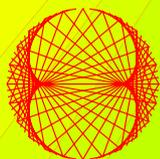


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The Stability of Electron Orbital Shells based on a Model of the Riemann-Zeta Function

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It is shown that the atomic number Z is prime at the beginning of the each s^1 , p^1 , d^1 , and f^1 energy levels of electrons, with some fluctuation in the actinide and lanthanide series. The periodic prime number boundary of s^1 , p^1 , d^1 , and f^1 is postulated to occur because of stability of Schrodinger's wave equation due to a fundamental relationship with the Riemann-Zeta function.

1 Introduction

It has been known that random matrix theory, and in particular a Gaussian Unitary Ensemble (GUE), can be used to solve the eigenvalue states of high- Z nuclei which would otherwise be computationally impossible. In 1972, Freeman Dyson and Hugh Montgomery of the University of Michigan realized that the values in the GUE matrix used in predicting energy levels of high- Z nuclei where similar to the spacing of zeros in the Riemann-Zeta function [1]. Prior to these discoveries, the use of approximation in traditional quantum mechanical models was well known and used, such as the Born-Oppenheimer method [2]. These approaches experienced problems at high- Z levels where many interacting factors made approximation difficult. The remaining question as to why the periodicity of zeros from the Riemann-Zeta function would match the spacing of energy levels in high- Z nuclei still remains a mystery, however.

It is the goal of this paper to explain the spacing of energy levels in electron orbital shells s^1 , p^1 , d^1 , and f^1 , where these designations represent the first electron to occupy the s , p , d and f shells. The first electron in each of these shells is an important boundary where new electron orbital shells are created in the atomic structure. The newly created shell is dependent upon the interaction of many electrons in the previous orbital shells that are filled much like the many body problems of gravitational masses. The first electron in each of the s , p , d and f shells is therefore hypothesized to represent a prime stability area where a new shell can form within the many-electron atom without significant perturbation to previous shells. With enough computational power the interaction of electrons in any combination of orbital shells can be computed through multiple manipulations of a system of Schrödinger's equations, but even the present numerical methods for this approach will use rough approximations due to the complexity of several non-linear equations and their solutions.

Choudhury and Pitchers [3] have used a configuration-interaction method of computation for many electron atoms where Schrödinger's equation is reduced to a system of lin-

ear homogenous equations. They then argue that the energy eigenvalues obtained by truncating this linear set of equations will converge, in the limit, to those of the original system. They show that this approximation holds true for two-electron atoms, but they note that variations start occurring for the three or more electron atom. These approximation methods are difficult enough for two or three electron atoms but for many-electron atoms the approximation methods are uncertain and are likely to introduce errors.

It is therefore proposed that the final result of the many-electron atom be first evaluated from the standpoint of the Riemann-Zeta function so that a simplifying method of working back to a valid system of Schrödinger's equations can hopefully be obtained. To justify this approach, the atoms for each of the newly filled s^1 , p^1 , d^1 , and f^1 shells are examined to show a potential relation between the spacing of the non-trivial zero solutions of the Riemann-Zeta function, where the argument s in the Zeta function that produces the zero lies on the critical line of $\text{Re}[s] = \frac{1}{2}$.

2 The Riemann-Zeta function

The Riemann-Zeta function takes the form:

$$\zeta(s) = 1 + \frac{1}{2^s} + \frac{1}{3^s} + \dots + \frac{1}{n^s} + \dots \quad (1)$$

Where $\zeta(s)$ is an alternating series function in powers of s as n terms go to infinity. The function $\zeta(s)$ is a single-valued, complex scalar function of s , much like the single complex variable of Schrödinger's wave equation. The addition of several Ψ solutions of Schrödinger's equation from many orbital electrons may be effectively modeled by (1), where additional $1/n^s$ terms in (1) contribute to the overall probability distribution of n interacting shells.

Hadamard and Vallée Poussin independently proved the prime number theorem in 1896 by showing that the Riemann-Zeta function $\zeta(s)$ has no zeros of the form $s = 1 + i\beta$, so that no deeper properties of $\zeta(s)$ are required for the proof of the prime number theorem. Thus the distribution of primes is intimately related to $\zeta(s)$.

Z number	Ln(Z number)	Z for S1 Shell	Ln (Z) from F	Number of S electrons at Z (x axis on graph)	Ln(Z) / (Number of S electrons) - y axis
1	0	1	0	0	undefined
3	1.098612289	3	1.098612289	1	1.098612289
5	1.609437912	11	2.397895273	2	1.198947636
11	2.397895273	19	2.944438979	3	0.98147966
13	2.564949357	37	3.610917913	4	0.902729478
19	2.944438979				
29	3.36729583				
31	3.433987204				
37	3.610917913				

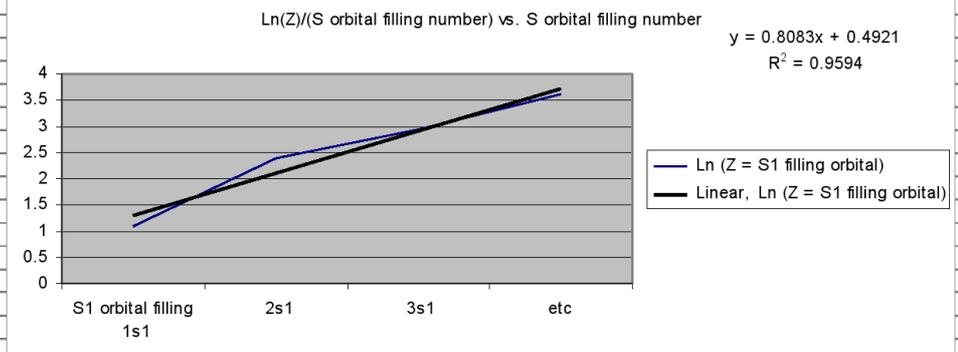


Table 2: $\ln(Z)$ for first s -orbital electron vs. Energy level (n).

3 The Periodic Table

From an analysis of the periodic table it is postulated that the stability of electronic shells s^1 , p^1 , d^1 , and f^1 follow a larger set of zeros which correlate to prime numbers from the Riemann-Zeta function.

If one examines the first and second periods of the periodic table as shown in Table 1, we find that the boundary of filling the first electron in the s , p , d and f shells of each quantum level designated as n is a stable zone that is indicated by a prime atomic number Z (the format in Table 1 is $Z: n\text{Level}^1$).

Where (repeat) indicates a repeat of the shell from a previous Z number and where the use of the format $[\text{Kr}].4d^4.5s^1$ shows the previous electronic formula of Krypton with the additional filling of the d and s shells so as to show the repeated s shell with different d -filling electrons. From the above data, both $Z = 57$ and $Z = 89$ begin a sequence of f shells filling before d shells ($Z = 57$ is the beginning of the Lanthanide series and $Z = 89$ is beginning of the Actinide series). In both the Lanthanide and Actinide series, the d shells that fill after the f shells are primes, explaining why only these d shells (beginning with $Z = 71$ for Lanthanide and $Z = 103$ for Actinide) are filled with primes because they would normally be f shells in the sequence if we looked strictly at observed spectroscopic data.

Notice that the prime Z numbers — 1, 3, 5, 11, 13, 19, 29, 31, 37, 41, 43, 47, etc. shows one consecutive set of primes $\{Z = 3, 5\}$, skips a prime $\{Z = 7\}$ then has two more consecutive primes $\{Z = 11, 13\}$. Note that at $Z = 13$ where we have skipped a prime ($Z = 7$), the ratio of $13/\ln(13)$ is 5.0

Z:	Shell
1:	$1s^1$
3:	$2s^1$
5:	$2p^1$
11:	$3s^1$
13:	$3p^1$
19:	$4s^1$
31:	$4p^1$
37:	$5s^1$
41:	$[\text{Kr}].4d^4.5s^1$ (repeat)
43:	$[\text{Kr}].4d^6.5s^1$ (repeat)
47:	$[\text{Kr}].4d^{10}.5s^1$ (repeat)
57:	$5d^1$
71:	$5d^1$ (repeat)
87:	$7s^1$
89:	$6d^1$
91:	$15f^1$
103:	$7p^1$

Table 1: Prime Atomic Numbers with respect to s^1 , p^1 , d^1 , and f^1 orbitals.

and $Z = 13$ is the fifth prime Z number with a valid p^1 shell. The sequence then skips one prime $\{Z = 17\}$, then has a valid prime at $\{Z = 19\}$ and it then skips another prime $\{Z = 23\}$, which follows five consecutive primes $\{Z = 29, 31, 37, 41, 43\}$. At this point we have skipped $Z = 7, 17,$ and 23 but when we look at $Z = 43$, we take the ratio of $43/\ln(43) = 11.4$ and note that $Z = 43$ is the 11th valid Z prime (with three Z numbers skipped). There appears to be a similar relationship between this data and the prime number theorem of $n/\ln(n)$,

but unlike the traditional prime number theorem where all primes are included, this data considers only valid Z primes (where the first shell is filled, s^1, p^1, d^1, f^1) with other primes skipped if they don't fill the first $s, p, d,$ or f shell. From this consideration, the linearity of these values is significant to the periodic table alone. Table 2 shows the first five s -orbital shells filled (to $5s^1$) plotted against the $\text{Ln}(Z)$ where Z is the associated atomic number for the valid s^1 shell. The number of valid s^1 shells also corresponds to the energy level n ($n = 1$ through $n = 5$ for $1s^1 - 5s^1$). The slope in Table 2 for just the s -shell is good with a linear relationship to $R^2 = 0.95$.

This sequence is also hypothesized to be similar to the distribution pattern of primes produced by finding the zeros on the critical line of the Riemann-Zeta function of (1). Based on the results of linearity in Table 2 there may be a relationship between the difference between valid s -shell orbitals (the Z numbers of skipped shells) versus the total number of shells, a further indication that Riemann-Zeta function could explain the prime orbital filling. There is also a similar prime number correlation for the nuclear energy levels where s, p, d, f and g shells begin on prime boundaries [4].

4 Conclusions

It is found by examining the Z number related to the $s^1, p^1, d^1,$ and f^1 shells of the periodic table that Z is prime for the first filling of s, p, d and f orbitals. It is also found that for shell filling of ns^1 , the logarithm of the prime number associated with Z is linear with respect to energy level n . This relationship is believed to correlate with the Riemann-Zeta function, a complex scalar function similar to the complex-scalar wave function of Schrödinger. The atomic Z primes that correspond to the $s^1, p^1, d^1,$ and f^1 shells is predicted to follow the distribution of primes that result from the non-trivial zeros of the Riemann-Zeta function.

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In-Depth Development of Classical Electrodynamics

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There is hope that a properly developed Classical Electrodynamics (CED) will be able to play a rôle in a unified field theory explaining electromagnetism, quantum phenomena, and gravitation. There is much work that has to be done in this direction. In this article we propose a move towards this aim by refining the basic principles of an improved CED. Attention is focused on the reinterpretation of the E-M potential. We use these basic principles to obtain solutions that explain the interactions between a constant electromagnetic field and a thin layer of material continuum; between a constant electromagnetic field and a spherical configuration of material continuum (for a charged elementary particle); between a transverse electromagnetic wave and a material continuum; between a longitudinal aether wave (dummy wave) and a material continuum.

1 Introduction

The development of Classical Electrodynamics in the late 19th and early 20th century ran into serious trouble from which Classical Electrodynamics was not able to recover (see R. Feynman's *Lectures on Physics* [1]: Volume 2, Chapter 28). According to R. Feynman, this development "ultimately falls on its face" and "It is interesting, though, that the classical theory of electromagnetism is an unsatisfactory theory all by itself. There are difficulties associated with the *ideas* of Maxwell's theory which are not solved by and not directly associated with quantum mechanics". Further in the book he also writes: "To get a *consistent* picture, we must imagine that something holds the electron together", and "the extra non-electrical forces are also known by the more elegant name, the Poincare stresses". He then concludes: "— there have to be other forces in nature to make a consistent theory of this kind". CED was discredited not only by R. Feynman but also by many other famous physicists. As a result the whole of theoretical physics came to believe in the impossibility of explaining the stability of electron charge by classical means, claiming defect in the classical principles. But this is not true.

We showed earlier [2, 3, 4] and further elaborate here that there is nothing wrong with the basic classical *ideas* that Maxwell's theory is based upon. It simply needs further development. The work [2] opens the way to the natural (without singularities) development of CED. In this work it was shown that Poincare's claim in 1906 that the "material" part of the energy-momentum tensor, "Poincare stresses", has to be of a "nonelectromagnetic nature" (see Jackson, [5]) is incorrect. It was shown that the definite material part is expressed only through current density (see formula (9) in [2]), and given a static solution: Ideal Particle, IP, see (19). The proper covariance of IP is manifest — the charges actually hold together and the energy inside an IP comes from the interior electric field (positive energy) and the interior charge density (negative energy, see formula (22) of [2]). The to-

tal energy inside an IP is zero, which means that the rest mass (total energy) corresponds to the vacuum energy only. The contributions to the "inertial mass" (linear momentum divided by velocity; R. Feynman called it "electromagnetic mass") can be calculated by making a Lorentz transformation and a subsequent integration. The total inertial mass is equal to the rest mass (which is in compliance with covariance) but the contributions are different: 4/3 comes from the vacuum electric field, 2/3 comes from the interior electric field, and -1 comes from the interior charge density. This is the explanation of the "anomalous factor of 4/3 in the inertia" (first found in 1881 by J. J. Thomson [5]).

Let us begin with Maxwell's equations:

$$j^i + \frac{c}{4\pi} F_{|k}^{ik} = 0, \quad j_{|k}^k = 0; \quad (1)$$

$$\operatorname{div} \vec{E} = \frac{4\pi}{c} j^0, \quad \oint_S \vec{E} \cdot d\vec{S} = \frac{4\pi}{c} \int_V j^0 dV; \quad (1a)$$

$$\operatorname{rot} \vec{H} = \frac{4\pi}{c} \vec{j} + \frac{1}{c} \frac{\partial \vec{E}}{\partial t}, \quad \oint_\Gamma \vec{H} \cdot d\vec{\Gamma} = \frac{1}{c} \int_S \left(4\pi \vec{j} + \frac{\partial \vec{E}}{\partial t} \right) \cdot d\vec{S}; \quad (1b)$$

$$\frac{1}{c} \frac{\partial j^0}{\partial t} + \operatorname{div} \vec{j} = 0, \quad \frac{\partial}{c \partial t} \int_V j^0 dV = \oint_S \vec{j} \cdot d\vec{S}. \quad (1c)$$

The other half of Maxwell's equations is

$$F_{|k}^{*ik} = 0, \quad F^{*ik} \equiv \frac{1}{2} e^{iklm} F_{lm}; \quad (2)$$

$$\operatorname{div} \vec{H} = 0, \quad \oint_S \vec{H} \cdot d\vec{S} = 0; \quad (2a)$$

$$\operatorname{rot} \vec{E} = -\frac{1}{c} \frac{\partial \vec{H}}{\partial t}; \quad \oint_\Gamma \vec{E} \cdot d\vec{\Gamma} = -\frac{1}{c} \int_S \frac{\partial \vec{H}}{\partial t} \cdot d\vec{S}. \quad (2b)$$

The equations are given in 4D form, 3D form, and in an integral form. Equation (1) represents the interaction law between the electromagnetic field and the current density. Equation (2) applies only to the electromagnetic field. This whole system, wherein equation (1c) is not included, is definite for the 6 unknown components of the electromagnetic field on the condition that the currents (all the components) are given. This is the first order PDE system, the characteristics of which are the wave fronts.

What kind of currents can be given for this system? Not only can continuous fields of currents be prescribed. A jump in a current density is a normal situation. We can even go further and prescribe infinite (but the space integral has to be finite) current density. But in this case we have to check the results. In other words, the system allows that the given current density can contain Dirac's delta-functions if none of the integrals in (1) and (2) goes infinite. But this is not the end. There exists an energy-momentum tensor that gives us the energy density in space. The space integral of that density also has to be finite. Here arises the problem. If we prescribe a point charge (3D delta-function) then the energy integral will be infinite. If we prescribe a charged infinitely thin string (2D delta-function) then the energy will also be infinite. But if we prescribe an infinitely thin surface with a finite surface charge density on it (1-d delta-function) then the energy integral will be finite. It appears that this is the only case that we can allow. But we have to remember that it is possible that a **disruption surface** (where the charge/current density can be infinite) can be present in our physical system. This kind of surface allows the electromagnetic field to have a jump across this surface (this very important fact was ignored in conventional CED — see below). It is also very important to understand that all these delta-functions for the charge distribution are at our discretion: we can prescribe them or we can “hold out”. If we choose to prescribe then we are taking on an additional responsibility. The major attempt to discredit CED (to remove any “obstacles” in the way of quantum theory) was right here. The detractors of CED (including celebrated names like R. Feynman in the USA and L. D. Landau in Russia but, remarkably, not A. Einstein) tried to convince us that a point charge is inherent to CED. With it comes the divergence of energy and the radiation reaction problem. This problem is solvable for the extended particle (which has infinite degrees of freedom) but is not solvable for the point particle. This is not an indication that the “classical theory of electromagnetism is an unsatisfactory theory by itself”. Rather this means that we should not use the point charge model (or charged string model). Only a charged closed surface model is suitable.

We have another serious problem in conventional electrodynamics. As we have shown below, the variation procedure of conventional CED results in the requirement that the electromagnetic field must be continuous across any disruption surface. That actually implies the impossibility of a surface

charge/current on a disruption surface. I changed the variation procedure of CED and arrived at a theory where the electromagnetic interaction (ultimately represented by Maxwell's equation (1)) is the only interaction. The so-called interaction term in the Lagrangian ($A^k j_k$) is abandoned. Also abandoned is the possibility introducing any other interactions (like the “strong” or “weak”). I firmly believe that all the experimental data for elementary particles, quantum phenomena, and gravitation can be explained starting only with the electromagnetic interaction (1).

What is the right expression for the energy-momentum tensor that corresponds to the system described by (1) and (2)? The classical principles require that this expression must be unique. Conventional electrodynamics provides us with the expression: $T^{ik} = \mu c u^i u^k \frac{ds}{dt}$ (for a “material” part containing free particles only: see Landau [6], formula 33.5) that contains density of mass, μ , and velocity only. No charge/current density is included. It seems that the mere presence of charge/current density has to contribute to the energy of the system. To correct the situation we took the simplest possible Lagrangian with charge density:

$$\Lambda = -\frac{1}{16\pi} g^{ab} g^{cd} F_{ac} F_{bd} - \frac{2\pi}{k_0^2 c^2} g^{ab} j_a j_b, \quad (3)$$

where k_0 is a new constant. No interaction term (like $A_k j^k$) is included.

2 Variation of metrics

Let us find the energy-momentum tensor that corresponds to the Lagrangian (3). The metric tensor in classical 4-space is $g_{ik} = \text{diag}[1, -1, -1, -1]$ (we assume $c = 1$). Let us consider an arbitrary variation of a metric tensor but on the condition that this variation does not introduce any curvature in space. This variation is:

$$\delta g_{ik} = \xi_{i|k} + \xi_{k|i}, \quad (4)$$

where ξ^k is an arbitrary but small vector. One has to use the mathematical apparatus of General Relativity to check that with the variation (3) the Riemann curvature tensor remains zero to first order. Assuming that the covariant components of the physical fields are kept constant (then the contravariant components will be varied as a result of the variation of the metric tensor, but we do not use them — see (3) for an explanation) we can calculate the variation of the action. The variation of the square root of the determinant of the metric tensor is: $\delta \sqrt{-g} = -\frac{1}{2} \sqrt{-g} g_{ik} \delta g^{ik}$ (this result can be found in textbooks on field theory). The variation of action becomes:

$$\begin{aligned} \delta S &= - \int \left\{ 2 \frac{\partial \Lambda}{\partial g^{ik}} - \Lambda g_{ik} \right\} \xi^{i|k} \sqrt{-g} d\Omega = \\ &= - \int T_{ik} \xi^{i|k} \sqrt{-g} d\Omega, \end{aligned} \quad (5)$$

where

$$T_{ik} = -\frac{1}{4\pi} g^{ab} F_{ia} F_{kb} + \frac{1}{16\pi} F_{ab} F^{ab} g_{ik} - \frac{4\pi}{k_0^2} j_i j_k + \frac{2\pi}{k_0^2} j_a j^a g_{ik}.$$

If our system consists of two regions that are separated by a closed disruption surface S then the above procedure has to be applied to each region separately. We can write: $T_{ik} \xi^{i|k} = (T_{ik} \xi^i)^{|k} - T_{ik}^{|k} \xi^i$. The 4D volume integrals over divergence (the first term) can be expressed through 3D hypersurface integrals according to the 4D theorem of Gauss. The integral over some remote closed surface becomes zero due to the smallness of T_{ik} on infinity (usually assumed). The integral over a 3D volume at t_1 and t_2 becomes zero due to the assumption: $\xi_i = 0$ at these times. What is left is:

$$\delta S = - \int T_{ik}^k \xi_i^i \sqrt{-g} d\Omega = \int_S (T_{i \text{ out}}^k - T_{i \text{ in}}^k) \xi^i dS_k + \int_{\text{in}} T_{i|k}^k \xi^i \sqrt{-g} d\Omega + \int_{\text{out}} T_{i|k}^k \xi^i \sqrt{-g} d\Omega.$$

Since ξ_i are arbitrary small functions (between t_1 and t_2), the requirement $\delta S = 0$ yields:

$$T_{|a}^{ia} = 0. \quad (6)$$

This condition has to be fulfilled for the inside and the outside regions separately. And the additional requirement on the disruption surface S ,

$$T^{ia} N_a, \quad (6a)$$

is continuous, where N_k is a normal to the surface.

