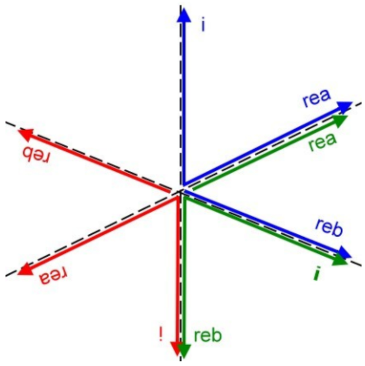

Fig. 9

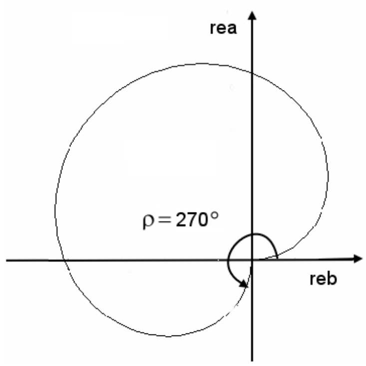

Fig. 10

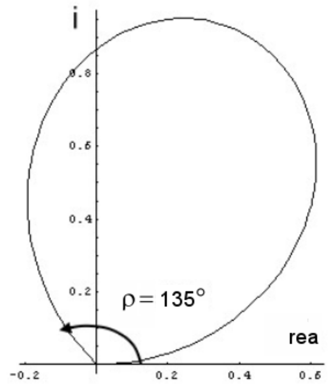

Fig. 11

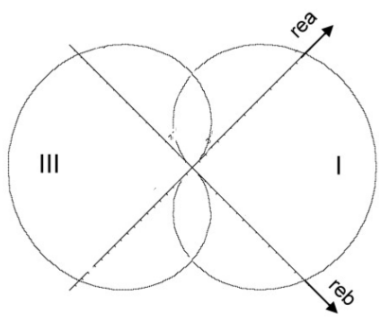

Fig. 12

with negative electric charge and a loop according to Fig. 11 is obtained for the u-quark with positive electric charge.

Antiquarks are characterized by an opposite electric charge so that the geometrical locus loop of the $\overline{d}$-quarks with positive electric charge is situated in a Gaussian plane and the geometrical locus loop of the $\overline{u}$-quark with negative electric charge is situated in the real plane.

5 Experiment of an order of elementary particles

Fig. 1 shows two vector groups (black and green) and Fig. 5 shows two hyperbolic spaces (blue and red); the vector groups and the hyperbolic spaces are each inter-linked in the coordinate origin. These presentations stem from the realisation that two quarks of the same type of each three-quark particle assume a particularly close bond. In the text this is called "dual-coordination", or briefly, "Zk". The third remaining quark of a three-quark particle is then called a "singular" quark. The different orientations of the quarks (**VP** and **EV**) with their spaces result in that the geometrical locus loops can stand at different angles relative to one another. A $\underline{dd}$-Zk for example has an angle zero between both ρ-rotation planes, see Fig. 12. The same applies to a $\underline{uu}$-Zk with angle zero between both ρ-rotation planes. Since the planes are positioned in parallel, the symbol || is used. If the rotation planes of two geometrical loci stand vertically on top of each other, the symbol $\perp$ is used. Table 1 is produced with this system.

Line	**ddd**	**ddu**	**duu**	**uuu**
A	$\underline{dd}_{\|\|} d \equiv e_e$	$\underline{dd}_{\|\|} u \equiv n^0$	$d_{\|\|} \underline{uu} \equiv p^+$	$\underline{uu}_{\|\|} u \equiv \Delta_\Delta$
B	$\underline{dd}_{\perp} d \equiv e^-$	$\underline{dd}_{\perp} u \equiv \nu_e$	$d_{\perp} \underline{uu} \equiv ?^+$	$\underline{uu}_{\perp} u \equiv \left(\Delta^{++}\right)$
C	$ddd \equiv \Delta^-$	$ddu \equiv \Delta^0$	$duu \equiv \Delta^+$	$uuu \equiv \Delta^{++}$

Table 1: The order of particles, sorted by quark flavours and the parallel || and vertical $\perp$ orientation of the geometrical loci.

The esteemed reader will be familiar with four of the spin $\frac{1}{2}\hbar$-particles (neutron n^0, proton p^+, electron e^- and neutrino ν_e) and, if applicable, the Δ-particles with spin $\frac{3}{2}\hbar$ from line C from high-energy physics. Because of the brevity of the present note the individual quark compositions will not be discussed. However, it is immediately evident that highly interesting consequences for the standard model of physics are obtained from the methodology of Table 1. This is evident on the examples of the electron and the neutrino, which, in the standard model, are considered as uniform particles, but here appear to be composed of quarks. In Parts II and IV of the publication the aspect of the electron composed of quarks is deepened. The structural nature of the quarks in the nucleons is another example for statements of these models that clearly go beyond the standard model.

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Models for Quarks and Elementary Particles — Part II: What is Mass?

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It is extremely productive to give the resultant vector (**EV**) from the outer vector product (Part I of this article series) a physical significance. The **EV** is assumed as electric flux ∉ with the dimensions [Vm]. Based on Maxwell's laws this develops into the idea of the magnetic monopole (MMP) in each quark. The MMP can be brought in relation with the Dirac monopole. The massless MMP is a productive and important idea on the one hand to recognise what mass is and on the other hand to develop the quark structure of massless photon (-likes) from the quark composition of the electron. Based on the experiments by Shapiro it is recognised that the sinusoidal oscillations of the quark can be spiralled in the photons. In an extreme case the spiralling of such a sinusoidal arc produces the geometric locus loop of a quark in a mass-loaded particle.

1 Introduction

Based on some characteristics of the photon mentioned in Part I [1], vectors are introduced to describe the quarks. The formal structures of the quarks (of the first generation) are presented with outer vector product, its angular movements and the corresponding space types. A first order of the elementary particles follows [2].

2 The magnetic monopole (MMP)

It is highly productive to give the vectors from the outer vector product (Part I of this series of papers) a physical meaning. Initially, the **EV** is assumed as electric flux ∉ with the dimensions [Vm].

A very good model for further considerations is given in [3] (see Fig. 7.128, p.398 therein), in which a changing electric field with an enclosing magnetic field is shown. For the present models this should be formulated as follows: a vector pair (**VP**) generates the **EV** issuing from the origin of a coordinate system, which **EV** is now identified with an electric flux ∉. When this flux is created, almost the entire electric flux ∉ based on Maxwell's laws creates the magnetic flux Φ located ring-shaped about the ∉-flux. With this linkage, the models are put on the basis of the QED mentioned in Part I. Feynman [22] calls the QED-theory the best available theory in natural sciences.

The electric source flux ∉ in turn comprises the toroidal magnetic flux Φ, (like the water of a fountain overflowing on all sides), whose maximum radius is designated MAGINPAR, which is illustrated with Fig. 1.

The ∉-**EV** with toroidal magnetic flux Φ is a substantial part of the description of a quark. With the coverage of the toroidal magnetic flux Φ through the electric source flux ∉ it is also an obvious explanation for the magnetic flux Φ not appearing outside the confinement under normal circumstances. The ∉-**EV** shown in Fig. 1 does not correspond to a dipole.

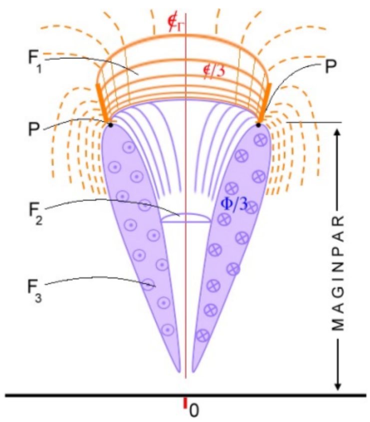

Fig. 1: Schematic section through the Φ- and ∉-fields of a (d-)quark. In the Φ-tube or funnel the ∉-field lines created in orange are not indicated. P designates the outer apex line of the Φ-flux which determines the MAGINPAR at the same time. Graphically, the configuration is also called "fountain". The symbolic $∉_\Gamma$-field line lies on the funnel longitudinal axis and is discussed in Part III.

With the latter, the fields shown would be simultaneously visible on two sides of the coordinate origin, while an ∉-flux trough would also have to appear opposite to a source flux ∉.

A **Zk** (see Part I) comprises two such ∉-source fluxes offset by 180° relative to each other which are merely like a dipole. A three-quark particle according to Table 1 (see Part I) comprises three ∉-source fluxes.

Dirac has stated the charge of the magnetic monopole according to Jackson ([3], p.319), as follows:

$$g^2 = \frac{1}{{}^e\alpha} \times \frac{n^2}{4} \times 4\pi\mu_0\hbar c\ [\mathrm{V^2 s^2}] \quad \text{or} \quad g = \frac{n}{2} \times \sqrt{\frac{4\pi\mu_0\hbar c}{{}^e\alpha}}\,,$$

$$g = 4.1357\times10^{-15}\ [\mathrm{V\,s}] \quad \text{with} \quad n = 1\,.$$

If this value is multiplied with double the value of the fine structure constant $2\,{}^e\alpha = 1/68.518$, it is identical to the value of the magnetic flux $\Phi = 6.03593\times10^{-17}$[Vs] of the present models. The dimension of g is likewise identical to the magnetic flux ϕ of the present models, (see [2] Chapter 8.1).

The electron or the electric unit charge of $q = = 1.60219\times10^{-19}$ [As] according to [1] Table 1 and according to [2] (Chapter 7 therein), consists of three d-quarks. Consequently the natural constant Φ does not stand for a quark either but for a "3QT", i.e. according to a first assumption for the three d-quarks of the electron. Imagining the electric flux $\notin$ and the toroidal magnetic flux Φ of a quark according to Fig. 1 the magnetic fluxes of a d-quark or of a u-quark amount to:

$$\Phi_d = \frac{\Phi}{3} = \frac{6.03593\times10^{-17}}{3} = 2.01198\times10^{-17}\ [\mathrm{Vs}]\,,$$

$$\Phi_u = \frac{2\Phi}{3} = 4.02396\times10^{-17}\ [\mathrm{Vs}]\,.$$

According to the present models these magnetic fluxes are the values of the magnetic monopoles (MMPs).

Obviously this means that we, and our entire world, also consist of the much sought-after MMPs.

The intensity of the interaction of the Dirac monopole is estimated extremely high. Since the MMP according to the present models is approximately $2\,{}^e\alpha$ smaller, the intensity of the interaction of the MMPs is substantially smaller as well. The force between two charged particles corresponds to the product of both charges:

$$\frac{g^2}{\Phi^2} = \frac{\left(4.1356\times10^{-15}\right)^2}{\left(6.03593\times10^{-17}\right)^2} = = \frac{\left(68.518\times6.03593\times10^{-17}\right)^2}{\left(6.03593\times10^{-17}\right)^2} = \frac{68.518^2}{1} = \frac{4695}{1}\,.$$

The charge quantity g determined by Dirac thus results in 4695 times greater a force between the charges g than between the fluxes Φ. A further reduction of the interaction obviously results through the $\frac{\Phi}{3}$ and $\frac{2\Phi}{3}$ fragments of the d- or u-quarks. These reduction factors are not the sole cause for the quite obviously much lower intensity of the interaction of the MMPs than assumed by Dirac. The probably decisive reduction factor is the construction of the quark sketched in Fig. 1, where the magnetic flux Φ of a quark is shielded to the outside by the electric flux $\notin$.

The literature sketches an MMP as follows:

- A constant magnetic field oriented to the outside on all sides (hedgehog) not allowing an approximation of additional MMPs;
- If two or more (anti-) MMPs attract one another, they are unable to assume a defined position relative to one another because of their point-symmetrical structure;
- The "literature MMP" is the logical continuation of the current world view of the "spheres" which is moderated through probability densities. Atoms are relatively "large spheres", nucleons are "very small spheres" therein, and the quarks would consequently be "even smaller spheres" in the nucleons and the electrons are allegedly point-like. The interactions between the "spheres" are secured by the bosons as photons or gluons.

The aspects of these models are:

- The idea of the "sphere chain" is exploded in these models since the swivelling and simultaneously pulsating MMPs act in all particles. Particles can be seen highly simplified as different constellations of MMPs;
- The idea of the "fountain" according to Fig. 1 contains the toroidal magnetic flux Φ as MMP;
- The structures brought about by the MMP are temporally, spatially and electromagnetically highly anisotropic and asymmetrical. Without this structure our world would not be possible. From this it can be concluded that the highly symmetrical "literature MMP" sketched above must not be seen as an elementary part of our world.

3 Some enigmas of the photon

(a) Why the photon has the electric and magnetic vectors positioned vertically to the direction of flight and vertically to each other is not answered in Part I;

(b) If the photon is created through "annihilation" of electron and positron as is well known and if the electron according to Table 1, Part I, has the quark structure $\underline{dd}\,{}_{\perp}d$, the question arises if the characterisation of the photon with the simple letter γ according to the standard model is correct;

(c) If electron and positron have a basic mass $m = 0.511$ MeV/c^2 why does the photon have the mass zero?

(d) Why does the wavelength of the light observed by us not fit to the Compton wavelength of the electrons emitting the light?

To solve the enigma, some courageous jumps have to be performed:

First jump: The photon consists of the same quark type as the electron, namely d according to Table 1 (of Part I);

Second jump: The photon contains its own anti-particle, i.e. consists of the quark types d and $\bar{d}$ according to the models;

Third jump: Both quark groups (3 d and 3 $\bar{d}$) oscillate by themselves with very similar basic frequencies. This is explained as follows:

The Compton wavelength of the electron (3 d) or that of the positron (3 $\bar{d}$) in each case results in a basic frequency of approximately 10^{20}Hz. Thus the photon has two very similar basic frequencies. The beat resulting from both frequencies has a wavelength or frequency which is greater and lower respectively by the factor 10^5 and with just under 10^{15}Hz is also in the visible range. The beat is the answer to Question (d) concerning the photon.

Some consequences of the courageous jumps:

(1) The photon must be seen as a composite **yet uniform** particle;

(2) Two frequencies in this uniform medium create a **beat**;

(3) According to Table 1 of Part I, Line B, there are three additional leptons in addition to the electron (or its anti-particle positron). It can be expected that from these leptons and each of their anti-particles composite **yet uniform** particles can be formed according to the same pattern as with the photon. These particles are called "photon-like" in the models.

In Table 1, the quark structure of the electron is introduced with $\underline{dd}\,{}_\perp\, d$. Using the anti-d-quark the positron has the same structure. If both elementary particles in the photon are connected it should be unsurprisingly expected that both structures can be found again in the photon. In addition to this it should be expected that both particles are closely connected with each other. This is expressed in that the two singular quarks of electron and positron in turn assume a close bond. In the models this is called "bond coordination" or "Bk" in brief and in the case of the photon $\underline{\underline{d\bar{d}}}$ as structural element. Consequently the overall structure of the photon appears as $\underline{dd}\,{}_\perp\,\underline{\underline{d\bar{d}}}\,{}_\perp\,\underline{\bar{d}\bar{d}}$. The overall photon-like structure of the neutrinos would be $\underline{dd}\,{}_\perp\,\underline{\underline{u\bar{u}}}\,{}_\perp\,\underline{\bar{d}\bar{d}}$, etc.

In contrast with the three-quark particles of Table 1 the photon-likes are six-quark particles. It is clear that the six-quark structure of the photon-likes has substantial consequences on the reaction equations of the weak interaction. This is reported in Part IV. The quark structure of the photon is the answer to Question (b) concerning the photon.

4 The "pioneering" experiments of Shapiro

Years after the discovery of the quark structure of the photons and long after the insight, as to what mass actually is, was gained, the experiments by Shapiro [5] were brought into relation with both. Here, the experiments by Shapiro are dealt with first in order to facilitate introduction to the subjects.

Towards the end of the nineteen-sixties, Shapiro observed a reduced speed of light c_M near the Sun. The cause is the "refractive index of the vacuum". Deviating from the interpretation through the standard model of physics and utilising new insights through these models the following is determined in a first jump:

Under the effect of directed electric fields the flat sinusoidal oscillation of the photon becomes helical (see [2], pages 167 and 179). This results in that at constant frequency the penetration points of the sine curve through the "x-axis" are situated closer together and the speed of the photon in direction of flight is no longer c but c_M.

Following this thought pattern it can be determined in a second jump: Under the effect of extremely strong highly directional electric fields the initially flat sinusoidal oscillation of the photon is spiralled to such an extent that the geometrical locus loops used for "stationary" particles appear (see [2], page 165ff and Fig. 2 and 3).

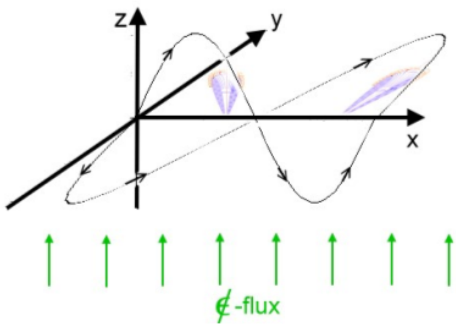

Fig. 2: A photon with initially flat sinusoidal arcs and with schematically sketched "fountain" runs vertically to the direction of an electric field while the arrows on the sinusoidal arcs indicate the sequence of the amplitude.

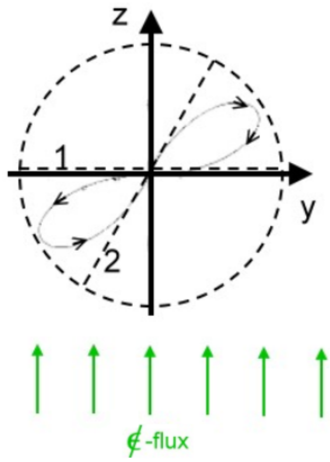

Fig. 3: Projection of the helically deformed initially horizontal and flat sinusoidal arcs of a photon according to Fig. 2 in the $y-z$ plane.

Looking at the helical sinusoidal oscillation in the direction of the x-axis a sinusoidal arc presents itself as a narrower or wider loop. If the loop is very narrow the progressive speed c_M of the photon is only a little smaller than c [5]. However if the loop is very wide the photon is practically unable to move. This means that the photon is then captured in an electron.

The extremely strong directional electric fields can be found in the source fields of the "fountain", Fig. 1, of the electron quarks. This means that an electron with suitable MAGINPAR is able to spiral the lateral sinusoidal oscillation of an approaching photon to such an extent that the lateral sinusoidal oscillation becomes a central-symmetrical sinusoidal oscillation. If the amplitudes or MAGINPAR of both particles fit to each other the photon is stored in the electron. This also means that an electron charged in this way — and that is every electron from our environment — has central-symmetrical sinusoidal oscillations of 3 d-quarks as well as stored 3 d- and 3 $\bar{d}$-groups of the photons.

It is now evident: the flat oscillation of the photon is converted to the radial oscillation in the electron or in the fermion through the extremely strong directional electric quark source fields. The geometrical locus loops developed from formal aspects which are shown in Part I for instance with Fig. 3 are sine curves or sinusoidal oscillations which are presented in polar coordinates for a centre each.

5 What is mass?

In Table 1 the neutron and the neutrino are positioned below each other. Both have the same types and quantities of quarks, however with different structural signs! The mass of the neutron almost amounts to 940 MeV, the mass of the neutrino according to the standard model below one eV. The electron and the positron each have a basic mass of 0.511 MeV, while the photon consisting of the same quarks has no mass at all.

Quite obviously, "mass" is not a characteristic of a quark. Mass is a characteristic which arises from the constellation of several quarks. Only certain elementary particles have mass! These include those where the MMPs perform central-symmetrical sinusoidal oscillations, e.g. the three-quark particles of Table 1. The amplitude of the central-symmetrical sinusoidal oscillations is practically identical with the MAGINPAR R. The magnitude of the MAGINPAR R is determined by the frequency ν via $\nu = \frac{c}{\lambda} = \frac{c}{X\pi R}$.

In [2] (page 164), mass is defined as follows:

$$\begin{aligned} m &= \frac{h}{c^2}\,\nu = \\ &= \frac{\Phi q}{2\,{}^e\alpha c^2}\,\nu = 7.3726\times10^{-51}\left[\frac{\mathrm{V\,A\,s^4}}{\mathrm{m^2}}\right]\times\nu\left[\frac{1}{\mathrm{s}}\right]. \end{aligned} \tag{1}$$

Conclusion: Mass is nothing other than the very, very frequent occurrence of the MMPs Φ at the coordinate centre of the particle in accordance with the frequency ν multiplied by the electric charge q divided by c^2 and also $2\,{}^e\alpha$. The constants jointly have the value 7.3726×10^{-51} [V A s^4/m^2]. These statements satisfy a desire of physics that has remained unanswered for a very long time. The masses of the mass-loaded elementary particles known to us that could only be experimentally measured in the past can be calculated from elementary quantities.

With a photon, the six quarks or MMPs involved describe a lateral movement along a line. The sinusoidal oscillations of the MMPs are not central-symmetrical. According to the definition such particles have no mass. The lateral movement is the answer to Question (c) regarding the photon.

The well-known relation of mass m [V A s^3/m^2] and inertia ${}^N\Theta$ [V A s^3/m] becomes visible by introducing the equations $h = {}^N h / 2\,{}^e\alpha$ and ${}^N h = {}^N\Theta \times c$. (See [2], Fig. 8.3a of Chapter 8.2.1 therein.) By this equation, equation (1) transforms to

$$m = \frac{{}^N\Theta\,\nu}{2\,{}^e\alpha c} \quad \text{or} \quad m = \frac{{}^N\Theta}{2\,{}^e\alpha X\pi R},$$

which is the short version of equation (8-II) on page 156 of [2].

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Electric Charge as a Form of Imaginary Energy

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Electric charge is considered as a form of imaginary energy. With this consideration, the energy of an electrically charged particle is a complex number. The real part is proportional to the mass, while the imaginary part is proportional to the electric charge. The energy of an antiparticle is given by conjugating the energy of its corresponding particle. Newton's law of gravity and Coulomb's law of electric force are classically unified into a single expression of the interaction between the complex energies of two electrically charged particles. Interaction between real energies (or masses) is the gravitational force. Interaction between imaginary energies (or electric charges) is the electromagnetic force. Since radiation is also a form of real energy, there are another two types of interactions between real energies: the mass-radiation interaction and the radiation-radiation interaction. Calculating the work done by the mass-radiation interaction on a photon, we can derive the Einsteinian gravitational redshift. Calculating the work done by the radiation-radiation interaction on a photon, we can obtain a radiation redshift. This study suggests the electric charge as a form of imaginary energy, so that classically unifies the gravitational and electric forces and derives the Einsteinian gravitational redshift.

1 Introduction

It is well known that mass and electric charge are two fundamental properties (inertia and electricity) of matter, which directly determine the gravitational and electromagnetic interactions via Newton's law of gravity [1] and Coulomb's law of electric force [2]. Mass is a quantity of matter [3], and the inertia of motion is solely dependent upon the mass. According to Einstein's energy-mass expression (or Einstein's first law) [4], mass is also understood as a form of real energy. The real energy is always positive. It cannot be destroyed but can be transferred from one form to another. Therefore, the mass is understood not only based on the gravitational interaction but also on the quantity of matter, the inertia of motion, and the energy

Electric charge has two varieties of either positive or negative. It appears always in association with mass to form positive or negative electrically charged particles with different masses. The interaction between electric charges, however, is independent of the mass. Positive and negative charges can annihilate or cancel each other and produce in pair with the total electric charges conserved. So far, the electric charge is understood only based on the electromagnetic interactions. Its own physics meaning of a pure electric charge is still unclear.

In this paper, the pure electric charge is suggested to be a form of imaginary energy. With this suggestion or idea of imaginary energy, we can express an electrically charged particle as a pack of certain amount of complex energy, in which the real part is proportional to the mass and the imaginary part is proportional to the electric charge. We can combine the gravitational and electromagnetic interactions between two electrically charged particles into the interaction between their complex energies. We can also naturally obtain the energy of an antiparticle by conjugating the energy of its corresponding particle and derive the Einsteinian gravitational redshift from the mass-radiation interaction, a type of interaction between real energies.

2 Electric charge — a form of imaginary energy

With the idea that the electric charge is a form of imaginary energy, total energy of a particle can be generally expressed as a complex number

$$E = E^M + iE^Q, \tag{1}$$

where $i = \sqrt{-1}$ is the imaginary number. The real energy $\mathrm{Re}(E) = E^M$ is proportional to the particle mass

$$E^M = Mc^2, \tag{2}$$

while the imaginary energy $\mathrm{Im}(E) = E^Q$ is proportional to the particle electric charge defined by

$$E^Q = \frac{Q}{\sqrt{G}}c^2 = \alpha E^M, \tag{3}$$

where G is the gravitational constant, c is the light speed, and α is the charge-mass ratio (or the imaginary-real energy ratio) defined by

$$\alpha \equiv \frac{E^Q}{E^M} = \frac{Q}{\sqrt{G}M}, \tag{4}$$

in the cgs unit system. The imaginary energy has the same sign as the electric charge has. Including the electric charge, we can modify Einstein's first law as

$$E = (1 + i\alpha)Mc^2. \tag{5}$$

The modulus of the complex energy is

$$|E| = \sqrt{1+\alpha^2}\,Mc^2. \tag{6}$$

For an electrically charged particle, the absolute value of α is a big number. For instance, proton's α is about 10^{18} and electron's α is about -2×10^{21}. Therefore, an electrically charged particle holds a large amount of imaginary energy in comparison with its real or rest energy. A neutral particle such as a neutron, photon, or neutrino has only a real energy.

3 Unification of Newton's law of gravity and Coulomb's law

Considering two pointlike electrically charged objects with masses M_1, M_2, electric charges Q_1, Q_2, and distance r, we can unify Newton's law of gravity and Coulomb's law of electric force by the following single expression of the interaction between complex energies

$$\vec{F} = -\,G\,\frac{E_1 E_2}{c^4 r^3}\,\vec{r}, \tag{7}$$

where E_1 is the energy of object one and E_2 is the energy of object two. Eq. (7) shows that the interaction between two particles is proportional to the product of their energies and inversely proportional to the square of the distance between them.

Replacing E_1 and E_2 by using the complex energy expression (1), we obtain

$$\vec{F} = -\,G\,\frac{M_1 M_2}{r^3}\,\vec{r} + \frac{Q_1 Q_2}{r^3}\,\vec{r} - i\,\sqrt{G}\,\frac{M_1 Q_2 + M_2 Q_1}{r^3}\,\vec{r} =$$

$$= \vec{F}_{MM} + \vec{F}_{QQ} + i\vec{F}_{MQ}\,. \tag{8}$$

The first term of Eq. (8) represents Newton's law for the gravitational interaction between two masses $\vec{F}_{MM}$. The second term represents Coulomb's law for the electromagnetic interaction between two electric charges $\vec{F}_{QQ}$. The third term is an imaginary force between the mass of one object and the electric charge of the other object $i\vec{F}_{MQ}$. This imaginary force is interesting and may play an essential role in adhering an electric charge on a mass or in combining an imaginary energy with a real energy. A negative imaginary force adheres a positive electric charge on a mass, while a positive imaginary force adheres a negative electric charge on a mass. Figure 1 sketches all of the interactions between two electrically charged particles as included in Eq. (8).

Electric charges have two varieties and thus three types of interactions: (1) repelling between positive electric charges $\vec{F}_{++}$, (2) repelling between negative electric charges $\vec{F}_{--}$, and (3) attracting between positive and negative electric charges $\vec{F}_{+-}$. Figure 2 shows the three types of the Coulomb interactions between two electric charges.

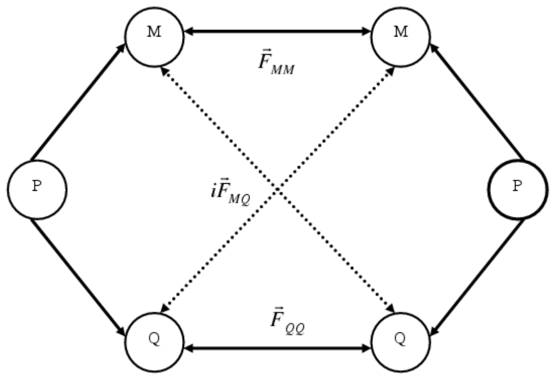

Fig. 1: Interactions between two electrically charged particles. They iclude (1) the gravitational force between masses, (2) the electric force between charges, and (3) the imaginary force between mass and charge.

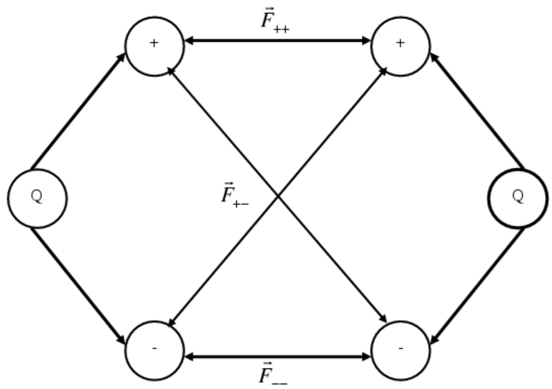

Fig. 2: Interactions between two electric charges. They include (1) repelling between two positive charges, (2) repelling between two negative charges, and (3) attraction between positive and negative charges.

4 Energy of antiparticles

The energy of an antiparticle [5, 6] is naturally obtained by conjugating the energy of the corresponding particle

$$E^* = \left(E^M + iE^Q\right)^* = E^M - iE^Q. \tag{9}$$

The only difference between a particle and its corresponding antiparticle is that their imaginary energies (thus their electric charges) have opposite signs. A particle and its antiparticle have the same real energy but have the sign-opposite imaginary energy.

In a particle-antiparticle annihilation process, their real energies completely transfer into radiation photon energies and their imaginary energies annihilate or cancel each other. Since there are no masses to adhere, the electric charges come together due to the electric attraction and cancel each other (or form a positive-negative electric charge pair (+,–)). In a particle-antiparticle pair production process, the radiation photon energies transfer to rest energies with a pair of imaginary energies, which combine with the rest energies to form a particle and an antiparticle.

To describe the energies of all particles and antiparticles, we can introduce a two-dimensional energy space. It is a complex space with two axes denoted by the real energy $\mathrm{Re}(E)$ and the imaginary energy $\mathrm{Im}(E)$. There are two phases in the energy space. In phase I, both real and imaginary energies are positive, while, in phase II, the imaginary energy is negative. Neutral particles including massless radiation photons are located on the real energy axis. Electrically charged particles are distributed between the real and imaginary energy axes. A particle and its antiparticle cannot be located in the same phase of the energy space.

5 Quantization of imaginary energy

The imaginary energy is quantized. Each electric charge quantum e (the electric charge of proton) has the following imaginary energy

$$E^e = \frac{e}{\sqrt{G}}\, c^2 \sim 1.67\times 10^{15}\ \text{ergs} \sim 10^{27}\ \text{eV}, \tag{10}$$

which is about 10^{18} times greater than proton's real energy (or the energy of proton's mass). Dividing the size of proton (10^{-15} cm) by proton's imaginary-real energy ratio (10^{18}), we obtain a scale length $l_Q = 10^{33}$ cm.

On the other hand, Kaluza-Klein theory geometrically unified the four-dimensional Einsteinian general theory of relativity and Maxwellian electromagnetic theory into a five-dimensional unification theory ([7–9] for the original studies, [10] for an extensive review, and [11, 12] for the field solutions). In this unification theory, the fifth dimension is a compact (one-dimensional circle) space with radius 10^{33} cm [13], which is about the order of l_Q obtained above. The reason why the fifth dimensional space is small and compact might be due to that the imaginary energy of an electrically charged particle is many orders of magnitude higher than its real energy. The charge is from the extra (or fifth) dimension [14], a small compact space. A pure electric charge is not measureable and is thus reasonably represented by an imaginary energy.

The imaginary energy of the electric charge quantum is about the thermal energy of the particle at a temperature $T_Q = 2E^e/k_B \sim 2.4\times 10^{31}$ °K. At this extremely high temperature, an electrically charged particle (e.g. proton) has a real energy in the same order of its imaginary energy. According to the standard big bang cosmology, the temperature at the grand unification era and earlier can be higher than about T_Q [15]. To have a possible explanation for the origin of the universe (or the origin of all the matter and energy), we suggest that a large electric charge such as 10^{46} Coulombs ($\sim 10^{76}$ ergs) was burned out, so that a huge amount of imaginary energies transferred into real energies at the temperature T_Q and above during the big bang of the universe. This suggestion gives a possible explanation for the origin of the universe from nothing to the real world in a process of transferring a large amount of imaginary energy (or electric charge) to real energy.

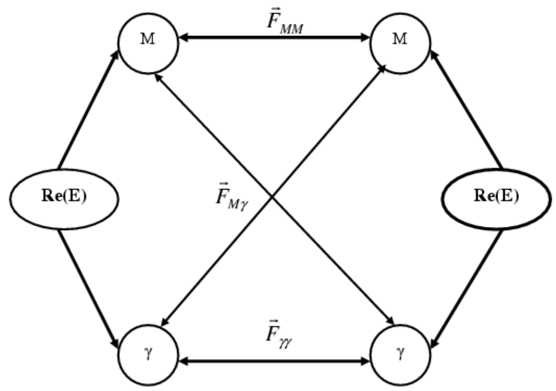

Fig. 3: Three types of gravitational interactions between real energies: (1) the mass-mass interaction, (2) the mass-radiation interaction, and (3) the radiation-radiation interaction.

6 Gravitational and radiation redshifts

Real energies actually have two components: matter with mass and matter without mass (i.e. radiation). The interactions between real energies may be referred as the gravitation in general. In this sense, we have three types of gravitations: (1) mass-mass interaction $\vec{F}_{MM}$, (2) mass-radiation interaction $\vec{F}_{M\gamma}$, and (3) radiation-radiation interaction $\vec{F}_{\gamma\gamma}$. Figure 3 sketches all these interactions between real energies.

The energy of a radiation photon is given by $h\nu$, where h is the Planck's constant and ν is the frequency of the radiation. According to Eq. (7), the mass-radiation interaction between a mass M and a photon γ is given by

$$\vec{F} = -\,G\,\frac{Mh\nu}{c^2 r^3}\,\vec{r}, \tag{11}$$

and the radiation-radiation interaction between two photons γ_1 and γ_2 is given by

$$\vec{F}_{\gamma\gamma} = -\,G\,\frac{(h\nu_1)(h\nu_2)}{c^4 r^3}\,\vec{r}. \tag{12}$$

Newton's law of gravity describes the gravitational force between two masses $\vec{F}_{MM}$. The Einsteinian general theory of relativity has successfully described the effect of matter (or mass) on the space-time and thus the interaction of matter on both matter and radiation (or photon). If we appropriately introduce a radiation energy-momentum tensor into the Einstein field equation, the Einsteinian general theory of relativity can also describe the effect of radiation on the space-time and thus the interaction of radiation on both matter and radiation.

When a photon of light travels relative to an object (e.g. the Sun) from $\vec{r}$ to $\vec{r} + d\vec{r}$, it changes its energy or frequency from ν to $\nu + d\nu$. The work done on the photon by the mass-radiation interaction ($\vec{F}_{M\gamma} \cdot d\vec{r}$) is equal to the photon energy change ($hd\nu$), i.e.,

$$-G\,\frac{Mh\nu}{c^2r^2}\,dr = hd\nu\,. \tag{13}$$

Eq. (13) can be rewritten as

$$\frac{d\nu}{\nu} = -\,\frac{GM}{c^2r^2}\,dr\,. \tag{14}$$

Integrating Eq. (14) with respect to r from R to ∞ and ν from ν_e to ν_o, we have

$$\ln\frac{\nu_o}{\nu_e} = -\,\frac{GM}{c^2R}\,, \tag{15}$$

where R is the radius of the object, ν_e is the frequency of the light when it is emitted from the surface of the object, ν_o is the frequency of the light when it is observed by the observer at an infinite distance from the object. Then, the redshift of the light is

$$Z_G = \frac{\lambda_o - \lambda_e}{\lambda_e} = \frac{\nu_e - \nu_o}{\nu_o} = \exp\left(\frac{GM}{c^2R}\right) - 1\,. \tag{16}$$

In the weak field approximation, it reduces

$$Z_G \simeq \frac{GM}{c^2R}\,. \tag{17}$$

Therefore, calculating the work done by the mass-radiation interaction on a photon, we can derive the Einsteinian gravitational redshift in the weak field approximation.

Similarly, calculating the work done on a photon from an object by the radiation-radiation gravitation $\vec{F}_{\gamma\gamma}$, we obtain a radiation redshift,

$$Z_\gamma = \frac{4GM}{15c^5}\,\sigma A T_c^4 + \frac{G}{c^5}\,\sigma A T_s^4\,, \tag{18}$$

where σ is the Stephan-Boltzmann constant, A is the surface area, T_c is the temperature at the center, T_s is the temperature on the surface. Here we have assumed that the inside temperature linearly decreases from the center to the surface. The radiation redshift contains two parts. The first term is contributed by the inside radiation. The other is contributed by the outside radiation. The redshift contributed by the outside radiation is negligible because $T_s \ll T_c$.

The radiation redshift derived here is significantly small in comparison with the empirical expression of radiation redshift proposed by Finlay-Freundlich [16]. For the Sun with $T_c = 1.5\times10^7$ °K and $T_s = 6\times10^3$ °K, the radiation redshift is only about $Z_\gamma = 1.3\times10^{-13}$, which is much smaller than the gravitational redshift $Z_G = 2.1\times10^{-6}$.

7 Discussions and conclusions

A quark has not only the electric charge but also the color charge [17, 18]. The electric charge has two varieties (positive and negative), while the color charge has three values (red, green, and blue). Describing both electric and color charges as imaginary energies, we may unify all of the four fundamental interactions into a single expression of the interaction between complex energies. Details of the study including the color charge will be given in the next paper.

Eq. (1) does not include the self-energy — the contribution to the energy of a particle that arises from the interaction between different parts of the particle. In the nuclear physics, the self-energy of a particle has an imaginary part [19, 20]. The mass-mass, mass-charge, and charge-charge interactions between different parts of an electrically charged particle will be studied in future.

As a summary, a pure electric charge (not observable and from the extra dimension) has been suggested as a form of imaginary energy. Total energy of an electrically charged particle is a complex number. The real part is proportional to the mass, while the imaginary part is proportional to the electric charge. The energy of an antiparticle is obtained by conjugating the energy of its corresponding particle. The gravitational and electromagnetic interactions have been classically unified into a single expression of the interaction between complex energies.

The interactions between real energies are gravitational forces, categorized by the mass-mass, mass-radiation, and radiation-radiation interactions. The work done by the mass-radiation interaction on a photon derives the Einsteinian gravitational redshift, and the work done by the radiation-radiation interaction on a photon gives the radiation redshift, which is significantly small in comparison with the gravitational redshift.

The interaction between imaginary energies is electromagnetic force. Since an electrically charged particle contains many order more imaginary energy than real energy, the interaction between imaginary energies are much stronger than that between real energies.

Overall, this study develops a new physics concept for electric charges, so that classically unifies the gravitational and electric forces and derives the Einsteinian gravitational redshift.

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An "Earth-Planet" or "Earth-Star" Couplet as a Gravitational Wave Antenna, wherein the Indicators are Microseismic Peaks in the Earth

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An "Earth-planet" or "Earth-star" couplet can be considered as a gravitational wave antenna. There in such an antenna a gravitational wave should lead to a peak in the microseismic background spectrum on the Earth (one of the ends of the antenna). This paper presents numerous observational results on the Earth's microseismic background. The microseismic spectrum, being compared to the distribution of the relative location of the nearest stars, found a close peak-to-peak correspondence. Hence such peaks can be a manifestation of an oscillation in the couplet "Earth-star" caused by gravitational waves arriving from the cosmos.

1 Introduction

Use the following simplest model. Focus on two gravitationally-connected objects such as the couplets "Earth-Moon", "Earth-Jupiter", "Earth-Saturn", "Earth-Sun", or "Earth-star" (a near star is meant). Such a couplet can be considered as a gravitational wave antenna. A gravitational wave, falling down onto such an antenna, should produce an oscillation in the system that leads to a peak in the microseismic background spectrum of the Earth (one of the ends of the antenna).

Gravitational waves radiated on different frequencies may have an origin in gravitationally unstable objects in the Universe. For instance, a gravitationally unstable cosmic cloud wherein a stellar form may be such a source. A mechanism which generates gravitational waves on a wide spectrum can be shown in such an example. There is a theorem: "if a system is in the state of unstable equilibrium, such a system can oscillatorily bounce at low frequencies in the stable area of the states; the frequency decreases while the system approaches the state of equilibrium (threshold of instability) with a finite wavenumber at zero frequency" [1, 2]. This theorem is applicable exactly to the case of the gravitational instability of the cosmic clouds. Such a gravitational instability is known as Jeans's instability, and leads to the process of the formation of stars [3]. In this process intense gravitational radiation should be produced. Besides the spectrum of the waves should be continuously shifting on low frequency scales as such a cloud approaches to the threshold of instability. Hence, gravitational waves radiated on the wide spectrum of frequencies should be presented in the Universe always as stellar creation process.

Hence, the peaks of the microseismic background on the Earth (if any observed), if correlated to the parameters of the "Earth-space body"system (such as the distance L between them), should manifest the reaction in the "Earth-space body" couplet of the gravitational waves arriving from the cosmos. The target of this study is the search for such correlation peaks in the microseismic background of the Earth.

2 Observations

Our observations were processed at the Seismic Station of Simpheropol University (Sevastopol, Crimea Peninsula), using a laser interferometer [4]. Six peaks were registered at 2.3 Hz, 1 Hz, 0.9 Hz, 0.6 Hz, 0.4 Hz, 0.2 Hz (see Fig. 1a and Fig. 1b). The graphs were drawn directly on the basis of the records made by the spectrum analyzer SK4-72. The spectrum analyzer SK4-72 accumulates output signals from an interferometer, then enhances periodic components of the signal relative to the chaotic components. 1,024 segregate records, 40-second length each, were averaged.

Many massive gravitating objects are located near the solar system at the distance of 1.3, 2.7, 3.5, 5, 8, and 11 parsecs. All the distances L between the Earth and these objects correspond to all the observed peaks (see Fig. 1a and Fig. 1b). The calculated distribution of the gravitational potential of the nearest stars is shown in Fig. 1c. Comparing Fig. 1a and Fig. 1b to Fig. 1c, we reveal a close similarity between the corresponding curves: each peak of Fig. 1a and Fig. 1b corresponds to a peak in Fig. 1c, and vice versa. Besides there are small deviations, that should be pointed out for clarity. For distances $L > 4$ parsecs the data were taken only for the brightest star, and the curve of the gravitational potential corresponding to this distance is lower than that for the $L < 4$ shown in the theoretical Fig. 1c. Another deviation is the presence of a uniform growth for the low-frequency background component in the experimental Fig. 1a and Fig. 1b, which doesn't appear in Fig. 1c. Such a uniform component of the microseismic background is usually described by the law $A_\omega \sim 1/\omega^2$ [5, 6].

*Posthumous publication prepared by Prof. Simon E. Shnoll (Institute of Theoretical and Experimental Biophysics, Russian Academy of Sciences, Pushino, Moscow Region, 142290, Russia), who was close to the author. E-mail of the submitter: shnoll@iteb.ru; shnoll@mail.ru. See Afterword for the biography and bibliography of the author, Prof. Vladimir A. Dubrovskiy (1935–2006).

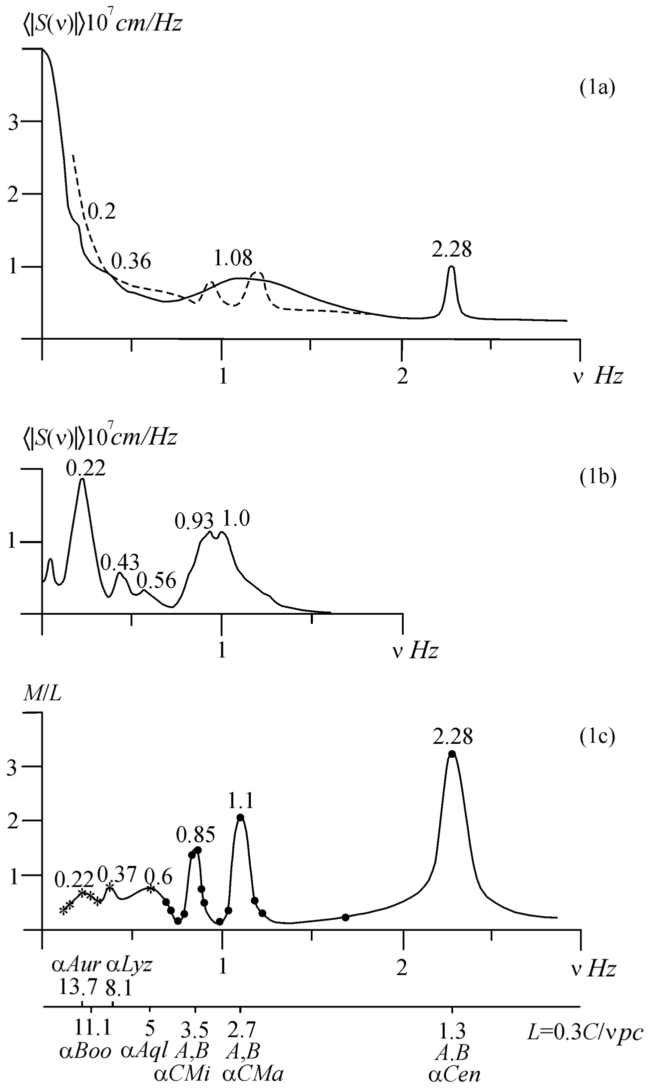

Fig. 1: The observed microseismic background (solid curve) after accumulation of the background signals from the interferometer output: Fig. 1a shows the range 0.1–5 Hz; Fig. 1b shows the range 0.1–2 Hz. The dotted curve of Fig. 1a shows the calculated distribution of the gravitational potential of the stars in common with the uniform part of the microseismic background. This dotted curve is normalized so that it is the same as that of the solid curve at 2.28 Hz. Fig. 1c shows the calculated distribution of the gravitational potential of the stars. The solid points correspond to all the nearest stars, a distance to which is $L < 4$ parsecs, and to all the brightest stars located at $L > 4$ parsecs. Masses M are expressed in the mass of the Sun. α Aur, α Lyz, etc. mean α stars of the constellations according to the astronomical notation [7, 8]. A, B sign for the components of the binaries. The numbers typed at the extrema are frequencies.

Moreover, the quantitative correlation between the frequency peaks and the distribution of the nearest stars is found. Namely, the sharpest peak at 2.28 Hz corresponds to the distance between the Earth and the nearest binary stars A and B, α Centaurus [7, 8]. The broader peak at 1 Hz (see Fig. 1a, and Fig. 1b) corresponds to the distances to the stars which are distributed over the range from 2.4 to 3.8 parsecs [7, 8]. The spectrum analyzer SK4-72 averages all the peaks in the range 2.4–3.8 parsecs into one broad peak near 1 Hz (Fig. 1a). At the same time the broad peak of Fig. 1a, being taken under detailed study, is shown to be split into two peaks (Fig. 1b) if the spectrum analyzer SK4-72 processes the frequency range from 0.1 to 2 Hz (the exaggeration of the frequency scale). This subdivision of the frequency range corresponds to the division of the group of stars located as far as in the range from 2.4 to 3.8 parsecs into two subgroups which are near 2.7 and 3.5 parsecs (Fig. 1c).

The distribution of the gravitational potential over the subgroups, in common with the uniform background spectrum, is shown by the dotted curve in Fig. 1a. We see therein both the quantitative and qualitative correlation between the frequent spectra of the microseismic background and the distribution of the gravitational potential in the subgroups.

The Sevastopol data correlation on the frequency spectra between the microseismic background and the distances between the Earth and the nearest stars are the same as the data registered in Arizona. The Sevastopol and Arizona data are well-overlapping with coincidence in three peaks [9].

It is possible to propose more decisive observations. Namely, it would be reasonable to look for peaks which could be corresponding to the Earth-Moon" ($\sim$240 MHz), "Earth-Sun" ($\sim$0.6 MHz), "Earth-Venus" ($\sim$0.3–2.2 MHz), "Earth-Jupiter" ($\sim$100–150 kHz), and "Earth-Saturn" ($\sim$58–72 kHz) antennae. Moreover, the peaks corresponding to Venus, Jupiter and Saturn should change their frequency in accordance with the change in the distance between the Earth and these planets during their orbital motion around the Sun. If such a correlation could be registered in an experiment, this would be experimentum crucis in support of the above presented results.

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Afterword by the Editor

In addition to the posthumous paper by Prof. Dubrovskiy, I should provide an explaination why we publish it in a form substantially trunscated to the originally version of the manuscript.

The originally Dubrovskiy manuscript, submitted by Prof. Simon E. Shnoll, was based on the preprint uloaded in 2001 into the Cornell arXiv.org, astro-ph/0106350. In that manuscript, aside for the experimental data presented in the current publication, Dubrovskiy tried to use the data as a verification to the Laplace speed of gravitation, which is many orders higher than the velocity of light. His belief in Laplace's theory unfortunately carried him into a few formally errors.

Laplace supposed such a speed as a result of his soltion of the gravitational two-body problem, which concerns the motion of two point particles that interact only with each other, due to gravity. In this problem a body A experiences that the force of gravitation which acts at that point where the body A is located in the moment. Because a body B (the source of the force) is distant from the body A and moves with respect to it with a velocity, there is incoincidence of two directions: the line connecting both bodies in the moment and the direction from the body A to that point where the body B was located, due to its motion, some time ago. What line is the location of the centre of gravity in such a system? If it is located in the first line, a force accelerating the body A should appear. If it is the second line, a non-compensated component of the momentum should appear in the body B, that is the breaking of the conservation law. As a result such a system becomes unstable anyway. This is a paradox of the two body problem of the 18th century. Using the mathematical methods accessed in the end of the 18th century, Laplace resolved this problem by introduction of the speed of gravitation, which should be, in the sample of the planets, at least ten orders higher than the velocity of light.

The contemporary Newtonian celestial mechanics resolves the two body problem with use of the methods of higher mathematics. This is a classical example, which shows that two bodies orbiting a common centre of gravity under specific conditions move along stable elliptic orbits so that they cannot leave the system or fall onto each other. This classical problem, known as the Kepler problem, is described in detail in §13 of *Short Course of Theoretical Physics. Mechanics. Electrodynamics* by Landau and Lifshitz (Nauka Publishers, Moscow, 1969).

The same situation takes a place in the General Theory of Relativity in a case where the physical conditions of the motion are close to the non-relativistic Newtonian mechanics. This problem is discussed in detail in §101 of *The Classical Theory of Fields* by Landau and Lifshitz (Butterworth-Heinemann, 1980). The mechanical energy and the moment of momentum of a two body system remain unchanged with only a small correction for the energy-momentum loss with gravitational radiation. In a system like the solar system the power of gravitational radiation, which is due to the orbiting planets, is nothing but only a few kilowatts. Therefore such a system is stable with the speed of gravitation equal to the velocity of light: the planets cannot leave the solar system or fall onto each other within a duration compared to the age of the Universe.

Due to the aforementioned reason, I substantially corrected the originally Dubrovskiy manuscript. I removed everything on the superluminal Laplace velocity of gravitation. I also corrected minor errors in the description of gravitational wave antennae.

I did it through the prior permission of Dr. Victor N. Sergeev (e-mail: svn@idg.chph.ras.ru), who was a close friend of Prof. Dubrovskiy and a co-author of many his works.

Dr. Sergeev is in contact with Prof. Shnoll. He read the corrected version of the manuscript, and agreed with the edition. Sergeev wrote, in a private letter of January 29, 2008: "... He [Dubrovskiy] considered the manuscript as a verification to his theory of gravitation where gravitational waves travel with a superluminal velocity. However the precense of a correlation of the microseismic spectra to the cosmic bodies, the result itself is important independent from interpretation given to it. Of course, it would be very good to publish this result. Besides, the edited version has nothing of those contradicting to the views of V. A. Dubrovskiy."

