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A New Theoretical Derivation of the Fine Structure Constant

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The present paper is devoted to a new derivation of the expression given already earlier for the fine structure constant α. This expression is exactly the same as that what we published several times since 1986. The equation $1/\alpha = \pi^4 \sqrt{2}\ m_{qm}/m_0$ (m_0 being the rest mass of the electron and m_{qm} the quantum-mechanical fraction of it) is precisely confirmed. The new derivation is based on relations for the energy density in the interior of a macroscopically resting electron within the framework of our standing wave model. This model is strongly supported by the present investigation. Two equations for the energy density inside of an electron were set equal, one of them is taken from classical electrodynamics, the other uses relations from quantum mechanics, special relativity theory and four-dimensional space. As the final theoretical equation for the fine structure constant is unchanged, the numerical value as published in 2008 is still maintained: $1/\alpha = 137.035\,999\,252$.

1 Introduction

In the fine structure constant $\alpha = e^2/\hbar c$ the constants of the electron charge e, Planck's constant h and the light velocity c are flowing together. These fundamental constants play a leading role in electrodynamics (ED), quantum mechanics (QM) and special relativity theory (SRT). Pauli [1] has called the explanation of the fine structure constant one of the most important problems of modern atomic physics. Mac Gregor 1971 [2] discussed α as an universal scaling factor. Here we present a new derivation for the fine structure constant obtained by equalizing two expressions for the energy density of the electromagnetic field inside the electron. One of these relations is based on ED, the other one is based on QM and SRT. In our opinion the new derivation is extraordinarily beautiful, simple and elegant.

We have developed a model of a macroscopically resting extended electron, called standing wave model. This model is based on the assumption that there is an internal energy flux along a closed curve of everywhere the same curvature. The energy flux takes place with velocity of light and is located on the surface of a sphere with radius r_m. The curve is denoted as spherical loop. It has an arc length $4\pi\rho_m$, where $\rho_m = r_m/\sqrt{2}$ is its radius of curvature and it consists of four semi circles. The internal motion produces the spin, magnetic moment and the electromagnetic field of the electron. In a set of publications [3–5] the authors have reported about these subjects.

Moreover, a study of the internal energy transport allowed us to derive a relation for the fine structure constant by investigating longitudinal and transversal standing waves inside of the electron. Here a new explanation of the fine structure constant is presented, also based on the standing wave model of the macroscopically resting electron but following a way which is essentially new.

We are convinced the new way of deriving α is of peculiar interest in understanding the structure of elementary particles. Therefore, we would like to open a discussion about our ideas and procedures.

2 Energy density based on electrodynamics

From classical electrodynamics applied to our standing wave model we were able to calculate the energy contributions of the electromagnetic field to the self-energy of an electron in the whole space. The energy flux is located on the surface of a sphere with the radius [5]

$$r_m = \frac{\hbar}{\sqrt{2} m_{qm} c}, \tag{1}$$

where m_{qm} denotes the quantum-mechanical fraction of the rest mass m_0 of the electron. Quantities which have a subscript m are related to the surface or the interior of a sphere with radius r_m and a subscript qm shall indicate that the corresponding quantity is related to quantum mechanics. Inside the sphere there are a transversal electric field with a field strength $\mathbf{E^t_m}$, and a magnetic field with a field strength $\mathbf{H_m}$. The absolute values of both field strengths are equal inside the sphere of radius r_m [5]:

$$\frac{e}{r_m^2} = |\mathbf{E^t_m}| = |\mathbf{H_m}|. \tag{2}$$

These fields are supposed to be homogeneous inside, i.e. the magnitudes of the field strenghts do not depend on the position. The volume of the sphere is given by

$$V_m = \frac{4}{3}\pi r_m^3. \tag{3}$$

The energy densities of the electric and magnetic fields can be taken from the field strength squares [6]:

$$u_E = \frac{1}{8\pi}|\mathbf{E^t_m}|^2 \tag{4}$$

$$u_H = \frac{1}{8\pi}|\mathbf{H_m}|^2. \tag{5}$$

The total energy density u_s of the electromagnetic field inside the electron is

$$u_s = u_E + u_H = \frac{2}{8\pi}\frac{e^2}{r_m^4}. \tag{6}$$

By integration over the sphere and using eq. (1) as well as the definition of the fine structure constant, the corresponding field energy is obtained

$$\begin{aligned} W_s &= \frac{2}{8\pi}\int_0^{r_m}\int_0^{\pi}\int_0^{2\pi}\frac{e^2}{r_m^4}r^2\sin\varphi\, d\theta\, d\varphi\, dr \\ &= \frac{2}{3}\frac{e^2}{2r_m} = \frac{2}{3}\frac{\alpha}{\sqrt{2}}m_{qm}c^2. \end{aligned} \tag{7}$$

The subscript s shall indicate that the corresponding quantities are related to the standing wave model.

3 Energy density based on QM, SRT and four dimensional space

We start from the three dimensional surface $S_{qm} = 2\pi^2 R^3$ of a four dimensional sphere (cf Schmutzer 1958 [7]). Choosing for the radius $R = \pi r_m$ there follows

$$S_{qm} = 2\pi^5 r_m^3. \tag{8}$$

The zero point energy inside this sphere is given by

$$W_{qm} = \frac{1}{2}\hbar\omega_0, \tag{9}$$

where ω_0 is the lowest possible, positive eigen frequency of the corresponding basic harmonic oscillator. According to the standing wave model this harmonic oscillator describes the electron. From the de Broglie relation

$$E = \hbar\omega_0 = m_0c^2 \tag{10}$$

there follows

$$W_{qm} = \frac{1}{2}m_0c^2, \tag{11}$$

and the energy density can be obtained from (8) and (11)

$$u_{qm} = \frac{W_{qm}}{S_{qm}} = \frac{m_0c^2}{4\pi^5 r_m^3}. \tag{12}$$

4 Fine structure constant

A calculation of the values of u_s and u_{qm} show that they are very close to each other. This stimulated us to set

$$u_s = u_{qm}. \tag{13}$$

Indeed, using (1), (6) and (12), we obtain

$$u_s = u_{qm} \Leftrightarrow \frac{e^2}{\hbar c} = \frac{1}{\sqrt{2}\pi^4}\frac{m_0}{m_{qm}}. \tag{14}$$

Now, using the definition of the fine structure constant, for the inverse of it there follows immediately

$$\frac{1}{\alpha} = \pi^4\sqrt{2}\frac{m_{qm}}{m_0}, \tag{15}$$

where m_0 denotes the rest mass of the electron and m_{qm} its quantum-mechanical fraction. Just the same relation has been found earlier in an other way [3–5]. There, we have shown that both, m_0 and m_{qm}, are depending on α. Solving equation (15) the latest theoretical value of the inverse fine structure constant is [5]

$$\frac{1}{\alpha} = 137.035\,999\,252. \tag{16}$$

This value has to be compared with the semi experimental value 137.035 999 084(51) obtained by combining theory and experiment of the anomalous magnetic moment of the electron [8], as well as with the value 137.035 999 074(44), which is the latest CODATA value [9] from 2010. Furthermore, the ratio m_0/m_{qm} is obtained to be

$$\frac{m_0}{m_{qm}} = 1.005\,263\,277. \tag{17}$$

If we replace (as an alternative) m_0 in (11) by m_{qm} and simultaneously e^2 in (6) by e_i^2 (e_i is the intrinsic or bare charge of the electron) then we have exactly the wonderful relation

$$\frac{\hbar c}{e_i^2} = \pi^4\sqrt{2} = 137.757\,257... \tag{18}$$

The equations (15) and (18) are identical if

$$\frac{m_0}{m_{qm}} = \frac{e^2}{e_i^2}. \tag{19}$$

5 Discussion and conclusions

The numerical value of the fine-structure constant α was often denoted to be a mystery, a magic number and an enigma. A lot of more or less obscure relations have been published with the aim to understand the origin, theoretical background and the numerical value of the fine structure constant, see for example the comprehensive compilation of Kragh 2003 [10]. Why a derivation like the present one has not been carried out earlier? Probably it was the lack of an accurate model of an extended electron. No such model was available, see for example Mac Gregor 1992 [11]. We are convinced that without an understanding of the geometry and inner dynamics of the electron, a consistent understanding of the fine structure constant will not be possible. The simplicity of the present explanation of the fine structure constant is really surprising. Nevertheless, a more detailed discussion and interpretation of the roots of the fine structure constant would be very desirable. So far it concerns the history it should be remarked that already König 1951 [12] found as a byproduct in a rather

complicated argumentation the same expression for α as we found here but without the factor m_{qm}/m_0. A difference between the theoretical and the experimental value of 0.53 % might be the reason that his paper, entitled "An electromagnetic wave picture of micro processes", have found very little attention.

We do not intend to give here a comprehensive discussion of the many aspects which are coupled with the fine structure constant. Several essays have been published devoted to to different aspects (Bahcall and Schmidt 1967 [13] (variation of α with time), Jehle 1972 [14] and 1977 [15] (flux quantization, loops, general discussion), Wilczek 2007 [16] (fundamental constants), Jordan 1939 [17] (cosmological constancy), Peik et al 2004 [18] (temporal limit), Dehnen et al. 1961 [19] (independence on gravitation field), Srianand et al. 2004 [20] (limits on time variation), Schönfeld 1996 [21] (self-energy analysis, see also [3–5])). We would like to remark and underline only two aspects of the present results: one is the exponent four at π which is obviously connected to the four dimensions of our world, the other is that the present result supports strongly the independence of the fine structure constant on time and space, i.e. expresses the cosmological constancy of alpha which was studied by theory and experiment in the last time. Naturally an experiment can give only an upper limit of time or position variation, compare [17–20].

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Redshift Adjustment to the Distance Modulus

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The distance modulus is derived from the logarithm of the ratio of observed fluxes of astronomical objects. The observed fluxes need to be corrected for the redshift as the ratio of observed to the emitted energy flux is proportional to the wavelength ratio of the emitted to observed light according to Planck's law for the energy of the photon. By introducing this redshift adjustment to the distance modulus, we find out that the apparent "acceleration" of the expansion of the Universe that was obtained from observations of supernovae cancels out.

1 Introduction

In the present study a redshift adjustment to the distance modulus was introduced. The rationale is that the observed fluxes of astronomical objects with respect to the emitting body are being reduced by the effect of redshift. According to Planck's law, the energy of the photon is inversely proportional to the wavelength of light; therefore, the ratio of observed to emitted fluxes should be multiplied by the wavelength ratio of emitted to observed light.

2 Model development

Below is shown the derivation of the redshift adjusted distance modulus.

Let us recall the derivation of the distance modulus. The magnitude as defined by Pogson [1] is:

$$m = -2.5 \log F + K, \tag{1}$$

where m is the magnitude, F the flux or brightness of the light source, and K a constant. The absolute magnitude is defined as the apparent magnitude measured at 10 parsecs from the source.

By definition, the brightness is a measure of the energy flux from an astronomical object and depends on distance. Therefore, a redshift correction to the flux is derived from Planck's law for the energy of the photon

$$E = \frac{h \cdot c}{\lambda}, \tag{2}$$

where E is the energy of the photon, h the Planck's constant, and λ the light wavelength.

The ratio of observed to emitted energy flux is derived from eq. (2), leading to

$$\frac{E_{obs}}{E_{emit}} = \frac{\lambda_{emit}}{\lambda_{obs}} = \frac{1}{1+z}, \tag{3}$$

where E_{obs} and E_{emit} are respectively the observed and emitted energy fluxes, λ_{obs} and λ_{emit} are respectively the observed and emitted light wavelengths, and z the redshift.

As light is emitted from a source, it is spread out uniformly over a sphere of area $4\pi d^2$. Excluding the redshift effect, the brightness – expressed in units of energy per time and surface area – diminishes with a relationship proportional to the inverse of square distance from the source of light. Therefore, taking into account the redshift effect, the following relationship is obtained for the brightness:

$$F_{obs} \propto \frac{L_{emit}}{d^2} \cdot \frac{E_{obs}}{E_{emit}}, \tag{4}$$

where L_{emit} is the emitted luminosity, and d the distance to the source of light.

Combining eq. (1), (3) and (4), we obtain

$$m = -2.5 \log \left(\frac{L_{emit}}{d^2 \cdot (1+z)} \right) + K. \tag{5}$$

And, because z is close to zero at 10 Parsec:

$$M = -2.5 \log \left(\frac{L_{emit}}{100} \right) + K, \tag{6}$$

where M is the absolute magnitude.

Hence, the redshift adjusted distance modulus, eq. (5) minus eq. (6) is:

$$m - M = -5 + 5 \log d + 2.5 \log(1+z) \tag{7}$$

with d in parsec, and log is the logarithm in base 10.

3 Discussion

In the present study the distance modulus was adjusted to take into account the effect of redshifts on the observed fluxes of astronomical objects. Evidence of an "accelerating" Universe expansion was established based on the observation of supernovae [2]. This result was obtained by detecting a deviation from linearity on the distance modulus versus redshift plot in log scale for supernovae. In order to account for the redshift adjustment, the adjusted distance modulus $m - M - 2.5log(1+z)$ should be plotted againt redshifts for the supernovae. A deviation of $m - M$ of about +0.5 magnitude was obtained at redshift 0.6. The redshift adjustment

$2.5log(1+z)$ is roughly equal to this deviation. By introducing the redshift adjusted distance modulus eq. (7) this deviation cancels out, and one may no longer conclude that the expansion of the Universe is accelerating.

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A Comment on arXiv:1110.2685

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This brief paper traces comments on the article [2]. This article, a preprint, has recently received an attention, raising errors related to the timing process within the OPERA Collaboration results in [1], that turns out to be a wrong route by which serious science should not be accomplished. A peer-reviewed status should be previously considered to assert that [2] claims a solution for the superluminal results in [1]. Within [2], it seems there is an intrinsical misconception within its claimed solution, since an intrinsical proper time reasoning leads to the assumption the OPERA collaboration interprets a time variation as a proper time when correcting time intervals between a GPS frame and the grounded baseline frame. Furthermore, the author of [2] seems to double radio signals, doubling the alleged half of the truly observed time of flight, since the Lorentz transformations do consider radio signals intrinsically by construction.

1 An intrinsical proper time reasoning? A misconception from the OPERA collaboration, or from the author of [2]? What is actually observed, τ_{clock}/γ?

The author of the article [2]* used, *ab initio*, the designation: *from the perspective of the clock...* Within the approach used by the author, via special relativity, the GPS frame of reference must use ***two*** distinct but synchronized clocks to tag the instants at A and B. The eq. (2) in [2] was, intrinsically, obtained via the Lorentz transformations for the neutrino events of departure from A and arrival to B, but this was not clearly specified within [2], being the construction of the Eq. (2) in [2] crudely accomplished under what would be being seen from the perspective of the clock, in the author of [2] words:

- ***From the perspective of the clock the detector at B moves towards location A at a speed v. And we find that the foton will reach the detector when the sum of the distances covered by the detector and the foton equals the original separation...***; [2].

This reasoning, *ab initio*, leads, as it very seems, to an intrinsical proper time reasoning under the perspective of what was being seen, locally, by the satellite at its very location. Let (x_A, t_A) and (x_B, t_B) be the spacetime events of departure and arrival of the neutrino in the baseline reference frame K, respectively. The time interval spent by the neutrino to accomplish the travel in the [2] GPS reference frame K' is:

$$\delta t' = \left(1 - v^2/c^2\right)^{-1/2} \left[(t_B - t_A) - \frac{v}{c^2}(x_B - x_A)\right], \qquad (1)$$

in virtue of the canonical Lorentz transformation for time in K' as a function of the spacetime coordinates in K, where v is the assumed boost of K' in relation to K in the baseline direction AB, c the speed of light in the empty space. With $\delta t = t_B - t_A$, $\delta x = x_B - x_A = S_{baseline}$, $\delta x = v_\nu \delta t$, where v_ν is the neutrino velocity along the AB direction, the eq. (1) reads:

$$\delta t' = \left(1 - v^2/c^2\right)^{-1/2} S_{baseline} \left(\frac{1}{v_\nu} - \frac{v}{c^2}\right). \qquad (2)$$

With $v_\nu = c$, $\gamma = \sqrt{1 - v^2/c^2}$, $\delta t' \stackrel{!}{=} \tau_{clock}$, as defined in [2], the eq. (2) here becomes the eq. (2) in [2]:

$$\tau_{clock} = \frac{\gamma S_{baseline}}{c + v} \Rightarrow c\tau_{clock} + v\tau_{clock} = \gamma S_{baseline}. \qquad (3)$$

But:

- $\delta t' \stackrel{!}{=} \tau_{clock}$ is not a proper time (it is a time interval measured by distinct clocks at different spatial positions in K'); hence: why would the OPERA collaboration correct $\delta t' \stackrel{!}{=} \tau_{clock}$ via $\delta t = \delta t'/\gamma$, as claimed via the eq. (5) in [2]?
- Such correction would be plausible if the events of departure and arrival of the neutrino had the same spatial coordinate $x'_A = x'_B$ in the GPS K' frame of reference, but it is not the case.

Hence, as asserted before, the claimed solution supposes an intrinsical proper time reasoning, but there is no reason for this, since the $\delta t'$ is not a proper time. Thus, the claimed solution turns out to be constructed on an erroneous correction. The correction that should be done by the OPERA Collaboration, if the [2] GPS reference frame was to be taken in consideration, would read:

$$\delta t = \left(1 - v^2/c^2\right)^{-1/2} \left[(t'_B - t'_A) + \frac{v}{c^2}(x'_B - x'_A)\right], \qquad (4)$$

and this correction would read: $\delta t = \delta t'/\gamma$, with the $\gamma = \sqrt{1 - v^2/c^2}$ defined in [2], ***if and only if***: $x'_B - x'_A = 0$, but it is not the case.

*The comments we raise here are related to the first version of [2], v1, uploaded to arXiv. Recently, the author uploaded an updated version, but the misconceptions seem to persist. The root of the arguments within [2] to obtain the alleged 64 ns seems to be flawed *ab initio*.

Furthermore, I would like to assert that, related to the K' reference frame, the frame taken by the author of [2] to explain the relevance of the GPS reference frame in terms of special relativity: the radio signals turn out to be irrelevant to be taken into consideration once the clocks within K' are synchronized, viz., the Lorentz transformations for events do consider radio signals intrinsically under the synchronization of clocks in a given reference frame. This said, the factor 2 the author uses to reach 64 ns seems misconcepted. Remembering, the τ_{clock} is the time interval in K', it is not a proper time interval, and this time interval totally accounts for the entire process of emission and detection of the neutrino at A and B, respectively, departure and arrival, from which there are not two corrections to be accomplished at the points A and B related to radio signals. The radio signals related to the events at A and B in the GPS reference frame in [2], K', were taken into consideration *ab initio*, in [2], since the clocks at A and B in this reference frame tagging the events of departure and arrival were previously synchronized by the very radio signals the author of [2] refers at the end of his article, due to the intrinsical use of the Lorentz transformations, *ab initio*, within the eq. (2) in [2], albeit the author of [2] had not written down his eq. (2) in [2] under a Lorentzian reasoning. Hence, once the Lorentz transformations provided the τ_{clock}, the radio signals should not be considered twice.

I would like to furtherly comment the root of misconceptions, by which the author of [2] seems to have carried his reasonings to raise his arguments. Related to my previous comments, as asseverated before (see footnote 1), these ones are related to the first version of the mentioned article uploaded to arXiv. The author uploaded an updated version, but the root of misconceptions persists within his primordial reasoning related to the Lorentz transformations. It very seems the author had in mind that the time interval to be corrected $\delta t' = \tau_{clock}$ (here, we continue to consider the notations within the first version of [2], since there are not substantial modifications throughout the updated version to avoid the criticisms raised) was a proper interval. Constructing his arguments, the author refers to what is observed in the satellite reference frame. Suppose, following the author of [2] reasonings, the satellite sends a radio signal to the event at A to see the departure of the neutrino when this radio signal is sent back to the satellite. Be t'_{ESA} (E denotes emission, S denotes satellite, and A denotes the location of the CERN at the instant, read in the satellite local clock, the neutrino starts the travel to Gran Sasso) the instant this signal is sent to reach the event of the neutrino departure; t'_{RSA} (R detotes reception) the instant the signal comes back to the satellite, read in the satellite local clock. These instants are related by:

$$t'_{RSA} = t'_{ESA} + 2d'_{SA}(t'_A)/c, \tag{5}$$

where $d'_{SA}(t'_A)$ is the distance between the satellite and the CERN location at A, at the instant the signal (radio signal) reaches A, viz., $d'_{SA}(t'_A)$ is the distance between the satellite and the CERN location at A at the instant t'_A the neutrino is sent to Gran Sasso in the satellite frame. Analogous reasoning related to the neutrino arrival at Gran Sasso, at B, leads to:

$$t'_{RSB} = t'_{ESB} + 2d'_{SB}(t'_B)/c, \tag{6}$$

where $d'_{SB}(t'_B)$ is the distance between the satellite and the Gran Sasso location at B, at the instant another signal previously sent by the satellite at instant t'_{ESB} read in the satellite local clock (another radio signal) reaches B, viz., $d'_{SB}(t'_B)$ is the distance between the satellite and the Gran Sasso location at B at the instant t'_B the neutrino arrives to Gran Sasso in the satellite frame. The instants t'_A and t'_B are respectively given by:

$$t'_A = \frac{t'_{ESA} + t'_{RSA}}{2}, \tag{7}$$

and:

$$t'_B = \frac{t'_{ESB} + t'_{RSB}}{2}. \tag{8}$$

From these relations, the proper time interval between the instants the satellite ***sees*** the events of departure and arrival, $t'_{RSB} - t'_{RSA}$, is given by:

$$t'_{RSB} - t'_{RSA} = t'_B - t'_A + \frac{d'_{SB}(t'_B)}{c} - \frac{d'_{SA}(t'_A)}{c}, \tag{9}$$

therefore, since $t'_B - t'_A = \delta t' = \tau_{clock}$, see my previous comments:

$$\tau_{clock} = t'_{RSB} - t'_{RSA} - \left(\frac{d'_{SB}(t'_B)}{c} - \frac{d'_{SA}(t'_A)}{c}\right), \tag{10}$$

from which: τ_{clock} ***does take into consideration*** the radio signals travelling, encapsulated within the time intervals within:

$$\tau_{signals} = \frac{d'_{SB}(t'_B)}{c} - \frac{d'_{SA}(t'_A)}{c}. \tag{11}$$

The problem within the reasonings of the author of [2] seems to be this author was thinking that τ_{clock} would be the proper interval related to what was being seen by the satellite, $t'_{RSB} - t'_{RSA}$. Hence, at the end of his article, this author applies a correction related to radio signals to account for the time interval $t'_B - t'_A$, but this process was already done when the author obtained $\delta t' = t'_B - t'_A$, viz., as said before within my previous comments, the Lorentz transformations have got radio signals intrinsically, by construction, to deal with events in spacetime. Thus, when the author of [2] applies the factor 2, this author seems to erroneously account for radio signals twice, and the factor 2 seems misconcepted. Even if the OPERA Collaboration had done the correction the author of [2] refers to, such discrepancy would be 32 ns, but not this value twice. The factor 2 seems to have not got logical explanation within the [2] reasoning, mostly being putted a fortiori.

2 Conclusions

Respectfully, the reasoning that led the author of [2] to the factor 2 is not clear. I think this reasoning should be putted under a fairly crystalline terms, as far as possible, in virtue of the importance given to this article, in virtue of the importance given to the subject. Furthermore, what would be being observed, $\delta t'/\gamma$ (this gamma is the original one used by the author of [2]), or this value twice? Why does not the author of [2] provide spacetime diagrams showing the process related to the radio signals that doubles the alleged half of the truly observed time of flight?

Concluding, it seems unlikely that the OPERA collaboration has misinterpreted a GPS time interval within the terms of [2].

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Spooky Action at a Distance or Action at a Spooky Distance?

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The paper demonstrates that the non-locality and non-reality of the quantum world are direct consequences of the concept of uncertainty. It is also shown that the analysis of states in the phase space entails the operator formalism of wave mechanics. While being well known that the uncertainty principle is a consequence of the commutation rules of operators, the paper shows that the reverse path is also possible; i.e. the uncertainty equations entails themselves the operators and wave equations of energy and momentum. The same theoretical approach has been eventually extended to infer significant results of the special relativity.

1 Introduction

Einstein never liked the weirdness and the conceptual limit of the quantum mechanics due to its probabilistic character; for instance, he disliked the incomplete knowledge about position and momentum of a particle, about all components of angular momentum and so forth. Paradoxically, just his theory of the specific heat and its explanation of the photoelectric effect were the strongest support to the energy quantization early introduced by Plank to explain the black body radiation. In fact to the quantum theory we owe not only the ability to explain weird experimental data, e.g. the dual wave/particle behavior of matter and the tunnel effect, but also important discoveries like the laser, the transistor and the superconductivity. Further experimental evidences recently obtained compelled however accepting besides its weird character other aspects even more counterintuitive of quantum behavior. Mostly important are in this respect the non-localism and non-realism: according to the former, exchange of information is allowed even between particles separated by a superluminal distance; according to the latter, the experimental measurements do not reveal preexisting properties of particles but concur to define themselves the measured properties. The EPR gedanken experiment [1] tried to overcome the conceptual incompleteness of quantum mechanics by hypothesizing "hidden variables" in the wave function, i.e. variables not accessible to experimental evidence but able to improve our extent of knowledge and to overcome the difficulty of a "spooky action at a distance" between correlated couples of particles. Yet, several experiments were able to exclude the existence of hidden variables while demonstrating instead non-local effects [2, 3]. The theoretical apparatus of quantum mechanics acknowledges the non-local behavior of the quantum particles through the concept of entanglement [4, 5]. This term was early introduced by Schrodinger [6] to describe the possibility of correlating quantum systems even though spatially separated; the most controversial point concerns of course the difficulty arising from the requirements of relativity. Even today the concept of entanglement has different interpretations: the most acknowledged point of view is the quantum superposition of states, according which two correlated particles share a single quantum state until a measurement is carried out. The quantum mechanics is founded on a set of mathematical rules, which however do not incorporate themselves since the beginning the non-locality and non-reality in its fundamental conceptual structure, in order to include and rationalize per se these effects. For this reason the EPR paper appears legitimate from a rational point of view, although in fact wrong from a physical point of view; indeed a separate theoretical tool, the Bell inequality [7], was necessary to evidence the inconsistency of the EPR attempt [8, 9]: the predictions of local realism on which is based the Bell inequality conflict with the results obtained in various experiments, e.g. [10, 11, 12]. It is worth noticing that no theoretical foundation of the wave mechanics can be considered really general without containing inherently the non-realism and non-localism of the quantum world. It is therefore interesting to examine in this respect the approach followed in previous papers [13, 14], where results consistent with that of wave mechanics have been inferred exploiting the following equations only

$$\Delta x \Delta p_x = n\hbar = \Delta\varepsilon\Delta t. \qquad (1,1)$$

The second equality is consequence of the first one defining formally $\Delta t = \Delta x/v_x$ and $\Delta\varepsilon = \Delta p_x v_x$, where v_x is the average velocity with which any particle travels through Δx; the equalities share the common number n of allowed states. The equations (1,1) do not require any assumption about the ranges, about the motion of the particle and even about its wave/corpuscle nature; this latter will be inferred as a corollary in section 6. The present paper aims to contribute some ideas about how to regard the non-locality and non-reality uniquely according to eqs. (1,1). For reasons that will be clear below, it is useful to introduce shortly in section 2 the way of exploiting these equations to infer the quantum angular momentum; the remarks at the end of this section, which has a preliminary worth, are essential to discuss subsequently the weirdness of the quantum world. Although the angular momentum has been already introduced in [13], its elucidation is so straightforward and elementary that it deser-

ves being shortly sketched here; in doing so, indeed, it introduces reference concepts that will be further developed in the following sections 3 and 4 that concern the non-reality and non-locality. Eventually, the connection between quantum theory and special relativity is also sketched in sections 5 and 7; the link between eqs. (1,1) and the operator formalism of wave mechanics is discussed in section 6.

2 The non-relativistic angular momentum

The non-relativistic quantization of the classical angular momentum M^2 and of one of its components M_w along an arbitrary direction defined by the unit vector $\mathbf{w}$ starts from the classical scalar $\mathbf{r} \times \mathbf{p} \cdot \mathbf{w}$; here $\mathbf{r}$ is the radial distance of any particle from the origin O of an arbitrary reference system R and $\mathbf{p}$ its momentum. For instance, this could be the case of an electron in the field of a nucleus centered in O. As introduced in [15], the positions

$$\mathbf{r} \rightarrow \Delta\mathbf{r} \qquad \mathbf{p} \rightarrow \Delta\mathbf{p} \tag{2,1}$$

enable the number l of quantum states to be calculated as a function of the ranges $\Delta\mathbf{r}$ and $\Delta\mathbf{p}$ of all local distances and momenta physically allowed to the particle. These ranges only, and not the random local values $\mathbf{r}$ and $\mathbf{p}$ themselves, are considered in the following. The first step yields $M_w = (\Delta\mathbf{r} \times \Delta\mathbf{p}) \cdot \mathbf{w} = (\mathbf{w} \times \Delta\mathbf{r}) \cdot \Delta\mathbf{p}$ and so $M_w = \Delta\mathbf{I} \cdot \Delta\mathbf{p}$, where $\Delta\mathbf{I} = \mathbf{w} \times \Delta\mathbf{r}$. If $\Delta\mathbf{p}$ and $\Delta\mathbf{I}$ are orthogonal, then $M_w = 0$; else, writing $\Delta\mathbf{I} \cdot \Delta\mathbf{p}$ as $(\Delta\mathbf{p} \cdot \Delta\mathbf{I}/\Delta I)\,\Delta I$ with $\Delta I = |\Delta\mathbf{I}|$, the component $\pm\Delta p_I = \Delta\mathbf{p} \cdot \Delta\mathbf{I}/\Delta I$ of $\Delta\mathbf{p}$ along $\Delta\mathbf{I}$ yields $M_w = \pm\Delta I \Delta p_I$. In turn this latter equation yields according to eqs. (1,1) $M_w = \pm l\hbar$, being l the usual notation for the number of states of the angular momentum; l is positive integer including zero. As expected, M_w is not a single valued function because of the uncertainties initially postulated for $\mathbf{r}$ and $\mathbf{p}$. One component of $\mathbf{M}$ only, e.g. along the z-axis, is knowable; repeating the same approach for the y and x components would trivially mean changing $\mathbf{w}$. Just this conclusion suggests that the average values $< M_x^2 >$, $< M_y^2 >$ and $< M_z^2 >$ should be equal; so the quantity of physical interest to describe the properties of quantum angular momentum is l, as a function of which M^2 is indeed inferred as well. Let us calculate these average components over the possible states summing $(l\hbar)^2$ from $-L$ to $+L$, where L is an arbitrary maximum value of l. Being by definition $< M_i^2 >= \sum_{l_i=-L}^{l_i=L} (\hbar l)^2/(2L+1)$, one finds $M^2 = \sum_{i=1}^{3} < M_i^2 >= L(L+1)\hbar^2$. Note that the mere physical definition of angular momentum is enough to find quantum results completely analogous to that of wave mechanics; any local detail of motion, like that of electron "orbit" around the nucleus, is utterly unnecessary. The quantization of the classical values appears merely introducing the delocalisation ranges into the definition of angular momentum and then exploiting eqs. (1,1). The reason of it is evident: after the steps (2,1), the unique information available comes from the uncertainty ranges of coordinates and momentum, rather than from the local values of these latter; then the quantities thereafter calculated concern the number of allowed states only, which have in fact the same physical meaning of the quantum number defined by the solution of the pertinent wave equation. An analogous approach shows that the non-relativistic hydrogenlike energy levels depend on a further integer n because of the radial uncertainty equation $\Delta p_\rho \Delta\rho = n\hbar$ of an electron from the nucleus [13]; again, even without specifying any local detail of motion, the numbers of states l and n related to the angular and radial uncertainties of the electron in the field of nucleus correspond to the respective quantum numbers that characterize the energy levels. This preliminary introduction on how to exploit eqs. (1,1) was included in the present paper to emphasize several points useful in the following, i.e.: (i) the replacements (2,1) that allow to exploit eqs. (1,1) are enough to plug the classical physical definition $\mathbf{r} \times \mathbf{p}$ of angular momentum into the quantum world; (ii) no hypothesis is necessary about the geometrical properties of motion of the particle nor about its wave/matter nature to infer the quantum result; (iii) trivial algebraic manipulations replace the solution of the pertinent wave equation; (iv) the information inferred through eqs. (1,1) only is fully consistent with that of the wave mechanics; (v) the local momentum and distance between the particles concerned in the "orbiting" system do not play any role in determining l; (vi) as found elsewhere, [15, 17], the number of allowed states plays actually the role of the quantum numbers of the operator formalism of wave mechanics; (vii) the amount of information accessible for the angular momentum is not complete like that expected in the classical physics; (viii) eqs. (1,1) rule out "a priori" any possibility of "hidden variables" that could in principle enhance our knowledge about M_w and M^2 in order to obtain a more complete description of the orbiting quantum system.

It is worth mentioning that the validity of the point (i) has been checked and extended in the papers [13, 14] also to more complex quantum systems like many electron atoms/ions and diatomic molecules. The fact that eqs. (1,1) efficiently replace the standard approach of wave mechanics has central interest for the topics introduced in following sections, especially as concerns the very important point (viii). In principle one could not exclude that the wave function, from which is extracted all physical information allowed about the quantum systems, could actually contain hidden variables; indeed this chance, reasonably suspected in the famous EPR paper, has been excluded later thanks to a separate theoretical tool only, the Bell inequality. In the present approach, instead, the quantization of angular momentum is more "transparent" in that it explicitly displays variables and steps that lead to the quantum result; in other words, the present approach excludes any possibility of hidden variables because it works with actual quantities inherent the mere definition of angular momentum only. In conclusion the present section aimed mostly to ensure that sensible results are obtained regarding the uncertainty as a fundamental principle of nature itself, rather than

as a by-product of the operator formalism of wave mechanics. It is necessary however to better understand eqs. (1,1). To ascertain "a posteriori" that these equations work well has no heuristic worth. Therefore, after having checked their validity, the remainder of the paper starts from a step behind them, i.e. to highlight the more profound physical basis rooted in the concept of space-time uncertainty.