We have found the unique definition of the energy-momentum tensor (5). If we want the action to be minimum with respect to the arbitrary variation of the metric tensor in flat space then (6) and (6a) should be satisfied. Let us rewrite the energy-momentum tensor in 3D form:

$$\left. \begin{aligned} T^{00} &= \frac{1}{8\pi} (E^2 + H^2) - \frac{2\pi}{k_0^2 c^2} [(j^0)^2 + (\vec{j})^2] \\ T^{11} &= \frac{1}{8\pi} (E^2 + H^2 - 2E_1^2 - 2H_1^2) - \frac{2\pi}{k_0^2 c^2} [(j^0)^2 - (\vec{j})^2 + 2(j^1)^2] \\ T^{01} &= \frac{1}{4\pi} (E_2 H_3 - E_3 H_2) - \frac{4\pi}{k_0^2 c^2} j^0 j^1 \\ T^{12} &= -\frac{1}{4\pi} (E_1 E_2 + H_1 H_2) - \frac{4\pi}{k_0^2 c^2} j^1 j^2 \end{aligned} \right\}. \quad (5a)$$

Notice that we have not used Maxwell's or any other field equations so far. It should also be noted that for the energy-momentum tensor (5), (5a) is not defined on the disruption surface itself, despite the fact that there can be a surface

charge/current on a surface (infinite volume density but finite surface density).

Going further, **we are definitely stating that Maxwell's equation (1) is a universal law that should be fulfilled in all space without exceptions. It defines the interaction between the electromagnetic field and the field of current density. This law cannot be subjected to any variation procedure.** Maxwell's equation (2) we will confirm later as a result of a variation; see formula (9). Substituting (5) in (6) and using Maxwell's equation (1) and the antisymmetry of F_{ik} , we obtain:

$$\left. \begin{aligned} j^a \left(\frac{k_0^2 c}{4\pi} F_{ai} + j_{a|i} - j_{i|a} \right) &= 0 \\ j^0 \left(\frac{k_0^2 c}{4\pi} \vec{E} + \nabla j^0 + \frac{1}{c} \frac{\partial \vec{j}}{\partial t} \right) + \\ &+ \vec{j} \times \left(\frac{k_0^2 c}{4\pi} \vec{H} - \text{rot } \vec{j} \right) &= 0 \end{aligned} \right\}. \quad (7)$$

This equation has to be fulfilled for the inside and outside regions separately because (6) is fulfilled separately in these regions. This is important. It is also important to realize that while the conservation of charge is fulfilled everywhere, including a disruption surface, the disruption surface itself is exempt from energy-momentum conservation (no surface energy, no surface tension). This arrangement is in agreement with the fact that we can integrate a delta-function (charge) but we cannot integrate its square (would be energy).

3 A new dynamics

Equation (7) we call a **Dynamics Equation**. It is a nonlinear equation. But it has to be fulfilled inside and outside the particle separately. This will allow us to reduce it to a linear equation inside these regions.

Definition: vacuum is a region of space where all the components of current density are zero.

Equation (7) is automatically satisfied in vacuum ($J^k = 0$). The other possibility ($J^k \neq 0$) will be the interior region of an elementary particle. The boundary between these regions will be a disruption surface. Inside the particle instead of (7) we have:

$$\left. \begin{aligned} \frac{k_0^2 c}{4\pi} F_{ai} + j_{a|i} - j_{i|a} &= 0 \\ \frac{k_0^2 c}{4\pi} \vec{E} + \nabla j^0 + \frac{1}{c} \frac{\partial \vec{j}}{\partial t} &= 0, \quad \frac{k_0^2 c}{4\pi} \vec{H} - \text{rot } \vec{j} = 0 \end{aligned} \right\}. \quad (7a)$$

All the solutions of equation (7a) are also solutions of the nonlinear equation (7). At present we know nothing about the solutions of (7) that do not satisfy (7a). Inside the elementary particle the dynamics equation (7) or (7a) describes, as we call it, a **Material Continuum**. A Material Continuum cannot be divided into a system of material points. The

Relativistic (or Newtonian) Dynamics Equation of CED, that describes the behavior of the particle as a whole, completely disappears inside the elementary particle. There is no mass, no force, no velocity or acceleration inside the particle. The field of current density j^k defines a kinematic state of the Material Continuum. A world line of current j^k is not a world line of a material point. That allows us to deny any causal connection between the points on this line. In consequence, j^k can be space-like as well as time-like. That is in no contradiction with the fact that the boundary of the particle cannot exceed the speed of light. Equation (7a) is linear and allows superposition of different solutions. Using (1) we can obtain:

$$\left. \begin{aligned} j_a^{k|a} - k_0^2 j^k &= 0; & j^k + k_0^2 j^k &= 0 \\ \Delta j^k - \frac{1}{c^2} \frac{\partial^2 j^k}{\partial t^2} + k_0^2 j^k &= 0 \end{aligned} \right\}. \quad (7b)$$

By equation (7) we have obtained something very important, but we are just on the beginning of a difficult and uncertain journey. Now the current density cannot be prescribed arbitrarily. Inside the particle it has to satisfy equation (7b). However, there are no provisions on the surface current density (if a surface current is different from zero then its density is necessarily expressed by a delta-function across the disruption surface).

4 The electromagnetic potential

Now we are going to vary the electromagnetic field F_{ik} in all the space, including a disruption surface. As usual, the variation is kept zero at t_1 and t_2 and also on a remote closed surface, at infinity. In this case the results of variation will be in force on the disruption surface itself. Still, we have to write the variation formulae for each region separately. We claim that equation (1) cannot be subjected to variation. It is the preliminary condition before any variation. In our system we have 10 unknown independent functions (4 functions in J_k and 6 functions in F_{ik}). These functions already have to satisfy 8 equations: 4 equations in (1) and 4 equations in (7). We have only 2 degrees of freedom left. We cannot vary F_{ik} by a straightforward procedure. Let us employ here the Lagrange method of indefinite factors. Let us introduce a modified Lagrangian:

$$\Lambda' = \Lambda + A^a \left(j_a + \frac{1}{4\pi} F_{ab}^{|b} \right), \quad (8)$$

where A^k are 4 indefinite Lagrange factors. Now we have $2 + 4 = 6$ degrees of freedom and we use them to vary F_{ik} . We have:

$$\begin{aligned} \delta S &= - \int \left\{ \frac{\partial \Lambda'}{\partial F_{ik}} \delta F_{ik} + \frac{\partial \Lambda'}{\partial F_{ik|l}} \delta F_{ik|l} \right\} dV_4 = \\ &- \int \left\{ \left(\frac{\partial \Lambda'}{\partial F_{ik|l}} \delta F_{ik} \right)_{|l} + \left[\frac{\partial \Lambda'}{\partial F_{ik}} - \left(\frac{\partial \Lambda'}{\partial F_{ik|l}} \right)_{|l} \right] \delta F_{ik} \right\} dV_4 = 0. \end{aligned}$$

The first term under integration is divergence and can be transformed to the hypersurface integral according to Gauss theorem. Since the variation is arbitrary, the square brackets term has to be zero in either case. It gives:

$$F_{ik} = A_{k|i} - A_{i|k}. \quad (9)$$

If V_4 is the inside region of the particle from t_1 to t_2 then the hypersurface integrals at t_1 and t_2 will be zero, but the hypersurface integral over the closed disruption surface will be

$$\frac{1}{4\pi} \int dt \oint (A^i g^{kl} - A^k g^{il})_{in} \delta F_{ik} dS_l.$$

If V_4 is the outside vacuum then the hypersurface integrals at t_1 and t_2 will be zero. The hypersurface integral over the remote closed surface will be zero, but the hypersurface integral over the disruption surface will be

$$-\frac{1}{4\pi} \int dt \oint (A^i g^{kl} - A^k g^{il})_{out} \delta F_{ik} dS_l.$$

These integrals will annihilate if the **potential A^k is continuous across the disruption surface**. The continuity of potential does not preclude the possibility of a surface charge/current and a jump of electromagnetic field as a consequence.

Claim: The variation procedure of conventional CED results in the impossibility of a surface charge/current on a disruption surface. The variation procedure of conventional CED begins with equation (9) replacing the electromagnetic field with a potential. It introduces the interaction term $A^k j_k$ in the Lagrangian and varies the potential δA^k . As a result of the least action it obtains Maxwell's equation (1). But it can be shown that the consideration of a disruption surface will produce the requirement of electromagnetic field continuity. This actually denies the possibility of a single layer surface charge/current (the double layers are not interesting and they will require the jump of potential and infinite electromagnetic field). Therefore, the conventional variation procedure is incorrect.

5 The physical meaning of potential

Now we learned that the electromagnetic potential, which was devoid of a physical meaning, has to be continuous across all the boundaries of disruption. This is a very important result. It allows me to reinterpret the physical meaning of potential. It is true that according to (9) we can add to the potential a gradient of some arbitrary function and the electromagnetic field won't change (gauge invariance). Yes, but this fact can be given another interpretation: **the potential is unique and it actually contains more information about physical reality than the electromagnetic field does**. To make the potential mathematically unique, besides initial data

and boundary conditions we need only to impose the conservation equation (formerly Lorenz gauge).

$$A^k|_k = 0, \quad A^k|_a = \frac{4\pi}{c} j^k, \quad \square A^k = -\frac{4\pi}{c} j^k. \quad (10)$$

This is true everywhere. Using (1), (7a), and (9) we can conclude that inside a material continuum the potential has to satisfy:

$$\left(A^k|_b - k_0^2 A^k \right)^{|i} - \left(A^i|_b - k_0^2 A^i \right)^{|k} = 0. \quad (7c)$$

If the equation:

$$A^k|_b - k_0^2 A^k = 0 \quad \text{or} \quad \square A^k + k_0^2 A^k = 0, \quad (11)$$

$$\square \equiv \Delta - \frac{\partial^2}{c^2 \partial t^2}$$

is satisfied then (7c) also satisfied. This type of equation is satisfied by the current density, see (7b). This equation can be called the ‘‘Generalized Helmholtz Equation’’. In static conditions (11) coincides with the Helmholtz equation. Equation (11) differs from the Klein-Gordon equation by the sign before the square of a constant.

The new interpretation of potential: A^0 represents the aether quantity (positive or negative), the 3-vector \vec{A} represents the aether current. All together: the potential uniquely describes the existing physical reality — the aether. In general, the interpretation of potential doubles the interpretation of current.

6 The implications of the re-interpretation of potential

Let us suppose that the potential is equal to a gradient of some function G , which we call a ‘‘dummy generator’’:

$$\left. \begin{aligned} A^k &= g^{k\alpha} G|_\alpha, \quad A_0 = \frac{1}{c} \frac{\partial G}{\partial t}, \quad \vec{A} = -\nabla G \\ G|_\alpha &= 0, \quad \Delta G - \frac{1}{c^2} \frac{\partial^2 G}{\partial t^2} = 0 \end{aligned} \right\}, \quad (12)$$

G has to be the solution of a homogeneous wave equation. However, there are no requirements for G on a disruption surface that we know of at present. But now we won’t say that G is devoid of a physical meaning (remember the mistake we made with potential).

What kind of a physical process is described here by the corresponding potential? There is no electromagnetic field and the energy-momentum tensor is equal to zero. These are the ‘‘dummy waves’’ — the longitudinal aether waves. These waves are physically significant only due to the boundary conditions on the disruption surfaces, which they affect. If this is the case, then G can be significant in physical experiment. It can be even unique under the laws (these laws are not completely clear) of another physical realm (realm of electromagnetic potential).

It is difficult to imagine an elementary particle without some oscillating electromagnetic field inside it. If we assume

that the oscillating field is present inside the particle then the boundary conditions may require the corresponding oscillating electromagnetic field in vacuum that surrounds the particle. It is easy to show that the energy of this vacuum electromagnetic field will be infinite. However, it is possible that in vacuum only waves of the scalar potential take care of the necessary boundary conditions. Since the potential is not present in the energy-momentum tensor (5), there won’t be any energy connected with it. **We are free to suggest that the massive elementary particles are the sources of these waves.** These waves are emitted continuously with an amplitude (or its square) that is proportional to the mass of the particle (this proposition seems to be reasonable). These waves are only outgoing waves. The incoming waves can only be plane incoherent waves (the spherical incoming coherent waves are impossible). We are not considering any incoming waves at this point.

We now show, by some examples, that the concept of the material continuum really works.

7 Obtaining solutions

Fortunately, all the equations for finding the solutions are linear. That allows us to seek a total solution as a superposition of the **particular solutions** which satisfy the equations and the boundary conditions separately. The only unlinear condition is (6a), which has to be fulfilled only on the disruption surface. Only the total solution can be used in (6a).

IP2 (Ideal Particle Second): Let us obtain the simplest static spherically symmetric solution with electric charge and electric field only. We have:

$$\left. \begin{aligned} A_{\text{in}}^0 &= \alpha (R_0(z) - R_0(z_1) + bz_1) \\ 0 \leq z \leq z_1, \quad j^0 &= \frac{k_0^2 c}{4\pi} \alpha R_0(z) \end{aligned} \right\}, \quad (13a)$$

$$\left. \begin{aligned} A_{\text{out}}^0 &= \alpha b \frac{z_1^2}{z}, \quad z_1 \leq z < \infty \\ b &\equiv \sqrt{R_0^2(z_1) + R_1^2(z_1)}, \quad z = k_0 r \end{aligned} \right\}, \quad (13b)$$

$$\left. \begin{aligned} E_{\text{in}}^r &= \alpha k_0 R_1(z), \quad 0 \leq z \leq z_1 \\ E_{\text{out}}^r &= \alpha k_0 b \frac{z_1^2}{z^2}, \quad z_1 \leq z < \infty \end{aligned} \right\}, \quad (13c)$$

$$Q_{\text{tot}} = \frac{\alpha}{k_0} z_1^2 b, \quad Q_{\text{surf}} = \frac{\alpha}{k_0} z_1^2 (b - R_1(z_1)), \quad (13d)$$

$$\begin{aligned} mc^2 &= \frac{\alpha^2}{2k_0} (-z_1^2 R_0(z_1) R_1(z_1) + z_1^3 R_0^2(z_1) + \\ &+ z_1^3 R_1^2(z_1)) = \frac{\alpha^2}{2k_0} (z_1 - \sin z_1 \cos z_1), \end{aligned} \quad (13e)$$

where $R_0(z)$ and $R_1(z)$ are spherical Bessel functions. In general, the electric field has a jump at the boundary of IP2. The position of the boundary z_1 is arbitrary, but only at $z_1 = n\pi$ (correspond to IP1) the surface charge is zero and the electric field continuous. The first term in the mass expression (with the minus sign) corresponds to the energy of the interior region of the particle. It can be positive or negative, depending on z_1 (at $z_1 = n\pi$ it is zero). The second and third terms together represent the vacuum energy, which is positive. The total energy/mass remains positive at all z_1 .

It was confirmed that IP is an unstable “equilibrium”. Given a small perturbation it will grow in time. We hope to find a stable solution among the more complicated solutions than IP. The first idea was to introduce a spin in a static solutions. Then we tried to introduce the steady-state oscillating solutions. It was confirmed that there exist oscillating solutions with oscillating potential in vacuum that does not produce any vacuum E-M field. Then we tried to introduce a spin that originates from the oscillating solutions. Also we tried to consider the cylindrically shaped particles that are moving with the speed of light (close to a photon, see [3]). All these attempts indicate that the boundary of a particle that separates the material continuum from vacuum is a key player in any solution.

8 The mechanism of interaction between a constant electric field and a static charge (simplified thin layer model)

The simplest solutions can be obtained in plane symmetry where all the physical quantities depend only on the third coordinate — z . Let us consider symmetry of the type, vacuum — material continuum — vacuum. The thin layer of material continuum from $z = 0$ to $z = a$ (a is of the order of the size of elementary particle) will represent a simplified model of an elementary particle. The boundaries at $z = 0$ and $z = a$ are deemed to be enforced by the particle and the whole deficit of energy or momentum on these boundaries is deemed to go directly to the particle. Actually, if we have a deficit of energy or momentum it means that we are missing a particular solution that brings this deficit to zero, according to (6a).

For further discussion we need to write down the integral form of the energy-momentum conservation:

$$\frac{\partial}{c \partial t} \int_V T^{m0} dV = - \oint_{\Sigma} T^{mq} d\Sigma_q, \quad (6b)$$

where V is a 3D volume (which is not moving — it is our choice), and Σ is a 3D closed surface around this volume (obviously also not moving). The index m can correspond to any coordinate, while the index q corresponds only to the terrestrial coordinates (1, 2, 3). If $m = 0$ then the left part of (6b) is the time rate of increase of the energy inside V . T^{0q} is the 3-dimensional Pointing vector (or the flow of energy through

a square unit per unit of time). If $m = 3$ (in the plane symmetry only one coordinate is of interest) then the left part of (6b) is the time rate of increase of the linear momentum of the volume V (actually it is a force applied to the volume V). T^{3q} is the 3-vector (in general q can be 1, 2, 3; in our case $q = 3$) of the flow of linear momentum through a square unit per unit of time. It is obvious that when static (or in a steady state) the left part of (6b) must be zero if there is no source/drain of energy/linear momentum inside the said volume.

Suppose the constant electric field in the first vacuum region is E . The scalar potential (aether quantity), the electric field, and the charge density are:

$$\left. \begin{aligned} \Phi_1 &= -Ez + C_1, & E_1 &= E \\ \Phi_2 &= -\frac{E}{k_0} \sin k_0 z + C_1 \cos k_0 z \\ C_1 &= \frac{4\pi Q + E(1 - \cos k_0 a)}{k_0 \sin k_0 a} \\ E_2 &= E \cos k_0 z + k_0 C_1 \sin k_0 z \\ \rho &= \frac{k_0^2}{4\pi} \Phi_2, & \Phi_3 &= -(E + 4\pi Q)(z - a) + C_2 \\ C_2 &= \frac{4\pi Q \cos k_0 a - E(1 - \cos k_0 a)}{k_0 \sin k_0 a} \\ E_3 &= E + 4\pi Q \end{aligned} \right\}. \quad (14)$$

Here the charge density is the solution of (7b) inside the second region. The potentials are the solutions of (10). All the physical quantities except ρ are continuous on the boundaries. That means that the jumps of the components of the energy-momentum tensor will be due to the jumps of the charge density only. The energy momentum tensor in this symmetry (and this particular case) is:

$$\left. \begin{aligned} T^{00} &= \frac{1}{8\pi} E^2 - \frac{2\pi}{k_0^2} \rho^2 = T^{11} = T^{22} \\ T^{03} &= 0, & T^{33} &= -\frac{1}{8\pi} E^2 - \frac{2\pi}{k_0^2 c} \rho^2 \end{aligned} \right\}. \quad (15)$$

There is no energy flow in this system, but there is a flow of linear momentum. In the first vacuum region it is: $T^{33} = -E^2/8\pi$. Then it jumps on the first and on the second boundaries:

$$\left. \begin{aligned} T^{33}(z = 0+) - T^{33}(z = 0-) &= -\frac{k_0^2 C_1^2}{8\pi} \\ T^{33}(z = a+) - T^{33}(z = a-) &= \frac{k_0^2 C_2^2}{8\pi} \\ \frac{k_0^2 C_1^2}{8\pi} - \frac{k_0^2 C_2^2}{8\pi} &= Q \left(E + \frac{4\pi Q}{2} \right) \end{aligned} \right\}. \quad (16)$$

After that it is: $T^{33} = -(E + 4\pi Q)^2/8\pi$. As we go from left to right the jump on the first boundary is negative. That

means that the small volume that includes the first boundary gets negative outside (we always consider the outside normal to the closed surface Σ) flow of linear momentum. That means that the volume itself, according to expression (6b), gets the positive rate of linear momentum, which is the force in the positive direction of the z -axis. The first boundary is pushed in the positive direction of the z -axis. The second boundary is also pushed, but in the negative direction of the z -axis. The difference is exactly equal to the force with which the field acts on a particle; see (16). We see that electric field does not act on a charge *per se* but only on a whole particle and only through its boundaries. This picture is true only at $t = 0$ because the missing particular solution that makes the appearance of “free” sources and drains most definitely will depend on time (the particle will begin to accelerate). This is the actual success of the proposed modification of CED.