In general, an idea about a free-mass gravitational wave antenna whose basis is set up by an "Earth-planet" or "Earth-star" couplet is highly original. No such an idea met in the science before Dubrovskiy. Moreover, the correlation of the microseismic oscillations to the distances found by him gives good chances that such a couplet can be used as a huge free-mass gravitational wave detector in the future. The interstellar distances are extremely larger to 5 mln. km of the basis of LISA — the Laser Interferometer Space Antenna planned by the European Space Agency to launch on the next decade. So the displacement effect in the Dubrovskiy mass-detector due the a falling gravitatuonal wave should be large that could result a microseismic activity in the Earth.

With such a fine result, this paper will leave fond memories of Prof. Dubrovskiy. May his memory live for ever!

Dmitri Rabounski, Editor-in-Chief
Progress in Physics

Vladimir A. Dubrovskiy (1935–2006)

Vladimir Anatolievich Dubrovskiy was born on March 20, 1935, in the formerly-known Soviet Union. In 1953–1959 he was a student in the Physics Department of Moscow University. Then he worked on the research stuff of the Academy of Sciences of URSS (now the Russian Academy of Sciences, RAS) all his life. During the first period, from 1959 to 1962, he was employed as a research scientist at the Institute of Mathematics in the Siberian Branch of the Academy of Sciences, where he worked on the physics of elementary particles. During the second period, from 1962 to 1965, he completed postgraduate education at the Institute of Applied Mechanics: his theme was a "quasi-classical approximation of the equations of Quantum Mechanics". During two decades, from 1966 to 1998, he worked at the Laboratory of Seismology of the Institute of the Physics of the Earth, in Moscow, where he advanced from a junior scientist to the Chief of the Laboratory. His main research at the Institute concerned the internal constitution and evolution of the Earth.

From 1972 to 1992 Dubrovskiy was the Executive Secretary of the "Intergovernmental Commission URSS-USA on the Prediction for Earthquakes". In 1986–1991 he was the Executive Secretary of the "Commission on the Constitution, Composition, and Evaluation of the Earth's Interior" by the Academy of Science of URSS and the German Research Foundation (Deutsche Forschungsgemeinschaft). In 1997 he was elected a Professor in the Department of Mechanics and Mathematics of Moscow University.

In the end of 1996, Dubrovskiy and all the people working with him at his Laboratory of Seismology were ordered for discharge from the Institute of the Physics of the Earth due to a conflict between Dubrovskiy and the Director of the Institute. Then, in February of 1997, Dubrovskiy accused the Director with repression in science like those against genetics during the Stalin regime, and claimed hungry strike. A month later, in March, his health condition had become so poor, forcing him to be hospitalized. (Despite the urgent medical treatment, his health didn't come back to him; he was still remaining very ill, and died nine years later.) All the story met a resonance in the scientific community. As a result, Dubrovskiy, in common with two his co-workers, was invited by another Institute of the Academy of Sciences, the Institute of Geospheres Dynamics in Moscow, where he worked from 1998 till death. He died on November 12, 2006, in Moscow.

Dubrovskiy authored 102 research papers published in scientific journals and the proceedings of various scientific conferences. A brief list of his scientific publications attached.

Main scientific legacy of V. A. Dubrovskiy

A five dimensional approach to the quasiclassical approach of the equations of Quantum Mechanics:

- Dubrovskiy V. A. and Skuridin G. A. Asymptotic decomposition in wave mechanics. *Magazine of Computational Mathematics and Mathematical Physics*, 1964, v. 5, no. 4.

The hypothesis on the iron oxides contents of the Earth's core:

- Dubrovskiy V. A. and Pan'kov V. L. On the composition of the Earth's core. *Izvestiya of the Academy of Sciences of USSR, Earth Phys.*, 1972, no. 7, 48–54.

Now this hypothesis has been verified by many scientists in their experimental and theoretical studies. A new idea is that the d-electrons of the transition elements (mainly iron), being under high pressure, participate with high activity in the formation of the additional covalent bindings. As a result the substances become dense, so the iron oxide FeO can be seen as the main part of the contents formation of the core of the Earth.

The theory of eigenoscillation of the elastic inhomogeneities:

- Dubrovskiy V. A. Formation of coda waves. In: *The Soviet-American Exchange in Earthquake Prediction. U.S. Geological Survey. Open-File Report*, 81–1150, 1981, 437–456.
- Dubrovskiy V. A. and Morochnik V. S. Natural vibrations of a spherical inhomogeneity in an elastic medium. *Izvestiya of the Academy of Sciences of USSR, Physics of the Solid Earth*, 1981, v. 17, no. 7, 494–504.
- Dubrovskiy V. A. and Morochnik V. S. Nonstationary scattering of elastic waves by spherical inclusion. *Izvestiya of the Academy of Sciences of USSR, Physics of the Solid Earth*, 1989, v. 25, no. 8, 679–685.

This presents the analytic solution of the boundary problem. The frequent equation is derived for both radial, torsional and spheroidal vibrations. A new method of solution for the diffraction problem is developed for a spherical elastic inclusion into an infinite elastic medium. The obtained analytical solution is checked by numerical computation. Formulae are obtained for the coda waves envelop in two limiting cases: single scattering and diffusion scattering. A frequency dependence on the quality factor is manifest through the corresponding dependance on the scattering cross-section.

The mechanism of the tectonic movements:

- Artemjev M. E., Bune V. J., Dubrovskiy V. A., and Kambarov N. Sh. Seismicity and isostasy. *Phys. Earth Planet. Interiors*, 1972, v. 6, no. 4, 256–262.
- Dubrovskiy V. A. Mechanism of tectonic movements. *Izvestiya of the Academy of Sciences of USSR, Physics of the Solid Earth*, 1986, v. 22, no. 1, 18–27.
- Dubrovskiy V. A., Sergeev V. N., and Fuis G. S. Generalized condition of isostasy. *Doklady of the Russian Academy of Sciences*, 1995, v. 342, no. 1.
- Dubrovskiy V. A. and Sergeev V. N. Physics of tectonic waves. *Izvestiya of the Russian Academy of Sciences, Physics of the Solid Earth*, 1997, v. 33, no. 10, 865–866.

This mechanism is seen to be at work in a "lithosphere-astenosphere" system which has the density inversion between the lithosphere and astenosphere. The substance of the elastic lithosphere is denser than that of the liquid astenosphere. A solution for the model of the elastic layer above the incompressible fluid with the density inversion is found. It is found that there is a nontrivial, unstable equilibrium on nonzero displacement of the elastic layer. The bifurcation point is characterized by a critical wavelength of the periodic disturbance. This wavelength is that of the wave disturbance when the Archimedian force reaches the elastic force of disturbance.

Two-level convection in Earth's mantle:

- Dubrovskiy V. A. Two-level convection in the Earth's mantle. *Doklady of the Russian Academy of Sciences*, 1994, v. 334, no. 1.
- Dubrovskiy V. A. Convective instability motions in the Earth's interiors. *Izvestiya of the Russian Academy of Sciences, Physics of the Solid Earth*, 1995, no. 9.

The mantle convection is considered at two levels: a convection in the lower mantle is the chemical-density convection due to the core-mantle boundary differentiation into the different compositionally light and heavy components, while the other convection is the heat-density convection in the"elastic lithosphere — fluid astenosphere" system. The last one manifests itself in different tectonic phenomena such as the tectonic waves, the oceanic plate tectonic and continental tectonic as a result of the density inversion in the "lithosphere-astenosphere" system. The lower mantle chemical convection gives the heat energy flow to the upper mantle heat convection.

Generation for the magnetic, electric and vortex fields in magnetohydrodynamics, electrohydrodynamics and vortex hydrodynamics:

- Dubrovskiy V. A. and Skuridin G. A. The propagation of small disturbances in magnetohydrodynamics. *Geomagnetism and Aeronomy*, 1965, v. 5, no. 2, 234–250.
- Dubrovskiy V. A. The equations of electrohydrodynamics and electroelasticity. *Soviet Physics Doklady*, v. 29(12), December 1984 (transl. from *Doklady Akademii Nauk URSS*, 1984, v. 279, 857–860).
- Dubrovskiy V. A. Conditions for magnetic field Generation. *Doklady Akadmii Nauk URSS*, 1986, v. 286, no. 1, 74–77.
- Dubrovskiy V. A. and Rusakov N. N. Mechanism of generation of an elastic field. *Doklady Akadmii Nauk URSS*, 1989, v. 306, no. 6, 64–67.

- Dubrovskiy V. A. On a relation between strains and vortices in hydrodynamic flows. *Doklady Physics*, 2000, v. 45, no. 2, 52–54 (transl. from *Doklady Akademii Nauk URSS*, 2000, v. 370, no. 6, 754–756.

A nonlinear system of the equations is obtained, which manifests a mutual influence between the motion of a dielectric medium and an electric field. This theory well-describes the atmospheric electricity, including ball lighting. The theory proves: the motion of a magnetohydrodynamical, electrohydrodynamical or hydrodynamical incompressible fluid is locally unstable everywhere relative to the disturbances of a vortex, magnetic or electric field. A mutual, pendulumlike conversion energy of the fluid flow and energy of a magnetic, electric or vortex field is possible. Two-dimensional motions are stable in a case where they are large enough. The magnetic restrain of plasma is impossible in three-dimensional case.

The elastic model of the physical vacuum:

- Dubrovskiy V. A. Elastic model of the physical vacuum. *Soviet Physics Doklady*, v. 30(5), May 1985 (translated from *Doklady Akademii Nauk URSS*, 1985, v. 283, 83–85.
- Dubrovskiy V. A. Measurments of the gravity waves velocity. arXiv: astro-ph/0106350.
- Dubrovskiy V. A. Relation of the microseismic background with cosmic objects. *Vestnik MGU* (Transactions of the Moscow University), 2004, no. 4.
- Dubrovskiy V. A. and Smirnov N. N. Experimental evaluation of the gravity waves velocity. In: *Proc. of the 54nd International Astronautical Congress*, September 29 — October 3, 2003, Bremen, Germany.

New variables in the theory of elasticity are used (e.g. the velocity, vortex, and dilation set up instead the velocity and stress used in the standard theory). This gives a new system of the equations describing the wave motion of the velocity, vortex and dilation. In such a model, transversal waves and longitudinal waves are associated to electromagnetic and gravitational waves respectively. Such an approach realizes the field theory wherein elementary particles are the singularities in the elastic physical vacuum.

A universal precursor for the geomechanical catastrophes:

- Dubrovskiy V. A. Tectonic waves. *Izvestiya of the Academy of Sciences of URSS, Earth Physics*, 1985, v. 21, no. 1, 20–23.
- Dubrovskiy V. A. and Dieterich D. Wave propagation along faults and the onset of slip instability. *EOS*, 1990, v. 71, no. 17, 635–636.
- Dubrovskiy V. A., McEvilly T. V., Belyakov A. S., Kuznetzov V. Y., and Timonov M. V. Borehole seismoacoustical emission study at the Parkfield prognosis range. *Doklady of the Russian Academy of Sciences*, 1992, v. 325, no. 4.
- Dubrovskiy V. A. and Sergeev V. N. The necessary precursor for a catastrophe. In: *Tectonic of Neogey: General and Regional Aspects*, GEOS, Moscow, 2001, v. 1, 222–226.

Unstable phenomena such as earthquakes can occur in a geomechanical system, if there is an unstable state of equilibrium in a set of critical geophysical parameters. There are two fields of the geophysical parameters, which correspond to the stable and unstable states. According to Dubrovskiy (1985) and also Dubrovskiy and Sergeev (2001), in the stable field of the parameters the geosystem has vibratory eigenmotions, where the frequencies tend to zero if the system approaches unstable equilibrium (during an earthquake occurrence, for instance). However the critical wavelength of the vibrations remains finite at zero frequency, and characterizes the size of the instability. Change in the eigenfrequencies affects the spectrum of seismoacoustic emission in an area surrounding an impending earthquake. Such a change indicates the fact that the geomechanical system is close to an unstable threshold, and the critical wavelength determines the energy and space dimensions of the developing instability sorce. Such an approach to the study of a systems in the state of unstable equilibrium is applicable to all system, whose behavior is described by hyperbolic equations in partial derivatives, i.e. not only geomechanical systems.

Remarks on Conformal Mass and Quantum Mass

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One shows how in certain model situations conformal general relativity corresponds to a Bohmian-Dirac-Weyl theory with conformal mass and Bohmian quantum mass identified.

The article [12] was designed to show relations between conformal general relativity (CGR) and Dirac-Weyl (DW) theory with identification of conformal mass $\hat{m}$ and quantum mass $\mathfrak{M}$ following [7, 9, 11, 25] and precision was added via [21]. However the exposition became immersed in technicalities and details and we simplify matters here. Explicitly we enhance the treatment of [7] by relating $\mathfrak{M}$ to an improved formula for the quantum potential based on [21] and we provide a specific Bohmian-Dirac-Weyl theory wherein the identification of CGR and DW is realized. Much has been written about these matters and we mention here only [1–7, 9–20, 23–28] and references therein. One has an Einstein form for GR of the form

$$S_{GR} = \int d^4x\,\sqrt{-g}\,(R - \alpha|\nabla\psi|^2 + 16\pi L_M) \tag{1.1}$$

(cf. [7, 22]) whose conformal form (conformal GR) is an integrable Weyl geometry based on

$$\begin{aligned}\hat{S}_{GR} &= \int d^4x\,\sqrt{-\hat{g}}\;e^{-\psi}\;\times\\ &\times\left[\hat{R} - \left(\alpha - \frac{3}{2}\right)|\hat{\nabla}\psi|^2 + 16\pi e^{-\psi}L_M\right] = \\ &= \int d^4x\,\sqrt{-\hat{g}}\left[\hat{\phi}\hat{R} - \left(\alpha - \frac{3}{2}\right)\frac{|\hat{\nabla}\hat{\phi}|^2}{\hat{\phi}} + 16\pi\hat{\phi}^2 L_M\right]\end{aligned} \tag{1.2}$$

where $\Omega^2 = \exp(-\psi) = \phi$ with $\hat{g}_{ab} = \Omega^2 g_{ab}$ and $\hat{\phi} = \exp(\psi) = \phi^{-1}$ (note $(\hat{\nabla}\psi)^2 = (\hat{\nabla}\hat{\phi})^2/(\hat{\phi})^2$). One sees also that (1.2) is the same as the Brans-Dicke (BD) action when $L_M = 0$, namely (using $\hat{g}$ as the basic metric)

$$S_{BD} = \int d^4x\,\sqrt{-\hat{g}}\left[\hat{\phi}\hat{R} - \frac{\omega}{\hat{\phi}}|\hat{\nabla}\hat{\phi}|^2 + 16\pi L_M\right]; \tag{1.3}$$

which corresponds to (1.2) provided $\omega = \alpha - \frac{3}{2}$ and $L_M = 0$. For (1.2) we have a Weyl gauge vector $w_a \sim \partial_a\psi = \partial_a\hat{\phi}/\hat{\phi}$ and a conformal mass $\hat{m} = \hat{\phi}^{-1/2}m$ with $\Omega^2 = \hat{\phi}^{-1}$ as the conformal factor above. Now in (1.2) we identify $\hat{m}$ with the quantum mass $\mathfrak{M}$ of [25] where for certain model situations $\mathfrak{M} \sim \beta$ is a Dirac field in a Bohmian-Dirac-Weyl theory as in (1.8) below with quantum potential Q determined via $\mathfrak{M}^2 = m^2\exp(Q)$ (cf. [10, 11, 21, 25] and note that $m^2 \propto T$ where $8\pi T^{ab} = (1/\sqrt{-g})(\delta\sqrt{-g}\,\mathfrak{L}_M/\delta g_{ab})$). Then $\hat{\phi}^{-1} = \hat{m}^2/m^2 = \mathfrak{M}^2/m^2 \sim \Omega^2$ for Ω^2 the standard conformal factor of [25]. Further one can write (**1A**) $\sqrt{-\hat{g}}\,\hat{\phi}\,\hat{R} = \hat{\phi}^{-1}\sqrt{-\hat{g}}\,\hat{\phi}^2\,\hat{R} = \hat{\phi}^{-1}\sqrt{-g}\,\hat{R} = (\beta^2/m^2)\sqrt{-g}\,\hat{R}$. Recall here from [11] that for $g_{ab} = \hat{\phi}\hat{g}_{ab}$ one has $\sqrt{-g} = \hat{\phi}^2\sqrt{-\hat{g}}$ and for the Weyl-Dirac geometry we give a brief survey following [11, 17]:

1. Weyl gauge transformations: $g_{ab} \to \tilde{g}_{ab} = e^{2\lambda}g_{ab}$; $g^{ab} \to \tilde{g}^{ab} = e^{-2\lambda}g^{ab}$ — weight e.g. $\Pi(g^{ab}) = -2$. β is a Dirac field of weight -1. Note $\Pi(\sqrt{-g}) = 4$;
2. Γ^c_{ab} is Riemannian connection; Weyl connection is $\hat{\Gamma}^c_{ab}$ and $\hat{\Gamma}^c_{ab} = \Gamma^c_{ab} = g_{ab}w^c - \delta^c_b w_a - \delta^c_a w_b$;
3. $\nabla_a B_b = \partial_a B_b - B_c\Gamma^c_{ab}$; $\nabla_a B^b = \partial_a B^b + B^c\Gamma^b_{ca}$;
4. $\hat{\nabla}_a B_b = \partial_a B_b - B_c\hat{\Gamma}^c_{ab}$; $\hat{\nabla}_a B^b = \partial_a B^b + B^c\hat{\Gamma}^b_{ca}$;
5. $\hat{\nabla}_\lambda g^{ab} = -2g^{ab}w_\lambda$; $\hat{\nabla}_\lambda g_{ab} = 2g_{ab}w_\lambda$ and for $\Omega^2 = \exp(-\psi)$ the requirement $\nabla_c g_{ab} = 0$ is transformed into $\hat{\nabla}_c\hat{g}_{ab} = \partial_c\psi\hat{g}_{ab}$ showing that $w_c = -\partial_c\psi$ (cf. [7]) leading to $w_\mu = \hat{\phi}_\mu/\hat{\phi}$ and hence via $\beta = m\hat{\phi}^{-1/2}$ one has $w_c = 2\beta_c/\beta$ with $\hat{\phi}_c/\hat{\phi} = -2\beta_c/\beta$ and $w^a = -2\beta^a/\beta$.

Consequently, via $\beta^2\hat{R} = \beta^2 R - 6\beta^2\nabla_\lambda w^\lambda + 6\beta^2 w^\lambda w_\lambda$ (cf. [11, 12, 16, 17]), one observes that $-\beta^2\nabla_\lambda w^\lambda = -\nabla_\lambda(\beta^2 w^\lambda) + 2\beta\partial_\lambda\beta w^\lambda$, and the divergence term will vanish upon integration, so the first integral in (1.2) becomes

$$I_1 = \int d^4x\,\sqrt{-g}\left[\frac{\beta^2}{m^2}R + 12\beta\partial_\lambda\beta w^\lambda + 6\beta^2 w^\lambda w_\lambda\right]. \tag{1.4}$$

Setting now $\alpha - \frac{3}{2} = \gamma$ the second integral in (1.2) is

$$\begin{aligned}I_2 &= -\gamma\int d^4x\,\sqrt{-\hat{g}}\;\hat{\phi}\;\frac{|\hat{\nabla}\hat{\phi}|^2}{|\hat{\phi}|^2} = \\ &= -4\gamma\int d^4x\,\sqrt{-\hat{g}}\;\hat{\phi}^{-1}\hat{\phi}^2\,\frac{|\hat{\nabla}\beta|^2}{\beta^2} = \\ &= -\frac{4\gamma}{m^2}\int d^4x\,\sqrt{-g}|\hat{\nabla}\beta|^2,\end{aligned} \tag{1.5}$$

while the third integral in the formula (1.2) becomes (**1B**) $16\pi\int\sqrt{-g}\,d^4x L_M$. Combining now (1.4), (1.5), and (**1B**) gives then

$$\begin{aligned}\hat{S}_{GR} = \frac{1}{m^2}\int d^4x\,\sqrt{-g}\,\big[\beta^2 R + 6\beta^2 w^\alpha w_\alpha + \\ + 12\beta\partial_\alpha\beta w^\alpha - 4\gamma|\hat{\nabla}\beta|^2 + 16\pi m^2 L_M\big].\end{aligned} \tag{1.6}$$

We will think of $\hat{\nabla}\beta$ in the form (**1C**) $\hat{\nabla}_\mu\beta = \partial_\mu\beta - w_\mu\beta = -\partial_\mu\beta$. Putting then $|\hat{\nabla}\beta|^2 = |\partial\beta|^2$ (1.6) becomes (recall $\gamma = \alpha - \frac{3}{2}$)

$$\hat{S}_{GR} = \frac{1}{m^2}\int d^4x\,\sqrt{-g}\,\times \\ \times\left[\beta^2 R + (3-4\alpha)|\partial\beta|^2 + 16\pi m^2 L_M\right]. \tag{1.7}$$

One then checks this against some Weyl-Dirac actions. Thus, neglecting terms $W^{ab}W_{ab}$ we find integrands involving $dx^4\,\sqrt{-g}$ times

$$-\beta^2 R + 3(3\sigma+2)|\partial\beta|^2 + 2\Lambda\beta^4 + \mathfrak{L}_M \tag{1.8}$$

(see e.g. [11,12,17,25]); the term $2\Lambda\beta^4$ of weight -4 is added gratuitously (recall $\Pi\left(\sqrt{-g}\right) = 4$). Consequently, omitting the Λ term, (1.8) corresponds to (1.7) times m^2 for $\mathfrak{L}_M \sim \sim 16\pi L_M$ and (**1D**) $9\sigma + 4\alpha + 3 = 0$. Hence one can identify conformal GR (without Λ) with a Bohmian-Weyl-Dirac theory where conformal mass $\hat{m}$ corresponds to quantum mass $\mathfrak{M}$.

REMARK 1.1. The origin of a β^4 term in (1.8) from $\hat{S}_{GR}$ in (1.2) with a term $2\sqrt{-\hat{g}}\hat{\Lambda}$ in the integrand would seem to involve writing (**1E**) $2\sqrt{-\hat{g}}\,\hat{\Lambda} = 2\sqrt{-\hat{g}}\,\hat{\phi}^2\Omega^4\hat{\Lambda} = 2\sqrt{-g}\,\beta^4\hat{\Lambda}/m^4$ so that Λ in (1.8) corresponds to $\hat{\Lambda}$. Normally one expects $\Lambda\sqrt{-g} \to \sqrt{-\hat{g}}\,\hat{\phi}^2\Lambda$ (cf. [2]) or perhaps $\Lambda \to \hat{\phi}^2\Lambda = \Omega^{-4}\Lambda = \hat{\Lambda}$. In any case the role and nature of a cosmological constant seems to still be undecided. ∎

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Gravitation and Electricity

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The equations of gravitation together with the equations of electromagnetism in terms of the General Theory of Relativity allow to conceive an interdependence between the gravitational field and the electromagnetic field. However the technical difficulties of the relevant problems have precluded from expressing clearly this interdependence. Even the simple problem related to the field generated by a charged spherical mass is not correctly solved. In the present paper we reexamine from the outset this problem and propose a new solution.

1 Introduction

Although gravitation and electromagnetism are distinct entities, the principles of General Relativity imply that they affect each other. In fact, the equations of electromagnetism, considered in the spacetime of General Relativity, depend on the gravitational tensor, so that the electromagnetic field depends necessarily on the gravitational potentials. On the other hand, the electromagnetism is involved in the equations of gravitation by means of the corresponding energy-momentum tensor, so that the gravitational potentials depend necessarily on the electromagnetic field. It follows that, in order to bring out the relationship between gravitation and electromagnetism, we must consider together the equations of electromagnetism, which depend on the gravitational tensor, and the equations of gravitation, which depend on the electromagnetic potentials. So we have to do with a complicated system of equations, which are intractable in general. Consequently it is very difficult to bring out in explicit form the relationship between gravitation and electromagnetism. However the problem can be rigorously solved in the case of the field (gravitational and electric) outside a spherical charged mass. The classical solution of this problem, the so-called Reissner-Nordström metric, involves mathematical errors which distort the relationship between gravitational and electric field. In dealing with the derivation of this metric, H. Weyl notices that "For the electrostatic potential we get the same formula as when the gravitation is disregarded" [5], without remarking that this statement includes an inconsistency: The electrostatic potential without gravitation is conceived in the usual spacetime, whereas the gravitation induces a non-Euclidean structure affecting the metrical relations and, in particular, those involved in the definition of the electrostatic potential. The correct solution shows, in fact, that the electrostatic potential depends on the gravitational tensor.

In the present paper we reexamine from the outset the problem related to the joint action of the gravitation and electromagnetism which are generated by a spherical charged source. We assume that the distribution of matter and charges is such that the corresponding spacetime metric is $S\Theta(4)$-invariant (hence also $\Theta(4)$-invariant), namely a spacetime metric of the following form [3, 4]

$$ds^2 = f^2 dx_0^2 + 2ff_1(xdx)dx_0 - \ell_1^2 dx^2 + \\ + \left(\frac{\ell_1^2 - \ell^2}{\rho^2} + f_1^2\right)(xdx)^2, \tag{1.1}$$

(where $f = f(x_0, \|x\|)$, $f_1 = f_1(x_0, \|x\|)$, $\ell_1 = \ell_1(x_0, \|x\|)$, $\ell = \ell(x_0, \|x\|)$, $\rho = \|x\|$).

It is useful to write down the components of (1.1):

$$g_{00} = f^2, \quad g_{0i} = g_{i0} = x_i f f_1\,,$$

$$g_{ii} = -\ell_1^2 + \left(\frac{\ell_1^2 - \ell^2}{\rho^2} + f_1^2\right)x_i^2\,,$$

$$g_{ij} = \left(\frac{\ell_1^2 - \ell^2}{\rho^2} + f_1^2\right)x_i x_j, \qquad (i, j = 1, 2, 3; i \neq j)\,,$$

the determinant of which equals $-f^2\ell^2\ell_1^4$. Then an easy computation gives the corresponding contravariant components:

$$g^{00} = \frac{\ell^2 - \rho^2 f_1^2}{f^2\ell^2}\,, \quad g^{0i} = g^{i0} = x_i \frac{f_1}{f\ell^2}\,,$$

$$g^{ii} = -\frac{1}{\ell_1^2} - \frac{1}{\rho_2}\left(\frac{1}{\ell^2} - \frac{1}{\ell_1^2}\right)x_i^2\,,$$

$$g^{ij} = -\frac{1}{\rho^2}\left(\frac{1}{\ell^2} - \frac{1}{\ell_1^2}\right)x_i x_j, \qquad (i, j = 1, 2, 3; i \neq j)\,.$$

Regarding the electromagnetic field, with respect to (1.1), it is defined by a skew-symmetrical $S\Theta(4)$-invariant tensor field of degree 2 which may be expressed either by its covariant components

$$\sum V_{\alpha\beta}\, dx_\alpha \otimes dx_\beta, \quad (V_{\alpha\beta} = -V_{\beta\alpha})\,,$$

or by its contravariant components

$$\sum V^{\alpha\beta} \frac{\partial}{\partial x_\alpha} \otimes \frac{\partial}{\partial x_\beta}, \quad (V^{\alpha\beta} = -V^{\beta\alpha})\,.$$

2 Electromagnetic field outside a spherical charged source. Vanishing of the magnetic field

According to a known result [2], the skew-symmetrical $S\Theta(4)$-invariant tensor field $\sum V_{\alpha\beta}dx_\alpha \otimes dx_\beta$ is the direct sum of the following two tensor fields:

(a) A $\Theta(4)$-invariant skew-symmetrical tensor field

$$q(x_0, \|x\|)(dx_0 \otimes F(x) - F(x) \otimes dx_0)\,,$$
$$\left(F(x) = \sum_{i=1}^{3} x_i dx_i\right),$$

which represents the electric field with components

$$\left.\begin{aligned} &V_{01} = -V_{10} = qx_1\,, \quad V_{02} = -V_{20} = qx_2\,, \\ &V_{03} = -V_{30} = qx_3\,. \end{aligned}\right\}; \quad (2.1)$$

(b) A purely $S\Theta(4)$-invariant skew-symmetrical tensor field

$$\begin{aligned} q_1(x_0, \|x\|)\big[&x_1(dx_2 \otimes dx_3 - dx_3 \otimes dx_2) + \\ &+ x_2(dx_3 \otimes dx_1 - dx_1 \otimes dx_3) + \\ &+ x_3(dx_1 \otimes dx_2 - dx_2 \otimes dx_1)\big]\,, \end{aligned}$$

which represents the magnetic field with components

$$\left.\begin{aligned} &V_{23} = -V_{32} = q_1x_1\,, \quad V_{31} = -V_{13} = q_1x_2\,, \\ &V_{12} = -V_{21} = q_1x_3\,. \end{aligned}\right\}. \quad (2.2)$$

Since the metric (1.1) plays the part of a fundamental tensor, we can introduce the contravariant components of the skew-symmetrical tensor field $\sum V_{\alpha\beta}dx_\alpha \otimes dx_\beta$ with respect to (1.1).

Proposition 2.1 *The contravariant components of the $S\Theta(4)$-invariant skew-symmetrical tensor field $\sum V_{\alpha\beta}dx_\alpha \otimes dx_\beta$ are defined by the following formulae:*

$$V^{01} = -V^{10} = -\frac{qx_1}{f^2\ell^2}\,, \quad V^{02} = -V^{20} = -\frac{qx_2}{f^2\ell^2}\,,$$
$$V^{03} = -V^{30} = -\frac{qx_3}{f^2\ell^2}\,,$$
$$V^{23} = -V^{32} = \frac{q_1x_1}{\ell_1^4}\,, \quad V^{31} = -V^{13} = \frac{q_1x_2}{\ell_1^4}\,,$$
$$V^{12} = -V^{21} = \frac{q_1x_3}{\ell_1^4}\,.$$

Proof. The componets V^{01} and V^{23}, for instance, result from the obvious formulae

$$\begin{aligned} V^{01} = \sum g^{0\alpha}g^{1\beta}V_{\alpha\beta} &= (g^{00}g^{11} - g^{01}g^{10})V_{01} + \\ &+ (g^{00}g^{12} - g^{02}g^{10})V_{02} + (g^{00}g^{13} - g^{03}g^{10})V_{03} + \\ &+ (g^{02}g^{13} - g^{03}g^{12})V_{23} + (g^{03}g^{11} - g^{01}g^{13})V_{31} + \\ &+ (g^{01}g^{12} - g^{02}g^{11})V_{12} \end{aligned}$$

and

$$\begin{aligned} V^{23} = \sum g^{2\alpha}g^{3\beta}V_{\alpha\beta} &= (g^{20}g^{31} - g^{21}g^{30})V_{01} + \\ &+ (g^{20}g^{32} - g^{22}g^{30})V_{02} + (g^{20}g^{33} - g^{23}g^{30})V_{03} + \\ &+ (g^{22}g^{33} - g^{23}g^{32})V_{23} + (g^{23}g^{31} - g^{21}g^{33})V_{31} + \\ &+ (g^{21}g^{32} - g^{22}g^{31})V_{12} \end{aligned}$$

after effectuating the indicated operations.

Proposition 2.2 *The functions $q = q(x_0, \rho)$, $q_1 = q_1(x_0, \rho)$, $(x_0 = ct,\ \rho = \|x\|)$, defining the components (2.1) and (2.2) outside the charged spherical source are given by the formulae*

$$q = \frac{\varepsilon f\ell}{\rho^3\ell_1^2}\,, \qquad q_1 = \frac{\varepsilon_1}{\rho^3}\,,$$
$$(\varepsilon = \text{const},\ \varepsilon_1 = \text{const.})$$

(The equations of the electromagnetic field are to be considered together with the equations of gravitation, and since these last are inconsistent with a punctual source, there exists a length $\alpha > 0$ such that the above formulae are valid only for $\rho \geqslant \alpha$.)

Proof. Since outside the source there are neither charges nor currents, the components (2.1), (2.2) are defined by the classical equations

$$\frac{\partial V_{\alpha\beta}}{\partial x_\gamma} + \frac{\partial V_{\beta\gamma}}{\partial x_\alpha} + \frac{\partial V_{\gamma\alpha}}{\partial x_\beta} = 0\,, \qquad (2.3)$$

$$\big(x_0 = ct,\ (\alpha, \beta, \gamma) \in \{(0,1,2), (0,2,3), (0,3,1), (1,2,3)\}\big),$$

$$\sum_{\beta=0}^{3} \frac{\partial}{\partial x_\beta}\left(\sqrt{-G}\,V^{\alpha\beta}\right) = 0\,, \qquad (2.4)$$

$$\big(\alpha = 0,1,2,3;\ \ G = -f^2\ell^2\ell_1^4\big).$$

Taking $(\alpha, \beta, \gamma) = (0,1,2)$, we have, on account of (2.3),

$$\frac{\partial(qx_1)}{\partial x_2} + \frac{\partial(q_1x_3)}{\partial x_0} - \frac{\partial(qx_2)}{\partial x_1} = 0$$

and since

$$\frac{\partial q}{\partial x_i} = \frac{\partial q}{\partial \rho}\frac{x_i}{\rho}\,, \quad (i = 1,2,3)\,,$$

we obtain

$$x_1x_2\frac{\partial q}{\partial \rho} - x_2x_1\frac{\partial q}{\partial \rho} + x_3\frac{\partial q_1}{\partial x_0} = 0\,,$$

whence $\dfrac{\partial q_1}{\partial x_0} = 0$, so that q_1 depends only on ρ, $q_1 = q_1(\rho)$.

On the other hand, taking $(\alpha, \beta, \gamma) = (1,2,3)$, the equation (2.3) is written as

$$\frac{\partial(q_1x_3)}{\partial x_3} + \frac{\partial(q_1x_1)}{\partial x_1} + \frac{\partial(q_1x_2)}{\partial x_2} = 0\,,$$

whence $3q_1 + \rho q_1' = 0$, so that $3\rho^2 q_1 + \rho^3 q_1' = 0$ or $(\rho^3 q_1)' = 0$ and $\rho^3 q_1 = \varepsilon_1 = \text{const}$ or $q_1 = \dfrac{\varepsilon_1}{\rho^3}$.

Consider now the equation (2.4) with $\alpha = 1$. Since $G = -f^2\ell^2\ell_1^4$,

$$V^{11} = 0\,,\;\; V^{10} = \frac{qx_1}{f^2\ell^2}\,,\;\; V^{12} = \frac{q_1x_3}{\ell_1^4}\,,\;\; V^{13} = -\frac{q_1x_2}{\ell_1^4}\,,$$

we have

$$\frac{\partial}{\partial x_0}\left(\frac{q\ell_1^2}{f\ell}\,x_1\right) + \frac{\partial}{\partial x_2}\left(\frac{q_1f\ell}{\ell_1^2}\,x_3\right) - \frac{\partial}{\partial x_3}\left(\frac{q_1f\ell}{\ell_1^2}\,x_2\right) = 0\,.$$

Because of

$$\frac{\partial}{\partial x_2}\left(\frac{q_1f\ell}{\ell_1^2}\,x_3\right) = \frac{x_3x_2}{\rho}\,\frac{\partial}{\partial\rho}\left(\frac{q_1f\ell}{\ell_1^2}\right) = \frac{\partial}{\partial x_3}\left(\frac{q_1f\ell}{\ell_1^2}\,x_2\right),$$

we obtain

$$x_1\frac{\partial}{\partial x_0}\left(\frac{q\ell_1^2}{f\ell}\right) = 0$$

so that $\frac{q\ell_1^2}{f\ell}$ depends only on ρ: $\frac{q\ell_1^2}{f\ell} = \varphi(\rho)$.

Now the equation (2.4) with $\alpha = 0$ is written as

$$\frac{\partial}{\partial x_1}(x_1\varphi(\rho)) + \frac{\partial}{\partial x_2}(x_2\varphi(\rho)) + \frac{\partial}{\partial x_3}(x_3\varphi(\rho)) = 0\,,$$

whence $3\varphi(\rho) + \rho\varphi'(\rho) = 0$ and $3\rho^2\varphi(\rho) + \rho^3\varphi'(\rho) = 0$ or $(\rho^3\varphi(\rho))' = 0$.

Consequently $\rho^3\varphi(\rho) = \varepsilon = \text{const}$ and $q = \frac{\varepsilon f\ell}{\rho^3\ell_1^2}$.

The meaning of the constants ε and ε_1:

Since the function q occurs in the definition of the electric field (2.1), it is natural to identify the constant ε with the electric charge of the source. Does a similar reasoning is applicable to the case of the magnetic field (2.2)? In other words, does the constant ε_1 represents a magnetic charge of the source? This question is at first related to the case where $\varepsilon = 0$, $\varepsilon_1 \neq 0$, namely to the case where the spherical source appears as a magnetic monopole. However, although the existence of magnetic monopoles is envisaged sometimes as a theoretical possibility, it is not yet confirmed experimentally. Accordingly we are led to assume that $\varepsilon_1 = 0$, namely that the purely $S\Theta(4)$-invariant magnetic field vanishes. So we have to do only with the electric field (2.1), which, on account of $q = \frac{\varepsilon f\ell}{\rho^3\ell_1^2}$, depends on the gravitational tensor (contrary to Weyl's assertion).

3 Equations of gravitation outside the charged source

We recall that, if an electromagnetic field

$$\sum V_{\alpha\beta}dx_\alpha \otimes dx_\beta, \quad (V_{\alpha\beta} = -V_{\beta\alpha})$$

is associated with a spacetime metric

$$\sum g_{\alpha\beta}dx_\alpha \otimes dx_\beta\,,$$

then it gives rise to an energy-momentum tensor

$$\sum W_{\alpha\beta}dx_\alpha \otimes dx_\beta$$

defined by the formulae

$$W_{\alpha\beta} = \frac{1}{4\pi}\left(\frac{1}{4}\,g_{\alpha\beta}\sum V_{\gamma\delta}V^{\gamma\delta} - \sum V_{\alpha\delta}V_{\beta}^{\cdot\,\delta}\right). \quad (3.1)$$

In the present situation, the covariant and contravariant components $V_{\gamma\delta}$ and $V^{\gamma\delta}$ are already known. So it remains to compute the mixed components

$$V_{\beta}^{\cdot\,\delta} = \sum g^{\gamma\delta}V_{\beta\gamma} = -\sum g^{\delta\gamma}V_{\gamma\beta} = -V^{\delta}_{\cdot\,\beta}\,.$$

Taking into account the vanishing of the magnetic field, an easy computation gives

$$\begin{aligned}
&V_0^{\cdot\,0} = \frac{\rho^2qf_1}{f\ell^2}\,,\quad V_0^{\cdot\,k} = -\frac{qx_k}{\ell^2}\,, && (k = 1,2,3)\,,\\
&V_k^{\cdot\,0} = -\frac{\ell^2-\rho^2f_1^2}{f^2\ell^2}\,qx_k\,, && (k = 1,2,3)\,,\\
&V_k^{\cdot\,k} = -\frac{qf_1}{f\ell^2}\,x_k^2\,, && (k = 1,2,3)\,,\\
&V_2^{\cdot\,3} = -\frac{qf_1}{f\ell^2}\,x_2x_3 = V_3^{\cdot\,2}\,,\\
&V_3^{\cdot\,1} = -\frac{qf_1}{f\ell^2}\,x_3x_1 = V_1^{\cdot\,3}\,,\\
&V_1^{\cdot\,2} = -\frac{qf_1}{f\ell^2}\,x_1x_2 = V_2^{\cdot\,1}\,.
\end{aligned}$$

It follows that

$$\begin{aligned}
\sum V_{\gamma\delta}V^{\gamma\delta} &= -\frac{2\rho^2q^2}{f^2\ell^2}\,,\\
\sum V_{0\delta}V_0^{\cdot\,\delta} &= -\frac{\rho^2q^2}{\ell^2}\,,\\
\sum V_{0\delta}V_1^{\cdot\,\delta} &= -\frac{\rho^2f_1q^2}{f\ell^2}\,x_1\,,\\
\sum V_{1\delta}V_2^{\cdot\,\delta} &= \frac{\ell^2-\rho^2f_1^2}{f^2\ell^2}\,q^2x_1x_2\,,\\
\sum V_{1\delta}V_1^{\cdot\,\delta} &= \frac{\ell^2-\rho^2f_1^2}{f^2\ell^2}\,q^2x_1^2\,,
\end{aligned}$$

and then the formula (3.1) gives the components W_{00}, W_{01}, W_{11}, W_{12} of the energy-momentum tensor. The other components are obtained simply by permuting indices.

Proposition 3.1 *The energy-momentum tensor associated with the electric field (2.1) is a $\Theta(4)$-invariant tensor defined by the following formulae*

$$\begin{aligned}
&W_{00} = E_{00}\,, && W_{0i} = W_{i0} = x_iE_{01}\,,\\
&W_{ii} = E_{11} + x_i^2E_{22}\,, && W_{ij} = W_{ji} = x_ix_jE_{22}\,,\\
& && (i,j = 1,2,3;\ i \neq j)\,,
\end{aligned}$$

where

$$E_{00} = \frac{1}{8\pi}\,\rho^2 f^2 E\,, \quad E_{01} = \frac{1}{8\pi}\,\rho^2 f f_1 E\,,$$

$$E_{11} = \frac{1}{8\pi}\,\rho^2 \ell_1^2 E\,, \quad E_{22} = \frac{1}{8\pi}(-\ell_1^2 - \ell^2 + \rho^2 f_1^2)E$$

with

$$E = \frac{q^2}{f^2\ell^2} = \frac{\varepsilon^2}{\rho^2 g^4}\,.$$

Regarding the Ricci tensor $R_{\alpha\beta}$, we already know [4] that it is a symmetric $\Theta(4)$-invariant tensor defined by the functions

$$Q_{00} = Q_{00}(t,\rho)\,, \quad Q_{01} = Q_{01}(t,\rho)\,,$$

$$Q_{11} = Q_{11}(t,\rho)\,, \quad Q_{22} = Q_{22}(t,\rho)$$

as follows

$$R_{00} = Q_{00}\,, \ \ R_{0i} = R_{i0} = Q_{01}x_i\,, \ \ R_{ii} = Q_{11} + x_i^2 Q_{22}\,,$$

$$R_{ij} = R_{ji} = x_i x_j Q_{22}\,, \quad (i,j = 1,2,3;\ i \neq j)\,.$$

So, assuming that the cosmological constant vanishes, we have to do from the outset with four simple equations of gravitation, namely

$$Q_{00} - \frac{R}{2}\,f^2 + \frac{8\pi k}{c^4}\,E_{00} = 0\,,$$

$$Q_{01} - \frac{R}{2}\,f f_1 + \frac{8\pi k}{c^4}\,E_{01} = 0\,,$$

$$Q_{11} + \frac{R}{2}\,\ell_1^2 + \frac{8\pi k}{c^4}\,E_{11} = 0\,,$$

$$Q_{11} + \rho^2 Q_{22} - \frac{R}{2}(\rho^2 f_1^2 - \ell^2) + \frac{8\pi k}{c^4}(E_{11} + \rho^2 E_{22}) = 0\,.$$

An additional simplification results from the fact that the mixed components of the electromagnetic energy-momentum tensor satisfy the condition $\Sigma W_\alpha^\alpha = 0$, and then the equations of gravitation imply (by contraction) that the scalar curvature R vanishes. Moreover, introducing as usual the functions $h = \rho f_1$, $g = \rho\ell_1$, and taking into account that $q = \dfrac{\varepsilon f\ell}{\rho^3\ell_1^2}$, we obtain

$$E = \frac{\varepsilon^2}{\rho^2 g^4}, \qquad E_{00} = \frac{\varepsilon^2}{8\pi}\frac{f^2}{g^4}\,, \qquad E_{01} = \frac{\varepsilon^2}{8\pi}\frac{f f_1}{g^4}\,,$$

$$E_{11} = \frac{\varepsilon^2}{8\pi}\frac{\ell_1^2}{g^4}\,, \quad E_{11} + \rho^2 E_{22} = \frac{\varepsilon^2}{8\pi}\frac{(-\ell^2 + h^2)}{g^4}\,,$$

so that by setting

$$\nu^2 = \frac{k}{c^4}\,\varepsilon^2,$$

we get the definitive form of the equations of gravitation

$$Q_{00} + \frac{\nu^2}{g^4}\,f^2 = 0\,, \tag{3.1}$$

$$Q_{01} + \frac{\nu^2}{g^4}\,f f_1 = 0\,, \tag{3.2}$$

$$Q_{11} + \frac{\nu^2}{g^4}\,\ell_1^2 = 0\,, \tag{3.3}$$

$$Q_{11} + \rho^2 Q_{22} + \frac{\nu^2}{g^4}(-\ell^2 + h^2) = 0\,. \tag{3.4}$$

4 Stationary solutions outside the charged spherical source

In the case of a stationary field, the functions Q_{00}, Q_{01}, Q_{11}, Q_{22} depend only on ρ and their expressions are already known [3, 4]

$$Q_{00} = f\left(-\frac{f''}{\ell^2} + \frac{f'\ell'}{\ell^3} - \frac{2f'g'}{\ell^2 g}\right), \tag{4.1}$$

$$Q_{01} = \frac{h}{\rho f}\,Q_{00}\,, \tag{4.2}$$

$$Q_{11} = \frac{1}{\rho^2}\left(-1 + \frac{g'^2}{\ell^2} + \frac{gg''}{\ell^2} - \frac{\ell' gg'}{\ell^3} + \frac{f'gg'}{f\ell^2}\right), \tag{4.3}$$

$$Q_{11} + \rho^2 Q_{22} = \frac{f''}{f} + \frac{2g''}{g} - \frac{f'\ell'}{f\ell} - \frac{2\ell' g'}{\ell g} + \frac{h^2}{f^2}\,Q_{00}\,. \tag{4.4}$$

On account of (4.2), the equation (3.2) is written as

$$\left(Q_{00} + \frac{\nu^2}{g^4}\,f^2\right) h = 0$$

so that it is verified because of (3.1).

Consequently it only remains to take into account the equations (3.1), (3.3), (3.4).

From (3.1) we obtain

$$\frac{\nu^2}{g^4} = -\frac{Q_{00}}{f^2}$$

and inserting this expression into (3.4) we obtain the relation

$$f^2(Q_{11} + \rho^2 Q_{22}) - (-\ell^2 + h^2)Q_{00} = 0$$

which, on account of (4.1) and (4.4), reduces, after cancelations, to the simple equation

$$\frac{g''}{g'} = \frac{f'}{f} + \frac{\ell'}{\ell}$$

which does not contain the unknown function h and implies

$$f\ell = cg', \quad (c = \text{const}). \tag{4.5}$$

Next, from (3.1) and (3.3) we deduce the equation

$$Q_{11} - \frac{Q_{00}}{f^2}\,\ell_1^2 = 0 \tag{4.6}$$

which does not contain the function h either.

Now, from (4.5) we find

$$f = \frac{cg'}{\ell}$$

and inserting this expression of f into (4.6), we obtain an equation which can be written as

$$\frac{d}{d\rho}\left(\frac{F'}{2g'}\right) = 0$$

with

$$F = g^2 - \frac{g^2 g'^2}{\ell^2}.$$

It follows that

$$F = 2A_1 g - A_2\,, \quad (A_1 = \text{const},\ A_2 = \text{const}),$$

and

$$g'^2 = \ell^2\left(1 - \frac{2A_1}{g} + \frac{A_2}{g^2}\right). \tag{4.7}$$

On account of (4.5), the derivative g' does not vanish. In fact $g' = 0$ implies either $f = 0$ or $\ell = 0$, which gives rise to a degenerate spacetime metric, namely a spacetime metric meaningless physically. Then, in particular, it follows from (4.7) that

$$1 - \frac{2A_1}{g} + \frac{A_2}{g^2} > 0\,.$$

The constant A_1, obtained by means of the Newtonian approximation, is already known:

$$A_1 = \frac{km}{c^2} = \mu\,.$$

In order to get A_2, we insert first

$$\frac{f'}{f} = \frac{g''}{g'} - \frac{\ell'}{\ell}$$

into (4.3) thus obtaining

$$\rho^2 Q_{11} = -1 + \frac{g'^2}{\ell^2} + \frac{2gg''}{\ell^2} - \frac{2\ell' g g'}{\ell^3}\,. \tag{4.8}$$

Next by setting

$$Q(g) = 1 - \frac{2A_1}{g} + \frac{A_2}{g^2}$$

we have

$$g' = \ell\sqrt{Q(g)}\,,$$

$$g'' = \ell'\sqrt{Q(g)} + \ell^2\left(\frac{A_1}{g^2} - \frac{A_2}{g^3}\right)$$

and inserting these expressions of g' and g'' into (4.8), we find

$$\rho^2 Q_{11} = -\frac{A_2}{g^2}\,.$$

The equation (3.3) gives finally the value of the constant A_2:

$$A_2 = \nu^2 = \frac{k\varepsilon^2}{c^4}\,.$$

It follows that the general stationary solution outside the charged spherical source is defined by two equations, namely

$$f\ell = c\,\frac{dg}{d\rho}\,, \tag{4.9}$$

$$\frac{dg}{d\rho} = \ell\sqrt{1 - \frac{2\mu}{g} + \frac{\nu^2}{g^2}}\,, \tag{4.10}$$

$$\left(\mu = \frac{km}{c^2}, \quad \nu = \frac{\sqrt{k}}{c^2}\,|\varepsilon|, \quad 1 - \frac{2\mu}{g} + \frac{\nu^2}{g^2} > 0\right).$$

The interdependence of the two fields, gravitational and electric, in now obvious: The electric charge ε, which defines the electric field, is also involved in the definition of the gravitational field by means of the term

$$\frac{\nu^2}{g^2} = \frac{k}{c^4}\left(\frac{\varepsilon}{g}\right)^2.$$

On the other hand, since

$$q = \frac{\varepsilon f \ell}{\rho^3 \ell_1^2} = \frac{c\varepsilon}{\rho g^2}\frac{dg}{d\rho}\,,$$

the components of the electric field:

$$\begin{aligned} V_{01} = -V_{10} = q x_1 &= \frac{c\varepsilon}{g^2}\frac{dg}{d\rho}\frac{x_1}{\rho} = \\ &= -\,c\varepsilon\frac{\partial}{\partial x_1}\left(\frac{1}{g}\right) = -c\frac{\partial}{\partial x_1}\left(\frac{\varepsilon}{g}\right), \end{aligned}$$

$$V_{02} = -V_{20} = q x_2 = -c\,\frac{\partial}{\partial x_2}\left(\frac{\varepsilon}{g}\right),$$

$$V_{03} = -V_{30} = q x_3 = -\,c\,\frac{\partial}{\partial x_3}\left(\frac{\varepsilon}{g}\right)$$

result from the electric potential:

$$\frac{\varepsilon}{g} = \frac{\varepsilon}{g(\rho)}$$

which is thus defined by means of the curvature radius $g(\rho)$, namely by the fundamental function involved in the definition of the gravitational field.