3 Non-realism and non-localism of eqs. (1,1)

Let us introduce a reference system R to define the ranges of eqs. (1,1). In the simplest 1D case, R is represented by an arbitrary axis where are defined two coordinates x_o and x_t with respect to an arbitrary origin O: the former describes the position of the range $\Delta x = x_t - x_o$ with respect to O, the latter describes its size. The postulated arbitrariness of size makes Δx consistent with the local coordinate x_o in the limit case $x_t \to x_o$ and with any other coordinate if is also allowed the limit size $\Delta x \to \infty$. If neither boundary coordinate is time dependent, then the section 2 and the papers [15, 16] show that this is all we need to know to define an observable physical property of the concerned quantum system: indeed, with the help of an analogous reasoning for the momentum range, this approach is enough to find the number of allowed states i.e. the quantum numbers that define the eigenvalues of the observable. If instead x_o and x_t are in general time dependent, then Δx expands or shrinks as a function of time, while possibly shifting with respect to O too, depending on how are mutually related the displacements of x_o and x_t. Actually the paper [15] shows that such a detailed information about how both of them displace with respect to O is physically redundant; all we need to know is the resulting $\Delta\dot{x}$ only. If Δx is an empty range, the chance of displacement in principle possible for x_o and x_t entails the presence of a force field within Δx; in the absence of a particle delocalized in it, however, this conclusion has a self-contained worth only that concerns a property of the the range itself in R. Instead consequences of physical interest are expected when a free particle is possibly therein delocalized; first of all because this presence requires itself highlighting the physical meaning of x_o and x_t to justify why these boundary coordinates, although remaining in principle completely arbitrary, can in fact include all values of dynamical variables allowed to the particle. Assume for instance two infinite potential barriers at x_o and x_t: if the size of the delocalization range changes from Δx_1 to Δx_2 during the time range $\Delta t = t_2 - t_1$, it means that necessarily the properties of the particle are affected during Δt as well; at the time t_1 the particle was constrained bouncing within Δx_1 with average frequency $\nu_1 = v_x \Delta x_1^{-1}$, at the time t_2 with average frequency $\nu_2 = v'_x \Delta x_2^{-1}$. The average displacement velocity v_x of the particle has been regarded different at the times t_1 and t_2 for sake of generality; however this fact is not essential, since $\Delta x_2 \neq \Delta x_1$ is enough to ensure $\nu_2 \neq \nu_1$. Hence the deformation of Δx as a function of time entails changing average displacement velocity, bouncing frequency of the particle and thus its momentum as well. To draw such a conclusion two essential elements have implemented the initial definition of delocalization range: the presence of a particle and the size change of Δx. Since however no assumption has been made about times and range sizes, nor about v_x and v'_x, these properties do not define themselves any state allowed to the particle; nothing about arbitrary range sizes, frequencies and velocities can be related to an integer number. Despite the intuitive fact that the particle dynamics has changed, n still appears unexplainable. This conclusion is important because, for the reasons introduced in section 2, just n entails the chance of measuring a physical observable of the particle. Overcoming this indeterminacy requires thus a further condition or constraint on ν_1 and ν_2, e.g. on the change of energy or momentum of the particle during the aforesaid time range. In effect, this condition is a crucial step to allow the transition from an unphysical "virtual" state towards an observable state: if for instance to define n concur the values of momentum or energy related to ν_1 and ν_2, then the sought number of states should correspondingly represent just the allowed eigenvalues of momentum or energy of the particle. The fact that a unique range is inadequate to define n, justifies reasonably the idea of introducing a further range ancillary to Δx able to represent in R the values of a second dynamical variable. Apart from this intuitive conclusion, it is necessary to explain why two arbitrary ranges of allowed dynamical variables are necessary to define the sought observable state of the particle. A reasonable idea is to examine the concept itself of measurement process. It is known that this concept is replaced in quantum mechanics by that of interaction, whose effect is to perturb the early state of the particle under test. The dynamical variables of the unperturbed free particle in R represent the initial boundary condition as a function of which is determined the effect of the interaction between particle and observer. Let the intensity of the local perturbation, whatever it might be, depend in general on the current local position and momentum of the particle; then the observer records an outcome somehow related to the boundary condition describing the particle before the measurement process. Since however the initial dynamical variables were unknown, they remain unpredictable and unknown after the measurement process as well; any correlation between initial and final state of the particle is impossible, simply because the former is in fact undefined. Renouncing "a priori" to know the local values of conjugate dynamical variables compels thus introducing ranges of their allowed values. Despite the lack of information about the sought correlation and kind of interaction, let us show that even so the concept of measurement allows defining the number of states, which in fact makes actual the properties of the particles. Regard to this purpose the aforesaid x_o and x_t respectively as coordinates of the particle before and after the measurement process; in agreement with eqs. (1,1), both are random, unknown and unpredictable, whereas du-

ring the interaction even intermediate values are expected to fall between these extremal boundaries. Considerations analogous to $x_t - x_o$ hold also for the conjugate momentum range $p_t - p_o$, whose boundary values p_o and p_t are related to the momentum of the particle before and after the measurement process. However $x_t - x_o$ and $p_t - p_o$, although fulfilling the requirements of both measurement process and eqs. (1,1), cannot be directly related themselves to Δx and Δp_x; the former are indeed uncorrelated and thus still unable to justify n, the central aim of the present discussion. Let us introduce thus the probabilities Π_x and Π_{p_x} that the values of both dynamical variables change during the measurement process in such a way that

$$x_t - x_o \rightarrow measurement \rightarrow \Delta x$$

$$p_t - p_o \rightarrow measurement \rightarrow \Delta p_x$$

where the usual notations Δx and Δp_x refer to ranges compliant with eqs. (1,1). This suggests writing

$$\Pi_x = \Delta x/(\Delta x + \Delta x'), \quad \Pi_{p_x} = \Delta p_x/(\Delta p_x + \Delta p'_x), \qquad (3,1)$$

where $\Delta x'$ and $\Delta p'_x$ are ancillary ranges consistent with the conditions $\Pi_x \rightarrow 0$ for $\Delta x \rightarrow 0$ and $\Pi_x \rightarrow 1$ for $\Delta x \rightarrow \infty$; analogous considerations hold of course for the momentum probability too. By definition therefore $\Delta x' > 0$ and $\Delta p'_x > 0$, in agreement with the idea that all ranges in the present model are positive. The physical meaning of $\Delta x'$ and $\Delta p'_x$ appears noting that initially, i.e. before defining n, space delocalization and momentum ranges are unrelated. Let us regard then $\Delta x + \Delta x' = x_t - x_o$ and $\Delta p_x + \Delta p'_x = p_t - p_o$ as the unperturbed early ranges, whose respective final sizes are just Δx and Δp_x of eqs. (1,1). So eqs. (3,1) concern the probability that the particle is eventually in Δx resulting after the measurement driven perturbation of the early $\Delta x + \Delta x'$, whereas an analogous explanation holds of course for Π_{p_x} as well. The total probability $\Pi_n = \Pi_x \Pi_{p_x}$ for space delocalization and momentum ranges fulfilling eqs. (1,1) is thus

$$\Pi_n = \Delta x \Delta p_x/(\Delta x \Delta p_x + \Delta x \Delta p'_x + \Delta p_x \Delta x' + \Delta p'_x \Delta x'). \quad (3,2)$$

In eq. (3,2) Π_n is expressed as a function of Δx and Δp_x that will bring us to eqs. (1,1) although starting from initial larger ranges still unrelated, whence the notation. First of all note that eq. (3,2) requires $(\Delta x/\sqrt{\Pi_n})(\Delta p_x/\sqrt{\Pi_n}) > \Delta x' \Delta p'_x$. Since all ranges appearing in this inequality are arbitrary, the left hand side can be shortly written as $\delta x \delta p_x$ whatever the specific values of $\Pi_x \neq 0$ and $\Pi_{p_x} \neq 0$ might be; these last positions are straightforward consequences of the previous considerations. Second, also note that the probability of quantum interest is the square root $\sqrt{\Pi_n} = \sqrt{\Pi_x \Pi_{p_x}}$ of that defined classically as ratio between favorable and total chances; this point will be further concerned in section 6. Third, by definition the product of ranges at right hand side of the inequality cannot be made equal to zero; this would contradict the concept of uncertainty, which must hold for any ranges of any size not simultaneously vanishing. So $\delta x \delta p_x > 0$ requires the existence of a value $const' > 0$ such that

$$\delta x \delta p_x > const' \qquad \Rightarrow \qquad \delta \varepsilon \delta t > const'. \qquad (3,3)$$

The second equation is obtained from the first likewise as in eqs. (1,1). This is in effect the uncertainty principle with the value of $const'$ of the order of the Plank constant; this inequality is then direct consequence of the probabilistic definition of eqs. (3,1) and supports the idea that the perturbation induced by the measurement process shrinks the initial uncorrelated ranges $\Delta x + \Delta x'$ and $\Delta p_x + \Delta p'_x$ to the correlated ones Δx and Δp_x of eqs. (1,1). The fact that eqs. (3,3) concern by definition observable states ensures that effectively $\sqrt{\Pi_n} \neq 0$. Eventually, together with eq. (3,2) must in principle exist also the probability

$$\Pi'_n = 1 - \Pi_n. \qquad (3,4)$$

Note that eq. (3,2) admits in principle $\Delta x' << \Delta x$ and $\Delta x' >> \Delta x$, together with analogous features of $\Delta p'_x$; so both limit probabilities can tend to 0 or to 1. Thus it is possible to regard eq. (3,2) as the effective chance of getting an eigenvalue from the measurement process and eq. (3,4) as that of not getting any eigenvalue. Both account for well known outcomes of wave mechanics, e.g.: (i) eq. (3,4) accounts for eigenvalues that actually do not exist, see for instance the previous conclusions about the x and y components of angular momentum once having determined M_z; (ii) when a quantum states is described by a superposition of several eigenfunctions, several eigenvalues exist whose respective actual occurrence is probabilistic, and so on. These chances must be inferred case by case when exploiting eqs. (1,1) through specific reasonings like that of section 2. The physical meaning of $\sqrt{\Pi_n}$ will also be shortly discussed in the next section 6; so eqs. (3,2) and (3,4) do not deserve further comments here. Now instead let us pose a question before proceeding on: why just shrinking and not expanding further the initial unrelated ranges? Apart from ther fact that the ranges are by definition all positive, the second chance would mean $\Delta x + \Delta x'$ and $\Delta p_x + \Delta p'_x$ defined by negative $\Delta x'$ and $\Delta p'_x$, which in turn would exclude the possibility of defining the probabilities Π_x and Π_{p_x} themselves. Besides this inconsistency, a plain consideration further clarifies the question. The measurement process tries to determine a physical property. Expanding the early unrelated ranges would mean decreasing our degree of knowledge about the particle, whose dynamical variables would oscillate within wider ranges of possible values; if so, the concept of measurement would be itself an oxymoron. Shrinking the early ranges, instead, is the best compromise offered by the nature to us during what we call "measurement process": while being forbidden the exact local values of the classical physics we must content ourselves,

at least, of reduced ranges of values for conjugate dynamical variables to which correspond however numbers of states. We must accept therefore the probabilities of eqs. (3,1) as the best we can get from a measurement process; this is what tells us the Heisenberg inequality just obtained from our probabilistic knowledge of the reality around us. To proceed further exploit again the arbitrariness of all ranges so far introduced in order to rewrite eq. (3,2) in various possible ways. In the first way $\Pi = \Delta x\Delta p_x/(\Delta x''\Delta p_x'')$, being $\Delta x''\Delta p_x'' \geq \Delta x\Delta p_x$ the sum of all addends at denominator. This suggests that $\Delta x\Delta p_x = \alpha const$, where *const* is a constant and α a parameter to be defined consistently with the actual product of the resulting uncertainties. Indeed this position allows writing in general

$$\Delta x''\Delta p_x'' = \alpha'' const, \quad \Delta x'''\Delta p_x''' = \alpha''' const \tag{3,5}$$

and so forth, depending on the values of the range products at left hand side. Let for instance be $\alpha''' \leq \alpha''$; eliminating *const* from these equations one finds $\Delta x'''\Delta p_x'''/(\Delta x''\Delta p_x'') = \alpha'''/\alpha''$ i.e. the sought form of Π_n. A further possibility of rewriting eq. (3,2) is $\Pi_n = \Delta x\Delta p_x/(4\Delta x^{\S}\Delta p_x^{\S})$ in the particular case where all terms at denominator of eq. (3,2) are equal to that here indicated with the unique notation $\Delta x^{\S}\Delta p_x^{\S}$; there is indeed no reason to discard also this chance, which must be therefore included in our definition of Π_n. Eventually, another consequence of the arbitrariness in defining $\Delta x'$ and thus $\Delta x''$ and $\Delta x'''$ of eqs. (3,5) must be taken into account: $\Delta x'$ could have been even rewritten itself as $\Delta x' = \Delta x^{\S} + \Delta x^{\S\S} + \cdots$, with several addends again arbitrary; in this case the number of addends at denominator of eq. (3,2) would have been any integer n rather than 4. All these requirements are easily included in the definition of Π_n simply putting $\alpha \equiv n$, so that eqs. (3,5) read $\Delta x''\Delta p_x'' = n''const$ and so forth with n arbitrary integer; in other words, n corresponds to the arbitrary number of possible subdivisions of the early ranges induced by the measurement process. This result effectively leads to both eqs. (1,1), which merely specify the value of *const* as that of $\hbar$. Note eventually that dividing more and more the initial interval $\Delta x'$ into an increasing number of intervals $\Delta x^{\S}$, $\Delta x^{\S\S}$, ... means considering smaller and smaller sized ranges, to which corresponds an increasing number n; since a smaller and smaller range actually tends to the limit of a local coordinate better and better defined, one realizes that $n \to \infty$ corresponds to the deterministic limit of the classical physics. Once more, the same holds for the other ranges. Since eqs. (1,1) are adequate to describe the existence of eigenvalues, one concludes that the measurement process is in fact consistent with the existence of experimental observables despite the initial uncertainties of both dynamical variables. Note that the reasoning above did not exploit any specific feature of the momentum; in other words, instead of the momentum range the reasoning could have identically exploited directly the perturbation of the velocity v_x of the particle under observation, i.e. a velocity range. The question about why we have in fact introduced just the momentum is irrelevant, as it rests merely on the particular choice of the physical dimension of *const*; regarding this latter as a product $const^{\S}m$, involving m times another constant, one would still find eqs. (3,5) with the form $\Delta x''\Delta p_x'' = n''const^{\S}m$ i.e. $\Delta x''\Delta v_x'' = n''const^{\S}$. Two further considerations are instead by far more relevant. The first is that eqs. (1,1) compel regarding any observable as the consequence of the measurement process itself, rather than as intrinsic feature of matter; no pre-existing state, and thus n, was indeed definable for the particle before the measurement. The conclusion that n characterizing the eigenvalues is consequence of the measurement process, rules the realism out of the quantum world. The second relevant feature of eqs.(1,1), which clearly appears recalling the results of section 2, concerns the localism. The particular example of the angular momentum has been introduced before any further consideration of central interest for the purposes of the present paper just to show that the local dynamical variables do not play any role in determining the observable properties of reality around us, as the experimental properties we measure are related to the eigenvalues and thus to the number of allowed states only. So the local values of dynamical variables become unphysical once accepting eqs. (1,1) to formulate quantum problems: nothing measurable corresponds to the local values. Hence, in lack of local information, the concept of distance is unphysical itself in the quantum world. For instance, in [15] the Newton and Coulomb forces between two interacting masses or charges have been inferred replacing the dependence on their classical distance x_{12}^{-2} with the dependence on Δx^{-2}: according to eqs. (1,1), the space range includes all possible local distances between the interacting particles whose coordinates fall within Δx. Regarded from this point of view, the EPR paradox is unphysical itself: it is impossible to define a superluminal distance conflicting with the exchange of information about the spin orientation of two particles arbitrarily apart each other. Whatever their distance might be, a range Δx including both of them certainly exists because its size is by definition arbitrary. Once regarding two particles within Δx, however, the concept of their local distance fails together with that of the respective local coordinates; in principle nobody knows or can measure how far they might actually be. For this reason it would be appropriate to describe the EPR gedankenexperiment as an action at a spooky distance, instead of a spooky action at a distance. Moreover the concept of entanglement appears itself implicitly inherent the present approach, as even particles at superluminal distance must behave consistently with their chance of being anywhere and thus of exchanging information as if they would actually be at very short distance. In this respect, just the quantum entanglement is itself the best demonstration of the correctness of the present point of view based exclusively on the eqs. (1,1), which thus exclude "a priori" both realism and localism from the quantum world; all this clearly appears in section 2. Also the Aharonov-Bohm effect is immediately understandable in the

frame of the present reasoning: an electrically charged particle is affected by an electro-magnetic field even when it is confined in a region where both electric and magnetic fields are zero. Actually it is here *and* there just like a wave propagating through, and thus filling, all available delocalization range. The previous considerations show indeed that regarding a quantum particle here *or* there is physically illusory; assigning a specific location is an idea arbitrarily and incorrectly extrapolated from the classical physics to the quantum world.

4 The Bell inequality

At this point, the exposition brings unavoidably into the mind the Bell inequality. The non-locality and non-reality of the results inferred from eqs. (1,1) suggest emphasizing the connection between the considerations of section 3 and the Bell inequality. To highlight this link let us rewrite the eqs. (1,1) as

$$\frac{\Delta x}{\Delta x_1}\frac{\Delta p_x}{\Delta p_1} = n, \qquad \frac{\Delta t}{\Delta t_1}\frac{\Delta \varepsilon}{\Delta \varepsilon_1} = n, \qquad n \geq 1, \tag{4,1}$$

where the subscript "1" means $n = 1$. In this way $\hbar$ does no longer appear explicitly in the expression of the number of states. Eqs. (4,1) appear therefore as an appropriate starting point to examine the relationship between eqs. (1,1) and Bell inequality, which has indeed general character not specifically related to the quantum theory. Considering for sake of brevity the first equation only (the second is indeed its straightforward consequence) and taking the logarithms of both sides one finds

$$\log\left(\frac{\Delta x}{\Delta x_1}\right) + \log\left(\frac{\Delta p_x}{\Delta p_1}\right) \geq 0. \tag{4,2}$$

This equation presents a formal analogy with the Bell-like inequality, [9]

$$N(A, B_{\rm n}) + N(B, C_{\rm n}) \geq N(A, C_{\rm n}), \tag{4,3}$$

where the subscript "n" stands for "not". Its demonstration is amazingly simple. Whatever the properties A, B and C might represent, the inequality $N(A, B_{\rm n}, C) + N(A_{\rm n}, B, C_{\rm n}) \geq 0$ expressing the sum of the respective numbers of occurrences/non-occurrences possible for A, B and C is self-evident. Add to both sides the sum $N(A, B_{\rm n}, C_{\rm n}) + N(A, B, C_{\rm n})$ expressing further numbers of occurrences/non-occurrences possible for B and C and note that terms like $N(A, B_{\rm n}, C) + N(A, B_{\rm n}, C_{\rm n})$ read actually $N(A, B_{\rm n})$; the notation emphasizes a resulting term no longer distinguished according to either property C, i.e. the sum including both chances allowed for C with the same A and $B_{\rm n}$ discriminates in fact the occurrences/non-occurrences of A and B only. So one infers immediately the inequality (4,3) that can be more expressively rewritten as

$$N_n(A, B_{\rm n}) + (N_n(B, C_{\rm n}) - N_n(A, C_{\rm n})) \geq 0 \tag{4,4}$$

with notations N_n for reasons that will be clear soon. Comparing the inequalities (4,2) and (4,4) requires emphasizing first of all what "not" stands for. In eqs. (3,1) the ranges $\Delta x'$ and $\Delta p'_x$ additional to Δx and Δp_x have been introduced to define the probability Π_x that after the measurement interaction the particle delocalization is described by Δx and no longer by $\Delta x + \Delta x'$, while an analogous idea holds also for Π_{p_x}; as we have shown, just the probabilities that both initial ranges shrink to new ranges fulfilling eqs. (1,1) entail the numbers of states n and thus the existence of the respective eigenvalues. This suggests that B and $B_{\rm n}$ describe respectively the chances of leaving the initial delocalization range unchanged or not after the perturbation induced by the observer, whereas C and $C_{\rm n}$ concern in an analogous way the momentum ranges of the particle. As regards A, it represents the existence of an eigenvalue of the particle; of course $A_{\rm n}$ means that delocalization and momentum ranges of the particle remain unchanged and so unrelated, thus not corresponding to any number of states. The notation N_n relates thus the inequality (4,4) to any possible eigenvalue. For instance: since n requires that are verified both favorable probabilities (3,1), it is reasonable to think that the various probabilities P_n corresponding to eq. (4,4) fulfill also the condition

$$P_n(A, B_{\rm n})P_n(A, C_{\rm n}) + P_n(A_{\rm n}, B)P_n(A_{\rm n}, C) = 1. \tag{4,5}$$

In effect, it is possible to normalize eq. (4,4) be means of an appropriate numerical factor in order to express the various numbers N_n of occurrences/non-occurrences through their respective probabilities P_n for one particle only. The first addend of eq. (4,5) represents the probability of getting an eigenvalue as a consequence of the measurement process, the second does not; in fact this idea was already introduced through the probabilities Π_n and Π'_n of eqs. (3,2) and (3,4). The sum of both chances that correspond to the Bell-like inequality

$$P_n(A, B_{\rm n}) + P_n(B, C_{\rm n}) - P_n(A, C_{\rm n}) \geq 0$$

must be of course equal to 1 in eq. (4,5). Let us try now to correlate term by term eqs. (4,2) and (4,4); the latter concerns directly the numbers of occurrences/non-occurrences leading to the n-th number of states allowed for one particle. This correlation yields

$$\Delta x = \Delta x_1 \exp\left(N_n(A, B_{\rm n})\right),$$

$$\Delta p_x = \Delta p_1 \exp\left(N_n(B, C_{\rm n}) - N_n(A, C_{\rm n})\right).$$

To verify if these equations can be simultaneously fulfilled, let us multiply them side by side; recalling that by definition $\Delta x_1 \Delta p_1 = \hbar$, one obtains

$$n = \exp\left(Q_n\right),$$

$$Q_n = N_n(A, B_{\rm n}) + N_n(B, C_{\rm n}) - N_n(A, C_{\rm n}) \geq 0. \tag{4,6}$$

So the result is that n must be equal just to the exponential of the number Q_n of occurrences/non-occurrences of the Bell-like inequality. It is clear however that in general the first equation (4,6) is false. Even admitting the chance that it is effectively verified for one among the possible numbers of states, say $n^{\S}$, by an appropriate value $Q_{n^{\S}}$, what about other numbers of states like for instance $n^{\S} - 1$ or $n^{\S} + 1$? It is clear that a hypothesis should be made on the respective $Q_{n^{\S}-1}$ and $Q_{n^{\S}+1}$. However the Bell-like inequality (4,3) does not prospect itself any indication about such a hypothesis, which therefore would require an "ad hoc" assumption valid for all arbitrary integers n progressively increasing from 1 by steps of 1 until to infinity. Note in this respect that the impossibility of eqs. (1,1) to fulfil the Bell-like inequality is in fact due to the quantization of n; if this latter could take any non-quantized value, then eq. (4,6) would be fulfilled in principle whatever Q_n might be. Hence is just the quantization of the eigenvalues that makes itself non-real and non-local the quantum world. In effect for $n \to \infty$ the number n approximates better and better a continuous variable of the classical physics, whence the realism and localism of the macroscopic classical world.

5 Uncertainty and special relativity

After having justified why the uncertainty ranges of position and momentum entail non-locality and non-reality, remains the concept of time and energy uncertainty to be better explained in the frame of such a conceptual context. Consider that also the time measurement requires a macroscopic apparatus, whose outcome is nothing else but the time of the observer. The question arises: is the observer time coincident with that of the particle? This question can be answered considering first that during the measurement process eqs. (1,1) apply to different reference systems, about which no hypothesis is made. Suppose that eqs. (1,1) refer to the particle; we must rewrite them as $\Delta x' \Delta p'_x = n'\hbar = \Delta\varepsilon' \Delta t'$ for the observer. Let R and R' be the respective reference systems; in both cases the ranges are completely arbitrary by definition, as concerns their sizes and analytical form. For instance it is not possible to establish if $\Delta x = x_o + v_x \Delta t$ or if $\Delta x = \sqrt{x_o^2 + (v_x \Delta t)^2}$ or anything else. The same holds also for the momentum range and for the energy range. Moreover n and n' are not assigned values, rather they are mere notations to indicate any integer unspecified and unspecifiable. So n and n' remain indistinguishable despite any integer of either reference system might turn into a different integer in the other reference system. Hence the arbitrariness of the analytical form of the ranges does not contradict the validity of eqs. (1,1) in different reference systems despite the chance of their possible size changes; the uncertainty equations (1,1) hold identically in R and in R', regardless of whether they refer to particle and observer in the respective reference systems. So, whatever the sizes of Δx of the particle and $\Delta x'$ of the observer might be, in principle eqs. (1,1) do not require that the time ranges Δt and $\Delta t'$ coincide. Recall now that the time range was introduced in section 1 to infer eqs. (1,1) through the positions $\Delta t = \Delta x / v_x$, which thus requires analogously $\Delta t' = \Delta x' / v'_x$, and note that both signs are allowed for the velocity components v_x and v'_x defined in R and R'. This means that with respect to the origin O of R we expect $\Delta x \pm v_x \Delta t = 0$ depending on whether the particle moves leftwards or rightwards. A possible position to summarize into a unique equation these chances regardless of either sign of v_x is $\Delta x^2 - v_x^2 \Delta t^2 = 0$; to this result corresponds of course an analogous expression in R', i.e. $\Delta x'^2 - v'^2_x \Delta t'^2 = 0$. Hence it is possible to write

$$\Delta x'^2 - v'^2_x \Delta t'^2 = 0 = \Delta x^2 - v_x^2 \Delta t^2. \qquad (5,1)$$

Both v_x and v'_x are reminiscent of the respective reference systems where they have been initially defined. Since no constraint is required for these velocities, both arbitrary by definition, the last equation allows replacing v_x and v'_x with any other values of velocity still defined in R and R'; so

$$\Delta x'^2 - v''^2_x \Delta t'^2 = \delta s^2_{v'',v'''} = \Delta x^2 - v'''^2_x \Delta t^2 \quad \delta s^2_{v'',v'''} \neq 0. \qquad (5,2)$$

Being unchanged the delocalization range sizes at right hand side, the interval $\delta s^2_{v'',v'''}$ is no longer equal to zero once having replaced v_x^2 with v'''^2_x; yet this does not hinder that this interval is still equal to the expression at left hand side if v'_x is replaced by another appropriate velocity v''_x also defined in R'; thus remains unchanged the analytical form of eqs. (5,1) and (5,2). In this way we have found a unique interval $\delta s^2_{v'',v'''}$ common to both reference systems R and R'. Yet this result is not a property of an interval defined by uncertainty ranges only, as it involves the presence of a particle through its displacement velocity; however it is interesting the fact that $\delta s^2_{v',v'''}$ does not require specific values of v''^2_x and v'''^2_x, which are indeed arbitrary like the ranges themselves. In the paper [15], was identified a velocity invariant in any reference system, called $v_x^{\max}$, i.e. the maximum average velocity with which any particle can displace in any Δx. This suggest the chance of expressing eqs. (5,2) just through this velocity, which will be called from now on c. If in particular we replace v''^2_x and v'''^2_x with c, then

$$\Delta x'^2_c - c^2 \Delta t'^2_c = \delta s^2_c = \Delta x_c^2 - c^2 \Delta t_c^2 \quad \delta s_c \neq 0. \qquad (5,3)$$

This result contains new delocalization ranges that can be chosen in order to generalize the previous result; this can be certainly done in agreement with this appropriate choice of the velocity, to which refers indeed the subscript c. In general eq. (5,3) holds for δs_c not necessarily equal to zero and represents a real step onwards with respect to eq. (5,2) because of the peculiar property of c, which is defined regardless of a specific reference system. The only quantities that depend on R are Δx_c and Δt_c that define δs_c regardless of the presence itself of any kind of particle thanks to the universal character of c. In conclusion, the present discussion allowed to find a

relationship that describes the form of an interval invariant in R and R', thus in any other reference system. Since this result has been obtained from eqs. (1,1), it is also compliant with the requirements of non-locality and non-reality previously introduced. The interval rule is a fundamental statement of special relativity, for instance it allows to infer the Lorentz transformations of space, time, momentum and energy [18]. However, apart from the formal analogy, the ranges introduced here have fully quantum physical meaning, i.e. they are uncertainty ranges; instead the ranges of relativity have the deterministic character of classical physics, i.e. they are defined as a function of selected local coordinates in principle exactly known. Therefore eq. (5,3) shows that even the relativity can be made compliant with the requirements of the quantum world provided that the local dynamical variables be discarded as done here and the macroscopic deterministic ranges take the physical meaning of uncertainty ranges. This crucial step, although abstractly simple, is certainly non-trivial as concerns the different way of regarding the conceptual basis of relativity. The next considerations concern just the consequences of this conclusion. From eq. (5,3) and according to eqs. (1,1) one infers, omitting for simplicity the subscripts c and x from now on but still intending that v is a component of average velocity along an arbitrary axis,

$$\frac{c^2\Delta t'^2}{c^2\Delta t^2} = \frac{(v/c)^2 - 1}{(v'/c)^2 - 1}, \qquad v = \Delta x/\Delta t, \qquad v' = \Delta x'/\Delta t'. \tag{5,4}$$

Putting in this equation $c \to \infty$, i.e. in the non-relativistic limit, $\Delta t' \to \Delta t$; as expected, without a finite light speed one finds the absolute time of Newton. Suppose now R and R' displacing each other at constant rate V such that in either of them, say in R, the particle is at rest. In the particular case $v = 0$, therefore, v' is just the rate V with which R displaces with respect to R'; of course it is also identically possible to put $v' = 0$, in which case $v = -V$. Since we have two equivalent ways to regard v and v', let us exploit for instance the first chance to find the transformation properties of the time range and the second chance for the space range; in the latter case it is convenient to put in eq. (5,3) $\delta s_c = 0$ to infer directly $c\Delta t_c = \Delta x_c$ and $c\Delta t'_c = \Delta x'_c$. One finds then

$$\Delta t' = \Delta t\Big(1 - (V/c)^2\Big)^{-1/2}, \qquad \Delta x'_c = \Delta x_c\Big(1 - (V/c)^2\Big)^{1/2}. \tag{5,5}$$

Actually the subscript c could have been omitted in the second equation; being arbitrary both time ranges of eq. (5,3), it holds in fact for any Δx and $\Delta x'$. The relevant remark is however that to time dilation corresponds length contraction in the primed reference system. It is also immediate to find the expressions of momentum and energy of a free particle. Let us consider first the following equalities obtained from eqs. (1,1) in the particular case $n = 1$

$$\Delta p^{(v)}\Delta x^{(v)} = \Delta t^{(v)}\Delta\varepsilon^{(v)} = \Delta t^{(c)}\Delta\varepsilon^{(c)} = \hbar,$$

$$\Delta t^{(c)} = \Delta t_{\min}, \qquad \Delta\varepsilon^{(c)} = \Delta\varepsilon_{\max}.$$

The superscripts emphasize the values taken by the velocity v in the various cases; the subscripts emphasize that when $v = c$ the traveling time is minimum whereas $\Delta\varepsilon$ is maximum, both consistently with $\hbar$ and with the arbitrary $\Delta p^{(v)}$ and $\Delta x^{(v)}$ describing a slower massive particle. These positions are important as they compel specifying how, in a given reference system, $\Delta p^{(v)}$ and $\Delta\varepsilon^{(v)}$ scale with respect to $\Delta p^{(c)}$ and $\Delta\varepsilon^{(c)}$ when $v < c$. Since $\Delta\varepsilon^{(c)} = cp_2^{(c)} - cp_1^{(c)}$, then $\varepsilon^{(c)} = cp^{(c)}$ by definition; here $\varepsilon^{(c)}$ and $p^{(c)}$ are random local values of energy and momentum within their own uncertainty ranges. For a slower massive particle $\Delta t^{(v)}$ and $\Delta\varepsilon^{(v)}$ scale like c/v and v/c with respect to $\Delta t^{(c)}$ and $\varepsilon^{(c)}$; hence, according to the former equality, $\varepsilon^{(v)} = \varepsilon^{(c)}v/c$ requires $p^{(v)}$ scaling with respect to $p^{(c)}$ like $cp^{(v)} = \varepsilon^{(c)}v/c$, i.e. $p^{(v)} = \varepsilon^{(c)}v/c^2$. Being $p^{(v)}$ and $\varepsilon^{(c)}$ random local quantities within the respective ranges, the functional relationship between any possible value of momentum and energy must be

$$p = \varepsilon v/c^2. \tag{5,6}$$

Momentum and energy of a free particle are constants both in classical physics and in special relativity. However eq. (5,6) is here a quantum result, which therefore must be accordingly handled. Let us admit that during a short time range δt even the energy of a free particle is allowed to fluctuate randomly by $\delta\varepsilon$. Eq. (5,6) is thus exploited to calculate the link between $\delta\varepsilon$ and related values of δp and δv during the time transient where the fluctuation allows the particle moving in altered way. Differentiating eq. (5,6) one finds $\delta\varepsilon = c^2\delta p/v - p(c/v)^2\delta v$: once having fixed p and v, this result defines the functional dependence of $\delta\varepsilon$ upon arbitrary δp and $\delta v = v_2 - v_1$ defined by two arbitrary values v_1 and v_1. Summing $\delta\varepsilon$ and eq. (5,6) one finds $\varepsilon + \delta\varepsilon = c^2(p + \delta p)/v - \varepsilon\delta v/v$. Note now that in general $\delta p\delta x = n\hbar$ reads identically $(\delta p)^2 = n\hbar\delta p/\delta x$, whereas in an analogous way $(\delta\varepsilon)^2 = n\hbar\delta\varepsilon/\delta t$. Regard in this way just the new ranges $\varepsilon + \delta\varepsilon$ and $p + \delta p$; putting $\delta x = v\delta t$ and replacing in the last expression to calculate $\delta(\varepsilon + \delta\varepsilon)/\delta t$, one finds

$$(n\hbar)^{-1}(\Delta\varepsilon)^2 = (n\hbar)^{-1}(\Delta pc)^2 - \varepsilon\delta\omega, \tag{5,7}$$

$$\Delta\varepsilon = \varepsilon + \delta\varepsilon, \qquad \Delta p = p + \delta p.$$

The last addend results because $v/\delta x$ has physical dimensions of a frequency ω, so that $\delta v/\delta x = \omega_2 - \omega_1$. Since $n\hbar\omega\delta\varepsilon = \delta(\varepsilon n\hbar\omega) - \varepsilon\delta(n\hbar\omega)$, replacing this identity in the last equation one finds $(\Delta\varepsilon)^2 = (\Delta pc)^2 + n\hbar\omega\delta\varepsilon - \delta(\varepsilon n\hbar\omega)$. Let us specify this result via the position

$$n\hbar\omega = \delta\varepsilon \tag{5,8}$$

which yields also $(\Delta\varepsilon)^2 - (\Delta pc)^2 = (\delta\varepsilon)^2 - \delta(\varepsilon\delta\varepsilon)$. At left hand side appear terms containing the ranges $\varepsilon + \delta\varepsilon$ and $p + \delta p$ only, at right hand side the ranges $\delta\varepsilon$ and δp only; so it is reasonable

to expect that the last equation splits into two equations linked by a constant energy ε_o

$$(\Delta\varepsilon)^2 - (\Delta pc)^2 = \varepsilon_o^2 = (\delta\varepsilon)^2 - \delta(\varepsilon\delta\varepsilon).$$

Indeed ε_o agrees with both of them just because it does not depend upon neither of them. Trivial manipulations show that the first equation yields

$$p = \pm\frac{\varepsilon_o v/c^2}{\sqrt{r_\varepsilon^2 - r_p^2(v/c)^2}}, \qquad \varepsilon = \pm\frac{\varepsilon_o}{\sqrt{r_\varepsilon^2 - r_p^2(v/c)^2}}, \quad (5,9)$$

$$r_p = 1 + \frac{\delta p}{p}, \qquad r_\varepsilon = 1 + \frac{\delta\varepsilon}{\varepsilon}.$$

As expected, eq. (5,6) results fulfilled even during the transient. The value of the constant ε_o is immediately found through the following boundary condition consequence of eq. (5,6)

$$\lim_{v\to 0}\frac{p}{v} = \frac{\varepsilon_{rest}}{c^2} = m. \qquad (5,10)$$

Then $\varepsilon_o^2 = \varepsilon_{rest}^2$. Eqs. (5,9) hold during the time transient allowing $\delta\varepsilon$; before and after that transient one must put $\delta\varepsilon = 0$ and $\delta p = 0$ which yields the "standard" Einstein momentum and energy of the particle, which are of course

$$\varepsilon_{Ein}^2 = c^2 p_{Ein}^2 + \varepsilon_{rest}^2, \qquad \varepsilon_{rest} = mc^2, \qquad (5,11)$$

$$p_{Ein} = \pm\frac{mv}{\sqrt{1-(v/c)^2}}, \qquad \varepsilon_{Ein} = \pm\frac{mc^2}{\sqrt{1-(v/c)^2}}.$$

It is easy now to calculate the energy and momentum gaps $\varepsilon - \varepsilon_{Ein}$ and $p - p_{Ein}$ during the time transient δt as a function of $\delta p/p$ and $\delta\varepsilon/\varepsilon$ as follows

$$\frac{mv}{\sqrt{r_\varepsilon^2 - r_p^2(v/c)^2}} - \frac{mv}{\sqrt{1-(v/c)^2}} = \frac{\hbar}{\delta l}, \qquad (5,12)$$

$$\frac{mc^2}{\sqrt{r_\varepsilon^2 - r_p^2(v/c)^2}} - \frac{mc^2}{\sqrt{1-(v/c)^2}} = \frac{\hbar}{\delta t}.$$

These equations, which are nothing else but the uncertainty equations of the fluctuation gaps, will be commented and exploited in section 7. The chance of obtaining the eqs. (5,6), (5,10) and (5,11) could be reasonably expected; in the paper [15] it was shown that eqs. (1,1) only are enough to infer the following corollaries: (i) equivalence of all inertial reference systems in describing the physical laws, (ii) existence of a maximum average displacement rate allowed for any particle in its delocalization range and (iii) invariance in all reference systems of such a maximum velocity. These corollaries are in fact the basic statements of special relativity. Five further remarks are crucial in this respect: (i) the mass m is not introduced here as the familiar concept of everyday common experience, rather the mass is inferred itself as a consequence of the uncertainty; (ii) the analytical expressions of energy and momentum have been obtained without need of any hypothesis additional to eqs. (1,1); (iii) the most representative formulas of special relativity are here obtained as straightforward consequences of the quantum uncertainty through trivial algebraic manipulations of eqs. (1,1) only; (iv) eqs. (5,11) are typical expressions of particle behaviour of matter, eq. (5,8) involves instead the wave behavior of matter too, because the frequency ω is a typical property of waves; unifying both properties into a unique equation leads to the well known relativistic formulas; (v) uncertainty ranges only appear in formulas coincident with that, well known, of the special relativity.

Note in this respect that the Einstein deterministic approach excludes the random fluctuation of velocity, energy and momentum, which is a typical quantum phenomenon; here instead the well known eqs. (5,11) are particular cases only of the more general eqs. (5,9) taking into account the possibility of fluctuations, in agreement with the fact that here the Einstein intervals here are actually quantum uncertainty ranges. Just this last statement opens the way to further considerations, carried out in section 7. Before exploiting the results of the present section, however, the next section 6 will concern a further topic previously introduced: the possibility of defining uncertainty sub-ranges included in larger ranges. The aim is to clarify the physical meaning of such a further way to regard the quantum uncertainty.

6 Uncertainty and operator formalism of wave mechanics

It is well known that the uncertainty principle is a consequence of the operator formalism of wave mechanics. This section aims to emphasize that the reverse path is also possible: here we show how to infer the momentum and energy wave equations starting from eqs. (1,1). This result is nontrivial: it emphasizes that the fundamental basis of the present theoretical approach leads also to the early wave equations from which has been developed the modern formulation of quantum mechanics. The uncertainty inherent Δx does not prevent to define in principle the probability $\Pi = \Pi(x,t)$ that the particle be in an arbitrary sub-range δx inside the total range

$$\frac{\delta x}{\Delta x} = \Pi, \qquad \delta x = x - x_o, \qquad \delta x \le \Delta x, \qquad (6,1)$$

provided that hold for δx the same uncertainty features of Δx; so no hypothesis is made about δx. Moreover x and x_o are both arbitrary and unknown likewise that of Δx; there is no chance of defining width or location of δx within Δx or distinguishing δx with respect to any other possible sub-range. In general Π is expected to depend on space coordinate and time; yet we consider first the explicit dependence of Π on x only, i.e. t is regarded as fixed parameter in correspondence to which are examined the properties of Π as a function of

x. Regard the width of δx variable, with x current coordinate and x_o constant. The couples of coordinates defining Δx and Δp_x are instead considered fixed. Eqs. (6,1) yield

$$\frac{1}{\Delta x} = \frac{\partial \Pi}{\partial x}, \qquad \Pi = \Pi(x, t). \tag{6,2}$$

Let Π and $1 - \Pi$ be the chances for the particle to be or not within δx and be n_+ and n_- the arbitrary numbers of states consistent with the respective probabilities. Putting

$$\delta x \Delta p = n_+ \hbar, \qquad (\Delta x - \delta x)\Delta p = n_- \hbar, \qquad n_+ + n_- = n, \tag{6,3}$$

then $n_+/n + n_-/n = 1$; also, eq. (6,3) yields the identity

$$(1 - \Pi)\Pi \Delta p^2 = n_- n_+ \hbar^2 \left(\frac{\partial \Pi}{\partial x}\right)^2. \tag{6,4}$$

Putting $n_+ n_- = n' + n''$, where n' and n'' are further arbitrary integers, eq. (6,4) splits as follows

$$\Pi \Delta p^2 = n' \hbar^2 \left(\frac{\partial \Pi}{\partial x}\right)^2, \tag{6,5a}$$

$$\Pi^2 \Delta p^2 = -n'' \hbar^2 \left(\frac{\partial \Pi}{\partial x}\right)^2. \tag{6,5b}$$

Since n_+ and n_- are by definition positive, at least one among n' and n'' or even both must be positive. Consider separately the possible signs of n' and n''.