9 The mechanism of interaction between a constant electric field and a static spherical charge

Here we will confirm that the thin layer treatment corresponds to the more accurate but more complicated spherical charge treatment. Suppose we have a constant electric field E directed along the z -axis in vacuum. Also we have a sphere of radius r_1 that separates the material continuum inside the sphere, from vacuum. The situation is static at $t = 0$. The potential in general has to satisfy the equation $A_{|k}^k = 0$ (10) everywhere, and equation (7c) inside the material continuum. This last equation, with 3rd derivatives, has to be satisfied strictly inside a material continuum and not on the disruption surface itself (where a single layer of charge/current density is possible and the charge/current density, $j^k = \frac{c}{4\pi} A_{|a}^k |^a$, can be infinite). In vacuum we have

$$A_{|a}^k |^a = 0. \quad (17)$$

Let us define a “dummy” potential by:

$$\left. \begin{aligned} D_{|k}^k = 0, \quad D^{i|k} - D^k |^i = 0, \\ \text{consequently: } D_{|a}^k |^a = 0. \end{aligned} \right\} \quad (18)$$

If we have a solution A^k of (10)+(7c) or a solution of (10)+(17) then $A^k + D^k$ will also be the solution of the same equations (it does not matter whether inside the material continuum or in vacuum).

Now we return to our particular case. The solution of (18) that we are interested in would be: $D^0 = \text{const}$. If there is no time dependence then (10) is satisfied for any A^0 if a vector potential is zero. Equation (7c) is a Laplace operator taken from a Helmholtz equation. The solutions of the Helmholtz equation being considered would be: $R_0(k_0 r)$ and $R_1(k_0 r) \cos \theta$ where R_n are the spherical Bessel functions. In vacuum we consider the solutions e/r , (where e is the total charge), $r \cos \theta$, and $(1/r^2) \cos \theta$. So, let us consider the

potential

$$\left. \begin{aligned} A_{\text{in}}^0 &= \alpha R_0(k_0 r) + \frac{e}{r_1} - \alpha R_0(k_0 r_1) \\ A_{\text{out}}^0 &= \frac{e}{r} + E \left(\frac{r_1^3}{r^2} - r \right) \cos \theta \end{aligned} \right\}. \quad (19)$$

It is continuous at $r = r_1$. The corresponding electric field and charge density will be,

$$\left. \begin{aligned} E_{r \text{ in}} &= \alpha k_0 R_1(k_0 r) \\ E_{r \text{ out}} &= \frac{e}{r^2} + E \left(1 + 2 \frac{r_1^3}{r^3} \right) \cos \theta \\ E_{\theta \text{ in}} &= 0, \quad E_{\theta \text{ out}} = E \left(\frac{r_1^3}{r^3} - 1 \right) \sin \theta \\ \rho &= \frac{\alpha k_0^2}{4\pi} R_0(k_0 r) \end{aligned} \right\}. \quad (20)$$

We see that the radial component of the electric field has a jump while the θ component is continuous. The surface charge density and the total surface charge are:

$$\left. \begin{aligned} 4\pi \rho_{\text{surf}} &= -E_{r \text{ in}}(r_1) + E_{r \text{ out}}(r_1) = \\ &= \frac{e}{r_1^2} - \alpha k_0 R_1(k_0 r_1) + 3E \cos \theta \\ Q_{\text{surf tot}} &= e - \alpha k_0 r_1^2 R_1(k_0 r_1) \end{aligned} \right\}. \quad (21)$$

We see that it does not matter what the relation is between the constants α and e , the surface of the particle has a “surface charge polarization” $3E \cos \theta$. Only this polarization will result in the net force on the charge. The polarization in the volume of the particle can be introduced using the solution $R_1(k_0 r) \cos \theta$. But this polarization won't change the net force (it can be introduced with any constant factor). We've made the corresponding calculations that support this statement. We do not present them here, for simplification.

The double radial component of the energy-momentum tensor will be:

$$\left. \begin{aligned} 8\pi T^{rr} &= E_{\theta}^2 - E_r^2 - \frac{16\pi^2}{k_0^2} \rho^2 \\ 8\pi T_{\text{surf in}}^{rr} &= -\alpha^2 k_0^2 (R_0^2(k_0 r_1) + R_1^2(k_0 r_1)) \\ 8\pi T_{\text{surf out}}^{rr} &= -\left(\frac{e^2}{r_1^4} + \frac{6e}{r_1^2} E \cos \theta + 9E^2 \cos^2 \theta \right) \\ T_{\text{surf in}}^{\theta r} &= T_{\text{surf out}}^{\theta r} = 0 \end{aligned} \right\}. \quad (22)$$

The force applied to the surface will be normal to the surface and equal to $T_{\text{surf in}} - T_{\text{surf out}}^{rr}$. This force is zero if $E = 0$. This case corresponds to the true static solution of

our equations with (6a) satisfied. This solution enforces the spherical boundary. If E is not zero, then we do not know the actual solution because (6a) is not satisfied. The actual solution will be not static. But we can calculate the force at the moment when E was “turned on”. To get the z component of this force we have to multiply the expression on $\cos \theta$. If we integrate this over the spherical surface then all the terms except the one with $\cos \theta$ are zero. The result of integration will be eE . This is exactly the force with which the electric field E acts on a charge e .

10 The transverse electromagnetic wave

Let us consider that the transverse electromagnetic wave is coming from the left and encounters the layer of material continuum. We expect to find the transmitted and reflected waves as well as the radiation pressure. “Behind” the transverse E-M wave we find that the transverse aether wave with only an x component (for x -polarized E-M wave) of the vector potential (aether current) is different from zero:

$$\left. \begin{aligned} {}_1A^1 &= \Phi_1^+ + \Phi_1^- \\ \Phi_1^+ &= F_1^+ e^{-ikz}, \quad \Phi_1^- = F_1^- e^{ikz} \\ {}_1E_1 &= -ik \cdot {}_1A^1, \quad {}_1H_2 = -ik \cdot (\Phi_1^+ - \Phi_1^-) \\ {}_2A^1 &= \Phi_2^+ + \Phi_2^- \\ \Phi_2^+ &= F_2^+ e^{-ik'z}, \quad \Phi_2^- = F_2^- e^{ik'z} \\ k &= \frac{\omega}{c}, \quad (k')^2 = k_0^2 + k^2 \\ {}_2E_1 &= -ik \cdot {}_2A^1, \quad {}_2H_2 = -ik' \cdot (\Phi_2^+ - \Phi_2^-) \\ j(z, t) &= \frac{ck_0^2}{4\pi} \cdot {}_2A^1 \\ {}_3A^1 &= F_3^+ e^{-ikz} \\ {}_3E_1 &= -ik \cdot {}_3A^1, \quad {}_3H_2 = -ik \cdot {}_3A^1 \end{aligned} \right\}, \quad (23)$$

where the prefixes to the fields always denote the number of the region (we did not attach indexes to the current density j because it is different from zero only in the second region). We assume that all the functions depend on t through the factor $\exp(i\omega t)$. In the first region the given incoming wave F_1^+ and some reflected wave F_1^- are present. In the second region two waves are present. They satisfy the equations:

$${}_2A^{1''} + k^2 \cdot {}_2A^1 = -\frac{4\pi}{c} j, \quad \frac{\partial}{\partial x} \cdot {}_2A^1 = 0. \quad (24)$$

On the boundaries the vector potential (aether current)

and its first derivative have to be continuous. We found that

$$\left. \begin{aligned} F_1^- &= -F_1^+ \frac{2ik_0^2 \sin(k'a)}{D} \\ F_3^+ e^{-ik'a} &= F_1^+ \frac{4kk'}{D} \\ D &\equiv (k+k')^2 e^{ik'a} - (k-k')^2 e^{-ik'a} \\ F_2^+ &= F_1^+ \frac{2k(k+k')}{D} e^{ik'a} \\ F_2^- &= F_1^+ \frac{2k(k'-k)}{D} e^{-ik'a} \end{aligned} \right\}. \quad (25)$$

Here we found the amplitudes of reflected and transmitted waves and the amplitudes of both waves in the second region (only F_1^+ is considered to be real and given).

We found previously that the energy-momentum tensor in a material continuum has the form (one-dimensional symmetry assumed):

$$\left. \begin{aligned} T^{00} &= \frac{1}{8\pi} (E^2 + H^2) - \frac{2\pi}{k_0^2 c^2} j^2 \\ T^{33} &= \frac{1}{8\pi} (E^2 + H^2) + \frac{2\pi}{k_0^2 c^2} j^2 \\ T^{11} &= \frac{1}{8\pi} (-E^2 + H^2) - \frac{2\pi}{k_0^2 c^2} j^2 = -T^{22} \\ T^{03} &= \frac{1}{4\pi} EH \end{aligned} \right\}. \quad (26)$$

Since we use complex numbers — we have to take the real parts of the physical values, multiply them and then take the time average. The result will be the real part of the product of the first complex amplitude on the conjugate of the second complex amplitude. The result in the second region is:

$$\left. \begin{aligned} \frac{2\pi}{k_0^2 c^2} j^2 &= F_1^{+2} \frac{k_0^2 k^2}{\pi |D|^2} \times \\ &\quad \times (k_0^2 + 2k^2 + k_0^2 \cos 2k'(a-z)) \\ T^{00} &= -F_1^{+2} \frac{2k^2}{\pi |D|^2} \times \\ &\quad \times (k_0^4 \cos 2k'(a-z) - k^2 (k_0^2 + 2k^2)) \\ T^{03} &= F_1^{+2} \frac{4k^4 k'^2}{\pi |D|^2} \\ T^{33} &= F_1^{+2} \frac{2k^2 k'^2}{\pi |D|^2} (k_0^2 + 2k^2) \end{aligned} \right\}. \quad (27)$$

The electric and magnetic fields are continuous in this system. The flow of energy appear to be independent of z in the second region. It is continuous on the boundaries (see (26); the currents are not included in T^{03}). This means that it

is constant through the whole system. The flow of linear momentum (T^{33}) is positive in the first region and then jumps up on the first boundary due to the jump of the current j . It means that the surface integral in (6b) is positive and the first boundary is losing linear momentum. The surface is pulled in the negative direction of the z -axis. But this pull is less than another pull due to the jump on the second boundary; this can be determined from (27). We consider $k'a \sim \frac{\pi}{4}$, but it will be true for any $k'a$ different from π . Notice also that at $k'a = \pi$ the reflected wave is zero as can be seen from (25). Thus, the material continuum will experience the force (through its boundaries) in the positive direction of the z -axis. The numerical value of this force can be calculated from the jumps and it is equal to the force that we usually calculate from the linear momentum of incident transmitted and reflected waves.

11 The longitudinal aether (dummy) wave

Let us consider a longitudinal aether wave travelling from the left, encountering the layer of material continuum. There are no electromagnetic fields that accompany this wave in vacuum. Not so inside the material continuum. We have:

$$\left. \begin{aligned} {}_1A^0 &= \Phi_1^+ + \Phi_1^- \\ \Phi_1^+ &= F_1^+ e^{-ikz}, \quad \Phi_1^- = F_1^- e^{ikz}, \quad k = \frac{\omega}{c} \\ {}_1A^3 &= \Phi_1^+ - \Phi_1^-, \quad {}_2A^0 = \Phi_2^+ + \Phi_2^- \\ \Phi_2^+ &= F_2^+ e^{-ik'z}, \quad \Phi_2^- = F_2^- e^{ik'z} \\ {}_2A^3 &= \frac{k}{k'} (\Phi_2^+ - \Phi_2^-), \quad (k')^2 = k_0^2 + k^2 \\ j^0(z, t) &= \frac{ck_0^2}{4\pi} (\Phi_2^+ + \Phi_2^-) \\ j^3(z, t) &= \frac{ck_0^2 k}{4\pi k'} (\Phi_2^+ - \Phi_2^-) \\ E_3 &= \frac{ik_0^2}{k'} (\Phi_2^+ - \Phi_2^-), \quad {}_3A^0 = {}_3A^3 = F_3^+ e^{-ikz} \end{aligned} \right\} \quad (28)$$

where we assume that all the functions depend on t through the factor $\exp(i\omega t)$. In the first region the given incoming wave F_1^+ and some reflected wave F_1^- are present (both are dummy waves). In the second region two waves are present. They satisfy the equations:

$$\left. \begin{aligned} {}_2A^{0''} + k^2 \cdot {}_2A^0 &= -\frac{4\pi}{c} j^0 \\ {}_2A^{3''} + k^2 \cdot {}_2A^3 &= -\frac{4\pi}{c} j^3 \\ ik \cdot {}_2A^0 + {}_2A^{3'} &= 0. \end{aligned} \right\} \quad (29)$$

To define all the waves we have to satisfy the conditions

on the boundaries. The scalar potential (aether quantity) and the vector potential (aether current) should be continuous across the boundaries. We found that

$$\left. \begin{aligned} \text{on } z = a : \quad F_2^+ e^{-ik'a} &= \frac{k+k'}{2k} F_3^+ e^{-ika} \\ F_2^- e^{ik'a} &= -\frac{k'-k}{2k} F_3^+ e^{-ika} \\ \text{on } z = 0 : \quad F_1^- &= F_1^+ \frac{2ik_0^2 \sin(k'a)}{D} \\ F_3^+ e^{-ika} &= F_1^+ \frac{4kk'}{D} \\ D &\equiv (k+k')^2 e^{ik'a} - (k'-k)^2 e^{-ik'a} \\ F_2^+ &= F_1^+ \frac{2k'(k+k')}{D} e^{ik'a} \\ F_2^- &= -F_1^+ \frac{2k'(k'-k)}{D} e^{-ik'a} \end{aligned} \right\} \quad (30)$$

Here we found the amplitudes of reflected and transmitted waves and the amplitudes of both waves in the second region (only F_1^+ is considered to be real).

From (28) we can calculate the derivatives:

$$\left. \begin{aligned} {}_1A^{0'} &= -ik \cdot {}_1A^3, \quad {}_2A^{0'} = -\frac{ik'^2}{k} \cdot {}_2A^3 \\ {}_1A^{3'} &= -ik \cdot {}_1A^0, \quad {}_2A^{3'} = -ik \cdot {}_2A^0 \end{aligned} \right\} \quad (28a)$$

We see that the aether current (A^3) has a continuous derivative while the derivative of aether quantity (A^0) has a jump at the boundaries. This means that there are surface charges associated with the boundaries.

We notice from (28) that the electric field, charge density, and current density are different from zero inside the second region. This means that the material continuum produces a kind of physical response to the energy-less dummy waves. We also found previously that the energy-momentum tensor in a material continuum has the form (one-dimensional symmetry assumed),

$$\left. \begin{aligned} T^{00} &= \frac{1}{8\pi} E^2 - \frac{2\pi}{k_0^2 c^2} (c^2 \rho^2 + j^2) \\ T^{11} &= -\frac{1}{8\pi} E^2 - \frac{2\pi}{k_0^2 c^2} (c^2 \rho^2 + j^2) \\ T^{22} = T^{33} &= \frac{1}{8\pi} E^2 - \frac{2\pi}{k_0^2 c^2} (c^2 \rho^2 - j^2) \\ T^{01} &= -\frac{4\pi}{k_0^2 c^2} c \rho j \end{aligned} \right\} \quad (31)$$

To actually calculate a time average of the energy-momentum tensor we have to take the real parts of the physical values, multiply them, and then take the time average. The

result will be the real part of the product of the first complex amplitude on the conjugate of the second complex amplitude. The result of calculation is,

$$\left. \begin{aligned} T^{33} &= -(F_1^+)^2 \frac{2k_0^4}{\pi|D|^2} (k_0^2 + 2k^2) \\ T^{03} &= -(F_1^+)^2 \frac{4k_0^2 k^2 k'^2}{\pi|D|^2} \\ T^{00} &= (F_1^+)^2 \frac{2k_0^6}{\pi|D|^2} \cos(2k'(a-z)) \end{aligned} \right\}. \quad (32)$$

The first two time averages of the tensor components appear to be independent of z . The energy density depends on z . All these tensor components are zero in both vacuum regions. This means that all of them jump at the boundaries.

On the first boundary the jump of T^{33} is negative. It means that the first boundary will be pushed to the right. On the second boundary the jump will be positive and the same by its absolute value (because T^{33} is constant inside the second region). The second boundary will be pushed in the negative direction of the z -axis with the same force — we have equilibrium — no “free” force.

On the first boundary the jump of T^{03} is negative. It means that the first boundary will be getting energy. On the second boundary the jump will be positive and the same by its absolute value (because T^{03} is constant inside the second region). The second boundary will be losing the same amount of energy — no “free” energy.

It appears that the particular solutions that we have carry energy and momentum from the second boundary to the first, while the missing particular solution carries them back. If we imagine that the energy and momentum can be lost on the way from the source to the drain then we get a free linear momentum directed to the source of dummy waves (gravitational force). Also we get a free energy for heating stars. This unconservation proposition can be quite real if we consider that we obtained the conservation of energy-momentum from the requirement of minimum action. In the real physical world the action may have a small jitter around the exact minimum. Obviously this jitter is very small so that it can revile itself only on a cosmic scale.

At the present time we hesitate in proceeding further from these results because the meaning of these results has still to be clarified.

12 De Broglie’s waves

Let us suppose, in addition (see Section 6), that the frequency of dummy waves (as well as the intensity) also proportional to the mass of the particle: $\omega = mc^2/\hbar$. The resting particles are present in abundance in the experimental arrangement itself. These resting particles can be partially synchronized in some proximity (the extent of this proximity is not known yet) of

any point inside the experimental device. We can expect some standing scalar waves of a dummy generator that can be experienced by the moving particle independently of the direction of motion. In this case we can explain De Broglie’s waves as beat frequency waves between the frequency of a resting particle and the Doppler shifted frequency of a moving particle. The rôle of the nonlinear device that is necessary to obtain the beat frequency wave, can be very well played by the boundary of the particle itself. This will explain “the wave properties of particles” by purely classical means, as first proposed in 1993 by Milo Wolff [7].

In the foregoing reformulation of conventional classical electrodynamics, we omitted the interaction term in the Lagrangian/Hamiltonian. Quantum Theory was undermined by this action. One should note that, historically, after the creation of quantum theory, there were attempts to legitimize the electromagnetic potential as a physically measurable value (see R. Feynman, [1]). Still, it is too early to try to find a classical basis for quantum theory, but the direction to go is that of the physical realm of the electromagnetic potential.

13 Conclusion

Probably it is not right to keep the disruption surface devoid from surface energy and surface tension. To introduce that correctly we have to consider some surface Lagrange density and add a surface integral to the action volume integral. That I hope to see in a future development.

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A Model of Electron-Positron Pair Formation

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The elementary electron-positron pair formation process is considered in terms of a revised quantum electrodynamic theory, with special attention to the conservation of energy, spin, and electric charge. The theory leads to a wave-packet photon model of narrow line width and needle-radiation properties, not being available from conventional quantum electrodynamics which is based on Maxwell's equations. The model appears to be consistent with the observed pair production process, in which the created electron and positron form two rays that start within a very small region and have original directions along the path of the incoming photon. Conservation of angular momentum requires the photon to possess a spin, as given by the present theory but not by the conventional one. The nonzero electric field divergence further gives rise to a local intrinsic electric charge density within the photon body, whereas there is a vanishing total charge of the latter. This may explain the observed fact that the photon decays on account of the impact from an external electric field. Such a behaviour should not become possible for a photon having zero local electric charge density.