Note that, among the functions occurring in the spacetime metric, only the function $h = \rho f_1$ does not appear in the equations (4.9) and (4.10). The problem does not require a uniquely defined h. Every differentiable function h satisfying the condition $|h| \leqslant \ell$ is allowable. And every allowable h gives rise to a possible conception of the time coordinate. Contrary to the Special Relativity, we have to do, in General Relativity, with an infinity of possible definitions of the time coordinate. In order to elucidate this assertion in the present situation, let us denote by ρ_1 the radius of the spherical stationary source, and consider a photon emitted radially from the sphere $\|x\| = \rho_1$ at an instant τ. The equation of motion of this photon, namely

$$f(\rho)dt + h(\rho)d\rho = \ell(\rho)d\rho$$

implies

$$\frac{dt}{d\rho} = \frac{-h(\rho) + \ell(\rho)}{f(\rho)}$$

whence $\tau = t - \psi(\rho)$ with

$$\psi(\rho) = \int_{\rho_1}^{\rho} \frac{-h(u) + \ell(u)}{f(u)}\, du\,.$$

For every value of $\rho \geqslant \rho_1$, $\pi(t,\rho) = t - \psi(\rho)$ is the instant of radial emission of a photon reaching the sphere $\|x\| = \rho$ at the instant t. The function $\pi(t,\rho)$ will be called *propagation function*, and we see that to each allowable h there corresponds a uniquely defined propagation function. Moreover each propagation function characterizes uniquely a conception of the notion of time. Regarding the radial velocity of propagation of light, namely

$$\frac{d\rho}{dt} = \frac{f(\rho)}{-h(\rho) + \ell(\rho)}\,,$$

it is not bounded by a barrier as in Special Relativity. In the limit case where the allowable h equals ℓ, this velocity becomes infinite.

This being said, we return to the equations (4.9) and (4.10) which contain the remaining unknown functions f, ℓ, g. Their investigation necessitates a rather lengthy discussion which will be carried out in another paper. At present we confine ourselves to note two significant conclusions of this discussion:

(a) Pointwise sources do not exist, so that the spherical source cannot be reduced to a point. In particular the notion of black hole is inconceivable;

(b) Among the solutions defined by (4.9) and (4.10), particularly significant are those obtained by introducing the radial geodesic distance

$$\delta = \int_0^{\rho} \ell(u)du\,.$$

Then we have to define the curvature radius $G(\delta) = g(\rho(\delta))$ by means of the equation

$$\frac{dG}{d\delta} = \sqrt{1 - \frac{2\mu}{G} + \frac{\nu^2}{G^2}}$$

the solutions of which need specific discussion according as $\nu^2 - \mu^2 > 0$ or $\nu^2 - \mu^2 = 0$ or $\nu^2 - \mu^2 < 0$. The first approach to this problem appeared in the paper [1].

We note finally that the derivation of the Reissner-Nordström metric contains topological errors and moreover identifies erroneously the fundamental function $g(\rho)$ with a radial coordinate. This is why the Reissner-Nordström metric is devoid of geometrical and physical meaning.

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Low-Lying Collective Levels in $^{224\text{-}234}$Th Nuclei

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The low-lying collective levels in $^{224\text{-}234}$Th isotopes are investigated in the frame work of the interacting boson approximation model (IBA-1). The contour plot of the potential energy surfaces, $V(\beta, \gamma)$, shows two wells on the prolate and oblate sides which indicate that all thorium nuclei are deformed and have rotational characters. The levels energy, electromagnetic transition rates B(E1) and B(E2) are calculated. Bending at angular momentum $I^+ = 20$ has been observed for ^{230}Th. Staggering effect has been calculated and beat patterns are obtained which indicate the existence of an interaction between the ground state band, (GSB), and the octupole negative parity band, (NPB). All calculated values are compared with the available experimental data and show reasonable agreement.

1 Introduction

The level schemes of $^{224\text{-}234}$Th isotopes are characterized by the existence of two bands of opposite parity and lie in the region of octupole deformations. The primary evidence for this octupole deformaton comes from the parity-doublet bands, fast electric transition $(E1)$ between the negative and positive parity bands and the low-lying 1^-, 0_2^+ and 2_2^+ excitation energy states. This kind of deformation has offered a real challenge for nuclear structure models. Even-even thorium nuclei have been studied within the frame work of the $Spdf$ interacting boson model [1] and found the properties of the low-lying states can be understood without stable octupole deformation. High spin states in some of these nuclei suggest that octupole deformation develops with increasing spin.

A good description of the first excited positive and negative parity bands of nuclei in the rare earth and the actinide region has achieved [2–4] using the interacting vector boson model. The analysis of the eigen values of the model Hamiltonian reveals the presence of an interaction between these bands. Due to this interaction staggering effect has reproduced including the beat patterns.

Shanmugam-Kamalahran (SK) model [5] for α-decay has been applied successfully to $^{226\text{-}232}$Th for studying their shapes, deformations of the parent and daughter nuclei as well as the charge distribution process during the decay. Also, a solution of the Bohr Hamiltonian [6] aiming at the description of the transition from axial octupole deformation to octupole vibrations in light actinides ^{224}Ra and ^{226}Th is worked out.The parameter free predictions of the model are in good agreement with the experimental data of the two nuclei, where they known to lie closest to the transition from octupole deformation to octupole vibrations in this region. A new frame-work for comparing fusion probabilities in reactions [7] forming heavy elements, ^{220}Th, eliminates both theoretical and experimental uncertinities, allowing insights into systematic behavior, and revealing previously hidden characteristics in fusion reactions forming heavy elements.

It is found that cluster model [8] succeeded in reproducing satisfactorily the properties of normal deformed ground state and super deformed excited bands [9, 10] in a wide range of even-even nuclei, $222 \leqslant A \geqslant 242$[11]. The calculated spin dependences [12] to the parity splitting and the electric multi pole transition moments are in agreement with the experimental data. Also, a new formula between half-lives, decay energies and microscopic density-dependent cluster model [13] has been used and the half-lives of cluster radioactivity are well reproduced.

A new imperical formula [14], with only three parameters, is proposed for cluster decay half-lives. The parameters of the formula are obtained by making least square fit to the available experimental cluster decay data. The calculated half-lives are compared with the results of the earlier proposed models models, experimental available data and show excellent agreement. A simple description of the cluster decay by suggesting a folding cluster-core interaction based on a self-consistant mean-field model [15]. Cluster decay in even-even nuclei above magic numbers have investigated.

Until now scarce informations are available about the actinide region in general and this is due to the experimental difficulties associated with this mass region. The aim of the present work is to:

(1) calculate the potential energy surfaces, $V(\beta, \gamma)$, and know the type of deformation exists;

(2) calculate levels energy, electromagnetic transition rates $B(E1)$ and $B(E2)$;

(3) study the relation between the angular momentum I, the rotational angular frequency $\hbar\omega$ and see if there any bending for any of thorium isotopes;

(4) calculate staggering effect and beat patterns to study the interaction between the $(+ve)$ and $(-ve)$ parity bands.

nucleus	EPS	$PAIR$	ELL	QQ	OCT	HEX	$E2SD(eb)$	$E2DD(eb)$
^{224}Th	0.2000	0.000	0.0081	−0.0140	0.0000	0.0000	0.2150	−0.6360
^{226}Th	0.2000	0.000	0.0058	−0.0150	0.0000	0.0000	0.2250	−0.6656
^{228}Th	0.2000	0.0000	0.0052	−0.0150	0.0000	0.0000	0.1874	−0.5543
^{230}Th	0.2000	0.0000	0.0055	−0.0150	0.0000	0.0000	0.1874	−0.5543
^{232}Th	0.2000	0.0000	0.0055	−0.0150	0.0000	0.0000	0.1820	−0.5384
^{234}Th	0.2000	0.0000	0.0063	−0.0150	0.0000	0.0000	0.1550	−0.4585

Table 1: Parameters used in IBA-1 Hamiltonian (all in MeV).

2 (IBA-1) model

2.1 Level energies

The IBA-1 model was applied to the positive and negative parity low-lying states in even-even $^{224\text{-}234}$Th isotopes. The proton, π, and neutron, ν, bosons are treated as one boson and the system is considered as an interaction between s-bosons and d-bosons. Creation $(s^\dagger d^\dagger)$ and annihilation $(s\tilde{d})$ operators are for s and d bosons. The Hamiltonian [16] employed for the present calculation is given as:

$$\begin{aligned} H &= EPS \cdot n_d + PAIR \cdot (P \cdot P) \\ &+ \frac{1}{2}\, ELL \cdot (L \cdot L) + \frac{1}{2}\, QQ \cdot (Q \cdot Q) \\ &+ 5OCT \cdot (T_3 \cdot T_3) + 5HEX \cdot (T_4 \cdot T_4)\,, \end{aligned} \tag{1}$$

where

$$P \cdot p = \frac{1}{2} \begin{bmatrix} \left\{ (s^\dagger s^\dagger)_0^{(0)} - \sqrt{5}(d^\dagger d^\dagger)_0^{(0)} \right\} x \\ \left\{ (ss)_0^{(0)} - \sqrt{5}(\tilde{d}\tilde{d})_0^{(0)} \right\} \end{bmatrix}_0^{(0)}, \tag{2}$$

$$L \cdot L = -10\,\sqrt{3}\left[(d^\dagger \tilde{d})^{(1)} x\, (d^\dagger \tilde{d})^{(1)} \right]_0^{(0)}, \tag{3}$$

$$Q \cdot Q = \sqrt{5} \begin{bmatrix} \left\{ (S^\dagger \tilde{d} + d^\dagger s)^{(2)} - \frac{\sqrt{7}}{2}\,(d^\dagger \tilde{d})^{(2)} \right\} x \\ \left\{ (s^\dagger \tilde{d} + + \tilde{d}s)^{(2)} - \frac{\sqrt{7}}{2}\,(d^\dagger \tilde{d})^{(2)} \right\} \end{bmatrix}_0^{(0)}, \tag{4}$$

$$T_3 \cdot T_3 = -\sqrt{7}\left[(d^\dagger \tilde{d})^{(2)} x\, (d^\dagger \tilde{d})^{(2)} \right]_0^{(0)}, \tag{5}$$

$$T_4 \cdot T_4 = 3\left[(d^\dagger \tilde{d})^{(4)} x\, (d^\dagger \tilde{d})^{(4)} \right]_0^{(0)}. \tag{6}$$

In the previous formulas, n_d is the number of boson; $P{\cdot}P$, $L \cdot L$, $Q \cdot Q$, $T_3 \cdot T_3$ and $T_4 \cdot T_4$ represent pairing, angular momentum, quadrupole, octupole and hexadecupole interactions between the bosons; EPS is the boson energy; and $PAIR$, ELL, QQ, OCT, HEX is the strengths of the pairing, angular momentum, quadrupole, octupole and hexadecupole interactions.

2.2 Transition rates

The electric quadrupole transition operator [16] employed in this study is given by:

$$\begin{aligned} T^{(E2)} &= E2SD \cdot (s^\dagger \tilde{d} + d^\dagger s)^{(2)} + \\ &+ \frac{1}{\sqrt{5}}\, E2DD \cdot (d^\dagger \tilde{d})^{(2)}\,. \end{aligned} \tag{7}$$

The reduced electric quadrupole transition rates between $I_i \to I_f$ states are given by

$$B(E_2, I_i - I_f) = \frac{[< I_f \parallel T^{(E_2)} \parallel I_i >]^2}{2I_i + 1}\,. \tag{8}$$

3 Results and discussion

3.1 The potential energy surface

The potential energy surfaces [17], $V(\beta,\gamma)$, for thorium isotopes as a function of the deformation parameters β and γ have been calculated using :

$$\begin{aligned} &E_{N_\Pi N_\nu}(\beta,\gamma) = \,<N_\pi N_\nu; \beta\gamma\, |H_{\pi\nu}| N_\pi N_\nu; \beta\gamma> \,= \\ &= \zeta_d(N_\nu N_\pi)\beta^2(1+\beta^2) + \beta^2(1+\beta^2)^{-2} \times \\ &\times \left\{ kN_\nu N_\pi [4 - (\bar{X}_\pi \bar{X}_\nu)\beta \cos 3\gamma] \right\} + \\ &+ \left\{ [\bar{X}_\pi \bar{X}_\nu \beta^2] + N_\nu(N_\nu - 1)\left(\frac{1}{10}\, c_0 + \frac{1}{7}\, c_2\right)\beta^2 \right\}, \end{aligned} \tag{9}$$

where

$$\bar{X}_\rho = \left(\frac{2}{7}\right)^{0.5} X_\rho \quad \rho = \pi \text{ or } \upsilon\,. \tag{10}$$

The calculated potential energy surfaces, $V(\beta,\gamma)$, for thorium series of isotopes are presented in Fig. 1. It shows that all nuclei are deformed and have rotational-like characters. The prolate deformation is deeper than oblate in all nuclei except ^{230}Th.The two wells on both oblate and prolate sides are equals and O(6) characters is expected to the nucleus. The energy and electromagnetic magnetic transition rates ratio are not in favor to that assumption and it is treated as a rotational-like nucleus.

I_i^+ I_f^+	^{224}Th	^{226}Th	^{228}Th	^{230}Th	^{232}Th	^{234}Th
0_1 Exp. 2_1	——-	6.85(42)	7.06(24)	8.04(10)	9.28(10)	8.00(70)
0_1 Theor. 2_1	4.1568	6.8647	7.0403	8.038	9.2881	8.0559
2_1 0_1	0.8314	1.3729	1.4081	1.6076	1.8576	1.6112
2_2 0_1	0.0062	0.0001	0.0044	0.0088	0.0105	0.0079
2_2 0_2	0.4890	0.8357	0.8647	1.0278	1.2683	1.1659
2_3 0_1	0.0127	0.0272	0.0211	0.0157	0.0122	0.0075
2_3 0_2	0.1552	0.0437	0.0020	0.0023	0.0088	0.0099
2_3 0_3	0.1102	0.0964	0.0460	0.0203	0.0079	0.0023
2_4 0_3	0.2896	0.4907	0.5147	0.6271	0.8048	0.7786
2_4 0_4	0.1023	0.0709	0.0483	0.0420	0.0385	0.0990
2_2 2_1	0.1837	0.1153	0.0599	0.0387	0.0286	0.0174
2_3 2_1	0.0100	0.0214	0.0211	0.0198	0.0178	0.0118
2_3 2_2	0.8461	1.0683	0.5923	0.2989	0.1538	0.0697
4_1 2_1	1.3733	2.0662	2.0427	2.2957	2.6375	2.2835
4_1 2_2	0.0908	0.1053	0.0764	0.0579	0.0445	0.0266
4_1 2_3	0.0704	0.0325	0.0104	0.0038	0.0018	0.0008
6_1 4_1	1.5696	2.2921	2.2388	2.4979	2.8606	2.4745
6_1 4_2	0.0737	0.0858	0.0685	0.0585	0.0493	0.0312
6_1 4_3	0.0584	0.0404	0.0198	0.0106	0.0061	0.0029
8_1 6_1	1.5896	2.3199	2.2720	2.5381	2.9105	2.5220
8_1 6_2	0.0569	0.0660	0.0554	0.0511	0.0466	0.0314
8_1 6_3	0.0483	0.0421	0.0256	0.0166	0.0109	0.0055
10_1 8_1	1.4784	2.2062	2.1948	2.4760	2.8586	2.4899
10_1 8_2	0.0448	0.0513	0.0438	0.0422	0.0407	0.0290

Table 2: Values of the theoretical reduced transition probability, $B(E2)$ (in $e^2\,b^2$).

I_i^- I_f^+	^{224}Th	^{226}Th	^{228}Th	^{230}Th	^{232}Th	^{234}Th
1_1 0_1	0.0428	0.0792	0.1082	0.1362	0.1612	0.1888
1_1 0_2	0.0942	0.0701	0.0583	0.0534	0.0515	0.0495
3_1 2_1	0.1607	0.1928	0.2209	0.2531	0.2836	0.3227
3_1 2_2	0.0733	0.0829	0.0847	0.0817	0.0768	0.0717
3_1 2_3	0.0360	0.0157	0.0054	0.0013	0.0002	0.0000
3_1 4_1	0.0233	0.0441	0.0652	0.0884	0.1150	0.1384
3_1 4_2	0.0170	0.0285	0.0371	0.0424	0.0460	0.0449
5_1 4_1	0.2873	0.3131	0.3363	0.3657	0.3946	—–
5_1 4_2	0.0787	0.0834	0.0868	0.0865	0.0835	—–
5_1 4_3	0.0160	0.0101	0.0051	0.0020	0.0006	—–
7_1 6_1	0.4178	0.4387	0.4581	0.4839	0.5100	—–
7_1 6_2	0.0732	0.0757	0.0798	0.0817	0.0812	—–
9_1 8_1	0.5532	0.5690	0.5848	0.6070	0.6301	—–
9_1 8_2	0.0639	0.0665	0.0707	0.0735	0.0748	—–

Table 3: Values of the theoretical reduced transition probability, $B(E1)$ (in $\mu\,e^2 b$).

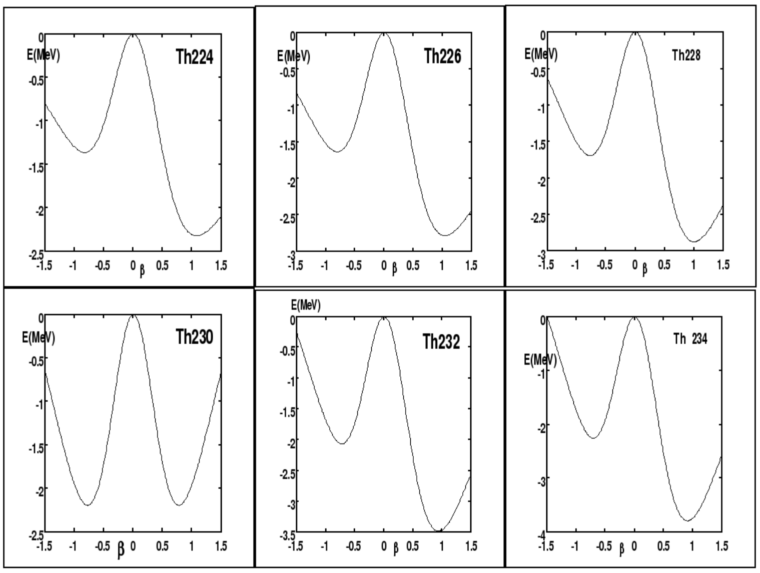

Fig. 1: Potential Energy surfaces for $^{224\text{-}234}$Th nuclei.

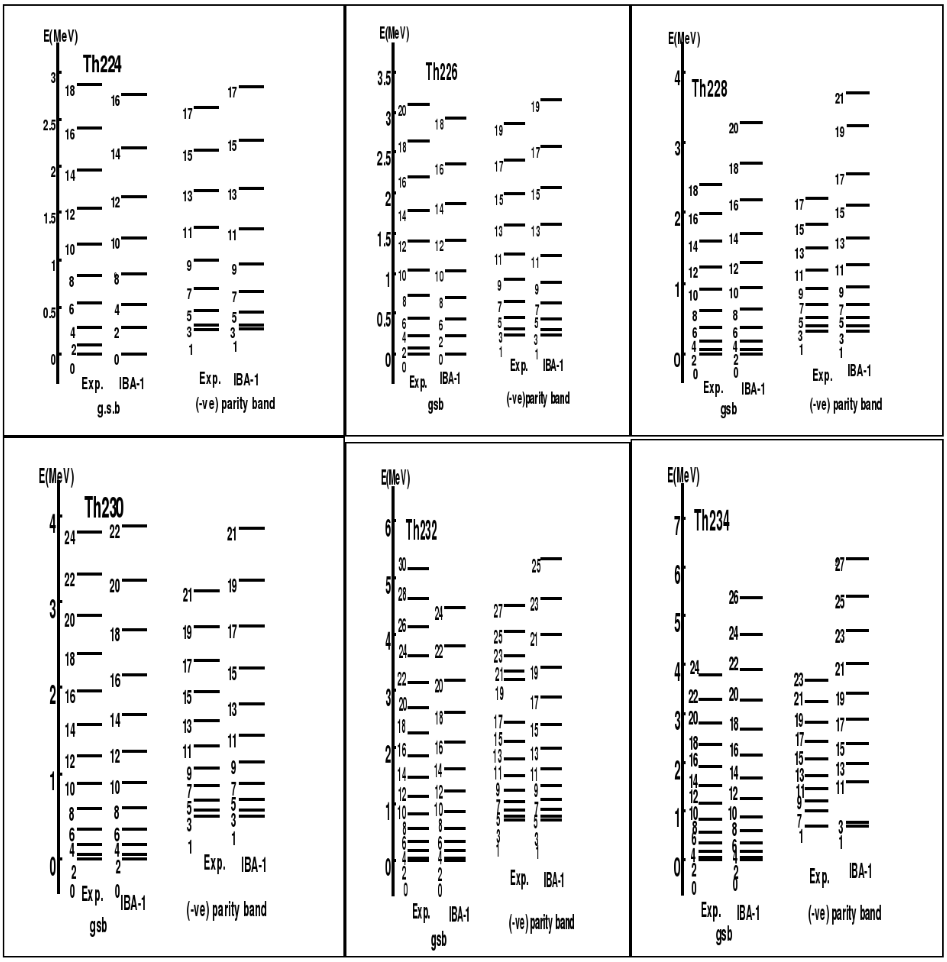

Fig. 2: Comparison between experimental (Exp.) and theoretical (IBA-1) energy levels in $^{224\text{-}234}$Th.

3.2 Energy spectra

IBA-1 model has been used in calculating the energy of the positive and negative parity low -lying levels of thorium series of isotopes. In many deformed actinide nuclei the negative parity bands have been established and these nuclei are considered as an octupole deformed. A simple means to examine the nature of the band is to consider the ratio R which for octupole band , $R \succ 1$, and defined as [18]:

$$R = \frac{E(I+3) - E(I-1)_{NPB}}{E(I) - E(I-2)_{GSB}}\,. \tag{11}$$

In the present calculations all values of R for thorium series of isotopes are $\succ 1$, and we treated them as octupole deformed nuclei.

A comparison between the experimental spectra [19–24] and our calculations, using values of the model parameters given in Table 1 for the ground and octupole bands, are illustrated in Fig. 2. The agreement between the calculated levels energy and their correspondence experimental values for all thorium nuclei are slightly higher especially for the higher excited states. We believe this is due to the change of the projection of the angular momentum which is due to band crossing and octupole deformation.

Unfortunately there is no enough measurements of electromagnetic transition rates $B(E2)$ or $B(E1)$ for these series of nuclei. The only measured $B(E2, 0_1^+ \rightarrow 2_1^+)$'s are presented, in Table's 2,3 for comparison with the calculated values. The parameters $E2SD$ and $E2DD$ used in the present calculations are determined by normalizing the calculated values to the experimentally known ones and displayed in Table 1.

For calculating $B(E1)$ and $B(E2)$ electromagnetic transition rates of intraband and interaband we did not introduce any new parameters. Some of the calculated values are presented in Fig. 3 and show bending at $N = 136$, 142 which means there is an interaction between the $(+ve)GSB$ and $(-ve)$ parity octupole bands.

The moment of inertia I and energy parameters $\hbar\omega$ are calculated using equations (12, 13):

$$\frac{2I}{\hbar^2} = \frac{4I-2}{\Delta E(I \rightarrow I-2)}\,, \tag{12}$$

$$(\hbar\omega)^2 = (I^2 - I + 1)\left[\frac{\Delta E(I \rightarrow I-2)}{(2I-1)}\right]^2. \tag{13}$$

All the plots in Fig. 4 show back bending at angular momentum $I^+ = 20$ for ^{230}Th. It means, there is a band crossing and this is confirmed by calculating staggering effect to these series of thorium nuclei. A disturbance of the regular band structure has observed not only in the moment of inertia but also in the decay properties.

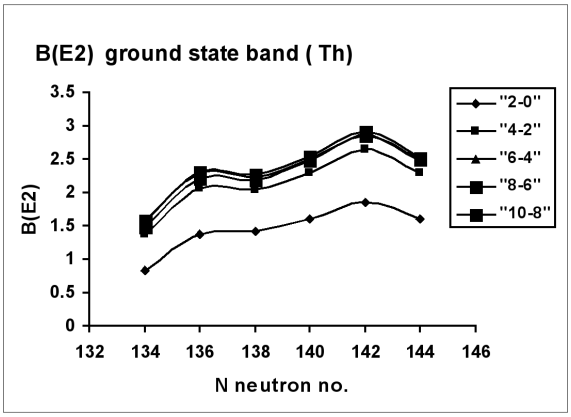

Fig. 3: The calculated $B(E2)$'s for the ground state band of $^{224\text{-}234}$Th isotopes.

3.3 The staggering

The presence of odd-even parity states has encouraged us to study staggering effect for $^{218\text{-}230}$Th series of isotopes [10, 12, 25, 26]. Staggering patterns between the energies of the GSB and the $(-ve)$ parity octupole band have been calculated, $\Delta I = 1$, using staggering function equations (14, 15) with the help of the available experimental data [19–24].

$$Stag(I) = 6\Delta E(I) - 4\Delta E(I-1) - 4\Delta E(I+1) + \Delta E(I+2) + \Delta E(I-2)\,, \tag{14}$$

with

$$\Delta E(I) = E(I+1) - E(I)\,. \tag{15}$$

The calculated staggering patterns are illustrated in Fig. 5, where we can see the beat patterns of the staggering behavior which show an interaction between the ground state and the octupole bands.

3.4 Conclusions

The IBA-1 model has been applied successfully to $^{224\text{-}234}$Th isotopes and we have got:

1. The ground state and octupole bands are successfully reproduced;
2. The potential energy surfaces are calculated and show rotational behavior to $^{224\text{-}234}$Th isotopes where they are mainly prolate deformed nuclei;
3. Electromagnetic transition rates $B(E1)$ and $B(E2)$ are calculated;
4. Bending for ^{230}Th has been observed at angular momentum $I^+ = 20$;
5. Staggering effect has been calculated and beat patterns are obtained which show an interaction between the ground state and octupole bands;

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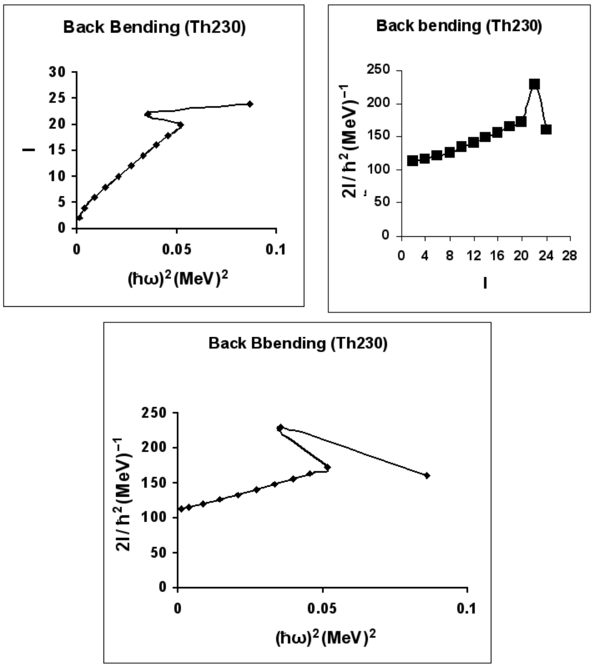

Fig. 4: Angular momentum I as a function of the rotational frequency $(\hbar\omega)^2$ and 2I/$\hbar^2$ as a function of $(\hbar\omega)^2$ for the GSB of ^{230}Th.

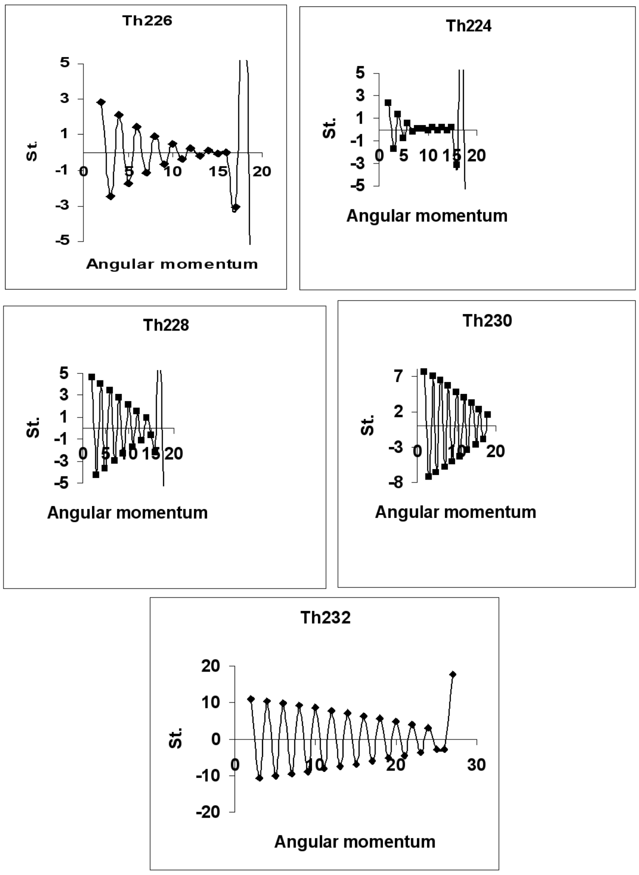

Fig. 5: $\Delta I = 1$, staggering patterns for the ground state and octupole bands of $^{224\text{-}232}$Th isotope.

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Correlated Detection of sub-mHz Gravitational Waves by Two Optical-Fiber Interferometers

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Results from two optical-fiber gravitational-wave interferometric detectors are reported. The detector design is very small, cheap and simple to build and operate. Using two detectors has permitted various tests of the design principles as well as demonstrating the first simultaneous detection of correlated gravitational waves from detectors spatially separated by 1.1 km. The frequency spectrum of the detected gravitational waves is sub-mHz with a strain spectral index $a = -1.4 \pm 0.1$. As well as characterising the wave effects the detectors also show, from data collected over some 80 days in the latter part of 2007, the dominant earth rotation effect and the earth orbit effect. The detectors operate by exploiting light speed anisotropy in optical-fibers. The data confirms previous observations of light speed anisotropy, earth rotation and orbit effects, and gravitational waves.

1 Introduction

Results from two optical-fiber gravitational-wave interferometric detectors are reported. Using two detectors has permitted various tests of the design principles as well as demonstrating the first simultaneous detection of correlated gravitational waves from detectors spatially separated by 1.1 km. The frequency spectrum of the detected gravitational waves is sub-mHz. As well as charactersing the wave effects the detectors also show, from data collected over some 80 days in the latter part of 2007, the dominant earth rotation effect and the earth orbit effect. The detectors operate by exploiting light speed anisotropy in optical-fibers. The data confirms previous observations [1–4, 6–10] of light speed anisotropy, earth rotation and orbit effects, and gravitational waves. These observations and experimental techniques were first understood in 2002 when the Special Relativity effects and the presence of gas were used to calibrate the Michelson interferometer in gas-mode; in vacuum-mode the Michelson interferometer cannot respond to light speed anisotropy [11, 12], as confirmed in vacuum resonant-cavity experiments, a modern version of the vacuum-mode Michelson interferometer [13]. The results herein come from improved versions of the prototype optical-fiber interferometer detector reported in [9], with improved temperature stabilisation and a novel operating technique where one of the interferometer arms is orientated with a small angular offset from the local meridian. The detection of sub-mHz gravitational waves dates back to the pioneering work of Michelson and Morley in 1887 [1], as discussed in [16], and detected again by Miller [2] also using a gas-mode Michelson interferometer, and by Torr and Kolen [6], DeWitte [7] and Cahill [8] using RF waves in coaxial cables, and by Cahill [9] and herein using an optical-fiber interferometer design, which is very much more sensitive than a gas-mode interferometer, as discussed later.

It is important to note that the repeated detection, over more than 120 years, of the anisotropy of the speed of light is not in conflict with the results and consequences of Special Relativity (SR), although at face value it appears to be in conflict with Einstein's 1905 postulate that the speed of light is an invariant in vacuum. However this contradiction is more apparent than real, for one needs to realise that the space and time coordinates used in the standard SR Einstein formalism are *constructed* to make the speed of light invariant wrt those special coordinates. To achieve that observers in relative motion must then relate their space and time coordinates by a Lorentz transformation that mixes space and time coordinates — but this is only an artifact of this formalism*. Of course in the SR formalism one of the frames of reference could have always been designated as the observable one. Such an ontologically real frame of reference, only in which the speed of light is isotropic, has been detected for over 120 years, yet ignored by mainstream physics. The problem is in not clearly separating a very successful mathematical formalism from its predictions and experimental tests. There has been a long debate over whether the Lorentz 3-space *and* time interpretation or the Einstein spacetime interpretation of observed SR effects is preferable or indeed even experimentally distinguishable.

What has been discovered in recent years is that a dynamical structured 3-space exists, so confirming the Lorentz interpretation of SR, and with fundamental implications for physics — for physics failed to notice the existence of the

*Thus the detected light speed anisotropy does not indicate a breakdown of Lorentz symmetry, contrary to the aims but not the outcomes of [13].

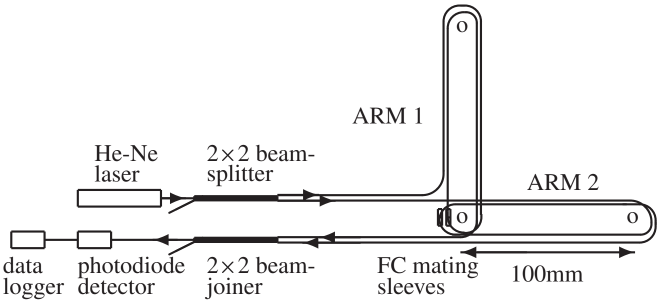

Fig. 1: Schematic layout of the interferometric optical-fiber light-speed anisotropy/gravitational wave detector. Actual detector is shown in Fig. 2. Coherent 633 nm light from the a He-Ne Laser is split into two lengths of single-mode polarisation preserving fibers by the 2×2 beam splitter. The two fibers take different directions, ARM1 and ARM2, after which the light is recombined in the 2×2 beam joiner, in which the phase differences lead to interference effects that are indicated by the outgoing light intensity, which is measured in the photodiode detector/amplifier (Thorlabs PDA36A or PDA36A-EC), and then recorded in the data logger. In the actual layout the fibers make two loops in each arm, but with excess lengths wound around one arm (not shown) — to reduce effective fiber lengths so as to reduce sensitivity. The length of one straight section is 100 mm, which is the center to center spacing of the plastic turners, having diameter = 52 mm, see Fig. 2. The relative travel times, and hence the output light intensity, are affected by the varying speed and direction of the flowing 3-space, by affecting differentially the speed of the light, and hence the net phase difference between the two arms.

Fig. 2: Photograph of a detector showing the optical fibers forming the two orthogonal arms. See Fig. 1 for the schematic layout. The 2×2 beam splitter and joiner (Thorlabs FC632-50B-FC) are the two small stainless steel cylindrical tubes. The two FC to FC mating sleeves (Thorlabs ADAFC1) are physically adjacent. The overall dimensions of the metal base plate are 160×160 mm. The 2×2 splitter and joiner each have two input and two output fibers, with one not used. Arm 2 is folded over the splitter and joiner, compared to the schematic layout. The interferometer shown costs approximately $400.

main *constituent* defining the universe, namely a dynamical 3-space, with quantum matter and EM radiation playing a minor role. This dynamical 3-space provides an explanation for the success of the SR Einstein formalism. It also provides a new account of gravity, which turns out to be a quantum effect [17], and of cosmology [16, 18–20], doing away with the need for dark matter and dark energy.

2 Dynamical 3-space and gravitational waves

Light-speed anisotropy experiments have revealed that a dynamical 3-space exists, with the speed of light being c, in vacuum, only wrt to this space: observers in motion "through" this 3-space detect that the speed of light is in general different from c, and is different in different directions*. The dynamical equations for this 3-space are now known and involve a velocity field $\mathbf{v}(\mathbf{r},t)$, but where only relative velocities are observable locally — the coordinates $\mathbf{r}$ are relative to a non-physical mathematical embedding space. These dynamical equations involve Newton's gravitational constant G and the fine structure constant α. The discovery of this dynamical 3-space then required a generalisation of the Maxwell, Schrödinger and Dirac equations. The wave effects already detected correspond to fluctuations in the 3-space velocity field $\mathbf{v}(\mathbf{r},t)$, so they are really 3-space turbulence or wave effects. However they are better known, if somewhat inappropriately, as "gravitational waves" or "ripples" in "spacetime". Because the 3-space dynamics gives a deeper understanding of the spacetime formalism we now know that the metric of the induced spacetime, merely a mathematical construct having no ontological significance, is related to $\mathbf{v}(\mathbf{r},t)$ according to [16, 18, 20]

$$ds^2 = dt^2 - \frac{(d\mathbf{r} - \mathbf{v}(\mathbf{r},t)dt)^2}{c^2} = g_{\mu\nu}dx^\mu dx^\nu. \qquad (1)$$

The gravitational acceleration of matter, and of the structural patterns characterising the 3-space, are given by [16, 17]

$$\mathbf{g} = \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} \qquad (2)$$

and so fluctuations in $\mathbf{v}(\mathbf{r},t)$ may or may not manifest as a gravitational force. The general characteristics of $\mathbf{v}(\mathbf{r},t)$ are now known following the detailed analysis of the experiments noted above, namely its average speed, and removing the earth orbit effect, is some 420±30 km/s, from direction RA = 5.5±2$^{\rm hr}$, Dec = 70±10°S — the center point of the Miller data in Fig. 12b, together with large wave/turbulence effects. The magnitude of this turbulence depends on the timing resolution of each particular experiment, and here we

*Many failed experiments supposedly designed to detect this anisotropy can be shown to have design flaws.

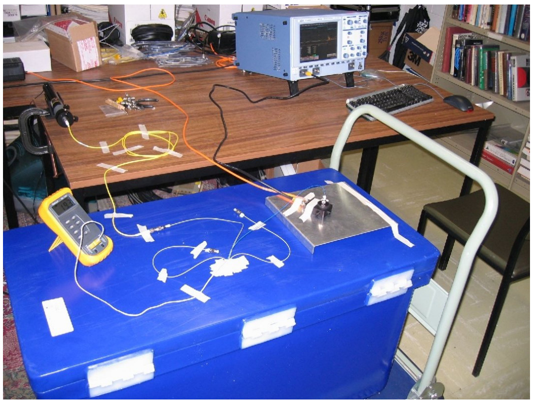

Fig. 3: (a) Detector 1 (D1) is located inside a sealed air-filled bucket inside an insulated container (blue) containing some 90 kg of water for temperature stabilisation. This detector, in the School of Chemistry, Physics and Earth Sciences, had an orientation of 5° anti-clockwise to the local meridian. Cylindrical He-Ne laser (Melles-Griot 0.5 mW 633 nm 05-LLR-811-230) is located on LHS of bench, while data logger is on RHS. Photodiode detector/pre-amplifier is located atop aluminium plate. (b) Detector 2 (D2) was located 1.1 km North of D1 in the Australian Science and Mathematics School. This detector had an orientation of 11° anti-clockwise to the local meridian. The data was logged on a PC running a PoScope USB DSO (PoLabs http://www.poscope.com).

Fig. 4: (a) Detectors are horizontally located inside an air-filled bucket. The plastic bag reduces even further any air movements, and thus temperature differentials. The blue crystals are silica gel to reduce moisture. (b) Bucket located inside and attached to bottom of the insulated container prior to adding water to the container.

characterise them at sub-mHz frequencies, showing that the fluctuations are very large, as also seen in [8].

3 Gravitational wave detectors

To measure $\mathbf{v}(\mathbf{r},t)$ has been difficult until now. The early experiments used gas-mode Michelson interferometers, which involved the visual observation of small fringe shifts as the relatively large devices were rotated. The RF coaxial cable experiments had the advantage of permitting electronic recording of the RF travel times, over 500m [6] and 1.5 km [7], by means of two or more atomic clocks, although the experiment reported in [8] used a novel technique that enable the coaxial cable length to be reduced to laboratory size*.

*The calibration of this technique is at present not well understood in view of recent discoveries concerning the Fresnel drag effect in optical fibers.

The new optical-fiber detector design herein has the advantage of electronic recording as well as high precision because the travel time differences in the two orthogonal fibers employ light interference effects, but with the interference effects taking place in an optical fiber beam-joiner, and so no optical projection problems arise. The device is very small, very cheap and easily assembled from readily available opto-electronic components. The schematic layout of the detector is given in Fig. 1, with a detailed description in the figure caption. The detector relies on the phenomenon where the 3-space velocity $\mathbf{v}(\mathbf{r},t)$ affects differently the light travel times in the optical fibers, depending on the projection of $\mathbf{v}(\mathbf{r},t)$ along the fiber directions. The differences in the light travel times are measured by means of the interference effects in the beam joiner. The difference in travel times is given by

$$\Delta t = k^2 \frac{L v_P^2}{c^3} \cos(2\theta), \tag{3}$$

where

$$k^2 = \frac{(n^2-1)(2-n^2)}{n}$$

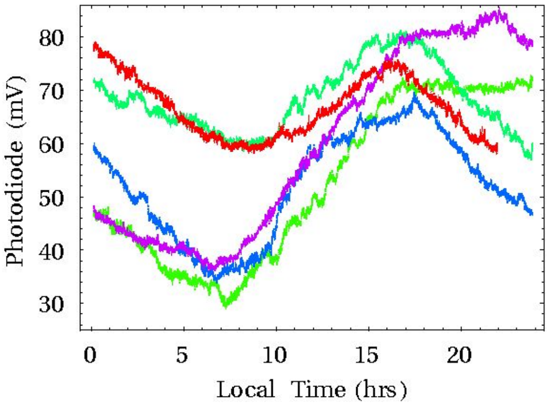

Fig. 5: D1 photodiode output voltage data (mV), recorded every 5 secs, from 5 successive days, starting September 22, 2007, plotted against local Adelaide time (UT = local time + 9.5 hrs). Day sequence is indicated by increasing hue. Dominant minima and maxima is earth rotation effect. Fluctuations from day to day are evident as are fluctuations during each day — these are caused by wave effects in the flowing space. Changes in RA cause changes in timing of min/max, while changes in magnitude are caused by changes in declination and/or speed. Blurring effect is caused by laser noise. Same data is plotted sequentially in Fig. 7a.

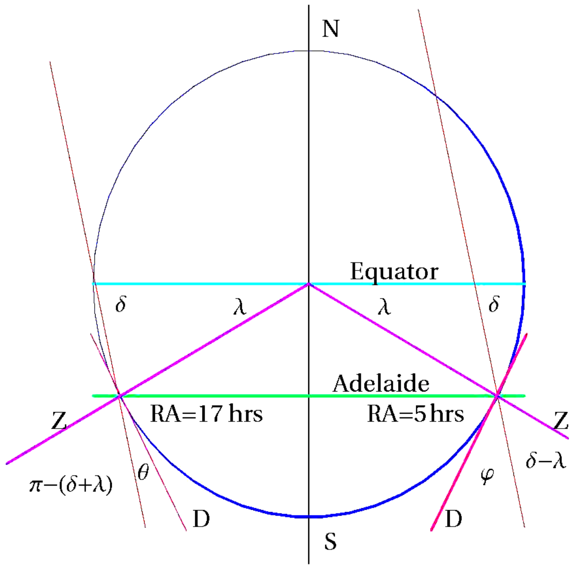

Fig. 6: Schematic of earth and spatial flow at approximate local sidereal times (RA) of 5 hrs and 17 hrs. The detector arms, D, of D1 and D2 are operated at small offset angles from the local meridian. The long straight lines indicate the spatial flow velocity vector, with declination δ. The large earth-rotation induced minima/maxima are caused by the inclination angle varying from a maximum ϕ to a minimum θ, respectively. Wave effects are changes in the velocity vector.

is the instrument calibration constant, obtained by taking account of the three key effects: (i) the different light paths, (ii) Lorentz contraction of the fibers, an effect depending on the angle of the fibers to the flow velocity, and (iii) the refractive index effect, including the Fresnel drag effect. Only if $n \neq 1$ is there a net effect, otherwise when $n = 1$ the various effects actually cancel. So in this regard the Michelson interferometer has a serious design flaw. This problem has been overcome by using optical fibers. Here $n = 1.462$ at 633 nm is the effective refractive index of the single-mode optical fibers (Fibercore SM600, temperature coefficient 5×10^{-2} fs/mm/C). Here $L \approx 200$ mm is the average effective length of the two arms, and $v_P(\mathbf{r}, t)$ is the projection of $\mathbf{v}(\mathbf{r}, t)$ onto the plane of the detector, and the angle θ is that of the projected velocity onto the arm.

The reality of the Lorentz contraction effect is experimentally confirmed by comparing the 2nd order in v/c Michelson gas-mode interferometer data, which requires account be taken of the contraction effect, with that from the 1st order in v/c RF coaxial cable travel time experiments, as in DeWitte [7], which does not require that the contraction effect be taken into account, to give comparable values for v.

For gas-mode Michelson interferometers $k^2 \approx n^2 - 1$, because then $n \approx 1^+$ is the refractive index of a gas. Operating in air, as for Michelson and Morley and for Miller, $n = 1.00029$, so that $k^2 = 0.00058$, which in particular means that the Michelson-Morley interferometer was nearly 2000 times less sensitive than assumed by Michelson, who used Newtonian physics to calibrate the interferometer — that analysis gives $k^2 = n^3 \approx 1$. Consequently the small fringe shifts observed by Michelson and Morley actually correspond to a light speed anisotropy of some 400 km/s, that is, the earth has that speed relative to the local dynamical 3-space. The dependence of k on n has been checked [11, 18] by comparing the air gas-mode data against data from the He gas-mode operated interferometers of Illingworth [3] and Joos [4].

The above analysis also has important implications for long-baseline terrestrial vacuum-mode Michelson interferometer gravitational wave detectors — they have a fundamental design flaw and will not be able to detect gravitational waves.

The interferometer operates by detecting changes in the travel time difference between the two arms, as given by (3). The cycle-averaged light intensity emerging from the beam joiner is given by

$$\begin{aligned} I(t) &\propto \left(\mathrm{Re}\left(\mathbf{E}_1 + \mathbf{E}_2 e^{i\omega(\tau+\Delta t)}\right)\right)^2 = \\ &= 2|\mathbf{E}|^2 \cos\left(\frac{\omega(\tau+\Delta t)}{2}\right)^2 \approx \\ &\approx a + b\Delta t\,. \end{aligned} \tag{4}$$

Here $\mathbf{E}_i$ are the electric field amplitudes and have the same value as the fiber splitter/joiner are 50%–50% types, and having the same direction because polarisation preserving fibers are used, ω is the light angular frequency and τ is a travel time difference caused by the light travel times not

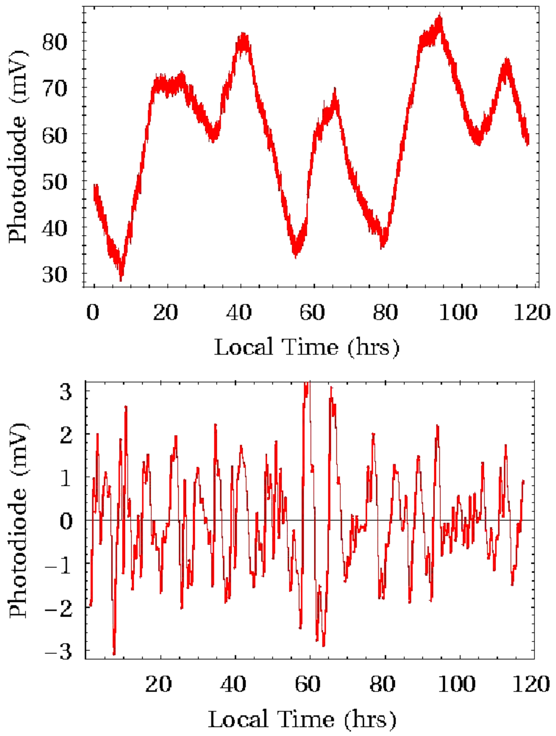

Fig. 7: (a) Plot of 5 days of data from Fig. 5 shown sequentially. The Fourier Transform of this data is shown in Fig. 12a. (b) Plot shows data after filtering out earth-rotation frequencies ($f < 0.025$ mHz) and laser noise frequencies ($f > 0.25$ mHz, $\text{Log}_{10}[0.25] = -0.6$). This shows wave/turbulence effects. Note that the magnitude of the wave component of the signal is some 10% of full signal in this bandwidth.

being identical, even when $\Delta t = 0$, mainly because the various splitter/joiner fibers will not be identical in length. The last expression follows because Δt is small, and so the detector operates, hopefully, in a linear regime, in general, unless τ has a value equal to modulo(T), where T is the light period. The main temperature effect in the detector, so long as a temperature uniformity is maintained, is that τ will be temperature dependent. The temperature coefficient for the optical fibers gives an effective fractional fringe shift error of $\Delta\tau/T = 3\times10^{-2}$/mm/C, for each mm of length difference. The photodiode detector output voltage $V(t)$ is proportional to $I(t)$, and so finally linearly related to Δt. The detector calibration constants a and b depend on k, τ and the laser intensity and are unknown at present.

4 Data analysis

The data is described in detail in the figure captions.

- Fig. 5 shows 5 typical days of data exhibiting the earth-rotation effect, and also fluctuations during each day and from day to day, revealing dynamical 3-space turbulence — essentially the long-sort-for gravitational waves. It is now known that these gravitational waves were first detected in the Michelson-Morley 1887 experiment [16], but only because their interferometer was operated in gas-mode. Fig. 12a shows the frequency spectrum for this data;
- Fig. 7b shows the gravitational waves after removing frequencies near the earth-rotation frequency. As discussed later these gravitational waves are predominately sub-mHz;
- Fig. 8 reports one of a number of key experimental tests of the detector principles. These show the two detector responses when (a) operating from the same laser source, and (b) with only D2 operating in interferometer mode. These reveal the noise effects coming from the laser in comparison with the interferometer signal strength. This gives a guide to the S/N ratio of these detectors;
- Fig. 9 shows two further key tests: 1st the time delay effect in the earth-rotation induced minimum caused by the detectors not being aligned NS. The time delay difference has the value expected. The 2nd effect is that wave effects are simultaneous, in contrast to the 1st effect. This is the first coincidence detection of gravitational waves by spatially separated detectors. Soon the separation will be extended to much larger distances;
- Figs. 10 and 11 show the data and calibration curves for the timing of the daily earth-rotation induced minima and maxima over an 80 day period. Because D1 is orientated away from the NS these times permit the determination of the Declination (Dec) and Right Ascension (RA) from the two running averages. That the running averages change over these 80 days reflects three causes (i) the sidereal time effect, namely that the 3-space velocity vector is related to the positioning of the galaxy, and not the Sun, (ii) that a smaller component is related to the orbital motion of the earth about the Sun, and (iii) very low frequency wave effects. This analysis gives the changing Dec and RA shown in Fig. 12b, giving results which are within 13° of the 1925/26 Miller results, and for the RA from the DeWitte RF coaxial cable results. Figs. 10a and 11a also show the turbulence/wave effects, namely deviations from the running averages;
- Fig. 12a shows the frequency analysis of the data. The fourier amplitudes, which can be related to the strain $h = v^2/2c^2$, decrease as f^a where the strain spectral index has the value $a = -1.4 \pm 0.1$, after we allow for the laser noise.

5 Conclusions

Sub-mHz gravitational waves have been detected and partially characterised using the optical-fiber version of a Michelson interferometer. The waves are relatively large and were

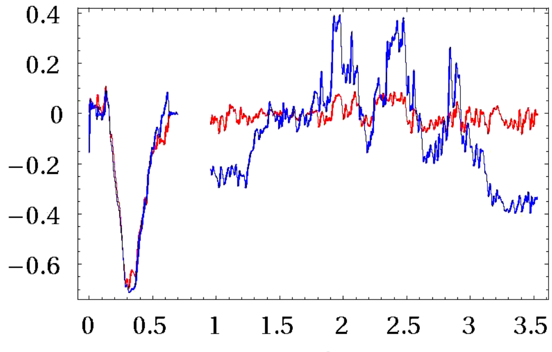

Fig. 8: Two tests of the detectors. (a) The left plot shows data from D1 and D2 when co-located, parallel and operating from the same laser. The data from one has been rescaled to match the data from the other, as they have different calibrations. Both detectors show a simultaneous gravitational wave pulse of duration ≈ 0.5 hrs. (b) The right plot shows data from D2 (blue) and from a direct feed of the common laser source to the photodiode detector of D1 (red), i.e bypassing the D1 interferometer. This data has been rescaled so that high frequency components have the same magnitude, to compensate for different feed amplitudes. The laser-only signal (red) shows the amplitude and frequency noise level of the laser. The signal from D2 (blue) shows the same noise effects, but with additional larger variations — these are wave effects detected by D2 operating in interferometer mode. This data shows that the laser noise is dominant above approximately 1 mHz.