Case (i) $n' > 0$ and $n'' < 0$. Eqs. (6,5) read also $\delta x \Delta p = (n'/n)\hbar$ and $\delta x^2 \Delta p^2 = |n''| \hbar^2$ because of eqs. (6,1) and (6,2). Moreover multiplying both sides of the latter by $|n''|$ and both sides of the former by $n^{\S} n/n'$, with $n^{\S}$ arbitrary integer, one finds

$$\delta x'' \Delta p = n'' \hbar, \qquad \delta x^{\S} \Delta p = n^{\S} \hbar,$$

where $\delta x'' = \sqrt{|n''|}\delta x$ and $\delta x^{\S} = (n^{\S} n/n')\delta x$. Also, $(n'/n)^2 = |n''|$ and $\Pi = |n''|/n'$. These results are mutually consistent for any integers at right hand sides, because are arbitrary not only n' and n'' but also δx; indeed the new uncertainty equations have an analogous form and physical meaning. Hence eqs. (6,5) do not exclude each other and are both acceptable; yet they are both formally analogous also to the initial eq. (1,1), the only difference being the size of their space uncertainty ranges only. In conclusion, being the sizes arbitrary by definition, this combination of signs of n' and n'' does not entails anything new with respect to eq. (1,1), and thus has no physical interest.

Case (ii) $n' < 0$ and $n'' > 0$. The right hand sides of both eqs. (6,5) have negative sign, so neither of them can have the same physical meaning of the initial eq. (1,1); they read $\Pi = -|n'|/n^2$ and $\Pi^2 = -n''/n^2$ because of eq. (6,2). Yet the result $\Pi = n''/|n'| = -|n'|/n^2$ is clearly absurd, so also this combination of signs has no physical interest.

Case (iii) $n' > 0$ and $n'' > 0$. Eqs. (6,5) are now physically different, because their ratio would entail Π negative. Thus these equations cannot be combined together, because of their different ways to describe the particle delocalized in Δx; they must be considered separately. Eq. (6,5a) is conceptually analogous to eq. (1,1); eq. (6,5b) excludes eq. (6,2) and admits the solution $\Pi = A' \exp(\pm i(x - x_o)\Delta p/\hbar \sqrt{n''})$, being A' the integration constant. Rewriting $\Pi = A \exp(\pm i\varphi \delta x/\Delta x)$ with $\varphi = n/\sqrt{n''}$, the probability Π inferred here significantly differs from Π of eq. (6,5a) despite the same notation; the former is indeed a complex function, the latter coincides instead with eq. (6,1). Both are however definable in principle.Thus eq. (6,5b) still retains the essential concept of delocalization within an arbitrary uncertainty range, yet without concerning itself the ability of regarding the particle as a corpuscle in any specific point of Δx.

The following discussion concerns the case (iii). To accept both eqs. (6,5) together, we must acknowledge their different form, i.e. their different way to describe the particle delocalization inside Δx. This dual outcome reveals however the inadequacy of regarding the particle as mere corpuscle delocalized somewhere in its uncertainty range, as required by eqs. (1,1). Despite the particle must be anyway randomly moving in Δx, eq. (6,5b) is incompatible with the corpuscle-like behaviour of eq. (6,5a). A further difficulty to regard together eqs. (6,5a) and (6,5b) is that Π defined by this latter is not real, as instead $\Pi^* \Pi = |const|^2$ does. Yet just this property suggests a possible way out from this difficulty, i.e. supposing that eq. (6,5b) requires a wave-like propagation of the particle: so $\Pi^* \Pi$ could stand for particle wave amplitude whereas A', in fact regarded here as $A_0 A(t)$ without contradicting any previous step, could define frequency and phase of the particle wave. This idea is confirmed rewriting the exponential $x\Delta p$ of Π as $t\Delta\varepsilon$ dividing and multiplying by an arbitrary velocity v in order that $\pm ix\Delta p/\hbar\sqrt{n''}$ turns into $\pm it\Delta\varepsilon/\hbar\sqrt{n''}$. So $A(t)$ results defined just by this requirement, i.e.

$$\Pi = A_0 \exp[\pm i(c_x(x - x_o)\Delta p + c_t(t - t_o)\Delta\varepsilon)/\hbar\sqrt{n''}], \tag{6,6}$$

being c_x and c_t arbitrary coefficients of the linear combination expressing the most general way to unify the space and time functions. Calculate $\partial^2\Pi/\partial x^2 = -(c_x\Delta p)^2\Pi$ to extract the real quantity $c_x\Delta p$ from Π, and then by analogy $\partial^2\Pi/\partial t^2 = -(c_t\Delta\varepsilon)^2\Pi$ too; eliminating Π between these equations and noting that by dimensional reasons $(c_x\Delta p/c_t\Delta\varepsilon)^2 = v^{-2}$, the result $\partial^2\Pi/\partial x^2 - v^{-2}\partial^2\Pi/\partial t^2 = 0$ confirms, whatever v might be, the wave-like character of particle delocalization provided by eq. (6,5b). A similar wave equation could not be inferred from eq. (6,5a), according which the physical properties of the particle are related directly to the probability Π of eq. (6,1); instead, owing to the complex form of Π resulting from eq. (6,5b), the physical properties of the wave are related to $\Pi^*\Pi$. It is possible to eliminate this discrepancy introducing the complex function $\sqrt{\Pi}$ in place of Π and rewriting eq. (6,5b) as a function of the former instead of the latter; this idea agrees with that already exploited to find eqs.

(3,3). Dividing both sides by Π, eq. (6,5b) reads

$$\left(\pm\hbar\frac{\partial\sqrt{\Pi}}{\partial x}\right)^2 = -\left(p^{\S}\sqrt{\Pi}\right)^2, \qquad p^{\S} = \pm\frac{\Delta p}{2\sqrt{n''}}. \quad (6,7)$$

The notation emphasizes that $p^{\S}$ does not depend on x and is not a range; being defined as solution of the differential equation (6,7) only, its value is not longer related to Δp, i.e. it is an eigenvalue of $\sqrt{\Pi}$. This is possible because n'' is arbitrary like Δp, which allows that the ratio $\Delta p/2\sqrt{n''}$ behaves as a well determined quantity specified just by $p^{\S}$, whose value and signs correspond to either component of momentum along the x-axis where are defined positive δx and Δx. Thus eq. (6,7) reads

$$\pm\frac{\hbar}{i}\frac{\partial\sqrt{\Pi}}{\partial x} = p^{\S}\sqrt{\Pi}, \qquad \sqrt{\Pi} = \sqrt{A}\sqrt{\exp(\pm i\varphi\delta x/\Delta x)}. \quad (6,8)$$

So $\sqrt{\Pi}\sqrt{\Pi^*}$ expresses the probability to find the particle so-mewhere in Δx. Write thus

$$\sqrt{\Pi}\sqrt{\Pi^*} = \pm\frac{\hbar}{ip^{\S}}\frac{\sqrt{\Pi^*}\partial\sqrt{\Pi}}{\partial x}.$$

The right hand side is real and yields $\sqrt{\Pi}\sqrt{\Pi^*} = \delta x_0/\Delta x = A_0$, being $\delta x_0 = A_0\hbar\varphi/2p^{\S}$. As a proper value of A_0 certainly exists such that $\delta x_0 \le \Delta x$, then $\sqrt{\Pi}\sqrt{\Pi^*}$ agrees with a concept of probability similar to that of the initial definition $\delta x/\Delta x$ of eq. (6,1); yet this latter is replaced in the last equation by a constant value, which entails thus equal probability to find the particle in any sub-range δx_0 regardless of its size and position within Δx. The physical meaning of this result is emphasized integrating both sides of eq. (6,8) with respect to x in the sub-range $\delta x_0 = x_{02} - x_{01}$, which yields

$$p^{\S} = \pm\left(\int_{x_{01}}^{x_{02}}\sqrt{\Pi}\sqrt{\Pi^*}dx\right)^{-1}\int_{x_{01}}^{x_{02}}\left(\sqrt{\Pi^*}\frac{\hbar}{i}\frac{\partial}{\partial x}\sqrt{\Pi}\right)dx. \quad (6,9)$$

The average value of momentum is thus equal to the eigenvalue expected for the steady motion of a free particle (Ehre-nfest's theorem), which suggests regarding $\delta x_0/\Delta x$ as average probability that the particle is in the sub-range δx_0. It is clearly convenient therefore to define A_0 in order that $\delta x_0 = \Delta x$ through $\int\sqrt{\Pi}\sqrt{\Pi^*}dx = 1$, i.e. the momentum eigenvalue concerns the certainty that the particle is really delocalized in the total range Δx. Being this latter arbitrary, it allows considering in general the particle from $-\infty$ to ∞. The physical information provided by eq. (6,5b) is thus really different from that of eq. (6,5a), although being unquestionable the consistency of eqs. (6,8) and (6,9) with the initial eq. (6,1) despite their different formulation: both come indeed from the same uncertainty equations (1,1). So it is not surprising that the uncertainty is still inherent $\sqrt{\Pi}$ and consistent with the eigenvalue $p^{\S}$. It is evident at this point that the results hitherto inferred concern just the basic ideas through which has been formulated the early quantum mechanics; it is enough to regard in general the wave functions in analogous way, e.g. as it is shown below for the energy eigenfunction. So, write $\psi = const\sqrt{\Pi}$ and $\psi^* = const\sqrt{\Pi^*}$ to define the probability density of the particle within the volume $\Delta x\Delta y\Delta z$; this is just the volume to normalize $\psi\psi^*$. Being the uncertainty ranges arbitrary, this probability density concerns actually the whole space allowed to the particle. The normalization constant is inessential for the purposes of the present paper and not explicitly concerned hereafter. The result of interest is that, after having introduced the probability Π of eq. (1,1), one finds two distinct equations concurrently inferred from the respective eqs. (6,5)

$$\Delta p^{\S}\Delta x^{\S} = n^{\S}\hbar, \quad (6,10a)$$

$$\frac{\hbar}{i}\frac{\partial\sqrt{\Pi}}{\partial x} = \pm p^{\S}\sqrt{\Pi}. \quad (6,10b)$$

Two comments about eqs. (6,10):

(i) eq. (6,10a) is conceptually equal to the initial eq. (1,1), from which it trivially differs because of the size of the uncertainty ranges and related number of states; (ii) eq. (6,10b) defines a differential equation that calculates an eigenvalue of momentum through the probability that the particle be in a given point of its allowed range $\Delta x^{\S}$.

Eq. (6,10a) does not consider explicitly the particle, but only its delocalization inside $\Delta x^{\S}$ and thus its phase space; the same holds also for the momentum, whence the positions (2,1) and the indistinguishability of identical particles whose specific properties are disregarded "a priori". The unique information available concerns indeed the number of states $n^{\S}$ consistent with $\Delta x^{\S}$ and $\Delta p^{\S}$ for any delocalized particle; nothing requires considering the local dynamical variables themselves. The point of view of eq. (6,10b) is different: it considers explicitly the sub-range δx through $\sqrt{\Pi}$ and thus, even without any hypothesis about size and position of δx within $\Delta x^{\S}$, concerns directly the particle itself through its properties $\sqrt{\Pi}\sqrt{\Pi^*}$ and $p^{\S}$; both these latter are explicitly calculated solving the differential equation. Yet the common derivation of both eqs. (6,10) from the initial eq. (1,1) shows that actually the respective ways to describe the particle must be consistent and conceptually equivalent, as in effect it has been verified in section 2. This coincidence evidences the conceptual link between properties of the particles and phase space; it also clarifies why the quantum eigenvalues do not depend on the current values of the dynamical variables of the particles, even though calculated solving the differential equation (6,10b). Initially Π was introduced in eq. (6,1) as mere function of uncertainty ranges and sub-ranges of the phase space; thereafter, however, it has taken through the steps from eqs. (6,2) to (6,10) the physical meaning of wave function $\sqrt{\Pi}$ of the particle defining the momentum eigenvalue $p^{\S}$, which involves the mass of the particle. Eq.

(6,10b) introduces the operator formalism of wave mechanics. The approach starting directly from eqs. (1,1) has therefore more general character than the latter, which starts just postulating eq. (6,10b) here found instead as a corollary: the basic reason is that eq. (6,10a) contains less information than eq. (6,10b). These equations can be now regarded together once having acknowledged the kind of information inferred from eqs. (1,1). On the one side eqs. (6,10) introduce the wave/corpuscle dual nature of particles: eq. (6,10a) admits that the particle is somewhere in Δx, even though renouncing to know exactly where because of the delocalization; eq. (6,10b) instead regards the particle as a wave propagating within Δx thus still delocalized but excluding in principle the unknown position of a material corpuscle. On the other side eqs. (6,10) confirm that properties of particles and properties of phase space must not be regarded separately, rather they are intrinsically correlated: just for this reason the results of section 2 show that the numbers of quantum states (properties of the phase space) coincide with the quantum numbers that define the eigenvalues (properties of the wave function of the particle). Further properties of $\sqrt{\Pi} = \psi$ could be easily found, e.g. the concept of parity or the fact that the arbitrariness of the coefficients c_x and c_t previously introduced in the early expression $\Pi = A_0 \exp[\pm i(c_x x\Delta p + c_t t\Delta\varepsilon)/\hbar\sqrt{n''}]$ allows to write the more general form for this equation

$$\Pi = \sum_j A_{0j} \exp[\pm i(c_{xj} x\Delta p_j + c_{tj} t\Delta\varepsilon_j)/\hbar\sqrt{n''_j}].$$

All these assertions are well known since the early birth of the quantum theory and do not need further consideration here for sake of brevity; their evolution brings the theory up to today's formulation. It is more interesting to examine the same problem considering the time instead of the space coordinate. The steps to find the energy operator are conceptually identical to those so far reported; yet one regards the probability for the particle to be in δx at the time t, i.e. Π is defined as ratio between the time range $\delta t = t - t_o$ spent within a fixed δx and the total time range $\Delta t = t_2 - t_1$ spent elsewhere within Δx. Let us write then $\Pi = \delta t/\Delta t$ at fixed coordinate x; eqs. (6,2) and (6,4) read now $\Delta t^{-1} = \partial\Pi/\partial t$ and $(1-\Pi)\Pi\Delta\varepsilon^2 = n_- n_+ \hbar^2 (\partial\Pi/\partial t)^2$. Replacing position and momentum with time and energy in eq. (6,2), eqs. (6,7) read

$$\left(\pm\hbar\frac{\partial\sqrt{\Pi}}{\partial t}\right)^2 = -\left(\varepsilon^{\S}\sqrt{\Pi}\right)^2, \qquad \varepsilon^{\S} = \pm\frac{\Delta\varepsilon}{2\sqrt{n''}}. \quad (6,11)$$

The second eq. (6,8) reads now $\sqrt{A}\sqrt{\exp(\pm i\varphi\delta t/\Delta t)}$, which however is disregarded here because it appears included in eq. (6,6); the first eq. (6,8) becomes

$$-\frac{\hbar}{i}\frac{\partial\sqrt{\Pi}}{\partial t} = \pm\varepsilon^{\S}\sqrt{\Pi}. \quad (6,12)$$

With the upper sign at right hand side of eq. (6,12), the classical Hamiltonian written with the help of eq. (6,8) is consistent with the result $\varepsilon^{\S} = p^{\S 2}/2m$ in the particular case of a free particle having mass m and momentum $p^{\S}$. Yet the lower sign, also allowed as a consequence of eq. (6,11), shows the possibility of states with negative energy as well. The couple of equations (6,10) turns into

$$\Delta t^{\S}\Delta\varepsilon^{\S} = n^{\S}\hbar, \quad (6,13a)$$

$$-\frac{\hbar}{i}\frac{\partial\sqrt{\Pi}}{\partial t} = \pm\varepsilon^{\S}\sqrt{\Pi}. \quad (6,13b)$$

For this couple of equations hold the same considerations carried out for the corresponding eqs. (6,10). This section has shown that the operator formalism of wave mechanics is consequence itself of the concept of uncertainty. On the one side this result explains why the properties of quantum particles can be obtained as shown in section 2 even without solving any wave equation. On the other side it appears clearly that both chances of describing the quantum world are nothing else but mirror consequences of the dual wave/corpuscle behavior of particles. All considerations so far carried out do not require knowing anything about the concerned uncertainty ranges.

7 Heuristic aspects of quantum special relativity

Let us introduce now some comments about eqs. (5,9) and (5,11) before exploiting eqs. (5,12). The momentum and energy equations during the quantum fluctuation transient rewritten identically as follows

$$p(t) = \pm\frac{mv_{eff}/r_p}{\sqrt{1-(v_{eff}/c)^2}}, \quad \varepsilon(t) = \pm\frac{mc^2/r_\varepsilon}{\sqrt{1-(v_{eff}/c)^2}}, \quad (7,1)$$

$$v_{eff} = r_p v/r_\varepsilon, \qquad r_p = r_p(t), \qquad r_\varepsilon = r_\varepsilon(t),$$

evidence that the Einstein quantities of eqs. (5,11) turn into new constant expressions calculated with an effective velocity and multiplied by the respective functions of time; the previous velocity v does not longer appear explicitly into the equations. If v_{eff} is regarded as a constant, then v turns into a time variable without contradicting the Einstein equations, whose deterministic character does not admit any fluctuation and requires a steady value of v; the fluctuation has been instead introduced by admitting the quantum meaning of $\delta\varepsilon$, δp and δv. The notation of eqs. (7,1) emphasizes that energy and momentum are functions of time during the transient; regarding r_ε and r_p like time variables is reasonable, because according to eqs. (5,9) $\delta\varepsilon$ and δp are related to r_ε and r_p during the fluctuation. The physical meaning of r_ε and r_p is that of describing the cycle of values of energy and momentum, whereas r_p/r_ε controls the range of transient values allowed for the velocity. To be more specific, any energy fluctuation is characterized by an initial time t_{in} where $\varepsilon(t_{in}) = \varepsilon_{Ein}$ that successively increases to $\varepsilon(t) > \varepsilon_{Ein}$ at $t > t_{in}$ and then decreases down to the initial value ε_{Ein} at the time t_{end}. Note now that during the fluctuation transient must hold the inequality $r_p < r_\varepsilon$; otherwise, being v arbitrary e.g. very close

to c, the chance $r_p > r_\varepsilon$ could entail $\varepsilon(t)$ imaginary although being real ε_{Ein}. This would actually mean that the fluctuation is not allowed to occur. Thanks to the former inequality, instead, v can increase in principle even beyond c while still keeping $v_{eff} < c$; this can happen during the time range between t_{in} and t_{end} without divergent or imaginary quantities because under square root of the transient formulas appears v_{eff} only. This point is easily verified noting that $\varepsilon(t)/p(t) = c^2/v$, as already emphasized in section 5. Thus it must be also true that $\varepsilon(t)^2 = c^2 p(t)^2 + (mc^2)^2$ likewise eq. (5,11). Trivial manipulations yield $(v/c)^2 = (r_\varepsilon^2 - 1)/(r_p^2 - 1)$; so if $r_\varepsilon > r_p$ then is even allowed a value $v_* > c$ without contradicting neither eqs. (5,5) nor (5,11) that describe a steady behavior of the particle. According to eqs. (5,7), $r_p < r_\varepsilon$ requires

$$\delta\varepsilon(t)/\delta p(t) > \varepsilon_{Ein}/p_{Ein}. \qquad (7,2)$$

From an intuitive point of view, the transient proceeds for an observer in the lab frame according to the following steps: (i) $r_p = r_\varepsilon = 1$ at $t = t_{in}$, i.e. hold eqs. (5,11) with a value of $v_{eff} = v < c$ uniquely fixed by the initial motion of the particle; (ii) when r_p and r_ε start changing at $t > t_{in}$, the value of v_{eff} is still constrained by $v_{eff} < c$ but now $v > v_{eff}$ according to the inequality (7,2); (iii) at a later time $t_* < t_{end}$ it could even happen that $v_* > c$, although still being $v_{eff} < c$; (iv) subsequently r_p and r_ε tend again to 1 when the fluctuation cycle ends at $t \to t_{end}$ while $p(t) \to p_{Ein}$ and $\varepsilon(t) \to \varepsilon_{Ein}$, i.e. $v \to v_{eff} < c$. Thanks to the concept of quantum fluctuation, therefore, the increase of velocity $v_* > c$ in the step (iii) does not involve directly the value of v appearing in the steady formulas of ε_{Ein} and p_{Ein}, as indeed it results in eqs. (5,12); so the superluminal step (iii) is in principle possible. However, what about the chance of detecting it experimentally? Certainly the answer is not found via eqs. (7,1), which describe local quantities at the random and unspecified time t; on the other hand, since the particle travels, t is related to a corresponding x, random and unspecified as well. Throughout this paper it has been emphasized that information of physical interest is obtainable through uncertainty ranges only; thus the considerations just carried out, based on time and space local coordinates, have worth only to guess and assess the possible behavior of the particle at any $t_{in} \le t \le t_{end}$ and better understand the physical results inferred by consequence. Coherently with the approach so far followed, we discard once again the local dynamical variables and pay attention to the respective uncertainty ranges only. Exploit thus eqs. (5,12) to get information comparable with the experience, putting $\delta t = t_{end} - t_{in}$ and δl equal to the distance across which is measured the velocity. In this way we can calculate an *average* velocity $\delta l/\delta t$ whose value depends upon how the experiment is carried out. If δt is shorter than the time τ for the particle to travel the distance δl, then the superluminal effect it is not detectable, because the fluctuation starts and ends while the particle is still traveling within δl; this means that the fluctuation is an event entirely occurring within a space delocalization range. Yet nothing is known about what happens within this uncertainty range. In this case, when considering the average velocity of the particle, we can only acknowledge that this latter is anyway smaller than c, whereas any information about any possible event allowed to occur within δl remains in fact unaccessible; moreover eqs. (5,12) do not have themselves physical meaning, as they attempt to get physical insight within an uncertainty range. If however δt is longer than τ, then the superluminal effect is at least in principle detectable without contradicting the previous reasoning, because now the fluctuation extends throughout all the range δl and beyond; it is no longer a local event hidden by the uncertainty. So if the average velocity is measured in these experimental conditions, i.e. with δl sufficiently short or δt sufficiently long, the superluminal effect is in principle detectable. Note in this respect that a small value of m in the second eq. (5,12) corresponds to a longer time at right hand side, so the inequality (7,2) is more easily fulfilled for a particle not too heavy than for a heavy particle; indeed the former typically travels with values of v closer to c than the latter for energy reasons and also entails a longer δt, so it could effectively overcome the superluminal transition threshold fulfilling more likely the condition $\delta t > \tau$. Once fulfilling these conditions, a light particle appears traveling the space range $\delta l = v_* \delta t$ at speed $v_* > c$ in the laboratory reference system even during a moderate energy fluctuation and without violating any principle of quantum special relativity formulated in section 5; indeed $\delta l/\delta t$ does not calculate v_{eff} but the average transient of v. As a clarifying comparison recall that $\delta\varepsilon$ does not violate the energy conservation, it is simply a temporary derogation to this latter allowed by the uncertainty principle only; why not should something similar happen also for the velocity, if this latter does not cause divergent or imaginary results? Anyway, for the comparison with the experiment are enough just the two equations (5,12) that relate in the laboratory frame the distance δl traveled by the particle to the time δt during which the transient is still in progress; their ratio, assumed physically consistent with the time length of the fluctuation transient, reads

$$\frac{\delta l}{\delta t} = \frac{\frac{mc^2}{\sqrt{r_\varepsilon^2 - (r_p v/c)^2}} - \frac{mc^2}{\sqrt{1-(v/c)^2}}}{\frac{mv}{\sqrt{r_\varepsilon^2 - (r_p v/c)^2}} - \frac{mv}{\sqrt{1-(v/c)^2}}} = c\frac{c}{v}.$$

Since $v < c$, then $\delta l/\delta t > c$, which demonstrates a superluminal particle transfer during the quantum fluctuation cycle. If for instance $v = 0.99c$ then $\delta l/\delta t = 1.01c$. Note that instead the speed of the photon $v = c$ remains identically, universally and invariantly equal to c. Eqs. (5,5) have been written through time and space uncertainty ranges only. The Einstein relativity specifies the time range $\Delta t = t - t_o$ through a current time coordinate t and a lower boundary $t_o = x_o V/c^2$; both times have a deterministic physical meaning. This last result could be easily guessed also here, thinking that even t_o must depend on V/c and must be related to

the corresponding x_o. Thus a value $V > c$ would change the signs of Δt and $\Delta t'$ in eq. (5,5), i.e. the concept itself of sequence "before" and "after". Apart from the fact that such a conclusion would be illusory in the present theoretical frame because the uncertainty discards "a priori" the local coordinates, it is also essential in this respect a further remark. As shown before, the lack of physical information about t and t_o and $t - t_o$ does not prevent to infer the relativistic formulas of energy and momentum: yet, even specifying $t_o = x_o V/c^2$, the possible time-reversal during the quantum fluctuation cycle does not affect any result previously obtained. First of all because actually this cycle has not been specified, i.e. exchanging t_{end} with t_{in} does not change any step of the previous reasoning; moreover if the cycle starts with an initial energy ε_{Ein} and ends with the same final energy ε_{Ein}, any discrimination between beginning and ending of the cycle seems unphysical. Therefore, since the possible time reversal should be a local effect concerning the quantum fluctuation only, all the conclusions hitherto obtained still hold. Also note that $\delta l/\delta t = \varepsilon_{Ein}/p_{Ein} = c^2/v$; so the inequality (7,2) reads $\delta\varepsilon/\delta p > \delta l/\delta t$ as well, i.e. $\delta\varepsilon/\delta l > \delta p/\delta t$: the left hand side represents the force acting on the particle due to its fluctuation driven energy gap along its path, the right hand side represents the force due to the momentum change during the fluctuation time length. Saying that the former is greater than the latter means an excess force with respect to the mere momentum change having fully quantum origin, necessarily due to nothing else but the fluctuation in the case of a free particle. It seems reasonable to assume that just this excess force justifies the superluminal effect. As expected, neither δl nor δt enter explicitly into the calculation of the velocity; the ratio between two uncertainty ranges provides of course an average value during the transient, which is in effect allowed in the frame of the present approach. It is interesting to emphasize that a given $\delta\varepsilon/\delta l$, related to the energy growing along the path traveled by the particle, could be at increasing δl not greater than $\delta p/\delta t$, related to the given fluctuation time length; this is because δl and δt are two independent quantities, the former related to the experimental apparatus, the latter to a feature of the fluctuation. If δl increases up to a larger value Δl such that $\delta\varepsilon/\Delta l < \delta p/\delta t$ the superluminal effect is not observable. Indeed this is just in line with the previous considerations recalling that: (i) the effect is detectable if at the end of the path of the particle within δl the fluctuation is still in progress; (ii) if instead the fluctuation cycle ends while the particle is still traveling inside δl, then it becomes an event occurring within an uncertainty range and thus, as such, unobservable. If the model is correct, this is what to expect imagining to increase the size of δl up to Δl: the same kind of observation should yield a positive outcome if carried out in the experimental situation (i), but certainly a negative outcome if carried out in the experimental situation (ii). This also suggests a possible way to verify the considerations just carried out: to detect the same velocity fluctuation event of not-heavy particles with two detectors located in two different laboratories. Although the concept of their respective "distances" from the source is illusory for the reasons introduced in sections 3, it remains nevertheless still true that different locations, wherever they might be, provide different chances for the uncertainty of revealing or hiding experimentally the superluminal transition. Thus the random occurring/non-occurring of the superluminal effect should not be ascribed to human experimental errors but to a further probabilistic weirdness of the quantum world.

8 Discussion

The ordinary formulation of quantum mechanics contains the classical physics as a limit case but needs this latter to be formulated [17]. Regarding instead eqs. (1,1) as expressions of a fundamental principle of nature, and not as mere by-products of the commutation rules of operators, this ambiguous link between classical and quantum physics is bypassed. Section 6 has shown that eqs. (1,1) entail as a corollary the operator formalism of wave mechanics; yet the present approach appears more general than that based on this latter. As shown in sections 4 and 5, it automatically introduces since the beginning the non-locality and non-reality into the description of quantum systems. In principle the quantum uncertainty does not prevent knowing exactly one dynamical variable only; being the size of all ranges arbitrary by definition, one must admit even the chance $\Delta x \to 0$ that means local position of a particle exactly known. The same reasoning holds separately for the momentum as well. Independent ranges however do not provide physical information on the observable properties of the quantum world. These observables require abandoning separate certainties independently allowed; the physical meaning of the ranges changes when considering together two conjugate dynamical variables, which also means discarding the classical realism and localism as well but gaining the eigenvalues. Does the moon exist regardless of whether one observes it? According to the approach sketched in section 2 this question should be better reformulated, for instance as follows: do the properties of the moon we know exist regardless of a possible observer? Yet if nobody observes the moon, nobody could define the properties "we know"; these latter are the outcomes of some kind of measurement, i.e. they are triggered themselves by a previous measurement interaction. Repeating this reasoning back in the time the conclusion is that before the first recording of light beam escaping from the moon nobody would even know the existence of the moon; in which case would become physically irrelevant the prospective physical properties of an object still to be discovered. In this sense it appears understandable that the properties we know exist when observations are carried out. Hence what we call moon is just the result of an interaction between an observer and an object sufficiently close to the Earth to be observable. As concerns the localism it is appro-

priate to think about an action at a spooky distance, since the local coordinates defining the distance are actually an arbitrary extrapolation to the quantum world of a classical way of thinking. This idea appeared since the early times of birth of quantum mechanics, when the deterministic concept of trajectory was irreversibly abandoned. The operator formalism requires a wave function of time and space coordinates; these latter identify in turn a region of space where however has physical meaning the mere probability density to find the particle only. Thus the wave function denies the classical meaning of the local coordinates, e.g. position and momentum or energy and time, as a function of which is however itself calculated. In this respect the present approach formulates an even more indeterministic and drastic view of the reality: to discard the local values since the beginning. In this sense, eqs. (1,1) seem a step ahead with respect to the operator formalism; even though seemingly more agnostic, they avoid handling the local variables to define and solve the appropriate wave equations from which are extracted the eigenvalues, i.e. the observables, in a probabilistic conceptual context. Here indeed we refuse "a priori" the physical usefulness of introducing time and space local coordinates and, in general, local quantities that do no longer appear in the eigenvalues; yet, even so the results are identical. This suggests that actually is the uncertainty the fundamental concept behind the results, a sort of essential information directly related to the knowledge we can afford; for instance, the arbitrariness of the quantum numbers of wave mechanics, due to the mathematical features of the solutions of differential equations, is replaced by that of the number of states; indeed the results show that the latter have a physical meaning identical to the former. Eqs. (1,1) provide these numbers since the beginning. This is the reason of the straightforward character of the present approach, which indeed does not require solving any differential equation but proceeds through trivial algebraic manipulations of the formulae. The arbitrariness seems a concept with negative valence, especially in science; yet it played an essential role in deriving eqs. (3,5) from eq. (3,2); on this step are based eqs. (1,1). The section 2 shows that these equations plug the classical definition of angular momentum into the quantum world thanks to two concepts: introducing the number of states and eliminating local information. The section 6 has shown why the indistinguishability of identical particles is a natural consequence of these premises; in the operator formalism instead it must be purposely introduced as a postulate and appropriately handled from a mathematical point of view, recall for instance the early Slater determinants. Moreover the section 4 has shown why the present approach entails inherently even the non-locality and the non-reality of the quantum world: while evidencing their link with the quantization of the physical observables, these weird features are automatically required by eqs. (1,1) through n. Eventually, let us emphasize that the present way of regarding the quantum world is compatible with the special relativity. The paper [15] has inferred its basic principles as corollaries, in section 7 some results particularly significant have been obtained: the invariant interval, the Lorentz transformations of time and length, the energy and momentum equations of a free particle, the rest energy of particle, the existence of antimatter and the concept of mass itself. The key idea underlying these results is the way to regard the relativistic intervals: to discard their deterministic definition, early introduced by Einstein, and regard them as uncertainty ranges. As shown before, this simple conceptual step is enough to plug into the quantum world even the special relativity. Moreover, the quantum way to infer the relativistic equations has opened the way to admit a typical quantum phenomenon, the energy fluctuation, able to account for unexpected effects otherwise precluded by the early deterministic basis of special relativity formulated by Einstein.

9 Conclusion

The approach based uniquely on eqs. (1,1) contains inherently the requirements of non-locality and non-reality that characterize the quantum world. This kind of approach is also consistent with the special relativity, whose basic statements were found as corollaries in previous paper.

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Thermodynamics and Scale Relativity

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It is shown how the fractal paths of SR = scale relativity (following Nottale) can be introduced into a TD = thermodynamic context (following Asadov-Kechkin).

1 Preliminary remarks

The SR program of Nottale et al (cf. [1]) has produced a marvelous structure for describing quantum phenomena on the QM type paths of Hausdorff dimension two (see below). Due to a standard Hamiltonian TD dictionary (cf. [2]) an extension to TD phenomena seems plausible. However among the various extensive and intensive variables of TD it seems unclear which to embelish with fractality. We avoid this feature by going to [3] which describes the arrow of time in connection with QM and gravity. This introduces a complex time (**1A**) $\tau = t - (i\hbar/2)\beta$ where $\beta = 1/kT$ with $k = k_B$ the Bolzmann constant and a complex Hamiltonian (**1B**) $\mathfrak{H} = \mathfrak{E} - (i\hbar\Gamma/2)$ where $\mathfrak{E}$ is a standard energy term, e.g. (**1C**) $\mathfrak{E} \sim (1/2)mv^2 + \mathfrak{W}(x)$. One recalls that complex time has appeared frequently in mathematical physics. We will show how fractality can be introduced into the equations of [3] without resorting to several complex variables or quaternions.

Thus from [3] one has equations

$$\mathfrak{H} = \mathfrak{E} - \left(\frac{i\hbar}{2}\Gamma\right);\ \tau = t - \frac{i\hbar}{2}\beta;\ [\mathfrak{E}, \Gamma] = [\mathfrak{H}, \mathfrak{H}^\dagger] = 0; \quad (1.1)$$

$$\Psi = exp^{-\frac{i\mathfrak{S}\tau}{\hbar}}\psi; P_n = \frac{w_n}{Z}; w_n = \rho_n exp^{[-\mathfrak{E}_n\beta + \Gamma_n t]};$$

$$i\hbar\partial_\tau\Psi = \mathfrak{H}\Psi; \Psi = \sum C_n\psi_n;$$

$$\mathfrak{H}_n = \mathfrak{E}_n - \frac{i\hbar}{2}\Gamma_n; [\mathfrak{H}, \mathfrak{H}^\dagger] = 0;$$

$$\mathfrak{E}\psi_n = \mathfrak{E}_n\psi_n; \Gamma\psi_n = \Gamma_n\psi_n; (\psi_n, \psi_k) = \delta_{nk}.$$

One could introduce another complex variable here, say j with $j^2 = -1$, but this can be avoided.

Now go to the SR theory and recall the equations

$$\frac{\hat{d}}{dt} = \frac{1}{2}\left(\frac{d_+}{dt} + \frac{d_-}{dt}\right) - \frac{i}{2}\left(\frac{d_+}{dt} - \frac{d_-}{dt}\right); \quad (1.2)$$

$$\mathcal{V} = \frac{\hat{d}x}{dt} = V - iU = \frac{1}{2}(v_+ + v_-) - \frac{i}{2}(v_+ - v_-);$$

$$\frac{\hat{d}}{dt} = \partial_t + \mathcal{V}\cdot\nabla - i\mathcal{D}\Delta;$$

$$\mathcal{H} = \frac{m}{2}\mathcal{V}^2 - im\mathcal{D}\nabla\cdot\mathcal{V} + \mathcal{W} = \frac{1}{2m}\mathcal{P}^2 - i\mathcal{D}\cdot\mathcal{P} + \mathcal{W}; \quad (1.3)$$

$$\mathcal{H} = \mathcal{V}\cdot\mathcal{P} - i\mathcal{D}\nabla\cdot\mathcal{P} - \mathcal{L};$$

$$\hat{\mathcal{V}} = \mathcal{V} - i\mathcal{D}\nabla; \quad (\partial_t + \hat{\mathcal{V}}\cdot\nabla)\mathcal{V} = -\frac{\nabla\mathcal{W}}{m}; \quad (1.4)$$

$$U = \mathcal{D}\nabla log(P); \quad P = |\psi|^2; \quad \psi = e^{i\Xi/2m\mathcal{D}};$$

$$\mathfrak{Q} = -2m\mathcal{D}^2\frac{\Delta\sqrt{P}}{\sqrt{P}}; \quad (1.5)$$

$$\mathcal{V} = -2i\mathcal{D}\nabla[log(\psi)]; \quad \Xi_0 = 2m\mathcal{D};$$

$$\mathcal{D}^2\Delta\psi + i\mathcal{D}\partial_t\psi - \frac{\mathcal{W}}{2m}\psi = 0; \quad (1.6)$$

$$\frac{dV}{dt} = \frac{F}{m} = U\cdot\nabla U + \mathcal{D}\Delta U.$$

This has been written for 3 space dimensions but we will restrict attention to a 1-D space based on x below.

We will combine the ideas in (1.1) and (1.2) in Section 2 below. Note here $\mathfrak{Q}$ is the QP = quantum potential (see e.g. [5–8] for background).

2 Combination and interaction

From (1.2)-(1.6) we see that the fractal paths in one space dimension have Hausdorff dimension 2 and we note that U in (1.2) is related to an osmotic velocity and completely determines the QP $\mathfrak{Q}$. Note that these equations (1.2)-(1.6) produce a macro-quantum mechanics (where $\mathcal{D} = \hbar/2m$ for QM). It is known that a QP represents a stabilizing organizational anti-diffusion force which suggests an important connection between the fractal picture above and biological processes involving life (cf. [1, 9–13]). We also refer to [14–16] for probabalistic aspects of quantum mechanics and entropy and recommend a number of papers of Agop et al (cf. [17]) which deal with fractality (usually involving Hausdorff dimension 2 or 3) in differential equations such as Ginzburg-Landau, Korteweg de-Vries, and Navier-Stokes; this work

includes some formulations in Weyl-Dirac geometry (Feoli-Gregorash-Papini-Wood formulation) involving superconductivity in a gravitational context.