1 Introduction

During the earliest phase of the expanding universe, the latter is imagined to be radiation-dominated, somewhat later also including particles such as neutrinos and electron-positron pairs. In the course of the expansion the "free" states of highly energetic electromagnetic radiation thus become partly "condensed" into "bound" states of matter as determined by Einstein's energy relation.

The pair formation has for a long time both been studied experimentally [1] and been subject to theoretical analysis [2]. When a high-energy photon passes the field of an atomic nucleus or that of an electron, it becomes converted into an electron and a positron. The orbits of these created particles form two rays which start within a very small volume and have original directions along the path of the incoming photon.

In this paper an attempt is made to understand the elementary electron-positron pair formation process in terms of a revised quantum electrodynamic theory and its application to a wave-packet model of the individual photon [3, 4, 5, 6]. The basic properties of the latter will be described in Section 2, the intrinsic electric charge distribution of the model in Section 3, the conservation laws of pair formation in Section 4, some questions on the vacuum state in Section 5, and the conclusions are finally presented in Section 6.

2 A photon model of revised quantum electrodynamics

The detailed deductions of the photon model have been reported elsewhere [3, 4, 5, 6] and will only be summarized here. The corresponding revised Lorentz and gauge invariant theory represents an extended version which aims beyond

Maxwell's equations. Here the electric charge density and the related electric field divergence are nonzero in the vacuum state, as supported by the quantum mechanical vacuum fluctuations and the related zero-point energy. The resulting wave equation of the electric field \mathbf{E} then has the form

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \nabla^2\right) \mathbf{E} + \left(c^2 \nabla + \mathbf{C} \frac{\partial}{\partial t}\right) (\text{div } \mathbf{E}) = 0, \quad (1)$$

which includes the effect of a space-charge current density $\mathbf{j} = \varepsilon_0 (\text{div } \mathbf{E}) \mathbf{C}$ that arises in addition to the displacement current $\varepsilon_0 \partial \mathbf{E} / \partial t$. The velocity \mathbf{C} has a modulus equal to the velocity c of light, as expressed by $C^2 = c^2$. The induction law still has the form

$$\text{curl } \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (2)$$

with \mathbf{B} standing for the magnetic field strength.

The photon model to be discussed here is limited to axisymmetric normal modes in a cylindrical frame (r, φ, z) where $\partial / \partial \varphi = 0$. A form of the velocity vector

$$\mathbf{C} = c(0, \cos \alpha, \sin \alpha) \quad (3)$$

is chosen under the condition $0 < |\cos \alpha| \ll 1$, such as not to get into conflict with the Michelson-Morley experiments, i.e. by having phase and group velocities which only differ by a very small amount from c . The field components can be expressed in terms of a generating function

$$G_0 \cdot G = E_z + (\cot \alpha) E_\varphi, \quad G = R(\rho) e^{i(-\omega t + kz)}, \quad (4)$$

where G_0 is an amplitude factor, $\rho = r/r_0$ with r_0 as a characteristic radial distance of the spatial profile, and ω and k standing for the frequency and wave number of a normal

mode. Such modes are superimposed to form a wave-packet having the spectral amplitude

$$A_k = \left(\frac{k}{k_0^2} \right) \exp \left[-z_0^2 (k - k_0)^2 \right], \quad (5)$$

where k_0 and $\lambda_0 = \frac{2\pi}{k_0} = \frac{c}{\nu_0}$ are the main wave number and wave length, and $2z_0$ represents the effective axial length of the packet. According to experimental observations, the packet must have a narrow line width, as expressed by $k_0 z_0 \gg 1$. The spectral averages of the field components in the case $|\cos \alpha| \ll 1$ are then

$$\bar{E}_r = -iE_0 R_5, \quad (6)$$

$$\bar{E}_\varphi = E_0(k_0 r_0)(\sin \alpha)(\cos \alpha) R_3, \quad (7)$$

$$\bar{E}_z = E_0(k_0 r_0)(\cos \alpha)^2 R_4 \quad (8)$$

and

$$\bar{B}_r = -\frac{1}{c} \frac{1}{\sin \alpha} \bar{E}_\varphi, \quad (9)$$

$$\bar{B}_\varphi = \frac{1}{c} (\sin \alpha) \bar{E}_r, \quad (10)$$

$$\bar{B}_z = \frac{1}{c} (\cos \alpha) \frac{R_8}{R_5} \bar{E}_r. \quad (11)$$

Here

$$R_3 = \rho^2 D_\rho R, \quad R_4 = R - R_3, \quad R_5 = \frac{d}{d\rho} (R - R_3), \quad (12)$$

$$R_8 = \left(\frac{d}{d\rho} + \frac{1}{\rho} \right) R_3, \quad D_\rho = \frac{d^2}{d\rho^2} + \frac{1}{\rho} \frac{d}{d\rho} \quad (13)$$

and

$$E_0 = e_0 \bar{f}, \quad e_0 = \frac{g_0 \sqrt{\pi}}{k_0^2 r_0 z_0}, \quad G_0 = g_0 (\cos \alpha)^2, \quad (14)$$

$$\bar{f} = [\cos(k_0 \bar{z}) + i \sin(k_0 \bar{z})] \exp \left[-\left(\frac{\bar{z}}{2z_0} \right)^2 \right], \quad (15)$$

where $\bar{z} = z - c(\sin \alpha) t$.

Choosing the part of the normalized generating function G which is symmetric with respect to the axial centre $\bar{z} = 0$ of the moving wave packet, the components $(\bar{E}_\varphi, \bar{E}_z, \bar{B}_r)$ become symmetric and the components $(\bar{E}_r, \bar{B}_\varphi, \bar{B}_z)$ antisymmetric with respect to the same centre. Then the integrated electric charge and magnetic moment vanish.

The equivalent total mass defined by the electromagnetic field energy and the energy relation by Einstein becomes on the other hand

$$m \cong 2\pi \frac{\varepsilon_0}{c^2} r_0^2 W_m e_0^2 \int_{-\infty}^{+\infty} f^2 d\bar{z}, \quad (16)$$

$$W_m = \int \rho R_5^2 d\rho,$$

where expression (15) has to be replaced by the reduced function

$$f = [\sin(k_0 \bar{z})] \exp \left[-\left(\frac{\bar{z}}{2z_0} \right)^2 \right] \quad (17)$$

due to the symmetry condition on G with respect to $\bar{z} = 0$. Finally the integrated angular momentum is obtained from the Poynting vector, as given by

$$\begin{aligned} s &\cong -2\pi \varepsilon_0 \int_{-\infty}^{+\infty} \int r^2 \bar{E}_r \bar{B}_z dr d\bar{z} = \\ &= 2\pi \frac{\varepsilon_0}{c} (\cos \alpha) r_0^3 W_s e_0^2 \int_{-\infty}^{+\infty} f^2 d\bar{z}, \end{aligned} \quad (18)$$

$$W_s = - \int \rho^2 R_5 R_8 d\rho.$$

Even if the integrated (total) electric charge of the photon body as a whole vanishes, there is on account of the nonzero electric field divergence a local nonzero electric charge density

$$\bar{\rho} = e_0 f \frac{\varepsilon_0}{r_0} \frac{1}{\rho} \frac{d}{d\rho} (\rho R_5). \quad (19)$$

Due to the factor $\sin(k_0 \bar{z})$ this density oscillates rapidly in space as one proceeds along the axial direction. Thus the electric charge distribution consists of two equally large positive and negative oscillating contributions of total electric charge, being mixed up within the volume of the wave packet.

To proceed further the form of the radial function $R(\rho)$ has now to be specified. Since the experiments clearly reveal the pair formation to take place within a small region of space, the incoming photon should have a strongly limited extension in its radial (transverse) direction, thus having the character of "needle radiation". Therefore the analysis is concentrated on the earlier treated case of a function R which is divergent at $\rho = 0$, having the form

$$R(\rho) = \rho^{-\gamma} e^{-\rho}, \quad \gamma > 0. \quad (20)$$

In the radial integrals of equations (16) and (18) the dominant terms then result in $R_8 \cong -R_5$ and

$$W_m = \int_{\rho_m}^{\infty} \rho R_5^2 d\rho = \frac{1}{2} \gamma^5 \rho_m^{-2\gamma}, \quad (21)$$

$$W_s = \int_{\rho_s}^{\infty} \rho^2 R_5^2 d\rho = \frac{1}{2} \gamma^5 \rho_s^{-2\gamma+1}, \quad (22)$$

where $\rho_m \ll 1$ and $\rho_s \ll 1$ are small nonzero radii at the origin $\rho = 0$. To compensate for the divergence of W_m and W_s when ρ_m and ρ_s approach zero, we now introduce the shrinking parameters

$$r_0 = c_r \cdot \varepsilon, \quad g_0 = c_g \cdot \varepsilon^\beta, \quad (23)$$

where c_r and c_g are positive constants and the dimensionless smallness parameter ε is defined by $0 < \varepsilon \ll 1$. From relations

(14)–(18), (21)–(23), the energy relation $mc^2 = h\nu_0$, and the quantum condition of the angular momentum, the result becomes

$$m = \pi^2 \frac{\varepsilon_0}{c^2} \gamma^5 \left(\frac{1}{k_0^2 z_0} \right)^2 c_g^2 \frac{\varepsilon^{2\beta}}{\rho_m^{2\gamma}} J_m = \frac{h}{\lambda_0 c}, \quad (24)$$

$$s = \pi^2 \frac{\varepsilon_0}{c} \gamma^5 \left(\frac{1}{k_0^2 z_0} \right)^2 c_g^2 c_r (\cos \alpha) \frac{\varepsilon^{2\beta+1}}{\rho_s^{2\gamma-1}} J_m = \frac{h}{2\pi} \quad (25)$$

with

$$J_m = \int_{-\infty}^{+\infty} f^2 d\bar{z} \cong z_0 \sqrt{2\pi}. \quad (26)$$

Here we are free to choose $\beta = \gamma \gg 1$ which leads to

$$\rho_s \cong \rho_m = \varepsilon. \quad (27)$$

The lower limits ρ_m , and ρ_s of the integrals (21) and (22) then decrease linearly with ε and with the radius r_0 . This forms a “similar” set of geometrical configurations, having a common shape which is independent of ρ_m , ρ_s , and ε in the range of small ε .

Taking $\hat{r} = r_0$ as an effective radius of the configuration (20), combination of relations (23)–(25) finally yields a photon diameter

$$2\hat{r} = \frac{\varepsilon \lambda_0}{\pi |\cos \alpha|} \quad (28)$$

being independent of γ . Thus the individual photon model becomes strongly needle-shaped when $\varepsilon \leq |\cos \alpha|$.

It should be observed that the photon spin of expression (25) disappears when $\text{div } \mathbf{E}$ vanishes and the basic relations reduce to Maxwell’s equations. This is also the case under more general conditions, due to the behaviour of the Poynting vector and to the requirement of a finite integrated field energy [3, 4, 5, 6].

3 The intrinsic electric charge distribution

We now turn to the intrinsic electric charge distribution within the photon wave-packet volume, representing an important but somewhat speculative part of the present analysis. It concerns the detailed process by which the photon configuration and its charge distribution are broken up to form a pair of particles of opposite electric polarity. Even if electric charges can arise and disappear in the vacuum state due to the quantum mechanical fluctuations, it may be justified as a first step to investigate whether the total intrinsic photon charge of one polarity can become sufficient as compared to the electric charges of the electron and positron.

With the present strongly oscillating charge density in space, the total intrinsic charge of either polarity can be estimated with good approximation from equations (17) and (19). This charge appears only within half of the axial extension of the packet, and its average value differs by the factor $\frac{2}{\pi}$ from

the local peak value of its sinusoidal variation. From equation (19) this intrinsic charge is thus given by

$$q = \frac{z_0}{\pi} \int_{\rho_q}^{\infty} 2\pi r \frac{\bar{\rho}}{f} dr = 2\sqrt{\pi} z_0 \varepsilon_0 \gamma^3 \frac{1}{k_0^2 z_0} c_g \frac{\varepsilon^\beta}{\rho_q^\gamma}, \quad (29)$$

where the last factor becomes equal to unity when $\beta = \gamma$ and the limit $\rho_q = \varepsilon$ for a similar set of geometrical configurations. Relations (29) and (24) then yield

$$q^2 = \frac{8}{\pi^3} \varepsilon_0 c^2 \gamma z_0 m = \frac{8}{\pi^3} \varepsilon_0 c h \gamma \frac{z_0}{\lambda_0} \cong \cong 45 \times 10^{-38} \gamma \frac{z_0}{\lambda_0} \quad (30)$$

and

$$\frac{q}{e} \cong 4.2 \left(\frac{\gamma z_0}{\lambda_0} \right)^{1/2}. \quad (31)$$

With a large γ and a small line width leading to $\lambda_0 \ll z_0$, the total intrinsic charge thus substantially exceeds the charge of the created particle pair. However, the question remains how much of the intrinsic charge becomes available during the disintegration process of the photon.

A much smaller charge would become available in a somewhat artificial situation where the density distribution of charge is perturbed by a 90 degrees phaseshift of the sinusoidal factor in expression (17). This would add a factor $2 \exp[-4\pi^2(z_0/\lambda_0)^2]$ to the middle and right-hand members of equation (30), and makes $q \gtrsim e$ only for extremely large values of γ and for moderately narrow line widths.

4 Conservation laws of pair formation

There are three conservation laws to be taken into account in the pair formation process. The first concerns the total energy. Here we limit ourselves to the marginal case where the kinetic energy of the created particles can be neglected as compared to the equivalent energy of their rest masses. Conservation of the total energy is then expressed by

$$mc^2 = \frac{hc}{\lambda_0} = 2m_e c^2. \quad (32)$$

Combination with equation (28) yields an effective photon diameter

$$2\hat{r} = \frac{\varepsilon h}{2\pi m_e c |\cos \alpha|}. \quad (33)$$

With $\varepsilon \leq |\cos \alpha|$ we have $2\hat{r} \leq 3.9 \times 10^{-13}$ m being equal to the Compton wavelength and representing a clearly developed form of needle radiation.

The second conservation law concerns the preservation of angular momentum. It is satisfied by the spin $\frac{h}{2\pi}$ of the photon in the capacity of a boson particle, as given by expression (25). This angular momentum becomes equal to the sum of the spin $\frac{h}{4\pi}$ of the created electron and positron being fermions. In principle, the angular momenta of the two

created particles could also become antiparallel and the spin of the photon zero, but such a situation would contradict all other experience about the photon spin.

The third conservation law deals with the preservation of the electric charge. This condition is clearly satisfied by the vanishing integrated photon charge, and by the opposite polarities of the created particles. In a more detailed picture where the photon disintegrates into the charged particles, it could also be conceived as a splitting process of the positive and negative parts of the intrinsic electric charge distributions of the photon.

Magnetic moment conservation is satisfied by having parallel angular momenta and opposite charges of the electron and positron, and by a vanishing magnetic moment of the photon [5, 6].

5 Associated questions of the vacuum state concept

The main new feature of the revised quantum electrodynamical theory of Section 2 is the introduction of a nonzero electric field divergence in the vacuum, as supported by the existence of quantum mechanical fluctuations. In this theory the values of the dielectric constant and the magnetic permeability of the conventional empty-space vacuum have been adopted. This is because no electrically polarized and magnetized atoms or molecules are assumed to be present, and that the vacuum fluctuations as well as superimposed regular phenomena such as waves take place in a background of empty space.

As in a review by Gross [7], the point could further be made that a "vacuum polarization" screens the point-charge-like electron in such a way that its effective electrostatic force vanishes at large distances. There is, however, experimental evidence for such a screening not to become important at the scale of the electron and photon models treated here. In the vacuum the electron is thus seen to be subject to scattering processes due to its full electrostatic field, and an electrically charged macroscopic object is also associated with such a measurable field. This would be consistent with a situation where the vacuum fluctuations either are small or essentially independent as compared to an external disturbance, and where their positive contributions to the local electric charge largely cancel their negative ones.

To these arguments in favour of the empty-space values of the dielectric constant and the magnetic permeability two additional points can also be added. The first is due to the Heisenberg uncertainty relation which implies that the vacuum fluctuations appear spontaneously during short time intervals and independently of each other. They can therefore hardly have a screening effect such as that due to Debye in a quasi-neutral plasma. The second point is based on the fact that static measurements of the dielectric constant and the magnetic permeability result in values the product of which becomes equal to the inverted square of the measured velocity of light.

6 Conclusions

The basis of the conservation laws in Section 4 is rather obvious, but it nevertheless becomes nontrivial when a comparison is made between conventional quantum electrodynamics based on Maxwell's equations on one hand and the present revised theory on the other. Thereby the following points should be observed:

- The needle-like radiation of the present photon model is necessary for understanding the observed creation of an electron-positron pair which forms two rays that start within a small region, and which have original directions along the path of the incoming photon. Such needle radiation does not come out of conventional theory [3, 4, 5, 6];
- The present revised theory leads to a nonzero spin of the photon, not being available from conventional quantum electrodynamics based on Maxwell's equations; [3, 4, 5, 6]. The present model is thus consistent with a photon as a boson which decays into two fermions;
- The nonzero divergence of the electric field in the present theory allows for a local nonzero electric charge density, even if the photon has a vanishing net charge. This may indicate how the intrinsic electric photon charges can form two charged particles of opposite polarity when the photon structure becomes disintegrated. Such a process is supported by the experimental fact that the photon decays into two charged particles through the impact of the electric field from an atomic nucleus or from an electron. This could hardly occur if the photon body would become electrically neutral at any point within its volume. Apart from such a scenario, the electromagnetic field configuration of the photon may also be broken up by nonlinear interaction with a strong external electric field;
- The present approach to the pair formation process has some similarity with the breaking of the stability of vacuum by a strong external electric field, as being investigated by Fradkin et al. [8].

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Ricci Flow and Quantum Theory

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We show how Ricci flow is related to quantum theory via Fisher information and the quantum potential.

1 Introduction

In [9, 13, 14] we indicated some relations between Weyl geometry and the quantum potential, between conformal general relativity (GR) and Dirac-Weyl theory, and between Ricci flow and the quantum potential. We would now like to develop this a little further. First we consider simple Ricci flow as in [35, 49]. Thus from [35] we take the Perelman entropy functional as **(1A)** $\mathfrak{F}(g, f) = \int_M (|\nabla f|^2 + R) \exp(-f) dV$ (restricted to f such that $\int_M \exp(-f) dV = 1$) and a Nash (or differential) entropy via **(1B)** $N(u) = \int_M u \log(u) dV$ where $u = \exp(-f)$ (M is a compact Riemannian manifold without boundary). One writes $dV = \sqrt{\det(g)} \prod dx^i$ and shows that if $g \rightarrow g + \text{sh}(g, h \in \mathcal{M} = \text{Riem}(M))$ then **(1C)** $\partial_s \det(g)|_{s=0} = g^{ij} h_{ij} \det(g) = (\text{Tr}_g h) \det(g)$. This comes from a matrix formula of the following form **(1D)** $\partial_s \det(A + B)|_{s=0} = (A^{-1} : B) \det(A)$ where $A^{-1} : B = a^{ij} b_{ji} = a^{ij} b_{ij}$ for symmetric B (a^{ij} comes from A^{-1}). If one has Ricci flow **(1E)** $\partial_s g = -2\text{Ric}$ (i.e. $\partial_s g_{ij} = -2R_{ij}$) then, considering $h \sim -2\text{Ric}$, one arrives at **(1F)** $\partial_s dV = -R dV$ where $R = g^{ij} R_{ij}$ (more general Ricci flow involves **(1G)** $\partial_t g_{ik} = -2(R_{ik} + \nabla_i \nabla_k \phi)$). We use now t and s interchangeably and suppose $\partial_t g = -2\text{Ric}$ with $u = \exp(-f)$ satisfying $\square^* u = 0$ where $\square^* = -\partial_t - \Delta + R$. Then $\int_M \exp(-f) dV = 1$ is preserved since **(1H)** $\partial_t \int_M u dV = \int_M (\partial_s u - Ru) dV = -\int_M \Delta u dV = 0$ and, after some integration by parts,

$$\begin{aligned} \partial_t N &= \int_M [\partial_t u (\log(u) + 1) dV + u \log(u) \partial_t dV] = \\ &= \int_M (|\nabla f|^2 + R) e^{-f} dV = \mathfrak{F}. \end{aligned} \quad (1.1)$$

In particular for $R \geq 0$, N is monotone as befits an entropy. We note also that $\square^* u = 0$ is equivalent to **(1I)** $\partial_t f = -\Delta f + |\nabla f|^2 - R$.