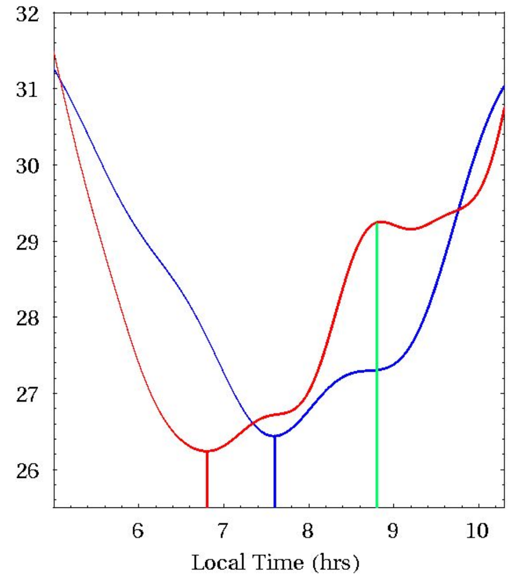

Fig. 9: Photodiode data (mV) on October 4, 2007, from detectors D1 (red plot) and D2 (blue plot) operating simultaneously with D2 located 1.1 km due north of D1. A low-pass FFT filter ($f \leqslant 0.25$ mHz, $\mathrm{Log}_{10}[f(\mathrm{mHz})] \leqslant -0.6$) was used to remove laser noise. D1 arm is aligned 5° anti-clockwise from local meridian, while D2 is aligned 11° anti-clockwise from local meridian. The alig nment offset between D1 and D2 causes the dominant earth-rotation induced minima to occur at *different* times, with that of D2 at $t = 7.6$ hrs *delayed* by 0.8 hrs relative to D1 at $t = 6.8$ hrs, as expected from Figs.10b and 11b for Dec $= 77°$. This is a fundamental test of the detection theory and of the phenomena. As well the data shows a *simultaneous* sub-mHz gravitational wave correlation at $t \approx 8.8$ hrs and of duration ≈ 1 hr. This is the first observed correlation for spatially separated gravitational wave detectors. Two other wave effects (at $t \approx 6.5$ hrs in D2 and $t \approx 7.3$ hrs in D1) seen in one detector are masked by the stronger earth-rotation induced minimum in the other detector.

first detected, though not recognised as such, by Michelson and Morley in 1887. Since then another 6 experiments [2,6–9], including herein, have confirmed the existence of this phenomenon. Significantly three different experimental techniques have been employed, all giving consistent results. In contrast vacuum-mode Michelson interferometers, with mechanical mirror support arms, cannot detect this phenomenon due to a design flaw. A complete characterisation of the waves requires that the optical-fiber detector be calibrated for speed, which means determining the parameter b in (4). Then it will be possible to extract the wave component of $\mathbf{v}(\mathbf{r}, t)$ from the average, and so identify the cause of the turbulence/wave effects. A likely candidate is the in-flow of 3-space into the Milky Way central super-massive black hole — this in-flow is responsible for the high, non-Keplerian, rotation speeds of stars in the galaxy.

The detection of the earth-rotation, earth-orbit and gravitational waves, and over a long period of history, demonstrate that the spacetime formalism of Special Relativity has been very misleading, and that the original Lorentz formalism is the appropriate one; in this the speed of light is not an invariant for all observers, and the Lorentz-Fitzgerald length contraction and the Lamor time dilation are real physical effects on rods and clocks in motion through the dynamical 3-space, whereas in the Einstein formalism they are transferred and attributed to a perspective effect of spacetime, which we now recognise as having no ontological significance — merely a mathematical construct, and in which the invariance of the speed of light is definitional — not observational.

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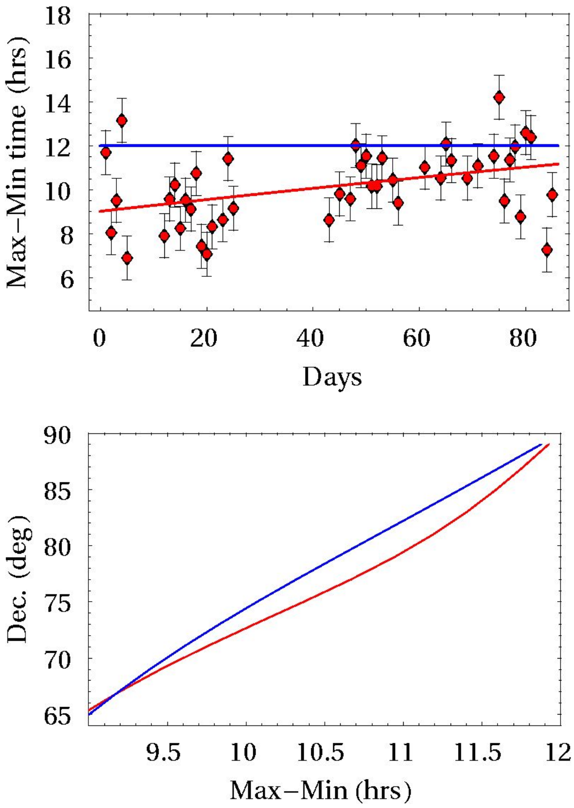


Fig. 10: (a) Time differences between maxima and minima each day from D1, from September 22 to December 16, 2007. Some days are absent due to data logger malfunction. The red curve shows a quadratic best-fit running average. If the detector arm was orientated along the meridian and there were no wave effects then one would see the horizontal line (blue) at 12 hrs. The data shows, however, that the running average has a time varying measure, from 9 hrs to 11 hrs over these days, caused by the orbital motion of the earth about the sun. Wave-effect fluctuations from day to day are also evident. This data in conjunction with the calibration curve in (b) permits a determination of the approximate Declination each day, which is used in the plot shown in Fig. 12b. (b) Declination calibration curve for D1 (red) and D2 (blue). From the orientation of the detector, with an offset angle of 5° for D1 anti-clockwise from the local meridian and 11° for D2 anti-clockwise from the local meridian, and the latitude of Adelaide, the offset angle causes the time duration between a minimum and a maximum to be different from 12 hrs, ignoring wave effects. In conjunction with the running average in Fig. 10a an approximate determination of the Declination on each day may be made without needing to also determine the RA and speed.

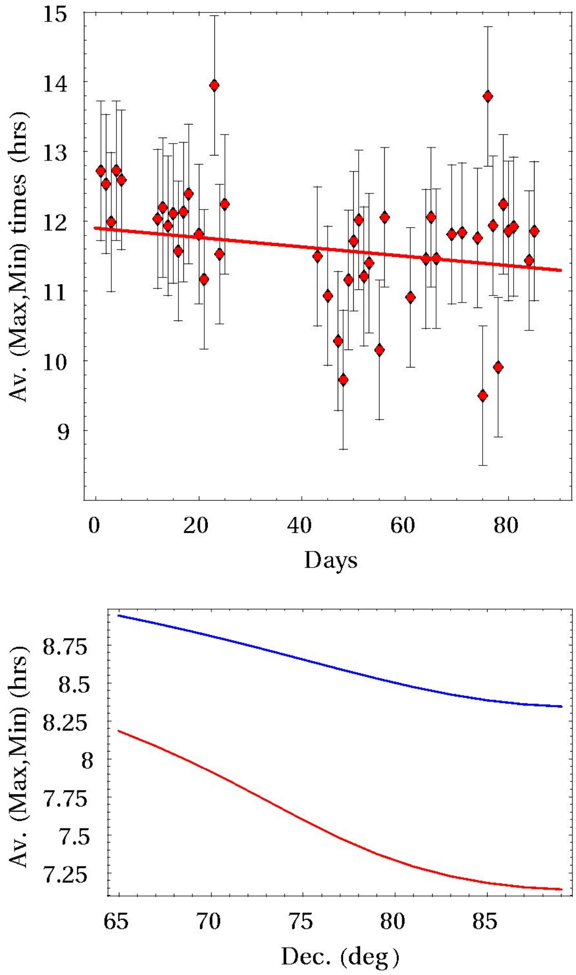
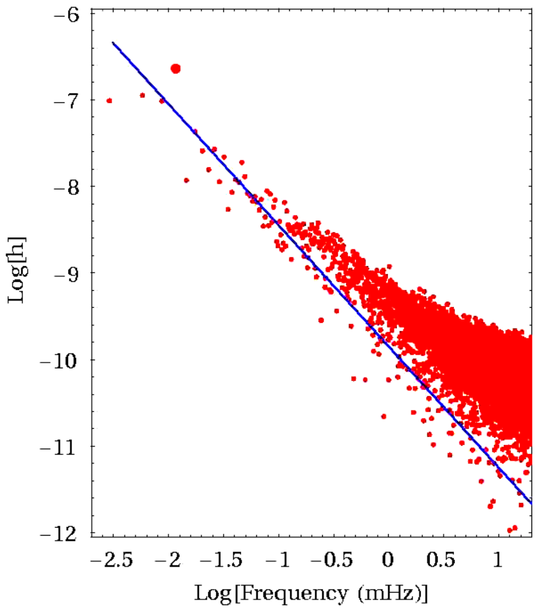


Fig. 11: (a) Time of average of minimum and maximum for each day. The linear best-fit line shows the trend line. Wave effects are again very evident. The decreasing trend line is cause by a combination of the sidereal effect, the earth orbit effect and very low frequency waves. This data may be used to determine the approximate RA for each day. However a correction must be applied as the arm offset angle affects the determination. (b) RA calibration curve for D1 (red) and D2 (blue). The detector offset angles cause the timing of the mid-point between the minima and maxima, ignoring wave effects, to be delayed in time, beyond the 6 hrs if detector were aligned NS. So, for example, if the Declination is found to be 70°, then this calibration curve gives 8 hrs for D1. This 8 hrs is then subtracted from the time in (a) to give the approximate true local time for the minimum to occur, which then permits computation of the RA for that day.



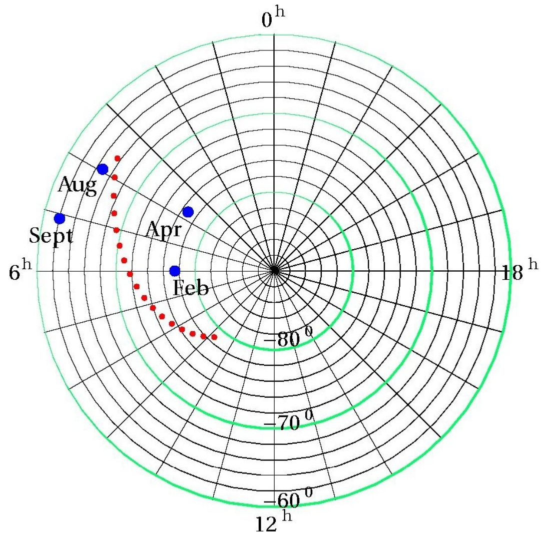


Fig. 12: (a) Log-Log plot of the frequency spectrum $|h(f)|$ of the data from the five days shown in Fig. 7a. $h(f)$ is the strain $v^2/2c^2$ at frequency f, normalised to $v = 400$ km/s at the 24 hr frequency. The largest component (large red point) is the 24 hr earth rotation frequency. The straight line (blue) is a trend line that suggests that the signal has two components — one indicated by the trend line having the form $|h(f)| \propto f^a$ with strain spectral index $a = -1.4 \pm 0.1$, while the second component, evident above 1 mHz, is noise from the laser source, as also indicated by the data in Fig. 8. (b) Southern celestial sphere with RA and Dec shown. The 4 blue points show the results from Miller [2] for four months in 1925/1926. The sequence of red points show the daily averaged RA and Dec as determined from the data herein for every 5 days. The 2007 data shows a direction that moves closer to the south celestial pole in late December than would be indicated by the Miller data. The new results differ by 10° to 13° from the corresponding Miller data points (the plot exaggerates angles). The wave effects cause the actual direction to fluctuate from day to day and during each day.

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Derivation of Maxwell's Equations Based on a Continuum Mechanical Model of Vacuum and a Singularity Model of Electric Charges

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The main purpose of this paper is to seek a mechanical interpretation of electromagnetic phenomena. We suppose that vacuum is filled with a kind of continuously distributed material which may be called $\Omega(1)$ substratum. Further, we speculate that the $\Omega(1)$ substratum might behave like a fluid with respect to translational motion of large bodies through it, but would still posses elasticity to produce small transverse vibrations. Thus, we propose a visco-elastic constitutive relation of the $\Omega(1)$ substratum. Furthermore, we speculate that electric charges are emitting or absorbing the $\Omega(1)$ substratum continuously and establish a fluidic source and sink model of electric charges. Thus, Maxwell's equations in vacuum are derived by methods of continuum mechanics based on this mechanical model of vacuum and the singularity model of electric charges.

1 Introduction

Maxwell's equations in vacuum can be written as [1]

$$\nabla \cdot \mathbf{E} = \frac{\rho_e}{\epsilon_0}, \tag{1}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{2}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{3}$$

$$\frac{1}{\mu_0} \nabla \times \mathbf{B} = \mathbf{j} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}, \tag{4}$$

where $\mathbf{E}$ is the electric field vector, $\mathbf{B}$ is the magnetic induction vector, ρ_e is the density field of electric charges, $\mathbf{j}$ is the electric current density, ϵ_0 is the dielectric constant of vacuum, μ_0 is magnetic permeability of vacuum, t is time, $\nabla = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}$ is the Hamilton operator.

The main purpose of this paper is to derive the aforementioned Maxwell equations in vacuum based on a continuum mechanics model of vacuum and a singularity model of electric charges.

The motivation for this paper was looking for a mechanism of electromagnetic phenomena. The reasons why new mechanical models of electromagnetic fields are interesting may be summarized as follows.

First, there exists various electromagnetic phenomena which could not be interpreted by the present theories of electromagnetic fields, e.g., the spin of an electron [1, 2], the Aharonov-Bohm effect [3, 4], etc. New theories of of electromagnetic phenomena may consider these problems from new sides.

Second, there exists some inconsistencies and inner difficulties in Classical Electrodynamics, e.g., the inadequacy of the Liéenard-Wiechert potentials [5–7]. New theories of electromagnetic phenomena may overcome such difficulties.

Third, there exists some divergence problems in Quantum Electrodynamics [8]. By Dirac's words, "I must say that I am very dissatisfied with the situation, because this so-called good theory does involve neglecting infinities which appear in its equations, neglecting them in an arbitrary way. This is just not sensible mathematics". New theories of electromagnetic phenomena may open new ways to resolve such problems.

Fourth, since the quantum theory shows that vacuum is not empty and produces physical effects, e.g., the Casimir effect [9–12], it is valuable to reexamine the old concept of electromagnetic aether.

Fifth, from the viewpoint of reductionism, Maxwell's theory of electromagnetic fields can only be regarded as a phenomenological theory. Although Maxwell's theory is a field theory, the field concept is different from that of continuum mechenics [13–16] due to the absence of a medium. Thus, from the viewpoint of reductionism, the mechanism of electromagnetic phenomena is still remaining an unsolved problem of physics [17].

Sixth, one of the puzzles of physics is the problem of dark matter and dark energy (refer to, for instance, [18–26]). New theories of electromagnetic phenomena may provide new ideas to attack this problem.

Finally, one of the tasks of physics is the unification of the four fundamental interactions in the Universe. New theories of electromagnetic phenomena may shed some light on this puzzle.

To conclude, it seems that new considerations for electromagnetic phenomena is needed. It is worthy keeping an open mind with respect to all the theories of electromagnetic phenomena before the above problems been solved.

Now let us briefly review the long history of the mechanical interpretations of electromagnetic phenomena.

According to E. T. Whittaker [17], Descartes was the first person who brought the concept of aether into science by suggested mechanical properties to it. Descartes believed that every physical phenomenon could be interpreted in the framework of a mechanical model of the Universe. William Watson and Benjamin Franklin (independently) constructed the one-fluid theory of electricity in 1746 [17]. H. Cavendish attempted to explain some of the principal phenomena of electricity by means of an elastic fluid in 1771 [17]. Not contented with the above mentioned one-fluid theory of electricity, du Fay, Robert Symmer and C. A. Coulomb developed a two-fluid theory of electricity from 1733 to 1789 [17].

Before the unification of both electromagnetic and light phenomena by Maxwell in 1860's, light phenomena were independent studied on the basis of Descartes' views for the mechanical origin of Nature. John Bernoulli introduced a fluidic aether theory of light in 1752 [17]. Euler believed in an idea that all electrical phenomena are caused by the same aether that moves light. Furthermore, Euler attempted to explain gravity in terms of his single fluidic aether [17].

In 1821, in order to explain polarisation of light, A. J. Frensnel proposed an aether model which is able to transmit transverse waves. After the advent of Frensnel's successful transverse wave theory of light, the imponderable fluid theories were abandoned. In the 19th century, Frensnel's dynamical theory of a luminiferous aether had an important influence on the mechanical theories of Nature [17]. Inspired by Frensnel's luminiferous aether theory, numerous dynamical theories of elastic solid aether were established by Stokes, Cauchy, Green, MacCullagh, Boussinesq, Riemann and William Thomson. (See, for instance, [17]).

Thomson's analogies between electrical phenomena and elasticity helped to James Clark Maxwell to establish a mechanical model of electrical phenomena [17]. Strongly impressed by Faraday's theory of lines of forces, Maxwell compared the Faraday lines of forces with the lines of flow of a fluid. In 1861, in order to obtain a mechanical interpretation of electromagnetic phenomena, Maxwell established a mechanical model of a magneto-electric medium. The Maxwell magneto-electric medium is a cellular aether, looks like a honeycomb. Each cell of the aether consists of a molecular vortex surrounded by a layer of idle-wheel particles. In a remarkable paper published in 1864, Maxwell established a group of equations, which were named after his name later, to describe the electromagnetic phenomena.

In 1878, G. F. FitzGerald compared the magnetic force with the velocity in a quasi-elastic solid of the type first suggested by MacCullagh [17]. FitzGerald's mechanical model of such an electromagnetic aether was studied by A. Sommerfeld, by R. Reiff and by Sir J. Larmor later [17].

Because of some dissatisfactions with the mechanical models of an electromagnetic aether and the success of the theory of electromagnetic fields, the mechanical world-view was removed with the electromagnetic world-view gradually. Therefore, the concepts of a luminiferous aether and an elastic solid aether were removed with the concepts of an electromagnetic aether or an electromagnetic field. This paradigm shift in scientific research was attributed to many scientists, including Faraday, Maxwell, Sir J. Larmor, H. A. Lorentz, J. J. Thomson, H. R. Hertz, Ludwig Lorenz, Emil Wiechert, Paul Drude, Wilhelm Wien, etc. (See, for instance, [17].)

In a remarkable paper published in 1905, Einstein abandoned the concept of aether [27]. However, Einstein's assertion did not cease the exploration of aether (refer to, for instance, [17,28–37,68,69]). Einstein changed his attitude later and introduced his new concept of aether [38, 39]. In 1979, A. A. Golebiewska-Lasta observed the similarity between the electromagnetic field and the linear dislocation field [28]. V. P. Dmitriyev have studied the similarity between the electromagnetism and linear elasticity since 1992 [32, 35, 37, 40]. In 1998, H. Marmanis established a new theory of turbulence based on the analogy between electromagnetism and turbulent hydrodynamics [34]. In 1998, D. J. Larson derived Maxwell's equations from a simple two-component solid-mechanical aether [33]. In 2001, D. Zareski gave an elastic interpretation of electrodynamics [36]. I regret to admit that it is impossible for me to mention all the works related to this field of history.

A. Martin and R. Keys [41–43] proposed a fluidic cosmonic gas model of vacuum in order to explain the physical phenomena such as electromagnetism, gravitation, Quantum Mechanics and the structure of elementary particles.

Inspired by the above mentioned works, we show that Maxwell's equations of electromagnetic field can be derived based on a continuum mechanics model of vacuum and a singularity model of electric charges.

2 Clues obtained from dimensional analysis

According to Descartes' scientific research program, which is based on his views about the mechanical origin of Nature, electromagnetic phenomena must be (and can be) interpreted on the basis of the mechanical motions of the particles of aether.

Therefore, all the physical quantities appearing in the theory of electromagnetic field should be mechanical quantities.

Thus, in order to construct a successful mechanical model of electromagnetic fields, we should undertake a careful dimensional analysis (refer to, for instance, [44]) for physical quantities in the theory of electromagnetism (for instance, electric field vector $\mathbf{E}$, magnetic induction vector $\mathbf{B}$, the density field of electric charges ρ_e, the dielectric constant of vacuum ϵ_0, the magnetic permeability of vacuum μ_0, etc.).

It is known that Maxwell's equations (1-4) in vacuum can also be expressed as [1]

$$\nabla^2\phi + \frac{\partial}{\partial t}(\nabla\cdot\mathbf{A}) = -\frac{\rho_e}{\epsilon_0}\,, \tag{5}$$

$$\nabla^2\mathbf{A} - \nabla(\nabla\cdot\mathbf{A}) - \mu_0\epsilon_0\frac{\partial}{\partial t}\left(\nabla\phi - \frac{\partial\mathbf{A}}{\partial t}\right) = -\mathbf{j}\,, \tag{6}$$

where ϕ is the scalar electromagnetic potential, **A** is the vector electromagnetic potential, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the Laplace operator.

In 1846, W. Thomson compared electric phenomena with elasticity. He pointed out that the elastic displacement **u** of an incompressible elastic solid is a possible analogy to the vector electromagnetic potential **A** [17].

Noticing the similarity between the Eq. (6) and the equation (39) of momentum conservation of elastic solids, it is natural to judge that vacuum is filled with a kind of elastic substratum. Further, we may say that the dimension of the electromagnetic vector potential **A** of such an elastic substratum is the same that of the displacement vector **u** of an elastic solid. Thus, the dimension of the vector electromagnetic potential **A** of the elastic substratum is $[L^0 M^0 T^0]$, where L, M and T stands for the dimensions of length, mass, and time, respectively. Therefore, we can determine the dimensions of the rest physical quantities of the theory of electromagnetism, for instance, the electric field vector **E**, the magnetic induction vector **B**, the electric charge q_e, the dielectric constant of vacuum ϵ_0, the magnetic permeability of vacuum μ_0, etc. For instance, the dimension of an electric charge q_e should be $[L^0 M^1 T^{-1}]$.

Inspired by this clue, we are going to produce, in the next Sections, an investigation in this direction.

3 A visco-elastic continuum model of vacuum

The purpose of this Section is to establish a visco-elastic continuum mechanical model of vacuum.

In 1845–1862, Stokes suggested that aether might behave like a glue-water jelly [45–47]. He believed that such an aether would act like a fluid on the transit motion of large bodies through it, but would still possessing elasticity to produce a small transverse vibration.

Following Stokes, we propose a visco-elastic continuum model of vacuum.

Assumption 1. *Suppose that vacuum is filled with a kind of continuously distributed material.*

In order to distinguish this material with other substratums, we may call this material as $\Omega(1)$ substratum, for convenience. Further, we may call the particles that constitute the $\Omega(1)$ substratum as $\Omega(1)$ particles (for convenience).

In order to construct a continuum mechanical theory of the $\Omega(1)$ substratum, we should take some assumptions based on the experimental data about the macroscopic behavior of vacuum.

Assumption 2. *We suppose that all the mechanical quantities of the $\Omega(1)$ substratum under consideration, such as the density, displacements, strains, stresses, etc., are piecewise continuous functions of space and time. Furthermore, we suppose that the material points of the $\Omega(1)$ substratum remain be in one-to-one correspondence with the material points before a deformation appears.*

Assumption 3. *We suppose that the material of the $\Omega(1)$ substratum under consideration is homogeneous, that is $\frac{\partial \rho}{\partial x} = \frac{\partial \rho}{\partial y} = \frac{\partial \rho}{\partial z} = \frac{\partial \rho}{\partial t} = 0$, where ρ is the density of the $\Omega(1)$ substratum.*

Assumption 4. *Suppose that the deformation processes of the $\Omega(1)$ substratum are isothermal. So we neglect the thermal effects.*

Assumption 5. *Suppose that the deformation processes are not influenced by the gradient of the stress tensor.*

Assumption 6. *We suppose that the material of the $\Omega(1)$ substratum under consideration is isotropic.*

Assumption 7. *We suppose that the deformaton of the $\Omega(1)$ substratum under consideration is small.*

Assumption 8. *We suppose that there are no initial stress and strain in the body under consideration.*

When the $\Omega(1)$ substratum is subjected to a set of external forces, the relative positions of the $\Omega(1)$ particles form the body displacement.

In order to describe the deformation of the $\Omega(1)$ substratum, let us introduce a Cartesian coordinate system $\{o, x, y, z\}$ or $\{o, x_1, x_2, x_3\}$ which is static relative to the $\Omega(1)$ substratum. Now we may introduce a definition to the displacement vector **u** of every point in the $\Omega(1)$ substratum:

$$\mathbf{u} = \mathbf{r} - \mathbf{r}_0, \tag{7}$$

where $\mathbf{r}_0$ is the position of the point before the deformation, while **r** is the position after the deformation.

The displacement vector may be written as $\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j} + u_3\mathbf{k}$ or $\mathbf{u} = u\mathbf{i} + v\mathbf{j} + w\mathbf{k}$, where **i**, **j**, **k** are three unit vectors directed along the coordinate axes.

The gradient of the displacement vector **u** is the relative displacement tensor $u_{i,j} = \frac{\partial u_i}{\partial x_j}$.

We decompose the tensor $u_{i,j}$ into two parts, the symmetric ε_{ij} and the skew-symmetric Ω_{ij} (refer to, for instance, [14, 48, 49])

$$u_{i,j} = \frac{1}{2}(u_{i,j} + u_{j,i}) + \frac{1}{2}(u_{i,j} - u_{j,i}) = \varepsilon_{ij} + \Omega_{ij}\,, \tag{8}$$

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})\,, \qquad \Omega_{ij} = \frac{1}{2}(u_{i,j} - u_{j,i})\,. \tag{9}$$

The symmetric tensor ε_{ij} manifests a pure deformation of the body at a point, and is known the strain tensor (refer to, for instance, [14,48,49]). The matrix form and the component notation of the strain tensor ε_{ij} are

$$\varepsilon_{ij} = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) & \frac{\partial v}{\partial y} & \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \\ \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) & \frac{1}{2}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) & \frac{\partial w}{\partial z} \end{pmatrix}, \tag{10}$$

$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{pmatrix} . \tag{11}$$

The strain-displacements equations come from Eq. (10)

$$\left.\begin{array}{ll} \varepsilon_{11} = \dfrac{\partial u}{\partial x}\,, & \varepsilon_{12} = \varepsilon_{21} = \dfrac{1}{2}\left(\dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x}\right) \\ \varepsilon_{22} = \dfrac{\partial v}{\partial y}\,, & \varepsilon_{23} = \varepsilon_{32} = \dfrac{1}{2}\left(\dfrac{\partial v}{\partial z} + \dfrac{\partial w}{\partial y}\right) \\ \varepsilon_{33} = \dfrac{\partial w}{\partial z}\,, & \varepsilon_{31} = \varepsilon_{13} = \dfrac{1}{2}\left(\dfrac{\partial w}{\partial x} + \dfrac{\partial u}{\partial z}\right) \end{array}\right\} . \tag{12}$$

For convenience, we introduce the definitions of the mean strain deviator ε_m and the strain deviator e_{ij} as

$$\varepsilon_m = \frac{1}{3}\left(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}\right), \tag{13}$$

$$e_{ij} = \varepsilon_{ij} - \varepsilon_m = \begin{pmatrix} \varepsilon_{11}-\varepsilon_m & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22}-\varepsilon_m & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33}-\varepsilon_m \end{pmatrix} . \tag{14}$$

When the $\Omega(1)$ substratum deforms, the internal forces arise due to the deformation. The component notation of the stress tensor σ_{ij} is

$$\sigma_{ij} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix} . \tag{15}$$

For convenience, we introduce the definitions of mean stress σ_m and stress deviator s_{ij} as

$$\sigma_m = \frac{1}{3}\left(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}\right), \tag{16}$$

$$s_{ij} = \sigma_{ij} - \sigma_m = \begin{pmatrix} \sigma_{11}-\sigma_m & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22}-\sigma_m & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33}-\sigma_m \end{pmatrix} . \tag{17}$$

Now let us turn to study the constitutive relation.

An elastic Hooke solid responds instantaneously with respect to an external stress. A Newtonian viscous fluid responds to a shear stress by a steady flow process.

In 19th century, people began to point out that fact that some materials showed a time dependence in their elastic response with respect to external stresses. When a material like pitch, gum rubber, polymeric materials, hardened cement and even glass, is loaded, an instantaneous elastic deformation follows with a slow continuous flow or creep.

Now this time-dependent response is known as viscoelasticity (refer to, for instance, [50–52]). Materials bearing both instantaneous elastic elasticity and creep characteristics are known as viscoelastic materials [51, 52]. Viscoelastic materials were studied long time ago by Maxwell [51–53], Kelvin, Voigt, Boltzamann [51, 52, 54], etc.

Inspired by these contributors, we propose a visco-elastic constitutive relation of the $\Omega(1)$ substratum.

It is natural to say that the constitutive relation of the $\Omega(1)$ substratum may be a combination of the constitutive relations of the Hooke-solid and the Newtonian-fluid.

For the Hooke-solid, we have the generalized Hooke law as follows (refer to, for instance, [14, 48, 49, 55]),

$$\sigma_{ij} = 2G\varepsilon_{ij} + \lambda\theta\delta_{ij}\,, \quad \varepsilon_{ij} = \frac{\sigma_{ij}}{2G} - \frac{3\nu}{Y}\,\sigma_m\delta_{ij}\,, \tag{18}$$

where δ_{ij} is the Kronecker symbol, σ_m is the mean stress, where Y is the Yang modulus, ν is the Poisson ratio, G is the shear modulus, λ is Lamé constant, θ is the volume change coefficient. The definition of θ is $\theta = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$.

The generalized Hooke law Eq. (18) can also be written as [55]

$$s_{ij} = 2G\,e_{ij}\,, \tag{19}$$

where s_{ij} is the stress deviator, e_{ij} is the strain deviator.

For the Newtonian-fluid, we have the following constitutive relation

$$\frac{de_{ij}}{dt} = \frac{1}{2\eta}\,s_{ij}\,, \tag{20}$$

where s_{ij} is the stress deviator, $\frac{de_{ij}}{dt}$ is the strain rate deviator, η is the dynamic viscocity.

The $\Omega(1)$ substratum behaves like the Hooke-solid during very short duration. We therefore differentiate both sides of Eq. (19), then obtain

$$\frac{de_{ij}}{dt} = \frac{1}{2G}\frac{ds_{ij}}{dt}\,. \tag{21}$$

A combination of Eq. (21) and Eq. (20) gives

$$\frac{de_{ij}}{dt} = \frac{1}{2\eta}\,s_{ij} + \frac{1}{2G}\frac{ds_{ij}}{dt}\,. \tag{22}$$

We call the materials behaving like Eq. (22) "Maxwell-liquid" since Maxwell established such a constitutive relation in 1868 (refer to, for instance, [50–53]).

Eq. (22) is valid only in the case of infinitesimal deformation because the presence of the derivative with respect to time. Oldroyd recognized that we need a special definition for the operation of derivation, in order to satisfy the principle of material frame indifference or objectivity [51, 56]. Unfortunately, there is no unique definition of such a differential operation fulfil the principle of objectivity presently [51].

As an enlightening example, let us recall the description [50] for a simple shear experiment. We suppose

$$\frac{d\sigma_t}{dt} = \frac{\partial\sigma_t}{\partial t}\,, \qquad \frac{de_t}{dt} = \frac{\partial e_t}{\partial t}\,, \tag{23}$$

where σ_t is the shear stress, e_t is the shear strain.

Therefore, Eq. (22) becomes

$$\frac{\partial e_t}{\partial t} = \frac{1}{2\eta}\,\sigma_t + \frac{1}{2G}\frac{\partial\sigma_t}{\partial t}\,. \tag{24}$$

Integration of Eq. (24) gives

$$\sigma_t = e^{-\frac{G}{\eta}t}\left(\sigma_0 + 2G\int_0^t \frac{de_t}{dt}\, e^{\frac{G}{\eta}}\, dt\right). \tag{25}$$

If the shear deformation is kept constant, i.e. $\frac{\partial e_t}{\partial t} = 0$, we have

$$\sigma_t = \sigma_0\, e^{-\frac{G}{\eta}t}. \tag{26}$$

Eq. (26) shows that the shear stresses remain in the Maxwell-liquid and are damped in the course of time.

We see that $\frac{\eta}{G}$ must have the dimension of time. Now let us introduce the following definition of Maxwellian relaxation time τ

$$\tau = \frac{\eta}{G}. \tag{27}$$

Therefore, using Eq. (27), Eq. (22) becomes

$$\frac{s_{ij}}{\tau} + \frac{ds_{ij}}{dt} = 2G\,\frac{de_{ij}}{dt}. \tag{28}$$

Now let us introduce the following hypothesis

Assumption 9. *Suppose the constitutive relation of the* $\Omega(1)$ *substratum satisfies Eq. (22).*

Now we can derive the the equation of momentum conservation based on the above hypotheses 9.

Let T be a characteristic time scale of an observer of the $\Omega(1)$ substratum. When the observer's time scale T is of the same order that the period of the wave motion of light, the Maxwellian relaxation time τ is a comparigly large number. Thus, the first term of Eq. (28) may be neglected. Therefore, the observer concludes that the strain and the stress of the $\Omega(1)$ substratum satisfy the generalized Hooke law.

The generalized Hooke law (18) can also be written as [14, 55]

$$\left.\begin{aligned}
\sigma_{11} &= \lambda\theta + 2G\varepsilon_{11}\\
\sigma_{22} &= \lambda\theta + 2G\varepsilon_{22}\\
\sigma_{33} &= \lambda\theta + 2G\varepsilon_{33}\\
\sigma_{12} &= \sigma_{21} = 2G\varepsilon_{12} = 2G\varepsilon_{21}\\
\sigma_{23} &= \sigma_{32} = 2G\varepsilon_{23} = 2G\varepsilon_{32}\\
\sigma_{31} &= \sigma_{13} = 2G\varepsilon_{31} = 2G\varepsilon_{13}
\end{aligned}\right\}, \tag{29}$$

where $\lambda = \frac{Y\nu}{(1+\nu)(1-2\nu)}$ is Lamé constant, θ is the volume change coefficient. By its definition, $\theta = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$.

The following relationship are useful

$$G = \frac{Y}{2(1+\nu)}, \qquad K = \frac{Y}{3(1-2\nu)}, \tag{30}$$

where K is the volume modulus.

It is known that the equations of the momentum conservation are (refer to, for instance, [14, 48, 49, 55, 57, 58]),

$$\frac{\partial\sigma_{11}}{\partial x} + \frac{\partial\sigma_{12}}{\partial y} + \frac{\partial\sigma_{13}}{\partial z} + f_x = \rho\,\frac{\partial^2 u}{\partial t^2}, \tag{31}$$

$$\frac{\partial\sigma_{21}}{\partial x} + \frac{\partial\sigma_{22}}{\partial y} + \frac{\partial\sigma_{23}}{\partial z} + f_y = \rho\,\frac{\partial^2 v}{\partial t^2}, \tag{32}$$

$$\frac{\partial\sigma_{31}}{\partial x} + \frac{\partial\sigma_{32}}{\partial y} + \frac{\partial\sigma_{33}}{\partial z} + f_z = \rho\,\frac{\partial^2 w}{\partial t^2}, \tag{33}$$

where f_x, f_y and f_z are three components of the volume force density $\mathbf{f}$ exerted on the $\Omega(1)$ substratum.

The tensor form of the equations (31-33) of the momentum conservation can be written as

$$\sigma_{ij,j} + f_i = \rho\,\frac{\partial^2 u_i}{\partial t^2}. \tag{34}$$

Noticing Eq. (29), we write Eqs. (31-33) as

$$2G\left(\frac{\partial\varepsilon_{11}}{\partial x} + \frac{\partial\varepsilon_{12}}{\partial y} + \frac{\partial\varepsilon_{13}}{\partial z}\right) + \lambda\frac{\partial\theta}{\partial x} + f_x = \rho\,\frac{\partial^2 u}{\partial t^2}, \tag{35}$$

$$2G\left(\frac{\partial\varepsilon_{21}}{\partial x} + \frac{\partial\varepsilon_{22}}{\partial y} + \frac{\partial\varepsilon_{23}}{\partial z}\right) + \lambda\frac{\partial\theta}{\partial y} + f_y = \rho\,\frac{\partial^2 v}{\partial t^2}, \tag{36}$$

$$2G\left(\frac{\partial\varepsilon_{31}}{\partial x} + \frac{\partial\varepsilon_{32}}{\partial y} + \frac{\partial\varepsilon_{33}}{\partial z}\right) + \lambda\frac{\partial\theta}{\partial z} + f_z = \rho\,\frac{\partial^2 w}{\partial t^2}. \tag{37}$$

Using Eq. (12), Eqs. (35-37) can also be expressed by means of the displacement $\mathbf{u}$

$$\left.\begin{aligned}
G\nabla^2 u + (G+\lambda)\,\tfrac{\partial}{\partial x}\left(\tfrac{\partial u}{\partial x} + \tfrac{\partial v}{\partial y} + \tfrac{\partial w}{\partial z}\right) + f_x = \rho\,\tfrac{\partial^2 u}{\partial t^2}\\
G\nabla^2 v + (G+\lambda)\,\tfrac{\partial}{\partial y}\left(\tfrac{\partial u}{\partial x} + \tfrac{\partial v}{\partial y} + \tfrac{\partial w}{\partial z}\right) + f_y = \rho\,\tfrac{\partial^2 v}{\partial t^2}\\
G\nabla^2 w + (G+\lambda)\,\tfrac{\partial}{\partial z}\left(\tfrac{\partial u}{\partial x} + \tfrac{\partial v}{\partial y} + \tfrac{\partial w}{\partial z}\right) + f_z = \rho\,\tfrac{\partial^2 w}{\partial t^2}
\end{aligned}\right\}. \tag{38}$$

The vectorial form of the aforementioned equations (38) can be written as (refer to, for instance, [14,48,49,55,57,58]),

$$G\nabla^2\mathbf{u} + (G+\lambda)\nabla(\nabla\cdot\mathbf{u}) + \mathbf{f} = \rho\,\frac{\partial^2\mathbf{u}}{\partial t^2}. \tag{39}$$

When no body force in the $\Omega(1)$ substratum, Eqs. (39) reduce to

$$G\nabla^2\mathbf{u} + (G+\lambda)\nabla(\nabla\cdot\mathbf{u}) = \rho\,\frac{\partial^2\mathbf{u}}{\partial t^2}. \tag{40}$$

From Long's theorem [48, 59], there exist a scalar function ψ and a vector function $\mathbf{R}$ such that $\mathbf{u}$ is represented by

$$\mathbf{u} = \nabla\psi + \nabla\times\mathbf{R} \tag{41}$$

and ψ and $\mathbf{R}$ satisfy the following wave equations

$$\nabla^2\psi - \frac{1}{c_l}\frac{\partial^2\psi}{\partial t^2} = 0, \tag{42}$$

$$\nabla^2\mathbf{R} - \frac{1}{c_t}\frac{\partial^2\mathbf{R}}{\partial t^2} = 0, \tag{43}$$

where c_l is the velocity of longitudinal waves, c_t is the velocity of transverse waves. The definitions of these two elastic wave velocities are (refer to, for instance, [48, 49, 57, 58]),

$$c_l = \sqrt{\frac{\lambda + 2G}{\rho}}, \quad c_t = \sqrt{\frac{G}{\rho}}. \tag{44}$$

ψ and $\mathbf{R}$ is usually known as the scalar displacement potential and the vector displacement potential, respectively.

4 Definition of point source and sink

If there exists a velocity field which is continuous and finite at all points of the space, with the exception of individual isolated points, then, usually, these isolated points are called velocity singularities. Point sources and sinks are examples of such velocity singularities.

Assumption 10. *Suppose there exists a singularity at a point $P_0 = (x_0, y_0, z_0)$ in a continuum. If the velocity field of the singularity at a point $P = (x, y, z)$ is*

$$\mathbf{v}(x, y, z, t) = \frac{Q}{4\pi r^2}\,\hat{\mathbf{r}}, \tag{45}$$

where $r = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}$, $\hat{\mathbf{r}}$ is the unit vector directed outward along the line from the singularity to this point $P = (x, y, z)$, we call such a singularity a point source in the case of $Q > 0$ or a point sink in the case of $Q < 0$. Here Q is called the strength of the source or sink.

Suppose that a static point source with the strength Q locates at the origin $(0, 0, 0)$. In order to calculate the volume leaving the source per unit of time, we may enclose the source with an arbitrary spherical surface S of the radius a. Calculation shows that

$$\oiint_S \mathbf{u}\cdot\mathbf{n}\,dS = \oiint_S \frac{Q}{4\pi a^2}\,\hat{\mathbf{r}}\cdot\mathbf{n}\,dS = Q\,, \tag{46}$$

where $\mathbf{n}$ is the unit vector directed outward along the line from the origin of the coordinates to the field point (x, y, z). Equation (46) shows that the strength Q of a source or sink evaluates the volume of the fluid leaving or entering a control surface per unit of time.

For the case of continuously distributed point sources or sinks, it is useful to introduce a definition for the volume density ρ_s of point sources or sinks. The definition is

$$\rho_s = \lim_{\triangle V\to 0} \frac{\triangle Q}{\triangle V}\,, \tag{47}$$

where $\triangle V$ is a small volume, $\triangle Q$ is the sum of the strengthes of all the point sources or sinks in the volume $\triangle V$.

5 A point source and sink model of electric charges

The purpose of this Section is to propose a point source and sink model of electric charges.

Let T be the characteristic time of a observer of an electric charge in the $\Omega(1)$ substratum. We may suppose that the observer's time scale T is very large to the Maxwellian relaxation time τ. So the Maxwellian relaxation time τ is a relatively small, and the stress deviator s_{ij} changes very slow. Thus, the second term in the left side of Eq. (28) may be neglected. For such an observer, the constitutive relation of the $\Omega(1)$ substratum may be written as

$$s_{ij} = 2\eta\,\frac{de_{ij}}{dt}\,. \tag{48}$$

The observer therefore concludes that the $\Omega(1)$ substratum behaves like a Newtonian-fluid on his time scale.

In order to compare fluid motions with electric fields, Maxwell introduced an analogy between sources or sinks and electric charges [17].

Einstein, Infeld and Hoffmann introduced an idea by which all particles may be looked as singularities in fields [60, 61].

Recently [62], we talked that the universe may be filled with a kind fluid which may be called "tao". Thus, Newton's law of gravitation is derived by methods of hydrodynamics based on a point sink flow model of particles.

R. L. Oldershaw talked that hadrons may be considered as Kerr-Newman black holes if one uses appropriate scaling of units and a revised gravitational coupling factor [63].

Inspired by the aforementioned works, we introduce the following

Assumption 11. *Suppose that all the electric charges in the Universe are the sources or sinks in the $\Omega(1)$ substratum. We define such a source as a negative electric charge. We define such a sink as a positive electric charge. The electric charge quantity q_e of an electric charge is defined as*

$$q_e = -\,k_Q\,\rho\,Q\,, \tag{49}$$

where ρ is the density of the $\Omega(1)$ substratum, Q is called the strength of the source or sink, k_Q is a positive dimensionless constant.

A calculation shows that the mass m of an electric charge is changing with time as

$$\frac{dm}{dt} = -\,\rho\,Q = \frac{q_e}{k_Q}\,, \tag{50}$$

where q_e is the electric charge quantity of the electric charge.

We may introduce a hypothesis that the masses of electric charges are changing so slowly relative to the time scaler of human beings that they can be treaten as constants approximately.

For the case of continuously distributed electric charges, it is useful to introduce the following definition of the volume density ρ_e of electric charges

$$\rho_e = \lim_{\triangle V\to 0} \frac{\triangle q_e}{\triangle V}\,, \tag{51}$$

where $\triangle V$ is a small volume, $\triangle q_e$ is the sum of the strengthes of all the electric charges in the volume $\triangle V$.

From Eq. (47), Eq. (49) and Eq. (51), we have

$$\rho_e = -\,k_Q \rho \rho_s\ . \tag{52}$$

6 Derivation of Maxwell's equations in vacuum

The purpose of this Section is to deduce Maxwell's equations based on the aforementioned visco-elastic continuum model of vacuum and the singularity model of electric charges.

Now, let us deduce the continuity equation of the $\Omega(1)$ substratum from the mass conservation. Consider an arbitrary volume V bounded by a closed surface S fixed in space. Suppose that there are electric charges continuously distributed in the volume V. The total mass in the volume V is

$$M = \iiint_V \rho\, dV\,, \tag{53}$$

where ρ is the density of the $\Omega(1)$ substratum.

The rate of the increase of the total mass in the volume V is

$$\frac{\partial M}{\partial t} = \frac{\partial}{\partial t}\iiint_V \rho\, dV\,. \tag{54}$$

Using the Ostrogradsky–Gauss theorem (refer to, for instance, [16, 64–67]), the rate of the mass outflow through the surface S is

$$\oiint_S \rho(\mathbf{v}\cdot\mathbf{n})\, dS = \iiint_V \nabla\cdot(\rho\,\mathbf{v})\, dV\,, \tag{55}$$

where $\mathbf{v}$ is the velocity field of the $\Omega(1)$ substratum.

The definition of the velocity field $\mathbf{v}$ is

$$v_i = \frac{\partial u_i}{\partial t}\,, \quad \text{or} \quad \mathbf{v} = \frac{\partial \mathbf{u}}{\partial t}\,. \tag{56}$$

Using Eq. (52), the rate of the mass created inside the volume V is

$$\iiint_V \rho\rho_s\, dV = \iiint_V -\frac{\rho_e}{k_Q}\, dV. \tag{57}$$

Now according to the principle of mass conservation, and making use of Eq. (54), Eq. (55) and Eq. (57), we have

$$\frac{\partial}{\partial t}\iiint_V \rho\, dV = \iiint_V -\frac{\rho_e}{k_Q}\, dV - \iiint_V \nabla\cdot(\rho\,\mathbf{v})\, dV. \tag{58}$$

Since the volume V is arbitrary, from Eq. (58) we have

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\,\mathbf{v}) = -\frac{\rho_e}{k_Q}\,. \tag{59}$$

According to Assumption 3, the $\Omega(1)$ substratum is homogeneous, that is $\frac{\partial\rho}{\partial x} = \frac{\partial\rho}{\partial y} = \frac{\partial\rho}{\partial z} = \frac{\partial\rho}{\partial t} = 0$. Thus, Eq. (59) becomes

$$\nabla\cdot\mathbf{v} = -\frac{\rho_e}{k_Q\,\rho}\,. \tag{60}$$

According to Assumption 11 and Eq. (50), the masses bearing positive electric charges are changing since the strength of a sink evaluates the volume of the $\Omega(1)$ substratum entering the sink per unit of time. Thus, the momentum of a volume element $\triangle V$ of the $\Omega(1)$ substratum containing continuously distributed electric charges, and moving with an average speed $\mathbf{v}_e$, changes. The increased momentum $\triangle\mathbf{P}$ of the volume element $\triangle V$ during a time interval $\triangle t$ is the decreased momentum of the continuously distributed electric charges contained in the volume element $\triangle V$ during a time interval $\triangle t$, that is,

$$\triangle\mathbf{P} = \rho\,(\rho_s \triangle V \triangle t)\,\mathbf{v}_e = -\frac{\rho_e}{k_Q}\triangle V \triangle t\,\mathbf{v}_e\,. \tag{61}$$

Therefore, the equation of momentum conservation Eq. (39) of the $\Omega(1)$ substratum should be changed as

$$G\nabla^2\mathbf{u} + (G+\lambda)\nabla(\nabla\cdot\mathbf{u}) + \mathbf{f} = \rho\,\frac{\partial^2\mathbf{u}}{\partial t^2} - \frac{\rho_e\mathbf{v}_e}{k_Q}\,. \tag{62}$$

In order to simplify the Eq. (62), we may introduce an additional assumption as

Assumption 12. *We suppose that the $\Omega(1)$ substratum is almost incompressible, or we suppose that θ is a sufficient small quantity and varies very slow in the space so that it can be treaten as $\theta = 0$.*

From Assumption 12, we have

$$\nabla\cdot\mathbf{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \theta = 0\,. \tag{63}$$

Therefore, the vectorial form of the equation of momentum conservation Eq. (62) reduces to the following form

$$G\nabla^2\mathbf{u} + \mathbf{f} = \rho\,\frac{\partial^2\mathbf{u}}{\partial t^2} - \frac{\rho_e\mathbf{v}_e}{k_Q}\,. \tag{64}$$

According to the Stokes-Helmholtz resolution theorem (refer to, for instance, [48, 57]), which states that every sufficiently smooth vector field may be decomposed into irrotational and solenoidal parts, there exist a scalar function ψ and a vector function $\mathbf{R}$ such that $\mathbf{u}$ is represented by

$$\mathbf{u} = \nabla\psi + \nabla\times\mathbf{R}. \tag{65}$$

Now let us introduce the definitions

$$\nabla\phi = k_E\,\frac{\partial}{\partial t}(\nabla\psi)\,, \quad \mathbf{A} = k_E\,\nabla\times\mathbf{R}\,, \tag{66}$$

$$\mathbf{E} = -\,k_E\,\frac{\partial\mathbf{u}}{\partial t}\,, \quad \mathbf{B} = k_E\,\nabla\times\mathbf{u}\,, \tag{67}$$

where ϕ is the scalar electromagnetic potential, $\mathbf{A}$ is the vector electromagnetic potential, $\mathbf{E}$ is the electric field intensity, $\mathbf{B}$ is the magnetic induction, k_E is a positive dimensionless constant.

From Eq. (65), Eq. (66) and Eq. (67), we have

$$\mathbf{E} = -\nabla\phi - \frac{\partial\mathbf{A}}{\partial t}, \quad \mathbf{B} = \nabla\times\mathbf{A} \tag{68}$$

and

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}\,, \tag{69}$$

$$\nabla \cdot \mathbf{B} = 0\,. \tag{70}$$

Based on Eq. (66) and noticing that

$$\nabla^2 \mathbf{u} = \nabla(\nabla \cdot \mathbf{u}) - \nabla \times (\nabla \times \mathbf{u})\,, \tag{71}$$

$$\nabla^2 \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla \times (\nabla \times \mathbf{A})\,, \tag{72}$$

and $\nabla \cdot \mathbf{u} = 0$, $\nabla \cdot \mathbf{A} = 0$, we have

$$k_E\, \nabla^2 \mathbf{u} = \nabla^2 \mathbf{A}\,. \tag{73}$$

Therefore, using Eq. (73), Eq. (64) becomes

$$\frac{G}{k_E}\, \nabla^2 \mathbf{A} + \mathbf{f} = \rho\, \frac{\partial^2 \mathbf{u}}{\partial t^2} - \frac{\rho_e \mathbf{v}_e}{k_Q}\,. \tag{74}$$

Using Eq. (72), Eq. (74) becomes

$$-\frac{G}{k_E}\, \nabla \times (\nabla \times \mathbf{A}) + \mathbf{f} = \rho\, \frac{\partial^2 \mathbf{u}}{\partial t^2} - \frac{\rho_e \mathbf{v}_e}{k_Q}\,. \tag{75}$$

Now using Eq. (68), Eq. (75) becomes

$$-\frac{G}{k_E}\, \nabla \times \mathbf{B} + \mathbf{f} = -\frac{\rho}{k_E}\frac{\partial \mathbf{E}}{\partial t} - \frac{\rho_e \mathbf{v}_e}{k_Q}\,. \tag{76}$$

It is natural to say that there are no other body forces or surface forces exerted on the $\Omega(1)$ substratum. Thus, we have $\mathbf{f} = 0$. Therefore, Eq. (76) becomes

$$\frac{k_Q G}{k_E}\, \nabla \times \mathbf{B} = \frac{k_Q \rho}{k_E}\frac{\partial \mathbf{E}}{\partial t} + \rho_e \mathbf{v}_e\,. \tag{77}$$

Now let us introduce the following definitions

$$\mathbf{j} = \rho_e \mathbf{v}_e\,, \qquad \epsilon_0 = \frac{k_Q \rho}{k_E}\,, \qquad \frac{1}{\mu_0} = \frac{k_Q G}{k_E}\,. \tag{78}$$

Therefore, Eq. (77) becomes

$$\frac{1}{\mu_0}\, \nabla \times \mathbf{B} = \mathbf{j} + \epsilon_0\, \frac{\partial \mathbf{E}}{\partial t}\,. \tag{79}$$

Noticing Eq. (67) and Eq. (78), Eq. (60) becomes

$$\nabla \cdot \mathbf{E} = \frac{\rho_e}{\epsilon_0}\,. \tag{80}$$

Now we see that Eq. (69), Eq. (70), Eq. (79) and Eq. (80) coincide with Maxwell's equations (1–4).