Now let us imagine that $\mathcal{W} \sim \mathfrak{W}$ and $V \sim v$ so that the energy terms in the real part of the SE arising from (1.2)-(1.6) will take the form

$$\mathfrak{E} \sim \frac{1}{2}mV^2 + \mathfrak{W} + \mathfrak{Q} \tag{2.1}$$

and we identify this with $\mathfrak{E}$ in the TD problem where

$$\mathfrak{Q} = -2m\mathcal{D}^2\frac{\Delta\sqrt{P}}{\sqrt{P}}; \quad P = |\Psi|^2. \tag{2.2}$$

One arrives at QM for $\mathcal{D} = \hbar/2m$ as mentioned above and one notes that the mean value $\bar{\mathfrak{E}}$ used in the analysis of [3] will now have the form

$$\bar{\mathfrak{E}} = \frac{1}{2}\int mV^2Pdx + \int |\mathfrak{W}|^2Pdx + \int \mathfrak{Q}Pdx \tag{2.3}$$

and the last term $\int \mathfrak{Q}Pdx$ has a special meaning in terms of Fisher information as developed in [5–7, 19–21]. In fact one has

$$\int \mathfrak{Q}Pdx = -2m\mathcal{D}^2\int \frac{\partial_x^2\sqrt{P}}{\sqrt{P}}Pdx = \tag{2.4}$$

$$= -\frac{\mathcal{D}^2}{2}\int\left[\frac{2P''}{P} - \left(\frac{P'}{P}\right)^2\right]Pdx = \frac{m\mathcal{D}^2}{2}\int\frac{(P')^2}{P}dx$$

In the quantum situation $\mathcal{D} = \hbar/2m$ leading to

$$\int \mathfrak{Q}Pdx = \frac{\hbar^2}{8m}\int\frac{(P')^2}{P}dx = \frac{\hbar^2}{8m}FI \tag{2.5}$$

where FI denotes Fisher information (cf. [7, 21]). And this term can be construed as a contribution from fractality.

One can now sketch very briefly the treatment of [3] based on (1.1). Thus one constructs a generalized QM (with arrow of time and connections to gravity for which we refer to [3]). The eigenvalues $\mathfrak{E}_n$, Γ_n, in (1.1) are exploited with

$$\rho_n = |C_n|^2; \quad P_n = \frac{w_n}{Z};$$

$$\Psi = \sum C_n\psi_n; \quad w_n = \rho_n e^{-\mathfrak{E}_n\beta + \Gamma_n t}. \tag{2.6}$$

One considers two special systems:

1. First let the eigenvectors Γ_n all be the same (decay free system) and then $w_n = \rho_n exp[-\mathfrak{E}_n\beta]$ which means that β is actually the inverse absolute temperature (multiplied by k_B) when $\mathfrak{E}_n$ is identified with the n-th energy level and the system is decay free.
2. Next let all the $\mathfrak{E}_n$ be the same so $w_n = \rho_n exp[-\Gamma_n t]$ and all the Γ_N have the sense of decay parameters if t is the conventional physical time.

Thus the solution space of the theory space can be decomposed into the direct sum of subspaces which have a given value of the energy or of the decay parameter. It is seen that for $\beta = constant$ the dynamical equation for the basis probabilities is

$$\frac{dP_n}{dt} = -(\Gamma_n - \bar{\Gamma})P_n; \quad \frac{d\bar{\Gamma}}{dt} = -D_\Gamma^2; \quad D_\Gamma^2 = \overline{(\Gamma - \bar{\Gamma})^2}. \tag{2.7}$$

From (2.7) one sees that $\bar{\Gamma}(t)$ is not increasing which means that the isothermal regime of evolution has an arrow of time, which is related to the average value of the decay operator. Thus P_n increases if $\bar{\Gamma} > \Gamma_n$ and decreases when $\bar{\Gamma} < \Gamma_n$. One can also show that in the general case of $\beta = \beta(t)$ the dynamical equations for the P_n have the form

$$\frac{dP_n}{dt} = -\left[\Gamma_n - \bar{\Gamma} + (\mathfrak{E}_n - \bar{\mathfrak{E}})\frac{d\beta}{dt}\right]P_n. \tag{2.8}$$

Here the specific function $d\beta/dt$ must be specified or extracted from a regime condition $f(t, \beta, \bar{\mathfrak{A}}(t, \beta)) = 0$ for some observable $\mathfrak{A}$ (e.g. $\bar{\mathfrak{E}} = constant$ is an adiabatic condition). In the adiabatic case for example when $\bar{\mathfrak{E}} = \sum_n \mathfrak{E}_n P_n = constant$ there results

$$\frac{d\beta}{dt} = -\frac{\overline{\mathfrak{E}T} - \bar{\mathfrak{E}}\bar{T}}{D_{\mathfrak{E}}^2} \tag{2.9}$$

where $D_{\mathfrak{E}}$ denotes a dispersion of the energy operator $\mathfrak{E}$. Using (2.8)-(2.9) one obtains

$$\frac{d\bar{\Gamma}}{dt} = -D_\Gamma^2\left[1 - \frac{(\overline{\mathfrak{E}T} - \bar{\mathfrak{E}}\bar{T})^2}{D_{\mathfrak{E}}^2 D_\Gamma^2}\right] \geq 0. \tag{2.10}$$

Subsequently classical dynamics is considered for $\hbar \to 0$ and connections to gravity are indicated with kinematically independent geometric and thermal times (cf. [3]).

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Van Aubel's Theorem in the Einstein Relativistic Velocity Model of Hyperbolic Geometry

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In this note, we present a proof to the Van Aubel Theorem in the Einstein Relativistic Velocity Model of Hyperbolic Geometry.

1 Introduction

Hyperbolic Geometry appeared in the first half of the 19^{th} century as an attempt to understand Euclid's axiomatic basis of Geometry. It is also known as a type of non-Euclidean Geometry, being in many respects similar to Euclidean Geometry. Hyperbolic Geometry includes similar concepts as distance and angle. Both these geometries have many results in common but many are different. There are known many models for Hyperbolic Geometry, such as: Poincaré disc model, Poincaré half-plane, Klein model, Einstein relativistic velocity model, etc. Here, in this study, we give hyperbolic version of Van Aubel theorem. The well-known Van Aubel theorem states that if ABC is a triangle and AD, BE, CF are concurrent cevians at P, then $\frac{AP}{PD} = \frac{AE}{EC} + \frac{AF}{FB}$ (see [1, p. 271]).

Let D denote the complex unit disc in complex z - plane, i.e.

$$D = \{z \in \mathbb{C} : |z| < 1\}.$$

The most general Möbius transformation of D is

$$z \to e^{i\theta}\frac{z_0 + z}{1 + \overline{z_0}z} = e^{i\theta}(z_0 \oplus z),$$

which induces the Möbius addition $\oplus$ in D, allowing the Möbius transformation of the disc to be viewed as a Möbius left gyrotranslation

$$z \to z_0 \oplus z = \frac{z_0 + z}{1 + \overline{z_0}z}$$

followed by a rotation. Here $\theta \in \mathbb{R}$ is a real number, $z, z_0 \in D$, and $\overline{z_0}$ is the complex conjugate of z_0. Let $Aut(D, \oplus)$ be the automorphism group of the grupoid $(D, \oplus)$. If we define

$$gyr : D \times D \to Aut(D, \oplus), gyr[a, b] = \frac{a \oplus b}{b \oplus a} = \frac{1 + a\overline{b}}{1 + \overline{a}b},$$

then is true gyrocommutative law

$$a \oplus b = gyr[a, b](b \oplus a).$$

A gyrovector space $(G, \oplus, \otimes)$ is a gyrocommutative gyrogroup $(G, \oplus)$ that obeys the following axioms:

1. $gyr[\mathbf{u}, \mathbf{v}]\mathbf{a} \cdot gyr[\mathbf{u}, \mathbf{v}]\mathbf{b} = \mathbf{a} \cdot \mathbf{b}$ for all points $\mathbf{a}, \mathbf{b}, \mathbf{u}, \mathbf{v} \in G$.
2. G admits a scalar multiplication, $\otimes$, possessing the following properties. For all real numbers $r, r_1, r_2 \in \mathbb{R}$ and all points $\mathbf{a} \in G$:

   (G1) $1 \otimes \mathbf{a} = \mathbf{a}$

   (G2) $(r_1 + r_2) \otimes \mathbf{a} = r_1 \otimes \mathbf{a} \oplus r_2 \otimes \mathbf{a}$

   (G3) $(r_1 r_2) \otimes \mathbf{a} = r_1 \otimes (r_2 \otimes \mathbf{a})$

   (G4) $\frac{|r| \otimes \mathbf{a}}{\|r \otimes \mathbf{a}\|} = \frac{\mathbf{a}}{\|\mathbf{a}\|}$

   (G5) $gyr[\mathbf{u}, \mathbf{v}](r \otimes \mathbf{a}) = r \otimes gyr[\mathbf{u}, \mathbf{v}]\mathbf{a}$

   (G6) $gyr[r_1 \otimes \mathbf{v}, r_1 \otimes \mathbf{v}] = I$
3. Real vector space structure $(\|G\|, \oplus, \otimes)$ for the set $\|G\|$ of onedimensional "vectors"

   $$\|G\| = \{\pm\|\mathbf{a}\| : \mathbf{a} \in G\} \subset \mathbb{R}$$

   with vector addition $\oplus$ and scalar multiplication $\otimes$, such that for all $r \in \mathbb{R}$ and $\mathbf{a}, \mathbf{b} \in G$,

   (G7) $\|r \otimes \mathbf{a}\| = |r| \otimes \|\mathbf{a}\|$

   (G8) $\|\mathbf{a} \oplus \mathbf{b}\| \leq \|\mathbf{a}\| \oplus \|\mathbf{b}\|$

Definition 1. *Let ABC be a gyrotriangle with sides a, b, c in an Einstein gyrovector space $(V_s, \oplus, \otimes)$, and let h_a, h_b, h_c be three altitudes of ABC drawn from vertices A, B, C perpendicular to their opposite sides a, b, c or their extension, respectively. The number*

$$S_{ABC} = \gamma_a a \gamma_{h_a} h_a = \gamma_b b \gamma_{h_b} h_b = \gamma_c c \gamma_{h_c} h_c$$

is called the gyrotriangle constant of gyrotriangle ABC (here $\gamma_{\mathbf{v}} = \frac{1}{\sqrt{1 - \frac{\|\mathbf{v}\|^2}{s^2}}}$ is the gamma factor).

(See [2, p. 558].)

Theorem 1. ***(The Gyrotriangle Constant Principle)*** *Let A_1BC and A_2BC be two gyrotriangles in a Einstein gyrovector plane $(\mathbb{R}^2_s, \oplus, \otimes)$, $A_1 \neq A_2$ such that the two gyrosegments A_1A_2 and BC, or their extensions, intersect at a point $P \in \mathbb{R}^2_s$. Then,*

$$\frac{\gamma_{|A_1P|}\,|A_1P|}{\gamma_{|A_2P|}\,|A_2P|} = \frac{S_{A_1BC}}{S_{A_2BC}}.$$

(See [2, p. 563].)

Theorem 2. ***(The Hyperbolic Theorem of Menelaus in Einstein Gyrovector Space)*** *Let $\mathbf{a}_1, \mathbf{a}_2$, and $\mathbf{a}_3$ be three non-gyrocollinear points in an Einstein gyrovector space $(V_s, \oplus, \otimes)$. If a gyroline meets the sides of gyrotriangle $\mathbf{a}_1\mathbf{a}_2\mathbf{a}_3$ at points $\mathbf{a}_{12}, \mathbf{a}_{13}, \mathbf{a}_{23}$, then*

$$\frac{\gamma_{\ominus\mathbf{a}_1\oplus\mathbf{a}_{12}}\,\|\ominus\mathbf{a}_1\oplus\mathbf{a}_{12}\|}{\gamma_{\ominus\mathbf{a}_2\oplus\mathbf{a}_{12}}\,\|\ominus\mathbf{a}_2\oplus\mathbf{a}_{12}\|}\cdot\frac{\gamma_{\ominus\mathbf{a}_2\oplus\mathbf{a}_{23}}\,\|\ominus\mathbf{a}_2\oplus\mathbf{a}_{23}\|}{\gamma_{\ominus\mathbf{a}_3\oplus\mathbf{a}_{23}}\,\|\ominus\mathbf{a}_3\oplus\mathbf{a}_{23}\|}\cdot$$

$$\frac{\gamma_{\ominus\mathbf{a}_3\oplus\mathbf{a}_{13}}\,\|\ominus\mathbf{a}_3\oplus\mathbf{a}_{13}\|}{\gamma_{\ominus\mathbf{a}_1\oplus\mathbf{a}_{13}}\,\|\ominus\mathbf{a}_1\oplus\mathbf{a}_{13}\|} = 1$$

(See [2, p. 463].)

Theorem 3. ***(The Gyrotriangle Bisector Theorem)*** *Let ABC be a gyrotriangle in an Einstein gyrovector space $(V_s, \oplus, \otimes)$, and let P be a point lying on side BC of the gyrotriangle such that AP is a bisector of gyroangle $\measuredangle BAC$. Then,*

$$\frac{\gamma_{|BP|}\,|BP|}{\gamma_{|PC|}\,|PC|} = \frac{\gamma_{|AB|}\,|AB|}{\gamma_{|AC|}\,|AC|}$$

(See [3, p. 150].) For further details we refer to the recent book of A. Ungar [2].

2 Main results

In this section, we prove Van Aubel's theorem in hyperbolic geometry.

Theorem 4. *If the point P does lie on any side of the hyperbolic triangle ABC, and BC meets AP in D, CA meets BP in E, and AB meets CP in F, then*

$$\frac{\gamma_{|AP|}\,|AP|}{\gamma_{|PD|}\,|PD|} = \frac{\gamma_{|BC|}\,|BC|}{2}\cdot\frac{\gamma_{|AE|}\,|AE|}{\gamma_{|EC|}\,|EC|}\cdot\frac{1}{\gamma_{|BD|}\,|BD|}+$$

$$\frac{\gamma_{|BC|}\,|BC|}{2}\cdot\frac{\gamma_{|FA|}\,|FA|}{\gamma_{|FB|}\,|FB|}\cdot\frac{1}{\gamma_{|CD|}\,|CD|}.$$

Proof. If we use the Menelaus's theorem in the h-triangles ADC and ABD for the h-lines BPE, and CPF respectively, then

$$\frac{\gamma_{|AP|}\,|AP|}{\gamma_{|PD|}\,|PD|} = \frac{\gamma_{|AE|}\,|AE|}{\gamma_{|EC|}\,|EC|}\cdot\frac{\gamma_{|BC|}\,|BC|}{\gamma_{|BD|}\,|BD|} \tag{1}$$

and

$$\frac{\gamma_{|AP|}\,|AP|}{\gamma_{|PD|}\,|PD|} = \frac{\gamma_{|FB|}\,|FB|}{\gamma_{|FA|}\,|FA|}\cdot\frac{\gamma_{|BC|}\,|BC|}{\gamma_{|CD|}\,|CD|} \tag{2}$$

From (1) and (2), we have

$$2\cdot\frac{\gamma_{|AP|}\,|AP|}{\gamma_{|PD|}\,|PD|} = \frac{\gamma_{|AE|}\,|AE|}{\gamma_{|EC|}\,|EC|}\cdot\frac{\gamma_{|BC|}\,|BC|}{\gamma_{|BD|}\,|BD|}+$$

$$\frac{\gamma_{|FA|}\,|FA|}{\gamma_{|FB|}\,|FB|}\cdot\frac{\gamma_{|BC|}\,|BC|}{\gamma_{|CD|}\,|CD|},$$

the conclusion follows. □

Corollary 1. *Let G be the centroid of the hyperbolic triangle ABC, and D, E, F are the midpoints of hyperbolic sides BC, CA, and AC respectively. Then,*

$$\frac{\gamma_{|AG|}\,|AG|}{\gamma_{|GD|}\,|GD|} = \frac{\gamma_{|BC|}\,|BC|}{2}\left[\frac{1}{\gamma_{|BD|}\,|BD|}+\frac{1}{\gamma_{|CD|}\,|CD|}\right]. \tag{3}$$

Proof. If we use theorem 4 for the triangle ABC and the centroid G, we have

$$\frac{\gamma_{|AG|}\,|AG|}{\gamma_{|GD|}\,|GD|} = \frac{\gamma_{|BC|}\,|BC|}{2}\cdot\frac{\gamma_{|AE|}\,|AE|}{\gamma_{|EC|}\,|EC|}\cdot\frac{1}{\gamma_{|BD|}\,|BD|}+$$

$$\frac{\gamma_{|BC|}\,|BC|}{2}\cdot\frac{\gamma_{|FA|}\,|FA|}{\gamma_{|FB|}\,|FB|}\cdot\frac{1}{\gamma_{|CD|}\,|CD|},$$

the conclusion follows. □

Corollary 2. *Let I be the incenter of the hyperbolic triangle ABC, and let the angle bisectors be AD, BE, and CF. Then,*

$$\frac{\gamma_{|AI|}\,|AI|}{\gamma_{|ID|}\,|ID|} = \frac{1}{2}\left[\frac{\gamma_{|AB|}\,|AB|}{\gamma_{|BD|}\,|BD|}+\frac{\gamma_{|AC|}\,|AC|}{\gamma_{|CD|}\,|CD|}\right]. \tag{4}$$

Proof. If we use theorem 3 for the triangle ABC, we have

$$\frac{\gamma_{|AE|}\,|AE|}{\gamma_{|EC|}\,|EC|} = \frac{\gamma_{|AB|}\,|AB|}{\gamma_{|BC|}\,|BC|}, \text{ and}$$

$$\frac{\gamma_{|AF|}\,|AF|}{\gamma_{|FB|}\,|FB|} = \frac{\gamma_{|AC|}\,|AC|}{\gamma_{|BC|}\,|BC|}. \tag{5}$$

If we use theorem 4 for the triangle ABC and the incenter I, we have

$$\begin{aligned}\frac{\gamma_{|AI|}\,|AI|}{\gamma_{|ID|}\,|ID|} = {} & \frac{\gamma_{|BC|}\,|BC|}{2}\cdot\frac{\gamma_{|AE|}\,|AE|}{\gamma_{|EC|}\,|EC|}\cdot\frac{1}{\gamma_{|BD|}\,|BD|}+\\ & \frac{\gamma_{|BC|}\,|BC|}{2}\cdot\frac{\gamma_{|FA|}\,|FA|}{\gamma_{|FB|}\,|FB|}\cdot\frac{1}{\gamma_{|CD|}\,|CD|}.\end{aligned} \tag{6}$$

From (5) and (6), we have

$$\frac{\gamma_{|AI|}\,|AI|}{\gamma_{|ID|}\,|ID|} = \frac{\gamma_{|BC|}\,|BC|}{2}\cdot\frac{\gamma_{|AB|}\,|AB|}{\gamma_{|BC|}\,|BC|}\cdot\frac{1}{\gamma_{|BD|}\,|BD|}+$$

$$\frac{\gamma_{|BC|}\,|BC|}{2}\cdot\frac{\gamma_{|AC|}\,|AC|}{\gamma_{|BC|}\,|BC|}\cdot\frac{1}{\gamma_{|CD|}\,|CD|},$$

the conclusion follows. □

The Einstein relativistic velocity model is another model of hyperbolic geometry. Many of the theorems of Euclidean geometry are relatively similar form in the Einstein relativistic velocity model, Aubel's theorem for gyrotriangle is an example in this respect. In the Euclidean limit of large s, $s \to \infty$, gamma factor γ_v reduces to 1, so that the gyroequality (1) reduces to the

$$\frac{|AP|}{|PD|} = \frac{|BC|}{2}\left[\frac{|AE|}{|EC|}\cdot\frac{1}{|BD|} + \frac{|FA|}{|FB|}\cdot\frac{1}{|CD|}\right]$$

in Euclidean geometry. We observe that the previous equality is a equivalent form to the Van Aubel's theorem of euclidian geometry.

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Rossi's Reactors – Reality or Fiction?

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A tabletop prototype of a new kind of nuclear device was demonstrated at the University of Bologna, several months ago. It generated thermal energy at the rate of 12 kW. A set of one hundred of such interconnected devices, able to generate energy at a much higher rate (up to 1000 kW) is said to be now commercially available. The inventor claims that the energy was produced via nuclear fusion of hydrogen and nickel. This note addresses conceptual difficulties associated with such interpretation. Experimental facts reported by the inventor seem to conflict with accepted knowledge. This, however, should not be a justification for the rejection of experimental data. Refutations and confirmations should be based on independently performed experiments.

1 Introduction

An interesting website, describing an ongoing research project, has been created by an Italian engineer Andrea Rossi [1]. He is the inventor of a tabletop device in which powdered nickel, mixed with common hydrogen, reported to generate thermal energy at the rate of 12 kW, for six months. A large percentage of nickel was said to be converted into copper, during that time. The device was recently demonstrated at the University of Bologna. The most obvious questions, raised by the reported features of the reactor are:

1. What lowers the coulomb barrier, between the atomic nuclei of hydrogen and nickel?
2. Is the reported accumulation of copper consistent with the well known half-lives of radioactive copper byproducts?
3. Is the measurable isotopic composition of nickel, in spent fuel, consistent with the amount of released energy?
4. The radiation level, outside the operating 12 kW reactor, was said to be comparable to that due to cosmic rays. Spent fuel, removed from the reactor, one hour after the shutdown, was found to be not radioactive [1]. How can these purported facts be explained?

Results from earlier experiments (2008 and 2009) are described in [2]. In one case the device was used to heat a "small factory" (probably two or three rooms) for one year.

2 Reported 2011 results

One demonstration of the device – January 14, 2011, at the University of Bologna – is described in [3–5]. Subsequent experiments – February 10, and March 29, 2011 – are described in [6–8]. In both cases the apparatus consisted of a cylinder containing nickel. Pure hydrogen was forced to flow through the hot nickel powder. The amount of powder was 100 grams [8, 9], or slightly more than one cubic inch, depending on the level of compression. Reactions between nickel and hydrogen turned out to be extremely exothermic, generating thermal energy at the rate of about 12.4 kW. This was 31 times higher than the rate at which electric energy was supplied, to operate the equipment [4].

In the February experiment the amount of thermal energy was determined from the flow rate of cooling water, and the difference between its input and output temperature. In the January experiment the water flow rate was slower; the entering water was a liquid, the escaping water was a vapor. The amount of thermal energy released was determined from the amount of liquid water (initially at 15 ^{o}C) transformed into 101 ^{o}C vapor. Rossi claims that most heat is produced from nuclear reactions:

$$p + Ni \rightarrow Cu,$$

where p is nothing but ionized hydrogen. This is very surprising because the temperature of hydrogen was below the melting point of nickel. Addressing this issue in [10] Rossi reported that about 30% of nickel was turned into copper, after six months of uninterrupted operation. A schematc diagram of the reactor, and additional details are in [11, 12].

Comment 1
Many physicists have studied fusion of protons with nickel nuclei. But their protons had much higher energies, such as 14.3 MeV [13]. Rossi's protons, by contrast, had very low energies, close to 0.04 eV. The probability of nuclear fusion, expressed in terms of measurable cross sections, is known to decrease rapidly when the energy is lowered. How can 0.04 eV protons fuse with nickel, whose atomic number is 28? Rossi is convinced that this is due a catalyst added to the powdered nickel. The nature of the catalyst has not been disclosed. This prevents attempts to replicate the experiments, or to discuss the topic theoretically. Secrecy might make sense in some business situations, but it is not consistent with scientific methodology.

Comment 2
How can 30% of nickel in Rossi's reactor be transmuted into

copper? This seems to be impossible, even if the coulomb barrier is somehow reduced to zero by his catalyst. To justify this let us focus on the ^{58}Ni and ^{60}Ni isotopes–they constitute 94.1% of the nickel initially loaded into the device. The reactions, by which copper is produced, from these isotopes, would be:

$$p + \ ^{58}Ni \rightarrow \ ^{59}Cu \quad \text{(half-life is 3.2 s)} \qquad \text{(A)}$$

and

$$p + \ ^{60}Ni \rightarrow \ ^{61}Cu \quad \text{(half-life is 3.3 h)} \qquad \text{(B)}$$

The reported amount of accumulated copper – 30% of the initial nickel being turned into copper, after six months of operation–would indeed be possible, via reactions (A) and (B), if the produced copper isotopes were stable, or had half-lives much longer than six months. But this is not the case, as shown above. The produced copper isotopes, ^{59}Cu and ^{61}Cu, rapidly decay into ^{59}Ni and ^{61}Ni. Each reaction, in other words, would lead to accumulation of these isotopes of nickel, not to accumulation of copper, as reported by Rossi. The accumulation of copper would practically stop after several half-lives. Note that ^{63}Cu and ^{65}Cu, if produced from fusion of protons with ^{62}Ni and ^{64}Ni, would be stable. But natural abundance of these isotopes of nickel, 3.63% and 0.92%, respectively, is too low to be consistent with the claimed accumulation of 30% of copper.

Comment 3

How much of the original ^{58}Ni should be destroyed, after six months of continuous operation, in order to generate thermal energy at the rate of 12 kW? Let us again assume that Coulomb barriers are somehow reduced to zero by Rossi's secret catalyst. The ^{58}Ni is 68% of the total. On that basis one can assume that 68% of 12 kW is due to the radioactive decay of ^{59}Cu, and its radioactive daughter, ^{59}Ni. Thus $P_1' = 0.68 \times 12 = 8.16$ kW. This is the thermal power. The nuclear power P_1 must be larger, because neutrinos and some gamma rays do escape from the vessel. As a rough estimate, assume that the nuclear power is

$$P_1 = 16 \text{ kW} = 16{,}000 \text{ J/s} = 10^{17} \text{ MeV/s}.$$

The excited ^{59}Cu, from the reaction (A), releases 3.8 MeV of energy, as one can verify using a table of known atomic masses. In the same way one can verify that the energy released from its radioactive daughter, ^{59}Ni, is 4.8 MeV. In other words, each transformation of ^{58}Ni into ^{60}Ni releases 3.8 + 4.8 = 8.6 MeV of nuclear energy.

The number of reactions (A) should thus be equal to $10^{17}/8.6 = 1.16 \times 10^{16}$ per second. Multiplying this result by the number of seconds in six months (1.55×10^7) one finds that the total number of destroyed ^{58}Ni nuclei is 1.80×10^{23}, or 17.4 grams. A similar estimate can be made for other initially present nickel isotopes. The overall conclusion is that the isotopic composition of nickel, after six months of operation, at the 12 kW level, would change drastically, if the reaction A were responsible for the heat produced in the reactor invented by Rossi.

The amount of ^{59}Ni, for example, would increase from 0% (natural abundance) to 17.4%. The amount of ^{58}Ni, on the other hand, would be reduced from 68% (natural abundance) to 50.6%. The isotopic composition of nickel in spent fuel was measured, according to [1], but results remain "privileged information".

Comment 4

The level of radioactivity, next to the reactor generating heat at the rate of 12 kW, was reported as not much higher than the natural background [5]. Is this consistent with reaction (A) being responsible for most of the heat? The answer is negative. How can this be justified? In the steady state the rate at which radioactive atoms, in this case ^{59}Cu, are decaying is the same as the rate at which they are produced. That rate, as shown in Comment 3, is 1.16×10^{16} atoms per second. In other words, the expected activity is

$$1.16 \times 10^{16}/3.7 \times 10^{10} = 313{,}000 \text{ Curies}.$$

The emitted radiation would include gamma rays of 1.3 MeV, able to escape. The level of radiation, next to the reactor, would depend on the wall thickness. It would certainly exceed the background by many orders of magnitude. Absence of excessive gamma radiation might be an indication that the reactions producing heat were different from the p+Ni fusion.

3 Addendum

Note that the reported fuel power density of 120 W/g would be at least ten times higher than in a fuel element of a nuclear reactor based on ^{235}U. What can be more desirable than higher safety and lower cost? Did Rossi really invent a new kind of nuclear reactor? Logical speculations, such as those above, are not sufficient to answer this question. Only independently performed experiments can do this.

Rossi's claims, if confirmed, would present a challenge to theoretical physicists. Physics, unlike mathematics, is based on confirmed experimental facts, not on axioms. Newly discovered facts often lead to improvements of accepted theories. Let's hope that Rossi's incredible results can be independently confirmed in the near future.

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Photon-assisted Spectroscopy of Dirac Electrons in Graphene

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The quantum Goos-Hanchen effect in graphene is investigated. The Goos-Hanchen phase shift is derived by solving the Dirac eigenvalue differential equation. This phase shift varies with the angle of incidence of the quasiparticle Dirac fermions on the barrier. Calculations show that the dependence of the phase shift on the angle of incidence is sensitive to the variation of the energy gap of graphene, the applied magnetic field and the frequency of the electromagnetic waves. The present results show that the conducting states in the sidebands is very effective in the phase shift for frequencies of the applied electromagnetic field. This investigation is very important for the application of graphene in nanoelectronics and nanophotonics.

1 Introduction

In recent years, the interest in novel device structures able to surmount the miniaturization limits imposed by silicon based transistors has led researchers to explore alternative technologies such as those originated in the field of semiconducting quantum dots, nanowire, graphene and carbon nanotubes [1, 2]. Graphene [3, 4] consists of a monolayer of carbon atoms forming a two-dimensional honeycomb lattice.

Graphene has been intensively studied due to its fascinating physical properties and potential applications in the field of nanoelectronics and another different field, for example, biosensor, hydrogen storage, and so on [5, 6]. In graphene, the energy bands touch the Fermi energy at six discrete points at the edges of the hexagonal Brillouin zone. Out of these six Fermi points, only two are inequivalent, they are commonly referred to as K and K´ points [7]. The quasiparticle excitation about K & K´ points obey linear Dirac like energy dispersion [8]. The presence of such Dirac like quasiparticle is expected to lead to a number of unusual electronic properties in graphene including relativistic quantum Hall effect [9], quasi-relativistic Klein tunneling [10, 11] and the lateral shift of these Dirac quasi-particles in graphene, which is known as Goos-Hanchen effect, Bragg reflector and wave guides [12–15]. The present paper is devoted to investigate the quantum Goos-Hanchen effect in graphene, taking into consideration the effect of electromagnetic waves of wide range of frequencies and magnetic field.

2 The Model

The transport of quasiparticle Dirac Fermions in monolayer graphene through a barrier of height, V_b, and width, d, is described by the following Dirac Hamiltonian, H_o, which is given as [4, 16]:

$$H_o = -i\hbar v_f \sigma \nabla + V_b, \tag{1}$$

where v_f is the Fermi velocity and $\sigma = (\sigma_x, \sigma_y)$ are the Pauli matrices. Since the graphene is connected to two leads and applying a top gate with gate voltage, V_g. Also, the transport of quasiparticle Dirac fermions are influenced by applying both magnetic field, B, and an electromagnetic field of amplitude, V_{ac}, and of wide range of frequencies, ω. So, accordingly Eq. (1) can be rewritten as follows:

$$H = -i\hbar v_f \sigma \nabla + V_b + eV_{sd} + eV_g + eV_{ac}\cos(\omega t) + \frac{\hbar eB}{2m^*}, \tag{2}$$

where V_{sd} is the bias voltage, $\hbar$ is reduced Planck's constant and m^* is the effective mass of quasiparticle Dirac fermions. Now, due to transmission of these quasi-particles Dirac fermions, a transition from central band to side-bands at energies [11, 17] $E \pm n\hbar\omega$, where n is an integer with values $0, \pm 1, \pm 2, \ldots$. The Dirac fermions Hamiltonian, H, (Eq. 2) operates in space of the two-component eigenfunction, Ψ, where Dirac eigenvalue differential equation is given by [11]:

$$H\Psi(r) = E\Psi(r), \tag{3}$$

where E is the scattered energy of quasi-particle Dirac fermions. The solution of Eq. (3) gives the following eigenfunctions [11,18]. The eigenfunction of incident quasi-particle Dirac fermions is

$$\Psi_{in}(r) = \sum_{n=1}^{\infty} J_n\left(\frac{eV_{ac}}{\hbar\omega}\right)[A + B], \tag{4}$$

where

$$A = \begin{pmatrix} 1 \\ se^{i\varphi} \end{pmatrix} \exp\left(i(k_x x + k_y y)\right),$$

$$B = r\begin{pmatrix} 1 \\ -se^{-i\varphi} \end{pmatrix} \exp\left(i(-k_x x + k_y y)\right),$$

J_n is the n^{th} order of Bessel function of first kind and the eigenfunction for the transmitted quasiparticle Dirac fermions through the barrier is given by:

$$\Psi_{tr}(r) = \sum_{n=1}^{\infty} J_n\left(\frac{eV_{ac}}{\hbar\omega}\right) t\begin{pmatrix} 1 \\ se^{i\varphi} \end{pmatrix} \exp\left(i(k_x x + k_y y)\right) \tag{5}$$

In Eqs. (4, 5), r and t are the reflection and transmission amplitude respectively and S = Sgn(E) is the signum function of E. The components of the wave vectors k_x and k_y outside the barrier are expressed in terms of the angle of incidence, φ, of the quasiparticles Dirac fermions as:

$$k_x = k_f \cos\varphi, \quad k_y = k_f \sin\varphi, \tag{6}$$

where k_f is the Fermi wave vector. The eigenfunction Ψ_b inside the region of the barrier is given by:

$$\Psi_b(r) = \sum_{n=1}^{\infty} J_n\left(\frac{eV_{ac}}{\hbar\omega}\right)[C + D], \tag{7}$$

where

$$C = \begin{pmatrix} \alpha \\ s'\beta e^{i\theta} \end{pmatrix} \exp\left(i(q_x x + k_y y)\right),$$

$$D = r\begin{pmatrix} \alpha \\ -s'\beta e^{-i\theta} \end{pmatrix} \exp\left(i(-q_x x + k_y y)\right),$$

$$q_x = (k_f'^2 - k_y^2)^{\frac{1}{2}}, \tag{8a}$$

and

$$\theta = \tan^{-1}\left(\frac{k_y}{q_x}\right) \tag{8b}$$

in which

$$k_f' = \frac{\sqrt{(V_b - \varepsilon)^2 - \dfrac{\varepsilon_g^2}{2}}}{\hbar v_f}, \tag{9}$$

where ε_g is the energy gap and ε is expressed as

$$\varepsilon = E - eV_g - n\hbar\omega - eV_{sd} - V_b + \frac{\hbar eB}{2m^*} \tag{10}$$

In Eq. (7), the parameters s', α, and β are given by:

$$s' = sgn\,(E - V_b) \tag{11}$$

$$\alpha = \sqrt{1 + \frac{\dfrac{s'\varepsilon_g}{2\hbar v_f}}{\sqrt{k_f'^2 + \dfrac{\varepsilon_g^2}{4(\hbar v_f)^2}}}} \tag{12}$$

This parameter, α, corresponds to K-point. Also, β is given by

$$\beta = \sqrt{1 - \frac{\dfrac{s'\varepsilon_g}{2\hbar v_f}}{\sqrt{k_f'^2 + \dfrac{\varepsilon_g^2}{4(\hbar v_f)^2}}}} \tag{13}$$

This parameter, β, corresponds to K'-point. Now, in order to find an expression for both the transmission coefficient, t, (Eq. 5) and the corresponding Goos-Hanchen phase shift, Φ, this is done by applying the boundary conditions at the boundaries of the barrier [11, 18]. This gives the transmission coefficient, t, as:

$$t = \sum_{n=1}^{\infty} J_n\left(\frac{eV_{ac}}{\hbar\omega}\right) \times \left[\frac{1}{\cos(q_x d) - F}\right], \tag{14}$$

where

$$F = i(s' s\gamma \sec(\varphi)\sec(\theta) + \tan(\varphi)\tan(\theta))\sin(q_x d)$$

and γ is expressed as:

$$\gamma = \frac{\sqrt{\dfrac{\varepsilon_g^2}{4(\hbar v_f)^2} + k_f'^2}}{k_f'} \tag{15}$$

The transmission coefficient, t, is related to the Goos-Hanchen phase shift, Φ, [12, 18] as:

$$t = \frac{e^{i\phi}}{f}, \tag{16}$$

where f is the Gaussian envelop of the shifted wave of quasiparticle Dirac fermions [12, 18, 19]. So, the expression for the phase shift is given by:

$$\Phi = \tan^{-1}\left[\frac{\sin(\theta)\sin(\varphi) + ss'\gamma}{\cos(\theta)\cos(\varphi)}\tan(q_x d)\right], \tag{17}$$

where d is the width of the barrier. We notice that the phase shift, Φ (Eq. 17) depends on the angle of incidence, ϕ of the quasiparticle Dirac fermion and on the barrier of height, V_b, and its width, d, and other parameters considered, for example, the energy gap, ε_g, the magnetic field, B, gate voltage, V_g, and the external pulsed photons of wide range of frequencies.

3 Results and Discussion

Numerical calculations are performed for phase shift, Φ, (Eq. 17) as shown below. For monolayer graphene, the values of both barrier height, V_b, and its width are respectively V_b = 120 meV and d = 80 nm [16, 18, 19]. Also, the value of the Fermi-velocity, v_f is approximately 10^6 m/s, and the effective mass of quasiparticle Dirac fermions is approximately m^* = 0.054 me [16, 18, 19]. The engineering of band gap of graphene generates a pathway for possible graphene-based nanoelectronics and nanophotonics devices. It is possible to open and tune the band gap of graphene by applying electric field [20] or by doping [21]. So, in our calculations we take the value of the energy gap of graphene to be ε_g = 0 eV, 0.03 eV, 0.05 eV [22].

The features of our results are the following:

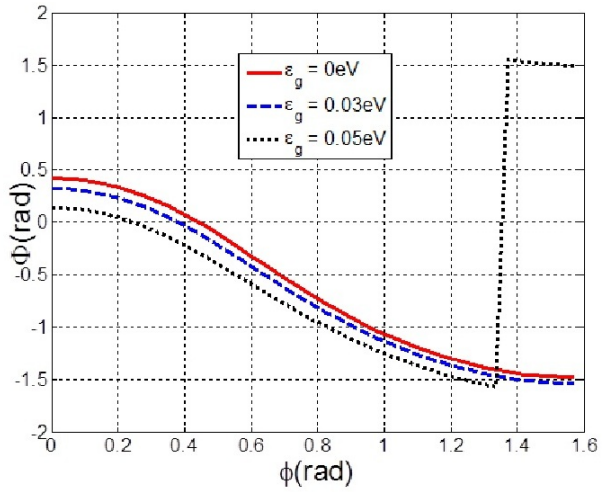

Fig. 1: The variation of Goos-Hanchen phase shift, Φ, with angle of incidence, ϕ, at different values of energy gap.