It was also noted in [49] that \mathfrak{F} is a Fisher information functional (cf. [8, 10, 24, 25]) and we showed in [13] that for a given 3-D manifold M and a Weyl-Schrödinger picture of quantum evolution based on [42, 43] (cf. also [4, 5, 6, 8, 9, 10, 11, 12, 16, 17, 51]) one can express \mathfrak{F} in terms of a quantum potential Q in the form **(1J)** $\mathfrak{F} \sim \alpha \int_M Q P dV + \beta \int_M |\vec{\phi}|^2 P dV$ where $\vec{\phi}$ is a Weyl vector and P is a probability distribution associated with a quantum mass density $\hat{\rho} \sim |\psi|^2$. There will be a corresponding Schrödinger

equation (SE) in a Weyl space as in [10, 13] provided there is a phase S (for $\psi = |\psi| \exp(iS/\hbar)$) satisfying **(1K)** $(1/m) \text{div}(P \nabla S) = \Delta P - R P$ (arising from $\partial_t \hat{\rho} - \Delta \hat{\phi} = -(1/m) \text{div}(\hat{\rho} \nabla S)$ and $\partial_t \hat{\rho} + \Delta \hat{\rho} - R \hat{\rho} = 0$ with $\hat{\rho} \sim P \sim u \sim |\psi|^2$). In the present work we show that there can exist solutions S of **(1K)** and this establishes a connection between Ricci flow and quantum theory (via Fisher information and the quantum potential). Another aspect is to look at a relativistic situation with conformal perturbations of a 4-D semi-Riemannian metric g based on a quantum potential (defined via a quantum mass). Indeed in a simple minded way we could perhaps think of a conformal transformation $\hat{g}_{ab} = \Omega^2 g_{ab}$ (in 4-D) where following [14] we can imagine ourselves immersed in conformal general relativity (GR) with metric \hat{g} and **(1L)** $\exp(Q) \sim \mathfrak{M}^2/m^2 = \Omega^2 = \hat{\phi}^{-1}$ with $\beta \sim \mathfrak{M}$ where β is a Dirac field and Q a quantum potential $Q \sim (\hbar^2/m^2 c^2) (\square_g \sqrt{\rho})/\sqrt{\rho}$ with $\rho \sim |\psi|^2$ referring to a quantum matter density. The theme here (as developed in [14]) is that Weyl-Dirac action with Dirac field β leads to $\beta \sim \mathfrak{M}$ and is equivalent to conformal GR (cf. also [8, 10, 36, 45, 46, 47] and see [28] for ideas on Ricci flow gravity).

REMARK 1.1. For completeness we recall (cf. [10, 50]) for $\mathfrak{L}_G = (1/2\chi) \sqrt{-g} R$

$$\begin{aligned} \delta \mathfrak{L}_G &= \frac{1}{2\chi} \left[R_{ab} - \frac{1}{2} g_{ab} R \right] \sqrt{-g} \delta g^{ab} + \\ &+ \frac{1}{2\chi} g^{ab} \sqrt{-g} \delta R_{ab}. \end{aligned} \quad (1.2)$$

The last term can be converted to a boundary integral if certain derivatives of g_{ab} are fixed there. Next following [7, 9, 14, 27, 38, 39, 40] the Einstein frame GR action has the form

$$S_{GR} = \int d^4 x \sqrt{-g} (R - \alpha (\nabla \psi)^2 + 16\pi L_M) \quad (1.3)$$

(cf. [7]) whose conformal form (conformal GR) is

$$\begin{aligned} \hat{S}_{GR} &= \int d^4 x \sqrt{-\hat{g}} e^{-\psi} \times \\ &\times \left[\hat{R} - \left(\alpha - \frac{3}{2} \right) (\hat{\nabla} \psi)^2 + 16\pi e^{-\psi} L_M \right] = \\ &= \int d^4 x \sqrt{-\hat{g}} \left[\hat{\phi} \hat{R} - \left(\alpha - \frac{3}{2} \right) \frac{(\hat{\nabla} \hat{\phi})^2}{\hat{\phi}} + 16\pi \hat{\phi}^2 L_M \right], \end{aligned} \quad (1.4)$$

where $\hat{g}_{ab} = \Omega^2 g_{ab}$, $\Omega^2 = \exp(\psi) = \phi$, and $\hat{\phi} = \exp(-\psi) = \phi^{-1}$. If we omit the matter Lagrangians, and set $\lambda = \frac{3}{2} - \alpha$, (1.4) becomes for $\hat{g}_{ab} \rightarrow g_{ab}$

$$\tilde{S} = \int d^4x \sqrt{-g} e^{-\psi} [R + \lambda(\nabla\psi)^2]. \quad (1.5)$$

In this form on a 3-D manifold M we have **exactly** the situation treated in [10, 13] with an associated SE in Weyl space based on **(1K)**. ■

2 Solution of (1K)

Consider now **(1K)** $(1/m)\text{div}(P\nabla S) = \Delta P - RP$ for $P \sim \sim \hat{\rho} \sim |\psi|^2$ and $\int P \sqrt{|g|} d^3x = 1$ (in 3-D we will use here $\sqrt{|g|}$ for $\sqrt{-g}$). One knows that $\text{div}(P\nabla S) = P\Delta S + \nabla P \cdot \nabla S$ and

$$\left. \begin{aligned} \Delta\psi &= \frac{1}{\sqrt{|g|}} \partial_m(\sqrt{|g|}\nabla\psi), \quad \nabla\psi = g^{mn}\partial_n\psi \\ \int_M \text{div}\mathbf{V} \sqrt{|g|} d^3x &= \int_{\partial M} \mathbf{V} \cdot \mathbf{ds} \end{aligned} \right\} \quad (2.1)$$

(cf. [10]). Recall also $\int P \sqrt{|g|} d^3x = 1$ and

$$\left. \begin{aligned} Q &\sim -\frac{\hbar^2}{8m} \left[\left(\frac{\nabla P}{P} \right)^2 - 2 \left(\frac{\Delta P}{P} \right) \right] \\ \langle Q \rangle_\psi &= \int PQ d^3x \end{aligned} \right\} \quad (2.2)$$

Now in 1-D an analogous equation to **(1K)** would be **(3A)** $(PS')' = P' - RP = F$ with solution determined via

$$\begin{aligned} PS' &= P' - \int RP + c \Rightarrow \\ \Rightarrow S' &= \partial_x \log(P) - \frac{1}{P} \int RP + cP^{-1} \Rightarrow \\ \Rightarrow S &= \log(P) - \int \frac{1}{P} \int RP + c \int P^{-1} + k, \end{aligned} \quad (2.3)$$

which suggests that solutions of **(1K)** do in fact exist in general. We approach the general case in Sobolev spaces à la [1, 2, 15, 22]. The volume element is defined via $\eta = \sqrt{|g|} dx^1 \wedge \dots \wedge dx^n$ (where $n = 3$ for our purposes) and $*$: $\wedge^p M \rightarrow \wedge^{n-p} M$ is defined via

$$\left. \begin{aligned} (*\alpha)_{\lambda_{p+1}\dots\lambda_n} &= \frac{1}{p!} \eta_{\lambda_1\dots\lambda_n} \alpha^{\lambda_1\dots\lambda_p} \\ (\alpha, \beta) &= \frac{1}{p!} \alpha_{\lambda_1\dots\lambda_p} \beta^{\lambda_1\dots\lambda_p} \end{aligned} \right\}, \quad (2.4)$$

$*1 = \eta$; $**\alpha = (-1)^{p(n-p)}\alpha$; $*\eta = 1$; $\alpha \wedge (*\beta) = (\alpha, \beta)\eta$. One writes now $\langle \alpha, \beta \rangle = \int_M (\alpha, \beta)\eta$ and, for (Ω, ϕ) a local chart we have **(2A)** $\int_M f dV = \int_{\phi(\Omega)} (\sqrt{|g|}f) \circ \phi^{-1} \prod dx^i$

($\sim \int_M f \sqrt{|g|} \prod dx^i$). Then one has **(2B)** $\langle d\alpha, \gamma \rangle = \langle \alpha, \delta\gamma \rangle$ for $\alpha \in \wedge^p M$ and $\gamma \in \wedge^{p+1} M$ where the codifferential δ on p-forms is defined via **(2C)** $\delta = (-1)^p *^{-1} d*$. Then $\delta^2 = d^2 = 0$ and $\Delta = d\delta + \delta d$ so that $\Delta f = \delta df = -\nabla^\nu \nabla_\nu f$. Indeed for $\alpha \in \wedge^p M$

$$(\delta\alpha)_{\lambda_1, \dots, \lambda_{p-1}} = -\nabla^\gamma \alpha_{\gamma, \lambda_1, \dots, \lambda_{p-1}} \quad (2.5)$$

with $\delta f = 0$ ($\delta : \wedge^p M \rightarrow \wedge^{p-1} M$). Then in particular **(2D)** $\langle \Delta\phi, \phi \rangle = \langle \delta d\phi, \phi \rangle = \langle d\phi, d\phi \rangle = \int_M \nabla^\nu \phi \nabla_\nu \phi \eta$.

Now to deal with weak solutions of an equation in divergence form look at an operator **(2E)** $Au = -\nabla(a\nabla u) \sim (-1/\sqrt{|g|}) \partial_m(\sqrt{|g|} a g^{mn} \nabla_n u) = -\nabla_m(a\nabla^m u)$ so that for $\phi \in \mathcal{D}(M)$

$$\begin{aligned} \int_M Au\phi dV &= -\int [\nabla_m(a g^{mn} \nabla_n u)] \phi dV = \\ &= \int a g^{mn} \nabla_n u \nabla_m \phi dV = \int a \nabla^m u \nabla_m \phi dV. \end{aligned} \quad (2.6)$$

Here one imagines M to be a complete Riemannian manifold with Sobolev spaces $H_0^1(M) \sim H^1(M)$ (see [1, 3, 15, 26, 29, 48]). The notation in [1] is different and we think of $H^1(M)$ as the space of L^2 functions u on M with $\nabla u \in L^2$ and H_0^1 means the completion of $\mathcal{D}(M)$ in the H^1 norm $\|u\|^2 = \int_M [|u|^2 + |\nabla u|^2] dV$. Following [29] we can also assume $\partial M = \emptyset$ with M connected for all M under consideration. Then let $H = H^1(M)$ be our Hilbert space and consider the operator $A(S) = -(1/m)\nabla(P\nabla S)$ with

$$B(S, \psi) = \frac{1}{m} \int P \nabla^m S \nabla_m \psi dV \quad (2.7)$$

for $S, \psi \in H_0^1 = H^1$. Then $A(S) = RP - \Delta P = F$ becomes **(2F)** $B(S, \psi) = \langle F, \psi \rangle = \int F\psi dV$ and one has **(2G)** $|B(S, \psi)| \leq c \|S\|_H \|\psi\|_H$ and $|B(S, S)| = \int P(\nabla S)^2 dV$. Now $P \geq 0$ with $\int P dV = 1$ but to use the Lax-Milgram theory we need here $|B(S, S)| \geq \beta \|S\|_H^2$ ($H = H^1$). In this direction one recalls that in Euclidean space for $\psi \in H_0^1(\mathbf{R}^3)$ there follows **(2H)** $\|\psi\|_{L^2}^2 \leq c \|\nabla\psi\|_{L^2}^2$ (Friedrich's inequality — cf. [48]) which would imply $\|\psi\|_H^2 \leq (c+1)\|\nabla\psi\|_{L^2}^2$. However such Sobolev and Poincaré-Sobolev inequalities become more complicated on manifolds and **(2H)** is in no way automatic (cf. [1, 29, 48]). However we have some recourse here to the definition of P , namely $P = \exp(-f)$, which basically is a conformal factor and $P > 0$ unless $f \rightarrow \infty$. One heuristic situation would then be to assume **(2I)** $0 < \epsilon \leq P(x)$ on M (and since $\int \exp(-f) dV = 1$ with $dV = \sqrt{|g|} \prod_1^3 dx^i$ we must then have $\epsilon \int dV \leq 1$ or $\text{vol}(M) = \int_M dV \leq (1/\epsilon)$). Then from **(2G)** we have **(2J)** $|B(S, S)| \geq \epsilon \|(\nabla S)\|^2$ and for any $\kappa > 0$ it follows: $|B(S, S)| + \kappa \|S\|_{L^2}^2 \geq \min(\epsilon, \kappa) \|S\|_{H^1}^2$. This means via Lax-Milgram that the equation

$$A(S) + \kappa S = -\frac{1}{m} \nabla(P\nabla S) + \kappa S = F = RP - \Delta P \quad (2.8)$$

has a unique weak solution $S \in H^1(M)$ for any $\kappa > 0$ (assuming $F \in L^2(M)$). Equivalently $(\mathbf{2K}) - \frac{1}{m}[P\Delta S + (\nabla P)(\nabla S)] + \kappa S = F$ has a unique weak solution $S \in H^1(M)$. This is close but we cannot put $\kappa = 0$. A different approach following from remarks in [29], pp. 56–57 (corrected in [30], p. 248), leads to an heuristic weak solution of $(\mathbf{1K})$. Thus from a result of Yau [53] if M is a complete simply connected 3-D differential manifold with sectional curvature $K < 0$ one has for $u \in \mathcal{D}(M)$

$$\begin{aligned} \int_M |\psi| dV &\leq (2\sqrt{-K})^{-1} \int_M |\nabla \psi| dV \Rightarrow \\ &\Rightarrow \int_M |\psi|^2 dV \leq c \int_M |\nabla \psi|^2 dV. \end{aligned} \quad (2.9)$$

Hence $(\mathbf{2H})$ holds and one has $\|\psi\|_{H^1}^2 \leq (1+c)\|\nabla \psi\|^2$. Moreover if M is bounded and simply connected with a reasonable boundary ∂M (e.g. weakly convex) one expects $(\mathbf{2L}) \int_M |\psi|^2 dV \leq c \int_M |\nabla \psi|^2 dV$ for $\psi \in \mathcal{D}(M)$ (cf. [41]). In either case $(\mathbf{2M}) |\bar{B}(S, S)| \geq \epsilon \|(\nabla S)^2\| \geq (c+1)^{-1} \epsilon \|S\|_{H_0^1}^2$ and this leads via Lax-Milgram again to a sample result

THEOREM 2.1 *Let M be a bounded and simply connected 3-D differential manifold with a reasonable boundary ∂M . Then there exists a unique weak solution of $(\mathbf{1K})$ in $H_0^1(M)$.*

REMARK 2.1. One must keep in mind here that the metric is changing under the Ricci flow and assume that estimates involving e.g. K are considered over some time interval. ■

REMARK 2.2. There is an extensive literature concerning eigenvalue bounds on Riemannian manifolds and we cite a few such results. Here $I_\infty(M) \sim \inf_\Omega (A(\partial\Omega)/V(\Omega))$ where Ω runs over (connected) open subsets of M with compact closure and smooth boundary (cf. [18, 19]). Yau's result is $I_\infty(M) \geq 2\sqrt{-K}$ (with equality for the 3-D hyperbolic space) and Cheeger's result involves follows $|\nabla \phi|_{L^2} \geq (1/2)I_\infty(M)\|\phi\|_{L^2} \geq \sqrt{-K}\|\phi\|_{L^2}$. There are many other results where e.g. $\lambda_1 \geq c(\text{vol}(M))^{-2}$ for M a compact 3-D hyperbolic manifold of finite volume (see [21, 34, 44] for this and variations). There are also estimates for the first eigenvalue along a Ricci flow in [33, 37] and estimates of the form $\lambda_1 \geq 3K$ for closed 3-D manifolds with Ricci curvature $R \geq 2K$ ($K > 0$) in [32, 33]. In fact Ling obtains $\lambda_1 \geq K + (\pi^2/\tilde{d}^2)$ where \tilde{d} is the diameter of the largest interior ball in nodal domains of the first eigenfunction. There are also estimates $\lambda_1 \geq (\pi^2/d^2)$ ($d = \text{diam}(M)$, $R \geq 0$) in [31, 52, 54] and the papers of Ling give an excellent survey of results, new and old, including estimates of a similar kind for the first Dirichlet and Neumann eigenvalues. ■

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A Possible General Approach Regarding the Conformability of Angular Observables with the Mathematical Rules of Quantum Mechanics

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The conformability of angular observables (angular momentum and azimuthal angle) with the mathematical rules of quantum mechanics is a question which still rouses debates. It is valued negatively within the existing approaches which are restricted by two amendable presumptions. If the respective presumptions are removed one can obtain a general approach in which the mentioned question is valued positively.

1 Introduction

In the last decades the pair of angular observables $L_z - \varphi$ (angular momentum — azimuthal angle) was and still is regarded as being unconformable to the accepted mathematical rules of Quantum Mechanics (QM) (see [1–24]). The unconformity is identified with the fact that, in some cases of circular motions, for the respective pair the Robertson-Schrödinger uncertainty relation (RSUR) is not directly applicable. That fact roused many debates and motivated various approaches planned to elucidate in an acceptable manner the missing conformability. But so far such an elucidation was not ratified (or admitted unanimously) in the scientific literature.

A minute inspection of the things shows that in the main all the alluded approaches have a restricted character due to the presumptions (P):

P_1 : Consideration of RSUR as a twofold reference element by: (i) proscription of its direct $L_z - \varphi$ descendant, and (ii) substitution of the respective descendant with some RSUR-mimic relations;

P_2 : Discussion only of the systems with sharp circular rotations (SCR).

But the mentioned presumptions are amendable because they conflict with the following facts (F):

F_1 : Mathematically, the RSUR is only a secondary piece, of limited validity, resulting from a generally valid element represented by a Cauchy Schwarz formula (CSF) (see down Section 4);

F_2 : From a natural physical viewpoint the $L_z - \varphi$ pair must be considered in connection not only with SCR but also with any orbital (spatial) motions (e.g. with the non-circular rotations (NCR), presented below in Section 3).

The above facts suggest that for the $L_z - \varphi$ problem ought to search new approaches, by removing the mentioned premises P_1 and P_2 . As we know until now such approaches were not promoted in the publications from the main stream of scientific literature. In this paper we propose a possible

general approach of the mentioned kind, able to ensure a natural conformability of the $L_z - \varphi$ pair with the prime mathematical rules of QM.

For distinguishing our proposal from the alluded restricted approaches, in the next Section we present briefly the respective approaches, including their main assertions and a set of unavoidable shortcomings which trouble them destructively. Then, in Section 3, we disclose the existence of two examples of NCR which are in discordance with the same approaches.

The alluded shortcomings and discordances reenforce the interest for new and differently oriented approaches of the $L_z - \varphi$ problem. Such an approach, of general perspective, is argued and detailed below in our Section 4. We end the paper in Section 5 with some associate conclusions.

2 Briefly on the restricted approaches

Certainly, for the history of the $L_z - \varphi$ problem, the first reference element was the Robertson Schrödinger uncertainty relation (RSUR) introduced [25, 26] within the mathematical formalism of QM. In terms of usual notations from QM the RSUR is written as

$$\Delta_{\psi} A \cdot \Delta_{\psi} B \geq \frac{1}{2} \left| \langle [\hat{A}, \hat{B}] \rangle_{\psi} \right|, \quad (1)$$

where $\Delta_{\psi} A$ and $\langle (\dots) \rangle_{\psi}$ signify the standard deviation of the observable A respectively the mean value of (\dots) in the state described by the wave function ψ , while $[\hat{A}, \hat{B}]$ denote the commutator of the operators \hat{A} and \hat{B} (for more details about the notations and validity regarding the RSUR 1, see the next Section).

The attempts for application of RSUR (1) to the case with $A = L_z$ and $B = \varphi$, i.e. to the $L_z - \varphi$ pair, evidenced the following intriguing facts.