7 Mechanical interpretation of electromagnetic waves

It is known that, from Maxwell's equations (1-4), we can obtain the following equations (refer to, for instance, [1])

$$\nabla^2 \mathbf{E} - \frac{1}{\mu_0 \epsilon_0}\frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{1}{\epsilon_0}\, \nabla \rho_e + \mu_0 \frac{\partial \mathbf{j}}{\partial t}\,, \tag{81}$$

$$\nabla^2 \mathbf{H} - \frac{1}{\mu_0 \epsilon_0}\frac{\partial^2 \mathbf{H}}{\partial t^2} = -\frac{1}{\mu_0}\, \nabla \times \mathbf{j}\,. \tag{82}$$

Eq. (81) and Eq. (82) are the electromagnetic wave equations with sources in the $\Omega(1)$ substratum. In the source free region where $\rho_e = 0$ and $\mathbf{j} = 0$, the equations reduce to the following equations

$$\nabla^2 \mathbf{E} - \frac{1}{\mu_0 \epsilon_0}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0\,, \tag{83}$$

$$\nabla^2 \mathbf{H} - \frac{1}{\mu_0 \epsilon_0}\frac{\partial^2 \mathbf{H}}{\partial t^2} = 0\,. \tag{84}$$

Eq. (83) and Eq. (84) are the electromagnetic wave equations without the sources in the $\Omega(1)$ substratum.

From Eq. (83), Eq. (84) and Eq. (78), we see that the velocity c_0 of electromagnetic waves in vacuum is

$$c_0 = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \sqrt{\frac{G}{\rho}}\,. \tag{85}$$

From Eq. (44) and Eq. (85), we see that the velocity c_0 of electromagnetic waves in the vacuum is the same as the velocity c_t of the transverse elastic waves in the $\Omega(1)$ substratum.

Now we may regard electromagnetic waves in the vacuum as the transverse waves in the $\Omega(1)$ substratum. This idea was first introduced by Frensnel in 1821 [17].

8 Conclusion

We suppose that vacuum is not empty and may be filled with a kind continuously distributed material called $\Omega(1)$ substratum. Following Stokes, we propose a visco-elastic constitutive relation of the $\Omega(1)$ substratum. Following Maxwell, we propose a fluidic source and sink model of electric charges. Thus, Maxwell's equations in vacuum are derived by methods of continuum mechanics based on this continuum mechanical model of vacuum and the singularity model of electric charges.

9 Discussion

Many interesting theoretical, experimental and applied problems can be met in continuum mechanics, Classical Electrodynamics, Quantum Electrodynamics and also other related fields of science involving this theory of electromagnetic phenomena. It is an interesting task to generalize this theory of electromagnetic phenomena in the static $\Omega(1)$ substratum to the case of electromagnetic phenomena of moving bodies.

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On the Geometry of Background Currents in General Relativity

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In this preliminary work, we shall reveal the intrinsic geometry of background currents, possibly of electromagnetic origin, in the space-time of General Relativity. Drawing a close analogy between the object of our present study and electromagnetism, we shall show that there exists an inherent, fully non-linear, conservative third-rank radiation current which is responsible for the irregularity in the curvature of the background space(-time), whose potential (generator) is of purely geometric origin.

1 Introduction

Herein we attempt to study, in a way that has never been fully explored before, the nature of background radiation fields from a purely geometric point of view. One may always expect that empty (matter-free) regions in a space(-time) of non-constant sectional curvature are necessarily filled with some kind of pure radiation field that may be associated with a class of null electromagnetic fields. As is common in practice, their description must therefore be attributed to the Weyl tensor alone, as the only remaining geometric object in emptiness (with the cosmological constant neglected). An in-depth detailed elaboration on the nature of the physical vacuum and emptiness, considering space(-time) anisotropy, can be seen in [6, 7].

Our present task is to explore the geometric nature of the radiation fields permeating the background space(-time). As we will see, the thrilling new aspect of this work is that our main stuff of this study (a third-rank background current and its associates) is geometrically non-linear and, as such, it cannot be gleaned in the study of gravitational radiation in weak-field limits alone. Thus, it must be regarded as an essential part of Einstein's theory of gravity.

Due to the intended concise nature of this preliminary work, we shall leave aside the more descriptive aspects of the subject.

2 A third-rank geometric background current in a general metric-compatible manifold

At first, let us consider a general, metric-compatible manifold $\mathbb{M}_D$ of arbitrary dimension D and coordinates x^α. We may extract a third-rank background current from the curvature as follows:

$$J_{\mu\nu\rho} = J_{\mu[\nu\rho]} = \nabla_\lambda R^\lambda{}_{\mu\nu\rho}\,,$$

where square brackets on a group of indices indicate anti-symmetrization (similarly, round brackets will be used to indicate symmetrization). Of course, ∇ is the covariant derivative, and, with $\partial_\mu = \frac{\partial}{\partial x^\mu}$,

$$R^\lambda{}_{\mu\nu\rho} = \partial_\nu \Gamma^\lambda_{\mu\rho} - \partial_\rho \Gamma^\lambda_{\mu\nu} + \Gamma^\sigma_{\mu\rho}\Gamma^\lambda_{\sigma\nu} - \Gamma^\sigma_{\mu\nu}\Gamma^\lambda_{\sigma\rho}$$

are the usual components of the curvature tensor R of the metric-compatible connection Γ whose components are given by

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2}\, g^{\lambda\sigma}\left(\partial_\sigma g_{\mu\nu} - \partial_\mu g_{\nu\sigma} + \partial_\nu g_{\sigma\mu}\right) + \Gamma^\lambda_{[\mu\nu]} - \\ - g^{\lambda\alpha}\left(g_{\mu\beta}\Gamma^\beta_{[\alpha\nu]} + g_{\nu\beta}\Gamma^\beta_{[\alpha\mu]}\right).$$

Here $g_{\mu\nu}$ are the components of the fundamental symmetric metric tensor g and $\Gamma^\lambda_{[\mu\nu]}$ are the components of the torsion tensor. The (generalized) Ricci tensor and scalar are then given, as usual, by the contractions $R_{\mu\nu} = R^\lambda{}_{\mu\lambda\nu}$ and $R = R^\mu{}_\mu$, respectively.

We may introduce the traceless Weyl curvature tensor W through the decomposition

$$R^\mu{}_{\alpha\beta\gamma} = K^\mu{}_{\alpha\beta\gamma} + \\ + \frac{1}{D-2}\left(\delta^\mu_\beta R_{\alpha\gamma} + g_{\alpha\gamma}R^\mu{}_\beta - \delta^\mu_\gamma R_{\alpha\beta} - g_{\alpha\beta}R^\mu{}_\gamma\right),$$

$$K^\mu{}_{\alpha\beta\gamma} = W^\mu{}_{\alpha\beta\gamma} + \\ + \frac{1}{(D-1)(D-2)}\left(\delta^\mu_\gamma g_{\alpha\beta} - \delta^\mu_\beta g_{\alpha\gamma}\right) R\,,$$

$$K_{\alpha\beta} = K_{(\alpha\beta)} = K^\mu{}_{\alpha\mu\beta} = -\frac{1}{D-2}\, g_{\alpha\beta} R\,,$$

for which $D > 2$. In particular, we shall take into account the following useful relation:

$$R^\mu{}_{\alpha\beta\gamma}R^{\alpha\beta\gamma\nu} = W^\mu{}_{\alpha\beta\gamma}W^{\alpha\beta\gamma\nu} + \\ + \frac{1}{D-2}\left(K^\mu{}_\alpha{}^\nu{}_\beta R^{\alpha\beta} + K^{\alpha\mu\beta\nu}R_{\alpha\beta} - K^{\alpha\beta\mu\nu}R_{[\alpha\beta]}\right) + \\ + \frac{1}{D-2}\left(2RR^{\mu\nu} + g^{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} - R^\mu{}_\alpha R^{\nu\alpha} - R_\alpha{}^\mu R^{\alpha\nu}\right) + \\ + \frac{2}{D-2}\left(R^{(\mu\alpha)}R^{[\nu}{}_{\alpha]} + R^{(\mu\alpha)}R_\alpha{}^\nu - R^\mu{}_\alpha R^{(\nu\alpha)}\right) - \\ - \frac{2}{(D-2)^2}\, RR^{\mu\nu}\,.$$

Now, for an arbitrary tensor field T, we have, as usual,

$$(\nabla_\beta \nabla_\alpha - \nabla_\alpha \nabla_\beta)\, T^{\mu\nu\cdots}_{\rho\sigma\cdots} =$$

$$= R^{\lambda}{}_{\rho\alpha\beta} T^{\mu\nu\cdots}_{\lambda\sigma\cdots} + R^{\lambda}{}_{\sigma\alpha\beta} T^{\mu\nu\cdots}_{\rho\lambda\cdots} + \cdots - R^{\mu}{}_{\lambda\alpha\beta} T^{\lambda\nu\cdots}_{\rho\sigma\cdots} -$$

$$- R^{\nu}{}_{\lambda\alpha\beta} T^{\mu\lambda\cdots}_{\rho\sigma\cdots} - 2\Gamma^{\lambda}_{[\alpha\beta]} \nabla_\lambda T^{\mu\nu\cdots}_{\rho\sigma\cdots}\,.$$

For a complete set of general identities involving the curvature tensor R and their relevant physical applications in Unified Field Theory, see [1–5].

At this point, we can define a second-rank background current density (field strength) f through

$$f^{\nu\rho} = f^{[\nu\rho]} = \nabla_\mu J^{\mu\nu\rho} = \nabla_\mu \nabla_\lambda R^{\lambda\mu\nu\rho} = -\nabla_{[\lambda}\, \nabla_{\mu]} R^{\lambda\mu\nu\rho}\,.$$

An easy calculation gives, in general,

$$f^{\mu\nu} = -\frac{1}{2}\left(R^{\mu}{}_{\alpha\beta\gamma} R^{\alpha\beta\gamma\nu} - R^{\nu}{}_{\alpha\beta\gamma} R^{\alpha\beta\gamma\mu}\right) -$$

$$- R_{[\alpha\beta]} R^{\alpha\beta\mu\nu} - \Gamma^{\alpha}_{[\rho\sigma]} \nabla_\alpha R^{\rho\sigma\mu\nu}\,.$$

In analogy to the electromagnetic source, we may define a first-rank current density through

$$j^{\mu} = \nabla_\nu f^{\mu\nu}\,.$$

Then, a somewhat lengthy but straightforward calculation shows that

$$\nabla_\mu j^{\mu} = R_{[\mu\nu]} f^{\mu\nu} + \Gamma^{\sigma}_{[\mu\nu]} \nabla_\sigma f^{\mu\nu}\,.$$

We may also define the field strength f through a sixth-rank curvature tensor F whose components are given by

$$F_{\mu\nu\rho\sigma\lambda\delta} = F_{[\mu\nu][\rho\sigma]\lambda\delta} =$$

$$= \frac{1}{2}\left(R_{\mu\delta\lambda\alpha} \overline{R}^{\alpha}{}_{\nu\rho\sigma} - R_{\nu\delta\lambda\alpha} \overline{R}^{\alpha}{}_{\mu\rho\sigma}\right) +$$

$$+ \frac{1}{2}\left(R_{\mu\rho\lambda\alpha} \overline{R}^{\alpha}{}_{\nu\delta\sigma} - R_{\nu\rho\lambda\alpha} \overline{R}^{\alpha}{}_{\mu\delta\sigma}\right) +$$

$$+ \frac{1}{2}\left(R_{\nu\sigma\lambda\alpha} \overline{R}^{\alpha}{}_{\mu\delta\rho} - R_{\mu\sigma\lambda\alpha} \overline{R}^{\alpha}{}_{\nu\delta\rho}\right) +$$

$$+ \frac{1}{2}\left(R_{\mu\lambda\delta\alpha} \overline{R}^{\alpha}{}_{\nu\rho\sigma} - R_{\nu\lambda\delta\alpha} \overline{R}^{\alpha}{}_{\mu\rho\sigma}\right) - R_{\mu\nu\delta\alpha} \overline{R}^{\alpha}{}_{\lambda\rho\sigma}\,.$$

where $\overline{R}_{\mu\nu\rho\sigma} = R_{\rho\sigma\mu\nu}$.

If we define a second-rank anti-symmetric tensor B by

$$B_{\mu\nu} = F_{\mu\nu\rho\sigma}{}^{\sigma\rho} =$$

$$= \frac{1}{2}\left(R_{\mu\alpha\beta\gamma} R^{\alpha\beta\gamma}{}_{\nu} - R_{\nu\alpha\beta\gamma} R^{\alpha\beta\gamma}{}_{\mu}\right) - R_{\mu\nu\alpha\beta} R^{[\alpha\beta]}\,,$$

we then obtain

$$f_{\mu\nu} = B_{\mu\nu} + \Gamma^{\alpha}_{[\rho\sigma]} \nabla_\alpha R^{\rho\sigma}{}_{\mu\nu}\,,$$

such that in the case of vanishing torsion, the quantities f and B are completely equivalent.

3 A third-rank radiation current relevant to General Relativity

Having defined the basic geometric objects of our theory, let us adhere to the standard Riemannian geometry of General Relativity in which the torsion vanishes, that is $\Gamma^{\lambda}_{[\mu\nu]} = 0$, and so the connection is the symmetric Levi-Civita connection. However, let us also take into account discontinuities in the first derivatives of the components of the metric tensor in order to take into account discontinuity surfaces corresponding to any existing background energy field. As we will see, we shall obtain a physically meaningful background current which is strictly conservative.

Now, in connection with the results of the preceding section, if we employ the simplified relation (which is true in the absence of torsion)

$$R^{\mu}{}_{\alpha\beta\gamma} R^{\alpha\beta\gamma\nu} = W^{\mu}{}_{\alpha\beta\gamma} W^{\alpha\beta\gamma\nu} +$$

$$+ \frac{1}{D-2}\left(K^{\mu}{}_{\alpha}{}^{\nu}{}_{\beta} R^{\alpha\beta} + K^{\alpha\mu\beta\nu} R_{\alpha\beta}\right) +$$

$$+ \frac{1}{D-2}\left(2RR^{\mu\nu} + g^{\mu\nu} R_{\alpha\beta} R^{\alpha\beta} - 4R^{\mu}{}_{\alpha} R^{\nu\alpha}\right) -$$

$$- \frac{2}{(D-2)^2}\, RR^{\mu\nu}\,,$$

as well as the relations

$$K^{\mu}{}_{\alpha\beta\gamma} K^{\alpha\beta\gamma\nu} = W^{\mu}{}_{\alpha\beta\gamma} W^{\alpha\beta\gamma\nu} + \frac{1}{(D-1)\,(D-2)^2}\, g^{\mu\nu} R^2,$$

$$K^{\mu\alpha\nu\beta} R_{\alpha\beta} = W^{\mu\alpha\nu\beta} R_{\alpha\beta} +$$

$$+ \frac{1}{(D-1)\,(D-2)}\left(R^{\mu\nu} - g^{\mu\nu} R\right) R\,,$$

we obtain the desired relation

$$f^{\mu\nu} = -\frac{1}{2}\left(W^{\mu}{}_{\alpha\beta\gamma} W^{\alpha\beta\gamma\nu} - W^{\nu}{}_{\alpha\beta\gamma} W^{\alpha\beta\gamma\mu}\right) -$$

$$- \frac{1}{D-2}\left(W^{\mu}{}_{\alpha}{}^{\nu}{}_{\beta} - W^{\nu}{}_{\alpha}{}^{\mu}{}_{\beta}\right) R^{\alpha\beta}.$$

If the metric tensor is perfectly continuous, it is obvious that

$$f^{\mu\nu} = 0\,.$$

In deriving this relation we have used the symmetry $W_{\mu\nu\rho\sigma} = W_{\rho\sigma\mu\nu}$. This shows that, in the presence of metric discontinuity, the field strength f depends on the Weyl curvature alone which is intrinsic to the background space(-time) only when matter and non-null electromagnetic fields are absent. We see that, in spaces of constant sectional curvature, we will strictly have $J^{\lambda\mu\nu} = 0$ and $f^{\mu\nu} = 0$ since the Weyl curvature vanishes therein. In other words, in the sense of General Relativity, the presence of background currents is responsible for the irregularity (anisotropy) in the curvature of the background space(-time). Matter, if not elementary particles, in this sense, can indeed be regarded as a form of perturbation with respect to the background space(-time).

Furthermore, it is now apparent that

$$J_{\mu\nu\rho} + J_{\nu\rho\mu} + J_{\rho\mu\nu} = 0 .$$

This relation is, of course, reminiscent of the usual Bianchi identity satisfied by the components of the Maxwellian electromagnetic field tensor.

Also, we obtain the conservation law

$$\nabla_\mu j^\mu = 0 .$$

which becomes trivial when the metric is perfectly continuous.

Hence, the formal correspondence between our present theory and the ordinary theory of electromagnetism may be completed, in the simplest way, through the relation

$$J_{\mu\nu\rho} = \nabla_\lambda R^\lambda{}_{\mu\nu\rho} = \nabla_\mu \Phi_{\nu\rho} ,$$

where the anti-symmetric field tensor Φ given by

$$\Phi_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$$

plays a role similar to that of the electromagnetic field strength. However, it should be emphasized that it exists in General Relativity's fully non-linear regime. In addition, it vanishes identically in the absence of curvature anisotropy. Interestingly, if one is willing to regard electromagnetism as a kind of non-linear gravity, one may alternatively regard Φ as being the complete equivalent of Maxwell's electromagnetic field strength. However, we shall not further pursue this interest here.

Furthermore, we obtain the relation

$$f_{\mu\nu} = \Box\, \Phi_{\mu\nu} ,$$

where $\Box = \nabla_\mu \nabla^\mu$. That is, the wave equation

$$\Box\, \Phi_{\mu\nu} = -\frac{1}{2}\left(W_{\mu\alpha\beta\gamma} W^{\alpha\beta\gamma}{}_\nu - W_{\nu\alpha\beta\gamma} W^{\alpha\beta\gamma}{}_\mu\right) - \\ - \frac{1}{D-2}\left(W_{\mu\alpha\nu\beta} - W_{\nu\alpha\mu\beta}\right) R^{\alpha\beta}.$$

In the absence of metric discontinuity, we obtain

$$\Box\, \Phi_{\mu\nu} = 0 .$$

Let us now introduce a vector potential ϕ such that the curl of which gives us the field strength f. Instead of writing $f_{\mu\nu} = \partial_\nu \phi_\mu - \partial_\mu \phi_\nu$ and instead of expressing the field strength f in terms of the Weyl tensor, let us write its components in the following equivalent form:

$$f_{\mu\nu} = -\frac{1}{2}\left(R_{\mu\alpha\beta\gamma} R^{\alpha\beta\gamma}{}_\nu - R_{\nu\alpha\beta\gamma} R^{\alpha\beta\gamma}{}_\mu\right) = \\ = \nabla_\nu \phi_\mu - \nabla_\mu \phi_\nu .$$

In order for the potential ϕ to be purely geometric, we shall have

$$\nabla_\nu \phi_\mu = -\frac{1}{2} R_{\mu\alpha\beta\gamma} R^{\alpha\beta\gamma}{}_\nu ,$$

from which an "equation of motion" follows somewhat effortlessly:

$$\frac{D\phi_\mu}{Ds} = -\frac{1}{2} R_{\mu\alpha\beta\gamma} R^{\alpha\beta\gamma}{}_\nu \frac{dx^\nu}{ds} ,$$

where $\frac{D\phi_\mu}{Ds} = \frac{dx^\nu}{ds} \nabla_\nu \phi_\mu$.

Note that, in the absence of metric discontinuity, the vector potential ϕ is a mere gradient of a smooth scalar field Θ: $\phi_\mu = \nabla_\mu \Theta$.

Now, it remains to integrate the equation

$$\partial_\nu \phi_\mu = -\frac{1}{2} R_{\mu\alpha\beta\gamma} R^{\alpha\beta\gamma}{}_\nu + \Gamma^\lambda_{\mu\nu} \phi_\lambda$$

by taking a closed contour P associated with the surface area dS spanned by infinitesimal displacements in two different directions, that is,

$$dS^{\mu\nu} = d_1 x^\mu d_2 x^\nu - d_1 x^\nu d_2 x^\mu .$$

An immediate effect of this closed-loop integration is that, by using the generally covariant version of Stokes' theorem and by explicitly assuming that the integration factor Z given by

$$Z^\rho{}_\mu = \frac{1}{2} \oiint_S \left(\nabla_\lambda \Gamma^\rho_{\mu\nu} - \nabla_\nu \Gamma^\rho_{\mu\lambda}\right) dS^{\lambda\nu} = \\ = \frac{1}{2} \oiint_S \left(R^\rho{}_{\mu\lambda\nu} + \Gamma^\rho_{\sigma\lambda}\Gamma^\sigma_{\mu\nu} - \Gamma^\sigma_{\mu\lambda}\Gamma^\rho_{\sigma\nu}\right) dS^{\lambda\nu} = \\ = \frac{1}{2} \oiint_S \left(R^\rho{}_{\mu\lambda\nu} + 2\Gamma^\rho_{\sigma\lambda}\Gamma^\sigma_{\mu\nu}\right) dS^{\lambda\nu}$$

does not depend on ϕ, the integral $\oint_P \Gamma^\lambda_{\mu\nu} \phi_\lambda dx^\nu$ shall indeed vanish identically.

Hence, we are left with the expression

$$\Delta\phi_\mu = -\frac{1}{2} \oint_P R_{\mu\alpha\beta\gamma} R^{\alpha\beta\gamma}{}_\nu dx^\nu .$$

By introducing a new integration factor X satisfying $X^{\alpha\beta\gamma} + X^{\beta\gamma\alpha} + X^{\gamma\alpha\beta} = 0$ as follows:

$$X^{\alpha\beta\gamma} = X^{[\alpha\beta]\gamma} = \oint_P R^{\alpha\beta\gamma}{}_\nu dx^\nu = \\ = \frac{1}{2} \oiint_S \left(\nabla_\mu R^{\alpha\beta\gamma}{}_\nu - \nabla_\nu R^{\alpha\beta\gamma}{}_\mu\right) dS^{\mu\nu} ,$$

we obtain, through direct partial integration,

$$\Delta\phi_\mu = -\frac{1}{2}\left(R_{\mu\alpha\beta\gamma} X^{\alpha\beta\gamma} - \int X^{\alpha\beta\gamma} dR_{\mu\alpha\beta\gamma}\right) .$$

Simplifying, by keeping in mind that $X = X\,(R, dR)$, we finally obtain

$$\Delta\phi_\mu = \frac{1}{2}\int R_{\mu\alpha\beta\gamma}\, dX^{\alpha\beta\gamma}\,.$$

The simplest desired result of this is none other than

$$\Delta\phi_\mu = \frac{1}{2}\, R_{\mu\alpha\beta\gamma} X^{\alpha\beta\gamma}\,,$$

which, expressed in terms of the Weyl tensor, the Ricci tensor, and the Ricci scalar, is

$$\Delta\phi_\mu = \frac{1}{2}\, W_{\mu\alpha\beta\gamma} X^{\alpha\beta\gamma} + \frac{1}{D-2}\left(X^{\alpha}{}_{\mu}{}^{\beta} R_{\alpha\beta} - X^{\alpha}{}_{\beta}{}^{\beta} R_{\alpha\mu}\right) + $$
$$+ \frac{1}{(D-1)\,(D-2)}\, X_{\mu}{}^{\alpha}{}_{\alpha} R\,.$$

Hence, through Einstein's field equation (i.e. through the energy-momentum tensor T)

$$R_{\mu\nu} = \pm\,\frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu} T\right)\,,$$

where G is Newton's gravitational constant and c is the speed of light, we may see how the presence of (distributed) matter affects the potential ϕ.

4 Final remarks

At this point, having outlined our study in brief, it remains to be seen whether our fully geometric background current may be associated with any type of conserved material current which is already known in the literature. It is also tempting to ponder, from a purely physical point of view, on the possibility that the intrinsic curvature of space(-time) owes its existence to null (light-like) electromagnetic fields or simply pure radiation fields.

In this case, let the null electromagnetic (pure radiation) field of the background space(-time) be denoted by φ, for which

$$\varphi_{\mu\nu}\varphi^{\mu\nu} = 0\,.$$

Then we may express the components of the Weyl tensor as

$$W_{\mu\nu\rho\sigma} = \varphi_{\mu\nu}\varphi_{\rho\sigma} - \varphi_{\mu\rho}\varphi_{\nu\sigma} + \varphi_{\mu\sigma}\varphi_{\nu\rho}\,,$$

such that the relation $W^{\rho}{}_{\mu\rho\nu} = 0$ is satisfied.

If this indeed is the case, then we shall have a chance to better understand how matter actually originates from such a pure radiation field in General Relativity. This will hopefully also open a new way towards the full geometrization of matter in physics.

Finally, as a pure theory of gravitation, the results in the present work may be compared to those given in [8] and [9], wherein, based on the theory of chronometric invariants [7], a new geometric formulation of gravity (which is fully equivalent to the standard form of General Relativity) is presented in a way very similar to that of the electromagnetic field, based solely on a second-rank anti-symmetric field tensor.

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The Asymptotic Approach to the Twin Paradox

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The argument of twins' asymmetry, essentially put forward in the common solution of the Twin Paradox, is revealed to be inoperative in some asymptotic situations in which the noninertial effects are insignificant. Consequently the respective solution proves itself as unreliable thing and the Twin Paradox is re-established as an open problem which require further investigations.

1 Introduction

Undoubtely that, in connection with Special Relativity, one of the most disputed subjects was (and still remaining) the so-called the *Twin Paradox*. Essentially this paradox consists in a contradiction between *time-dilatation* (relativistic transformations of time intervals) and the simple belief in symmetry regarding the ageing degrees of two relatively moving twins. The idea of time-dilattation is largely agreed in scientific literature (see [1–5] and references therein) as well as in various (more or less academic) media. However the experimental convincingness of the respective idea still remains a subject of interest even in the inverstigations of the last decades (see for examples [6–9]).

It is notable the fact that, during the last decades, the disputes regarding the Twin Paradox were diminished and disimulated owing to the *common solution* (CS), which seems to be accredited with a great and unimpeded popularity. In the essence, CS argues that the twins are in completely asymmetric ageing situations due to the difference in the noninertial effects which they feel. Such noninertial effects are connected with the nonuniform motion of only one of the two twins. Starting from the mentioned argumentation, without any other major and credible proof, CS states that the Twin Paradox is nothing but an apparent and fictitious problem.

But even in the situations considered by CS a kind of symmetry between the twins can be restored if the noninertial effects are adequately managed. Such a management is possible if we take into account an asymptotic situation when the motions of the traveling twin is prevalently uniform or, in addition, the nonuniform motions are symmetrically present for both of the twins. Here we will see that the existence of the mentioned asymptotic situations have major consequences/implications for the reliability of CS. Our search is done in the Special Relativity approach (without appeals to General Relativity). This is because we consider such an approach to be sufficiently accurate/adequate for the situations under discussion.

In the end we shall conclude that the existence of the alluded asymptotic situations invalidate the CS and restores the Twin Paradox as a real (non-apparent, non-fictitious) and open problem which requires further investigations.

2 Asymptotic situations in which the noninertial effects are insignificant

In order to follow our project let us reconsider, in a quantitative manner, the twins arrangement used in CS. We consider two twins A and B whose proper reference frames are K_A and K_B respectively. The situations of the two twins A and B are reported in comparison with an inertial reference frame K.

2.1 Discusions about an asymptotic asymmetric situation

Within the framework of a first approach, we consider the twin A remaining at rest in the coordinate origin O of the frame K while the twin B moves forth and back along the positive part of the x-axis of K. The motion of B passes throgh the points O, M, N and P whose x-positions are: $x_O = 0$, $x_M = D$, $x_N = D + L$, $x_P = 2D + L$. The motion starts and finishes at O, while P is a turning point — i.e. the velocity of B is zero at O and P . Along the segments OM and NP the motion is nonuniform (accelerated or decelerated) with a time t dependent velocity $v(t)$. On the other hand, along the segment MN, the motion is uniform with the velocity of $v_0 =$ const. In the mentioned situation K_A coincides with K, while K_B moves (nonuniformly or uniformly) with respect to K. The time variables describing the degrees of ageeing of the two twins will be indexed respective to A and B. Also the mentioned time variables will be denoted respective to τ and t as they refer to the proper (intrinsic) time of the considered twin or, alternatively, to the time measured (estimed) in the reference frame of the other twin. The infinitesimal or finite intervals of τ and t will be denoted by $d\tau$ and dt respectively by $\Delta\tau$ and Δt.

With the menioned specifications, according to the relativity theory, for the time interval from the start to the finish of the motion of the twin B, one can write the relations

$$\Delta\tau_A = (\Delta t_B)_n + (\Delta t_B)_u \ , \tag{1}$$

$$\Delta\tau_B = \int\limits_{(\Delta t_B)_n} \sqrt{1 - \frac{v^2\,(t_B)}{c^2}} \cdot dt_B + (\Delta t_B)_u \sqrt{1 - \frac{v_0^2}{c^2}} \ . \tag{2}$$

In these equations the indices n and u refer to the nonuniform respectively uniform motions, while c denotes the light velocity. In (2) it was used the fact (accepted in the relativity theory [10]) that instantaneously, at any moment of time, an arbitrarily moving reference frame can be considered as inertial. Because $v(t_B) \leqslant v_0 \leqslant c$, from (1) and (2) a formula follows:

$$\Delta\tau_A > \Delta\tau_B\,. \tag{3}$$

On the other hand, in the framework of a simple conception (naive belief) these two twins must be in symmetric ageing when B returns at O. This means that, according to the respective conception, the following supposed relation (s.r) have to be taken into account

$$\Delta\tau_A = \Delta\tau_B \qquad \text{(s.r.)}\,. \tag{4}$$

Moreover, for the same simple conception, by invoking the relative character of the twins' motion, the roles of A and B in (3) might be (formally) inverted. Then one obtains another supposed relation, namely

$$\Delta\tau_A < \Delta\tau_B \qquad \text{(s.r.)}\,. \tag{5}$$

This obvious disagreement between the relativistic formula (3) and the supposed relations (4) and (5) represents just the Twin Paradox.

For resolving of the Twin Paradox, CS invokes [1–3] (as essential and unique argument) the assertion that the twins ageing is completely asymmetric. The respective assertion is argued with an idea that, in the mentioned arrangement of twins, B feels non-null noninertial effects during its nonuniform motions, while A, being at rest, does not feel such effects. Based on the alluded argumentation, without any other major and credible proof, CS rejects the supposed relations (4) and (5) as unfounded and fictitious. Then, according to CS only the relativistic formula (3) must be regarded as a correct relation. Consequently CS inferes the conclusion: the Twin Paradox is nothing but a purely and apparent fictitious problem.

But now we have to notify the fact that CS does not approach any discussion on the comparative importance (significance) in the Twin Paradox problem of the respective nonuniform and uniform motions. Particularly, it is not taken into discussions the asymptotic situatios where, comparatively, the effects of the noninertial motions become insignificant. Or, it is clear that, as it is considered by CS, the asymmetry of the twins is generated by the nonuniform motions, while the uniform motions have nothing to do on the respective asymmetry. That is why we discuss that the alluded comparative importance is absolutely necessary. Moreover such a discussion should refer (in a quantitative manner) to the comparative value/ratio of L and D. This is because

$$(\Delta t_B)_u = \frac{2L}{v_0}\,, \tag{6}$$

while, on the other hand, $(\Delta t_B)_n$ depends on D, — e.g. when the nonuniformity of B motions is caused by constant forces, the relativity theory gives

$$(\Delta t_B)_n = \frac{4v_0 D}{c^2\left(1-\sqrt{1-\frac{v_0^2}{c^2}}\,\right)}\,. \tag{7}$$

Then with the notation $\eta = \frac{D}{L}$ one obtains

$$\frac{(\Delta t_B)_n}{(\Delta t_B)_u} = \eta\,\frac{2v_0^2}{c^2\left(1-\sqrt{1-\frac{v_0^2}{c^2}}\,\right)} \approx 4\eta \quad (\text{for } v_0 \ll c)\,. \tag{8}$$

This means that, in the mentioned circumstances, the ratio $\eta = \frac{D}{L}$ has a property which gives a quantitative description to the comparative importance (significance) of the respective nonuniform and uniform motions. It is natural to consider η as the bearing the mentioned property in the circumstances that are more general than those refered in (7) and (8). That is why we will conduct our discussions in terms of the parameter η.

Specification: The quantities D and v_0 are considered as bing nonnull and constant, while L is regarded as an adjustable quantity. So we can consider situations where $\eta \ll 1$ or even where $\eta \to 0$.

Now let us discuss the cases where $\eta \ll 1$. In such a case the twin B moves predominantly uniform, and the noninertiel effects on it are prevalently absent. The twins' positions are prevalently symmeric or even become asymptotically symetric when $\eta \to 0$. That is why we regard/denote the respective cases as asymptotic situations. In such situations the role of the accelerated motions (and of associated noninertial effects) becomes insignifiant (negligible).

These just alluded situations should be appreciated by consideration (prevalently or even asymptotically) of Einstein's posulate of relativity, which states [3] that the inertial frames of references are equivalent to each other, and they cannot be distinguished by means of investigation of physical phenomena. Such an appreciation materializes itself in the relations

$$\left.\begin{aligned} &\Delta\tau_A \approx \Delta\tau_B\,, \qquad (\eta \ll 1) \\ &\lim_{\eta\to 0}\Delta\tau_A = \lim_{\eta\to 0}\Delta\tau_B \end{aligned}\right\}. \tag{9}$$

Also, from (1) and (2) one obtains

$$\Delta\tau_B \approx \Delta\tau_A\sqrt{1-\frac{v_0^2}{c^2}} < \Delta\tau_A\,, \qquad (\eta \ll 1)\,. \tag{10}$$

By taking into account the mentioned Einstein postulate in (10), the roles of A and B might be inverted and one finds

$$\Delta\tau_A \approx \Delta\tau_B\sqrt{1-\frac{v_0^2}{c^2}} < \Delta\tau_B\,, \qquad (\eta \ll 1)\,. \tag{11}$$

Note that, in the framework of the discussed case, the relations (9) and (11) are not supposed (or fictitious) pieces as (4) and (5) are, but they are true formulae like (10). This means that for $\eta \ll 1$ the mentioned arrangement of the twins leads to a set of incompatible relations (9)–(11). Within CS the respective incompatibility cannot be avoided by any means.

2.2 Discusions about an asymptotic completely symmetric situation

Now let us consider a new arrangement of the twins as follows. The twin B preserves exactly his situation previously presented. On the other hand, the twin A moves forward and backward in the negative part of the x-axis in K, symmetric to as B moves with respect to the point O. All the mentioned notations remain unchanged as the above. Evidently that, in the framework of the new arrangement, the situations of these two twins A and B, as well as their proper frames K_A and K_B, are completely symmetric with respect to K. From this fact, for the time intervals between start and finish of the motions, it results directly the relation

$$\Delta\tau_A = \Delta\tau_B\,. \tag{12}$$

In addition, for asymptotic situations where $\eta \ll 1$, one obtains

$$\Delta\tau_A = \Delta\tau_B \approx \frac{2L}{v_0}\sqrt{1-\frac{v_0^2}{c^2}}\,, \qquad (\eta \ll 1)\,. \tag{13}$$

On the other hand, by taking into account Einstein's postulate of relativity, similarly to the relations (10) and (11) for the new arrangement in the asymptotic situations (i.e., where $\eta \ll 1$ and the noninertial effects are insignificant), one finds

$$\Delta\tau_B \approx \Delta\tau_A\sqrt{1-\frac{w_0^2}{c^2}} < \Delta\tau_A\,, \qquad (\eta \ll 1)\,, \tag{14}$$

$$\Delta\tau_A \approx \Delta\tau_B\sqrt{1-\frac{w_0^2}{c^2}} < \Delta\tau_B\,, \qquad (\eta \ll 1)\,, \tag{15}$$

with

$$w_0 = \frac{2v_0}{1+\frac{v_0^2}{c^2}}\,. \tag{16}$$

It should be noted that fact that, with respect to the relativity theory, the relations (12), (14) and (15) are true formulae: they are not supposed and/or fictitious. On the other hand, one finds that the mentioned relations are incompatible to each other. The respective incompatibility cannot be resolved or avoided in a rational way by CS whose solely major argument is the asymmetry of the twins.

3 Some final comments

The above analysed facts show that, in the mentioned "asymptotic situations", the noninertial effects are insignificant for the estimation of the time intervals evaluated (felt) by the two twins. Consequently in such situations the inertial-noninertial asymmetry between such two twins cannot play a significant role. Therefore the respective asymmetry cannot be considered a reliable proof in the resolving of the Twin Paradox. This means that the CS loses its essential (and solely) argument. So, the existence of the above mentioned asymptotic situations appears as a true incriminating test for CS.

Regarding to its significance and implications, the mentioned test has to be evaluated/examined concurently with the "approvingly ilustrations" invoked and preached by the supporters of CS. At this point it seems to be of some profit to remind the Feynmann's remark [11] that, in fact, a conception/theory is invalidated (proved to be wrong) by the real and irrefutable existence of a single incriminating test, indifferently of the number of approving illustrations. Some scientists consider that such a test must be only of experimental but not of theoretical nature. We think that the role of such tests can be played also by theoretical consequences rigurously derived from a given conception. So thinking, it is easy to see that for CS the existence of the above discussed asymptotic situations has all the characteristics of an irrefutable incriminating test. The respecrtive existence invalidate the CS which must be abandoned as a wrong and unreliable approach of the Twin Paradox.

But even if CS is abandoned the incompatibility regardig the relations (9)–(11) or/and the formulae (12), (14) and (15) remains as an unavoidable and intriguing fact. Then what is the significance and importance of the respective fact? We think that it restores the Twin Paradox as an authentic unsolved problem which is still waiting for further investigations. Probably that such investigations will involve a large variety of facts/arguments/opinions.

In connection to the alluded further investigations the following first question seems to be non-trivially interesting: can the investigations on the Twin Paradox be done in a credible manner withot troubling the Special Theory of Relativity? If a negative answer, a major importance goes to the second question: can the Twin Paradox, restored as mentioned above, be an incriminating test for the Special Theory of Relativity, in the sense of the previously noted Feynmann's remark, or not? Can the second question be connected to the "sub-title" of the volume mentioned in the reference [9], or not?

This paper was prepared on the basis of an earlier manuscript of mine [12].

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A New Detector for Perturbations in Gravitational Field

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The paper presents design, principles of operation, and examples of registrations carried out by original device developed and constructed by V. N. Smirnov. The device manifested the possibility to register very weak gravitational perturbations of non-seismic kind both from celestial bodies and from the internal processed in the terrestrial globe.

Given all hypotheses of the possible, do choice for such one which doesn't limit your further thinking on the studied phenomenon.

J. C. Maxwell

1 Introduction

At present day, we have many working properly gravitational wave detectors such as LIGO (USA), GEO-600 (Great Britain and Germany), VIRGO (Italy), TAMA-300 (Japan), miniGRAIL (the Netherlands) and so on. The physical principles of measurement, on a basis of which all the detectors work, lie in the theory of deviation of two particles in the field of a falling gravitational wave meant as a wave of the space metric (so called deformation gravitational waves [1, 2]). The first of such devices was a solid-body (resonant) detector — a 1.500 kg aluminum pig, which is approximated by two particles connected by an elastic force (spring). It was constructed and armed in the end of 1960's by Joseph Weber, the pioneer of these measurements [3–5]. Later there were constructed also free-mass gravitational detectors, built on two mirrors, distantly located from each other and equipped by a laser range-finder to measure the distance between them. Once a gravitational wave falls onto both solid-body or free-mass detector, the detector should have smallest deformation that could be registered as piezo-effect in a solid-body detector or the change of the distance between the mirrors in a free-mass detector [1]. For instance, LIGO (USA) is a free-mass detector, while miniGRAIL (the Netherlands) is a solid-body detector built on a 65-cm metallic sphere, cooled down to liquid Helium. (A spherical solid-body detector is especially good, because it easily registers the direction of the falling gravitational wave that manifests the source of the gravitational radiation.) A device similar to miniGrail will soon be launched at Saõ-Paolo, Brasil. Moreover, it is projected a "big Grail" which mass expects to be 110 tons.

As supposed, the sources of gravitational radiation should be the explosions of super-novae, stellar binaries, pulsars, and the other phenomena in the core of which lies the same process: two masses, which rotate round the common centre of inertia, loose the energy of gravitational interaction with time so shorten the distance between them; the lost energy of gravitational interaction exceeds into space with gravitational radiation [1]. In the same time, we may expect the sources of gravitational radiation existing in not only the far cosmos, but also in the solar system and even in the Earth. The nearest cosmic source of gravitational waves should be the system Earth-Moon. Besides, even motion of tectonic masses should generate gravitational radiation. Timely registering gravitational radiation produced by such tectonic masses, we could reach a good possibility for the prediction of earthquakes.

Here we represent a device, which could be considered as a gravitational wave detector of a new kind, which is a resonant-dynamic system. The core of such a detector is a rotating body (made from metal or ceramics) in the state of negative acceleration. Besides the advantage of the whole system is that is gives a possibility for easy registration of the direction of the gravitation wave moved through it.

2 The dynamical scheme of the device

Fig. 1 shows the dynamical scheme of the device, where the rotating body is a 200 g cylindrical pig made from brass and shaped as a cup (it is marked by number 1). The rotor is fixed up on the axis of a micro electrical motor of direct current (number 2). In the continuation of the axis 3 of the motor a thick disc made from aluminum (number 4) is located; the other side of the disc is painted by a light-absorbing black color ink, except of the small reflecting sector 5. There over the disc, an azimuth circle 6 is located, it is for orientation of the device to the azimuth coordinates (they can further be processed into the geographical coordinates of the sources of a registered signal, or the celestial coordinates of it if it is located in the cosmos). The azimuth circle has a optical pair consisting of semiconductor laser as emitter and photodiode as receiver 7. A laser beam, reflected by the sector 5, acts onto the photodiode. The electrical motor 2 is fixed on the rectangular magnetic platform 8, which is suspended by the strong counter-field 0.3 Tesla of the stationary fixed magnet 9. There between the magnetic platforms an inductive detector 10 is located.

We consider the functional dependencies between the elements of this device. The rotor 1 turns into rotation by the

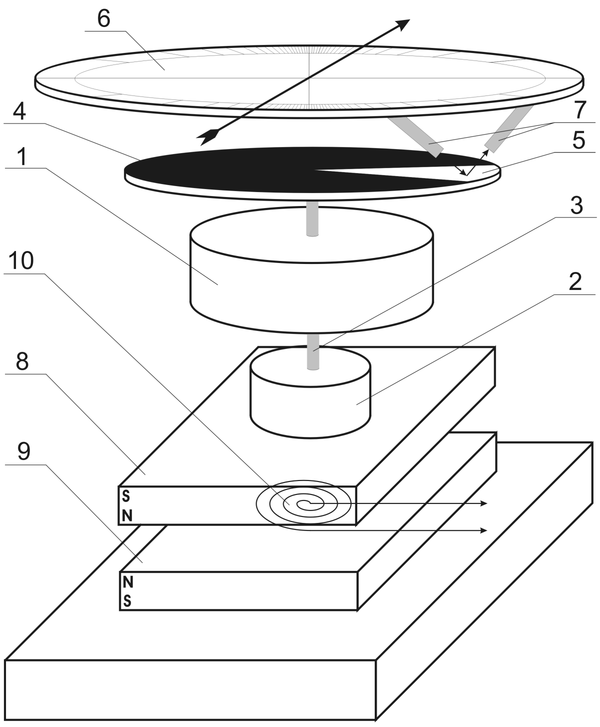

Fig. 1: Dynamical scheme of the device.

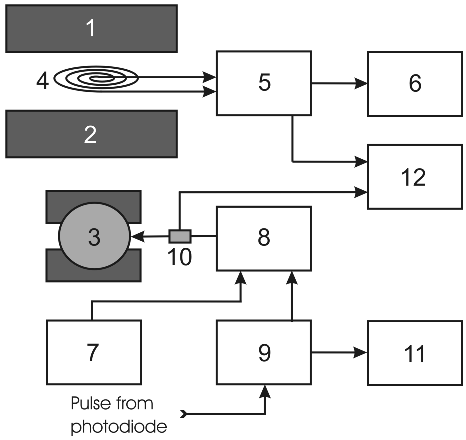

Fig. 2: Diagram of the control block.

electric motor to 4.000 rpm; the disc 4 rotates synchronically with the rotor. Once the reflected laser beam falls onto the photodiode, it produces an electric current. The pulse signal, produced by the photodiode, goes into the control electronic block which produces a rectangular pulse of voltage with the regulated duration in the scale from 1.5 to 4.0 μsec. Next time these impulses go into the input of the motor driver. If the output of the driver had a stable voltage with the polarity $(+, -)$, the inverts to $(-, +)$ in the moment when the electric pulse acts. For this moment the motor's rotation is under action of a negative acceleration: the rotation is braking for a short time. During the braking a reverse pulse current is inducted in the motor circuit, that is a "braking current" appears a form of which is under permanent control on the screen of an control oscilloscope.

Fig. 2 shows the block diagram of the control block. There are: the rectangular magnetic platform 1, in common with the rotor and the motor 3 located on it; the stationary fixed magnetic platform 2; the inductive detector 4; selective amplifier 5 working in the range from 10 Hz to 20 kHz; plotter 6; the source of the power for the electrical motor (number 7); the driver 8; the electronic block for processing of the electric pulse coming out from the photodiode (number 9); the inductive detector of the pulse current (number 10); the indicator of the angular speed of the rotor (number 11); oscilloscope 12.

At the end of braking pulse finishes (if to be absolutely exact — on falling edge of pulse) the electrical motor rotating with inertia re-starts, so a positive acceleration appears in the system. The starting pulse is due to the strong starting currents in the power supply circuit. According to Ampere's law, the occurred starting current leads to a mechanical impact experienced by the electrical motor armature (it is the necessary condition for the work of the whole device as a detector of gravitational perturbation). During the rotor's rotation, the whole spectrum of the low frequent oscillations produced by this mechanical impact are transferred to the mechanical platform 1, which induces electromotive force on the detector 4. This signal is transferred to the selective amplifier 5, wherein a corresponding harmonic characterizing the rotor's state is selected. This harmonic, converted into analogous signal, is transferred to the plotter 6.

3 The peculiarities of the experiment

The impulsive mechanical impact experienced by the motor armature is actually applied to the centre of the fixation of the rotor at the axis of electrical motor. The rotor, having a form of cylindrical resonator, reach excitation with low frequency due to this impact. In order to increase the excitation effect, a brass bush seal was set up on the motor's axis: the contact surface between the axis and the rotor became bigger than before that. As a result in the rotor a standing sonar wave occurs which has periodically excited, while all the time between the excitations it dissipates energy. The rotor, as a low frequent resonator, has its own resonant frequency, which was measured with special equipment by the method of the regulated frequent excitation and laser diagnostics. (The necessity to know the resonator frequency of the rotor proceeded from the requirement to choose the frequency of its rotation and also the frequency of its excitation.

Effect produced in the rotor due to a gravitational perturbation consist of the change of the period of its rotation that leads to the change in the initially parameters of the whole system: the shift of the operating point on frequency response function of selective amplifier and also the signal's amplitude changed at the output of the selective amplifier. Besides the

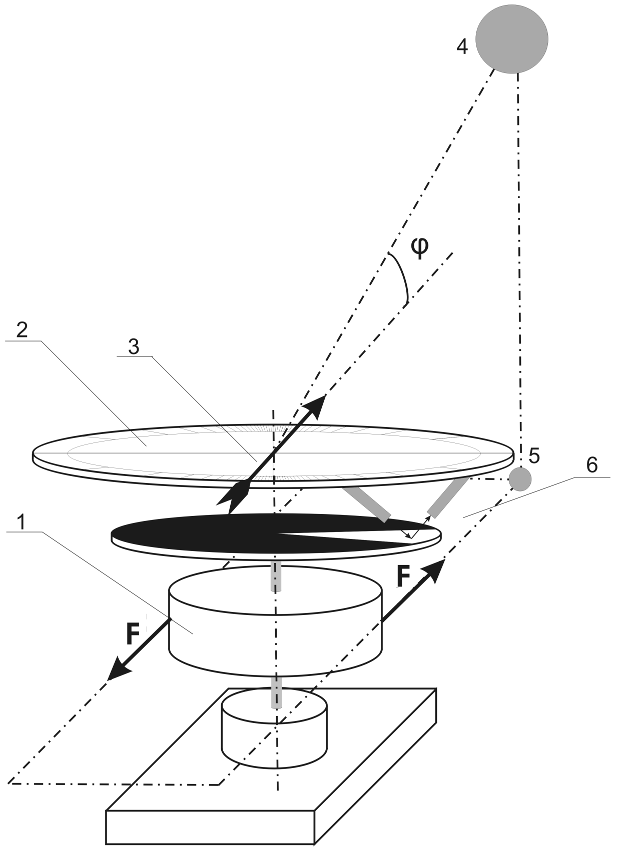

Fig. 3: Diagram of the device orientation at the supposed source of gravitational perturbation.

change of the angular speed of the rotation, due to the momentum conservation law, produces a reaction in the magnetic platform. Because the magnetic has rectangular form, the magnetic field between the platforms 1 and 2 (see Fig. 2) is non-uniform so the derivative of the density of the magnetic flow is substantial. All these lead to the fast change in the level of the signal's amplitude, and are defining the sensitivity of the whole device.

Plotter registered such a summarized change of the signal's amplitude.

Thus the sensitivity of the device is determined by the following parameters: (1) the choice for the required resonant frequency of the rotor; (2) the choice of the angular speed of its rotation; (3) the duration of the braking pulse; (4) the choice for the information sensor which gives information about the rotor; (5) the factor of the orientation of the device at the supposed source of gravitational perturbation.

The vector of the device orientation is the direction of the impulsive braking force F or, that is the same, the negative acceleration vector applied to the rotor. In the moment of braking there a pair of forces F appears, which are applied to the rotor. The plane where the forces act is the antennae parameter of the system. Fig. 3 represent a fragment of the device, where 1 is the rotor, 2 is the azimuth circle, 3 are the indicators of direction, where the angular scale has the origin of count (zero degree) pre-defined to the Southern pole. If we suppose that the source of gravitational perturbation (it is pictured by gray circle, 4) is a cosmic object, the device should be oriented to the projection of this source onto the horizontal plane (this projection is marked by number 5, and pictured by small gray circle). The plane 6 is that for the acting forces of braking.