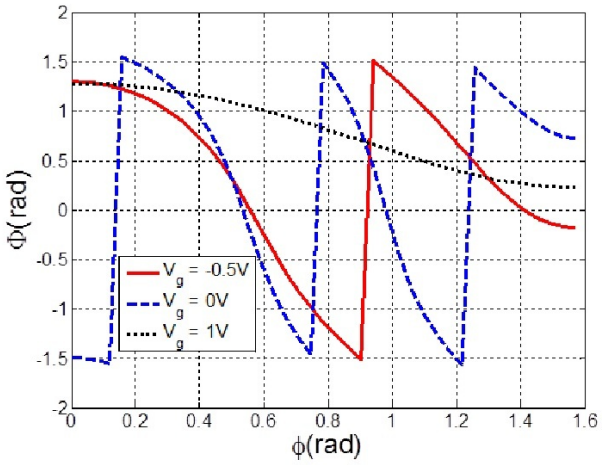

Fig. 2: The variation of Goos- Hanchen phase shift, Φ, with angle of incidence, ϕ, at different values of gate voltage.

1. Fig. 1, shows the dependence of the Goos-Hanchen phase shift, Φ, on the angle of incidence ϕ at different values of energy gap, ε_g. As shown from the figure that the phase shift, Φ, decreases as the angle of incidence, ϕ, increase for the considered values of the energy gap, ε_g. It must be noticed that for ε_g = 0.05eV, for angle of incidence $\phi \approx$1.335 rad, the phase shift, Φ , increases from -1.571 rad to 1.549 rad and then slightly decreases. This result shows the strong dependence of Goos-Hanchen phase shift on the engineered band gap of graphene [18, 23]. This result shows that the phase shift, Φ , can be enhanced by certain energy gap at the Dirac points.
2. Fig. 2 shows the dependence of the phase shift, Φ, on the angle of incidence, ϕ , at different values of the gate voltage, V_g. As shown from the figure that for large values of gate voltage, V_g, for example, V_g = 1V, the phase shift, Φ , decreases as the angle of incidence, ϕ, increase and phase shift takes only positive values. While for values of V_g = 0V or V_g = -0.5V, the value of phase shift oscillates between negative and positive values. It is well known that the tunneling of quasiparticle Dirac fermions could be controlled by changing the barrier height, V_b, this could be easily implemented by applying a gate voltage, V_g, to graphene [11,24–26].
3. Fig. 3 shows the dependence of the phase shift, Φ, on the angle of incidence, ϕ ,at different values of magnetic field, B. As shown from the figure that for B = 0.5 T, the phase shift decreases gradually as the angle of incidence, ϕ , increases to value Φ = 1.335 rad and then increases to Φ = 1.549 rad at ϕ = 1.374 rad and very slightly decreases. While for values B = 5 T and 10 T the value of the phase shift, Φ, is negative and decreases up to *Phi* = -1.561 rad when ϕ = 0.8635rad (when B = 5T) and then increases to Φ = 1.529 rad when ϕ = 0.902 rad and then decreases as the angle of incidence increases. For B = 10 T, the value of phase shift is negative and decreases as the value of ϕ increases up to ϕ = 0.432 rad and increases up to Φ = 1.547 rad and ϕ = 0.471 rad and decreases as the angle of incidence increases. This result shows that how a magnetic field modifies the transport of quasiparticle Dirac fermions in graphene with certain barrier height and certain energy gap [26].
4. Fig. 4 shows the variation of the phase shift, Φ, at different values of frequencies, ν , of the pulsed electromagnetic field. As shown from the figure, for higher frequencies 400 THz, 800 THz and 1000 THz, the value of the phase shift, Φ, decreases as the angle of incidence increases. We notice that in this range of frequencies, the value of phase shift is negative. While for microwave frequencies, MW = 300 GHz the value of the phase shift, Φ, decreases as the angle of incidence increases up to ϕ = 1.021 rad and then the phase shift increases up to Φ = 1.55 rad and ϕ = 1.06 rad and then decreases gradually.

This result shows that the conducting states in the side bands can be effective in the Goos-Hanchen phase shift for a certain frequency of the applied electromagnetic signal [27]. This result is very important for tailoring graphene for photonic nano-devices.

The present results show that the Goos-Hanchen phase shift can be modulated by both intrinsic parameters, for example, the barrier height, the energy gap and the extrinsic parameters, for example, magnetic field and the induced photons of electromagnetic field. The present research is very important for the applications of graphene in different nanoelectronics and nanophotonic devices.

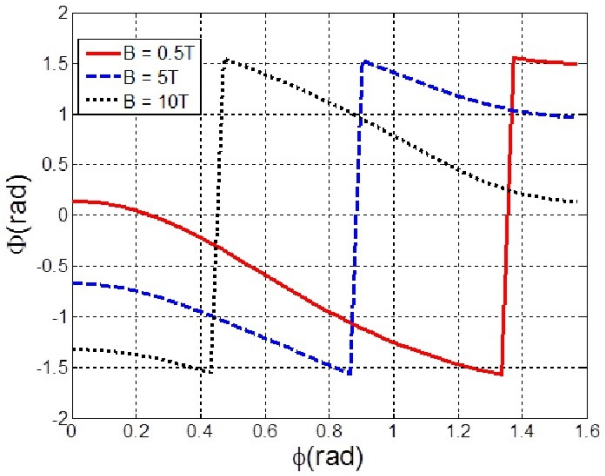

Fig. 3: The variation of Goos-Hanchen phase shift, Φ, with angle of incidence, ϕ, at different values of magnetic field.

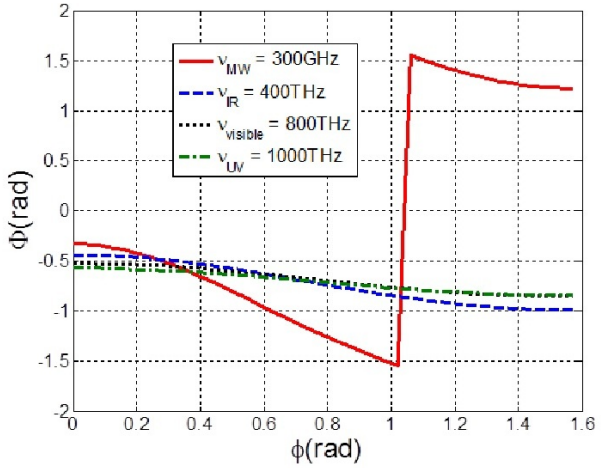

Fig. 4: The variation of Goos-Hanchen phase shift, Φ, with angle of incidence, ϕ, at different values of electromagnetic wave frequencies.

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Coherent Spin Polarization in an AC-Driven Mesoscopic Device

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The spin transport characteristics through a mesoscopic device are investigated under the effect of an AC-field. This device consists of two-diluted magnetic semiconductor (DMS) leads and a nonmagnetic semiconducting quantum dot. The conductance for both spin parallel and antiparallel alignment in the two DMS leads is deduced. The corresponding equations for giant magnetoresistance (GMR) and spin polarization (SP) are also deduced. Calculations show an oscillatory behavior of the present studied parameters. These oscillations are due to the coupling of photon energy and spin-up & spin-down subbands and also due to Fano-resonance. This research work is very important for spintronic devices.

1 Introduction

The field of semiconductor spintronics has attracted a great deal of attention during the past decade because of its potential applications in new generations of nanoelectronic devices, lasers, and integrated magnetic sensors [1, 2]. In addition, magnetic resonant tunneling diodes (RTDs) can also help us to more deeply understand the role of spin degree of freedom of the tunneling electron and the quantum size effects on spin transport processes [3–5]. By employing such a magnetic RTD, an effective injection of spin-polarized electrons into nonmagnetic semiconductors can be demonstrated [6]. A unique combination of magnetic and semiconducting properties makes diluted magnetic semiconductors (DMSs) very attractive for various spintronics applications [7, 8]. The II-VI diluted magnetic semiconductors are known to be good candidates for effective spin injection into a non-magnetic semiconductor because their spin polarization can be easily detected [9, 10]. The authors investigated the spin transport characteristics through mesoscopic devices under the effect of an electromagnetic field of wide range of frequencies [11–14].

The aim of the present paper is to investigate the spin transport characteristics through a mesoscopic device under the effect of both electromagnetic field of different frequencies and magnetic field. This investigated device is made of diluted magnetic semiconductor and semiconducting quantum dot.

2 The Model

The investigated mesoscopic device in the present paper is consisted of a semiconducting quantum dot connected to two diluted magnetic semiconductor leads. The spin-transport of electrons through such device is conducted under the effect of both electromagnetic wave of wide range of frequencies and magnetic effect. It is desired to deduce an expression for spin-polarization and giant magnetoresistance. This is done as follows:

The Hamiltonian, H, describing the spin transport of electrons through such device can be written as:

$$\begin{aligned} H &= -\frac{\hbar^2}{2m^*}\frac{d^2}{dx^2} + eV_{sd} + eV_g + E_F + V_b \\ &\quad + eV_{ac}\cos(\omega t) \pm \frac{1}{2}g\mu_B\sigma B + \frac{N^2e^2}{2C} \pm \sigma h_o, \end{aligned} \tag{1}$$

where m^* is the effective mass of electron, $\hbar$ is the reduced Planck's constant, V_{sd} is the source-drain voltage (bias voltage), V_g is the gate voltage, E_F is the Fermi-energy, V_b is the barrier height at the interface between the leads and the quantum dot, V_{ac} is the amplitude of the applied AC-field with frequency ω, g is the Landé factor of the diluted magnetic semiconductor, μ_B is Bohr magneton, B is the applied magnetic field, σ-Pauli matrices of spin, and h_o is the exchange field of the diluted magnetic semiconductor. In eq. (1), the term $(N^2e^2/2C)$ represents the Coulomb charging energy of the quantum dot in which e is the electron charge, N is the number of electrons tunneled through the quantum dot, and C is the capacitance of the quantum dot. So, the corresponding Schrödinger equation for such transport is

$$H\psi = E\psi, \tag{2}$$

with the solution for the eigenfunction, $\psi(x)$, in the corresponding regions of the device can be expressed as [15]:

$$\psi(x) = \begin{cases} \left[A_1 e^{ik_1 x} + B_1 e^{-ik_1 x}\right] J_n\left(\frac{eV_{ac}}{\hbar\omega}\right) e^{-in\omega t}, x < 0 \\ \left[A_2 Ai(\rho(x)) + B_2 Bi(\rho(x))\right] J_n\left(\frac{eV_{ac}}{\hbar\omega}\right) \\ \times e^{-in\omega t}, 0 < x < d \\ A_3 e^{ik_2 x} J_n\left(\frac{eV_{ac}}{\hbar\omega}\right) e^{-in\omega t}, x > d \end{cases} \tag{3}$$

where $Ai(\rho(x))$ is the Airy function and its complement is $Bi(\rho(x))$ [16]. In eqs. (3), the parameter $J_n(eVac/\hbar\omega)$ represents the n^{th} order Bessel function of the first kind. The

solutions of eqs. (3) must be generated by the presence of the different side-bands "n" which come with phase factor $e^{-in\omega t}$ [11–14], and d represents the diameter of the quantum dot. Also, the parameters k_1, k_2 and $\rho(x)$ in eqs. (3) are:

$$k_1 = \sqrt{\frac{2m^*}{\hbar^2}\left(E + n\hbar\omega + V_b + \sigma h_o\right)}, \tag{4}$$

n = 0, ± 1, ± 2, ± 3 ...

$$k_2 = \sqrt{\frac{2m^*}{\hbar^2}(V_b + eV_{sd} + eV_g + E_F + \frac{N^2e^2}{2C} + n\hbar\omega \pm \frac{1}{2}g\mu_B B\sigma \pm \sigma h_o)} \tag{5}$$

and

$$\rho(x) = \frac{d}{eV_{sd}\Phi}\left(E_F + V_b + eV_{sd}\left(\frac{x}{d}\right) + eV_g + \frac{N^2e^2}{2C} + \frac{1}{2}g\mu_B B\sigma + E\right) \tag{6}$$

in which Φ is given by

$$\Phi = \sqrt[3]{\frac{\hbar^2 d}{2m^* eV_{sd}}}. \tag{7}$$

Now, the tunneling probability, $\Gamma(E)$, could be obtained by applying the boundary conditions to the eigenfunctions (eq. (3)) and their derivative at the boundaries of the junction [11–14]. We get the following expression for the tunneling probability, $\Gamma(E)$, which is:

$$\Gamma(E) = \sum_{n=1}^{\infty} J_n^2\left(\frac{eV_{ac}}{\hbar\omega}\right)\cdot\left\{\frac{4k_1k_2}{\pi^2\Phi^2}\left[\alpha^2k_1^2k_2^2 + \beta^2{m^*}^2k_1^2\right]^{-1}\right\}, \tag{8}$$

where α and β are given by:

$$\alpha = Ai(\rho(0))\cdot Bi(\rho(d)) - Bi(\rho(0))\cdot Ai(\rho(d)), \tag{9}$$

and

$$\beta = \frac{1}{\Phi m^*}\left[Ai(\rho(0))\cdot Bi'(\rho(d)) - Bi(\rho(0))\cdot Ai'(\rho(d))\right], \tag{10}$$

where $Ai'(\rho(x))$ is the first derivative of the Airy function and $Bi'(\rho(x))$ is the first derivative of its complement. Now, the conductance, G, of the present device is expressed in terms of the tunneling probability, $\Gamma(E)$, through the following equation as [11–14, 17]:

$$G = \frac{2e^2}{h}\sin(\varphi)\int_{E_F}^{E_F+n\hbar\omega} dE\left(-\frac{\partial f_{FD}}{\partial E}\right)\cdot\Gamma(E), \tag{11}$$

where ϕ is the phase of the scattered electrons and the factor $(-\partial f_{FD}/\partial E)$ is the first derivative of the Fermi-Dirac distribution function and it is given by:

$$\left(-\frac{\partial f_{FD}}{\partial E}\right) = (4k_BT)^{-1}\cosh^{-2}\left(\frac{E - E_F + n\hbar\omega}{2k_BT}\right), \tag{12}$$

where k_B is the Boltzmann constant and T is the absolute temperature. The spin polarization, SP, and giant magnetoresistance, GMR, are expressed in terms of the conductance, G, as follows [18]:

$$Sp = \frac{G_{\uparrow\uparrow} - G_{\uparrow\downarrow}}{G_{\uparrow\uparrow} + G_{\uparrow\downarrow}}, \tag{13}$$

and

$$GMR = \frac{G_{\uparrow\uparrow} - G_{\uparrow\downarrow}}{G_{\uparrow\uparrow}}, \tag{14}$$

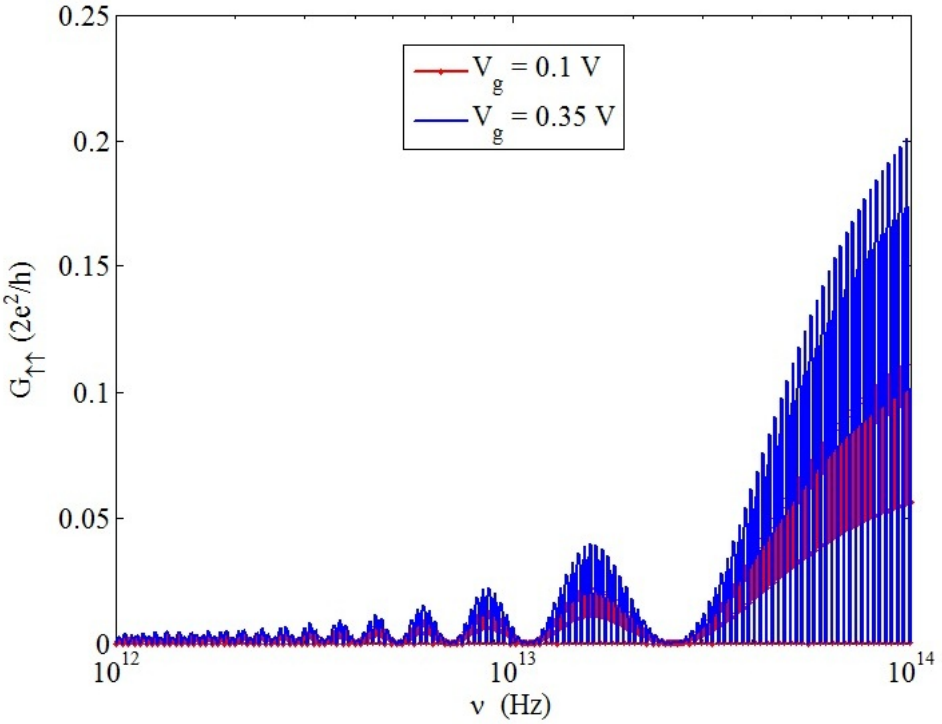
where $G_{\uparrow\uparrow}$ is the conductance when the magnetization of the two diluted magnetic-semiconductor leads are in parallel alignments, while $G_{\uparrow\downarrow}$ is the conductance for the case of antiparallel alignment of the magnetization in the leads. The indicator ↑ corresponds to spin up and also ↓ corresponds to spin down.

3 Results and Discussion

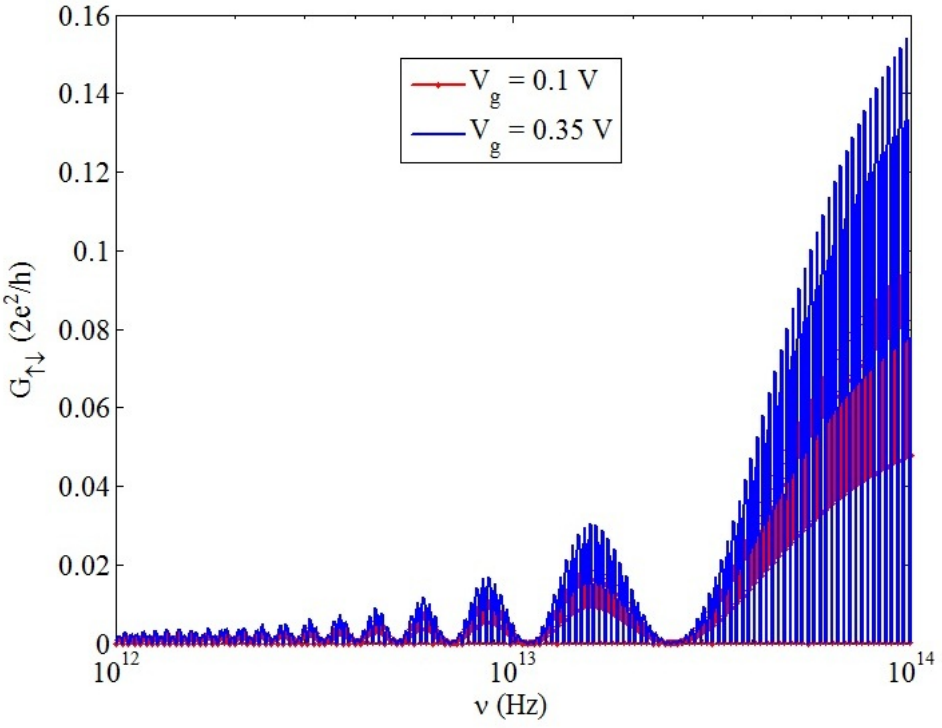
Numerical calculations to eqs. (11, 13 and 14), taking into consideration the two cases for parallel and antiparallel spins of quasiparticles in the two leads. In the present calculations, we take the case of quantum dot as GaAs and the two leads as diluted magnetic semiconductors GaMnAs. The values for the quantum dot are [11–14, 19–21]: E_F = 0.75 eV, C = 10^{-16} F and d = 2 nm, V_b = 0.3 eV. The value of the exchange field, h_o, for GaMnAs is -1 eV and g = 2 [18–22].

The features of the present results:

1. Figs. 1a, 1b show the variation of the conductance with the induced photon of the frequency range $10^{12} - 10^{14}$ Hz. The range of frequency is in the infra-red range at different values of gate voltage, V_g. Fig. 1a is for the case of the parallel alignment of spin in the two diluted magnetic semiconductor leads, while Fig. 1b for antiparallel case. As shown from these figures that an oscillatory behavior of the conductance with the frequency for the two cases. It must be noted the peak height of the conductance (for the two cases) increases as the frequency of the induced photons. Also, the trend of the dependence is a Lorentzian shape for each range of frequencies. These results are due to photon-spin-up and spin-down subbands coupling. This coupling will be enhanced as the frequency of the induced photon increases.
2. Fig. 2a shows the variation of the giant magnetoresistance, GMR, with the frequency of the induced photon at different values of gate voltage, V_g. As shown from the figure, random oscillations of GMR with random peak heights. GMR attains a maximum value ∼ 30%

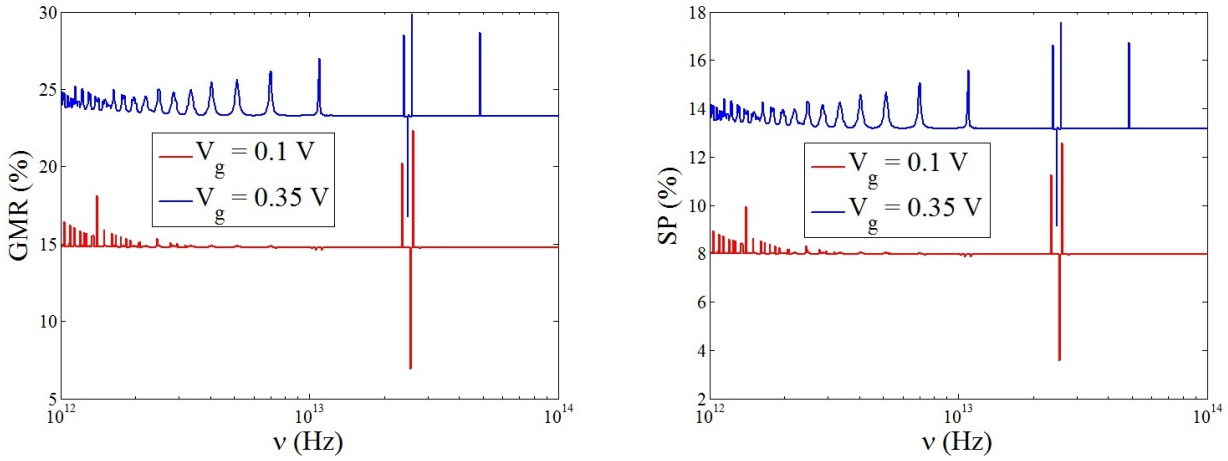


(a)



(b)

Fig. 1: The variation of conductance with frequency at two different gate voltages for (a) parallel spin alignment and (b) antiparallel spin alignment.



(a) (b)

Fig. 2: The variation of (a) GMR and (b) SP with frequency at two different gate voltages.

at $\nu = 2.585\times10^{13}$ Hz ($V_g = 0.35$ V) and GMR attains a maximum value ~ 22% at $\nu = 2.615\times 10^{13}$ Hz ($V_g = 0.1$ V).

3. Fig. 2b shows the variation of the spin polarization, SP, with the frequency of the induced photon at different values of gate voltage, V_g. As shown from the figure, random oscillations of spin polarization with random peak heights. SP attains a maximum value ~ 17.6% at $\nu = 2.585\times10^{13}$ Hz ($V_g = 0.35$ V), and also SP attains a maximum value ~ 12.6% at $\nu = 2.615\times10^{13}$ Hz ($V_g = 0.1$ V).

These random oscillations for both GMR & SP might be due to spin precession and spin flip of quasiparticles which are influenced strongly as the coupling between the photon energy and spin-up & spin-down subbands in quantum dot.

Also, these results show that the position and line shape of the resonance are very sensitive to the spin relaxation rate of the tunneled quasiparticles [23,24] through the whole junction.

In general, the oscillatory behavior of the investigated physical quantities might be due to Fano-resonance as the spin transport is performed from continuum states of dilute magnetic semiconductor leads to the discrete states of non-magnetic semiconducting quantum dots [14, 25].

So, our analysis of the spin polarization and giant magnetoresistance can be potentially useful to achieve a coherent spintronic device by optimally adjusting the material parameters. The present research is practically very useful in digital storage and magneto-optic sensor technology.

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The Upper Limit of the Periodic Table of Elements Points out to the "Long" Version of the Table, Instead of the "Short" One

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Herein we present an analysis of the internal constitution of the "short" and "long" forms of the Periodic Table of Elements. As a result, we conclude that the second (long) version is more correct. We also suggest a long version of the Table consisting of 8 periods and 18 groups, with the last (heaviest) element being element No. 155, which closes the Table.

1 Introduction

Many research papers have been written about the discovery of the Periodic Law of Elements. Many spectacular versions of this law have likewise been suggested. However the main representation of this law is still now a two-dimensional table consisting of cells (each single cell manifests a single element). The cells are joined into periods along the horizontal axis (each row represents a single period), while the cells are joined into groups along the vertical axis (each column represents a single group). The resulting system is represented in three different forms: the "short version" (short-period version); the "long version" (long-period version); and the "super-long version" (extended version), wherein each single period occupies a whole row.

Our task in this paper is the consideration of the first two versions of the Periodic System.

There are hundreds of papers discussing the different versions of the Periodic Table, most of whom have been suggested by Mark R. Leach [1].

To avoid any form of misunderstanding of the terminology, we should keep in mind that, in each individual case, the Periodic Law sets up the fundamental dependence between the numerical value of the atomic nucleus and the properties of the element, while the Periodic System shows how we should use this law in particular conditions. The Periodic Table is a graphical manifestation of this system.

On March 1, 1869, Dmitri Mendeleev suggested the first "long" version of his Table of Elements. Later, in December of 1970, he published another, "short" version of the Table. His theory was based on atomic masses of the elements. Therefore, he formulated the Periodic Law as follows:

"*Properties of plain bodies, and also forms and properties of compounds of the elements, have a periodic dependence on the numerical values of the atomic masses of the elements*".

After the internal constitution of each individual atom had been discovered, this formulation was changed to:

"*Properties of plain substances, and also forms and properties of compounds of the elements, have a periodic dependence from the numerical value of the electric charge of the respective nucleus*".

All elements in the Periodic Table have been numbered, beginning with number one. These are the so-called atomic numbers. Further, we will use our data about the upper limit of the Periodic Table [2–4], when continuing both the short and long versions of the Table upto their natural end, which is manifested by element No. 155.

2 The short version of the Periodic Table

2.1 The Periods

The Periodic System of Elements is presented with the Periodic Table (see Table 1), wherein the horizontal rows are known as Periods. The first three Periods are referred to as "short ones", while the last five — "long ones". The elements are distributed in the Periods as follows: Period 1 — by 2 elements, Periods 2 and 3 — by 8 elements in each, Periods 4 and 5 — by 18 elements in each, Periods 6 and 7 — by 32 elements in each, Period 8 — by 37 elements. Herein we mean that Period 7 is full upto its end, while Period 8 has been introduced according to our calculation. Each single Period (except for Hydrogen) starts with an alkaline metal and then ends with a noble gas. In Periods 6 and 7, within the numbers 58–71 and 90–103, families of Lanthanoids and Actinides are located, respectively. They are placed on the bottom of the Table, and are marked by stars. Chemical properties of Lanthanides are similar to each other: for instance, they all are "reaction-possible" metals — they react with water, while producing Hydroxide and Hydrogen. Proceeding from this fact we conclude that Lanthanides have a very manifested horizontal analogy in the Table. Actinides, in their compounds, manifest more different orders of oxidation. For instance, Actinium has the oxidation order +3, while Uranium — only +3, +4, +5, and +6. Experimentally studying chemical properties of Actinides is a very complicate task due to very high instability of their nuclei. Elements of the same Period have very close numerical values of their atomic masses, but different physical and chemical properties. With these, and depending on the length of the particular Period — each small one consists of one row, while each long one consists of two rows (the upper even row, and the lower odd row), — the rate of change of the properties is smoother and slower

in the second case. In the even rows of the long Periods (the rows 4, 6, 8, and 10 of the Table), only metals are located. In the odd rows of the long Periods (these are the rows 5, 7, and 9), properties of the elements change from left to right in the same row as well as those of the typical elements of the Table.

The main sign according to which the elements of the long Periods are split into two rows is their oxidation order: the same numerical values of it are repeated in the same Period with increase of atomic mass of the elements. For instance, in Period 4, the oxidation order of the elements from K to Mn changes from +1 to +7, then a triad of Fe, Co, Ni follows (they are elements of an odd row), after whom the same increase of the oxidation order is observed in the elements from Cu to Br (these are elements of an odd row). Such distribution of the elements is also repeated in the other long Periods. Forms of compounds of the elements are twice repeated in them as well. As is known, the number of each single Period of the Table is determined by the number of electronic shells (energetic levels) of the elements. The energetic levels are then split into sub-levels, which differ from each one by the coupling energy with the nucleus. According to the modern reference data, the number of the sub-levels is n, but not bigger than 4. However, if taking Seaborg's suggestion about two additional Periods of 50 elements in each into account, then the ultimate high number of the electrons at an energetic level, according to the formula $N = 2n^2$, should be 50 (under $n = 5$). Hence, the quantum mechanical calculations require correction.

2.2 The Groups

The Periodic Table of Elements contains 8 Groups of the elements. The Groups are numbered by Roman numbers. They are located along the vertical axis of the Table. Number of each single Group is connected with the oxidation order of the elements consisting it (the oxidation number is manifested in the compounds of the elements). As a rule, the positive highest oxidation order of the elements is equal to the number of that Group which covers them. An exception is Fluorine: its oxidation number is -1. Of the elements of Group VIII, the oxidation order +8 is only known for Osmium, Ruthenium, and Xenon. Number of each single Group depends on the number of the valence electrons in the external shell of the atom. However it is equally possible to Hydrogen, due to the possibility of its atom to loose or catch the electron, to be equally located in Group I or Group VII. Rest elements in their Groups are split into the *main* and *auxiliary* sub-groups. Groups I, II, II include the elements of the left side of all Periods, while Groups V, VI, VII — the elements located in the right side. The elements which occupy the middle side of the long Periods are known as the transferring elements. They have properties which differ from the properties of the elements of the short Periods. They are considered, separately, as Groups IVa, Va, VIa, VIII, which include by three elements of each respective long Period Ib, IIb, IIIb, IVb. The main sub-groups consist of the typical elements (the elements of Periods 2 and 3) and those elements of the long Periods which are similar to them according to their chemical properties. The auxiliary sub-groups consist of only metals — the elements f the long Periods. Group VIII differs from the others. Aside for the main sub-group of Helium (noble gases), it contains three shell sub-groups of Fe, Co, and Ni. Chemical properties of the elements of the main and auxilary subgroups differ very much. For instance, in Group VII, the main sub-group consists of non-metals F, Cl, Br, I, At, while the auxiliary subgroup consists of metals Mn, Tc, Re. Thus, the sub-groups join most similar elements of the Table altogether. Properties of the elements in the sub-groups change, respectively: from up to down, the metalic properties strengthen, while the non-metalic properties become weak. It is obvious that the metalic properties are most expressed on Fr then on Cs, while the non-metalic properties are most expressed on F then on O [5].

2.3 Electron configuration of the atoms, and the Periodic Table

The periodic change of the properties of the elements by increase of the ordinal number is explained as the periodic change of their atoms' structure, namely by a number of electrons at their outer energetic levels. Elements are divided into *seven periods* (*eight according to our dates*) in accordance with energetic levels in electron shells. The electron shell of Period 1 contains one energetic level, Period 2 contains two energetic levels, Period 3 — three, Period 4 — 4, and so on. Every Period of the Periodic System of Elements begins with elements whose atoms, each, have one electron at the outer level, and ends with elements whose atoms, each, have at the outer shell 2 (for Period 1) or 8 electrons (for all subsequent Periods). Outer shells of elements (Li, Na, Ka, Rb, Cs); (Be, Mg, Ca, Sr); (F, Cl, Br, I); (He, Ne, Ar, Kr, Xe) have a similar structure. The number of the *main sub-Groups* is determined by the maximal number of elements at the energetic level which equals 8. The number of common elements (elements of auxiliary sub-Groups) is determined by maximal electrons at d-sub-level, and it equals 10 for every large Period (see Table 2).

As far as one of auxiliary sub-Groups of the Periodic Table of Elements contains at once three common elements with similar chemical properties (so called triads Fe-Co-Ni, Ru-Rh-Pd, Os-Ir-Pt), then the number, as of common sub-Groups as main ones, equals 8. The number of Lanthanoids and Actinides placed at the foot of the Periodic Table as independent rows equals the maximum number of electrons at the f-Sublevel in analogy with common elements, i.e. it equals 14.

A Period begins with an element the atom of which contains one s-electron at the outer level: this is hydrogen in Pe-

Group → Period ↓	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
1	1 H																	2 He
2	3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	*	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	**	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Uut	114 Uuq	115 Uup	116 Uuh	117 Uus	118 Uuo
8	119 Uue	120 Ubn	***															
	155																	
Lanthanides *				57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
Actinides **				89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr
g-elements ***				121 Ubu	122 Ubb	123 Ubt	124 Ubq											
f-elements				125	126	127	128	129	130	131	132	133	134	135	136	137	138	139
Ubb-series				140	141	142	143	144	145	146	147	148	149	150	151	152	153	154

Table 1: The standard (long) version of the Periodic Table of Elements.

Period	Row	a I b						aVIIb	a VIII b		
1	1	H 1	a II b	a III b	a IV b	a V b	a VI b	(H)	He 2		
2	2	Li 3	Be 4	B 5	C 6	N 7	O 8	F 9	Ne 10		
3	3	Na 11	Mg 12	Al 13	Si 14	P 15	S 16	Cl 17	Ar 18		
4	4	K 19	Ca 20	Sc 21	Ti 22	V 23	Cr 24	Mn 25	Fe 26	Co 27	Ni 28
	5	Cu 29	Zn 30	Ga 31	Ge 32	As 33	Se 34	Br 35	Kr 36		
5	6	Rb 37	Sr 38	Y 39	Zr 40	Nb 41	Mo 42	Tc 43	Ru 44	Rh 45	Pd 46
	7	Ag 47	Cd 48	In 49	Sn 50	Sb 51	Te 52	I 53	Xe 54		
6	8	Cs 55	Ba 56	La* 57	Hf 72	Ta 73	W 74	Re 75	Os 76	Ir 77	Pt 78
	9	Au 79	Hg 80	Tl 81	Pb 82	Bi 83	Po 84	At 85	Rn 86		
7	10	Fr 87	Ra 88	Ac† 89	Rf 104	Db 105	Sg 106	Bh 107	Hs 108	Mt 109	Ds 110
	11	Rg 111	Uub 112	Uut 113	Uuq 114	Uup 115	Uuh 116	Uus 117	Uuo 118		

Lanthanides (the upper row) and Actinides (the lower row)

Ce 58	Pr 59	Nd 60	Pm 61	Sm 62	Eu 63	Gd 64	Tb 65	Dy 66	Ho 67	Er 68	Tm 69	Yb 70	Lu 71
Th 90	Pa 91	U 92	Np 93	Pu 94	Am 95	Cm 96	Bk 97	Cf 98	Es 99	Fm 100	Md 101	No 102	Lr 103

Period 8

8	12	119 Uue	120 Ubn	121 Ubu	122 Ubb	123 Ubt	124 Ubq	125 Ubp	126 Ubh	127 Ubs	128 Ubo
	13	129 Ube	130 Utn	131 Utu	132 Utb	133 Utt	134 Utq	135 Utp	136 Uth		
	14	137 Uts	138 Uto	139 Ute	140 Uqn	141 Uqu	142 Uqb	143 Uqt	144 Uqq	145 Uqp	146 Uqh
	15	147 Uqs	148 Uqo	149 Uqe	150 Upn	151 Upu	152 Upb	153 Upt	154 Upq		
	16	**155**									

121-124—g-elements
125-138---f-elements
155-Upp (Unpentpentium)—Last element

Table 2: The suggested (short) version of the Periodic Table of Elements, up to No. 155.

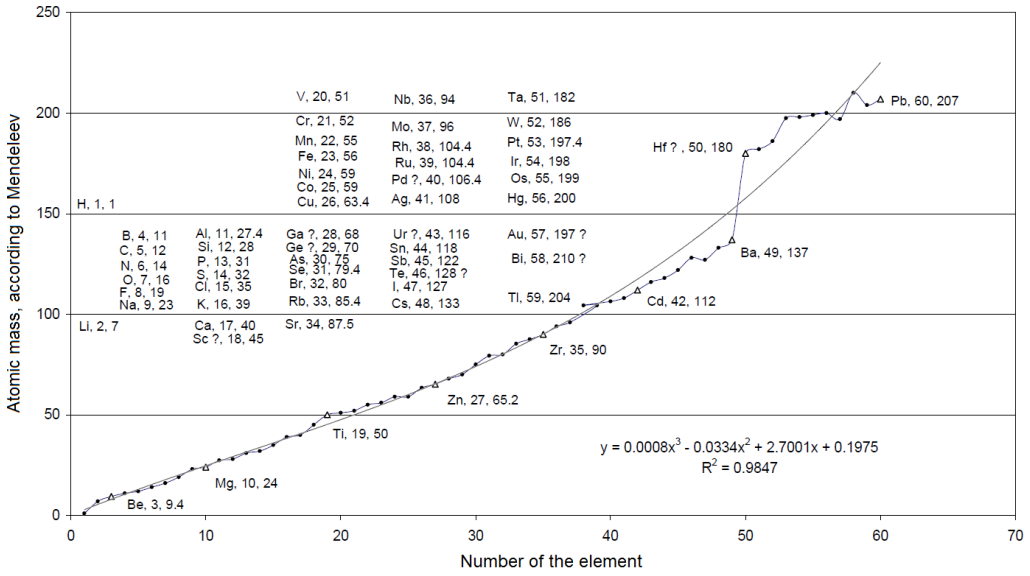

Fig. 1: Experience of the System of the Elements, based on their atomic mass (the table, according to Mendeleev). Dependence of the atomic mass from the number of the elements (the graphs, according to the suggested formulation). The triangles mean the beginning of each Period.