On the one hand, according to the usual procedures of QM [27], the observables L_z and φ should be described by the conjugated operators

$$\hat{L}_z = -i\hbar \frac{\partial}{\partial \varphi}, \quad \hat{\varphi} = \varphi. \quad (2)$$

respectively by the commutation relation

$$[\hat{L}_z, \hat{\varphi}] = -i\hbar. \quad (3)$$

So for the alluded pair the RSUR (1) requires for its direct descendant the relation

$$\Delta_\psi L_z \cdot \Delta_\psi \varphi \geq \frac{\hbar}{2}. \quad (4)$$

On the other hand this last relation is explicitly inapplicable in cases of angular states regarding the systems with sharp circular rotations (SCR). The respective inapplicability is pointed out here bellow.

As examples with SCR can be quoted : (i) a particle (bead) on a circle, (ii) a 1D (or fixedaxis) rotator and (iii) non-degenerate spatial rotations. One finds examples of systems with spatial rotations in the cases of a particle on a sphere, of 2D or 3D rotators and of an electron in a hydrogen atom respectively. The mentioned rotations are considered as non-degenerate if all the specific (orbital) quantum numbers have well-defined (unique) values. The alluded SRC states are described by the following wave functions taken in a φ — representation

$$\psi_m(\varphi) = (2\pi)^{-\frac{1}{2}} e^{im\varphi} \quad (5)$$

with the stipulations $\varphi \in [0, 2\pi)$ and $m = 0, \pm 1, \pm 2, \dots$. The respective stipulations are required by the following facts. Firstly, in cases of SRC the angle φ is a ordinary polar coordinate which must satisfy the corresponding mathematical rules regarding the range of definition [28]. Secondly, from a physical perspective, in the same cases the wave function $\psi(\varphi)$ is enforced to have the property $\psi(0) = \psi(2\pi - 0) : = \lim_{\varphi \rightarrow 2\pi - 0} \psi(\varphi)$.

For the alluded SRC one finds

$$\Delta_\psi L_z = 0, \quad \Delta_\psi \varphi = \frac{\pi}{\sqrt{3}}. \quad (6)$$

But these expressions for $\Delta_\psi L_z$ and $\Delta_\psi \varphi$ are incompatible with relation (4).

For avoiding the mentioned incompatibility many publications promoted the conception that in the case of $L_z - \varphi$ pair the RSUR (1) and the associated procedures of QM do not work correctly. Consequently it was accredited the idea that formula (4) must be proscribed and replaced by adjusted $\Delta_\psi L_z - \Delta_\psi \varphi$ relations planned to mime the RSUR (1). So, along the years, a lot of such mimic relations were proposed. In the main the respective relations can be expressed in one of the following forms:

$$\frac{\Delta_\psi L_z \cdot \Delta_\psi \varphi}{a(\Delta_\psi \varphi)} \geq \hbar \left| \langle b(\varphi) \rangle_\psi \right|, \quad (7)$$

$$\Delta_\psi L_z \cdot \Delta_\psi f(\varphi) \geq \hbar \left| \langle g(\varphi) \rangle_\psi \right|, \quad (8)$$

$$(\Delta_\psi L_z)^2 + \hbar^2 (\Delta_\psi u(\varphi))^2 \geq \hbar^2 \langle v(\varphi) \rangle_\psi^2, \quad (9)$$

$$\Delta_\psi L_z \cdot \Delta_\psi \varphi \geq \frac{\hbar}{2} |1 - 2\pi |\psi(2\pi - 0)||. \quad (10)$$

In (7)–(9) by a, b, f, g, u and v are denoted various adjusting functions (of $\Delta_\psi \varphi$ or of φ), introduced in literature by means of some circumstantial (and more or less fictitious) considerations.

Among the relations (7)–(10) of some popularity is (8) with $f(\varphi) = \sin \varphi$ (or $= \cos \varphi$) respectively $g(\varphi) = [\hat{L}_z, f(\hat{\varphi})]$. But, generally speaking, none of the respective relations is agreed unanimously as a suitable model able to substitute formula (4).

A minute examination of the facts shows that, in essence, the relations (7)–(10) are troubled by shortcomings revealed in the following remarks (**R**):

R₁ : The relation (10) is correct from the usual perspective of QM (see formulas 18 and 25 in the next Secion). But the respective relation evidently does not mime the RSUR (1) presumed as standard within the mentioned restricted approaches of $L_z - \varphi$ problem;

R₂ : Each replica from the classes depicted by (7)–(10) were planned to harmonize in a mimic fashion with the same presumed reference element represented by RSUR (1). But, in spite of such plannings, regarded comparatively, the respective replicas are not mutually equivalent;

R₃ : Due to the absolutely circumstantial considerations by which they are introduced, the relations (7)–(9) are in fact ad hoc formulas without any direct descendance from general mathematics of QM. Consequently the respective relations ought to be appreciated by taking into account sentences such are:

“In ... science, ad hoc often means the addition of corollary hypotheses or adjustment to a ... scientific theory to save the theory from being falsified by compensating for anomalies not anticipated by the theory in its unmodified form. ... Scientists are often suspicious or skeptical of theories that rely on ... ad hoc adjustments” [29].

Then, if one wants to preserve the mathematical formalism of QM as a unitary theory, as it is accredited in our days, the relations (7)–(9) must be regarded as unconvincing and inconvenient (or even prejudicial) elements;

R₄ : In fact in relations (7)–(9) the angle φ is substituted more or less factitiously with the adjusting functions a, b, f, g, v or u . Then in fact, from a natural perspective of physics, such substitutions, and consequently the respective relations, are only mathematical artifacts. But, in physics, the mathematical artifacts burden the scientific discussions by additions of extraneous entities (concepts, assertions, reasonings, formulas) which are not associated with a true information regarding the real world. Then, for a good efficiency of the discussions, the alluded additions ought to be evaluated by taking into account the principle of parsimony: *“Entities should not be multiplied unneces-*

sarily" (known also [30, 31] as the "Ockham's Razor" slogan). Through such an evaluation the relations (7)–(9) appear as unnecessary exercises which do not give real and useful contributions for the elucidation of the $L_z - \varphi$ problem.

In our opinion the facts revealed in this Section offer a minimal but sufficient base for concluding that as regards the $L_z - \varphi$ problem the approaches restricted around the premises P_1 and P_2 are unable to offer true and natural solutions.

3 The discordant examples with non-circular rotations

The discussions presented in the previous Section regard the situation of the $L_z - \varphi$ pair in relation with the mentioned SCR. But here is the place to note that the same pair must be considered also in connection with other orbital (spatial) motions which differ from SCR. Such motions are the non-circular rotations (NCR). As examples of NCR we mention the quantum torsion pendulum (QTP) respectively the degenerate spatial rotations of the systems mentioned in the previous Section (i.e. a particle on a sphere, 2D or 3D rotators and an electron in a hydrogen atom). A rotation (motion) is degenerate if the energy of the system is well-specified while the non-energetic quantum numbers (here of orbital nature) take all permitted values.

From the class of NCR let us firstly refer to the case of a QTP which in fact is a simple quantum oscillator. Indeed a QTP which oscillates around the z -axis is characterized by the Hamiltonian

$$\hat{H} = \frac{1}{2I} \hat{L}_z^2 + \frac{1}{2} J\omega^2 \varphi^2. \quad (11)$$

Note that in this expression φ denotes the azimuthal angle whose range of definition is the interval $(-\infty, \infty)$. In the same expression appears \hat{L}_z as the z -component of angular momentum operator defined also by (2). The other symbols J and ω in (11) represent the QTP momentum of inertia respectively the frequency of torsional oscillations. The Schrödinger equation associated to the Hamiltonian (11) shows that the QTP have eigenstates described by the wave functions

$$\psi_n(\varphi) = \psi_n(\xi) \propto \exp\left(-\frac{\xi^2}{2}\right) \mathcal{H}_n(\xi), \quad \xi = \varphi \sqrt{\frac{J\omega}{\hbar}}, \quad (12)$$

where $n = 0, 1, 2, 3, \dots$ signifies the oscillation quantum number and $\mathcal{H}_n(\xi)$ stand for Hermite polynomials of ξ . The eigenstates described by (12) have energies $E_n = \hbar\omega(n + \frac{1}{2})$. In the states (12) for the observables L_z and φ associated with the operators (2) one obtains the expressions

$$\Delta_\psi L_z = \sqrt{\hbar J\omega \left(n + \frac{1}{2}\right)}, \quad \Delta_\psi \varphi = \sqrt{\frac{\hbar}{J\omega} \left(n + \frac{1}{2}\right)}, \quad (13)$$

which are completely similar with the corresponding ones for the $x - p$ pair of a rectilinear oscillator [27]. With the expres-

sions (13) for $\Delta_\psi L_z$ and $\Delta_\psi \varphi$ one finds that in the case of QTP the $L_z - \varphi$ pair satisfies the proscribed formula (4).

From the same class of NCR let us now refer to a degenerate state of a particle on a sphere or of a 2D rotator. In such a state the energy is $E = \hbar^2 l(l+1)/2J$ where the orbital number l has a well-defined value ($J =$ moment of inertia). In the same state the magnetic number m can take all the values $-l, -l+1, \dots, -1, 0, 1, \dots, l-1, l$. Then the mentioned state is described by a wave function of the form

$$\psi_l(\theta, \varphi) = \sum_{m=-l}^l c_m Y_{lm}(\theta, \varphi). \quad (14)$$

Here θ and φ denote polar respectively azimuthal angles ($\theta \in [0, \pi], \varphi \in [0, 2\pi)$), $Y_{lm}(\theta, \varphi)$ are the spherical functions and c_m represent complex coefficients which satisfy the normalization condition $\sum_{m=-l}^l |c_m|^2 = 1$. With the expressions (2) for the operators \hat{L}_z and $\hat{\varphi}$ in a state described by (14) one obtains

$$(\Delta_\psi L_z)^2 = \sum_{m=-l}^l |c_m|^2 \hbar^2 m^2 - \left[\sum_{m=-l}^l |c_m|^2 \hbar m \right]^2, \quad (15)$$

$$\begin{aligned} (\Delta_\psi \varphi)^2 = & \sum_{m=-l}^l \sum_{r=-l}^l c_m^* c_r (Y_{lm, \varphi} Y_{lr}) - \\ & - \left[\sum_{m=-l}^l \sum_{r=-l}^l c_m^* c_r (Y_{lm, \varphi} Y_{lr}) \right]^2, \quad (16) \end{aligned}$$

where (f, g) is the scalar product of the functions f and g .

By means of the expressions (15) and (16) one finds that in the case of alluded NCR described by the wave functions (14) it is possible for the proscribed formula (4) to be satisfied. Such a possibility is conditioned by the concrete values of the coefficients c_m .

Now is the place for the following remark

R₅ : As regards the $L_z - \varphi$ problem, due to the here revealed aspects, the NCR examples exceed the bounds of the presumptions P_1 and P_2 of usual restricted approaches. That is why the mentioned problem requires new approaches of general nature if it is possible.

4 A possible general approach and some remarks associated with it.

A general approach of the $L_z - \varphi$ problem, able to avoid the shortcomings and discordances revealed in the previous two Sections, must be done by starting from the prime mathematical rules of QM. Such an approach is possible to be obtained as follows. Let us appeal to the usual concepts and notations of QM. We consider a quantum system whose state (of orbital nature) and two observables A_j ($j = 1, 2$) are described by the wave function ψ respectively by the operators \hat{A}_j . As usually with (f, g) we denote the scalar product of the functions

f and g . In relation with the mentioned state, the quantities $\langle A_j \rangle_\psi = (\psi, \hat{A}_j \psi)$ and $\delta_\psi \hat{A}_j = \hat{A}_j - \langle \hat{A}_j \rangle_\psi$ represent the mean (expected) value respectively the deviation-operator of the observable A_j regarded as a random variable. Then, by taking $A_1 = A$ and $A_2 = B$, for the two observables can be written the following Cauchy-Schwarz relation:

$$\left(\delta_\psi \hat{A} \psi, \delta_\psi \hat{A} \psi \right) \left(\delta_\psi \hat{B} \psi, \delta_\psi \hat{B} \psi \right) \geq \left| \left(\delta_\psi \hat{A} \psi, \delta_\psi \hat{B} \psi \right) \right|^2. \quad (17)$$

For an observable A_j regarded as a random variable the quantity $\Delta_\psi A_j = \left(\delta_\psi \hat{A}_j \psi, \delta_\psi \hat{A}_j \psi \right)^{1/2}$ represents its standard deviation. From (17) it results directly that the standard deviations $\Delta_\psi A$ and $\Delta_\psi B$ of the observables A and B satisfy the relation

$$\Delta_\psi A \cdot \Delta_\psi B \geq \left| \left(\delta_\psi \hat{A} \psi, \delta_\psi \hat{B} \psi \right) \right|, \quad (18)$$

which can be called *Cauchy-Schwarz formula* (CSF). Note that CSF (18) (as well as the relation (17) is always valid, i.e. for all observables, systems and states. Add here the important observation that the CSF (18) implies the restricted RSUR (1) only in the cases when the two operators $\hat{A} = \hat{A}_1$ and $\hat{B} = \hat{A}_2$ satisfy the conditions

$$\left(\hat{A}_j \psi, \hat{A}_k \psi \right) = \left(\psi, \hat{A}_j \hat{A}_k \psi \right), \quad j = 1, 2; \quad k = 1, 2. \quad (19)$$

Indeed in such cases one can write the relation

$$\begin{aligned} & \left(\delta_\psi \hat{A} \psi, \delta_\psi \hat{B} \psi \right) = \\ & = \frac{1}{2} \left(\psi, \left(\delta_\psi \hat{A} \cdot \delta_\psi \hat{B} \psi + \delta_\psi \hat{B} \cdot \delta_\psi \hat{A} \right) \psi \right) - \\ & - \frac{i}{2} \left(\psi, i \left[\hat{A}, \hat{B} \right] \psi \right), \end{aligned} \quad (20)$$

where the two terms from the right hand side are purely real and imaginary quantities respectively. Therefore in the mentioned cases from (18) one finds

$$\Delta_\psi A \cdot \Delta_\psi B \geq \frac{1}{2} \left| \left\langle \left[\hat{A}, \hat{B} \right] \right\rangle_\psi \right|. \quad (21)$$

i.e. the well known RSUR (1). The above general framing of RSUR (1)/(21) shows that for the here investigated question of $L_z - \varphi$ pair it is important to examine the fulfilment of the conditions (19) in each of the considered cases. In this sense the following remarks are of direct interest.

R₆ : In the cases described by the wave functions (5) for $L_z - \varphi$ pair one finds

$$\left(\hat{L}_z \psi_m, \hat{\varphi} \psi_m \right) = \left(\psi_m, \hat{L}_z \hat{\varphi} \psi_m \right) + i \hbar, \quad (22)$$

i.e. a clear violation in respect with the conditions (19);

R₇ : In the cases associated with the wave functions (12) and (14) for $L_z - \varphi$ pair one obtains

$$\left(\hat{L}_z \psi_n, \hat{\varphi} \psi_n \right) = \left(\psi_n, \hat{L}_z \hat{\varphi} \psi_n \right), \quad (23)$$

$$\begin{aligned} & \left(\hat{L}_z \psi_l, \hat{\varphi} \psi_l \right) = \left(\psi_l, \hat{L}_z \hat{\varphi} \psi_l \right) + \\ & + i \hbar \left\{ 1 + 2 \operatorname{Im} \left[\sum_{m=-l}^l \sum_{r=-l}^l c_m^* c_r \hbar m (Y_{lm}, \hat{\varphi} Y_{lr}) \right] \right\}, \end{aligned} \quad (24)$$

(where $\operatorname{Im} [\alpha]$ denotes the imaginary part of α);

R₈ : For any wave function $\psi(\varphi)$ with $\varphi \in [0, 2\pi)$ and $\psi(2\pi - 0) = \psi(0)$ it is generally true the formula

$$\left| \left(\delta_\psi \hat{L}_z \psi, \delta_\psi \hat{\varphi} \psi \right) \right| \geq \frac{\hbar}{2} |1 - 2\pi |\psi(2\pi - 0)| |, \quad (25)$$

which together with CSF (18) confirms relation (10).

The things mentioned above in this Section justify the following remarks

R₉ : The CSF (18) is an ab origine element in respect with the RSUR (1)/(21). Moreover, (18) is always valid, independently if the conditions (19) are fulfilled or not;

R₁₀ : The usual RSUR (1)/(21) are valid only in the circumstances strictly delimited by the conditions (19) and they are false in all other situations;

R₁₁ : Due to the relations (22) in the cases described by the wave functions (5) the conditions (19) are not fulfilled. Consequently in such cases the restricted RSUR (1)/(21) are essentially inapplicable for the pairs $L_z - \varphi$. However one can see that in the respective cases, mathematically, the CSF (18) remains valid as a trivial equality $0 = 0$;

R₁₂ : In the cases of NCR described by (12) the $L_z - \varphi$ pair satisfies the conditions (19) (mainly due to the relation (23)). Therefore in the respective cases the RSUR (1)/(21) are valid for L_z and φ ;

R₁₃ : The fulfilment of the conditions (19) by the $L_z - \varphi$ pair for the NCR associated with (14) depends on the annulment of the second term in the right hand side from (24) (i.e. on the values of the coefficients c_m). Adequately, in such a case, the correctness of the corresponding RSUR (1)/(21) shows the same dependence;

R₁₄ : The result (25) points out the fact that the adjusted relation (10) is only a secondary piece derivable from the generally valid CSF (18);

R₁₅ : The mimic relations (7)–(9) regard the cases with SCR described by the wave functions (5) when φ plays the role of polar coordinate. But for such a role [28] in order to be a unique (univocal) variable φ must be defined naturally only in the range $[0, 2\pi)$. The same range is considered in practice for the normalization of the wave functions (5). Therefore, in the cases under discussion the derivative with respect to φ refers to the mentioned range. Particularly for the extremities of the interval $[0, 2\pi)$ it has to operate with backward respectively forward derivatives. So in the alluded SCR cases

the relations (2) and (3) act well, with a natural correctness. The same correctness is shown by the respective relations in connection with the NCR described by the wave functions (12) or (14). In fact, from a more general perspective, the relations (2) and (3) regard the QM operators \hat{L}_z and $\hat{\varphi}$. Therefore they must have unique forms — i.e. expressions which do not depend on the particularities of the considered situations (e.g. systems with SCR or with NCR);

R₁₆ : The troubles of RSUR (1) regarding $L_z - \varphi$ pair are directly connected with the conditions (19). Then it is strange that in almost all the QM literature the respective conditions are not taken into account adequately. The reason seems to be related with the nowadays dominant Dirac's $\langle bra|$ and $|ket\rangle$ notations. In the respective notations the terms from the both sides of (19) have a unique representation namely $\langle \psi | \hat{A}_j \hat{A}_k | \psi \rangle$. The respective uniqueness can entail confusion (unjustified supposition) that the conditions (19) are always fulfilled. It is interesting to note that systematic investigations on the confusions/surprises generated by the Dirac's notations were started only recently [32]. Probably that further efforts on the line of such investigations will bring a new light on the conditions (19) as well as on other QM questions.

The ensemble of things presented above in this Section appoints a possible general approach for the discussed $L_z - \varphi$ problem and answer to a number of questions associated with the respective problem. Some significant aspects of the respective approach are noted in the next Section.

5 Conclusions

The facts and arguments discussed in the previous Sections guide to the following conclusions (**C**):

- C₁** : For the $L_z - \varphi$ pair the relations (2)–(3) are always viable in respect with the general CSF (18). That is why, from the QM perspective, for a correct description of questions regarding the respective pair, it is not at all necessary to resort to the mimetic formulas (7)–(10). Eventually the respective formulas can be accounted as ingenious exercises of pure mathematical facture. An adequate description of the mentioned kind can be given by taking CSF (18) and associated QM procedures as basic elements;
- C₂** : In respect with the conjugated observables L_z and φ the RSUR (1)/(21) is not adequate for the role of reference element for normality . For such a role the CSF (18) is the most suitable. In some cases of interest the respective CSF degenerates in the trivial equality $0 = 0$;
- C₃** : In reality the usual procedures of QM, illustrated above by the relations (2), (3), (17) and (18), work well and

without anomalies in all situations regarding the $L_z - \varphi$ pair. Consequently with regard to the conceptual as well as practical interests of science the mimic relations like (7)–(9) appear as useless inventions.