4 Experimental results

Here are typical experimental results we got on the device over a years of investigations.

The fact that such a device works as an antenna permits to turn it so that it will be directed in exact at the selected space objects in the sky or the earthy sources located at different geographical coordinates.

First, we were looking for the gravitational field perturbations due to the tectonic processes that could be meant the predecessors of earthquakes. Using the geographic map of the tectonic breaks, we set up an experiment on the orientation of the device to such breaks. Despite the fact that exact measurement of such directions is possible by a system of a few devices (or in that case where the device is located in area of a tectonic brake), the measurement of the azimuth direction by our device was as precise as $\pm 2°$. The azimuthal directions were counted with respect to the South pole. All measurement represented on the experimental diagrams (Fig. 4–9) are given with Moscow time, because the measurement were done at Moscow, Russia. The period of the rotation of the gyro changed in the range from 75 μsec to 200 μsec during all the measurements produced on: the rises and sets of the planets of the solar system (including the Moon) and also those of the Sun; the moments when the full moon and new moon occurs; the solar and lunar eclipses; the perihelion and aphelion of the Earth, etc. In some experiments (Fig. 6) extremely high gravitational perturbations were registered, during which the period of the rotation of the gyro was changed till 400 μsec and even more (the duration of such extremely high perturbations was 5–10 minutes on the average). Further we found a correlation of the registered signals to the earthquakes. The correlation showed: the perturbations of the earthy gravitational field, registered by our device, predesecced the earthquakes in the range from 3 to 15 days in the geographic areas whereto the device was directed (Fig. 4–6).

Examples of records in Fig. 7–8 present transit of Venus through the disc of the Sun (Fig. 7) and solar eclipse at Moscow, which occur at November 03, 2005.

Aside for such single signals as presented in Fig. 4–8, our device registered also periodic signals. The periodic signals were registered twice a year, in October and May, that are two points in the chord of the orbit of the Earth which connects the directions to Taurus and Virgo. The time interval between

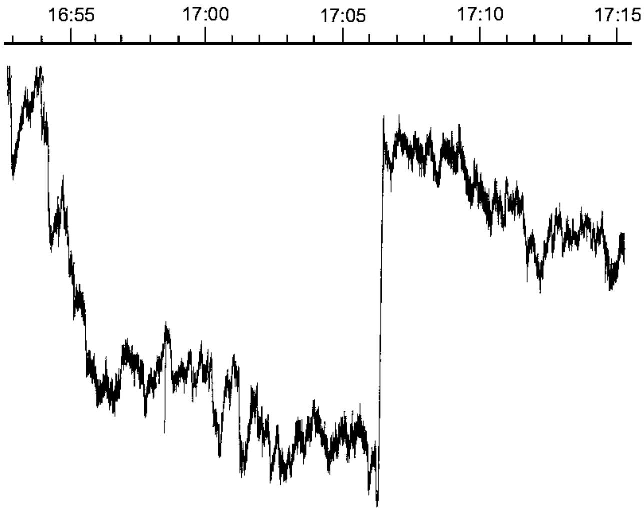

Fig. 4: June 30, 2005. The azimuth of the signal is ∼53° to East. The predecessing signal of the earthquake in the Indian Ocean near Sumatra Island, Indonesia, July 05, 2005.

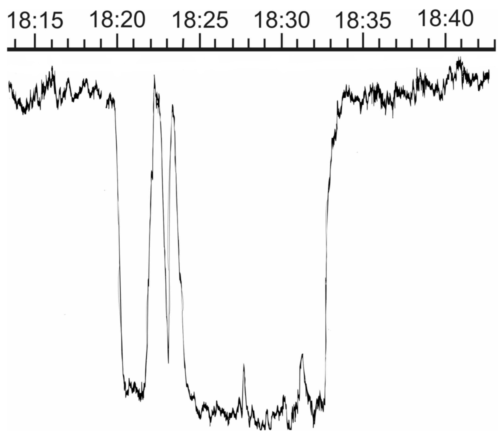

Fig. 5: March 29, 2006. The azimuth of the signal is ∼9° to East: the predecessor of the earthquake in the Western Iran, which occurred on April 02, 2006.

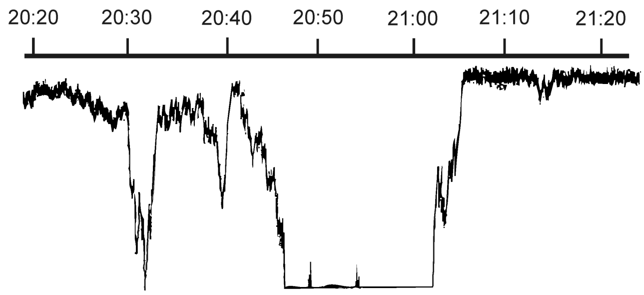

Fig. 6: May 05, 2007. A high altitude gravitational perturbation. The azimuth of the signal is 122° to West. The central states of the USA became under action of 74 destructing tornados two days later, on May 08, 2007.

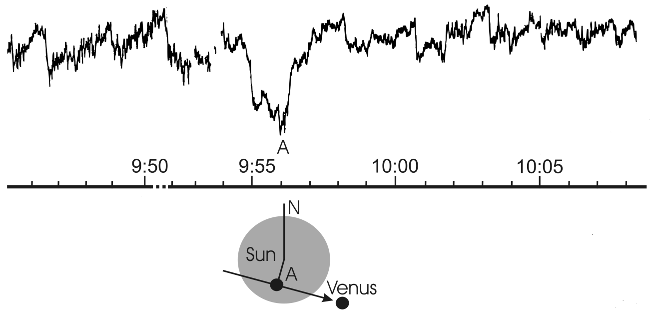

Fig. 7: June 08, 2004. Transit of Venus through the disc of the Sun, $09^{\mathrm{h}}51^{\mathrm{min}}$.

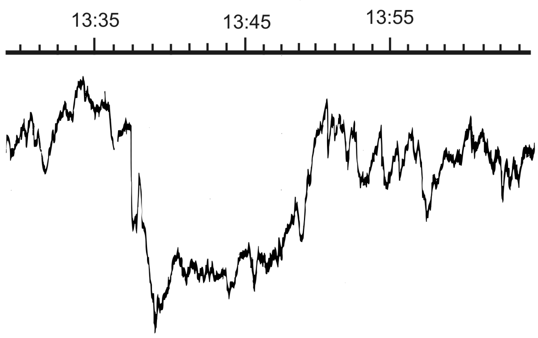

Fig. 8: November 03, 2005. The solar eclipse at Moscow city, Russia. The eclipse phase is ∼0.18.

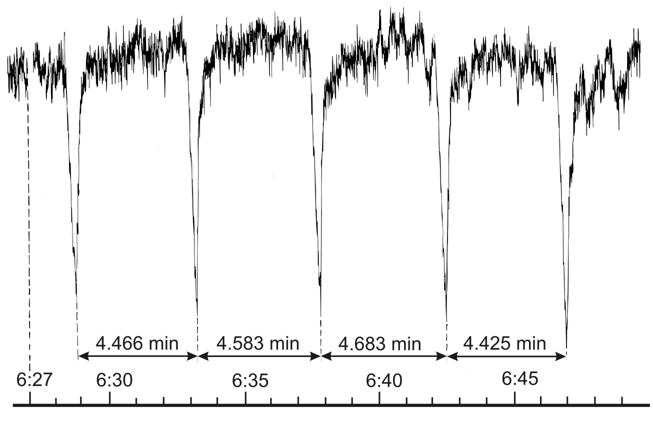

Fig. 9: May 31, 2003. Periodical signals.

the signals growing with the motion of the Earth along its orbit during 5 days then deceased. A fragment of the graph is represented in Fig. 9.

It should be noted that when Joseph Weber claimed about a gravitational wave signal registered with his solid-body detector [3–5], he pointed out that fact that the solely registered signal came from Taurus.

5 Conclusion

The core of the device is a rotating body (in our case it is a rotating brass resonator), which sensitivity to gravitational radiation lies in its excitation expected in the field of a falling gravitational wave. Despite the physical state of the gyro-resonator corresponds, in main part, to the wave-guide solid-body gyros, its internal construction and the principles of work are substantially different from those [6].

The device manifested the possibility to register gravitational perturbations of non-seismic kind from the internal processed in the terrestrial globe, and locate the terrestrial coordinates of the sources of the perturbations.

An auxiliary confirmation of such a principle for the registration of gravitational perturbation is that fact that one of the gyros CMG-3 working on board of the International Space Station "experienced an unusual high vibration" on March 28, 2005 (it was registered by the space station commander Leroy Chiao and the astronaut Salizhan Sharipov [7]), in the same time when a huge earthquake occurred near Nias Island (in the shelf of Indian Ocean, close to Sumatra, Indonesia).

This device is a really working instrument to be used for the aforementioned tasks. In the same time, a lack of attention to it brakes the continuation of the experiments till the stop of the whole research program in the near future.

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Third Quantization in Bergmann-Wagoner Theory

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We present the third quantization of Bergmann-Wagoner scalar-tensor and Brans Dicke solvable toy models. In the first one we used an exponential cosmological term, for the second one we considered vanishing cosmological constant. In both cases, it is found that the number of the universes produced from nothing is very large.

1 Introduction

The Wheeler-DeWitt (WDW) equation is a result of quantization of a geometry and matter (second quantization of gravity), in this paper we consider the third quantization of a solvable inflationary universe model, i.e., by analogy with the quantum field theory, it can be done the second quantization of the universe wavefunction ψ expanding it on the creation and annihilation operators (third quantization) [1]. Because in the recent years there has been a great interest in the study of scalar-tensor theories of gravitation, owing that of the unified theories [2, 3], we choose to work with the most general scalar-tensor theory examined by Bergmann and Wagoner [4, 5], in this theory the Brans-Dicke parameter ω and cosmological function λ depend upon the scalar gravitational field ϕ. The Brans-Dicke theory can be obtained setting $\omega = \text{const}$ and $\lambda = 0$.

The WDW equation is obtained by means of canonical quantization of Hamiltonian H according to the standard canonical rule, this leads to a difficulty known as the problem of time [6]. Also, this equation has problems in its probabilistic interpretation. In the usual formulation of quantum mechanics a conserved positive-definite probability density is required for a consistent interpretation of the physical properties of a given system, and the universe in the quantum cosmology perspective, do not satisfied this requirement, because the WDW equation is a hyperbolic second order differential equation, there is no conserved positive-definite probability density as in the case of the Klein-Gordon equation, an alternative to this, is to regard the wavefunction as a quantum field in minisuperspace rather than a state amplitude [7].

The paper is organized as follows. In Section 2 we obtain the WDW equation. In Section 3 we show third quantization of the universe wavefunction using two complete set of modes for the most easy choice of factor ordering. Finally, Section 4 consists of conclusions.

2 Canonical formalism

Our starting point is the action of Bergmann-Wagoner scalar tensor theory

$$S = \frac{1}{l_p^2}\int_M \sqrt{-g}\left[\phi R^{(4)} - \frac{\omega(\phi)}{\phi}\, g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} + 2\,\phi\,\lambda(\phi)\right] d^4x + \\ + \frac{2}{l_p^2}\int_{\partial M}\sqrt{h}\,\phi\, h_{ij}K^{ij}\, d^3x\,, \tag{1}$$

where $g = \det(g_{\mu,\nu})$, $\phi(t)$ is the conventional real scalar gravitational field, while l_p is the Planck length and $\lambda(\phi)$ is the cosmological term. The quantity $R^{(4)}$ is the scalar curvature of the Friedmann-Robertson-Walker theory, which is given, according to the theory, by

$$R^{(4)} = -\frac{6k}{a^2} - 6\frac{\dot{a}^2}{N^2a^2} - 6\frac{\ddot{a}}{N^2a} + 6\frac{\dot{a}\dot{N}}{N^3a}\,. \tag{2}$$

The second integral in (1) is a surface term involving the induced metric h_{ij} and second fundamental form K^{ij} on the boundary, needed to cancel the second derivatives in $R^{(4)}$ when the action is varied with the metric and scalar field, but not their normal derivatives, fixed on the boundary. Substituting (2) in (1) and integrating with respect to space coordinates, we have

$$S = \frac{1}{2}\int\left[-Nka\phi + \frac{a\phi}{N}\,\dot{a}^2 + \frac{a^2}{N}\,\dot{a}\dot{\phi} - \right. \\ \left. - \frac{N\omega(\phi)}{6\phi}\,a^3\dot{\phi}^2 + \frac{N}{3}\,a^3\phi\lambda(\phi)\right]dt\,, \tag{3}$$

where dot denotes time derivative with respect to the time t, now introducing a new time $d\tau = \phi^{\frac{1}{2}}dt$ and the following independent variables

$$\alpha = a^2\phi\cosh\int\left(\frac{2\omega(\phi)+3}{3}\right)^{\frac{1}{2}}\frac{d\phi}{\phi}\,, \tag{4}$$

$$\beta = a^2\phi\sinh\int\left(\frac{2\omega(\phi)+3}{3}\right)^{\frac{1}{2}}\frac{d\phi}{\phi}\,, \tag{5}$$

$$\lambda(\phi) = 3\phi\left[\Lambda_1\cosh\int\left(\frac{2\omega(\phi)+3}{3}\right)^{\frac{1}{2}}\frac{d\phi}{\phi} + \right. \\ \left. + \Lambda_2\sinh\int\left(\frac{2\omega(\phi)+3}{3}\right)^{\frac{1}{2}}\frac{d\phi}{\phi}\right], \tag{6}$$

where Λ_1 and Λ_2 are constants, with gauge $N=1$, then action (3) transforms into a symmetric form

$$S=\frac{1}{2}\int\left[\frac{1}{4}\left(\alpha'^2-\beta'^2\right)+\Lambda_1\alpha+\Lambda_2\beta-k\right]d\tau, \quad (7)$$

here prime denotes time derivative with respect to τ. The Hamiltonian of the system is

$$H=2\pi_\alpha^2-2\pi_\beta^2+\frac{1}{2}(k-\Lambda_1\,\alpha-\Lambda_2\beta). \quad (8)$$

After canonical quantization of H, the WDW equation is

$$\Big[\partial_\alpha^2+A\alpha^{-1}\partial_\alpha-\partial_\beta^2-B\beta^{-1}\partial_\beta+ + \frac{1}{4}(\Lambda_1\,\alpha+\Lambda_2\beta-k)\Big]\psi(\alpha,\beta)=0, \quad (9)$$

where A and B are ambiguity ordering parameters. The general universe wavefunction for this model can be given in terms of Airy functions.

3 Third quantization

The procedure of the universe wavefunction ψ quantization is called third quantization, in this theory we consider ψ as an operator acting on the state vectors of a system of universes and can be decomposed as

$$\hat{\psi}(\alpha,\beta)=\hat{C}_i\,\psi_i^+(\alpha,\beta)+\hat{C}_i^\dagger\,\psi_i^-(\alpha,\beta)\,, \quad (10)$$

where $\psi_i^\pm(\alpha,\beta)$ form complete orthonormal sets of solutions to WDW equation. This is in analogy with the quantum field theory, where $\hat{C}_i$ and $\hat{C}_i^\dagger$ are creation and annihilation operators. Thus, we expect that the vacuum state in a third quantized theory is unstable and creation of universes from the initial vacuum state takes place. In this view, the variable α plays the role of time, and variable β the role of space. $\psi(\alpha,\beta)$ is interpreted as a quantum field in the minisuperspace.

We assume that the creation and annihilation operators of universes obey the standard commutation relations

$$\left[\,C(s),C^\dagger(s')\,\right]=\delta(s-s'), \quad (11)$$

$$\left[\,C(s),C(s')\,\right]=\left[\,C^\dagger(s),C^\dagger(s')\,\right]=0\,. \quad (12)$$

The vacuum state $|0\rangle$ is defined by

$$C(s)|0\rangle \quad \text{for} \quad \forall\, C\,, \quad (13)$$

and the Fock space is spanned by $C^\dagger(s_1)C^\dagger(s_2)...|0\rangle$. The field $\psi(\alpha,\beta)$ can be expanded in normal modes ψ_s as

$$\psi(\alpha,\beta)=\int_{-\infty}^{+\infty}\left[\,C(s)\,\psi_s(\alpha,\beta)+C^\dagger(s)\,\psi_s^*(\alpha,\beta)\right]ds\,, \quad (14)$$

here, the wave number s is the momentum in Planck units and is very small.

3.1 General model

Let us consider the quantum model (9) for the most easy factor ordering $A=B=0$, with $\Lambda_2=0$ and closed universe $k=1$. Then, the WDW equation becomes

$$\left[\partial_\alpha^2-\partial_\beta^2+\frac{1}{4}(\Lambda_1\alpha-1)\right]\psi(\alpha,\beta)=0, \quad (15)$$

the third-quantized action to yield this equation is

$$S_{3Q}=\frac{1}{2}\int\left[(\partial_\alpha\psi)^2-(\partial_\beta\psi)^2-\frac{1}{4}(\Lambda_1\alpha-1)\psi^2\right]d\alpha\,d\beta\,, \quad (16)$$

this action can be canonically quantized and we impose the equal time commutation relations

$$\left[\,i\,\partial_\alpha\psi(\alpha,\beta),\psi(\alpha,\beta')\,\right]=\delta(\beta-\beta')\,, \quad (17)$$

$$\left[\,i\,\partial_\alpha\psi(\alpha,\beta),i\,\partial_\alpha\psi(\alpha,\beta')\,\right]=0\,, \quad (18)$$

$$\left[\,\psi(\alpha,\beta),\psi(\alpha,\beta')\,\right]=0\,. \quad (19)$$

A suitable complete set of normalized positive frequency solutions to equation (15) are:

$$\psi_s^{out}(\alpha,\beta)=\frac{e^{is\beta}}{(16\Lambda_1)^{\frac{1}{16}}}\Big\{\mathrm{Ai}\left[(2\Lambda_1)^{-\frac{2}{3}}(1-4s^2-\Lambda_1\alpha)\right]+ + i\,\mathrm{Bi}\left[(2\Lambda_1)^{-\frac{2}{3}}(1-4s^2-\Lambda_1\alpha)\right]\Big\}, \quad (20)$$

and

$$\psi_s^{in}(\alpha,\beta)=\frac{\sqrt{2}\,e^{is\beta}}{(16\Lambda_1)^{\frac{1}{16}}}\times \times\Big\{e^{\frac{(1-4s^2)^{\frac{3}{2}}}{3\Lambda_1}}\mathrm{Ai}\left[(2\Lambda_1)^{-\frac{2}{3}}(1-4s^2-\Lambda_1\alpha)\right]+ + \frac{i}{2}\,e^{-\frac{(1-4s^2)^{\frac{3}{2}}}{3\Lambda_1}}\mathrm{Bi}\left[(2\Lambda_1)^{-\frac{2}{3}}(1-4s^2-\Lambda_1\alpha)\right]\Big\}, \quad (21)$$

$\psi_s^{out}(\alpha,\beta)$ and $\psi_s^{in}(\alpha,\beta)$ can be seen as a positive frequency out going and in going modes, respectively, and these solutions are orthonormal with respect to the Klein-Gordon scalar product

$$\langle\psi_s,\psi_{s'}\rangle=i\int\psi_s\,\overset{\leftrightarrow}{\partial}_\beta\,\psi_{s'}^*\,d\beta=\delta(s-s')\,. \quad (22)$$

The expansion of $\psi(\alpha,\beta)$ in terms of creation and annihilation operators for the in-mode and out-mode is

$$\psi(\alpha,\beta)=\int\left[C_{in}(s)\,\psi_s^{in}(\alpha,\beta)+ + C_{in}^\dagger(s)\,\psi_r^{in*}(\alpha,\beta)\right]ds\,, \quad (23)$$

and

$$\psi(\alpha,\beta)=\int\left[C_{out}(s)\,\psi_s^{out}(\alpha,\beta)+ + C_{out}^\dagger(s)\,\psi_r^{out*}(\alpha,\beta)\right]ds\,. \quad (24)$$

As both sets (20) and (21) are complete, they are related to each other by the Bogoliubov transformation defined by

$$\psi_s^{out}(\alpha,\beta) = \int \left[C_1(s,r)\,\psi_r^{in}(\alpha,\beta) + \right. \\ \left. + \; C_2(s,r)\,\psi_r^{in*}(\alpha,\beta) \right] dr\,, \qquad (25)$$

and

$$\psi_s^{in}(\alpha,\beta) = \int \left[C_1(s,r)\,\psi_r^{out}(\alpha,\beta) + \right. \\ \left. + \; C_2(s,r)\,\psi_r^{out*}(\alpha,\beta) \right] dr\,. \qquad (26)$$

Then, we obtain that the Bogoliubov coefficients $C_1(s,r) = \delta(s-r)C_1(s)$ and $C_2(s,r) = \delta(s+r)C_2(s)$ are

$$C_1(s,r) = \delta(s-r)\frac{1}{\sqrt{2}} \times \\ \times \left(\frac{1}{2}\, e^{-\frac{(1-4s^2)^{\frac{3}{2}}}{3\Lambda_1}} + e^{\frac{(1-4s^2)^{\frac{3}{2}}}{3\Lambda_1}} \right), \qquad (27)$$

and

$$C_2(s,r) = \delta(s+r)\frac{1}{\sqrt{2}} \times \\ \times \left(\frac{1}{2}\, e^{-\frac{(1-4s^2)^{\frac{3}{2}}}{3\Lambda_1}} + e^{\frac{(1-4s^2)^{\frac{3}{2}}}{3\Lambda_1}} \right). \qquad (28)$$

The coefficients $C_1(s,r)$ and $C_2(s,r)$ are not equal to zero. Thus, two Fock spaces constructed with the help of the modes (20) and (21) are not equivalent and we have two different third quantized vacuum states (voids): the in-vacuum $|\,0,\,in\,\rangle$ and out-vacuum $|\,0,\,out\,\rangle$ (which are the states with no Friedmann Robertson Walker-like universes) defined by

$$C_{in}(s)\;|\;0, in\rangle = 0 \quad \text{and} \quad C_{out}(s)\;|\;0, out\,\rangle = 0, \qquad (29)$$

where $s \in \mathbf{R}$. Since the vacuum states $|\,0,\,in\,\rangle$ and $|\,0,\,out\,\rangle$ are not equivalent, the birth of the universes from nothing may have place, where nothing is the vacuum state $|\,0, in\,\rangle$. The average number of universes produced from nothing, in the s-tn mode $N(s)$ is

$$N(s) = \Big\langle 0, in\;|\;C^{\dagger}_{out}(s)\,C_{out}(s)\;|\;0, in \Big\rangle\,, \qquad (30)$$

as follows from equation (25) we get

$$N(s) = \frac{1}{2}\left(\frac{1}{2}\, e^{-\frac{(1-4s^2)^{\frac{3}{2}}}{3\Lambda_1}} - e^{\frac{(1-4s^2)^{\frac{3}{2}}}{3\Lambda_1}} \right)^2, \qquad (31)$$

considering Coleman's wormhole mechanism [8] for the vanishing cosmological constant and the constraint $\Lambda_1 \leqslant \frac{1}{8}\pi \times 10^{-120}\, m_p^4$, with $|s| \ll 1$, then the number of state $N(s)$ is

$$N(s) \approx \frac{1}{2}\, e^{\frac{2}{3\Lambda_1}(1-4s^2)^{\frac{3}{2}}}\,. \qquad (32)$$

This result from third quantization shows that the number of the universes produced from nothing is exponentially large.

3.2 Particular model

An interesting model derived from Bergmann Wagoner action (1) with $\omega(\phi) = \omega_0 = \text{const}$, $\lambda(\phi) = 0$ and $N = 1$ is the Brans-Dicke theory

$$S = \frac{1}{l_p^2}\int \sqrt{-g}\left[\phi R^{(4)} - \frac{\omega(\phi)}{\phi}\, g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} + 2\phi\lambda(\phi) \right] dt\,. \quad (33)$$

By means of new variables

$$x = \ln(a^2\phi)\,, \qquad y = \ln \phi^{\frac{1}{\rho}}\,, \qquad dt = a d\tau\,, \qquad (34)$$

where $\rho^2 = \frac{3}{2\omega_0+3}$, action (33) transforms into

$$S = \frac{1}{2}\int \left[\frac{x'^2}{4} - \frac{y'^2}{4} - 1 \right] e^x d\tau\,, \qquad (35)$$

the WDW equation for this model is

$$\left[x^{-A}\partial_x(x^A\partial_x) - \partial_y^2 - \frac{e^{2x}}{4} \right] \psi(x,y) = 0\,, \qquad (36)$$

the ambiguity of factor ordering is encoded in the A parameter. The third quantized action to yield the WDW equation (36) is

$$S_{3Q} = \frac{1}{2}\int \left[(\partial_x\psi)^2 - (\partial_y\psi)^2 + \frac{e^{2x}}{4}\,\psi^2 \right] dx\, dy\,. \quad (37)$$

Again, in order to quantize this toy model, we impose equal time commutation relations given by (17–19), and by means of normal mode functions ψ_p we can expand the field $\psi(x,y)$. A suitable normalized out-mode function with positive frequency for large scales, is

$$\psi_p^{out}(x,y) = \frac{1}{2\sqrt{2}}\, e^{-\frac{\pi}{2}|p|}\, H^{(2)}_{-q}\, \frac{ie^x}{2}\, e^{ipy}, \qquad (38)$$

where $H^{(2)}_{-q}$ is a Hankel function and $q = -i\,|p|$. The normalized in-mode function is

$$\psi_p^{in}(x,y) = \frac{e^{\frac{\pi}{2}|p|}}{2\sinh^{\frac{1}{2}}(\pi\,|p|)}\, J_q\, \frac{ie^x}{2}\, e^{ipy}, \qquad (39)$$

where J_q is a first class Bessel function. In the classically allowed regions the positive frequency modes correspond to the expanding universe [9]. As both wavefunctions (38) and (39) are complete, they are related to each other by a Bogoliubov transformation. The corresponding coefficients are

$$C_1(p,q) = \delta(p-q)\frac{1}{\sqrt{1 - e^{-2\pi|p|}}}\,, \qquad (40)$$

and

$$C_2(p,q) = \delta(p+q)\frac{1}{\sqrt{e^{2\pi|p|} - 1}}\,. \qquad (41)$$

The coefficients $C_1(p)$ and $C_2(p)$ are not equal to zero and satisfy the probability conservation condition

$$|\,C_1(p)\,|^2 - |\,C_2(p)\,|^2 = 1\,.$$

In this way, it can be constructed two not equivalent Fock spaces by means of (38) and (39). These two different third quantized vacuum states, the in-vacuum $|\,0,\,in\rangle$ and out-vacuum $|\,0,\,out\rangle$ are defined by (29). The average number of universes created from nothing, i.e., the in-vacuum in the p-th $N(p)$, is

$$\begin{aligned} N(p) &= \langle 0, in \mid C^{\dagger}_{out}(p)\, C_{out}(p) \mid 0, in\rangle = \\ &= |\, C_2(p)\,|^2 = \\ &= \frac{1}{e^{2\pi|p|}-1}\,. \end{aligned} \tag{42}$$

This expresion corresponds to a Planckian distribution of universes.

4 Conclusions

By means of a suitable choice of lapse function and independent variables, we have solved the WDW equation in the Bergmann-Wagoner gravitational theory for a cosmological function of the form $\lambda(\phi) = \Lambda_1 \cosh[2y(\phi)] + \Lambda_2 \sinh[2y(\phi)]$, this kind of cosmological term is important because of new scenario of extended inflation [10]. Also, we have studied on the third quantization of Bergmann-Wagoner and Brans-Dicke models, in which time is related by the scalar factor of universe and the space coordinate is related with the scalar field. The universe is created from stable vacuum obtained by the Bogoliubov-type transformation just as it is in the quantum field theory.

One of the main results of third quantization is that the number of universes produced from nothing is exponentially large. We calculated the number density of the universes creating fron nothing and found that the initial state $|\;0, in\rangle$ is populated by a Planckian distribution of universes.

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Gravity Model for Topological Features on a Cylindrical Manifold

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A model aimed at understanding quantum gravity in terms of Birkhoff's approach is discussed. The geometry of this model is constructed by using a winding map of Minkowski space into a $\mathbb{R}^3 \times S^1$-cylinder. The basic field of this model is a field of unit vectors defined through the velocity field of a flow wrapping the cylinder. The degeneration of some parts of the flow into circles (topological features) results in inhomogeneities and gives rise to a scalar field, analogous to the gravitational field. The geometry and dynamics of this field are briefly discussed. We treat the intersections between the topological features and the observer's 3-space as matter particles and argue that these entities are likely to possess some quantum properties.

1 Introduction

In this paper we shall discuss a mathematical construction aimed at understanding quantum gravity in terms of Birkhoff's twist Hamiltonian diffeomorphism of a cylinder [1]. We shall also use the idea of compactification of extra dimensions due to Klein [2]. To outline the main idea behind this model in a very simple way, we can reduce the dimensionality and consider the dynamics of a vector field defined on a 2-cylinder $\mathbb{R}^1 \times S^1$. For this purpose we can use the velocity field $u(x,\tau)$ of a two-dimensional flow of ideal incompressible fluid moving through this manifold.

Indeed, the dynamics of the vector field $u(x,\tau)$ with the initial condition $u(x,0)$ is defined by the evolution equation

$$\delta \int_{\Delta\tau} \int_{\Delta x} dx \wedge u(x,\tau) d\tau \to 0\,, \tag{1.1}$$

where we use the restriction of the vector field onto an arbitrary cylinder's element; $\Delta\tau$ is the evolution (time) interval, and Δx is an arbitrary segment of the cylinder's element. In other words, we assume the variation of the integral of the mass carried by the flow through the segment during a finite time interval to be vanishing. That is, as a result of the field evolution, $u(x,0) \to u(x,\infty)$, the functional of the flow mass approaches to its maximal value. If, at the initial moment of time, the regular vector field $u(x,0)$ corresponds to a unit vector forming an angle φ with the cylinder's element, then the evolution of this field is described by the equation

$$\delta \int_{\Delta\tau} \int_{\Delta x} dx \wedge u(x,\tau) d\tau = \\ = \delta \int_{\Delta\tau} \int_{\Delta x} \sin\varphi(\tau) dx d\tau = \cos\varphi(\tau) \Delta\tau \Delta x \to 0\,. \tag{1.2}$$

Therefore, the case of $\varphi(0)=0$ corresponds to the absolute instability of the vector field. During its evolution, $u(x,0) \to u(x,\infty)$, the field is relatively stable at $0 < \varphi(\tau) < \frac{\pi}{2}$, achieving the absolute stability at the end of this evolution, when $\varphi(\infty) = \frac{\pi}{2}$. If, additionally, we fix the vector field $u(x,\tau)$ at the endpoints of the segment Δx by imposing some boundary conditions on the evolution equation (1.1), we would get the following dynamical equation:

$$\delta \int_{\Delta\tau} \int_{\Delta x} dx \wedge u(x,t) d\tau = 0\,. \tag{1.3}$$

Let some flow lines of the vector field $u(x,\tau)$ be degenerated into circles (topological features) as a result of the absolute instability of the field and fluctuations during the initial phase of its evolution. Since the dynamics of such topological features is described by (1.3), the features would tend to move towards that side of Δx where the field $u(x,\tau)$ is more stable. Thus, the topological features serve as attraction points for each other and can be used for modelling matter particles (mass points).

We must emphasise that the plane (x,τ), in which our variational equations are defined, has the Euclidean metric. That is, in the case of the Euclidean plane (x,ϕ) wrapping over a cylinder we can identify the azimuthal parameter ϕ with the evolution parameter τ. By choosing the observer's worldline coinciding with a cylinder's element we can speak of a classical limit, whereas by generalising and involving also the azimuthal (angular) parameter we can speak of the quantisation of our model. So, when the observer's worldline is an arbitrary helix on the cylinder, the variational equation (1.3) reads

$$\delta \int_{\Delta x_0} \int_{\Delta x_1} dx_1 \wedge g(x) dx_0 = 0\,, \tag{1.4}$$

where the varied is the vector field $g(x)$ defined on the pseudo-Euclidean plane (x_0, x_1) oriented in such a way that one of its isotropic lines covers the cylinder-defining circle and the other corresponds to a cylinder's element. In this case we can speak of a relativistic consideration. If the observer's worldline corresponds to a curved line orthogonal to the flow lines of the vector field $g(x)$, where $g^2(x) > 0$, then we have to use the variational equation defined on a two-dimensional pseudo-Riemann manifold M induced by the vector field

$g(x)$, namely,

$$\delta \int_{\Delta M} g^2(x') \sqrt{-\det g_{ij}}\, dx'_0 \wedge dx'_1 = 0\,, \tag{1.5}$$

where $\Delta M = \Delta x'_0 \times \Delta x'_1$ is an arbitrary region of the manifold M; $x'_0(\phi)$ is the flow line of the vector field $g(x)$ parameterised by the angular coordinate ϕ; $x'_1(r)$ is the spatial coordinate on the cylinder (orthogonal to the observer worldline) parameterised by the Euclidean length r; g_{ij} is the Gram matrix corresponding to the pair of tangent vectors $\left(\frac{dx'_0}{d\phi}, \frac{dx'_1}{dr}\right)$. In this case the dynamics of the vector field is described through the geometry of its flow lines [3–5].

Thus, we can say that our approach to the dynamics of the vector field is based on maximisation of the mass carried by the flow [6, 7], which is not exactly what is typically used in the ergodic theory [8–10]. However, this principle is likely to be related to the the minimum principle for the velocity field [13–15], which is a special case of the more general principle of minimum or maximum entropy production [11, 12].

Before a more detailed discussion of this model we have to make a few preliminary notes. First, throughout this paper we shall use a somewhat unconventional spherical coordinates. Namely, latitude will be measured modulo 2π and longitude – modulo π. In other words, we shall use the following spherical $(\rho, \varphi, \theta_1, \ldots, \theta_{n-2})$ to Cartesian $(x_1, \ldots, x_n)$ coordinate transformation in $\mathbb{R}^n$:

$$\begin{aligned}
x_1 &= \rho \cos\varphi, \\
x_2 &= \rho \sin\varphi \cos\theta_1, \\
x_3 &= \rho \sin\varphi \sin\theta_1, \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
x_{n-1} &= \rho \sin\varphi\ \ldots\ \sin\theta_{n-3} \cos\theta_{n-2}, \\
x_n &= \rho \sin\varphi\ \ldots\ \sin\theta_{n-3} \sin\theta_{n-2},
\end{aligned}$$

where $0 \leqslant \rho < \infty$, $0 \leqslant \varphi < 2\pi$ and $0 \leqslant \theta_i < \pi$. We shall also be interpreting the projective space RP^n as the space of centrally symmetric lines in $\mathbb{R}^{n+1}$, that is, as a quotient space $\mathbb{R}^{n+1}\backslash\{0\}$ under the equivalence relation $x \sim r\,x$, where $r \in \mathbb{R}\backslash\{0\}$.

2 The geometry of the model

We can describe the geometry of our model in terms of the mapping of the Euclidean plane into a 2-sphere, S^2, by winding the former around the latter. We can also use similar winding maps for the pseudo-Euclidean plane into a cylinder, $\mathbb{R} \times S^1$, or a torus, $S^1 \times S^1$. More formally this could be expressed in the following way [16]. Take the polar coordinates (φ, ρ) defined on the Euclidean plane and the spherical coordinates (θ, ϕ) on a sphere. We can map the Euclidean plane into sphere by using the congruence classes modulo π and 2π. That is,

$$\theta = |\varphi| \bmod \pi, \quad \phi = |\pm \pi\rho| \bmod 2\pi, \tag{2.1}$$

where the positive sign corresponds to the interval $0 \leqslant \varphi < \pi$ and negative — to the interval $\pi \leqslant \varphi < 2\pi$. If the projective lines are chosen to be centrally symmetric then the Euclidean plane can be generated as the product $RP^1 \times \mathbb{R}$. Here the components of $\mathbb{R}$ are assumed to be Euclidean, i.e., rigid and with no mirror-reflection operation allowed. Similarly, we can define a space based on unoriented lines in the tangent plane to the sphere. Therefore, the sphere can be generated by the product $RP^1 \times S^1$, the opposite points of the circle being identified with each other. In this representation all centrally symmetric Euclidean lines are mapped as

$$\mathbb{R} \to S^1 : e^{i\pi x} = e^{\pm i\pi\rho} \tag{2.2}$$

by winding them onto the corresponding circles of the sphere.

The winding mapping of Euclidean space onto a sphe-re can be extended to any number of dimensions. Here we are focusing mostly on the case of Euclidean space, $\mathbb{R}^3$, generated as the product $RP^2 \times \mathbb{R}$ and also on the case of a 3-sphere generated as $RP^2 \times S^1$. In both cases we assume the Euclidean rigidity of straight lines and the identification of the opposite points on a circle. Euclidean space, $\mathbb{R}^3$, can be mapped into a sphere, S^3, by the winding transformation analogous to (2.1). Indeed, for this purpose we only have to establish a relation between the length of the radius-vector in Euclidean space and the spherical coordinate (latitude) measured modulo 2π. The relevant transformations are as follows:

$$\theta_1 = \vartheta\,, \quad \theta_2 = |\varphi| \bmod \pi, \quad \phi = |\pm\pi\rho| \bmod 2\pi, \tag{2.3}$$

where the sign is determined by the quadrant of φ.

Let (e_0, e_1) be an orthonormal basis on a pseudo-Euclidean plane with coordinates (x_0, x_1). Let the cylindrical coordinates of $\mathbb{R} \times S^1$ be (ϕ, r). Then the simplest mapping of this pseudo-Euclidean plane to the cylinder would be

$$\phi = |\pi(x_0 + x_1)| \bmod 2\pi, \quad r = x_0 - x_1\,. \tag{2.4}$$

That is, the first isotropic line is winded here around the cylinder's cross-section (circle) and the second line is identified with the cylinder's element. In this way one can make a correspondence between any non-isotropic (having a non-zero length) vector in the plane and a point on the cylinder. For instance, if a vector x having coordinates (x_0, x_1) forms a hyperbolic angle φ with the e_0 or $-e_0$, then

$$\phi = |\pm\pi e^{-\varphi}\rho| \bmod 2\pi = |\pi(x_0 + x_1)| \bmod 2\pi. \tag{2.5}$$

If this vector forms the hyperbolic angle φ with the e_1 or $-e_1$, then

$$r = \pm e^{\varphi}\rho = x_0 - x_1\,, \tag{2.6}$$

where $\varphi = -\ln\left|\frac{x_0 + x_1}{\rho}\right|$; $\rho = |(x_0 + x_1)(x_0 - x_1)|^{1/2}$.

By analogy, one can build a winding map of the pseudo-Euclidean plane into the torus, with the only difference that in the latter case the second isotropic line is winded around the longitudinal (toroidal) direction of the torus.

Now let us consider a 6-dimensional pseudo-Euclidean space $\mathbb{R}^6$ with the signature $(+,+,+,-,-,-)$. In this case the analogue to the cylinder above is the product $\mathbb{R}^3 \times S^3$, in which the component $\mathbb{R}^3$ is Euclidean space. In order to wind the space $\mathbb{R}^6$ over the cylinder $\mathbb{R}^3 \times S^3$ we have to take an arbitrary pseudo-Euclidean plane in $\mathbb{R}^6$ passing through the (arbitrary) orthogonal lines x_k, x_p that belong to two Euclidean subspaces $\mathbb{R}^3$ of the space $\mathbb{R}^6$. Each plane (x_k, x_p) has to be winded onto a cylinder with the cylindrical coordinates (ϕ_k, r_p); the indices k, p correspond to the projective space RP^2. We can take all the possible planes and wind them over the corresponding cylinders. The mapping transformation of the pseudo-Euclidean space $\mathbb{R}^6$ into the cylinder $\mathbb{R}^3 \times S^3$ is similar to the expressions (2.5) and (2.6):

$$\begin{aligned} \phi_k &= |\pm \pi e^{-\varphi}\rho| \bmod 2\pi = \\ &= |\pi(x_k + x_p)| \bmod 2\pi\,, \end{aligned} \tag{2.7}$$

$$r_p = \pm e^{\varphi}\rho = x_k - x_p\,. \tag{2.8}$$

By fixing the running index k and replacing it with zero we can get the winding map of the Minkowski space $\mathbb{R}^4$ into the cylinder $\mathbb{R}^3 \times S^1$, which is a particular case (reduction) of (2.7) and (2.8). Conversely, by winding $\mathbb{R}^3$ over a 3-sphere, S^3, we can generalise the case and derive a winding map from $\mathbb{R}^6$ into $S^3 \times S^3$.

Let us consider the relationship between different orthonormal bases in the pseudo-Euclidean plane, which is winded over a cylinder. It is known that all of the orthonormal bases in a pseudo-Euclidean are equivalent (i.e., none of them can be chosen as privileged). However, by defining a regular field c of unit vectors on the pseudo-Euclidean plane it is, indeed, possible to get such a privileged orthonormal basis (c, c_1). In turn, a non-uniform unitary vector field $g(x)$, having a hyperbolic angle $\varphi(x)$ with respect to the field c, would induce a non-orthonormal frame $(g'(x), g_1'(x))$. Indeed, if we assume that the following equalities are satisfied:

$$\begin{aligned} \pi &= |\pm \pi e^{-\varphi}\rho(e^{\varphi}g)| \bmod 2\pi = \\ &= |\pm \pi e^{-\varphi}\rho(g')| \bmod 2\pi, \end{aligned} \tag{2.9}$$

$$\pm 1 = \pm e^{\varphi}\rho(e^{-\varphi}g_1) = \pm e^{\varphi}\rho(g_1')\,, \tag{2.10}$$

we can derive a non-orthonormal frame $(g'(x), g_1'(x))$ by using the following transformation of the orthonormal frame $(g(x), g_1(x))$:

$$g'(x) = e^{\varphi}g(x), \quad g_1'(x) = e^{-\varphi}g_1(x). \tag{2.11}$$

Then the field $g(x)$ would induce a 2-dimensional pseudo-Riemann manifold with a metric tensor $\{g'_{ij}\}$ $(i, j = 0, 1)$, which is the same as the Gram matrix corresponding to the system of vectors $(g'(x), g_1'(x))$. A unitary vector field $g(x)$ defined in the Minkowski space winded onto the cylinder $\mathbb{R}^3 \times S^1$ would induce a 4-dimensional pseudo-Riemann manifold. Indeed, take the orthonormal frame (g, g_1, g_2, g_3) derived by hyperbolically rotating the Minkowski space by the angle $\varphi(x)$ in the plane $(g(x), c)$. Then the Gram matrix g'_{ij} $(i, j = 0, 1, 2, 3)$ corresponding to the set of vectors $\{e^{\varphi}g, e^{-\varphi}g_1,\ g_2, g_3\}$ would be related to the metric of the pseudo-Riemann manifold. Note, that, since the determinant of the Gram matrix is unity [17, 18], the induced metric preserves the volume. That is, the differential volume element of our manifold is equal to the corresponding volume element of the Minkowski space.

3 The dynamics of the model

As we have already mentioned in Section 1, the dynamics of the velocity field $u(x, \tau)$ of an ideal incompressible fluid on the surface of a cylinder $\mathbb{R}^3 \times S^1$ can be characterised by using the minimal volume principle, i.e., by assuming that the 4-volume of the flow through an arbitrary 3-surface $\Sigma \subset \mathbb{R}^3$ during the time T is minimal under some initial and boundary conditions, namely:

$$\delta \int_0^T \int_\Sigma dV \wedge u(x, \tau)\, d\tau = 0\,, \tag{3.1}$$

where dV is the differential volume element of a 3-surface Σ. This is also equivalent to the minimal mass carried by the flow through the measuring surface during a finite time interval.

In a classical approximation, by using the winding projection of the Minkowski space into a cylinder $\mathbb{R}^3 \times S^1$, we can pass from the dynamics defined on a cylinder to the statics in the Minkowski space. Let the global time t be parameterised by the length of the flow line of the vector field c in the Minkowski space corresponding to some regular vector field on the cylinder and let the length of a single turn around the cylinder be h. Let us take in the Minkowski space a set of orthogonal to c Euclidean spaces $\mathbb{R}^3$ in the Minkowski space. The distance between these spaces is equal to hz, where $z \in \mathbb{Z}$. The projection of this set of spaces into the cylinder is a three-dimensional manifold, which we shall refer to as a global measuring surface. Then we can make a one-to-one correspondence between the dynamical vector field $u(x, \tau)$ and the static vector field $g(x)$, defined in the Minkowsky space. Thus, in a classical approximation there exists a correspondence between the minimisation of the 4-volume of the flow $u(x, \tau)$ on the cylinder and the minimisation of the 4-volume of the static flow defined in the Minkowski space by the vector field $g(x)$, namely:

$$\delta \int_0^{x_0} \int_{\Sigma'} dV \wedge g(x)\, dx_0 = 0\,, \tag{3.2}$$

where the first basis vector e_0 coincides with the vector c, and the 3-surfaces, Σ', lie in the Euclidean sub-spaces orthogonal to the vector c. Let $\{(c_i)\} = (c_0, c_1, c_2, c_3)$ be an orthonormal basis in $\mathbb{R}^4$ such that $c_0 = c$. Let the reference frame bundle be such that each non-singular point of $\mathbb{R}^4$ has a corresponding non-orthonormal frame $(g_i(x)) = (g_0, g_1, g_2, g_3)$, where $g_0 = g(x)$, $g_1 = c_1$, $g_2 = c_2$, $g_3 = c_3$. Let us form a matrix $\{g_{ij}\}$ of inner products (c_i, g_j) of the basis vectors $\{c_i\}$ and the frame $\{g_i\}$. The absolute value of its determinant, $\det(g_{ij})$, is equal to the volume of the parallelepiped formed by the vectors (g_0, g_1, g_2, g_3). It is also equal to the scalar product, $(g(x), c)$. On the other hand, the equation $(g(x), c)^2 = |\det G(x)|$ holds for the Gram matrix, $G(x)$, which corresponds to the set of vectors $\{g_i(x)\}$ [21]. Then, according to the principle (3.2), the vector field $g(x)$ satisfies the variational equation

$$\delta \int_{\Omega} (g(x), c)\, dx^4 = \delta \int_{\Omega} |\det G(x)|^{\frac{1}{2}}\, dx^4 = 0\,, \tag{3.3}$$

where dx^4 is the differential volume element of a cylindrical 4-region Ω of the Minkowski space, having the height T. The cylinder's base is a 3-surface Σ with the boundary condition $g(x) = c$. In order to derive the differential equation satisfying the integral variational equation (3.3), we have to find the elementary region of integration, Ω. Let $\Delta\pi$ be an infinitesimal parallelepiped spanned by the vectors $\Delta x_0, \Delta x_1, \Delta x_2, \Delta x_3$, with ω being a tubular neighbourhood with the base spanned by the vectors $\Delta x_1, \Delta x_2, \Delta x_3$. This (vector) tubular neighbourhood is filled in with the vectors $|\Delta x_0| g(x)$ obtained from the flow lines of the vector field $g(x)$ by increasing the natural parameter (the pseudo-Euclidean length) by the amount $|\Delta x_0|$. Then the localisation expression of the equation (3.3) gives [19]:

$$\delta \int_{\Delta\pi} |\det G(x,t)|^{\frac{1}{2}}\, dx^4 = \delta\, \mathrm{Vol}\, \omega = 0\,. \tag{3.4}$$

Since the field lines of a nonholonomy vector field $g(x)$ are nonparallel even locally, any variation of such a field (i.e, the increase or decrease of its nonholonomicity) wo-uld result in a non-vanishing variation of the volume $\mathrm{Vol}\,\omega$. Conversely, in the case of a holonomy field its variations do not affect the local parallelism, so that the holonomicity of the field $g(x)$ appears to be the necessary condition for the zero variation of $\mathrm{Vol}\,\omega$. Given a vector field $g(x)$ with an arbitrary absolute value, the sufficient conditions for the vanishing variation of the volume of the tubular neighbourhood ω are the potentiality of this field and the harmonic character of its potential. In terms of differential forms these conditions correspond to a simple differential equation:

$$d \star g(x) = 0\,, \tag{3.5}$$

where d is the external differential; $\star$ is the Hodge star operator; $g(x) = d\varphi(x)$; and $\varphi(x)$ is an arbitrary continuous and smooth function defined everywhere in the Minkowski space, except for the singularity points (topological features). Substituting the unitary holonomy fi-eld $g(x) = k(x) d\varphi(x)$ in (3.5), where $k(x) = 1/|d\varphi(x)|$, we shall find that the unitary vector field $g(x)$ must satisfy the minimum condition for the integral surfaces of the co-vector field dual to $g(x)$. In this case the magnitude of the scalar quantity $\varphi(x)$ will be equal to the hyperbolic angle between the vectors $g(x)$ and c. We can also note that the potential vector field $g(x) = d\varphi(x)$ represented by the harmonic functions $\varphi(x)$ is the solution to the following variational equation:

$$\delta \int_0^T \int_{\Sigma} \left[\left(\frac{\partial \varphi(x,t)}{\partial t} \right)^2 - \nabla^2 \varphi(x,t) \right] dx^3\, dt = 0\,, \tag{3.6}$$

in which Σ is a region in Euclidean space of the "global" observer; the function $\varphi(x,t)$ is defined in the Minkowski space. Thus, the stationary scalar field $\varphi(x)$ induced by a topological feature in the global space is identical to the Newtonian gravitational potential of a mass point.

We have to bear in mind that the space of a "real" observer is curved, since the line for measuring time and the surface for measuring the flux is defined by the vector field $g(x)$, and not by the field c as in the case of the global observer. Therefore, if we wish to derive a variational equation corresponding to the real observer, we have to define it on the pseudo-Riemann manifold M induced in the Minkowski space by the holonomy field $g(x)$, whose flux is measured through the surfaces orthogonal to its flow lines and whose flow lines serve for measuring time. The metric on M is given by the Gram matrix of four tangent vectors, one of which corresponds to the flow line $x'_0(\phi)$ parameterised by the angular coordinate of the cylindrical manifold, and the three others are tangent to the coordinate lines of the 3-surface $x'_1(r), x'_2(r), x'_3(r)$ parameterised by the Euclidean length. The following variational equation holds for an arbitrary region ΔM of M:

$$\delta \int_{\Delta M} g^2(x')\, dV = 0 \tag{3.7}$$

(under the given boundary conditions) where dV is the differential volume element of M. Note that the norm of the vector $g(x)$ coincides with the magnitude of the volume-element deformation of the pseudo-Riemann ma-nifold, which allows making the correspondence between our functional and that of the Hilbert-Einstein action.

Returning to the global space, let us consider some properties of the vector field $g(x)$. Let a point in the Minkowski space has a trajectory $X(\tau)$ and velocity $\dot{X}$. Its dynamics is determined by the variational equation:

$$\delta \int_0^T \big(g(x), \dot{X}\big)\, d\tau = 0\,. \tag{3.8}$$

The varied here is the trajectory $X(\tau)$ in the Minkowski space where the vector field $g(x)$ is defined and where the absolute time τ plays the role of the evolution parameter. For small time intervals the integral equation (3.8) can be reduced to

$$\delta\big(g(x), \dot{X}\big) = 0\,, \tag{3.9}$$

which is satisfied by the differential equation

$$\ddot{X} = g(X)\,. \tag{3.10}$$

Taking the orthogonal projection $\xi(\tau) = \mathrm{pr}_{\mathbb{R}^3} X(\tau)$ of the trajectory of a given topological feature in Euclidean space of the global observer, as well as the projection $\nabla\varphi(X) = \mathrm{pr}_{\mathbb{R}^3} g(X)$ of the vector field $g(x)$ at the point $X(\tau)$ gives a simple differential equation

$$\ddot{\xi}(\tau) = \nabla\varphi(x), \tag{3.11}$$

which (as in Newtonian mechanics) expresses the fact that the acceleration of a mass point in an external gravitational field does not depend on the mass.