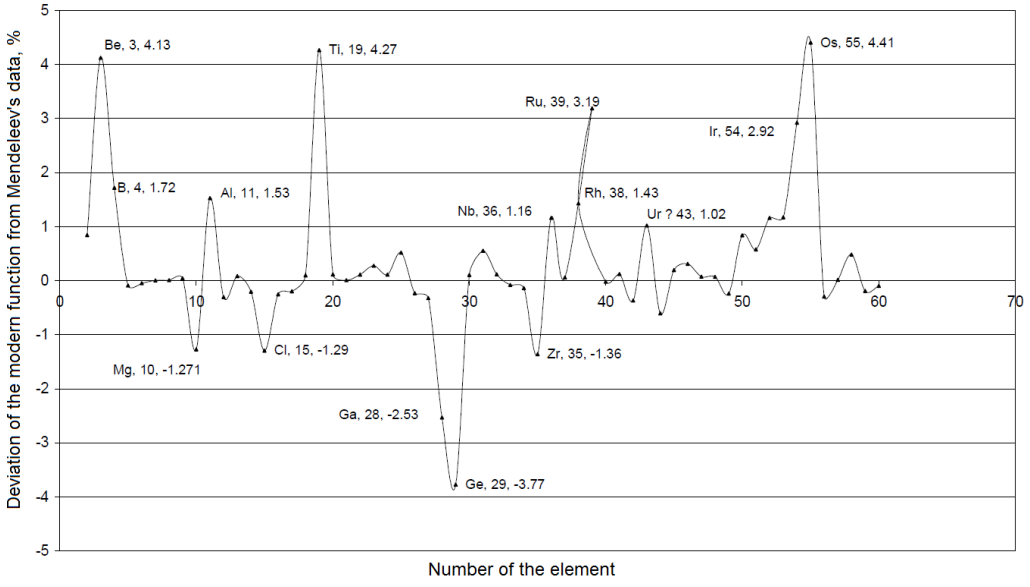

Fig. 2: Deviation of the modern (suggested) dependence of the atomic mass from the number of the elements from Mendeleev's data.

riod 1, and alkaline metals in the others. A Period ends with precious gas: helium ($1s^2$) in Period 1.

Detailed studies of the structure of an atom are not the aim of our paper, therefore we draw common conclusions concerning the corresponding locations of **elements in blocks**:

1. **s-elements**: electrons fill s-sub-shells of the outer level; two first elements of every Period are related to them;
2. **p-elements**: electrons fill p-sub-shells of the outer level; six last elements of every Period are related to them;
3. **d-elements**: electrons fill s-sub-shells of the outer level; they are elements of inserted decades of big Periods placed between s- and p-elements (they are called also common elements);
4. **f-elements**: electrons fill f-sub-shells; they are Lanthanoids and Actinides.

3 Drawbacks of the short version and advantages of the long version of the Periodic Table

The "short" form of the Table was cancelled officially by IUPAC in 1989. But it is still used in Russian information and educational literature, must probably, according to a tradition. But it follows by detailed consideration that it contains some moot points.

In particular, Group VIII contains in the common Group, together with precious gases (the main sub-Group), triads of elements, which have precisely expressed the properties of metals. The contradiction here is that the triad Fe, Co, Ni is near families of platinum metals although their properties differ from the properties of Groups of iron. Group I contains alkaline metals having very strong chemical activity, but simultaneously the sub-Group "b" contains copper, silver and gold which have not these properties but possess excellent electric conductivity. Besides gold, silver and platinum, metals have very weak chemical activity.

Group VII, where nearby halogens such metals as manganese, technezium and renium are placed, is also incorrect, because in the same Group two sub-Groups of elements possessing absolutely different properties are collected.

The "short" Table is sufficiently informative but it is difficult in terms of use due to the presence of the "long" and the "short" Groups, i.e. the small and big Periods divided by even and odd lines. It is very difficult to place f-elements inside eight Groups.

The "long" form of the Table consisting of 18 Groups was confirmed by IUPAC in 1989. Defect characteristics of the "short" Table were removed here: the sub-Groups are excepted, Periods consist of one stitch, elements are composed of blocks, the families of iron and platinum metals have disappeared, and so on.

The known Periodic Table consisting of 118 elements and 7 Periods where our dates for Period 8 are added must contain: 17 s-elements, 42 p-elements, 50 d-elements, 42 f-elements, and 4 g-elements.

The number 17 for s-elements follows from the fact that two of them are in Group I and Group II of Period 8, while element No. 155 (the last s-element, 17-th) is in Period 9 and Group I (the sole) closes the Table.

The extended Table consisting of blocks containing the number of elements calculated by us is published in [4].

3.1 From the Periodic Law to the Hyperbolic Law and the upper limit of the Periodic System

A note by Mendeleev, in March of 1869, was published and sent in Russian and French to scientists, titled "Experience of Systems of Elements Founded on Their Atomic Weights and Chemical Similarity" (with "atomic weight" to be understood as "atomic mass" here and in the future). This date is considered as the discovery date of the periodic law of chemical elements. The author dedicated the next two years to the work in this direction, which was a correction of atomic masses, an elaboration of studies about the periodical properties of elements, about the rôle of Groups, of big and small Periods, as well as about the places of chemical combinations in the Table. As a result, "Mendeleev's Natural System of Elements" which was the first periodic table of chemical elements was published in the first edition of his book "The Foundations of Chemistry", in 1871.

It is necessary to note that the dates published in the table of "Experience of Systems of Elements Founded on Their Atomic Weights and Chemical Similarity" permits us to use them in order to prove the correctness of Mendeleev's work.

The comprehensive table based the book "Experience of System of Element Found on Their Atomic Weight and Chemical Similarity", in terms of the dependence of each atomic mass on the number of the corresponding element, has been built by us and showed on Fig. 1. Because then it was not known yet that the ordinal number of each element characterizes its charge, it was simply the case that an element possessing a minimal mass was allowed to be designed as No. 1, and this order is conserved in the future: the next, in terms of mass, element will be designated as No. 2, the third as No. 3, and so on. Thus the ordinal number, which was attributed to the element after the theory of the atom was constructed has here another numerical value — symbolizing order of priority. The Table on Fig. 1 is the same as the one composed by Mendeleev, and the elements and the numbers are placed as the points on the arc where the triangles designate the beginning of the Periods. As is clear, the arc goes smoothly, preceding the elements and the atomic mass ~100, and after that it deviates preceding Ba. The trend line equation can be easily described by the multinomial of the third degree, i.e. by $R^2 = 0.9847$, in spite of a strong jump in the region of Lantanides. It should be noted that the part of the arc preceding Ba has $R^2 = 0.999$. It means that the direction

of the trend line after Ba reflects correctly the further course of our dependence, which allows us to calculate easily the atomic weights of other elements.

It should be noted that the trend line of the curve constructed according to contemporary dates has $R^2 = 0.9868$. In order that compare the dependence of the atomic mass from the ordinal number according to contemporary dates and the dates of Mendeleev the graph of was constructed (see Fig. 2). As is clear, the maximal deviations (3–4%) are observed for 6 cases, (1–1.5%) — for 8 cases, the others are placed lower. Because the common number of elements is 60, this spread is negligible for the those time.

As follows from the indicated dates, Mendeleev showed by means of his works concerning the Periodic Law that it is true for 60–70 elements, opening the way for the extension of the Table up to No. 118.

But our studies of the Periodic Table distinctly show that a hyperbolic law takes place in it. The law determines the upper limit of the Table through element No. 155. This fact is indisputable and it is justified by numerous publications.

4 Conclusion

If it was allowed in the 1950s that a maximum value of an ordinal number in Periodic Table could not exceed the value $Z = 110$ due to a spontaneous division of the nucleus, then in the 1960s theoreticians proposed the hypothesis that the atomic nucleus could have anomalously high stability. Seaborg called these regions "islands of stability" in a "sea of instability". He hoped for a possible synthesis of superelements inside these regions, **"... but until [now] the problem of the upper bound of the Periodic System [remains] unsolved"** (and so: at that time)!

Since in order to solve any problem it is necessary to know a final goal and to define its bounds, we have realized experimental studies and constructed a mathematical apparatus for the determination of the upper bound of the Periodic Table. According to our calculations, the last element is estimated and its location is determined: Period 9, Group I, with atomic mass of 411.66 (approximately), for which $Z = 155$. The earlier-proposed extended tables by Seaborg for 168 and 216 elements simply cannot be realized, because **only 155 elements can be in the Table, in its entirety**.

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Kepler-16 Circumbinary System Validates Quantum Celestial Mechanics

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We report the application of quantum celestial mechanics (QCM) to the Kepler-16 circumbinary system which has a single planet orbiting binary stars with the important system parameters known to within one percent. Other gravitationally bound systems such as the Solar System of planets and the Jovian satellite systems have large uncertainties in their total angular momentum. Therefore, Kepler-16 allows us for the first time to determine whether the QCM predicted angular momentum *per mass* quantization is valid.

1 Introduction

We report a precision test of quantum celestial mechanics (QCM) in the Kepler-16 circumbinary system that has planet-b orbiting its two central stars at a distance of 0.70 AU from the barycenter. QCM, proposed in 2003 by H.G. Preston and F. Potter [1] as an extension of Einstein's general theory of relativity, predicts angular momentum *per mass* quantization states for bodies orbiting a central mass in all gravitationally bound systems with the defining equation in the Schwarzschild metric being

$$\frac{L}{\mu} = m\frac{L_T}{M_T}. \qquad (1)$$

Here μ is the mass of the orbiting body with orbital angular momentum L and M_T is the total mass of the bound system with total angular momentum L_T. We calculate that the quantization integer m = 10, an amazing result with about a 1% uncertainty. Note that in all systems tested, we assume that the orbiting bodies have been in stable orbits for at least a 100 million years.

Kepler-16 is the first solar system type for which the total mass and the total angular momentum are *both* known accurately enough to allow a test of the angular momentum per mass quantization condition to within a few percent. The advantage this system has over all others is that the binary stars in revolution at its center contribute more than 99.5% of the system's total angular momentum. Moreover, more orbiting bodies may be detected in the future to provide the acid test of the theory because our precision result should improve.

2 Brief Review

Contrary to popular statements in the literature about planetary orbital angular momentum, the angular momentum of the Oort Cloud dominates the total angular momentum of the Solar System, being about 60 times the angular momentum of the orbiting planets, but its value has high uncertainty. The Jovian planets have differential internal rotations which bring their angular momentum uncertainties to more than 10% also. The Earth-Moon and Pluto-Charon systems have known values and a fit can be made to m = 65 and m = 9, respectively, but the application of the Schwarzschild metric is questionable in systems for which a reduced mass must be used. In addition, there is not another orbiting body for prediction purposes.

The Mars-Phobos-Deimos system offers a test of the angular momentum condition. We find that m = 61 for Phobos and m = 97 for Deimos, with uncertainties less than about 4%. The Schwarzschild metric is a good approximation here but the integers are very large and therefore somewhat unsatisfactory for a definitive test. We would prefer to find a system for which the m values that fit are small integers, if possible.

We have applied the equation to many multiplanet exosystems and found that the fits all predict additional undetected angular momentum. Such solar systems can be expected to have an additional planet and/or the equivalent of an Oort Cloud that contributes significant orbital angular momentum. Examples include: Kepler-18, HR 8799, HIP 57274, Gliese 581, 55Cnc, Kepler-11, PSR 1257, HD 10180, HD 125612, HD 69830, 47 Uma, and 61 Vir.

Other confirmed circumbinary systems with one or two known planets are either dominated by the planetary angular momentum or the planets contribute about 50%, rendering their fits unsuitable for a precision test: HW Virginis, NNSerpentis, and DP Leonis.

Our original article [1] contains the derivation of QCM from the general relativistic Hamilton-Jacobi equation and its new gravitational wave equation for any metric. Our first application, to the Solar System without knowledge of the Oort Cloud angular momentum, predicted that all the planetary orbits should be within the Sun's radius! Subsequently, we learned about the Oort Cloud and were able to produce two excellent QCM linear regression fits with $R^2 > 0.999$ for m sets (1) 2,3,4,5,9,13,19,24,28; (2) 3,4,5,6,11,15,21,26,30. Therefore, we predict a total angular momentum for the Solar System $L_{SS} \approx 1.9 \times 10^{45}$ kg m^2/s with the planets contributing only $L_{pl} = 3.1 \times 10^{43}$ kg m^2/s.

Several follow-up articles verify its application to galaxies without requiring 'dark matter' for gravitational lensing by the galaxy quantization states [2], the quantization state of baryonic mass in clusters of galaxies [3], and how the cosmo-

logical redshift is interpreted as a gravitational redshift that agrees with the accelerated expansion of the Universe [4]. That is, QCM applied to the Universe with the interior metric dictates that every observer at distance r from the source sees the light originating from an effective negative potential $V(r) \approx -kr^2 c^2/[2(1-kr^2)^2]$, meaning the clocks run slower at the distant source.

In the Schwarzschild metric the QCM wave equation reduces to a Schrödinger-like equation that predicts quantization states for the angular momentum per mass and for the energy per mass. There is no Planck's constant per se but instead each system has its unique constant $H = L_T/M_T c$, a characteristic distance for the gravitationally bound system. Important physical quantities can be related to H and the Schwarzschild radius. In the single free particle limit, such as a free electron, the QCM equation reduces to the standard quantum mechanical Schrödinger equation. Note that QCM is not quantum gravity.

3 The Kepler-16 System

We have been waiting about 10 years for a gravitationally bound system for which its total angular momentum per total mass is known to about 1%. Finally, in September, 2011, the Kepler-16 system was reported [5] with two stars, star A and star B, separated by 0.22 AU and a planet called planet-b orbiting their barycenter at 0.70 AU. The list below provides the important physical parameters of this system.

Star A:

Mass = 0.6897 ± 0.0035 solar masses

Orbital radius = 0.05092 ± 0.00027 AU

Period = 41.079220 ± 0.000078 days

Angular momentum = (1.4247 ± 0.0170) x 10^{44} m^2/s

Star B:

Mass = 0.20255 ± 0.00066 solar masses

Orbital radius = 0.17339 ± 0.00115 AU

Period = 41.079220 ± 0.000078 days

Angular momentum = (4.8514 ± 0.0632) x 10^{44} m^2/s

planet-b:

Mass = 0.333 ± 0.0016 Jupiter masses

Orbital radius = 0.7048 ± 0.0011 AU

Period = 228.776 ± 0.037 days

Angular momentum = (2.2479 ± 0.1080) x 10^{42} m^2/s

Kepler-16 system:

L_T/M_T = (3.517± 0.011) x 10^{14} m^2/s

L_b/M_b = (3.555± 0.036) x 10^{15} m^2/s

Note that although the planet mass value has about a 5% uncertainty, this large uncertainty is excluded from the equation because the planet mass divides out in L_b/μ_b. Our result for the QCM angular momentum per mass quantization integer is

$$m = 10.1 \pm 0.1. \quad (2)$$

Therefore, we have determined that planet-b is in the $m = 10$ quantization state with a maximum uncertainty of less than 2%. In Einstein's general theory of relativity and in Newtonian gravitation there is no a priori reason for m to be an integer, so its value could have been anywhere.

4 Comments

As good as this result has been, the acid test for QCM is yet to come. We need to detect at least one more planet in the Kepler-16 system to determine whether the QCM prediction leads to its correct angular momentum value, i.e., an integer multiple of L_T/M_T equal to the classical value at radius r.

Assuming that QCM passes the acid test, we wish to point out that the existence of quantization states of angular momentum per mass and energy per mass are important concepts for the formation of stars, planets, solar systems, galaxies, and clusters of galaxies. Models ignoring QCM will be incomplete and will need speculative inventions such as dark matter and perhaps dark energy to preserve traditional incomplete approaches toward 'understanding' these gravitational systems.

An additional gravitational test of QCM would be a laboratory experiment with a slowly rotating attractor mass producing a repulsive effect to counteract the Newtonian attraction at specific rotation frequencies for the given separation distance to the affected mass. We are in the process of searching for this behavior.

Acknowledgements

One of us (F. P.) thanks physicist Gregory Endo of Endo Engineering for recently suggesting that we apply QCM to a binary star system.

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Nuclear Structure of $^{122\text{-}134}$Xe Isotopes

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The potential energy surfaces, $V(\beta,\gamma)$, for a series of Xenon isotopes $^{122-134}$Xe have been calculated. The relatively flat potential to ^{130}Xe and energy ratio $E^+_{4_1}/E^+_{2_1} = 2.2$ show $E(5)$ symmetry to the nucleus which is laying in the transition region from γ-soft to vibrational characters. The interacting boson approximation model ($IBA-1$) has been used in calculating levels energy and electromagnetic transition probabilities $B(E2)'s$. Back bending is observed for $^{122-130}$Xe. The calculated values are compared to the available experimental data and show reasonable agreement.

1 Introduction

The chain of $^{122-134}$Xe isotopes is of great interest because of the existence of transitional nuclei where the nuclear structure changes from rotational to vibrational shapes. Many authors studied this area of isotopes experimentally and theoretically.

Experimentally, the mass of $^{122-134}$Xe isotopes [1] were detected on line using mass separator ISOLDE/CERN while the lifetimes of the low lying states in $^{122-134}$Xe were measured using Doppler-Shift [2] technique.

Theoretically, many authors studied this series of isotopes useing different theoretical models as algebric $sp(4)$ shell model [3], cranked Strutinsky method [4], relativistic mean field theory [5, 6], isospin-dependent lattice gas model [7, 8], general Bohr Hamiltonian [9], quadrupole-quadrupole plus pairing model [10], cranked Hartree-Fock-Bogoliubov model [11, 12] and interacting boson approximation model [13, 17]. They reported:

1. the reduced transition probabilities for Yrast spectra up to $I^+ = 10$;
2. the existance of shape transitions as well as $E(5)$ and $X(5)$ symmetry nuclei,
3. the occurrence of backbending in $^{122-130}$Xe nuclei, and
4. M1 transition probabilities between the mixed-symmetry and fully symmetric states.

2 Interacting Boson Approximation Model

The IBA-1 model [18] was applied to the positive parity low-lying states in even-even $^{122-134}$Xe isotopes. The proton, π, and neutron, ν, bosons are treated as one boson and the system is considered as an interaction between s-bosons and d-bosons. Creation ($s^\dagger d^\dagger$) and annihilation ($s\tilde{d}$) operators are for s and d bosons. The Hamiltonian employed for the present calculation is given as:

$$\begin{aligned} H &= EPS\cdot n_d + PAIR\cdot(P\cdot P)+ \\ &\quad +\tfrac{1}{2}\,ELL\cdot(L\cdot L)+\tfrac{1}{2}\,QQ\cdot(Q\cdot Q)+ \\ &\quad +5\,OCT\cdot(T_3\cdot T_3)+5HEX\cdot(T_4\cdot T_4), \end{aligned} \tag{1}$$

where

$$P\cdot p = \frac{1}{2}\begin{bmatrix} \left\{(s^\dagger s^\dagger)^{(0)}_0 - \sqrt{5}(d^\dagger d^\dagger)^{(0)}_0\right\} x \\ \left\{(ss)^{(0)}_0 - \sqrt{5}(\tilde{d}\tilde{d})^{(0)}_0\right\} \end{bmatrix}^{(0)}_0, \tag{2}$$

$$L\cdot L = -10\sqrt{3}\left[(d^\dagger\tilde{d})^{(1)} x\,(d^\dagger\tilde{d})^{(1)}\right]^{(0)}_0, \tag{3}$$

$$Q\cdot Q = \sqrt{5}\begin{bmatrix} \left\{(S^\dagger\tilde{d}+d^\dagger s)^{(2)} - \dfrac{\sqrt{7}}{2}(d^\dagger\tilde{d})^{(2)}\right\} x \\ \left\{(s^\dagger\tilde{d}+\tilde{d}s)^{(2)} - \dfrac{\sqrt{7}}{2}(d^\dagger\tilde{d})^{(2)}\right\} \end{bmatrix}^{(0)}_0, \tag{4}$$

$$T_3\cdot T_3 = -\sqrt{7}\left[(d^\dagger\tilde{d})^{(2)} x\,(d^\dagger\tilde{d})^{(2)}\right]^{(0)}_0, \tag{5}$$

$$T_4\cdot T_4 = 3\left[(d^\dagger\tilde{d})^{(4)} x\,(d^\dagger\tilde{d})^{(4)}\right]^{(0)}_0. \tag{6}$$

In the previous formulas, n_d is the number of bosons; $P\cdot P$, $L\cdot L$, $Q\cdot Q$, $T_3\cdot T_3$ and $T_4\cdot T_4$ represent pairing, angular momentum, quadrupole, octupole and hexadecupole interactions between the bosons; EPS is the boson energy; and $PAIR$, ELL, QQ, OCT, HEX are the strengths of the pairing, angular momentum, quadrupole, octupole and hexadecupole interactions.

3 Results and discussion

3.1 The potential energy surfaces, (PESs)

The PESs [19], $V(\beta,\gamma)$, for Xenon isotopes as a function of the deformation parameters β and γ have been calculated using :

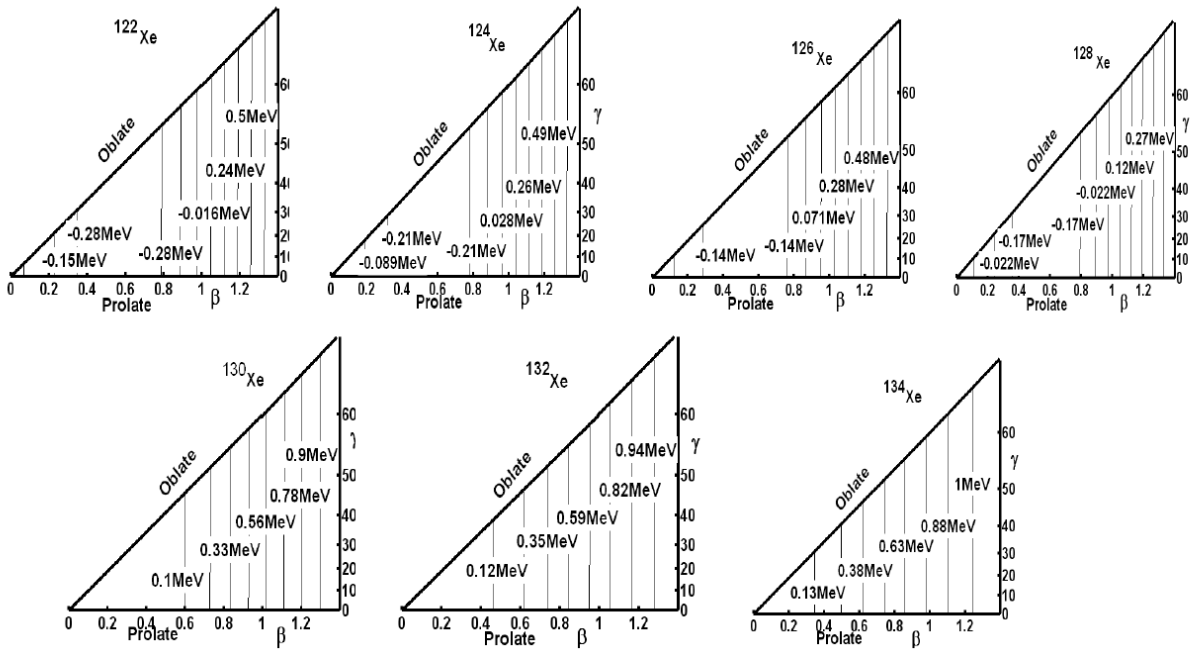

Fig. 1: Contour plot of the potential energy surfaces for $^{122-134}$Xe nuclei.

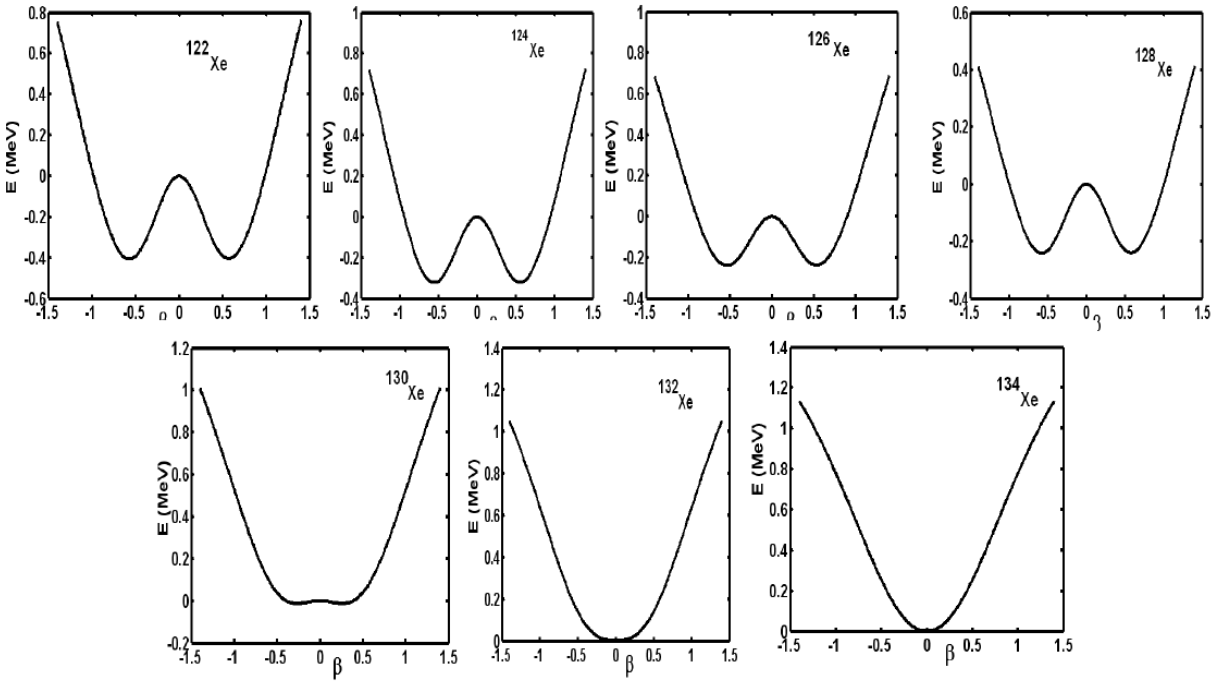

Fig. 2: Potential energy surfaces for $^{122-134}$Xe nuclei at $\gamma = 0^o$ (Prolate) and $\gamma = 60^o$ (Oblate).

nucleus	*EPS*	*PAIR*	*ELL*	*QQ*	*OCT*	*HEX*	$E2SD(eb)$	$E2DD(eb)$
^{122}Xe	0.4700	0.0000	0.0216	−0.0200	0.0000	0.00000	0.1390	−0.4112
^{124}Xe	0.4680	0.0000	0.0216	−0.0200	0.0000	0.0000	0.1280	−0.3786
^{126}Xe	0.4490	0.0000	0.0216	−0.0200	0.0000	0.0000	0.1260	−0.3727
^{128}Xe	0.4720	0.0000	0.0216	−0.0200	0.0000	0.0000	0.1410	−0.4171
^{130}Xe	0.5420	0.0000	0.0216	−0.0200	0.0000	0.0000	0.1500	−0.4437
^{132}Xe	0.6450	0.0000	0.0216	−0.0200	0.0000	0.0000	0.1460	−0.4319
^{134}Xe	0.8020	0.0000	0.0216	−0.0200	0.0000	0.0000	0.1480	−0.4378

Table 1: Parameters used in IBA-1 Hamiltonian (all in MeV).

I_i^+ I_f^+	^{122}Xe	^{124}Xe	^{126}Xe	^{128}Xe	^{130}Xe	^{132}Xe	^{134}Xe
$0_1{}^*Exp.\ 2_1$	1.40(6)	0.96(6)	0.770(25)	0.750(40)	0.65(5)	0.460(30)	0.34(6)
0_1 Theo. 2_1	1.4038	0.9651	0.7691	0.7575	0.6575	0.4684	0.3451
$2_1\ 0_1$	0.2808	0.1930	0.1538	0.1515	0.1315	0.0937	0.0690
$2_2\ 0_1$	0.0057	0.0033	0.0022	0.0015	0.0007	0.0002	0.0001
$2_2\ 0_2$	0.1552	0.0979	0.0741	0.0684	0.0567	0.0412	0.0343
$2_3\ 0_1$	0.0009	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000
$2_3\ 0_2$	0.1640	0.1278	0.1047	0.1077	0.0926	0.0583	0.0298
$2_3\ 0_3$	0.0465	0.0248	0.0161	0.0133	0.0113	0.0091	0.0086
$2_4\ 0_3$	0.0766	0.0355	0.0198	0.0121	0.0064	0.0025	——
$2_4\ 0_4$	0.1031	0.0886	0.0784	0.0867	0.0839	0.0683	——
$4_1\ 2_1$	0.5297	0.3583	0.2787	0.2650	0.2186	0.1447	0.0941
$4_1\ 2_2$	0.0487	0.0316	0.0239	0.0227	0.0194	0.0145	0.0124
$4_1\ 2_3$	0.0737	0.0562	0.0452	0.0456	0.0386	0.0240	0.0122
$6_1\ 4_1$	0.6735	0.4529	0.3448	0.3183	0.2482	0.1465	0.0714
$6_1\ 4_2$	0.0476	0.0326	0.0254	0.0259	0.0244	0.0198	0.0182
$6_1\ 4_3$	0.0563	0.0428	0.0337	0.0332	0.0261	0.0127	——
$8_1\ 6_1$	0.7369	0.4875	0.3586	0.3139	0.2199	0.0979	——
$8_1\ 6_2$	0.0409	0.0290	0.0230	0.0246	0.0248	0.0214	——
$8_1\ 6_3$	0.0438	0.0319	0.0237	0.0210	0.0127	——	——
$10_1\ 8_1$	0.7363	0.4717	0.3269	0.2567	0.1362	——	——
$10_1\ 8_2$	0.0347	0.0252	0.0202	0.0223	0.0237	——	——

Table 2: Theoretically calculated reduced transition probabilities, $B(E2)'s$ in $e^2\,b^2$. *Ref. [27]

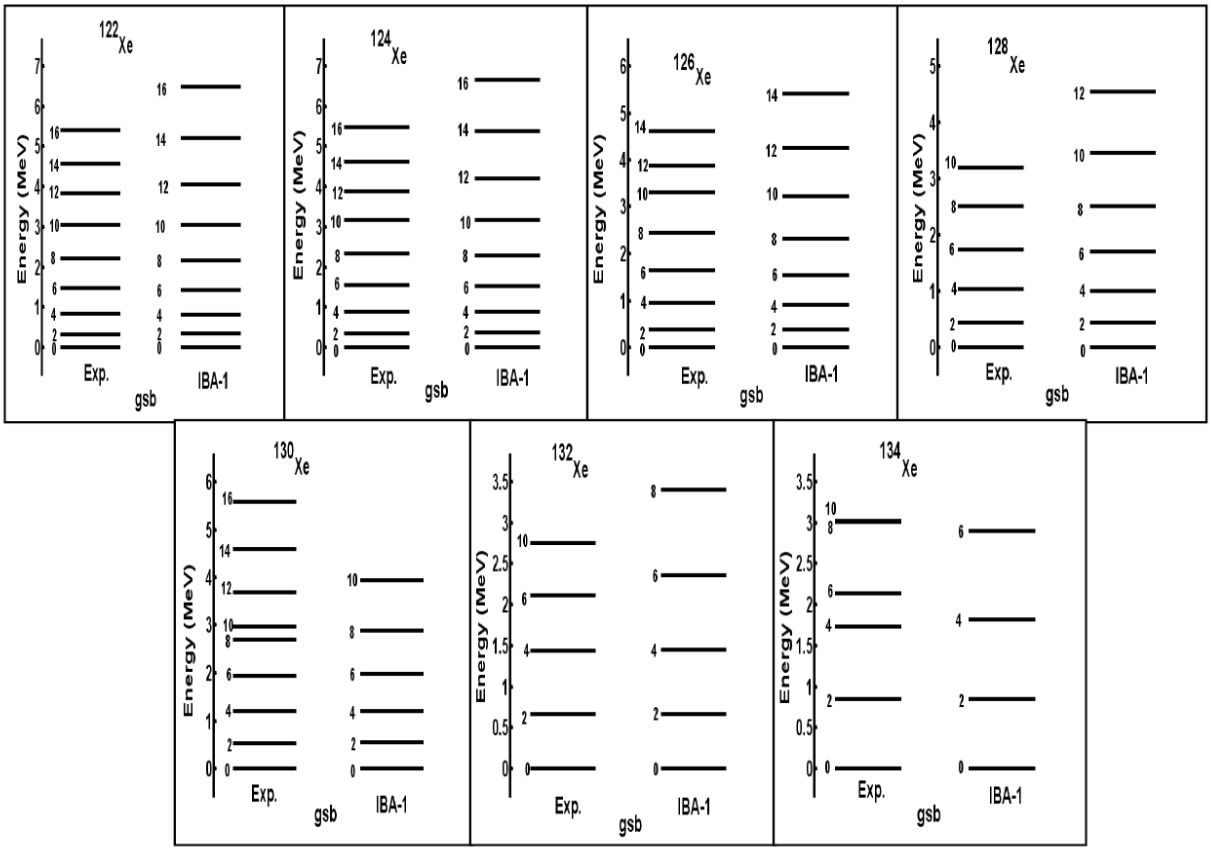

Fig. 3: Comparison between experimental [20–26] and theoretical (IBA) energy levels.

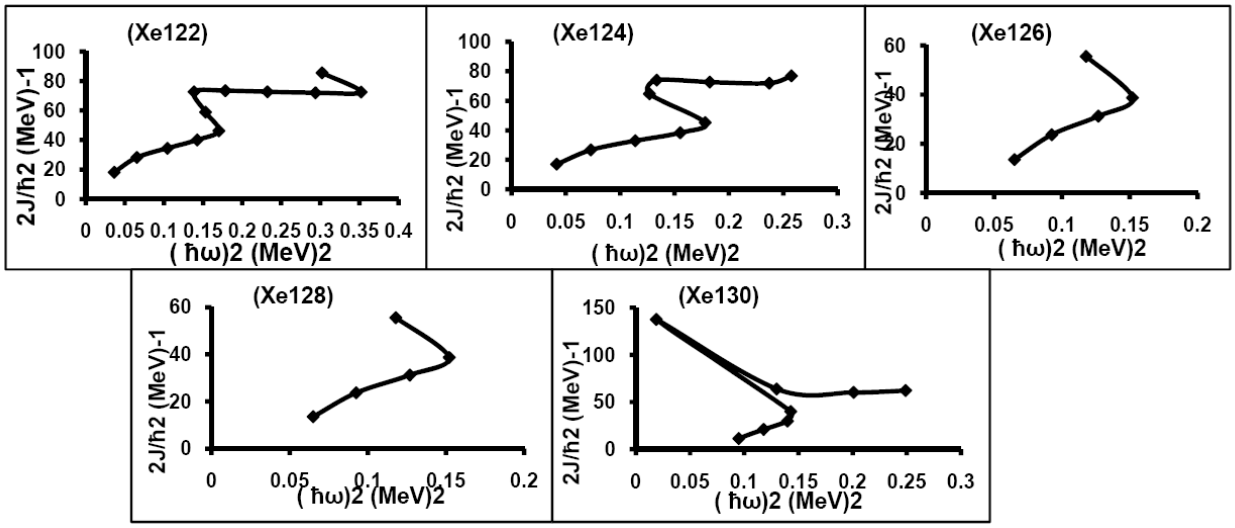

Fig. 4: Back bending in $^{122-134}$Xe isotopes.

$$
\begin{aligned}
E_{N_\Pi N_\nu}(\beta,\gamma) &= <N_\pi N_\nu;\beta\gamma|H_{\pi\nu}|N_\pi N_\nu;\beta\gamma> = \\
&= \zeta_d(N_\nu N_\pi)\beta^2(1+\beta^2)+\beta^2(1+\beta^2)^{-2}\times \\
&\times\left\{kN_\nu N_\pi[4-(\bar{X}_\pi\bar{X}_\nu)\beta\cos 3\gamma]\right\}+ \\
&+\left\{[\bar{X}_\pi\bar{X}_\nu\beta^2]+N_\nu(N_\nu-1)\left(\frac{1}{10}\,c_0+\frac{1}{7}\,c_2\right)\beta^2\right\},
\end{aligned}
\tag{7}
$$

where

$$
\bar{X}_\rho=\left(\frac{2}{7}\right)^{0.5}X_\rho,\quad \rho=\pi\,or\nu\,. \tag{8}
$$

The calculated PESs, $V(\beta,\gamma)$, for Xenon series of isotopes are presented in Fig. 1 and Fig. 2. They show that $^{122-128}$Xe nuclei are deformed and the two wells on both oblate and prolate sides are nearly equals and O(6) characters is expected to these nuclei. ^{130}Xe has flat potential energy, Fig. 2, which indicates that the nucleus is $E(5)$ symmetry and confirmed by the energy ratio $R=E^+_{4_1}/E^+_{2_1}=2.2$ as well as it is laying also in the transition from γ- unstable, $O(6)$, to vibrational, $U(5)$, nuclei while, $^{132,134}Xe$ are vibrational like nuclei.

3.2 Energy spectra and transition rates

IBA-1 model has been used in calculating the energy of the positive parity low-lying levels of Xenon series of isotopes. Comparison between the experimental spectra [20–26] and our calculations, using values of the model parameters given in Table 1, are illustrated in Fig. 3. The agreement between the low-laying calculated energy levels and their corresponding experimental values is fairly good but for higher states theoretical values are slightly higher. We believe that is due to the change of the projection of the angular momentum which may be due to band crossing and change in angular momentum.

The electric quadrupole transition operator [18] employed in this study is given by:

$$
T^{(E2)}=E2S\,D\cdot(s^\dagger\tilde{d}+d^\dagger s)^{(2)}+\frac{1}{\sqrt{5}}\,E2DD\cdot(d^\dagger\tilde{d})^{(2)}\,. \tag{9}
$$

The reduced electric quadrupole transition rates between $I_i\to I_f$ states are given by

$$
B(E_2,I_i-I_f)=\frac{[<I_f\parallel T^{(E_2)}\parallel I_i>]^2}{2I_i+1}\,. \tag{10}
$$

Unfortunately there is no enough measurements of electromagnetic transition rates $B(E2)$ for these series of nuclei. The only measured $B(E2,0^+_1\to 2^+_1)$'s are presented, in Table 2 for comparison to the calculated values. The parameters $E2S\,D$ and $E2DD$, displayed in Table 1, are used in the present calculation of the transition rates $B(E2)'s$ and then normalized to the experimentally known ones [27]. In our calculations we did not introduce any new parameters.

3.3 Back bending

The moment of inertia J and energy parameters $\hbar\omega$ are calculated [28]using equations (11, 12):

$$
\frac{2J}{\hbar^2}=\frac{4I-2}{\Delta E(I\to I-2)}\,, \tag{11}
$$

$$
(\hbar\omega)^2=(I^2-I+1)\left[\frac{\Delta E(I\to I-2)}{(2I-1)}\right]^2\,. \tag{12}
$$

The plots in Fig. 4 show back bending for $^{122-126}$Xe at $I^+=10$ while at $I^+=12$ for ^{128,130}Xe and this is in agreement with the work done by other authors [29]. Back bending in Xenon isotopes in higher states is explained [10] as due to partial rotational alignment of a pair of neutrons in the $1h_{1/2}$ neutron orbit near the Fermi surface.