Now we wish to add the following observations (**O**):

O₁ : Mathematically the relation (17) is generalisable in the form

$$\det \left[\left(\delta_{\psi} \hat{A}_j \psi, \delta_{\psi} \hat{A}_k \psi \right) \right] \geq 0 \quad (26)$$

where $\det [\alpha_{jk}]$ denotes the determinant with elements α_{jk} and $j = 1, 2, \dots, r$; $k = 1, 2, \dots, r$ with $r \geq 2$. Such a form results from the fact that the quantities $\left(\delta_{\psi} \hat{A}_j \psi, \delta_{\psi} \hat{A}_k \psi \right)$ constitute the elements of a Hermitian and non-negatively defined matrix. Nevertheless, comparatively with (17), the generalisation (26) does not bring supplementary and inedited features regarding the conformability of observables $L_z - \varphi$ with the mathematical rules of QM;

O₂ : We consider [34, 42] that the above considerations about the problem of $L_z - \varphi$ pair can be of some non-trivial interest for a possible revised approach of the similar problem of the pair $N - \phi$ (number-phase) which is also a subject of controversies in recent publications (see [4, 11, 12, 13, 35, 36, 37, 38, 39] and References therein);

O₃ : Note that we have limited this paper only to mathematical aspects associated with the RSUR (1), without incursions in debates about the interpretations of the respective RSUR. Some opinions about those interpretations and connected questions are given in [40, 41, 42]. But the subject is delicate and probably that it will rouse further debates.

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A Unified Field Theory of Gravity, Electromagnetism, and the Yang-Mills Gauge Field

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In this work, we attempt at constructing a comprehensive four-dimensional unified field theory of gravity, electromagnetism, and the non-Abelian Yang-Mills gauge field in which the gravitational, electromagnetic, and material spin fields are unified as intrinsic geometric objects of the space-time manifold \mathbb{S}_4 via the connection, with the generalized non-Abelian Yang-Mills gauge field appearing in particular as a sub-field of the geometrized electromagnetic interaction.

1 Introduction

In our previous work [1], we developed a semi-classical conformal theory of quantum gravity and electromagnetism in which both gravity and electromagnetism were successfully unified and linked to each other through an “external” quantum space-time deformation on the fundamental Planck scale. Herein we wish to further explore the geometrization of the electromagnetic field in [1] which was achieved by linking the electromagnetic field strength to the torsion tensor built by means of a conformal mapping in the evolution (configuration) space. In so doing, we shall in general disregard the conformal mapping used in [1] and consider an arbitrary, very general torsion field expressible as a linear combination of the electromagnetic and material spin fields.

Herein we shall find that the completely geometrized Yang-Mills field of standard model elementary particle physics, which roughly corresponds to the electromagnetic, weak, and strong nuclear interactions, has a more general form than that given in the so-called rigid, local isospace.

We shall not simply describe our theory in terms of a Lagrangian functional due to our unease with the Lagrangian approach (despite its versatility) as a truly fundamental physical approach towards unification. While the meaning of a particular energy functional (to be extremized) is clear in Newtonian physics, in present-day space-time physics the choice of a Lagrangian functional often appears to be non-unique (as it may be concocted arbitrarily) and hence devoid of straightforward, intuitive physical meaning. We shall instead, as in our previous works [1–3], build the edifice of our unified field theory by carefully determining the explicit form of the connection.

2 The determination of the explicit form of the connection for the unification of the gravitational, electromagnetic, and material spin fields

We shall work in an affine-metric space-time manifold \mathbb{S}_4 (with coordinates x^μ) endowed with both curvature and torsion. As usual, if we denote the symmetric, non-singular, fun-

damental metric tensor of \mathbb{S}_4 by g , then $g_{\mu\lambda}g^{\nu\lambda} = \delta_\mu^\nu$, where δ is the Kronecker delta. The world-line s is then given by the quadratic differential form $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$. (The Einstein summation convention employed throughout this work.)

As in [1], for reasons that will be clear later, we define the electromagnetic field tensor F via the torsion tensor of space-time (the anti-symmetric part of the connection Γ) as follows:

$$F_{\mu\nu} = 2 \frac{mc^2}{e} \Gamma_{[\mu\nu]}^\lambda u_\lambda,$$

where m is the mass (of the electron), c is the speed of light in vacuum, and e is the electric charge, and where $u^\mu = \frac{dx^\mu}{ds}$ are the components of the tangent world-velocity vector whose magnitude is unity. Solving for the torsion tensor, we may write, under very general conditions,

$$\Gamma_{[\mu\nu]}^\lambda = \frac{e}{2mc^2} F_{\mu\nu} u^\lambda + S_{\mu\nu}^\lambda,$$

where the components of the third-rank material spin (chirality) tensor 3S are herein given via the second-rank anti-symmetric tensor 2S as follows:

$$S_{\mu\nu}^\lambda = S_{\mu}^\lambda u_\nu - S_{\nu}^\lambda u_\mu.$$

As can be seen, it is necessary that we specify the following orthogonality condition:

$$S_{\mu\nu} u^\nu = 0,$$

such that

$$S_{\mu\nu}^\lambda u_\lambda = 0.$$

We note that 3S may be taken as the intrinsic angular momentum tensor for microscopic physical objects which may be seen as the points in the space-time continuum itself. This way, 3S may be regarded as a microspin tensor describing the internal rotation of the space-time points themselves [2]. Alternatively, 3S may be taken as being “purely material” (entirely non-electromagnetic).

The covariant derivative of an arbitrary tensor field T is given via the asymmetric connection Γ by

$$\begin{aligned} \nabla_\lambda T_{\rho\sigma\dots}^{\mu\nu\dots} &= \partial_\lambda T_{\rho\sigma\dots}^{\mu\nu\dots} + \Gamma_{\alpha\lambda}^\mu T_{\rho\sigma\dots}^{\alpha\nu\dots} + \Gamma_{\alpha\lambda}^\nu T_{\rho\sigma\dots}^{\mu\alpha\dots} + \dots - \\ &- \Gamma_{\rho\lambda}^\alpha T_{\alpha\sigma\dots}^{\mu\nu\dots} - \Gamma_{\sigma\lambda}^\alpha T_{\rho\alpha\dots}^{\mu\nu\dots} - \dots, \end{aligned}$$

where $\partial_\lambda = \frac{\partial}{\partial x^\lambda}$. Then, as usual, the metricity condition $\nabla_\lambda g_{\mu\nu} = 0$, or, equivalently, $\partial_\lambda g_{\mu\nu} = \Gamma_{\mu\nu\lambda} + \Gamma_{\nu\mu\lambda}$ (where $\Gamma_{\mu\nu\lambda} = g_{\mu\rho} \Gamma_{\nu\lambda}^\rho$), gives us the relation

$$\Gamma_{\mu\nu}^\lambda = \frac{1}{2} g^{\lambda\rho} (\partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu} + \partial_\mu g_{\nu\rho}) + \Gamma_{[\mu\nu]}^\lambda - g^{\lambda\rho} (g_{\mu\sigma} \Gamma_{[\rho\nu]}^\sigma + g_{\nu\sigma} \Gamma_{[\rho\mu]}^\sigma).$$

Hence we obtain, for the connection of our unified field theory, the following explicit form:

$$\begin{aligned} \Gamma_{\mu\nu}^\lambda &= \frac{1}{2} g^{\lambda\rho} (\partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu} + \partial_\mu g_{\nu\rho}) + \\ &+ \frac{e}{2mc^2} (F_{\mu\nu} u^\lambda - F_{\mu}^\lambda u_\nu - F_{\nu}^\lambda u_\mu) + \\ &+ S_{\mu\nu}^\lambda - g^{\lambda\rho} (S_{\mu\rho\nu} + S_{\nu\rho\mu}), \end{aligned}$$

where

$$\Delta_{\mu\nu}^\lambda = \frac{1}{2} g^{\lambda\rho} (\partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu} + \partial_\mu g_{\nu\rho})$$

are the components of the usual symmetric Levi-Civita connection, and where

$$\begin{aligned} K_{\mu\nu}^\lambda &= \frac{e}{2mc^2} (F_{\mu\nu} u^\lambda - F_{\mu}^\lambda u_\nu - F_{\nu}^\lambda u_\mu) + S_{\mu\nu}^\lambda - \\ &- g^{\lambda\rho} (S_{\mu\rho\nu} + S_{\nu\rho\mu}) \end{aligned}$$

are the components of the contorsion tensor in our unified field theory.

The above expression for the connection can actually be written alternatively in a somewhat simpler form as follows:

$$\begin{aligned} \Gamma_{\mu\nu}^\lambda &= \frac{1}{2} g^{\lambda\rho} (\partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu} + \partial_\mu g_{\nu\rho}) + \\ &+ \frac{e}{2mc^2} (F_{\mu\nu} u^\lambda - F_{\mu}^\lambda u_\nu - F_{\nu}^\lambda u_\mu) + 2S_{\mu}^\lambda u_\nu. \end{aligned}$$

At this point, we see that the geometric structure of our space-time continuum is also determined by the electromagnetic field tensor as well as the material spin tensor, in addition to the gravitational (metrical) field.

As a consequence, we obtain the following relations (where the round brackets on indices, in contrast to the square ones, indicate symmetrization):

$$\begin{aligned} \Gamma_{(\mu\nu)}^\lambda &= \Delta_{\mu\nu}^\lambda - \frac{e}{2mc^2} (F_{\mu}^\lambda u_\nu + F_{\nu}^\lambda u_\mu) + S_{\mu}^\lambda u_\nu + S_{\nu}^\lambda u_\mu, \\ \varphi_\mu &= K_{\mu\lambda}^\lambda = 2\Gamma_{[\mu\lambda]}^\lambda = \frac{e}{mc^2} F_{\mu\lambda} u^\lambda. \end{aligned}$$

We also have

$$\gamma_\mu = \Gamma_{\mu\lambda}^\lambda = \Delta_{\mu\lambda}^\lambda + \frac{e}{mc^2} F_{\mu\lambda} u^\lambda,$$

in addition to the usual relation

$$\Gamma_{\lambda\mu}^\lambda = \Delta_{\lambda\mu}^\lambda = \partial_\mu (\ln \sqrt{\det(g)}).$$

At this point, we may note that the spin vector φ is always orthogonal to the world-velocity vector as

$$\varphi_\mu u^\mu = 0.$$

In terms of the four-potential A , if we take the electromagnetic field tensor to be a pure curl as follows:

$$F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu = \bar{\nabla}_\nu A_\mu - \bar{\nabla}_\mu A_\nu,$$

where $\bar{\nabla}$ represents the covariant derivative with respect to the symmetric Levi-Civita connection alone, then we have the following general identities:

$$\begin{aligned} \partial_\lambda F_{\mu\nu} + \partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} &= \bar{\nabla}_\lambda F_{\mu\nu} + \bar{\nabla}_\mu F_{\nu\lambda} + \bar{\nabla}_\nu F_{\lambda\mu} = 0, \\ \nabla_\lambda F_{\mu\nu} + \nabla_\mu F_{\nu\lambda} + \nabla_\nu F_{\lambda\mu} &= \\ &= -2 \left(\Gamma_{[\mu\nu]}^\rho F_{\lambda\rho} + \Gamma_{[\nu\lambda]}^\rho F_{\mu\rho} + \Gamma_{[\lambda\mu]}^\rho F_{\nu\rho} \right). \end{aligned}$$

The electromagnetic current density vector is then given by

$$J^\mu = -\frac{c}{4\pi} \nabla_\nu F^{\mu\nu}.$$

Its fully covariant divergence is then given by

$$\nabla_\mu J^\mu = -\frac{c}{4\pi} \nabla_\mu (\Gamma_{[\rho\sigma]}^\mu F^{\rho\sigma}).$$

If we further take $J^\mu = \rho_{em} u^\mu$, where ρ_{em} represents the electromagnetic charge density (taking into account the possibility of a magnetic charge), we see immediately that our electromagnetic current is conserved if and only if $\bar{\nabla}_\mu J^\mu = 0$, as follows

$$\begin{aligned} \nabla_\mu J^\mu &= \partial_\mu J^\mu + \Gamma_{\lambda\mu}^\mu J^\lambda = \\ &= \bar{\nabla}_\mu J^\mu + \frac{e}{mc^2} F_{\lambda\mu} J^\lambda u^\lambda = \bar{\nabla}_\mu J^\mu. \end{aligned}$$

In other words, for the electromagnetic current density to be conserved in our theory, the following conditions must be satisfied (for an arbitrary scalar field Φ):

$$J^\mu = -\frac{c}{4\pi} \Gamma_{[\rho\sigma]}^\mu F^{\rho\sigma},$$

$$\Gamma_{[\mu\nu]}^\lambda = \delta_\mu^\lambda \partial_\nu \Phi - \delta_\nu^\lambda \partial_\mu \Phi.$$

These relations are reminiscent of those in [1]. Note that we have made use of the relation $(\nabla_\mu \nabla_\nu - \nabla_\nu \nabla_\mu) \Phi = 2\Gamma_{[\mu\nu]}^\lambda \nabla_\lambda \Phi$.

Now, corresponding to our desired conservation law for electromagnetic currents, we can alternatively express the connection as

$$\Gamma_{\mu\nu}^\lambda = \Delta_{\mu\nu}^\lambda + 2 (g^{\lambda\rho} g_{\mu\nu} \partial_\rho \Phi - \delta_\nu^\lambda \partial_\mu \Phi).$$

Contracting the above relation, we obtain the simple relation $\Gamma_{\mu\lambda}^\lambda = \Delta_{\mu\lambda}^\lambda - 6\partial_\mu \Phi$. On the other hand, we also have the relation $\Gamma_{\mu\lambda}^\lambda = \Delta_{\mu\lambda}^\lambda + \frac{e}{mc^2} F_{\mu\lambda} u^\lambda$. Hence we see that Φ

is a constant of motion as

$$\partial_\mu \Phi = -\frac{e}{6mc^2} F_{\mu\nu} u^\nu,$$

$$\frac{d\Phi}{ds} = 0.$$

These two conditions uniquely determine the conservation of electromagnetic currents in our theory.

Furthermore, not allowing for external forces, the geodesic equation of motion in \mathbb{S}_4 , namely,

$$\frac{Du^\mu}{Ds} = u^\nu \nabla_\nu u^\mu = 0,$$

must hold in \mathbb{S}_4 in order for the gravitational, electromagnetic, and material spin fields to be genuine intrinsic geometric objects that uniquely and completely build the structure of the space-time continuum.

Recalling the relation $\Gamma_{(\mu\nu)}^\lambda = \Delta_{\mu\nu}^\lambda - \frac{e}{2mc^2} (F_{\mu\nu}^\lambda u_\nu + F_{\nu\mu}^\lambda u_\mu) + S_{\mu\nu}^\lambda u_\nu + S_{\nu\mu}^\lambda u_\mu$, we obtain the equation of motion

$$\frac{du^\mu}{ds} + \Delta_{\nu\rho}^\mu u^\nu u^\rho = \frac{e}{mc^2} F_{\nu\mu}^\mu u^\nu,$$

which is none other than the equation of motion for a charged particle moving in a gravitational field. This simply means that our relation $F_{\mu\nu} = 2 \frac{mc^2}{e} \Gamma_{[\mu\nu]}^\lambda u_\lambda$ does indeed indicate a valid geometrization of the electromagnetic field.

In the case of conserved electromagnetic currents, we have

$$\frac{du^\mu}{ds} + \Delta_{\nu\rho}^\mu u^\nu u^\rho = -6 g^{\mu\nu} \partial_\nu \Phi.$$

3 The field equations of the unified field theory

The (intrinsic) curvature tensor R of \mathbb{S}_4 is of course given by the usual relation

$$(\nabla_\nu \nabla_\mu - \nabla_\mu \nabla_\nu) V_\lambda = R_{\lambda\mu\nu}^\rho V_\rho - 2\Gamma_{[\mu\nu]}^\rho \nabla_\rho V_\lambda,$$

where V is an arbitrary vector field. For an arbitrary tensor field T , we have the more general relation

$$(\nabla_\nu \nabla_\mu - \nabla_\mu \nabla_\nu) T_{\rho\sigma\cdots}^{\alpha\beta\cdots} = R_{\rho\mu\nu}^\lambda T_{\lambda\sigma\cdots}^{\alpha\beta\cdots} + R_{\sigma\mu\nu}^\lambda T_{\rho\lambda\cdots}^{\alpha\beta\cdots} + \cdots - R_{\lambda\mu\nu}^\alpha T_{\rho\sigma\cdots}^{\lambda\beta\cdots} - R_{\lambda\mu\nu}^\beta T_{\rho\sigma\cdots}^{\alpha\lambda\cdots} - \cdots - 2\Gamma_{[\mu\nu]}^\lambda \nabla_\lambda T_{\rho\sigma\cdots}^{\alpha\beta\cdots}.$$

Of course,

$$R_{\lambda\mu\nu}^\rho = \partial_\mu \Delta_{\lambda\nu}^\rho - \partial_\nu \Delta_{\lambda\mu}^\rho + \Gamma_{\lambda\nu}^\sigma \Gamma_{\sigma\mu}^\rho - \Gamma_{\lambda\mu}^\sigma \Gamma_{\sigma\nu}^\rho.$$

If we define the following contractions:

$$R_{\mu\nu} = R_{\mu\lambda\nu}^\lambda,$$

$$R = R_{\mu}^\mu,$$

then, as usual,

$$R_{\mu\nu\rho\sigma} = C_{\mu\nu\rho\sigma} + \frac{1}{2} (g_{\mu\rho} R_{\nu\sigma} + g_{\nu\sigma} R_{\mu\rho} - g_{\mu\sigma} R_{\nu\rho} - g_{\nu\rho} R_{\mu\sigma}) + \frac{1}{6} (g_{\mu\sigma} g_{\nu\rho} - g_{\mu\rho} g_{\nu\sigma}) R,$$

where C is the Weyl tensor. Note that the generalized Ricci tensor (given by its components $R_{\mu\nu}$) is generally asymmetric.

Let us denote the usual Riemann-Christoffel curvature tensor by \bar{R} , i.e.,

$$\bar{R}_{\lambda\mu\nu}^\rho = \partial_\mu \Delta_{\lambda\nu}^\rho - \partial_\nu \Delta_{\lambda\mu}^\rho + \Delta_{\lambda\nu}^\sigma \Delta_{\sigma\mu}^\rho - \Delta_{\lambda\mu}^\sigma \Delta_{\sigma\nu}^\rho.$$

The symmetric Ricci tensor and the Ricci scalar are then given respectively by $\bar{R}_{\mu\nu} = \bar{R}_{\mu\lambda\nu}^\lambda$ and $\bar{R} = \bar{R}_{\mu}^\mu$.

Furthermore, we obtain the following decomposition:

$$R_{\lambda\mu\nu}^\rho = \bar{R}_{\lambda\mu\nu}^\rho + \bar{\nabla}_\mu K_{\lambda\nu}^\rho - \bar{\nabla}_\nu K_{\lambda\mu}^\rho + K_{\lambda\nu}^\sigma K_{\sigma\mu}^\rho - K_{\lambda\mu}^\sigma K_{\sigma\nu}^\rho.$$

Hence, recalling that $\varphi_\mu = K_{\mu\lambda}^\lambda = 2\Gamma_{[\mu\lambda]}^\lambda$, we obtain

$$R_{\mu\nu} = \bar{R}_{\mu\nu} + \bar{\nabla}_\lambda K_{\mu\nu}^\lambda - K_{\mu\rho}^\lambda K_{\lambda\nu}^\rho - \bar{\nabla}_\nu \varphi_\mu + 2K_{\lambda\mu}^\lambda \varphi_\lambda,$$

$$R = \bar{R} - 2\bar{\nabla}_\mu \varphi^\mu - \varphi_\mu \varphi^\mu - K_{\mu\nu\lambda} K^{\mu\lambda\nu}.$$

We then obtain the following generalized Bianchi identities:

$$R_{\lambda\mu\nu}^\rho + R_{\mu\nu\lambda}^\rho + R_{\nu\lambda\mu}^\rho = -2(\partial_\nu \Gamma_{[\lambda\mu]}^\rho + \partial_\lambda \Gamma_{[\mu\nu]}^\rho + \partial_\mu \Gamma_{[\nu\lambda]}^\rho + \Gamma_{\sigma\lambda}^\rho \Gamma_{[\mu\nu]}^\sigma + \Gamma_{\sigma\mu}^\rho \Gamma_{[\nu\lambda]}^\sigma + \Gamma_{\sigma\nu}^\rho \Gamma_{[\lambda\mu]}^\sigma),$$

$$\nabla_\lambda R_{\mu\nu\rho\sigma} + \nabla_\rho R_{\mu\nu\sigma\lambda} + \nabla_\sigma R_{\mu\nu\lambda\rho} = 2 \left(\Gamma_{[\rho\sigma]}^\alpha R_{\mu\nu\alpha\lambda} + \Gamma_{[\sigma\lambda]}^\alpha R_{\mu\nu\alpha\rho} + \Gamma_{[\lambda\rho]}^\alpha R_{\mu\nu\alpha\sigma} \right),$$

$$\nabla_\mu \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right) = 2g^{\mu\nu} \Gamma_{[\rho\mu]}^\lambda R_{\nu\lambda}^\rho + \Gamma_{[\rho\sigma]}^\lambda R^{\rho\sigma\nu}{}_\lambda,$$

in addition to the standard Bianchi identities

$$\bar{R}_{\lambda\mu\nu}^\rho + \bar{R}_{\mu\nu\lambda}^\rho + \bar{R}_{\nu\lambda\mu}^\rho = 0,$$

$$\bar{\nabla}_\lambda \bar{R}_{\mu\nu\rho\sigma} + \bar{\nabla}_\rho \bar{R}_{\mu\nu\sigma\lambda} + \bar{\nabla}_\sigma \bar{R}_{\mu\nu\lambda\rho} = 0,$$

$$\bar{\nabla}_\mu \left(\bar{R}^{\mu\nu} - \frac{1}{2} g^{\mu\nu} \bar{R} \right) = 0.$$

(See [2–4] for instance.)