4 Some implications

Let us consider some implications of our model for a real observer in a classical approximation (by the real observer we mean the reference frame of a topological feature). First, we can note that a real observer moving uniformly along a straight line in the Minkowski space cannot detect the "relative vacuum" determined by the vector c and, hence, cannot measure the global time t. By measuring the velocities of topological features (also uniformly moving along straight lines) our observer would find that for gauging space and time one can use an arbitrary unitary vector field c' defined on the Minkowski space. Therefore, the observer would conclude that the notion of spacetime should be relative. It is seen that the real observer can neither detect the unitary vector field $g(x)$ nor its deviations from the vector c. However, it would be possible to measure the gradient of the scalar (gravitational) field and detect the pseudo-Riemann manifold induced by $g(x)$.

Indeed, in order to gauge time and distances in different points of space (with different magnitudes of the scalar field) one has to use the locally orthonormal basis $\{g'_i\}$ defined on the 4-dimensional pseudo-Riemann manifold with its metric tensor $\{g'_{ij}\}$. Thus, for the real observer, the deformations of the pseudo-Euclidean space could be regarded as if induced by the scalar field. Locally, the deformations could be cancelled by properly accelerating the mass point (topological feature), which implies that its trajectory corresponds to a geodesics of the manifold.

We can see that the dynamics of a topological feature in our model is identical to the dynamics of a mass point in the gravitational field. Indeed, the scalar field around a topological feature is spherically symmetric. At distance r from the origin the metric will be $e^{2\varphi}dt^2 - e^{-2\varphi}dr^2$, which corresponds to the metric tensor of the gravitational field of a point mass, given $e^{2\varphi} \approx 1 + 2\varphi$ for small φ. If $\varphi = H\tau$, i.e., hyperbolic angle φ linearly depends from the evolutionary parameter τ, then we can compare the constant H with the cosmological factor.

Let us now consider some quantum properties of our model. Let the absolute value of the vector field c be a continuous function $|c(x)|$ in the Minkowski space. Then the angular velocity of the flow will be:

$$\dot{\phi}(x) = \frac{d\phi(x)}{dt} = \frac{\pi}{h}|c(x)|\,, \tag{4.1}$$

where the angular function $\phi(x)$ can be identified with the phase action of the gauge potential in the observer space. On the other hand, it is reasonable to associate the angular velocity $X(\tau)$ of the topological feature with the Lagrangian of a point mass in the Minkowski space:

$$\dot{\phi}(X) = \frac{d\phi(X)}{d\tau} = \frac{\pi}{h}\,L(x)\,. \tag{4.2}$$

Let us consider the random walk process of the topological feature in the cylinder space $\mathbb{R}^3 \times S^1$. Let a probability density function $\rho(x)$ be defined on a line, such that $\rho(x)$,

$$\int_{-\infty}^{+\infty} \rho(x)\,dx = 1\,. \tag{4.3}$$

Let us calculate the expectation value for the random variable $e^{i\pi x}$, which arises when a line is compactified into a circe:

$$M(e^{i\pi x}) = \int_{-\infty}^{+\infty} \rho(e^{i\pi x})\,dx = \int_{-\infty}^{+\infty} e^{i\pi x}\rho(x)\,dx = pe^{i\pi\alpha}. \tag{4.4}$$

Here the quantity $pe^{i\pi\alpha}$ can be called the complex probability amplitude. It characterises two parameters of the random variable distribution, namely, the expectation value itself, $e^{i\pi\alpha}$, and the probability density, p, i.e. the magnitude of the expectation value. If $\rho(x) = \delta(\alpha)$, then $M(e^{i\pi x}) = 1 \cdot e^{i\pi\alpha}$. Conversely, if $\rho(x)$ is uniformly distributed along the line then the expectation value is $M(e^{i\pi x}) = 0$. It follows from these considerations that a distribution in $\mathbb{R}^3$ of a complex probability amplitude is related to random events in the cylinder space $\mathbb{R}^3 \times S^1$.

In order to specify the trajectories $X(\tau)$ in the Minkowski space with an external angular potential $\phi(x)$ we shall use the procedure proposed by Feynman [22]. Let the probabilistic behaviour of the topological feature be described as a Markov random walk in the cylinder space $\mathbb{R}^3 \times S^1$. An elementary event in this space is a free passage. In the Minkowski space such an event is characterised by two random variables, duration, $\Delta\tau$, and the random path vector, ΔX, whose projection into Euclidean space of the absolute observer is $\Delta\xi$. The ratio $\frac{\Delta\xi}{\Delta\tau}$ is a random velocity vector, $\dot{\xi}$. On the other hand, the free passage of a topological feature corresponds to an increment in the phase angle $\Delta\phi(X) = \dot{\phi}(X)\Delta\tau$ (phase action) in the cylinder space $\mathbb{R}^3 \times S^1$.

Let the probability distribution of the phase action has an exponential form, say, $\rho(\Delta\phi) = e^{-\Delta\phi}$ (neglecting the normalisation coefficient). Then, the corresponding probability density for the random variable $e^{i\Delta\phi}$ will be

$$\rho(e^{i\Delta\phi}) = e^{-\Delta\phi}e^{i\Delta\phi}. \tag{4.5}$$

Using the properties of a Markov chain [20], we can derive the probability density for an arbitrary number of random walks:

$$\rho(e^{i\phi}) = \prod_{0}^{T} e^{-\dot{\phi} d\tau} e^{i\dot{\phi} d\tau}. \tag{4.6}$$

To get the expectation value of the random variable $e^{i\phi}$ we have to sum up over the all possible trajectories, that is, to calculate the quantity

$$M(e^{i\phi}) = \sum \prod_{0}^{T} e^{-\dot{\phi} d\tau} e^{i\dot{\phi} d\tau}. \tag{4.7}$$

It is known that any non-vanishing variation of the phase action has a vanishing amplitude of the transitional probability and, on the contrary, that the vanishing variation corresponds to a non-vanishing probability amplitude [23–25]. Then it is seen that the integral action corresponding to the topological feature must be minimal. It follows that the "probabilistic trap" of a random walk [26] in the cylinder space $\mathbb{R}^3 \times S^1$ is determined by the variational principle — the same that determines the dynamics of a mass point in classical mechanics.

5 Conclusions

In conclusion, we have made an attempt to describe the dynamics of spacetime (as well as of matter particles) in terms of the vector field defined on a cylindrical manifold and based on the principle of maximum mass carried by the field flow. The analysis of the observational implications of our model sheds new light on the conceptual problems of quantum gravity.

Still many details of our model are left unexplored. For example, it would be instructive to devise the relationship between the vector field $g(x)$ and the 4-potential of electromagnetic field $A(x)$ and to consider the local perturbations of $g(x)$ as gravitons or/and photons. We also expect that the most important properties of our model would be revealed by extending it to the cylindrical manifold $\mathbb{R}^3 \times S^3$. In particular, we hope that within such an extended version of our framework it would be possible to find a geometric interpretation of all known gauge fields. It is also expected that studying the dynamics of the minimal unit vector field on a 7-sphere should be interesting for cosmological applications of our approach.

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Kaluza-Klein-Carmeli Metric from Quaternion-Clifford Space, Lorentz' Force, and Some Observables

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It was known for quite long time that a quaternion space can be generalized to a Clifford space, and vice versa; but how to find its neat link with more convenient metric form in the General Relativity theory, has not been explored extensively. We begin with a representation of group with non-zero quaternions to derive closed FLRW metric [1], and from there obtains Carmeli metric, which can be extended further to become 5D and 6D metric (which we propose to call Kaluza-Klein-Carmeli metric). Thereafter we discuss some plausible implications of this metric, beyond describing a galaxy's spiraling motion and redshift data as these have been done by Carmeli and Hartnett [4, 5, 6]. In subsequent section we explain Podkletnov's rotating disc experiment. We also note possible implications to quantum gravity. Further observations are of course recommended in order to refute or verify this proposition.

1 Introduction

It was known for quite long time that a quaternion space can be generalized to a Clifford space, and *vice versa*; but how to find its neat link to more convenient metric form in the General Relativity theory, has not been explored extensively [2].

First it is worth to remark here that it is possible to find a flat space representation of quaternion group, using its algebraic isomorphism with the ring division algebra [3, p.3]:

$$E_i E_j = -\delta_{ij} + f_{ijk} E_k \,. \tag{1}$$

Working for R^{dim}, we get the following metric [3]:

$$ds^2 = dx_\mu dx^\mu, \tag{2}$$

imposing the condition:

$$x_\mu x^\mu = R^2. \tag{3}$$

This rather elementary definition is noted here because it was based on the choice to use the square of the radius to represent the distance (x_μ), meanwhile as Riemann argued long-time ago it can also been represented otherwise as the square of the square of the radius [3a].

Starting with the complex $n = 1$, then we get [3]:

$$q = x_0 + x_1 E_1 + x_2 E_2 + x_3 E_3 \,. \tag{4}$$

With this special choice of x_μ we can introduce the special metric [3]:

$$ds^2 = R^2(\delta_{ij}\partial\Phi_i\partial\Phi_j). \tag{5}$$

This is apparently most direct link to describe a flat metric from the ring division algebra. In the meantime, it seems very interesting to note that Trifonov has shown that the geometry of the group of nonzero quaternions belongs to closed FLRW metric. [1] As we will show in the subsequent Section, this approach is more rigorous than (5) in order to describe neat link between quaternion space and FLRW metric.

We begin with a representation of group with non-zero quaternions to derive closed FLRW metric [1], and from there we argue that one can obtain Carmeli 5D metric [4] from this group with non-zero quaternions. The resulting metric can be extended further to become 5D and 6D metric (which we propose to call Kaluza-Klein-Carmeli metric).

Thereafter we discuss some plausible implications of this metric, beyond describing a galaxy's spiraling motion and redshift data as these have been done by Carmeli and Hartnett [4–7]. Possible implications to the Earth geochronometrics and possible link to coral growth data are discussed. In the subsequent Section we explain Podkletnov's rotating disc experiment. We also note a possible near link between Kaluza-Klein-Carmeli and Yefremov's Q-Relativity, and also possible implications to quantum gravity.

The reasons to consider this Carmeli metric instead of the conventional FLRW are as follows:

- One of the most remarkable discovery from WMAP is that it reveals that our Universe seems to obey Euclidean metric (see Carroll's article in *Nature*, 2003);
- In this regards, to explain this observed fact, most arguments (based on General Relativity) seem to agree that in the edge of Universe, the metric will follow Euclidean, because the matter density tends to approaching zero. But such a proposition is of course in contradiction with the basic "assumption" in GTR itself, i.e. that the Universe is *homogenous isotropic* everywhere, meaning that the matter density should be the same too in the edge of the universe. In other words, we need a new metric to describe the *inhomogeneous isotropic* spacetime.

$$g_{\alpha\beta} = \begin{pmatrix} \tau(\eta)(\frac{\dot{R}}{R})^2 & 0 & 0 & 0 \\ 0 & -\tau(\eta) & 0 & 0 \\ 0 & 0 & -\tau(\eta)\sin^2(\chi) & 0 \\ 0 & 0 & 0 & -\tau(\eta)\sin^2(\chi)\sin^2(\vartheta) \end{pmatrix}. \tag{6}$$

- Furthermore, from astrophysics one knows that spiral galaxies do not follow Newtonian potential exactly. Some people have invoked MOND or modified (Post-) Newton potential to describe that deviation from Newtonian potential [8, 9]. Carmeli metric is another possible choice [4], and it agrees with spiral galaxies, and also with the redshift data [5–7].
- Meanwhile it is known, that General Relativity is strictly related to Newtonian potential (Poisson's equation). All of this seems to indicate that General Relativity is only applicable for some limited conditions, but it may not be able to represent the *rotational aspects of gravitational phenomena*. Of course, there were already extensive research in this area of the generalized gravitation theory, for instance by introducing a torsion term, which vanishes in GTR [10].

Therefore, in order to explain spiral galaxies' rotation curve and corresponding "dark matter", one can come up with a different route instead of invoking a kind of strange matter. In this regards, one can consider dark matter as a property of the metric of the spacetime, just like the precession of the first planet is a property of the spacetime in General Relativity.

Of course, there are other methods to describe the *inhomogeneous* spacetime, see [15, 16], for instance in [16] a new differential operator was introduced: $\frac{\delta}{\delta\tau} = \frac{1}{H_o}\frac{1}{c}\frac{\delta}{\delta t}$, which seems at first glance as quite similar to Carmeli method. But to our present knowledge Carmeli metric is the most consistent metric corresponding to generalized FLRW (derived from a quaternion group).

Further observations are of course recommended in order to refute or verify this proposition.

2 FLRW metric associated to the group of non-zero quaternions

The quaternion algebra is one of the most important and well-studied objects in mathematics and physics; and it has natural Hermitian form which induces Euclidean metric [1]. Meanwhile, Hermitian symmetry has been considered as a method to generalize the gravitation theory (GTR), see Einstein paper in *Ann. Math.* (1945).

In this regards, Trifonov has obtained that a natural extension of the structure tensors using nonzero quaternion bases will yield formula (6). (See [1, p.4].)

Interestingly, by *assuming* that [1]:

$$\tau(\eta)\left(\frac{\dot{R}}{R}\right)^2 = 1\,, \tag{7}$$

then equation (6) reduces to closed FLRW metric [1, p.5]. Therefore one can say that closed FLRW metric is neatly associated to the group of nonzero quaternions.

Now consider equation (7), which can be rewritten as:

$$\tau(\eta)(\dot{R})^2 = R^2. \tag{8}$$

Since we choose (8), then the radial distance can be expressed as:

$$dR^2 = dz^2 + dy^2 + dx^2. \tag{9}$$

Therefore we can rewrite equation (8) in terms of (9):

$$\tau(\eta)(d\dot{R})^2 = (dR)^2 = dz^2 + dy^2 + dx^2, \tag{10}$$

and by defining

$$\tau(\eta) = \tau^2 = \frac{1}{H_0^2(\eta)} = \frac{1}{\alpha(H_0^2)^n}\,. \tag{11}$$

Then we can rewrite equation (10) in the form:

$$\tau(\eta)(d\dot{R})^2 = \tau^2(dv)^2 = dz^2 + dy^2 + dx^2, \tag{12}$$

or

$$-\tau^2(dv)^2 + dz^2 + dy^2 + dx^2 = 0\,, \tag{13}$$

which is nothing but an original Carmeli metric [4, p.3, equation (4)] and [6, p.1], where H_0 represents Hubble constant (by setting $\alpha = n = 1$, while in [12] it is supposed that $\alpha = 1.2$, $n = 1$). Further extension is obviously possible, where equation (13) can be generalized to include the $(i\,cdt)$ component in the conventional Minkowski metric, to become (Kaluza-Klein)-Carmeli 5D metric [5, p.1]:

$$-\tau^2(dv)^2 + dz^2 + dy^2 + dx^2 + (icdt)^2 = 0\,. \tag{14}$$

Or if we introduce equation (13) in the general relativistic setting [4, 6], then one obtains:

$$ds^2 = \tau^2(dv)^2 - e^{\xi}\cdot dr^2 - R^2\cdot(d\vartheta^2 + \sin^2\vartheta\cdot d\phi^2). \tag{15}$$

The solution for (15) is given by [6, p.3]:

$$\frac{dr}{dv} = \tau\cdot\exp\left(-\frac{\xi}{2}\right)\,, \tag{16}$$

which can be written as:

$$\frac{d\dot{r}}{dr} = \frac{dv}{dr} = \tau^{-1}\cdot\exp\left(\frac{\xi}{2}\right)\,. \tag{17}$$

This result implies that there shall be a metric deformation, which may be associated with astrophysics observation, such as the possible AU differences [11, 12].

Furthermore, this proposition seems to correspond neatly to the Expanding Earth hypothesis, because [13]:

"In order for expansion to occur, the moment of inertia constraints must be overcome. An expanding Earth would necessarily rotate more slowly than a smaller diameter planet so that angular momentum would be conserved." (Q.1)

We will discuss these effects in the subsequent Sections.

We note however, that in the original Carmeli metric, equation (14) can be generalized to include the potentials to be determined, to become [5, p.1]:

$$ds^2 = \left(1 + \frac{\Psi}{\tau^2}\right) \tau^2 (dv)^2 - dr^2 + \left(1 + \frac{\Phi}{c^2}\right) c^2 dt^2, \quad (18)$$

where

$$dr^2 = dz^2 + dy^2 + dx^2. \quad (19)$$

The line element represents a spherically symmetric *inhomogeneous* isotropic universe, and the expansion is a result of the spacevelocity component. In this regards, metric (18) describes *funfbein* ("five-legs") similar to the standard Kaluza-Klein metric, for this reason we propose the name Kaluza-Klein-Carmeli for all possible metrics which can be derived or extended from equations (8) and (10).

To observe the expansion at a definite time, the $(i\,cdt)$ term in equation (14) has been ignored; therefore the metric becomes "*phase-space*" Minkowskian. [5, p.1]. (A similar phase-space Minkowskian has been considered in various places, see for instance [16] and [19].) Therefore the metric in (18) reduces to (by taking into consideration the isotropic condition):

$$dr^2 + \left(1 + \frac{\Psi}{\tau^2}\right) \tau^2 (dv)^2 = 0\,. \quad (20)$$

Alternatively, one can suppose that in reality this assumption may be reasonable by setting $c \to 0$, such as by considering the metric for the phonon speed c_s instead of the light speed c; see Volovik, etc. Therefore (18) can be rewritten as:

$$\begin{aligned} ds^2_{phonon} = \left(1 + \frac{\Psi}{\tau^2}\right) \tau^2 (dv)^2 - dr^2 + \\ + \left(1 + \frac{\Phi}{c_s^2}\right) c_s^2\, dt^2. \end{aligned} \quad (21)$$

To summarize, in this Section we find out that not only closed FLRW metric is associated to the group of nonzero quaternions [1], but also the same group yields Carmeli metric. In the following Section we discuss some plausible implications of this proposition.

3 Observable A: the Earth geochronometry

One straightforward implication derived from equation (8) is that the ratio between the velocity and the radius is directly proportional, regardless of the scale of the system in question:

$$\left(\frac{\dot{R}}{R}\right)^2 = \tau(\eta)^{-1}, \quad (22)$$

or

$$\left(\frac{R_1}{\dot{R}_1}\right) = \left(\frac{R_2}{\dot{R}_2}\right) = \sqrt{\tau(\eta)}\,. \quad (23)$$

Therefore, one can say that there is a direct proportionality between the *spacevelocity* expansion of, let say, Virgo galaxy and the Earth geochronometry. Table 1 displays the calculation of the Earth's radial expansion using the formula represented above [17]:

Therefore, the Earth's radius increases at the order of ~ 0.166 cm/year, which may correspond to the decreasing angular velocity (Q.1). This number, albeit very minute, may also correspond to the Continental Drift hypothesis of A. Wegener [13, 17]. Nonetheless the reader may note that our calculation was based on Kaluza-Klein-Carmeli's phase-space *spacevelocity* metric.

Interestingly, there is a quite extensive literature suggesting that our Earth experiences a continuous deceleration rate. For instance, J. Wells [14] described a increasing day-length of the Earth [14]:

"It thus appears that the length of the day has been increasing throughout geological time and that the number of days in the year has been decreasing. At the beginning of the Cambrian the length of the day would have been 21^{h}." (Q.2)

Similar remarks have been made, for instance by G. Smoot [13]:

"In order for this to happen, the lunar tides would have to slow down, which would affect the length of the lunar month. ... an Earth year of 447 days at 1.9 Ga decreasing to an Earth year of 383 days at 290 Ma to 365 days at this time. However, the Devonian coral rings show that the *day is increasing by 24 seconds every million years,* which would allow for an expansion rate of about 0.5% for the past 4.5 Ga, all other factors being equal." (Q.3)

Therefore, one may compare this result (Table 1) with the increasing day-length reported by J. Wells [13].

4 Observable B: the Receding Moon from the Earth

It is known that the Moon is receding from the Earth at a constant rate of $\sim$ 4cm/year [17, 18].

Using known values: $G = 6.6724 \times 10^{-8}$ cm^2/(g $\cdot$ sec^2) and $\rho = 5.5 \times 10^6$ g/m^3, and the Moon's velocity $\sim$7.9 km/sec, then one can calculate using known formulas:

$$\mathrm{Vol} = \frac{4}{3}\,\pi \cdot (R + \Delta R)^3, \quad (24)$$

$$M + \Delta M = \mathrm{Vol} \cdot \rho\,, \quad (25)$$

$$r + \Delta r = \frac{G \cdot (M + \Delta M)}{v^2}\,, \quad (26)$$

where r, v, M each represents the distance from the Moon to the Earth, the Moon's orbital velocity, and the Earth's mass,

Nebula	Radial velocity (mile/s)	Distance (10^3 kly)	Ratio (10^{-5} cm/yr)	the Earth dist. (R, km)	Predicted the Earth exp. (ΔR, cm/year)
Virgo	750	39	2.617	6371	0.16678
Ursa Mayor	9300	485	2.610	6371	0.166299
Hydra	38000	2000	2.586	6371	0.164779
Bootes 2	86000	4500	2.601	6371	0.165742
Average			2.604		0.1659

Table 1: Calculation of the radial expansion from the Galaxy velocity/distance ratio. Source: [17].

respectively. Using this formula we obtain a prediction of the Receding Moon at the rate of 0.00497 m/year. This value is around 10% compared to the observed value 4 cm/year.

Therefore one can say that this calculation shall take into consideration other aspects. While perhaps we can use other reasoning to explain this discrepancy between calculation and prediction, for instance using the "*conformal brane*" method by Pervushin [20], to our best knowledge this effect has neat link with the known paradox in astrophysics, i.e. the observed matter only contributes around ∼1–10% of all matter that is supposed to be "there" in the Universe.

An alternative way to explain this discrepancy is that there is another type of force different from the known Newtonian potential, i.e. by taking into consideration the expansion of the "surrounding medium" too. Such a hypothesis was proposed recently in [21]. But we will use here a simple argument long-time ago discussed in [22], i.e. if there is a force other than the gravitational force acting on a body with mass, then it can be determined by this equation [22, p.1054]:

$$\frac{d(mv_0)}{dt} = F + F_{gr}\,, \tag{27}$$

where v_0 is the velocity of the particle relative to the absolute space [22a]. The gravitational force can be defined as before:

$$F_{gr} = -\,m\,\nabla V\,, \tag{28}$$

where the function V is solution of Poisson's equation:

$$\nabla^2 V = 4\pi K\mu\,, \tag{29}$$

and K represents Newtonian gravitational constant. For system which does not obey Poisson's equation, see [15].

It can be shown, that the apparent gravitational force that is produced by an aether flow is [22]:

$$F_{gr} = m\,\frac{\partial v}{\partial t} + m\,\nabla\left(\frac{v^2}{2}\right) - mv_0 \times \nabla \times v + v\,\frac{dm}{dt}\,, \tag{30}$$

which is an extended form of Newton law:

$$\vec{F} = \frac{d}{dt}\left(\vec{m}\vec{v}\right) = m\left(\frac{d\vec{v}}{dt}\right) + v\left(\frac{d\vec{m}}{dt}\right). \tag{31}$$

If the surrounding medium be equivalent to Newton's theory, this expression shall reduce to that given in (27). Supposing the aether be irrotational relative to the particular system of the coordinates, and $m = \text{const}$, then (29) reduces [22]:

$$F_{gr} = -m\left(-\frac{\partial v}{\partial t} - \nabla\left(\frac{v^2}{2}\right)\right), \tag{32}$$

which will be equivalent to equation (27) only if:

$$\nabla V = \frac{\partial v}{\partial t} + \nabla\left(\frac{v^2}{2}\right). \tag{33}$$

Further analysis of this effect to describe the Receding Moon from the Earth will be discussed elsewhere. In this Section, we discuss how the calculated expanding radius can describe (at least partially) the Receding Moon from the Earth. Another possible effect, in particular the deformation of the surrounding medium, shall also be considered.

5 Observable C: Podkletnov's rotation disc experiment

It has been discussed how gravitational force shall take into consideration the full description of Newton's law. In this Section, we put forth the known equivalence between Newton's law (31) and Lorentz' force [23], which can be written (supposing m to be constant) as follows:

$$\vec{F} = \frac{d}{dt}\left(\gamma\vec{m}\vec{v}\right) = \gamma m\left(\frac{d\vec{v}}{dt}\right) = q\left(\vec{E} + \frac{1}{c}\,\vec{v}\times\vec{B}\right), \tag{34}$$

where the relativistic factor is defined as:

$$\gamma = \pm\sqrt{\frac{1}{1-\beta^2}}\,. \tag{35}$$

while we can expand this equation in the cylindrical coordinates [23], we retain the simplest form in this analysis. In accordance with Spohn, we define [24]:

$$E = -\nabla A\,. \tag{36}$$

$$B = \nabla \times A\,. \tag{37}$$

For Podkletnov's experiment [26–28], it is known that there in a superconductor $E = 0$ [25], and by using the mass m in lieu of the charge ratio $\frac{e}{c}$ in the right hand term of (34) called the "gravitational Lorentz force", we get:

$$m\left(\frac{d\vec{v}}{dt}\right) = \frac{m}{\gamma}\left(\vec{v}\times\vec{B}\right) = \frac{1}{\gamma}\left(\vec{p}\times\vec{B}\right). \tag{38}$$

Let us suppose we conduct an experiment with the weight $w = 700\,\text{g}$, the radius $r = 0.2\,\text{m}$, and it rotates at $f = 2\,\text{cps}$ (cycle per second), then we get the velocity at the edge of the disc as:

$$v = 2\pi \cdot fr = 2.51 \text{ m/sec}, \tag{39}$$

and with known values for $G = 6.67\times10^{-11}$, $c \simeq 3\times10^{8}$ m/sec, $M_{earth} = 5.98\times10^{24}$ kg, $r_{earth} = 3\times10^{6}$ m, then we get:

$$F_{gr} = \frac{G}{c^2 r} Mv \approx 3.71\times10^{-9} \text{ newton/kgm sec.} \tag{40}$$

Because $B = F/\text{meter}$, then from (39), the force on the disc is given by:

$$F_{disc} = \vec{B}_{earth} \cdot \vec{p}_{disc} \approx B_{earth} \cdot \left(m\,\frac{c}{\gamma} \right). \tag{41}$$

High-precision muon experiment suggests that its speed can reach around $\sim 0.99\,c$. Let us suppose in our disc, the particles inside have the speed $0.982\,c$, then $\gamma^{-1} = 0.1889$. Now inserting this value into (40), yields:

$$\begin{aligned} F_{disc} = (3.71\times10^{-9}) \cdot (0.7) \cdot (3\times10^{8}) \cdot 0.189 &= \\ = 0.147 \text{ newton} = 14.7 \text{ gr.} \end{aligned} \tag{42}$$

Therefore, from the viewpoint of a static observer, the disc will get a mass reduction as large as $\frac{14.7}{700} = 2.13\%$, which seems quite near with Podkletnov's result, i.e. the disc can obtain a mass reduction up to 2% of the static mass.

We remark here that we use a simplified analysis using Lorentz' force, considering the fact that superconductivity may be considered as a relativistic form of the ordinary electromagnetic field [25].

Interestingly, some authors have used different methods to explain this apparently bizarre result. For instance, using Tajmar and deMatos' [29] equation: $\gamma_0 = \frac{a\Omega}{2} = \frac{0.2\cdot 2}{2} = 0.2$. In other words, it predicts a mass reduction around $\sim\frac{0.2}{9.8} = 2\%$, which is quite similar to Podkletnov's result.

Another way to describe those rotating disc experiments is by using simple Newton law [33]. From equation (31) one has (by setting $F = 0$ and because $g = \frac{dv}{dt}$):

$$\frac{dm}{dt} = -\frac{m}{v}\,g = -\frac{m}{\omega R}\,g\,, \tag{43}$$

Therefore one can expect a mass reduction given by an angular velocity (but we're not very how Podkletnov's experiment can be explained using this equation).

We end this section by noting that we describe the rotating disc experiment by using Lorentz' force in a rotating system. Further extension of this method in particular in the context of the (extended) Q-relativity theory, will be discussed in the subsequent Section.

6 Possible link with Q-Relativity. Extended 9D metric

In the preceding Section, we have discussed how closed FLRW metric is associated to the group with nonzero quaternions, and that Carmeli metric belongs to the group. The only problem with this description is that it neglects the directions of the velocity other than against the x line.

Therefore, one can generalize further the metric to become [1, p.5]:

$$-\,\tau^2 (dv_R)^2 + dz^2 + dy^2 + dx^2 = 0\,, \tag{44}$$

or by considering each component of the velocity vector [23]:

$$\begin{aligned} (i\,\tau\,dv_X)^2 + (i\,\tau\,dv_Y)^2 + (i\,\tau\,dv_Z)^2 &+ \\ + dz^2 + dy^2 + dx^2 &= 0\,. \end{aligned} \tag{45}$$

From this viewpoint one may consider it as a generalization of Minkowski's metric into biquaternion form, using the modified Q-relativity space [30, 31, 32], to become:

$$ds = (dx_k + i\,\tau\,dv_k)\,q_k. \tag{46}$$

Please note here that we keep using definition of Yefremov's quaternion relativity (Q-relativity) physics [30], albeit we introduce dv instead of dt in the right term. We propose to call this metric *quaternionic Kaluza-Klein-Carmeli metric*.

One possible further step for the generalization this equation, is by keep using the standard Q-relativistic dt term, to become:

$$ds = (dx_k + i\,c\,dt_k + i\,\tau\,dv_k)\,q_k\,, \tag{47}$$

which yields 9-Dimensional extension to the above quaternionic Kaluza-Klein-Carmeli metric. In other words, this generalized 9D KK-Carmeli metric is seemingly capable to bring the most salient features in both the standard Carmeli metric and also Q-relativity metric. Its prediction includes plausible time-evolution of some known celestial motion in the solar system, including but not limited to the Earth-based satellites (albeit very minute). It can be compared for instance using Arbab's calculation, that the Earth accelerates at rate 3.05 arcsec/cy^2, and Mars at 1.6 arcsec/cy^2 [12]. Detailed calculation will be discussed elsewhere.

We note here that there is quaternionic multiplication rule which acquires the compact form [30–32]:

$$1q_k = q_k 1 = q_k\,, \quad q_j q_k = -\delta_{jk} + \varepsilon_{jkn} q_n\,, \tag{48}$$

where δ_{kn} and ε_{jkn} represent 3-dimensional symbols of Kronecker and Levi-Civita, respectively [30]. It may also be worth noting here that in 3D space Q-connectivity has clear geometrical and physical treatment as movable Q-basis with behavior of Cartan 3-frame [30].

In accordance with the standard Q-relativity [30, 31], it is also possible to write the dynamics equations of Classical Mechanics for an inertial observer in the constant Q-basis, as follows:

$$m\,\frac{d^2}{dt^2}\,(x_k q_k) = F_k q_k\,. \tag{49}$$

Because of the antisymmetry of the connection (the generalized angular velocity), the dynamics equations can be written in vector components, by the conventional vector notation [30, 32]:

$$m\left(\vec{a}+2\vec{\Omega}\times\vec{v}+\vec{\Omega}\times\vec{r}+\vec{\Omega}\times(\vec{\Omega}\times\vec{r})\right)=\vec{F}\,, \tag{50}$$

which represents known types of classical acceleration, i.e. the linear, the Coriolis, the angular, and the centripetal acceleation, respectively.

Interestingly, as before we can use the equivalence between the inertial force and Lorentz' force (34), therefore equation (50) becomes:

$$\begin{aligned} m\left(\frac{d\vec{v}}{dt}+2\vec{\Omega}\times\vec{v}+\vec{\Omega}\times\vec{r}+\vec{\Omega}\times(\vec{\Omega}\times\vec{r})\right) &= \\ = q_{\otimes}\left(\vec{E}+\frac{1}{c}\,\vec{v}\times\vec{B}\right), \end{aligned} \tag{51}$$

or

$$\begin{aligned} \left(\frac{d\vec{v}}{dt}\right) = \frac{q_{\otimes}}{m}\left(\vec{E}+\frac{1}{c}\,\vec{v}\times\vec{B}\right) - \\ -\frac{2\vec{\Omega}\times\vec{v}+\vec{\Omega}\times\vec{r}+\vec{\Omega}\times(\vec{\Omega}\times\vec{r})}{m}\,. \end{aligned} \tag{52}$$

Please note that the variable q here denotes electric charge, not quaternion number.

Therefore, it is likely that one can expect a new effects other than Podkletnov's rotating disc experiment as discussed in the preceding Section.

Further interesting things may be expected, by using (34):

$$\begin{aligned} \vec{F} = m\left(\frac{d\vec{v}}{dt}\right) = q\left(\vec{E}+\frac{1}{c}\,\vec{v}\times\vec{B}\right) \Rightarrow \\ \Rightarrow m\,(d\vec{v}) = q\left(\vec{E}+\frac{1}{c}\,\vec{v}\times\vec{B}\right)dt\,. \end{aligned} \tag{53}$$

Therefore, by introducing this Lorentz' force instead of the velocity into (44), one gets directly a plausible extension of Q-relativity:

$$ds=\left[dx_k+i\,\tau\,\frac{q}{m}\left(\vec{E}_k+\frac{1}{c}\,\vec{v}_k\times\vec{B}_k\right)dt_k\right]q_k\,. \tag{54}$$

This equation seems to indicate how a magnetic wormhole can be induced in 6D Q-relativity setting [16, 19]. The reason to introduce this proposition is because there is known link between magnetic field and rotation [34]. Nonetheless further experiments are recommended in order to refute or verify this proposition.

7 Possible link with quantum gravity

In this Section, we remark that the above procedure to derive the closed FLRW-Carmeli metric from the group with nonzero quaternions has an obvious advantage, i.e. one can find Quantum Mechanics directly from the quaternion framework [35]. In other words, one can expect to put the gravitational metrical (FLRW) setting and the Quantum Mechanics setting in equal footing. After all, this may be just a goal sought in "quantum gravity" theories. See [4a] for discussion on the plausible quantization of a gravitational field, which may have observable effects for instance in the search of extrasolar planets [35a].

Furthermore, considering the "phonon metric" described in (20), provided that it corresponds to the observed facts, in particular with regards to the "surrounding medium" vortices described by (26–29), one can say that the "surrounding medium" is comprised of the phonon medium. This proposition may also be related to the superfluid-interior of the Sun, which may affect the Earth climatic changes [35b]. Therefore one can hypothesize that the signatures of quantum gravity, in the sense of the quantization in gravitational large-scale phenomena, are possible because the presence of the phonon medium. Nonetheless, further theoretical works and observations are recommended to explore this new proposition.

8 Concluding remarks

In the present paper we begun with a representation of a group with non-zero quaternions to derive closed FLRW metric [1], and we obtained Carmeli 5D metric [4] from this group. The resulting metric can be extended further to become 5D and 6D metric (called by us *Kaluza-Klein-Carmeli metric*).

Thereafter we discussed some plausible implications of this metric. Possible implications to the Earth geochronometrics and possible link to the coral growth data were discussed. In subsequent Section we explained Podkletnov's rotating disc experiment. We also noted possible neat link between Kaluza-Klein-Carmeli metric and Yefremov's Q-Relativity, in particular we proposed a further extension of Q-relativity to become 9D metric. Possible implications to quantum gravity, i.e. possible observation of the quantization effects in gravitation phenomena was also noted.

Nonetheless we do not pretend to have the last word on some issues, including quantum gravity, the structure of the aether (phonon) medium, and other calculations which remain open. There are also different methods to describe the Receding Moon or Podkletnov's experiments. What this paper attempts to do is to derive some known gravitational phenomena, including Hubble's constant, in a simplest way as possible, without invoking a strange form of matter. Furthermore, the Earth geochronometry data may enable us to verify the cosmological theories with unprecedented precision.

Therefore, it is recommended to conduct further observations in order to verify and also to explore the implications of our propositions as described herein.

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The Palindrome Effect

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As initially experimental material of this paper serves sets of histograms built on the base of short samples which provided the daily time series of the α-decay rate fluctuations and the p-n junction current fluctuations. Investigations of the histograms similarity revealed the palindrome effect, which is: two sets of histograms built on the base of two consecutive 12-hours time series are most similar if one set of the histograms is rearranged in inverse order, and the start time of the series is exact six hours later the local noon.

1 Introduction

As was shown in our previous works, the similarity of histograms built on the base of short samples of the time series of fluctuations measured on the processes of different nature, changes the regularly with time. These changes can be characterized by different periods equal the solar (1440 min) and sidereal (1436 min) days, several near 27-day periods, and yearly periods [1–5]. At different geographical locations the shapes of the histograms are similar to each other with high probability for the coincident moments of the local time [6]. Also it was found the dependence of the histogram patterns on the spatial directions of outgoing α-particles [5] and the motion specific to the measurement system [7]. Aforementioned phenomena led us to an idea that the histogram patterns can be dependent on also the sign of the projection obtained from the velocity vector of the measurement system projected onto the Earth's orbital velocity vector. As was found, this supposition is true.

2 The method

A raw experimental data we used for this paper were sets of the histograms built on the base of short samples which provided the daily time series of ^{239}Pu α-decay rate fluctuations and the p-n junction current fluctuations. The experimental data processing and histogram sets analyzing are given in details in [1, 2].

We use the daily time series of fluctuations in the study. Every time series started six hours later the local noon. After the data acquisition, we divided the 24-hours record into two 12-hours ones. On the base of these two consecutive 12-hours time series two sets of histograms (so-called "direct sets") were obtained for further analysis. The sign of the measurement system's velocity projected onto the Earth's orbital velocity is positive for one set, while the sign is negative for the other. Proceeding from the direct sets, by rearranging in inverse order, we obtained two "inverse" sets of histograms.

The histograms themselves were built on the base of the 60 of 1-sec measurements. So, one histogram durations was 1 min, while the 12-hours time series we used in the present work formed the sets consisting of 720 such histograms. The similarity of the histogram was studied for couplets ("direct-direct" and "direct-inverse") along the 720-histogram sets. Here we present the results in the form of interval distribution: the number of similar pairs of the histograms is present as a function on the time interval between them.

3 Experimental results

Fig. 1 shows the interval distributions for two couplets of the sets built on the base of the daily time series of ^{239}Pu α-decay rate fluctuations, obtained on April 23, 2004. The left diagram, Fig. 1a, shows the interval distribution for the "direct-inverse" histogram sets. From the right side of the diagram, we get the "direct-direct" histogram sets.

A peak shown in Fig. 1a means that the histograms with the coincident numbers in the "direct-inverse" sets are similar with very high probability. These sets of similar histograms constitute about 20% from the total number (720) of the pairs. In contrast to the "direct-inverse" sets, the interval distributions in the "direct-direct" histogram sets (Fig. 1b) achieve only 5% of the total number of the pairs for the same zero interval.

We call the *palindrome effect** such a phenomenon, where two sets of the histograms built on the base of two consecutive 12-hours time series are most similar in the case where one of the sets is rearranged in inverse order, while the daily record starts six hours later the local noon.

The palindrome effect doesn't depend from the annual motion of the Earth. This effect is actually the same for all the seasons. This statement is illustrated by Fig. 2, where the palindrome effect is displayed for the measurements carried out on the autumnal equinox, September 22–23, 2005.

*This comes from the Greek word παλὶνδρομος, which means *there and back*.

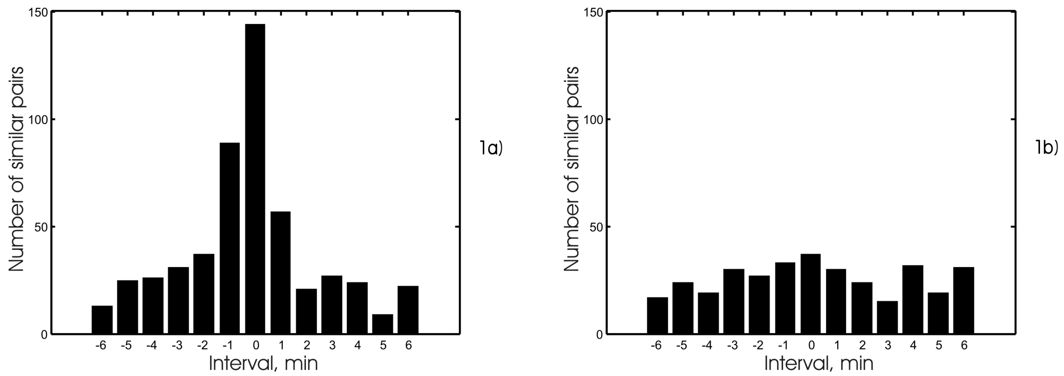

Fig. 1: The palindrome effect in the daily time series of the ^{239}Pu α-decay rate fluctuations, registered on April 23, 2004. The interval distribution for the "direct-inverse" histogram sets are shown in Fig. 1a, while those for the "direct-direct" histograms sets are shown in Fig. 1b.

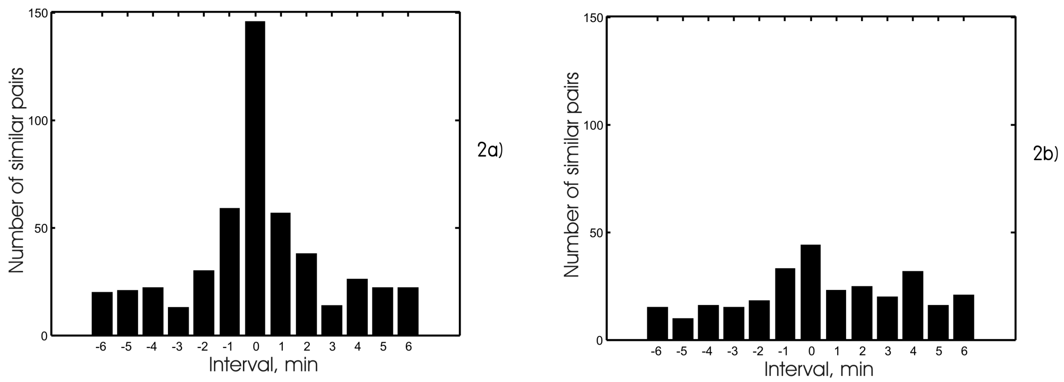

Fig. 2: The palindrome effect in the daily time series of the ^{239}Pu α-decay rate fluctuations, registered on the autumnal equinox, September 22-23, 2005. The interval distribution for the "direct-inverse" histogram sets are shown in Fig. 1a, while those for the "direct-direct" histogram sets are shown in Fig. 2b.

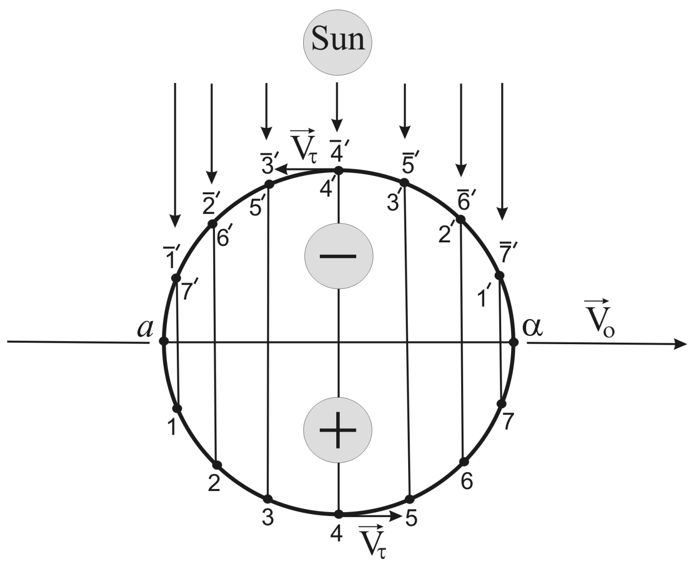

Fig. 3: The palindrome effect.

As easy to see, Fig. 2a and Fig. 2b are similar to Fig. 1a and Fig. 1b respectively. Similarly to Fig. 1 and Fig. 2, the interval distribution was obtained also for the winter and summer solstice.

The aforementioned results mean that, for different locations of the Earth in its circumsolar orbit, we have the same appearance of the palindrome effect.

4 Discussions

It is important to note that the 12-hours time series used in the present work were measured in such a way that the projection of the tangential velocity vector V_τ (Fig. 3) of the measurement system (which is due to the rotatory motion of the Earth) onto the vector of the orbital velocity of the Earth V_o has the same sign. So, two moments of time or, in another word, two singular points a and α exist in the 24-hours daily circle where the sign of the projection changes. The sign of the projection is showed in Fig. 3 by gray circles. The palindrome effect can be observed, if the 12-hours time series start exact at the moments a and α. For the aforementioned results, these moments are determined within a 1-min accuracy by zero peaks shown in Fig. 1–2.

A special investigation on the time series measured within the 20-min neighborhood of the a and α moments was carry out with use of a semiconductor source of fluctuations (fluctuations of p-n junction current). The interval distribution obtained on the base of two sets of the 2-sec histograms constructed from this time series showed these moments to within the 2-sec accuracy. If we get a symmetric shift of the start-point of the time series relative to the a and α points, we find that the peak on the interval distribution (like those shown in Fig. 1–2) has the same time shift relative to zero interval.

The importance of two singular points a and α for the palindrome effect leads us to an idea about the significance of the tangential velocity vector V_τ and its projection onto the vector V_o. If consider the numerical value of the projection, we see that the set $1'$–$7'$ is symmetric to the $\bar{7}$–$\bar{1}$. In such a case the interval distributions (a) and (b) in Fig. 1–2 should be the same. Because they are different in real, just given supposition is incorrect. We also can consider our measurement system as oriented. In this case the 1 and $1'$ histograms should be the same. This means that zero peaks should be located in the "direct-direct" interval distributions, and be absent on the "direct-inverse" one. As seen in Fig. 1–2, this is not true.

On the other hand, it is possible to formulate a supposition which is qualitatively agreed with the obtained experimental results. This supposition is as follows. There is an external influence unshielded by the Earth, and this influence is orthogonal to V_o. In such a case the inversion of one set of the histograms is understood, and leads to the interval distributions like those of Fig. 1-2. As easy to see, in such an inversion rearrange order of the histograms, the histograms whose location is the same orthogonal line have the same numbers. This is because we have zero-peak in the "direct-inverse" interval distribution.

The origin of such lines can be the Sun. The only problem in this case is the orbital motion of the Earth. We cannot be located in the same line after 24-hours. As probable, we should suppose that this structure of the lines, which are orthogonal to V_o, moves together with the Earth.

Now we continue this bulky research on the palindrome effect. Detailed description of new results and the verifications to the aforementioned suppositions will be subjected in forthcoming publications.

Acknowledgements

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On the Second-Order Splitting of the Local-Time Peak

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The paper presents experimental investigations of a local-time peak splitting right up to a second-order splitting. The splitting pattern found in the experiments has a fractal structure. A hypothesis about the possibility of high order splitting is proposed. The obtained experimental result leads to a supposition that the real space possesses a fractal structure.

1 Introduction

The main subject of this paper is a local-time effect, which is one of manifestations of the phenomenon of macroscopic fluctuations. The essence of this phenomenon is that the pattern (shape) of histograms, which are built on the base of short samples of the time series of the fluctuations measured in the processes of different nature, are non-random. Many-years of investigations of such histograms carried out by the method of macroscopic fluctuations [1] revealed a variety of phenomena [2–4]. The most important among the phenomena is the local-time effect [5–8].

The local-time effect consists of the high probability of the similarity of the histogram pairs, which are divided by a time interval equal to the local-time difference between the points of measurement. This effect was registered in the scale of distances from the maximal distance between the locations of measurement which are possble on the Earth's surface (about 15,000 km) to the distances short as 1 meter. Besides, this effect can be observed on the processes of very different nature [2–4].

The idea of a typical local-time experiment is illustrated by Fig. 1. There in the picture we have two spaced sources of fluctuations 1 and 2, which are fixed on the distance L between them, and synchronously moved with a velocity V in such a way that the line which connects 1 and 2 is parallel to the vector of the measurement system's velocity V. In this case, after a time duration Δt_0

$$\Delta t_0 = \frac{L}{V}, \tag{1}$$

the source of fluctuations 1 appear in the same position that the source 2 was before. In Fig. 1 these new places are presented as $1'$ and $2'$. According to the local-time effect, coincident spatial positions cause similar histograms patterns. In the interval distribution built on the base of the measurements carried out by the system displayed in Fig. 1 (the number of similar pairs of the histograms as a function of the time interval between them), a single peak in the interval Δt_0 is observed.

In our previous works [6-8], we showed that there within

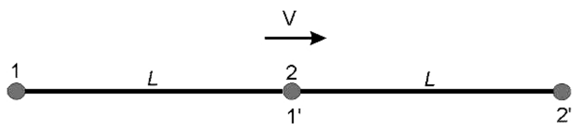

Fig. 1: This diagram illustrates the appearance of the local-time effect.

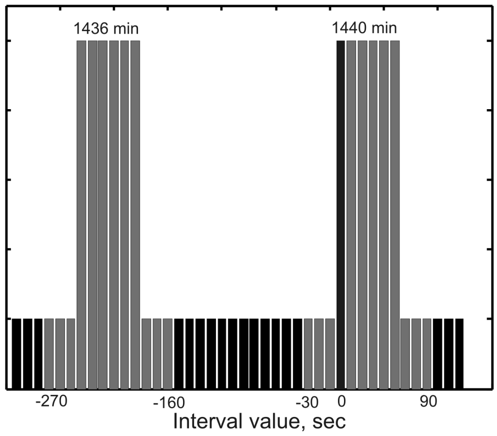

Fig. 2: The sketch of the solar (1440 min) and stellar (1446 min) splitting of the daily period.

the time resolution enhancement (with use histograms, shortest in time) the local-time peak splits onto two sub-peaks. It was found that the ratio between the splitting Δt_1 and the local-time value Δt_0 is $k = 2.78\times10^{-3}$. This numerical value is equal, with high accuracy, to the ratio between the daily period splitting 240 sec and the daily period value $T = 86400$ sec [7, 8]. This equality means that the local-time effect and the daily period are originated in the same phenomenon. From this viewpoint, the daily period can be considered as the maximum value of the local-time effect, which can be observed on the Earth.