4 Conclusions

The IBA-1 model has been applied successfully to $^{122-134}$Xe isotopes and we have got:

1. The ground state bands are successfully reproduced;
2. The potential energy surfaces are calculated and show $O(6)$ characters to $^{122-128}$Xe isotopes where the prolate and oblate depths are equal;
3. Flat potential energy to ^{130}Xe and energy ratios confirmed that the nucleus is an $E(5)$ symmetry;
4. ^{132,134}Xe nuclei show vibrational-like characters;
5. Electromagnetic transition rates, $B(E2)'s$, are calculated, then normalized to experimental B(E2, 0_1-2_1) values and then compared to the available data, and
6. Back bending for $^{122-126}$Xe have been observed at angular momentum $I^+=10$ and at $I^+=12$ for ^{128,130}Xe.

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The Crucial Role of Multi-Configuration States of Bound Fermions

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The structure of a bound state of several Dirac particles is discussed. Relying on solid mathematical arguments of the Wigner-Racah algebra, it is proved the a non-negligible number of configurations is required for a description of this kind of systems. At present, the main results are not widely known and this is the underlying reason for the phenomenon called the proton spin crisis.

1 Introduction

Once upon a midnight dreary,
while I pondered weak and weary,
Over many a quaint and curious
volume of forgotten lore... [1].

The main objective if this work is to prove that the multi-configuration structure of a bound state of several Dirac particles plays an extremely important role. The existence of such a multi-configuration structure was already known many decades ago [2, 3] and early electronic computers were used for providing a numerical proof of this issue [4]. (Note that the first edition of [2] was published in 1935.) Unfortunately, this scientific evidence has not found its way to contemporary textbooks of physics and has become a kind of a forgotten lore. For example, [5] uses a single configuration and remarks that the error is about 5 per cent [5, see a comment on p. 234]. Here [6, see p. 116] is a notable exception. The paper proves the main points of this issue and shows its far reaching meaning and its relevance to physical problems that are still unsettled. In doing so the paper aims to make a contribution to the correction of this situation.

It is well known that quantum mechanics explains the Mendeleev periodic table of chemical elements. The shell structure of electrons provides an easy interpretation of chemical properties of noble gases (a full shell), halogens (a full shell minus 1), alkali metals (a full shell + 1) etc. The standard explanation of the Mendeleev periodic table uses a single configuration for a description of the electronic states of each chemical element. Thus, for example, the helium and the lithium atoms are described by the $1s^2$ and $1s^2 2s$ configurations, respectively. At this point the following problem arises: Does the unique configuration structure of an atomic ground state make an acceptable description of its quantum mechanical system or is it just a useful pedagogical explanation of the Mendeleev periodic table? The answer to this problem certainly must be obtained from a mathematical analysis of the quantum mechanical state of systems that contain more than one electron. By describing an outline of this task, the present work proves beyond any doubt that an atomic state of more than one electron has a multi-configuration structure and that no single configuration dominates the system.

The conclusion stated above has two important aspects. First, It is clear that a correct understanding of the structure of any fundamental physical system is a vital theoretical asset for every physicist. Next, it turns out that the lack of an adequate awareness of this physical evidence has already caused the phenomenon called the "proton spin crisis" [7] which haunts the particle physics community for decades. The measurements published in [7] show that quarks carry a very small portion of the proton's spin and this evidence has been regarded as a surprise. Now, it is shown in this work that the multiconfiguration structure found in atomic states is not a specific property of the Coulomb interaction. Thus, it is expected to be also found in any bound state of three spin 1/2 quarks, like it is found in bound states of several spin 1/2 electrons. For this reason, one can state that if the experiment described in [7] would have shown that *quarks carry the entire proton's spin then this result should have been regarded as a real crisis of fundamental quantum mechanical principles.*

In this work, units where $\hbar = c = 1$ are used. The second section contains a brief description of the main properties of a bound state of several Dirac particles that are required for the discussion. The underlying mathematical reasons for the multiconfiguration structure of states are discussed in the third section. Some aspects of the results are pointed out in the last section.

2 General Arguments

The main objective of this work is to find a reliable mathematical method for describing the ground state of a bound system of spin 1/2 particles. Applying Wigner's analysis of the Poincare group [8, 9], one concludes that the total mass (namely, energy) and the total spin are good quantum numbers. Thus, one assumes that an energy operator (namely, a Hamiltonian) exists. For this reason, one can construct a Hilbert space of functions that can be used for describing the given system as an eigenfunction of the Hamiltonian. Evidently, in the system's rest frame, an energy eigenfunction has the time dependent factor $\exp(-iEt)$. This factor can be removed and the basis of the Hilbert space contains time independent functions.

The fact that every relatively stable state has a well de-

fined total spin J can be used for making a considerable simplification of the problem. Thus, one uses a basis for the Hilbert space that is made of functions that have the required spin J and ignores all functions that do not satisfy this condition. Evidently, a smaller Hilbert space reduces the amount of technical work needed for finding the Hamiltonian's eigenfunctions. An additional argument holds for systems whose state is determined by a parity conserving interaction, like the strong and the electromagnetic interactions. Thus, one can use functions that have a well defined parity and build the Hilbert space only from functions that have the required parity. This procedure makes a further simplification of the problem.

The notion of a configuration of a system of several Dirac particles is a useful mathematical tool that satisfies the two requirements stated above [2, see p. 113] and [10, see p. 245]. A configuration is written in the form of a product of single particle wave functions describing the corresponding radial and orbital state of each particle belonging to the system (the m quantum number is ignored). For atomic systems a non-relativistic notation is commonly used and the values of the nl quantum numbers denote a configuration, like $1s^2 2s^1$. In relativistic cases the variables nlj [10, see p. 245] are used. In the latter case, the variables nj^π (here π denotes parity and it takes the values ± 1) is an equivalent notation for a relativistic configuration because $l = j \pm 1/2$ and the numerical parity of the l-value of a Dirac spinor upper part defines the single particle's parity. (This work uses the nj^π notation.) Evidently, any acceptable configuration must be consistent with the Pauli exclusion principle.

For any given state where the total spin J and parity are given, one can use configurations that are consistent with J and the product of the single-particle parity equals the parity of the system. The total angular momentum J is obtained from an application of the law of vector addition of angular momentum [2, see p. 56] and [10, see p. 95]. Here the triangular condition holds [10, see p. 98]. Thus, for example, an acceptable configuration for the two-electron 0^+ ground state of the helium atom must take the form $n_1 j_1^{\pi_1}\, n_2 j_2^{\pi_2}$, where $j_1 = j_2$ and $\pi_1 = \pi_2$. Similarly, a description of a 2-electron state where $J^\pi = 3^+$ *cannot* contain a configuration of the form $n_1 \frac{1}{2}^+\, n_2 \frac{3}{2}^+$, because the two J values 1/2 and 3/2 can only yield a total $J = 1$ or $J = 2$.

At this point the structure of the relevant Hilbert space is known. It is made of configurations that satisfy certain requirements. This is one of the useful properties of using configurations - the relevant Hilbert space is smaller because many configurations can be ignored due to the total spin and parity requirements. Obviously, a smaller Hilbert space indicates shorter computational efforts. Thus, the framework needed for the analysis is established. The problem of finding how many configurations are required for an acceptable description of an atomic state is discussed in the following section.

3 The Multi-Configuration Structure of Atomic States

The purpose of this section is to outline a proof that shows why a bound state of several electrons takes the form of a linear combination of terms, each of which belongs to a specific configuration. For this purpose, the Hamiltonian matrix is constructed for a Hilbert space whose basis is made of functions that take a configuration form. Evidently, non-vanishing off-diagonal matrix elements prove that the required state is a linear combination of configurations. It is shown that this property holds even for the simplest atomic state of more than one electron, namely the $J^\pi = 0^+$ ground state of the 2-electron Helium atom.

It is explained in the previous section that the required Hilbert space contains functions that have the given total spin and parity. The form of a two electron function is written as follows

$$\chi(\mathbf{r}_1, \mathbf{r}_2) = F_i(r_1) F_k(r_2) (j_1^{\pi_1}\, j_2^{\pi_2} JM). \tag{1}$$

Here, $F_i(r_1)$, $F_k(r_2)$ denote radial functions of the appropriate electron, j_1, j_2, π_1, π_2 denote the single particle spin and parity of the electrons, respectively, J is the total spin obtained by using the appropriate Clebsch-Gordan coefficients [2, 10] and M denotes the magnetic quantum number of the total angular momentum,

Let us use the principles described in the previous section and try to find the structure of the helium atom ground state. Thus, due to the triangular rule [10, see p. 98] and in order to be consistent with $J = 0$, we must use configurations where $j_1 = j_2$. Similarly, in order to have an even total parity, we must use configurations where the two electrons have the same parity. Thus, the required Hilbert space contains functions of the following form

$$\chi(\mathbf{r}_1, \mathbf{r}_2) = F_i(r_1) F_k(r_2) (j^\pi j^\pi 00), \tag{2}$$

where j is a positive number of the form $j = n + 1/2$, n is an integer and $\pi = \pm 1$.

The angular parts of any two different functions of (2) are orthogonal. Hence, off-diagonal matrix elements of any pure radial operator vanish. Since the following discussion is focused on finding off-diagonal matrix elements of the Hamiltonian, radial coordinates and radial operators are not always shown explicitly in expressions.

At this point one can use a given Hamiltonian and construct its matrix. Before doing this assignment one has to find a practical procedure that can be used for overcoming the infinite number of configurations that can be obtained from the different values of n, j and π. For this purpose one organizes the configurations of (2) in an ascending order of j and examines a Hilbert subspace made of the first N_0 functions, where N_0 is a positive integer. Here a finite Hamiltonian matrix is obtained and one can diagonalize it, find the smallest eigenvalue E_0 and its associated eigenfunction Ψ_0. The quantities

found here represent an approximation for the required solution. Let this approximate solution be denoted in this form

$$\{E_0, \Psi_0\}. \tag{3}$$

In order to evaluate the goodness of this approximation, one replaces N_0 by $N_1 = N_0 + 1$ and repeats the procedure. The new solution $\{E_1, \Psi_1\}$ is a better approximation because it relies on a larger Hilbert subspace. The difference between these solutions provides an estimate for the goodness of the solutions obtained. This procedure can be repeated for an increasing value of N_i. Thus, if a satisfactory approximation is reached for a certain value of N_i then one may terminate the calculation and use the solution obtained from this procedure as a good approximation to the accurate solution.

Now we are ready to examine the Hamiltonian's matrix elements. This examination demonstrates the advantage of using configurations as a basis for the Hilbert space. Thus, the angular part of the kinetic energy of each electron takes the form found for the hydrogen atom and only diagonal matrix elements do not vanish. The same result is obtained for the spherically symmetric radial potential operator Ze^2/r of the nucleus. It follows that off-diagonal matrix elements can be obtained only from the interaction between the two electrons. (This quantity does not exist for the one electron hydrogen atom and for this reason, each of the hydrogen atom eigenfunctions takes the form of a unique configuration.) In a full relativistic case the two-electron interaction takes the form of Breit interaction [11, see p. 170]. which contains the instantaneous ordinary Coulomb term and a velocity-dependent term. The existence and the results of the Hamiltonian's off-diagonal matrix elements are the main objective of this discussion and it is shown below that for this purpose the examination of the relatively simple Coulomb term is enough.

Thus, one has to write the $1/r_{12}$ operator in a form that is suitable for a calculation that uses the single particle independent variables $\mathbf{r}_1, \mathbf{r}_2$ of the configurations (2). This objective is achieved by carrying out a tensor expansion of the interaction [10, see p. 208]. For the specific case of the Coulomb interaction, one obtains [12, see p. 114]

$$\frac{1}{r_{12}} = \sum_{k=0}^{\infty} \frac{r_<^k}{r_>^{k+1}} P_k(\cos\theta_{12}). \tag{4}$$

Here $r_<$ and $r_>$ denote the smaller and the larger values of r_1 and r_2, respectively and θ_{12} is the angle between them. $P_k(\cos\theta_{12})$ is the Legendre polynomial of order k. At this point one uses the addition theorem for spherical harmonics [10, see p. 113]

$$P_k(\cos\theta_{12}) = \frac{4\pi}{2k+1} \sum_{m=-k}^{k} (-1)^m Y_{k,-m}(\theta_1, \phi_1) Y_{k,m}(\theta_2, \phi_2) \tag{5}$$

and obtains an expansion of the appropriate Legendre polynomial $P_k(\cos\theta_{12})$ of (4) in terms of spherical harmonics that depend on single particle angular variables. This analysis shows how matrix elements can be obtained for a Hilbert space whose basis is made of functions that are an appropriate set of configurations.

At this point the wave functions of the Hilbert space basis as well as the Hamiltonian operator depend on the radial and the angular coordinates of single particle functions. The main objective of this section is to explain why the electronic states are described as a linear combination of configurations. It is shown above that the configurations of the Hilbert space basis are eigenfunctions of the operators representing the kinetic energy and the interaction with the spherically symmetric potential of the nucleus. Hence, the discussion is limited to the two particle operator (4) that depends on the expansion (5).

Let us find, for example, the off-diagonal matrix element of the configurations $((1\frac{1}{2}^+)^2 00)$ and $((2\frac{3}{2}^-)^2 00)$ of the Hilbert space basis (2). Consider the 2-electron Coulomb interaction obtained for the upper (large) component of the Dirac spinor. Thus, $\frac{1}{2}^+$ is a spatial s-wave and $\frac{3}{2}^-$ is a spatial p-wave. The Wigner-Racah algebra provides explicit formulas for expressions that depend on the angular coordinates. Now, as stated above, the main objective of the discussion is to show that off-diagonal matrix elements do not vanish. For this purpose, only the main points of the calculation are written and readers can use explicit reference for working out the details.

The formal form of the angular component of the off-diagonal matrix element is

$$H_{ij} = < j_1 j_2 JM | \frac{1}{r_{12}} | j'_1 j'_2 JM > . \tag{6}$$

Here j_1, j_2 of the ket are angular momentum values of the first and the second electron, respectively and they are coupled to a total J, M. The bra has an analogous structure. In the particular case discussed here $J = M = 0$ and (6) takes the form

$$H_{ij} = < \tfrac{1}{2}\tfrac{1}{2}00 | \frac{1}{r_{12}} | \tfrac{3}{2}\tfrac{3}{2}00 > . \tag{7}$$

The following points describe the steps used in the calculation of (7).

1. The Wigner-Eckart theorem shows that (6) can be cast into a product of a *Wigner 3j symbol* and a *reduced matrix element* [10, see p. 117]
2. In (4), the expansion (5) of $1/r_{12}$ is a *scalar product of two tensors* [10, see p. 128].
3. The reduced matrix element of such a scalar product can be put in the form of a product of a *Racah coefficient* and two reduced matrix elements that depend on the first and the second electron, respectively [10, see p. 129].
4. Each of these reduced matrix elements takes the form $< sl\,j \| Y_k \| sl'\,j' >$ where sl denote single particle spin and spatial angular momentum that are coupled to the

particle's total angular momentum j. In the specific case discussed here it is $< \frac{1}{2}0\frac{1}{2}\|Y_1\|\frac{1}{2}1\frac{3}{2} >$. The value of the last expression can be readily obtained as a product of a square root of an integer and a Wigner $3j$ symbol [10, see p. 521]. The final value is

$$< \tfrac{1}{2}0\tfrac{1}{2}\|Y_1\|\tfrac{1}{2}1\tfrac{3}{2} >= \frac{-2}{\sqrt{4\pi}}. \quad (8)$$

This discussion shows that the Hamiltonian's off diagonal matrix elements do not vanish for the $J = 0$ ground state of the He atom. It means that a single configuration does not describe accurately this state. The next step is to carry out an explicit calculation and find out how good is the usage of a single configuration. This task has already been carried out [4] and it was proved that the description of the ground state of the He atom requires many configurations. Here radial and angular excitations take place and no single configuration plays a dominant role.

4 Discussion

Several aspects of the conclusion obtained in the previous section are discussed below.

Intuitively, the multiconfiguration structure of the ground state may be regarded as a mistake. Indeed, the ground state takes the lowest energy possible. Hence, how can a mixture of a lower energy state and a higher energy state yield a combined state whose energy is lower than either of the two single mono-configuration states? The answer to this question relies on a solid mathematical basis. Thus, a diagonalization of a Hermitian matrix reduces the lowest eigenvalue and increases the highest eigenvalue [12, see e.g. pp. 420–423]. Hence, *for a Hermitian matrix, any off-diagonal matrix element increases the difference between the corresponding diagonal elements*. It means that the smaller diagonal element decreases and the larger diagonal element increases. Since the Hamiltonian is a Hermitian operator, one concludes that if the Hilbert space basis yields a non-diagonal Hamiltonian matrix then the lowest eigenvalue "favors" eigenfunctions that are a linear combination of the Hilbert space basis functions.

It is shown in the previous section that the non-vanishing off-diagonal matrix elements rely on the two body Coulomb interaction between electrons. Thus, the tensor expansion of the interaction (4) casts the 2-body Coulomb interaction into a series of Legendre polynomials where $cos\theta_{12}$ is the polynomial's argument. Evidently, any physically meaningful interaction depends on the distance between the interacting particles. Hence, an expansion in terms of the Legendre polynomials can be obtained. This expansion proves that the mathematical procedure described in the previous section has a comprehensive validity [10, see p. 208]. Thus, what is found in the previous section for electrons in the He atom ground state also holds for quarks in the proton. Moreover, the proton is an extremely relativistic system of quarks and, as such, its spin-dependent interactions are expected to be quite strong. Evidently, spin dependent interactions make a contribution to off-diagonal matrix elements. On the basis of this conclusion, one infers that the proton's quark state must be described by a linear combination of many configurations.

A polarized proton experiment has been carried out where the instantaneous spin direction of quarks was measured [7]. The measurements have shown that *the total quark spin constitutes a rather small fraction of the proton's spin.* This result is in a complete agreement with the mathematical analysis carried out above. Thus, the relativistic proton dynamics indicates that the jj-coupling provides a better approach (and this is the reason for the usage of this notation here). In each quark configuration, spin and spatial angular momentum are coupled to a total single particle j-value and the Clebsch-Gordan coefficients determine the portion of spin-up and spin-down of the quark. Next, The relativistic quark state indicates that, unlike the case of the hydrogen atom, the lower part of the Dirac spinor of quarks is quite large. As is well known, if in the upper part of a Dirac spinor is $l = j \pm 1/2$ then its lower part is $l = j \mp 1/2$. Hence, different Clebsch-Gordan coefficients are used for the upper and the lower parts of the Dirac spinor. Furthermore, in different configurations, different Clebsch-Gordan coefficients are used for the single particle coupling of the three quarks to the total proton's spin and the overall weight of the spin-up and spin-down components takes a similar value. This argument indicates that the outcome of [7] is quite obvious and that if the experiment would have yielded a different conclusion where *quarks carry the entire proton's spin then this result should have been regarded as a real crisis of fundamental quantum mechanical principles.* This discussion also shows that the quite frequently used description of the results of [7] as "the proton spin crisis" is unjustified.

Computers are based on quantum mechanical processes that take place in solid state devices. Hence, it is clear that people who have established the laws of quantum mechanics had no access to the computational power of computers. For this reason, several approximations have been contrived in order to get an insight into atomic structure. A method that deals with configurations is called *central field approximation* [5, see p. 225]. Here, for every electron, the actual field of all other electrons is replaced by an approximate spherically symmetric radial field. Evidently, as explained in the third section, such a radial field does not cause a configuration mixture and, in this approximation, a single configuration is used for describing atomic states. This approach is frequently used in a description of the Mendeleev's periodic table [5, see pp. 240–247].

However, even in the early days of quantum mechanics, the central field approximation has been regarded as an approximation and people have constructed mathematical tools for treating the multi-configuration atomic structure which is known as the Wigner-Racah algebra of angular momentum. These mathematical tools have been used in the early days of

electronic computers [4] and the result is quite clear: *many configurations are required even for the simplest case of the ground state $J = 0$ of the 2-electron He atom and no single configuration plays a dominant role.* Today, this outcome is still known [6, see p. 116] but unfortunately not widely known. Thus, [6] is based on lectures delivered in a chemistry department. On the other hand, the birth and the long duration of the idea concerning *the proton spin crisis* prove that this fundamental physical issue is indeed not widely known. This paper has been written for the purpose of improving the present status.

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Discovery of Uniformly Expanding Universe

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Saul Perlmutter and the Brian Schmidt – Adam Riess teams reported that their Friedmann-model GR-based analysis of their supernovae magnitude-redshift data revealed a new phenomenon of "dark energy" which, it is claimed, forms 73% of the energy/matter density of the present-epoch universe, and which is linked to the further claim of an accelerating expansion of the universe. In 2011 Perlmutter, Schmidt and Riess received the Nobel Prize in Physics "for the discovery of the accelerating expansion of the Universe through observations of distant supernovae". Here it is shown that (i) a generic model-independent analysis of this data reveals a uniformly expanding universe, (ii) their analysis actually used Newtonian gravity, and finally (iii) the data, as well as the CMB fluctuation data, does not require "dark energy" nor "dark matter", but instead reveals the phenomenon of a dynamical space, which is absent from the Friedmann model.

1 Introduction

Observational determination of the time evolution of the scale factor $a(t)$ of the universe is fundamental to understanding the dynamics of the universe. Measurement [1, 2] of supernovae magnitude-redshifts provided that critical data, and it is a simple procedure to determine $a(t)$ from that data. A secondary process is then to test different dynamical theories of the universe against that data. However this did not happen, and not for the 1st time in the history of astronomy was one predetermined theory forced into the data fitting.

The 1st example was Ptolemy's fitting of his geocentric model of the solar system to the Babylonian planetary orbit data. This then required, and correctly so, that the orbits have epicycle components. This model persisted for some 1400 years, until the heliocentric model replaced the geocentric model, and for which the epicycle phenomenon then evaporated - it was merely an artifact of the incorrect geocentric model. It now appears that a similar confusion of data and model has reappeared in analysing the supernovae data, for again a simple and manifestly inadequate model of the universe, namely Newtonian gravity (NG), has been used. A generic model-independent analysis of the data reveals that the universe is undergoing a uniform expansion, see sect.2. However use of the Newtonian gravity model has resulted in a new collection of model-induced artifacts, namely "dark energy", "dark matter", and a claim that the universe expansion is accelerating. These artifacts also disappear once we use a model that replaces Newtonian gravity.

It is usually argued that General Relativity (GR) in the form of the Friedmann equation is superior to NG, and it was the Friedmann equation that was used in analysing the supernovae data [1, 2]. However in sect.3 we derive the Friedmann equation from NG in a few simple steps. This happens because GR was constructed as a generalisation of NG, and reduces to NG in the limit of low matter densities and low speeds. Alternatively, in sect.4, we show in a few simple steps, that the dynamical 3-space theory of space and gravity yields a uniformly expanding universe, and so dispenses with the "dark energy" and "dark matter" artifacts. The implication here, and in previous analyses of the dynamics of space itself, shows that NG is a flawed model of gravity, even at the level of laboratory measurements of G, bore-hole g anomalies, galactic rotation, and so on. So the Friedmann equation is based upon a flawed theory. This is in fact a major outcome of the observations of supernova events, and needs to be understood.

2 Model Independent Analysis Reveals Uniform Expansion

The scale factor $a(t) = r(t)/r(t_0)$; ($a(t_0) \equiv 1$ by definition), where $r(t)$ are galactic separations on a sufficiently large scale, and t_0 is the present moment age of the universe. It describes the time evolution of the universe assuming a homogeneous and isotropic description. In principle it may be directly extracted from magnitude-redshift data without the use of any particular dynamical model for $a(t)$. The redshift is $z = 1/a(t) - 1$, and the Hubble function is $H(t) = \dot{a}/a$. We define $H(z)$ by changing variables from t to z. A dimensionless luminosity distance is given by (see appendix)

$$d_L(z) = (1 + z) \int_0^z \frac{H_0 dz'}{H(z')}. \quad (1)$$

$d_L(z)$ takes account of the reduced photon flux and energy loss caused by the expansion. Then the magnitude-redshift observables are computable from $a(t)$

$$\mu(z) = 5 \log_{10} d_L(z) + m, \quad (2)$$

where m is determined by the intrinsic brightness of the SNe Ia supernova. In principle this can be inverted to yield $a(t)$, without reference to any dynamical theory for $a(t)$. A simple

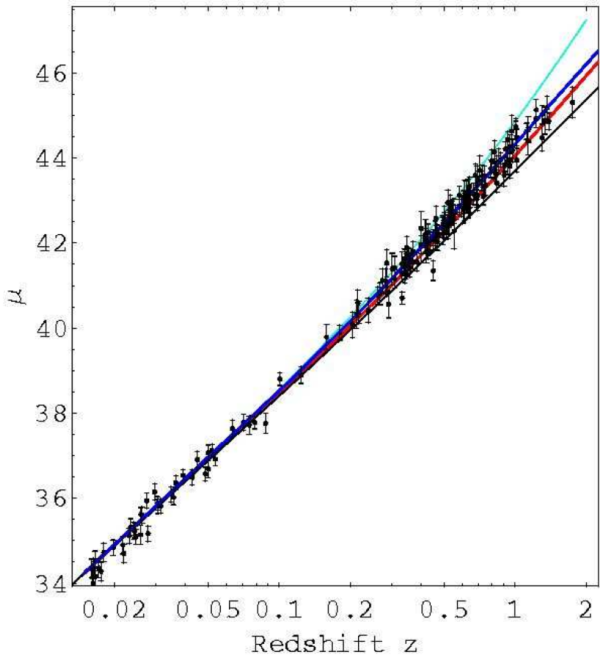

Fig. 1: Supernovae magnitude-redshift data. Upper curve (light blue) is "dark energy" only $\Omega_\Lambda = 1$. Next curve (blue) is best fit of "dark energy"-"dark-matter" $\Omega_\Lambda = 0.73$. Lowest curve (black) is "dark matter" only $\Omega_\Lambda = 0$. 2nd lowest curve (red) is generic uniformly expanding universe.

first analysis of the data tries a uniform expansion $a(t) = t/t_0$, which involves one parameter $t_0 = 1/H_0$, which sets the time scale. Fig.1 shows that this uniform expansion (shown by red plot) gives an excellent account of the data. We conclude that the supernovae magnitude-redshift data reveals a uniformly expanding universe. So why did [1, 2] report an accelerating expansion for the universe? The answer, according to the Nobel Prize briefing notes, is because "the evolution of the Universe is described by Einstein's theory of general relativity" [3]. To the contrary we argue that the data should be used to test possible theories of the universe, as in the usual scientific method, and not *a priori* demand that one theory, with *ad hoc* adjustments, be defined to be the only correct theory.

3 Newtonian Gravity Universe Model

The analysis in [1, 2] used the GR-based Friedmann equation for $a(t)$

$$\dot{a}^2 = \frac{8}{3}\pi G a(t)^2 \rho(t), \tag{3}$$

where $\rho(t)$ is the matter/energy density. However this equation follows trivially from Newtonian gravity. Consider a uniform density of matter moving radially with speed $v(r, t)$, at distance r, away from an origin. The kinetic + gravitational potential energy, with total energy E, of a test particle of mass m is given by

$$\frac{1}{2}mv^2 - \frac{GmM(r)}{r} = E, \tag{4}$$

where $M(r) = \frac{4}{3}\pi r^3 \rho$ is the mass enclosed within radius r - this follows simply from Newton's Inverse Square Law. Using $r(t) = a(t)r_0, v = \dot{r}$ and the so-called critical case $E = 0$, immediately gives (3). The reason for this simple derivation is that GR was constructed as a generalisation of NG that reduces to NG in the limit of low speeds and matter densities. So the Friedmann equation inherits all of the known failures of NG. As well the redshift z is a Doppler shift, caused by the motion of the source relative to the observer. Consider then some of the implications of (3): (i) if $\rho = 0$, i.e. no matter, then there is no expanding universe possible: $\dot{a} = 0$. This arises because (3) is about the effects of matter-matter gravitational attraction, and without matter there are no gravitational effects. (ii) (3) is not about the expansion of space, for it arises from NG in which matter moves through a Euclidean and unchanging space, (iii) (3) requires, at $t = t_0$, that

$$H_0^2 = \frac{8}{3}\pi G \rho_c, \tag{5}$$

where ρ_c is the so-called critical density. However (5) is strongly violated by the data: the observed baryonic matter density is some 20 times smaller than ρ_c, and so ρ must be padded out to satisfy (5), and (iv) (3) does not posses uniformly expanding solutions, unless $\rho \sim 1/a^2$, a form not considered in [1, 2]. To fit the data [1, 2] used the restricted *ad hoc* form

$$\rho(a) = (\frac{\Omega_M}{a^3} + \Omega_\Lambda)\rho_c, \tag{6}$$

where Ω_Λ is the "dark energy" composition parameter, and Ω_M is the "matter" composition parameter. There is no theoretical underpinning for this "dark energy". The above $H_0 - \rho_c$ (5) relationship requires that $\Omega_\Lambda + \Omega_M = 1$, resulting in a two parameter model: H_0 and Ω_Λ. Fitting the data, by solving (3), and then using (1) and (2), gives $\Omega_\Lambda = 0.73$, and so $\Omega_M = 0.27$. This fitting is shown in Fig. 1. Essentially $\Omega_\Lambda = 0.73$ is the value for which NG best mimics a uniformly expanding universe, despite its inherent weakness as a model of a universe. The known baryonic matter density, corresponding to $\Omega_m = 0.05$, then requires that $\Omega_M - \Omega_m = 0.22$ be interpreted as the "dark matter" composition. However (3) has another strange feature, namely that $a(t)$, as a consequence of the "dark energy" parametrisation, possess an exponential component: neglecting Ω_M, which becomes increasingly valid into the future we get

$$a(t) \sim e^{H_0 \sqrt{\Omega_\Lambda} t}. \tag{7}$$

The Nobel Prize for Physics in 2011 was awarded for the discovery of this "accelerated expansion of the universe", despite the fact that the model-independent analysis in sect. 2 shows no such effect.

4 Dynamical Space Universe Model

A newer dynamical model of space describes the velocity of this structured space, relative to an observer using coordinate system $\mathbf{r}$ and t, by [5]

$$\nabla\cdot\left(\frac{\partial\mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v}\right)+\frac{\alpha}{8}\left((trD)^2-tr(D^2)\right)+$$
$$+\frac{\delta^2}{8}\nabla^2\left((trD)^2-tr(D^2)\right)+...=-4\pi G\rho$$

$$\nabla\times\mathbf{v}=\mathbf{0},\quad D_{ij}=\frac{\partial v_i}{\partial x_j}. \tag{8}$$

The 1st term involves the Euler constituent acceleration, while the $\alpha-$ and $\delta-$ terms contain higher order derivative terms. This dynamical theory is conjectured to arise from a derivative expansion of a quantum foam theory of space. Laboratory, geophysical and astronomical data show that α is the fine structure constant, while δ appears to be a very small Planck-like length. Quantum theory determines the "gravitational" acceleration of quantum matter to be, as a quantum wave refraction effect,

$$\mathbf{g}=\frac{\partial\mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v}+(\nabla\times\mathbf{v})\times\mathbf{v}_R-\frac{\mathbf{v}_R}{1-\dfrac{\mathbf{v}_R^2}{c^2}}\frac{1}{2}\frac{d}{dt}\left(\frac{\mathbf{v}_R^2}{c^2}\right)+..., \tag{9}$$

where $\mathbf{v}_R=\mathbf{v}_0-\mathbf{v}$ is the velocity of matter relative to the local space. Substituting the Hubble form $\mathbf{v}(\mathbf{r},t)=H(t)\mathbf{r}$, and then $H(t)=\dot{a}/a$, we obtain

$$4a\ddot{a}+\alpha\dot{a}^2=-\frac{16}{3}\pi G a^2\rho. \tag{10}$$

This has a number of key features: (i) even when $\rho=0$, i.e. no matter, $a(t)\neq 0$ and monotonically increasing. This is because the space itself is a dynamical system, and the (small) amount of actual baryonic matter merely slightly slows that expansion, as the matter dissipates space. As well relation (5) no longer applies, and so there is no "critical density", (ii) the redshift z is no longer a Doppler shift; now it is caused by the expansion of the space removing energy from photons. Because of the small value of $\alpha=1/137$, the α term only plays a significant role in extremely early epochs, but only if the space is completely homogeneous*. In the limit $\rho\to 0$ and neglecting the α term, we obtain the solution $a(t)=t/t_0$. This uniformly expanding universe solution is exactly the form directly determined in sect.2 from the supernovae data. It requires neither "dark energy" nor "dark matter" – these effects have evaporated, and are clearly revealed as nothing more than artifacts of the NG model. The "accelerating expansion of the universe" in the future has also disappeared.

*Keeping the α term we obtain $a(t)=(t/t_0)^{1/(1+\alpha/4)}$

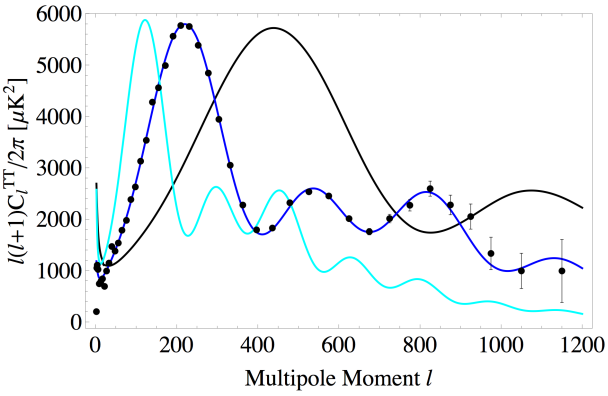

Fig. 2: CMB angular power spectrum for (i) $\Omega_\Lambda=1$ (light blue curve), (ii) = 0.73 (dark blue curve), and (iii) = 0 (black curve), confirming that the background space is uniformly expanding.

5 CMB Fluctuations

Another technique for determining the expansion rate of the universe is to use the Cosmic Microwave Background (CMB) temperature angular fluctuation spectrum. This spectrum is computed as a perturbation of the plasma relative to an assumed homogeneous background universe dynamical model. The background model used is the Friedmann equation (3). We show in Fig. 2 the angular fluctuation power spectrum from CAMB (Code for Anisotropies in the Microwave Background), [6, 7], for the same three values $\Omega_\Lambda=0$, 0.73 and 1, as also used in Fig. 1. However, as already noted in sect. 3, this homogeneous background dynamics is merely a Newtonian gravity model, with "dark energy" and "dark matter" used to pad out the critical density and mimic a uniform expansion. The Newtonian model and the dynamical 3-space model give the same age for the universe, 13.7 Gyr, as they both describe the same uniform expansion rate, with the minor variations in the Newtonian model expansion rate cancelling out. However they give different decoupling times, 0.38 Myr for the Newtonian model and 1.4 Myr for the dynamical 3-space. So it is important to note that the decoupling time is very model dependent.

6 Conclusions

The supernovae magnitude-redshift data is of great significance to cosmology. It reveals, using a model-independent analysis, that the universe is undergoing a uniform expansion. This represents a major challenge to theories of the universe, particularly as GR does not have such solutions. We have also noted that GR, via the Friedmann equation, is nothing more than Newtonian gravity applied to the gravitational force between matter, essentially with galaxies as that matter. To mimic the uniform expansion the canonical value $\Omega_\Lambda=0.73$ emerges by fitting the NG model to either the data, or more revealingly, by fitting to the dynamical 3-space the-

ory. However the *ad hoc* introduction of the "dark energy" parameter results in a spurious accelerating expansion. These spurious effects, "dark energy", "dark matter", and "accelerating expansion", are reminiscent of Ptolemy's epicycles when an incorrect model of the solar system was forced to fit the data, rather than using the data to test different models of the solar system. This recurring failure to use the scientific method resulted, in both cases, in deeply wrong theories being embellished and promoted as orthodoxy, with astronomers now committing major resources to "explaining" these new epicycles. The dynamical 3-space theory has been extensively tested, from bore hole g anomalies, to supermassive black holes and cosmic filaments. It gives a uniformly expanding universe without the introduction of any *ad hoc* parameters, and disagrees in general with Newtonian gravity, even in the low matter density, low speed limits, while nevertheless reproducing the NG restricted successes within the solar system. Introducing "dark matter" and "dark energy" amounts to the belief that Newton had correctly and completely described space and gravity some 300 years ago, requiring only the identification of new matter/energy. The supernova data is informing us that this is not so [8]. The use of the *ad hoc* parametrisation in (6) is not sufficiently general to give an unbiased fitting procedure, forcing an exponential growth term which is not present in the data.

7 Acknowledgments

We acknowledge the use of the Legacy Archive for Microwave Background Data Analysis (LAMBDA). We also acknowledge use of the CAMB (Lewis et al. 2000) package.

8 Appendix: Luminosity Distance

To extract $a(t)$ we need to describe the relationship between the cosmological observables: the apparent energy-flux magnitudes and redshifts, and in a model independent manner. We use the dynamical space formalism, although the result, in (1) & (15), is generic and was used in [1, 2]. First we take account of the reduction in photon count caused by the expanding 3-space, as well as the accompanying reduction in photon energy. To that end we first determine the distance travelled by the light from a supernova event before detection. Using a choice of embedding-space coordinate system, with $r = 0$ at the location of a supernova event at time t_1, the speed of light relative to this embedding space frame is $c + v(r(t; t_1), t)$, i.e. c wrt the space itself, where $r(t; t_1)$ is the photon embedding-space distance from the source. Then the distance travelled by the light at time t, after emission at time t_1, is determined implicitly by

$$r(t; t_1) = \int_{t_1}^{t} dt'(c + v(r(t'; t_1), t'), \tag{11}$$

which has the solution, on using $v(r, t) = H(t)r$,

$$r(t; t_1) = ca(t) \int_{t_1}^{t} \frac{dt'}{a(t')}. \tag{12}$$

This distance gives directly the surface area $4\pi r(t; t_1)^2$ of the expanding sphere and so the decreasing photon count per unit area on that surface. With $t \to t_0$ (and then dropping t_0 in the notation), $a(t_0) = 1$ and $a(t_1) = 1/(1 + z(t_1))$ we obtain

$$r(z) = c \int_0^z \frac{dz'}{H(z')}. \tag{13}$$

However because of the expansion the flux of photons is reduced by the factor $1/(1 + z)$ simply because they become spaced further apart by the expansion. The photon flux is then given by $\mathcal{F}_P = \mathcal{L}_P/4\pi(1 + z)r(z)^2$ where $\mathcal{L}_P$ is the source photon-number luminosity. However usually the energy flux is measured, and the energy of each photon is reduced by the factor $1/(1 + z)$ because of the redshift. Then the energy flux is, in terms of the source energy luminosity $\mathcal{L}_E$: $\mathcal{F}_E = \mathcal{L}_E/4\pi(1 + z)^2 r(z)^2 \equiv \mathcal{L}_E/4\pi r_L(z)^2$ which defines the effective energy-flux luminosity distance $r_L(z)$. Then the energy-flux luminosity effective distance is

$$r_L(z) = (1 + z)r(z) = c(1 + z) \int_0^z \frac{dz'}{H(z')} \tag{14}$$

The dimensionless "energy-flux" luminosity effective distance is then given by

$$d_L(z) = (1 + z) \int_0^z \frac{H_0 dz'}{H(z')}. \tag{15}$$

For the uniformly expanding universe $H(z) = (1 + z)H_0$ and $d_L(z) = (1 + z) \ln(1 + z)$.

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LETTERS TO PROGRESS IN PHYSICS

On the Epistemological Nature of Genius and Individual Scientific Creation

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This brief exposition summarizes a universally over-arching deepening of the epistemology of aesthetics (especially as regards the nature of Genius) as outlined in a particular section of the Author's work on an all-embracing, post-Kantian epistemological theory of Reality and the Universe called "The Surjective Monad Theory of Reality" (SMTR), which generalizes, in the utmost ontological sense, Kantianism, phenomenology, and a paradigm of Reality called "Reflexive Monism" (RM).

Most people, both eruditically trained and untrained, are profoundly mistaken in their belief about the nature of Genius, especially in relation to the mere prevalence of talent and the dominant structure of pedantry (i.e., a dominant world-paradigm of mass-education, as opposed to authentic individual education), the epistemological nature of the so-called "scientific research", and the entire psychologism thereof. By "psychologism", we mean an ultimately solipsistic, super-tautological basis that manages to present science and scientific-technological progress (let alone revolution in the sciences), among others, to the world at large in the image of a homogeneously working contingency of non-independent scientists, political factors, and industrial games, as opposed to single creative individuals in the profoundest sense.

Such a semi-popular image replete with "democratic-spiritism" (not to be confused with democracy in and of itself), which easily captures unassuming, aspiring talents into the underlying system, cannot be denuded for what it is, what it is not, and what is universally, utterly other than it, except by (advances in) epistemology. Until then, the utmost critical attitude towards the world of informative representations (e.g., in the sense of Wittgenstein), if not the most universal nature of philosophy, science, and art, is found among individual epistemic geniuses alone — who know just "what is what" absolutely independently of all "otherness".

In the sense of the post-Kantian epistemological theory of Reality outlined in [1], Genius is indeed not even a "superlative of talent" and is separated from all else by an entire world of noumena. In terms of the ontological, multi-teleological reality alluded to therein, which embraces also the eidetic-noumenal "surject" (or "qualon", which is beyond mere "omnijectivity" and "inter-subjectivity") in addition to the usual reflection ("object"), projection ("subject"), and annihilation ("abject") in a certain domain of epistemological dimensionality ("prefect"), Genius is said to be "noumenal-reflective" ("surjective"), while talent is termed "reflective-projective" ("phenomenal-reflexive"). Thus, by itself, the said epistemological framework qualifies itself as being post-Hegelian in its sector of dialectics: by the very presence of "surjection", Genius is beyond the usual triplicity of thesis, anti-thesis, and synthesis — and so beyond all multiplicity-dependent, contingent, linear progression.

The universal logic (i.e., meta-logic) thereof, by which our epistemological meta-structure surpasses Kantian philosophy and Socratic-Hegelian dialectics entirely is four-fold, anholonomic, and asymmetric in that the general surjective representation of a universal entity, as regards its "place" in Reality, is as follows:

(without, within, within-the-within, without-the-without).

Thus, for a given complete ontological entity A (and not merely a phenomenologically abstract and concrete entity), there exists the following four-fold eidetic representation:

$$\{\mathrm{A}\} = \{\mathrm{A},\ \text{non-A},\ \text{non-non-A},\ \text{none of these}\}\,.$$

The above, being "twice-qualified ontological", is not to be confused with both four-fold phenomenological Buddhist logic (of phenomena embedded in infinite contingency) and Whiteheadian process philosophy. Rather, the first two elements, i.e., A ("without") and non-A ("within") are of the phenomenological level (in the self-dual concrete and abstract sense): given an object of contemplation ("without"), it is impossible to discern its causal, formative "interior" ("within") without considering the abstract contingency (interconnectedness) of all possible phenomenal existents; while the last two ontologically, surjectively denote Universality ("within-the-within") and Reality ("without-the-without"), respectively. These four constituents are hereby called "ontological categories" for simplicity. Therefore, an entity or instance is called "universal" if and only if it is "four-fold eidetically qualified", and not just "two-fold phenomenologically qualified".

That which is surely universally qualified as such is the Universe itself, for which we have the following representation:

$$\{\text{the Universe}\} = \{\text{the Material Universe, the Abstract Universe, the Universe-in-itself, Reality}\}\,.$$

Meanwhile, for Thought itself, we have

$$\{\text{Thought}\} = \{\text{Thought, Anti-Thought, Unthought, Reality}\}\,,$$

i.e., the Universe-in-itself corresponds to Unthought (not to be confused arbitrarily with "irrationality") in the sense that the Universe as Unthought is a direct presentation ("surdetermination") of Reality and not a mere (phenomenological-reflective) representation, rendering Reality unthinkable in the first place, and so it is beyond both the Material Universe and the Abstract Universe, which are the domains of the traditional sciences (with respect to which, therefore, progress always seems endlessly "infinite"). Note that, especially when an arbitrary "thought" other than a "truly universal thought" (peculiar to Genius) is considered, "thought" and "anti-thought" always exist in a single phenomenological contingency while their directions of causality ("momenta") differ.

This way, the Cartesian dictum, "I think therefore I am", should be replaced by a twice-qualified ontological thinker (and universal observer) as follows: "I think therefore I am, I am not, I am not-not, and none of these".

Accordingly, Reality is such that: 1. It is One-Singular and cannot be reduced to Unreality simply because "Reality-in-itself does not mingle with Unreality" in the first place, whether by necessity or by chance (i.e., unlike arbitrary phenomenological entities mingling across time and space), for otherwise (noumenal and phenomenal) "things", even the Universe itself, would cease to exist "as one and at once" (at one "Now") — and both Reality and Unreality too would be Not —, which is absurd in a four-fold manner: before, during, after, and without time. 2. It contains "things" and yet these "things" contain it not, not merely in the spatio-temporal sense but in the sense that Reality, as Moment, always precedes and surpasses "things" behind, within, and ahead of them, and "none of these at all". 3. The "distance", i.e., meta-logical foliage, between the four ontological categories is thus asymmetric and anholonomic: phenomenally approaching Reality (M) from the transitive entirety of phenomena (O) will be substantially different from approaching such phenomenal entirety (O) directly from Reality (M). In other words: $\{OM\} \neq \{MO\}$. 4. There exists a meta-logical exception in that there are surjective instances with respect to which Reality is their exception just as they are Reality's exceptions (singularities) everywhere in the Universe, i.e., they, unlike others, exist in sheer eidetic-noumenal symmetry with Reality and the Universe. Such an instance is none other than Genius. 5. In the surjective-deterministic sense of Reality, there exists an ultimate observer in the twice-qualified ontological sense of Genius, as opposed to an arbitrary observer: whether or not a leaf falls in a forest with apparently no observer around, it still falls simply because the Universe, in its capacity as an ultimate observer, observes it. This is because the universal meta-structure is such that the Universe is without both "inside" and "outside" with respect to the (noumenal) entirety of the laws of Nature. This saves both common-sense objectivity while, up to such non-arbitrary ontological qualification, keeping intact the unification of observers and observables as found in both quantum mechanics and the monad formalism of General Relativity (e.g., of Abraham Zelmanov). Otherwise, without such universal determination, one is left with mere surrealism and omnijectivity, which, as we have said, can in no way be a direct presentation of Reality-in-itself.

All that, in a word, is symbolically-noumenally written in a single "Reality equation" as follows:

$$\mathbf{M{:}\ N\big(U(g,\, dg)\big) \sim S}$$

where M stands for Reality (Reality-in-itself, "Being-qua-Being"), N for the Qualic Monad (Reality's entirely pre-reflexive, self-singular presentation of itself, i.e., with or without the Universe and reflective world-foliages, or "Multiverse"), U for the noumenal Universe (the Universe-in-itself), (g, dg) for Surjectivity and infinite self-differentiation (isomorphic to Genius — which is none other than surjective, archetypal insight and motion — and the "interior" of the Universe), and S for Suchness (Eidos).

Thus, by "Universe" — in this truly qualified sense of Reality — we always mean "Such Universe", where "Such" is "Twice-That/There" (in terms of the phenomenal "without" and the noumenal "without-the-without") and "Universe" is "Twice-This/Here" (in terms of the phenomenal "within" and the noumenal "within-the-within").

In this epistemology, the Universe — in the likeness of Reality itself — is therefore most tangible and most elusive at once: it is "that which draws near from farness and draws far from nearness". It takes Genius to truly comprehend this as it is, for the relationship between the Universe and Genius in this respect is like that between the entire cosmos and the monopolar meta-particle.

Such is how our framework generalizes Kantianism (and what not) by the presence of the self-singular monad ("surject" or "qualon", i.e., the ultimate pre-reflexive singularity) free of the inconsistent inner state of "singularity in and of multiplicity" when it comes to phenomenologically defining traditional "Kantian oneness" (due to which Kantianism ultimately fails to distinguish between — or simply transcend — "a thing-in-itself" and "another thing-in-itself", let alone between all noumena). In addition, it also effortlessly surpasses the analytical rigor of Wittgensteinian logic and eradicates all discrepancies between "essentialism" and "existentialism" on a highest possible ontological level.

As such, Genius belongs to a self-singular nature (self-constitution) of not just psychological thought, but also of Reality itself, independently of the entire contingency (and, often, over-determination) of tautologically constructed world-representations by the majority of sentient beings. Such strictly individual determination, of Genius, is thus called "surjective". This, while talent is always info-cognitively co-dependent on the entirety of prevailing contingencies, i.e., on the way a specific world is represented by them as "multiple intelligences" (through theses and anti-theses).

In other words, with respect to the Universe, Genius is Reality's very exception just as Reality is the very exception of Genius. Just as Reality is One-Singular beyond reducibility and reflexivity (mere reflection and projection), so is Genius, and so is the "mirror", i.e., the mirror in which the surjective instance of Genius appears: the Universe itself. As such, unlike the case of talent, there is indeed no such a thing as "mathematical genius", "physical genius", "philosophical genius", "musical genius", etc. as people are commonly, partially, phenomenally used to these terms. Rather, Genius is always universal and, by that very universality, it is solitary and chanceless: such is the nature of universal creation known as art, which is the quintessence (sine qua non) of genuine philosophical, artistic, and scientific creation.

In physics especially, the universal weight of an instance of scientific creation by an individual of Genius inevitably differs from the rest of physicists simply because the former moves — without residue and mere chance — as an epistemically solitary artist at the very universal level of "science-in-itself", and thus at the Universal Moment, by whose act the artist is immensely self-rewarded without even seeking recognition other than the necessity to move as the Universe categorically moves from the noumenal category to the phenomenal domain, while at best the latter is merely tautologically interested in "the problems that are important according to others" — ever at the risk of genuine originality (although, as we have seen, Genius is not a matter of merely being situational, but of the pan-Kierkegaardian infinite single-mindedness of "I cannot do otherwise", in contrast to talent).

Hence, silently in the face of Reality, Genius happens to the Universe as much as the Universe happens to it, while others can hardly notice, let alone imbibe, this epistemological degree of universal solitariness.

That is, to paraphrase Einstein somehow,

> "True science, if not art itself, consists in the following: apply yourself entirely and fearlessly to what deeply interests you the most, and not simply to what others — no matter who — are interested in, as this is between you and the Universe, not you and people. This is because every true philosopher (or profound thinker and creator), who truly understands his own moments, has his own Kant".

Of course, depending on the epistemological dimensionality of a given human endeavor or science, there are instances where "working as a group" is important and essential to progress (e.g., medicine, experimental psychology, and engineering). But in fundamental abstract sciences, as fundamental as they are in relation to art and philosophy, there should be no excuse as to the arbitrary, non-epistemological "peer-group treatment" and "machination" to which true individual geniuses are often subject, precisely because such individuals alone carry the very archetype of Universality and Revolution, which is absolutely not a matter of societal training and progress. Intrinsically, such an individual may indeed refuse the entirety of conventions of a particular society of people and their agendas in order to infinitely eye the noumenal-creative "science-in-itself", instead of just participating in "big scientism" and its often excessive relative loudness.

For instance, aside from the creation of fundamental theories or mathematical methods, the eminent general relativist who spear-headed the Soviet cosmological school, Abraham Zelmanov, is said to have regarded writing mere academic articles as a "waste of time" [5]. Also Einstein himself is known to have principally disregarded the anonymous "peer-review" system prevalent in the American system, as opposed to the way things were done rather transparently, epistemologically, and dialectically in Europe at the time his theories flourished: so long as there are no mathematical and other fundamental flaws in a submitted scientific thesis containing some genuine novelty, a corresponding anti-thesis would simply be presented by the scientific editor(s), and thereafter a common synthesis should likely be reached by both the individual scientist and the universally capable editor(s): such is the epistemologically universal way of disseminating novel scientific ideas and progress, and of championing true academic freedom, as greatly opposed to all superficial excuses (especially those made by fallible, anonymous observers). It was also Einstein's single-mindedness which made him unable to accept "quantum theory as Copenhagen sees it", strongly believing in a more deterministic (geometric) fashion thereof — a "fate" he shared with even de Broglie (who envisioned a kind of hidden "thermostat medium" in quantum physics) and Bohm (with his hidden-variable quantum theory), among others.

This, while mere "crackpots" are easily seen in broad daylight for themselves, and yet Genius is not even visible in the blazing sun of the day as in the mirrorless depths of the night — unless by way of sheer deliberation on the part of the individual of Genius himself. Indeed, of this — and after a lengthy, peripheral epistemic discourse and logical ascension — Wittgenstein himself would have said, "Up there, I am senseless: you must understand me senselessly". (See, e.g., [6]; during his entire solitary life, Wittgenstein only cared to produce two condensed philosophical works — each being a self-complete fundamental treatise written in a very unorthodox style — instead of writing mere philosophical "documentaries".)

However, the situation with "Genius and people" is rather helpless in any age due to the anholonomic, asymmetric nature of Genius — and the entire Universe itself — with respect to the rest of otherness, of which individuals of Genius are acutely conscious: just as the distance between Reality and "things" is not the same as that between "things" and Reality, as we have seen, the distance between Genius and people is not the same as that between people and Genius. Thus, mere sense-projection often only makes things worse.

To understand Genius, one must understand the noumenal Universe within its very own solitary instant, while most people, merely existing in groups and in definite contingency of both stances of the "dogmatist" (of objective dogmatism) and the "relativist" (of subjective relativism), are still far away from such cognizance, not just in the phenomenal-progressive sense, but in the entire ontological-noumenal sense. Still, one must know the noumenal even better than Kant himself understood it (and his entire epistemology), hence the phrase, "to understand Kant is to simply surpass him, there is no other way". Needless to say, the same seems to hold for most known physical theories as well — such as relativity and quantum theory, — especially in terms of the truly epistemological-universal construction of quantum gravity and unified field theories.

Indeed, while some of the known geniuses of the past are rather belatedly celebrated by people today (only to superficially project themselves on the past and to aggrandize their own sense of historical continuity as such), they always tend to neglect the geniuses of the present. This is precisely because they themselves, no matter how talented and bright, are not geniuses and have no substantial resemblance with them whatsoever: they are merely the product of the age. It is in this rather secluded Schopenhauerian-Weiningerian sense and infinite, silent understanding that Genius, more than others, embraces tragedy willingly: he is absolutely not the product of the age in the first place and he suffers most intuitively amidst people.

Hence, in any cosmic epoch, the so-called "Renaissance" is that infinitely solitary period of Genius before everyone else is capable of naming it, and not merely its subsequent, timely crumbs as received by a particular culture (society). It is the "mysterious" (as Einstein would have called it), not "public space".

A man of Genius is simply a universal volunteer on the canvas of Reality, without ulterior motives whatsoever, and without him, Reality would never "archetypally act upon itself" in and of the Universe: as such, he is most capable of infinite differentiation ("noema" and creation) peculiar to his singular Genus alone. Such Genus ("Kudos") is transcendent — not simply parallel or anti-parallel — with respect to all species.

As long as the four-fold logic behind Reality, the Universe, the manifold world-imagery, and Genius is not realized, an "objective dogmatist" will always fall into a "subjective relativist" (and mere sophist) soon enough, and vice versa, for the horizon-forming duality of phenomenological things remains as such, according to traditional "two-dimensional" (or "two-and-a-half" at most) eruditic logic. Such, then, only serves to yield a fallible observer, of whom Genius has no need whatsoever. In this sense, art is indeed most suitable to most geniuses than is academic science, precisely due to the more solitary noumenal-epistemological nature (richness) of art and its practicality at large. But, whenever such a universal mind appears in scientific territories, one must intimate the art of it all, without any "sophisticated pretention" whatsoever, rather than simply dismiss the emergent qualic unorthodoxy peculiar to Genius (for, as history has shown, such only results in one's shameful chagrin in the face of Reality, whether immediately or eventually), of which that one has no true understanding whether in short or at length. (In this respect, one can simply imagine Kant and Goethe — rather than Euler and Gauss — doing some particular sciences, apart from philosophy and art, and the predictable neglect and cold calculation of those who feel their territories have been violated. Fortunately, this particular case involving the two men and the rest of the world does not seem to have taken place.)

Undoubtedly, the foregoing epistemological discourse fully capable of mirroring "worlds", "anti-worlds", and "non-worlds", (by "world", of course we also mean "thought" or "paradigm") from the universal standpoint of Reality itself, is particularly relevant to the championing of scientific human rights as outlined in [2] as well as to the importance of aprioristic and dialectical thinking in physics (and science in general) as reflected, e.g., in [3] and [4].

All that — the Universe itself — is inevitably opposed to mere communalism, especially in the post-modern era of "big scientism".

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LETTERS TO PROGRESS IN PHYSICS

From the Chloride of Tungsten to the Upper Limit of the Periodic Table of Elements

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Experimental study of the physical chemical properties and the technology of manufacturing chemically clean hexachloride of tungsten has led to unexpected results. It was found that each element of the Periodic Table of Elements has its own hyperbola in the graph "molecular mass — content of the element". The hyperbolas differ according to the atomic mass of the elements. Lagrange's theorem shows that the tops of the hyperbolas approach to an upper limit. This upper limit means the heaviest element, which is possible in the Table. According to the calculation, its atomic mass is 411.66, while its number is 155.

1 Introduction

In the early 1960's, I and my research group worked in the Department of Rare, Radioactive Metals and Powder Metallurgy at Moscow Institute of Steel and Alloys, Russia. We looked for a better technology of manufacturing the chemically clean hexachlorid of tungsten (WCl_6) through chlorination of ferrotungsten. Then, in the 1970's, I continued this experimental research study at the Baikov Institute of Metallurgy, Russian Academy of Sciences.

Our main task in this experimental search was to obtain a purely oxygen-free product. Because the raw material we worked with was resented as a many-component gaseous mix, we studied behaviour of the vaporous medleys during filtering them by saline method, distillation, and rectification. As a result, the percent of mass of the metal we have obtained in vaporous medley was 99.9% for W, 20.0% for Mo, 2.0% for Fe [1–3].

After cleaning the obtained condensate with the aforementioned methods, we have found a small inclusion of the chloride compound of tungsten in it. This chloride compound of tungsten differs from the hexachloride of tungsten in colour and the boiling temperature, which was 348°C for WCl_6, 286°C for WCl_5, and 224°C for $WOCl_4$ [4]. The cleaned hexachloride of tungsten recovers to the powder metallic state by hydrogen in the boiling layer, in plasma, precipitates as a thin cover on a base in use. It is used for manufacturing alloys with other metals through metalthermic method, etc. [5].

2 Results

In development of this technology, it was found that the theoretical (expected) results of the chemical analysis of the vaporous medleys do not match the experimental results for a little. This occurred due to some quantity of WO_2Cl_2 and $WOCl_4$ obtained in the process, which were used further for manufacturing a high clean WO_3 [6]. In order to keep control on the product of the chemical reactions, we have drawn dependencies of the content of tungsten, chlorine, and oxygen in the compounds (per one gram-atom of each element). This is necessary because, for example, the common quantity of the chloride of tungsten in chlorides is presented with a broken line (see Fig. 1) whose mathematical equation is impossible. As was found, after our Fig. 1, the arc of the content of tungsten is presented with an equilateral hyperbola $Y = K/X$ wherein its different compounds (in particular WO_3) are located. In analogy to this graph, the respective arcs were obtained for chlorine and oxygen, which appeared as hyperbolas as well.

Further checking for the possibility of creating similar functions for the other chemical elements manifested the fact that each element of the Periodic Table of Elements has its own hyperbola, which differs from the others according to the atomic mass of the element. As an example, Fig. 2 shows the hyperbolas created for the elements of Group 2, including the hypothetical elements No.126 and No.164. As is known, an equilateral hyperbola is symmetric with respect to the bisector of the angle XOY in the first quarter. Besides, the bisector coincides with the real axis, while the point of intersection of it with the hyperbola (the top point) is determined as the square root from $K(X_0 = Y_0)$. Respectively, for instance, the top point of the hyperbola of beryllium (atomic mass 9.0122) is located at $X_0 = Y_0 = 3.00203$.

In chemistry, it is commonly assumed to calculate the quantity of a reacted element in the parts of unit. Therefore, the hyperbola of each element begins from the mass of the element and $Y = 1$. From here, through Lagrange's theorem, we calculate the top of the hyperbola of beryllium: $X = 60.9097$, $Y = 0.14796$. Comparing the obtained coordinates, it is easy to see that $X/X_0 = 20.2895$ and $Y_0/Y = 20.2895$, which is the inverse proportionality with a respective scaling coefficient. Tangent of the angle of inclination of the real axis in the other (scaled) coordinates is $Y/X = 0.14796/60.9097 = 0.00242917$. The scaling coefficient al-

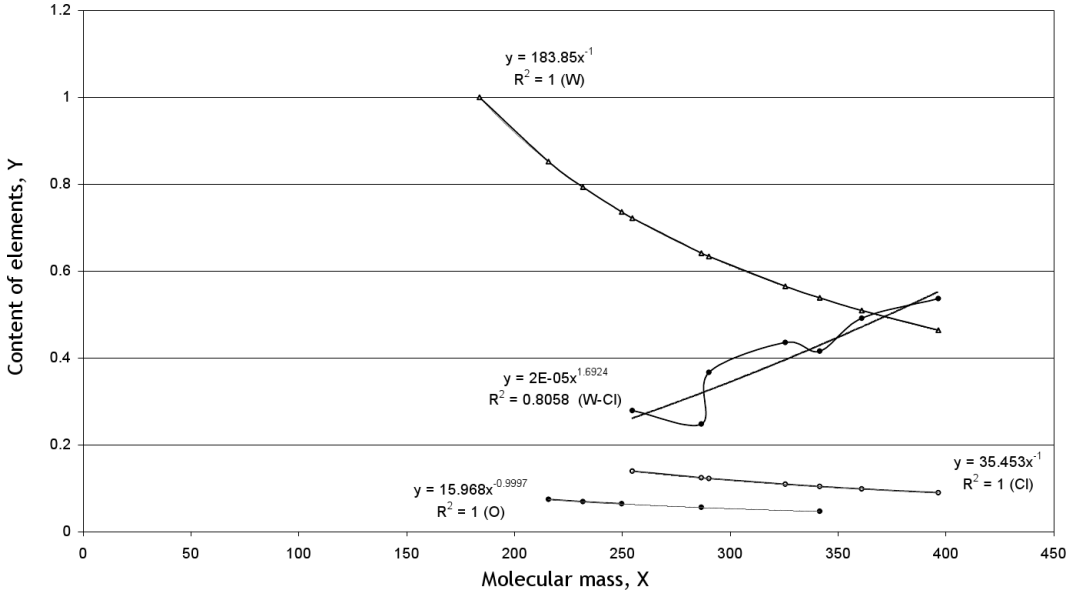

Fig. 1: The common quantity of the chloride of tungsten in chlorides.

lowed us to create a line joining the tops of the hyperbolas, located in the real axis (see Fig. 3). This is a straight crossing the line $Y = 1$, where the atomic and molecular masses of an element described by the hyperbolas are equal to each other ($K = X$). This is only possible if the origin of the hyperbola and its top meet each other at a single point where the content Y takes maximal numerical value (according to the equation $Y = K/X$). Atomic mass of this «ultimate» element, determined by the crossing point, was calculated with use of the scaling coefficient and the tangent of inclination of the real axis: $X = Y/\tan\alpha = 1/0.00242917 = 411.663243$. This calculated element is the last (heaviest of all theoretically possible elements) in the Periodic Table of Elements because Y cannot exceed 1. The second important characteristic of the element – its atomic number – was calculated through the equation of the exponent $Y = 1.6089\exp^{1.0993x}$ ($R^2 = 0.9966$). The calculated number of the last element is 155. With use of these equations, the respective parameters of all other elements of the Periodic Table can be calculated, including in the interval of super-heavy elements No.114–No.155 [7, 8].

3 Discussion

We see that on the basis of the initially experimental studies of the chloride of tungsten, a new law was found in the Periodic Table of Elements. This is the hyperbolic law, according to which the content Y of any element (per 1 gram-atom) in any chemical compound of a molecular mass X can be described by the equation of the positive branches of an equilateral hyperbola of the kind $Y = K/X$ (where $Y \leqslant 1$ and $K \leqslant X$). The hyperbolas of the respective chemical elements lie in the order of the increasing nuclear charge, and have a common real axis which meets their tops. The tops, with distance from the origin of the coordinates, approach the location $Y = 1$ and $K = X$ wherein atomic mass takes its maximally possible numerical value, which indicates the last (heaviest) element of the Periodic Table.

It should be noted that the new dependencies we pointed out here have provided not only better conditions of applied research, but also a possibility for re-considering our views on the conditions of synthesis of super-heavy elements. If already in 2003 theoretical physicists discussed properties of elements with number near 400 whose nuclei contain until 900 neutrons each [9], in February 2009, after primary publication of our studies, they discuss the elements with numbers not higher than 150–200 [10].

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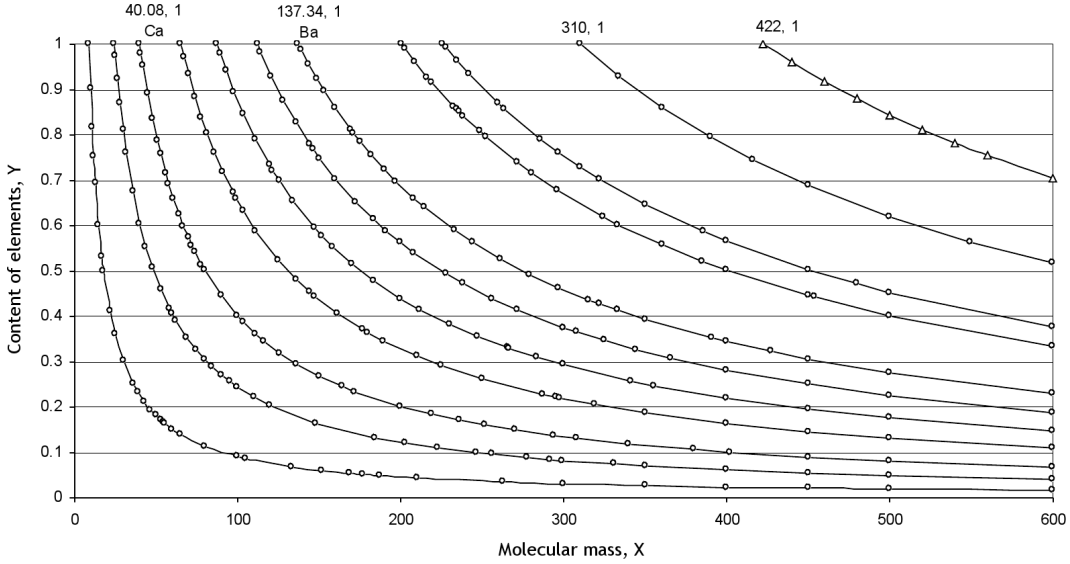

Fig. 2: The hyperbolas created for the elements of Group 2, including the hypothetical elements No.126 and No.164.

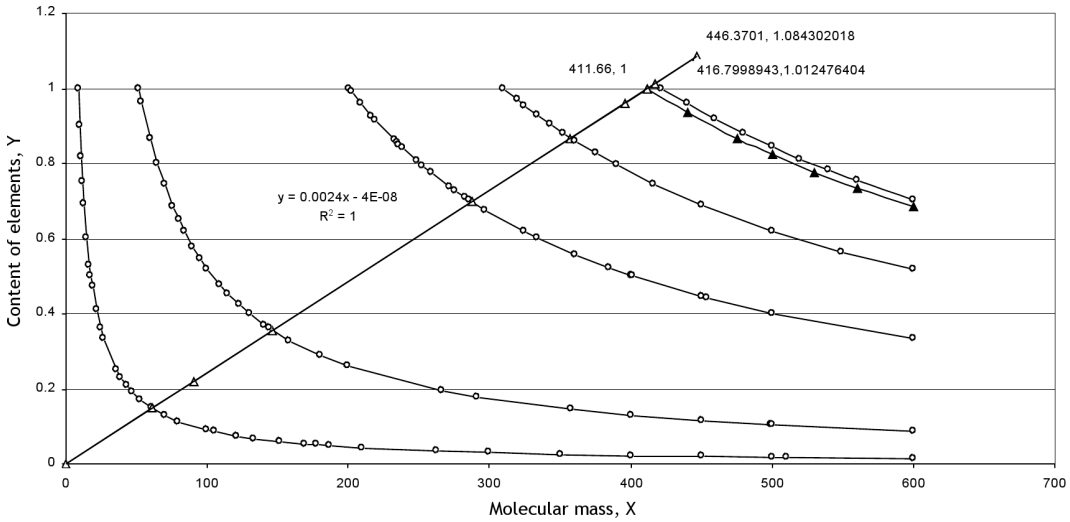

Fig. 3: The upper limit of the Periodic Table of Elements.

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LETTERS TO PROGRESS IN PHYSICS

Superluminal Physics and Instantaneous Physics as New Trends in Research

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In a similar way as passing from Euclidean Geometry to Non-Euclidean Geometry, we can pass from Subluminal Physics to Superluminal Physics, and further to Instantaneous Physics. In the lights of two consecutive successful CERN experiments with superluminal particles in the Fall of 2011, we believe that these two new fields of research should begin developing.

1 Introduction

Let's start by recalling the history of geometry in order to connect it with the history of physics.

Then we present the way of S-denying a law (or theory) and building a spectrum of spaces where the same physical law (or theory) has different forms, then we mention the S-multispace with its multistructure that may be used to the Unified Field Theory by employing a *multifield*.

It is believed that the S-multispace with its multistructure is the best candidate for 21st century *Theory of Everything* in any domain.

2 Geometry's history

As in Non-Euclidean Geometry, there are models that validate the hyperbolic geometric and of course invalidate the Euclidean geometry, or models that validate the elliptic geometry and in consequence they invalidate the Euclidean geometry and the hyperbolic geometry.

Now, we can mix these geometries and construct a model in which an axiom is partially validated and partially invalidated, or the axiom is only invalidated but in multiple different ways [1]. This operation produces a degree of negation of an axiom, and such geometries are hybrid. We can in general talk about the *degree of negation of a scientific entity* P, where P can be a theorem, lemma, property, theory, law, etc.

3 S-denying of a theory

Let's consider a physical space S endowed with a set of physical laws L, noted by (S, L), such that all physical laws L are valid in this space S.

Then, we construct another physical space (or model) S_1 where a given law has a different form, afterwards another space S_2 where the same law has another form, and so on until getting a spectrum of spaces where this law is different.

We thus investigate spaces where anomalies occur [2].

4 Multispace theory

In any domain of knowledge, multispace (or S-multispace) with its multistructure is a finite or infinite (countable or uncountable) union of many spaces that have various structures. The spaces may overlap [3].

The notions of multispace (also spelt multi-space) and multistructure (also spelt multi-structure) were introduced by the author in 1969 under his idea of hybrid science: combining different fields into a unifying field (in particular combinations of different geometric spaces such that at least one geometric axiom behaves differently in each such space), which is closer to our real life world since we live in a heterogeneous multispace. Today, this idea is accepted by the world of sciences. S-multispace is a qualitative notion, since it is too large and includes both metric and non-metric spaces.

A such multispace can be used for example in physics for the Unified Field Theory that tries to unite the gravitational, electromagnetic, weak and strong interactions by constructing a *multifield* formed by a gravitational field united with an electromagnetic field united with a weak-interactions field and united with a strong-interactions field.

Or in the parallel quantum computing and in the mu-bit theory, in multi-entangled states or particles and up to multi-entangles objects.

We also mention: the algebraic multispaces (multi-groups, multi-rings, multi-vector spaces, multi-operation systems and multi-manifolds, also multi-voltage graphs, multi-embedding of a graph in an n-manifold, etc.) or structures included in other structures, geometric multispaces (combinations of Euclidean and Non-Euclidean geometries into one space as in S-geometries), theoretical physics, including the Relativity Theory [4], the M-theory and the cosmology, then multi-space models for p-branes and cosmology, etc.

The multispace is an extension of the neutrosophic logic and set, which derived from neutrosophy. Neutrosophy (1995) is a generalization of dialectics in philosophy, and takes into consideration not only an entity <A> and its opposite <antiA> as dialectics does, but also the neutralities <neutA> in between. Neutrosophy combines all these three <A>, <antiA>, and <neutA> together. Neutrosophy is a metaphilosophy.

Neutrosophic logic (1995), neutrosophic set (1995), and

neutrosophic probability (1995) have, behind the classical values of truth and falsehood, a third component called indeterminacy (or neutrality, which is neither true nor false, or is both true and false simultaneously — again a combination of opposites: true and false in indeterminacy).

Neutrosophy and its derivatives are generalizations of the paradoxism (1980), which is a vanguard in literature, arts, and science, based on finding common things to opposite ideas (i.e. combination of contradictory fields).

5 Physics history and the future

a) With respect to the size of space there are: *Quantum Physics* which is referring to the subatomic space, the *Classical Physics* to our intuitive living space, while *Cosmology* to the giant universe;

b) With respect to the direct influence: the *Locality*, when an object is directly influenced by its immediate surroundings only, and the *Nonlocality*, when an object is directly influenced by another distant object without any interaction mediator;

c) With respect to the speed: the *Newtonian Physics* is referred to low speeds, the *Theory of Relativity* to subluminal speeds near to the speed of light, while *Superluminal Physics* will be referred to speeds greater than c, and *Instantaneous Physics* to instantaneous motions (infinite speeds).

A physical law has a form in Newtonian physics, another form in Relativity Theory, and different form at Superluminal theory, or at Infinite (Instantaneous) speeds — as above in the S-Denying Theory spectrum.

We get new physics at superluminal speeds and other physics at a very very big speed ($v \gg c$) speeds or at instantaneous (infinite) traveling.

At the beginning we have to extend physical laws and formulas to superluminal traveling and afterwards to instantaneous traveling.

For example, what/how would be Doppler effect if the motion of an emitting source relative to an observer is greater than c, or $v \gg c$ (much greater than c), or even at instantaneous speed?

Also, what addition rule should be used for superluminal speeds?

Then little by little we should extend existing classical physical theories from subluminal to superluminal and instantaneous traveling.

For example: if possible how would the Theory of Relativity be adjusted to superluminal speeds?

Lately we need to found a general theory that unites all theories at: law speeds, relativistic speeds, superluminal speeds, and instantaneous speeds — as in the S-Multispace Theory.

6 Conclusion

Today, with many contradictory theories, we can reconcile them by using the S-Multispace Theory.

We also propose investigating new research trends such as Superluminal Physics and Instantaneous Physics. Papers in these new fields of research should be e-mailed to the author by July 01, 2012, to be published in a collective volume.

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LETTERS TO PROGRESS IN PHYSICS

A More Elegant Argument that *P≠NP*

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In April 2011, Craig Alan Feinstein published a paper in *Progress in Physics* entitled "An elegant argument that $P \neq NP$". Since then, Craig Alan Feinstein has discovered how to make that argument much simpler. In this letter, we present this argument.

In April 2011, I published a paper in *Progress in Physics* entitled "An elegant argument that $P \neq NP$" [1]. Since then, I have discovered how to make that argument much simpler. In this letter, I present this argument.

Consider the following problem: Let $\{s_1, \ldots, s_n\}$ be a set of n integers and t be another integer. We want to determine whether there exists a subset of $\{s_1, \ldots, s_n\}$ for which the sum of its elements equals t. We shall consider the sum of the elements of the empty set to be zero. This problem is called the SUBSET-SUM problem [2].

Let $k \in \{1, \ldots, n\}$. Then the SUBSET-SUM problem is equivalent to determining whether there exist sets $I^+ \subseteq \{1, \ldots, k\}$ and $I^- \subseteq \{k+1, \ldots, n\}$ such that

$$\sum_{i \in I^+} s_i = t - \sum_{i \in I^-} s_i \, .$$

There is nothing that can be done to make this equation simpler. Then since there are 2^k possible expressions on the left-hand side of this equation and 2^{n-k} possible expressions on the right-hand side of this equation, we can find a lower-bound for the worst-case running-time of an algorithm that solves the SUBSET-SUM problem by minimizing $2^k + 2^{n-k}$ subject to $k \in \{1, \ldots, n\}$.

When we do this, we find that $2^k + 2^{n-k} = 2^{\lfloor n/2 \rfloor} + 2^{n-\lfloor n/2 \rfloor} = \Theta(\sqrt{2^n})$ is the solution, so it is impossible to solve the SUBSET-SUM problem in $o(\sqrt{2^n})$ time with a deterministic and exact algorithm. This lower-bound is tight [1]. And this conclusion implies that $P \neq NP$ [2].

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