In our recent work [8], we suggested that the sub-peaks of the local-time peak can also be split with resolution enhancement, and, in general, we can expect an n-order splitting with the value Δt_n

$$\Delta t_n = k^n\, \Delta t_0\,, \qquad n = 1, 2, 3, \ldots \tag{2}$$

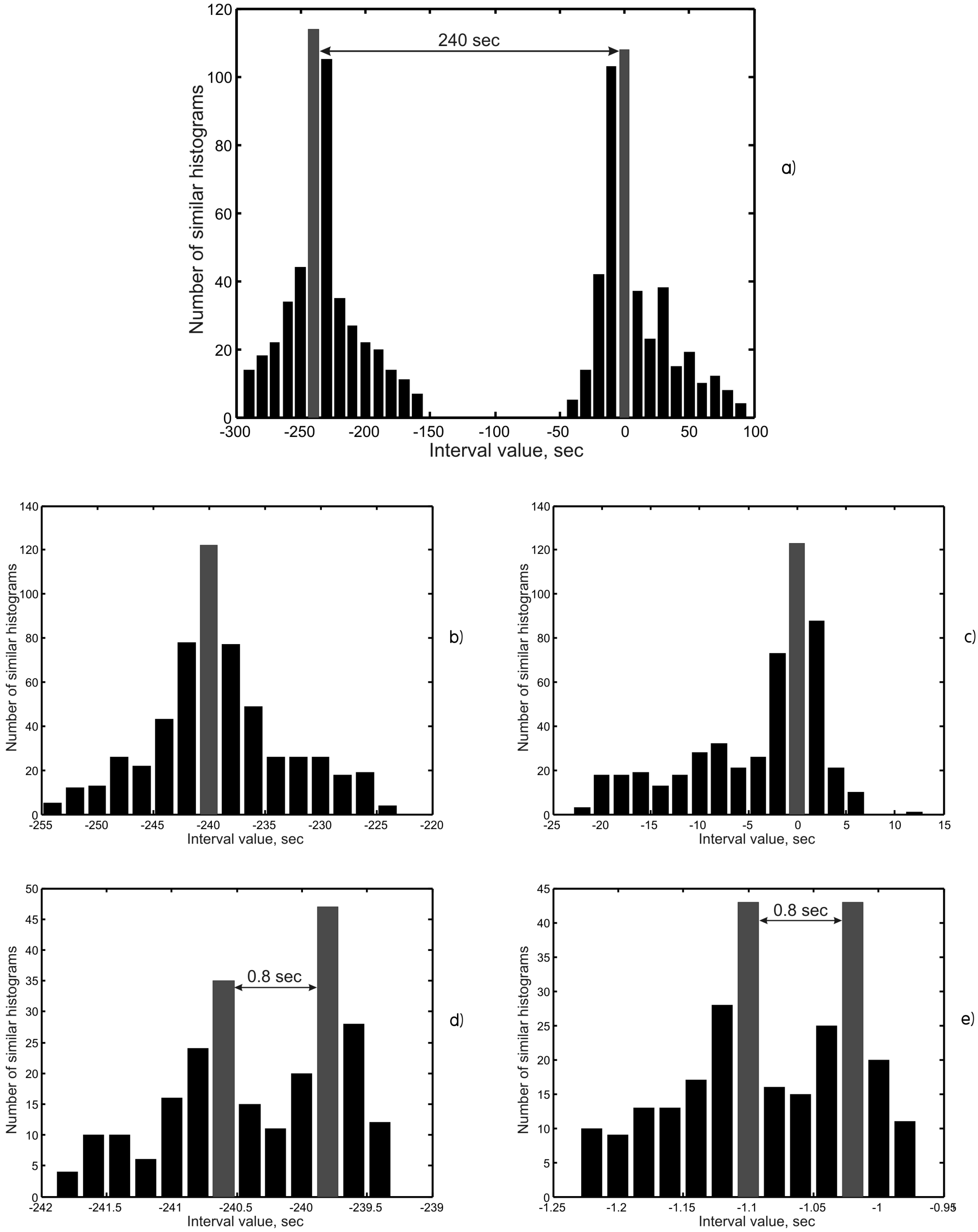

Fig. 3: The interval distributions for the 10-sec histograms (a), 2-sec histograms (b)–(c), and 0.2-sec histograms (d–e).

Preliminary results obtained in [8] verified this suggestion in part. The present work provides further investigation on the second-order splitting of the local-time peak.

As easy to see, from (2), every subsequent value of the local-time peak splitting Δt_n needs more than two orders of resolution enhancement. Therefore, most easy way to study Δt_n is to use the maximum value of Δt_0. Such a value, as stated above, is the daily period $\Delta t_0 = 86400$ sec.

2 Experimental results

To study the second-order splitting of the daily period, we use the known positions of the "solar peak" (1440 min) and the "sidereal peak" (1436 min), which are the first-order splittings of the daily period. The peaks are schematically displayed in Fig. 2. To find the position of the second-order splitting peaks, we used the method of consecutive refinements of the positions of the solar and sidereal peaks. The peaks displayed in Fig. 2 are determined with one-minute accuracy. Since the positions of the solar and stellar peaks are well-known, we can study its closest neighborhood by shortest (to one minute) histograms. In Fig. 2, such a neighborhood is displayed by gray bars (they mean 10-sec histograms). After obtaining the intervals distribution for the 10-sec histograms, the procedure was repeated, while the rôle of the 1-min histograms was played by the 10-sec histograms, and those were substituted for the 2-sec histograms. After this, the procedure was on the 2-sec and 0.2-sec histograms.

Zero interval (Fig. 2) marked by black colour corresponds to the start-point of the records. We used two records, started in the neighboring days at the same moments of the local time. So, the same numbers of histograms were divided by the time interval equal to the duration of solar day: 86400 sec. The interval values shown in Fig. 2 are given relative to zero interval minus 86400 sec.

The time series of the fluctuations in a semiconductor diode were registered on November 2–4, 2007. Each of the measurement consisted of two records with a length of 50000 and 19200000 points measured with the sampling rate 5 Hz and 8 kHz. On the base of these time series, we built the sets of the 10-sec, 2-sec, and 0.2-sec histograms. We used these sets in our further analysis.

The 10-second set of histograms was built on the base of the records, obtained with the sampling rate 5 Hz. Each 10-sec histogram was built from 50 points samples of the time series of fluctuation. The 2-sec and 0.2-sec sets were built on the base of the 50-point samples of the 25 Hz and 250 Hz time series (they were recount from the 8 kHz series).

It is important to note, that the solar day duration is not equal exactly to 86400 sec, but oscillates along the year. Such oscillations are described by the time equation [8]. To provide our measurements, we choose the dates when the time equation has extrema. Due to this fact, the day duration for all the measurements can be considered as the same, and we can average the interval distributions obtained on the base of the time series measured on November 2–4, 2007.

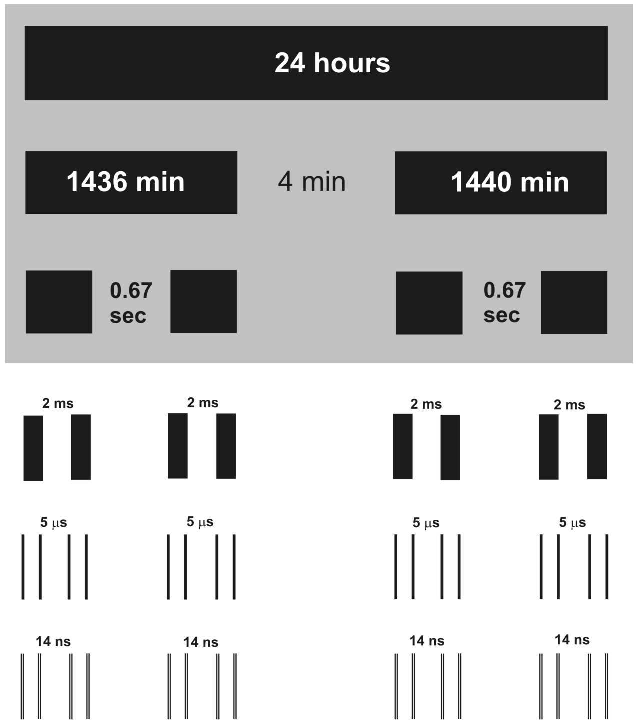

Fig. 4: The daily period splitting. Gray color marks the experimentally found splittings. The splitting displayed below were calculated on the base of the formula (2) for $n = 3 \ldots 5$.

The interval distributions obtained after the comparison of the histograms are given in Fig. 3. The upper graph, Fig. 3a, displays the interval distribution for the 10-sec histograms. As follows from Fig. 3a, the interval distribution in the neighborhood of the 1-min peaks consists of two sharp peaks (displayed by gray bars) which are separated by a time interval of 240 ± 10 sec giving the positions of the solar and sidereal peaks with 10-sec accuracy.

The interval distributions in the neighborhood of the 10-sec peaks (Fig. 3a), for the 2-sec histograms, are displayed in Fig. 3b–3c. Gray bars in Fig. 3b–3c correspond to the new positions of the solar and sidereal peaks with 2-sec accuracy. Considering the neighboring of the 2-sec peaks (Fig. 3b–3c) to the 0.2-sec histograms, we obtain the interval distributions displayed in Fig. 3d–3e. As easy to see, instead of the more precise position of the 2-sec solar and sidereal peaks, we obtain the splitting of the aforementioned peaks onto two couplets of new distinct peaks. So, from Fig. 3d–3e, we state the second-order splitting of the daily period.

3 Discussion

On the base of the formula (2), for $n = 2$ with use of $\Delta t_0 = 86400$ sec and $k = 2.78 \times 10^{-3}$ for the second-order splitting Δt_2, we get the value $\Delta t_2 = 0.67$ sec. From the experimentally obtained interval distribution (Fig. 3d–3e), we have $\Delta t_2 = 0.8 \pm 0.2$ sec. So, the experimental value agrees with the theoretical estimations made on the base of the formula (2).

Such an agreement leads us to a suppositon that there is a high-order splitting, which can be obtained from the formula (2). In Fig. 4, we marked by gray colour the experimentally found splitting. The splitting displayed below was calculated on the base of (2) for $n = 3 \ldots 5$, $\Delta t_0 = 86400$ sec, and $k = 2.78 \times 10^{-3}$. This splitting will be a subject of our further studies, and only this splitting is accessed to be studies now. For $n > 5$, we will need to get measurements with a sampling rate of about 7.5 THz. Such a sampling rate is out of the technical possibilities for now.

As was stated in Introduction, the local-time effect exists in the scale from the maximal distances, which are possible on the Earth's surface, to the distances close to one meter. Besides, the local-time effect doesn't depend on the origin (nature) of the fluctuating process. In the case, where the spatial basis of the measurements is about one meter, the time required for obtaining of the long-length time series (that is sufficient for further analysis) is about 0.5 sec. Any external influences of geophysical origin, which affect the sources of the fluctuations synchronously, cannot be meant a source of the experimentally obtained results. Only the change we have is the changing of the spatial position due to the motion of whole system with a velocity V originated in the rotatory motion of the Earth (see formula 1). From this, we can conclude that the local-time effect originates in the heterogeneity of the space itself. The results presented in Fig. 3 lead us to a supposition that such an heterogeneity has a fractal structure.

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LETTERS TO PROGRESS IN PHYSICS

On the Explanation of the Physical Cause of the Shnoll Characteristic Histograms and Observed Fluctuations

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Interpretations are given herein regarding the very visionary and important Pu-239 histogram work of Shnoll, and calling attention to background research which was not fully described in that paper. In particular, this Letter gives results of our theoretical and experimental research of gravitational anomalies during total solar eclipses and planetary line-up, and compares interpretations of the data with the work of Shnoll.

I am writing this Letter-to-the-Editor in reference to the very luminary paper authored by S. Shnoll [1], published in this journal, because of the far-reaching impact of the implications of this paper in describing nature, and because I have corresponded scientifically with the author on the subject of his repeat-pattern histogram work [2, 3] since 2001 when I first conveyed to Shnoll that his very meritorious radioactive decay findings of periodicities was an element of a larger and more ubiquitous external-particle net-transfer-of-momenta model and theory in which the origin of gravity due to collision-induced phenomena, was the initial cornerstone [4]. At that time Shnoll reported that the cause of the periodicities in his radioactive decay histograms was unknown but must be due to "profound cosmophysical phenomena" [2, 3]. The cited references within [1] do not convey the full background of the work leading to that paper [1] which Shnoll refers to as a survey, but in my opinion is far beyond a simple review-of-the-literature paper, and is instead a very significant archival work. Additionally, within the text itself [1] there are no references to the private communications References 40 and 41 cited in the list of references in [1]. For these above reasons, I wish to clarify various elements of the paper [1].

I also advised Shnoll in 2003 and 2004 to search out his earlier Pu-239 alpha decay data that were taken at the time of a total solar eclipse [5], doing so because I was impressed with his work during 2003 on characteristic histograms during the New Moon, observed simultaneously independent of location and latitude [6]. As stated, although my work, and that of colleague, Frank Lucatelli, is referenced as private communications in Shnoll's paper [1], as Refs. 40 and 41, those references are not cited in the text, but instead only in the bibliography, and thus most readers would be unaware of our input into Shnoll's paper of [1]. I also conveyed to Dr. Shnoll our own work whereby at my request, colleagues had measured a dip in the radioactive decay of Co-60 in southeastern Kansas, and in Po-210 in the Boston area at the time of the total solar eclipse of 4 December, 2002, when the "umbra" passed closest to the isotope sources [7]. We predicted that this effect would be observed based on the data of Allais [8], and of Saxl and Allen [9] showing decreases in gravity associated with the eclipses of 1954 and 1959, and the eclipse of 1970, respectively, and also based on the dip in gravity which I observed using a dual Newton-cradle pendula system during the planetary line-up of Earth-Sun-Jupiter's/magnetosphere-Saturn on 18 May 2001 [10]. This prediction was based on my postulate that if gravity were a result of external particle impingement on mass particles, then the other three axiomatic "forces" should also depend upon, or be influenced by the external particle flux.

In this Letter-to-the-Editor, I wish to address points regarding Dr. Shnoll's interpretation of his decades of data, and of the data of others.

Shnoll has conducted very excellent collimator studies, which showed that when the collimator was pointed north toward the pole star, the near-daily-periods in the repeat histogram patterns of Pu-239 decay were *not* observed, contrasting the data showing repeat histograms when the collimators were oriented east, and when they were oriented west. Shnoll interprets these data stating that … "Such a dependence, in its turn, implies a sharp anisotropy of space." I suggest that a better and more correct manner to interpret these data is in terms of the Earth-Moon-Sun system, spinning and orbiting in the east-west ecliptic plane interrupting, through capture and/or scattering, elementary particles (probably neutrinos) that would otherwise impinge upon the radioactive source and perturb the weak interaction in unstable nuclei. This is not a proof of heterogeneity and anisotropy of space time in a general sense, but indication of celestial body orbits that exist in the general plane of the ecliptic — the external particles being omnidirectional, and the heterogeneity arising generally from supernovae explosions and their consequences. Shnoll earlier in the paper rightfully states, referring to daily, monthly, and yearly periods in repeat forms of the histograms, that "All these periods imply the dependence of the obtained histogram pattern on two factors of rotation — (1) rotation of the Earth around its axis, and (2) movement of the Earth along its circumsolar orbit", thus supporting the above explanation. Shnoll alludes to this explanation by stating that a heterogeneity in the gravitational field results from the existence of "mass thicknesses" of celestial bodies, and this then must relate to the capture cross-section between nucleons of the mass bodies. Stanley [11] has described in detail the properties of mass that relate to gravity, and treated mathematically the flux of externally impinging neutrinos [11] as related to gravitational interactions. Shnoll invokes a "wave interference" and relates it to a gravitational effect (which associates with our use of interruption and capture, but in our case the phenomenon is particle-based rather than wave-based).

In Section 10 of [1], the author describes the observations of characteristic histogram patterns for the occurrence of the New Moon, and the total solar eclipse. The author writes that the specific patterns do not "depend on position on the Earth's surface where the Moon's shadow falls during the eclipse or the New Moon." We have found, however, that the decrease in gravity signature during a total solar eclipse *does* depend upon the latitude of the location of totality and of the measurements [12], and this is clearly proven in comparing the different data signatures during eclipses in different locations, most notably the work of Wang et al. [13, 14] during the eclipse of March 1997 in China. The work of Stanley and Vezzoli [12] has been able to mathematically describe from first principles the detailed gravimeter data of Wang et al. [13, 14] for the above eclipse, including the parabolic dips in gravity at first contact, and at last contact. The dependence upon latitude of the location of the measurements and of totality is due to the elastic scattering properties of the three-body problem. Shnoll then interprets the overall data in association with the fractality of space-time — a conclusion that we have also reached in our gravity research [11, 15] and that is also described very recently by Loll [16]. Shnoll notes that he also observes a chirality in histograms, which we have shown is fundamental in the nature of materials and the aggregation of mass to form compounds [17].

It is interesting to note that in [1], Shnoll concludes that there is a spatial heterogeneity on the scale of 10^{-13} cm. This is the value that we calculate for the inter-neutrino spacing of the neutrino flux, corresponding to a collision cross-section with nucleons of $\sim 10^{-38}$ cm^2, and a particle density 3.7×10^{28}–10^{34} particles per cm^3.

Our work, and our interpretation of the Shnoll work [1–3], and many other works by Shnoll, correlates very well with the positron annihilation work of Vikin [19] showing that the production of positronium from Na-22 undergoes a maximum near the time of the New Moon, and a minimum near the time of the Full Moon. At the time of the New Moon, the Earth laboratory (whether measurements are of gravitational interactions or of radioactive decay phenomena) faces in the general direction of the line of the Moon and the Sun for a short period of the day, and then rotates such that the laboratory faces free and open space and distant stars during the duration of the day, so that a large complement of neutrinos falls uninterrupted onto the measuring device; also neutrinos that are emitted by the Sun may be scattered by the Moon to affect the data. During the Full Moon, however, the Earth laboratory is always between the Moon and the Sun, and hence the overall collision physics is considerably different.

Shnoll sums the interpretation of the work that he describes within [1] by stating "Taken together, all these facts can mean that we deal with narrowly directed wave fluxes", which he refers to as beams that are more narrow than the aperture of the collimators of the apparatus (0.9 mm). Our model and theory of gravity [11] is based on a flux of particles, and the "narrow beam" is interpreted due to very low-angle elastic scattering of external particles by the nucleons of the celestial bodies [11, 12], particularly the Moon (near body in [12]) and Sun (far body), such that some particles never reach the detecting apparatus such as pendula, gravimeter, or radioactive source-detector system.

Fundamental to Shnoll's work is his assertion that these periodic characteristic histograms relate to a wide variety of phenomena ranging from bio-chemical phenomena, to the noise in a gravitational antenna, to alpha decay. This is in agreement with my own work and that of others, and I have found that anomalies in gravity, radioactive decay of Po-210 (and Co-60), and changes in plant growth, orientation, and physiology, as well as embryonic centriole-centriole separation phenomena, and even DNA and its sheathing H_2O, are affected by the Earth-Moon-Sun relationship [10–12, 14, 17, 19, 20]. It has been shown by Gershteyn et al. [21] that the value of G varies at least 0.054% with the orientation of the torsion pendula masses with the stars, and that G is periodic over the sidereal year [21] — this periodicity arguing for a strong link between the Shnoll radioactive decay data and gravity. Furthermore the Shnoll work [1] cites the possibility of a space-time anisotropy in a preferential direction, and refers to the drift of the solar system toward the constellation Hercules. Our theoretical work in collision-induced gravity shows that G is a function of collision cross-section of the neutrino-nucleon interaction [11], and experimental work indicates that G is a function also of at least temperature, phase, and shape [10, 22]. Our very recent experimental work determined that $G = 6.692\times10^{-11}$ cubic meters per kg sec^2 [15] which compares very favorably with the slightly earlier work of Fixler et al [22] using precision a interferrometric method in conjunction with cold Cs atoms and a known Pb mass, yielding $G = 6.693\times10^{-11}$ cubic meters per kg sec^2 — these values being considerably larger than the normally utilized value of 6.67×10^{-11}. These data are in accord with an increasing trend in G that could possibly be related to other trends such as that cited by Shnoll [1].

Shnoll reports [1] that the subject histograms have a fine structure that shows what he refers to as "macroscopic fluctuations". We have reported gravitational fluctuations [10] that appear at random, and are associated with time intervals of ∼0.13 sec, indicating another correlation between gravity data and radioactive decay data. The gravitational fluctuations that we detected were observed in the form of two Newton cradle pendula dwelling near each other for prolonged periods of time, but occurring in an unpredictable manner. We tentatively correlated these events with signals arriving from supernovae events that had occurred somewhere in the vastness of the universe. We also had detected on 27 August 2001 a peak in the radioactive decay of our Po-210 source, far in excess of two-sigma Poisson statistics, and later correlated with the arrival of radiation from supernovae explosion SN 2001 dz in UGC 47, emitting energy in all neutrinos of the order of 10^{46} joules.

All of the above points to the ubiquity of a model of nature based on elementary impinging momentum-transferring external particles that can be interrupted by mass particles, rather than nature being based on the conventional four axiomatic forces and their respective field theory. Furthermore, in an external particle based model for gravity, there is no need to invoke a purely mathematical "fabric" to space-time curvilinearity according to geodesics or warping, nor is it necessary to invoke Riemanian space, nor Minkowski space, but instead space-time is considered to be of a fractal geometry, and the trajectory of mass particles and photons through space is curved because of collisions with neutrinos (WIMPS). Although the collision cross-section of the neutrino with the photon is extremely low, the flux density of the neutrino in our region of the universe is extremely high, and we postulate that the bending of light is due to that interaction. It seems that astrophysics is now poised to affirm modifications to Einstein's theory of General Relativity, and this is not unexpected in that many recent findings have indicated that gravity is quantized [15, 16, 24–26]. Understanding the nature and details of this quantization is one of the very major challenges and objectives in physics of this new century.

See also [27] for corroboration by private communication of periodic behavior of radioactive decay data during New Moon.

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27. In private communications, Professor N. Goleminov (Moscow) had corresponded data to me in in the summer of 2006, showing that the standard deviation in his radioactive decay data was oscillatory, giving a minimum at the time of the New Moon, and a maxima in standard deviation occurring prior the New Moon, and another maxima subsequent to the New Moon. In the light of what I have conveyed in this Letter, Goleminov's work can be interpreted to a scattering by the moon of particles emanating from the Sun (during the New Moon), that would otherwise affect the radioactive decay data and cause a higher standard deviation at times other than the New Moon period. This interpretation would also be allied to Shnoll [1] using the term "interference" of flux. Goleminov's data also correlates indirectly with the positron annihilation periodicity work cited in this Letter.

LETTERS TO PROGRESS IN PHYSICS

Reply to the Letter by Gary C. Vezzoli

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In a letter published by Dr. Vezzoli in the current issue of your journal, he claims priority back to 2001 for an explanation to certain gravitational phenomena, which were first recorded by me and my co-workers at my laboratory. He claims priority to me on the basis of the fact that he shared his results and plans with me in 2001 in private communication. However, I and my co-workers understood the phenomena in the same terms as much as 20 years before that, in the 1980's, and discussed by us in numerous publications during the 1980's, in the Soviet (now Russian) scientific journals. I provide a list of my early publications, refuting Dr. Vezzoli's claim to priority.

Dear Sir,

I refer a letter published by Dr. Vezzoli in the current issue of your journal he claims priority back to 2001 for an explanation to certain gravitational phenomena, which were first recorded by me and my co-workers at my laboratory. Clearly, Dr. Vezzoli is mistaken to think that he was the first person to propose, in 2001, an explanation of the gravitational phenomena recorded by me and my co-workers, at my laboratory. We in fact understood the phenomena in the same terms as much as 20 years before that, in the 1980's, as numerous publications [1–17] testify. For instance, an explanation of the experiments was given by me in 1989 at the International Congress on Geo-Cosmic Relations, in Amsterdam [4, 5]. This explanation was repeated in the other papers, published by us in 1989, 1995, and 2001. Our data, obtained during solar eclipses, began with the eclipse of July 31, 1981, when a large series of measurements was processed by 30 experimentalists connected to my laboratory, located at 10 geographical points stretching from the Atlantic to the Pacific (Sakhalin Island) along the corridor of the eclipse. We got more than 100,000 single measurements of the speed of chemical reactions during that eclipse. Our results were published in 1985 and 1987 [2, 3]. Since 1981 we processed measurements obtained during many solar and lunar eclipses, and also Full Moon and New Moon phases. The results were published in part only because a detailed analysis was required. In 1989 I published a paper wherein I claimed an observed change in the form of histograms obtained from a radioactive decay which was dependent upon the position of the Moon over the horizon [6]. This effect was observed at different geographical points. In the same paper [6] I suggested a gravitational origin of the observed effects.

I was pleased by the fact that a suggestion similar to that of mine was given by our American colleagues (Dr. Vezzoli, Dr. Lucatelli, and others), 20 years subsequent to me. This is despite that fact that their conclusions were made on the basis of scanty experimental data, in contrast to our own.

Dr. Vezzoli's claim to priority in this research, and hence his intellectual property, is I feel due to the following circumstance: the absence of information in the West about most publications made by us during the 1980's in the Soviet (now Russian) scientific journals.

My belief is that I, being a purely experimental physicist, should represent neither theoretical interpretations of the observed phenomena nor hypotheses on the subject given by the other authors. They may do that in their own papers; such a policy would be most reasonable from any standpoint.

Unfortunately, no definite theoretical explanation of the phenomenon we observed [1–16] was published in the scientific press until now. The authors of a series of papers, published in 2001 in *Biophysics*, v. 46, no. 5, presented different hypotheses on the subject. Not one of those hypotheses resulted in a calculation which could be verified by experiment.

I am responsible for a huge volume of experimental data, resulting from decades of continuous experimental research carried out by myself and dozens of my co-workers. I wouldn't like to dilute the data with a survey on hypotheses and theoretical propositions given by the theoretical physicists. Frankly speaking, I have no obligation to give such a survey. I am prepared to provide references to published papers on the subject, if it is suitable according to contents. However I feel that it is wrong to refer any information obtained in private communications before they publish their views on their own account.

I give below a list of my early publications, which refute the claim made by Dr. Vezzoli. Even a cursory inspection of the publications reveals the fact that the information provided to me by Dr. Vezzoli and Dr. Lucatelli wasn't news to me. I do not wish to be embroiled in any quarrel with them. However, having the list of my early publications, it would be strange to raise the issue of priority.

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LETTERS TO PROGRESS IN PHYSICS

The Earth Microwave Background (EMB), Atmospheric Scattering and the Generation of Isotropy

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In this work, the presence of substantial microwave power in the atmosphere of the Earth is discussed. It is advanced that this atmospheric microwave power constitutes pools of scattered photons initially produced, at least in substantial part, by the $\sim$3 K microwave background. The existence of these microwave pools of photons can serve to explain how the Earth, as an anisotropic source, is able to produce an Earth Microwave Background (EMB) at $\sim$3 K which is isotropic.

The $\sim$3 K microwave background [1] has always been associated with the primordial universe [2]. Conversely, I have advanced an oceanic origin for this signal [3–7], a scenario supported by Rabounski and Borissova [8–10]. The Earth has an anisotropic surface comprised of water and solid matter. However, the microwave background is isotropic. As a result, if the Earth is the emitter of the $\sim$3 K signal [1], isotropy must be achieved by scattering oceanic photons in the atmosphere.

Initially, I invoked a Compton process in the atmosphere in order to generate isotropy from an anisotropic oceanic source [3]. Yet, given the nature of the scattering required and the energies involved, such a mechanism is not likely. I therefore proposed that Mie scattering should be present [4]. Finally, I discussed both Rayleigh and Mie scattering [6]. Rayleigh scattering should be more important at the lower frequencies, while directional Mie scattering would prevail at the higher frequencies [6].

Currently [2], the microwave background is believed to be continuously striking the Earth from all spatial directions. Under steady state, any photon initially absorbed by the atmosphere must eventually be re-emitted, given elastic interactions. Since the incoming microwave background is isotropic [1, 2], then even scattering effects associated with absoption/ emission should not reduce the signal intensity on the ground, because of steady state [6]. Thus, there should be no basis for signal attenuation by the atmosphere, as I previously stated [6]. Nonetheless, current astrophysical models of the atmosphere assume that such attenuations of the microwave background occur [i.e. 11, 12]. These models also appear to neglect atmospheric scattering [i.e. 11, 12].

I have mentioned that scattering processes are a central aspect of the behavior of our atmosphere at microwave frequencies [6]. In addition, since steady state assumptions should hold, any scattering of radiation, should build up some kind of reservoir or pool of scattered photons in the atmosphere. Scattering is known to become more pronounced with increasing frequencies. Consequently, larger photon reservoirs might be seen at the shorter wavelengths.

It is known that the atmosphere interferes with the measurements of the microwave background [11]. However, this interference has been attributed to atmospheric emissions [i.e. 11, 12], not to scattering. Experimental measurements have demonstrated that atmospheric emissions increase substantially with frequency [11, 12]. For instance, emissions attributed to the water continuum tend to increase with the square of the frequency [12]. Atmospheric contributions are so pronounced at the elevated frequencies, that they can contribute in excess of 15 K to the microwave background temperature measurements at wavelengths below 1 cm [see table 4.2 in ref 11]. At a wavelength of 23.2 cm, Penzias and Wilson [1] obtained a 2.3 K contribution to their measurement just from the atmosphere [see table 4.2 in ref 11]. Even at a wavelength of 75 cm, an atmospheric contribution of 1K can be expected [see table 4.2 in ref 11]. Atmospheric modeling used in microwave background studies confirms the increase in interference with frequency and its decrease with altitude [i.e. 11, 12].

A pronounced increase in emission with frequency is expected if scattering is present. As such, it is reasonable to postulate that astrophysics is dealing with scattering in this instance [6], not with simple emission [11, 12]. Microwave background measurements at the elevated frequencies are therefore primarily complicated not by a lack of absolute signal, as I previously believed [6], but rather, by the tremendous interference from the scattered signal reservoirs in the atmosphere. In order to eliminate this effect, we are therefore forced to study the elevated frequencies from mountains top or at higher altitudes using balloons, rockets and satellites [11].

Should the microwave background arise from the universe [2], the atmosphere of the Earth would still generate the same reservoirs of scattered radiation. The atmosphere cannot distinguish whether a photon approaches from space [2] or from the oceanic surface [6]. Thus, establishing the presence of the scattered pool of photons in the atmosphere cannot reconcile, by itself, whether the microwave background originates from the cosmos, or from the oceans. Nonetheless, since a steady state process is involved, if a ∼3 K signal is indeed produced by the oceans, then a ∼3 K signal will be detected, either on Earth [1] or above the atmosphere [13]. The Planckian nature of this signal will remain unaltered precisely because of steady state. This is a key feature of the steady state regimen. Importantly, experimental measures of emission [11, 12] do confirm that substantial microwave power appears to be stored in the scattering reservoirs of the atmosphere. Consequently, a mechanism for creating isotropy from an anisotropic oceanic signal [5] is indeed present for the oceanic ∼3 K Earth Microwave Background.

Dedication: This work is dedicated to my three sons, Jacob, Christophe and Luc.

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LETTERS TO PROGRESS IN PHYSICS

Reply to the "Certain Conceptual Anomalies in Einstein's Theory of Relativity" and Related Questions

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This paper answers twelve most common questions on the basics of Einstein's theory of relativity. The answers remove most key problems with a real, solid understanding of the theory.

Since its inception, *Progress in Physics*, has maintained the importance of freedom of expression in science [1]. As a result, the journal has sometimes published works even though the editorial staff differred either with the premise or with the conclusions of a paper. The editorial board maintains that it is best to disseminate works, rather than to unknowlingly suppress seminal ideas. The validity of all scientific arguments will eventually be discovered. For this reason, the journal strongly upholds the rights of individual scientists relative to publication. At the same time, many questions focusing on fundamental aspects of Einstein's theory of relativity have been submitted to the journal. Most of these letters were not published as they were concieved by authors who did not properly grasp the concepts outlined within the classic textbooks on this subject, such as *The Classical Theory of Fields* by Landau and Lifshitz [2] and others [3].

Recently, the editorial board made the decision to publish a work by Stephen J. Crothers [4] even though some questions remained relative to its basic premise. We chose to move to publication for two reasons. First, Crothers is a capable scientist who has already demonstrated substantial insight into General Relativity [5]. Indeed, the editorial board has written in support of these ideas [6]. Second, the journal has received substantial correspondance from both amateurs and established scientists. These letters have focused on perceived problems with Einstein's theory of relativity. The editors therefore feels compelled to address these concerns, both relative to Crothers [4] and to other serious scientists who had previously worked, with success, on numerous applications of the theory of relativity.

In general, the correspondance we have received has expressed doubt concerning the validity of some key points in Einstein's theory. We found that these questions originated in the fact that the scientists asking the questions were educated as physicists, while the base of Einstein's theory is Riemannian geometry. It is therefore not suprising that some confusion might arise. The meaning of Einstein's theory is the geometrization of physics, the expression of all physics through the geometrical properties of the four-dimensional pseudo-Riemannian space (the basic space-time of the theory of relativity) or its extensions. Many physicists came to the theory of relativity from the other fields of physics; they learned Einstein's theory through brief courses which gave the theory in its historical sense, often with artifically introduced principles and postulates. When the meaning of Einstein's theory, the geometrization of physics, was finally understood through the joint intellectual powers of Albert Einstein and Marcel Grossmann, all the physical principles came out from the consideration; they all became covered by the particular properties of the geometry within four-dimensional pseudo-Riemannian space. Such a "historical" approach, which is very common in most brief courses on the theory of relativity for physicists, often carries a student away with speculations on the principles and postulates, instead of studying Riemannian geometry itself. As a result, serious physicists erred relative to simple questions which remained open after their brief education. Only a small minority of physicists, who devoted their life to understanding the theory of relativity, were lucky enough to be able to study the special (more advanced) courses on this subject.

Here we collected twelve of the most common questions on the basics of Einstein's theory, asked by the readers and some of our colleagues. We hope the answers will remove most key problems with a real, solid understanding of the theory.

First. Naturally, each term in Einstein's equations in emptiness (i.e. with zero right-hand-side) vanishes. This is due to that fact that, in such a case, the scalar curvature is zero $R = 0$, so Einstein's equations become the vanishing condition for Ricci's tensor: $R_{\alpha\beta} = 0$. In the same time, Ricci's tensor $R_{\alpha\beta}$ isn't a number, but a 2nd-rank tensor whose components are 16 (only 10 of whom are independent). The formula $R_{\alpha\beta} = 0$, i.e. Einstein's equations in emptiness, means 10 different differential equations with zero elements on the right-hand-side. These are differential equations with respect to the components of the fundamental metric tensor $g_{\alpha\beta}$: each of 10 equations $R_{\alpha\beta} = 0$ is expressed in the terms containing the components of $g_{\alpha\beta}$ and their derivatives according to the definition of Ricci's tensor $R_{\alpha\beta}$. Nothing more. (With non-zero elements on the right-hand-side, these would be Einstein's equations in a space filled with distributed matter, e.g. electromagnetic field, dust, liquid, etc. In such a case these would be 10 differential equations with a free term.)

Therefore the vanishing of each term of Einstein's equations in emptiness doesn't matter with respect to the validity of the equations in both general and particular cases.

Second. A common mistake is that a gravitational field is described by Einstein's equations. In fact, a gravitational field is described not by Einstein's equations, but the components of the fundamental metric tensor $g_{\alpha\beta}$ of which only 10 are substantial (out of 16). To find the components, we should solve a system of 10 Einstein's equations, consisting of $g_{\alpha\beta}$ and their derivatives: the differential equations with zero right-hand-side (in emptiness) or non-zero right-hand-side (with distributed matter).

Third. The condition $R_{\alpha\beta} = 0$ doesn't mean flateness, the pseudo-Euclidean space ($g_{00} = 1$, i.e. the absence of gravitational fields), but only emptiness (see the first point that above). Only a trivial case means flatness when $R_{\alpha\beta} = 0$.

Fourth. A mass, the source of a gravitational field, is contained in the time-time component g_{00} of the fundamental metric tensor $g_{\alpha\beta}$: the gravitational potential expresses as $\mathrm{w} = c^2\sqrt{1-g_{00}}$. Therefore Einstein's equations in emptiness, $R_{\alpha\beta} = 0$, satisfy a gravitational field produced by a mass ($g_{00} \neq 1$). The right-hand-side terms (the energy-momentum tensor $T_{\alpha\beta}$ of matter and the λ-term which describes physical vacuum) describe distributed matter. There is no contradiction between Einstein's equations in emptiness and the equivalence principle.

Fifth. In the case of geometrized matter, the most known of which are isotropic electromagnetic fields (such fields are geometrized due to Rainich's condition and Nortvedt-Pagels' condition), the energy-momentum tensor of the field expresses itself through the components of the fundamental metric tensor. In such a case, we can also construct Einstein's equations containing only the "geometrical" left-hand-side by moving all the right side terms (they consist of only $g_{\alpha\beta}$ and their functions) to the left-hand-side so the right-hand-side becomes zero. But such equations aren't Einstein's equations in emptiness because $R_{\alpha\beta} \neq 0$ therein.

Sixth. Minkowski's space, the basic space-time of Special Relativity, permits test-masses, not point-masses. A test-mass is one which is so small that the gravitational field produced by it is so negligible that it doesn't have any effect on the space metric. A test-mass is a continous body, which is approximated by its geometrical centre; it has nothing in common with a point-mass whose density should obviously be infinite.

The four-dimensional psedo-Riemannian space with Minkowski's signature $(+---)$ or $(-+++)$, the space-time of General Relativity, permits continuosly gravtating masses (such a mass can be approximated by the centre of its gravity) and test-masses which move in the gravitational field. No point-masses are present in the space-time of both Special Relativity and General Relativity.

Seventh. Einstein's theory of relativity doesn't work on infinite high density. According to Einstein, the theory works on densities up to the nuclear density. When one talks about a singular state of a cosmological solution, one means a so-called singular object. This is not a point, but a compact object with a finite radius and high density close to the nuclear density. Infinite high density may occur on the specific conditions within a finite radius (this is described in the modern relativistic cosmology [7]), but Einstein's theory does consider only the states before and after that transit, when the density lowers to that in atomic nuclei. Such a transit itself is out of consideration in the framework of Einstein's theory.

Eighth. Einstein's pseudotensor isn't the best solution for elucidating the energy of a gravitational field, of course. On the other hand, the other solutions proposed to solve this problem aren't excellent as well. Einstein's pseudotensor of the energy of a gravitational field permits calculation of real physical problems; the calculation results meet experiment nicely. See, for intance, Chapter XI of the famous *The Classical Theory of Fields* by Landau and Lifshitz [2]. This manifests the obvious fact that Einstein's pseudotensor, despite many drawbacks and problems connected to it, is a good approximation which lies in the right path.

Bel's tensor of superenergy, which is constructed in analogy to the tensor of the electromagnetic field, is currently the best of the attempts to solve the problem of the energy of the gravitational field in a way different from that of Einstein. See the original publications by Louis Bel [8]. More can be found on Bel's tensor in Debever's paper [9] and also in Chapter 5 of *Gravitational Waves in Einstein's Theory* by Zakharov [10].

Besides Bel's tensor, a few other solutions were proposed to the problem of the energy of the gravitational field, with less success. Einstein's theory of relativity isn't fosilized, rather it is under active development at the moment.

Nineth. Another very common mistake is the belief that Einstein's equations have no dynamical solution. There are different dynamical solutions, Peres' metric for instance [11]. Peres's metric, one of the empty space metrics, being applied to Einstein's equations in emptiness (which are $R_{\mu\nu} = 0$), leads to a solely harmonic condition along the x^1 and x^2 directions. One can read all these in detail, for instance, in Chapter 9 of the well-known book *Gravitational Waves in Einstein's Theory* by Zakharov [10].

Tenth. The main myths about Einstein's theory proceed in a popular misconception claiming the principal impossibility of an exceptional (absolute) reference frame in the theory of relativity. This is naturally impossible in the space-time of Special Relativity (Minkowski's space, which is the four-dimensional pseudo-Euclidean space with Minkowski's signature) due to that fact that, in such a space, all space-time (mixed) components g_{0i} of the fundamental metric tensor are zero (the space is free of rotation), and also all non-zero components of the metric are independent from time (the space deformation is zero). This however isn't true in the space-time of General Relativity which is pseudo-Riemannian, so any components of the metric can be non-zero therein. It was shown already in the 1940's, by Abraham Zelmanov, a prominent scientist in the theory of relativity and cosmology, that the space-time of General Relativity permits absolute reference frames connected to the anisotropy of the fields of the space rotation or deformation of the whole Universe, i. e. connected to globally polarized (dipole-fit) fields which are as a global background gyro. See Chapter 4 in his book of 1944, *Chronometric Invariants* [7], for detail.

Eleventh. Another popular myth claims that an experiment, which manifests the anisotropy of the distribution of the velocity of light, is in contradiction to the basics of the theory of relativity due to the world-invariance of the velocity of light. This myth was also completely shattered [12]. According to the theory of physical observables in General Relativity [7], the observable velocity of light lowers from the world-invariance of the velocity by the gravitational potential and the linear velocity of the space rotation at the point of observation. The vector of the observable velocity of light directed towards an attracting body is carried into the direction of our motion in the space. As a result, the distribution of the vectors of the velocity of light beams has a preferred direction in space, depending on the motion, despite the fact that the world-invariance of the velocity of light remains unchanged. In such a case the field of the observable velocities of light is distributed anisotropically. If the space is free from rotation and gravitation (for instance, Minkowski's space of Special Relativity), the anisotropic effect vanishes: the spatial vectors of the observable velocity of light are distributed equally in all directions in the three-dimensional space. The anisotropic effect hence is due to only General Relativity. Here is nothing contradictory to the basics of Einstein's theory.

Twelfth. About Friedmann's models of a homogeneous universe, including the Big Bang scenario. It was already shown in the 1930's [7] that Friedmann's models have substantial drawbacks both in its principal and mathematical approaches. Friedmann's models are empty (free of distributed matter), homogeneous, and isotropic. They were only the first, historical step made by the scientists in the attempt to create physically and mathematically valid models of relativistic cosmology. There are hundreds of thousands of solutions to Einsten's equations. True relativistic cosmology should be stated by models of an inhomogeneous, anisotropic universe, which meet the real physical conditions of the cosmos, and can be applied to only a local volume, not the whole Universe [7]. A classification of the cosmological models, which are theoretically thinkable on the basis of Einstein's equations, was given in the 1940's. See Chapter 4 of *Chronometric Invariants* by Zelmanov [7], for detail. Many different cosmological scenarios are listed there, including such exotics as the transits through the states of infinite rarefraction and infinite density on a finite volume (that is possible under special physical conditions). The Big Bang model, the model of expansion of a compact object of a finite radius and nuclear density, where the space is free of gravitating bodies, rotation, and deformation, is just one of many. Aside for this model, many other models of an expanding universe can be conceived on the basis of the solutions of Einstein's equations.

Relativistic cosmology is based on the time functions of the density, volume and others obtained from solutions to Einstein's equations. Therefore, only those states are under consideration, which are specific to Einstein's equations (they work up to only the nuclear density). Relativistic cosmology points out only the possibility of the state of infinite density as a theoretically extrem of the density function, while the equations of the theory are valid up to only the nuclear density. It is a very common mistake that Einstein's theory studies the state of infinite density, including a singular point-state.

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LETTERS TO PROGRESS IN PHYSICS

Rational Thinking and Reasonable Thinking in Physics

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The usual concept of space and time, based on Aristotle's principle of contemplation of the world and of the absoluteness of time, is a product of rational thinking. At the same time, in philosophy, rational thinking differs from reasonable thinking; the aim of logic is to distinguish finite forms from infinite forms. Agreeing that space and time are things of infinity in this work, we shall show that, with regard to these two things, it is necessary to apply reasonable thinking. Spaces with non-Euclidean geometry, for example Riemannian and Finslerian spaces, in particular, the space of the General Theory of the Relativity (four-dimensional pseudo-Riemannian geometry) and also the concept of multi-dimensional space-time are products of reasonable thinking. Consequently, modern physical experiment not dealing with daily occurrences (greater speeds than a low speed to the velocity of light, strong fields, singularities, etc.) can be covered only by reasonable thinking.

In studying the microcosm, the microcosm or any extreme conditions in physics, we deal with neo-classical, unusual physics. For example, the uncertainty principle in quantum physics and the relativity principle in relativistic physics are really unusual to our logic. We may or may not desire such things, but we shall agree with physical experiments in which there is no exact localization of micro-particles or in which, in all inertial systems, light has the same speed and, hence, time is not absolute. Our consent with such experiments, the results of which are illogical from the view-point of ordinary consciousness, means that we accept to start to operate at another level of consciousness which is distinct from the level of consciousness necessary for the acceptance of experimental results of classical physics. The fundamental difference consists of the human consciousness at such a new level which operates with other categories — forms of infinity.

The world is a thing of infinity. Hence, a logic which includes forms of infinity is necessary for its cognition. The logic in itself considers the thinking in its activity and in its product. This product shall then be used by all sciences. The one and only philosophy, underlining that problem of logic is to distinguish finite forms from infinite forms, and to show some necessity to consider thinking in its activity. This activity is supra-sensory activity; though it may look like sensual perception, such as contemplation. Therefore the content of logic is the supra-sensory world and in studying it we will stay (i.e., remain) in this world. Staying in this world, we find the universal. For instance, the general laws of the motion of planets, are invisible (they are not "written in the sky") and inaudible; they exist only as a process of activity of our thinking. Hence, we arrive at Hegel's slogan "what is reasonable, is real" [1] by which the status of thinking is raised to the status of truth. As a result, it is possible not only to assume that our real world has a tie with unusual geometries, but, in fact, it is true.

From this point of view, it is possible to agree with many mathematicians [2–6], that Euclid could direct natural sciences. In another way, at the same time, he could have taken not space as primary concept, but time.

Aristotle, having proclaimed the general principle of a world-contemplation of motions occurring simultaneously [7], has come to a conclusion (which is only natural to that epoch) that the duration of any phenomenon does not depend on a condition of rest or motion of a body in which this motion is observed, i.e. time is absolute and does not depend on the observer. This principle satisfied requirements of the person for the cognition of the world for such a very time. Why? Because, what is reasonable, is necessarily real. In reasoning itself, there is everything that it is possible to find in experience. Aristotle said, "There is nothing existent in (man's) experience that would not be in reason". Hence, in reasoning, there exist many constructions which can be adjusted to the experience.

Prior to the beginning of the 20th century, the Aristotle's principle of contemplation of world was sufficient for understanding our experiencing the world. The experiment of Michelson-Morley on measuring the velocity of light had not yet surfaced. This experiment appeared only later when there also appeared other experiments confirming relativity theory and quantum mechanics. The new principle of the contemplation of the world, explaining these experiments, has proclaimed things, which are "monstrous" from the point of view of rational thinking. Instead of time, it is the velocity of light which turns out to be the absolute magnitude. The observed duration of events (the perception of time) depends on the rest and motion of the observer. The understanding of this fact hasn't come from rational thinking, but from reasonable thinking. Rational thinking, which can explain only finite things, has become insufficient for a crucial explanation of new experimental data. Only reasonable thinking can realize such infinite things as, for example, the world, time, space. And only reasonable thinking can understand Aristotle's question whether time (related to that which divides the past and the future) is uniform or not, whether time remains always identical and invariable, or whether it constantly changes. Strict rational thinking protests against such a question, but reasonable thinking answers it. Furthermore, it depends on the level of our thinking (the level of consciousness of the observer). One may object: it depends not on one's level of consciousness, but from one's level of physical experiment. But experiment itself depends on the level of our knowledge and therefore depends on the level of our consciousness. Any principle of contemplation of the world exists in our reasoning. Our reasoning the chooses necessary principle for a concrete case. Really, our reasoning is infinite.

As it is known, after the experiments confirming relativity theory our relation to the real world has changed. Riemannian geometry has played a huge role in understanding the structure of physical reality. It was a victory of "reason over mind". Relativity theory and Riemannian geometry (and its special case — pseudo-Euclidian geometry of Minkowski's space which is the basis of the Special Theory of Relativity) are products of reasoning.

We ask ourselves, why is there no unusual geometry related to the ordinary representation of the observer? This results from the fact that in life, in usual experiment, we deal with small speeds and weak fields. In such conditions, the differences among geometries are insignificant. As a simple example, in seeing that bodies are in motion as a result of some action-force, our mind has decided, that it will be carried out in any case. That is, motion is force. It is an example of naive thinking. Newton's first law has finished with this kind of knowledge because, as it became known at some later stage in the history of physics, bodies can move with constant velocity without influence of any force. There are many such examples. Perhaps, among various possible representations, one may further revise the geometries of Lobachevsky, Riemann, and Finsler.

In receiving abnormal results, the mind will treat them somehow, but not in the direction of revision of "obvious" geometrical properties. Thus, if we can overcome the resistance of the mind and reconsider "obvious" things, then our thinking can reproduce from itself new sensations and contemplations.

For example, let's consider multi-dimensional time. Within the limits of existenting models that assume multi-dimensional time, there is a set of the parallel worlds (various spatial sections intersecting each other at the same point of a given space-time). It is like a set of possible states of a body in Euclidean space. Let's notice, that our reason at all does not resist to this new sensation in order to construct a new principle of the contemplation of the world.

Even if concepts of multi-dimensional space and time, constructed via reasonable thinking, demand confirmation by physical experiment (which at present seems far-fetched), it is still possible to confirm it in other ways. As Hegel has spoken, experience is done for the cognition of phenomena but not for the cognition of truth itself. One experience is not enough for the cognition of truth. Empirical supervision gives us numerous identical perceptions. However, generality is something different from a simple set. This generality is found only by means of reasoning.

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LETTERS TO PROGRESS IN PHYSICS

A Blind Pilot: Who is a Super-Luminal Observer?

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This paper discusses the nature of a hypothetical super-luminal observer who, as well as a real (sub-light speed) observer, perceives the world by light waves. This consideration is due to that fact that the theory of relativity permits different frames of reference, including light-like and super-luminal reference frames. In analogy with a blind pilot on board a supersonic jet aeroplane (or missile), perceived by blind people, it is concluded that the light barrier is observed in the framework of only the light signal exchange experiment.

We outline a few types of the frames of reference which may exist in the space-time of General Relativity — the four-dimensional pseudo-Riemannian space with Minkowski's signature $(+---)$ or $(-+++)$. Particles, including the observer himself, that travel at sub-luminal speed ("inside" the light cone), bear real relativistic mass. In other words, the particles, the body of reference and the observer are in the state of matter commonly referred to as "substance". Therefore any observer whose frame of reference is one of this kind is referred to as a *sub-luminal speed observer*, or as a *substantial observer*.

Particles and the observer that travel at the speed of light (i. e. over the surface of a light hypercone) bear zero rest-mass $m_0 = 0$ but their relativistic mass (mass of motion) is nonzero $m \neq 0$. They are in the light-like state of matter. In other words, such an observer accompanies the light. We therefore call such an observer a *light-like observer*.

Accordingly, we will call particles and the observer that travel at a super-luminal speed *super-luminal* particles and observer respectively. They are in the state of matter for which rest-mass is definitely zero $m_0 = 0$ but the relativistic mass is imaginary.

It is intuitively clear who a sub-luminal speed observer is: this term requires no further explanation. The same more or less applies to a light-like observer. From the point of view of a light-like observer the world around looks like a colourful system of light waves. But who is a super-light observer? To understand this let us give an example.

Imagine a new supersonic jet aeroplane (or missile) to be commissioned into operation. All members of the ground crew are blind, and so is the pilot. Thus we may assume that all information about the surrounding world the pilot and the members of the ground crew gain is from sound, that is, from transverse waves traveling in air. It is sound waves that build a picture that those people will perceive as their "real world".

The aeroplane takes off and begins to accelerate. As long as its speed is less than the speed of sound in air, the blind members of the ground crew will match its "heard" position in the sky to the one we can see. But once the sound barrier is overcome, everything changes. The blind members of the ground crew will still perceive the speed of the plane equal to the speed of sound regardless of its real speed. The speed of propagation of sound waves in air will be the *maximum speed of propagation of information*, while the real supersonic jet plane will be beyond their "real world", in the world of "imaginary objects", and all its properties will be imaginary too. The blind pilot will hear nothing as well. Not a single sound will reach him from his past reality and only local sounds from the cockpit (which also travels at the supersonic speed) will break his silence. Once the speed of sound is overcome, the blind pilot leaves the subsonic world for a new supersonic one. From his new viewpoint (the supersonic frame of reference) the old subsonic fixed world that contains the airport and the members of the ground crew will simply disappear to become a realm of "imaginary quantities".

What is light? — Transverse waves that run across a certain medium at a constant speed. We perceive the world around through eyesight, receiving light waves from other objects. It is waves of light that build our picture of the "truly real world".

Now imagine a spaceship that accelerates faster and faster to eventually overcome the light barrier at still growing speed. From the purely mathematical viewpoint this is quite possible in the space-time of General Relativity. For us the speed of the spaceship will be still equal to the speed of light whatever is its real speed. For us the speed of light will be the maximum speed of propagation of information, and the real spaceship for us will stay in another "unreal" world of super-light speeds where all properties are imaginary. The same is true for the spaceship's pilot. From his viewpoint, overcoming the light barrier brings him into a new super-light world that becomes his "true reality". And the old world of sub-light speeds is banished to the realm of "imaginary reality".

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