Furthermore, we can now obtain the following explicit expression for the curvature tensor R :

$$R_{\lambda\mu\nu}^\rho = \partial_\mu \Delta_{\lambda\nu}^\rho - \partial_\nu \Delta_{\lambda\mu}^\rho + \Delta_{\lambda\nu}^\sigma \Delta_{\sigma\mu}^\rho - \Delta_{\lambda\mu}^\sigma \Delta_{\sigma\nu}^\rho + \frac{e}{2mc^2} \left\{ (\partial_\mu F_{\lambda\nu} - \partial_\nu F_{\lambda\mu}) u^\rho + (\partial_\nu F_{\mu}^\rho - \partial_\mu F_{\nu}^\rho) u_\lambda + u_\mu \partial_\nu F_{\lambda}^\rho - u_\nu \partial_\mu F_{\lambda}^\rho + F_{\lambda\nu} \partial_\mu u^\rho - F_{\lambda\mu} \partial_\nu u^\rho + F_{\mu}^\rho \partial_\nu u_\lambda - F_{\nu}^\rho \partial_\mu u_\lambda + (\partial_\nu u_\mu - \partial_\mu u_\nu) F_{\lambda}^\rho + (F_{\lambda\nu} u^\sigma - F_{\lambda}^\sigma u_\nu - F_{\nu}^\sigma u_\lambda) \Delta_{\sigma\mu}^\rho + (F_{\sigma\mu} u^\rho - F_{\sigma}^\rho u_\mu - F_{\mu}^\rho u_\sigma) \Delta_{\lambda\nu}^\sigma - (F_{\lambda\mu} u^\sigma - F_{\lambda}^\sigma u_\mu - F_{\mu}^\sigma u_\lambda) \Delta_{\sigma\nu}^\rho \right\}$$

$$\begin{aligned}
& - (F_{\sigma\nu} u^\rho - F_{\sigma}^{\rho} u_{\nu} - F_{\nu}^{\rho} u_{\sigma}) \Delta_{\lambda\mu}^{\sigma} + \frac{e}{2m c^2} (F_{\lambda\nu} F_{\sigma\mu} - \\
& - F_{\lambda\mu} F_{\sigma\nu}) u^{\sigma} u^{\rho} + \frac{e}{2m c^2} (F_{\lambda\mu} F_{\sigma}^{\rho} - F_{\lambda\sigma} F_{\mu}^{\rho}) u_{\nu} u^{\sigma} + \\
& + \frac{e}{2m c^2} (F_{\lambda\sigma} F_{\nu}^{\rho} - F_{\lambda\nu} F_{\sigma}^{\rho}) u_{\mu} u^{\sigma} + \frac{e}{2m c^2} (F_{\mu}^{\sigma} F_{\sigma\nu} - \\
& - F_{\nu}^{\sigma} F_{\sigma\mu}) u_{\lambda} u^{\rho} + \frac{e}{2m c^2} (F_{\sigma\nu} F_{\mu}^{\rho} - F_{\sigma\mu} F_{\nu}^{\rho}) u_{\lambda} u^{\sigma} + \\
& + \frac{e}{2m c^2} F_{\lambda}^{\sigma} F_{\sigma\nu} u_{\mu} u^{\rho} + \frac{e}{2m c^2} F_{\nu}^{\sigma} F_{\sigma}^{\rho} u_{\lambda} u_{\mu} - \\
& - \frac{e}{2m c^2} F_{\lambda}^{\sigma} F_{\sigma\mu} u_{\nu} u^{\rho} - \frac{e}{2m c^2} F_{\mu}^{\sigma} F_{\sigma}^{\rho} u_{\lambda} u_{\nu} + \\
& + \frac{e}{2m c^2} (F_{\lambda\mu} F_{\nu}^{\rho} - F_{\lambda\nu} F_{\mu}^{\rho}) \} + \Omega_{\lambda\mu\nu}^{\rho},
\end{aligned}$$

where the tensor Ω consists of the remaining terms containing the material spin tensor 2S (or 3S).

Now, keeping in mind that $\Gamma_{(\mu\nu)}^{\lambda} = \Delta_{\mu\nu}^{\lambda} - \frac{e}{2m c^2} (F_{\mu}^{\lambda} u_{\nu} + F_{\nu}^{\lambda} u_{\mu}) + S_{\mu\nu}^{\lambda} + S_{\nu\mu}^{\lambda}$ and also $\gamma_{\mu} = \Gamma_{\mu\lambda}^{\lambda} = \Delta_{\mu\lambda}^{\lambda} + \frac{e}{m c^2} F_{\mu\lambda} u^{\lambda}$, and decomposing the components of the generalized Ricci tensor as $R_{\mu\nu} = R_{(\mu\nu)} + R_{[\mu\nu]}$, we see that

$$\begin{aligned}
R_{(\mu\nu)} &= \partial_{\lambda} \Gamma_{(\mu\nu)}^{\lambda} - \frac{1}{2} (\partial_{\nu} \gamma_{\mu} + \partial_{\mu} \gamma_{\nu}) + \Gamma_{(\mu\nu)}^{\lambda} \gamma_{\lambda} - \\
& - \frac{1}{2} (\Gamma_{\mu\lambda}^{\rho} \Gamma_{\rho\nu}^{\lambda} + \Gamma_{\nu\lambda}^{\rho} \Gamma_{\rho\mu}^{\lambda}),
\end{aligned}$$

$$\begin{aligned}
R_{[\mu\nu]} &= \partial_{\lambda} \Gamma_{[\mu\nu]}^{\lambda} - \frac{1}{2} (\partial_{\nu} \gamma_{\mu} - \partial_{\mu} \gamma_{\nu}) + \Gamma_{[\mu\nu]}^{\lambda} \gamma_{\lambda} - \\
& - \frac{1}{2} (\Gamma_{\mu\lambda}^{\rho} \Gamma_{\rho\nu}^{\lambda} - \Gamma_{\nu\lambda}^{\rho} \Gamma_{\rho\mu}^{\lambda}).
\end{aligned}$$

In particular, we note that

$$\begin{aligned}
R_{[\mu\nu]} &= \partial_{\lambda} \Gamma_{[\mu\nu]}^{\lambda} - \frac{1}{2} (\partial_{\nu} \gamma_{\mu} - \partial_{\mu} \gamma_{\nu}) + \Gamma_{[\mu\nu]}^{\lambda} \gamma_{\lambda} - \\
& - \frac{1}{2} (\Gamma_{\mu\lambda}^{\rho} \Gamma_{\rho\nu}^{\lambda} - \Gamma_{\nu\lambda}^{\rho} \Gamma_{\rho\mu}^{\lambda}) = \\
& = \partial_{\lambda} \Gamma_{[\mu\nu]}^{\lambda} + \Gamma_{\lambda\rho}^{\rho} \Gamma_{[\mu\nu]}^{\lambda} + \Gamma_{\mu\lambda}^{\rho} \Gamma_{[\nu\rho]}^{\lambda} - \Gamma_{\nu\lambda}^{\rho} \Gamma_{[\mu\rho]}^{\lambda} + \\
& + \frac{1}{2} (\partial_{\nu} \gamma_{\mu} - \partial_{\mu} \gamma_{\nu}) = \nabla_{\lambda} \Gamma_{[\mu\nu]}^{\lambda} + \frac{1}{2} (\partial_{\nu} \gamma_{\mu} - \partial_{\mu} \gamma_{\nu}).
\end{aligned}$$

Hence we obtain the relation

$$\begin{aligned}
R_{[\mu\nu]} &= \frac{e}{2m c^2} \left(F_{\mu\nu} \nabla_{\lambda} u^{\lambda} + \frac{D F_{\mu\nu}}{D s} \right) + \nabla_{\lambda} S_{\mu\nu}^{\lambda} + \\
& + \frac{1}{2} (\partial_{\nu} \gamma_{\mu} - \partial_{\mu} \gamma_{\nu}),
\end{aligned}$$

where $\frac{D F_{\mu\nu}}{D s} = u^{\lambda} \nabla_{\lambda} F_{\mu\nu}$. More explicitly, we can write

$$\begin{aligned}
R_{[\mu\nu]} &= \frac{e}{2m c^2} \left(F_{\mu\nu} \nabla_{\lambda} u^{\lambda} + \frac{D F_{\mu\nu}}{D s} + (\partial_{\nu} F_{\lambda\mu} - \right. \\
& \left. - \partial_{\mu} F_{\lambda\nu}) u^{\lambda} + F_{\lambda\mu} \partial_{\nu} u^{\lambda} - F_{\lambda\nu} \partial_{\mu} u^{\lambda} \right) + \nabla_{\lambda} S_{\mu\nu}^{\lambda}.
\end{aligned}$$

It is therefore seen that, in general, the special identity

$$\partial_{\lambda} R_{[\mu\nu]} + \partial_{\mu} R_{[\nu\lambda]} + \partial_{\nu} R_{[\lambda\mu]} = 0$$

holds only when $\nabla_{\mu} u^{\mu} = 0$, $\frac{D F_{\mu\nu}}{D s} = 0$, and $\nabla_{\lambda} S_{\mu\nu}^{\lambda} = 0$.

We are now in a position to generalize Einstein's field equation in the standard theory of general relativity. The usual Einstein's field equation is of course given by

$$\begin{aligned}
\bar{G}_{\mu\nu} &= \bar{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \bar{R} = k T_{\mu\nu}, \\
\bar{\nabla}_{\mu} \bar{G}^{\mu\nu} &= 0,
\end{aligned}$$

where \bar{G} is the symmetric Einstein tensor, T is the energy-momentum tensor, and $k = \pm \frac{8\pi G}{c^4}$ is Einstein's coupling constant in terms of the Newtonian gravitational constant G . Taking $c = 1$ for convenience, in the absence of pressure, traditionally we write

$$\bar{G}^{\mu\nu} = k \left(\rho_m u^{\mu} u^{\nu} + \frac{1}{4\pi} \left(F_{\lambda}^{\mu} F^{\nu\lambda} - \frac{1}{4} g^{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} \right) \right),$$

where ρ_m is the material density and where the second term on the right-hand-side of the equation is widely regarded as representing the electromagnetic energy-momentum tensor.

Now, with the generalized Bianchi identity for the electromagnetic field, i.e., $\nabla_{\lambda} F_{\mu\nu} + \nabla_{\mu} F_{\nu\lambda} + \nabla_{\nu} F_{\lambda\mu} = -2 \left(\Gamma_{[\mu\nu]}^{\rho} F_{\lambda\rho} + \Gamma_{[\nu\lambda]}^{\rho} F_{\mu\rho} + \Gamma_{[\lambda\mu]}^{\rho} F_{\nu\rho} \right)$, at hand, and assuming the "isochoric" condition $\frac{D \rho_m}{D s} = -\rho_m \nabla_{\mu} u^{\mu} = 0$ ($\rho_m \neq 0$), we obtain

$$\nabla_{\nu} \bar{G}^{\mu\nu} = k g^{\mu\nu} \left(\Gamma_{[\rho\sigma]}^{\lambda} F_{\nu\lambda} + \Gamma_{[\sigma\nu]}^{\lambda} F_{\rho\lambda} + \Gamma_{[\nu\rho]}^{\lambda} F_{\sigma\lambda} \right) F^{\rho\sigma}.$$

In other words,

$$\nabla_{\nu} \bar{G}^{\mu\nu} = k \left(2 g^{\mu\nu} \Gamma_{[\sigma\nu]}^{\lambda} F_{\rho\lambda} F^{\rho\sigma} - \frac{1}{4\pi} F_{\nu}^{\mu} J^{\nu} \right).$$

This is our first generalization of the standard Einstein's field equation, following the traditional ad hoc way of arbitrarily adding the electromagnetic contribution to the purely material part of the energy-momentum tensor.

Now, more generally and more naturally, using the generalized Bianchi identity $\nabla_{\mu} \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right) = 2 g^{\mu\nu} \Gamma_{[\rho\mu]}^{\lambda} R_{\lambda}^{\rho} + \Gamma_{[\rho\sigma]}^{\lambda} R^{\rho\sigma\nu}_{\lambda}$, we can obtain the following fundamental relation:

$$\begin{aligned}
\nabla_{\mu} \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right) &= \frac{e}{m c^2} \left(F_{\mu}^{\nu} R_{\lambda}^{\mu} + \frac{1}{2} F_{\mu\rho} R^{\mu\rho\nu}_{\lambda} \right) u^{\lambda} + \\
& + 2 S_{\mu\lambda}^{\nu} R^{\mu\lambda} + S_{\mu\rho}^{\lambda} R^{\mu\rho\nu}_{\lambda}.
\end{aligned}$$

Alternatively, we can also write this as

$$\begin{aligned}
\nabla_{\mu} \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right) &= \frac{e}{m c^2} \left(F_{\mu}^{\nu} R_{\lambda}^{\mu} + \frac{1}{2} F_{\mu\rho} R^{\mu\rho\nu}_{\lambda} \right) u^{\lambda} + \\
& + S_{\mu\rho} R^{\rho\mu} u^{\nu} - S_{\rho}^{\lambda} u_{\mu} R^{\mu\rho\nu}_{\lambda} + (S_{\mu}^{\nu} R^{\rho\mu} - S_{\mu}^{\lambda} R^{\mu\rho\nu}_{\lambda}) u_{\rho}.
\end{aligned}$$

Now, as a special consideration, let Σ be the "area" of a three-dimensional space-like hypersurface representing matter in \mathbb{S}_4 . Then, if we make the following traditional choice

for the third-rank material spin tensor 3S :

$$S^{\mu\nu\lambda} = \iiint_{\Sigma} \rho_m (x^\lambda T^{\mu\nu} - x^\nu T^{\mu\lambda}) d\Sigma,$$

where now T is the total asymmetric energy-momentum tensor in our theory, we see that, in the presence of matter, the condition $S^{\mu\nu\lambda} = 0$ implies that

$$T^{[\mu\nu]} = -\frac{1}{2} (x^\mu \nabla_\lambda T^{\lambda\nu} - x^\nu \nabla_\lambda T^{\lambda\mu}).$$

In this special case, we obtain the simplified expression

$$\nabla_\mu \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right) = \frac{e}{mc^2} \left(F_{\mu}^{\nu} R_{\lambda}^{\mu} + \frac{1}{2} F_{\mu\rho} R^{\mu\rho\nu} \right) u^\lambda.$$

If we further assume that the sectional curvature $\Psi = \frac{1}{12} R$ of \mathbb{S}_4 is everywhere constant in a space-time region where the electromagnetic field (and hence the torsion) is absent, we may consider writing $R_{\mu\nu\rho\sigma} = \Psi (g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho})$ such that \mathbb{S}_4 is conformally flat ($C_{\mu\nu\rho\sigma} = 0$), and hence $R_{\mu\nu} = 3\Psi g_{\mu\nu}$ and $R_{[\mu\nu]} = 0$. In this case, we are left with the simple expression

$$\nabla_\nu \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right) = -\frac{eR}{6mc^2} F_{\nu}^{\mu} u^\nu.$$

This is equivalent to the equation of motion

$$\frac{du^\mu}{ds} + \Delta_{\nu\rho}^{\mu} u^\nu u^\rho = -\frac{6}{R} \nabla_\nu \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right).$$

4 The minimal Lagrangian density of the theory

Using the general results from the preceding section, we obtain

$$R = \bar{R} + \frac{e^2}{4m^2 c^4} F_{\mu\nu} F^{\mu\nu} - \frac{e^2}{m^2 c^4} F_{\mu\lambda} F_{\nu}^{\mu} u^\lambda u^\nu - \frac{2e}{m c^2} \bar{\nabla}_\mu f^\mu + 2S_{\mu\nu} S^{\mu\nu} - K_{\mu(\nu\lambda)} K^{\mu(\nu\lambda)},$$

for the curvature scalar of \mathbb{S}_4 . Here $f^\mu = F_{\nu}^{\mu} u^\nu$ can be said to be the components of the so-called Lorentz force.

Furthermore, we see that

$$K_{\mu(\nu\lambda)} K^{\mu(\nu\lambda)} = \frac{e^2}{m^2 c^4} F_{\mu\nu} F^{\mu\nu} + 2S_{\mu\nu} S^{\mu\nu} - \frac{2e}{m c^2} F_{\mu\nu} S^{\mu\nu} - \frac{e^2}{2m^2 c^4} F_{\mu\lambda} F_{\nu}^{\mu} u^\lambda u^\nu.$$

Hence we obtain

$$R = \bar{R} - \frac{e^2}{2m^2 c^4} F_{\mu\nu} F^{\mu\nu} - \frac{2e}{m c^2} (\bar{\nabla}_\mu f^\mu + F_{\mu\nu} S^{\mu\nu}) - \frac{e^2}{2m^2 c^4} F_{\mu\lambda} F_{\nu}^{\mu} u^\lambda u^\nu.$$

The last two terms on the right-hand-side of the expression can then be grouped into a single scalar source as fol-

lows:

$$\phi = -\frac{2e}{m c^2} (\bar{\nabla}_\mu f^\mu + F_{\mu\nu} S^{\mu\nu}) - \frac{e^2}{2m^2 c^4} F_{\mu\lambda} F_{\nu}^{\mu} u^\lambda u^\nu.$$

Assuming that ϕ accounts for both the total (material-electromagnetic) charge density as well as the total energy density, our unified field theory may be described by the following action integral (where the $L = R \sqrt{\det(g)}$ is the minimal Lagrangian density):

$$I = \iiint\!\!\!\int R \sqrt{\det(g)} d^4x = \iiint\!\!\!\int \left(\bar{R} - \frac{e^2}{2m^2 c^4} F_{\mu\nu} F^{\mu\nu} + \phi \right) \sqrt{\det(g)} d^4x.$$

In this minimal fashion, gravity (described by \bar{R}) appears as an emergent phenomenon whose intrinsic nature is of electromagnetic and purely material origin since, in our theory, the electromagnetic and material spin fields are nothing but components of a single torsion field.

5 The non-Abelian Yang-Mills gauge field as a sub-torsion field in \mathbb{S}_4

In \mathbb{S}_4 , let there exist a space-like three-dimensional hypersurface Θ_3 , with local coordinates X^i (Latin indices shall run from 1 to 3). From the point of view of projective differential geometry alone, we may say that Θ_3 is embedded (immersed) in \mathbb{S}_4 . Then, the tetrad linking the embedded space Θ_3 to the enveloping space-time \mathbb{S}_4 is readily given by

$$\omega_\mu^i = \frac{\partial X^i}{\partial x^\mu}, \quad \omega_i^\mu = (\omega_\mu^i)^{-1} = \frac{\partial x^\mu}{\partial X^i}.$$

Furthermore, let N be a unit vector normal to the hypersurface Θ_3 . We may write the parametric equation of the hypersurface Θ_3 as $H(x^\mu, d) = 0$, where d is constant. Hence

$$N^\mu = \frac{g^{\mu\nu} \partial_\nu H}{\sqrt{g^{\alpha\beta} (\partial_\alpha H) (\partial_\beta H)}}, \quad N_\mu N^\mu = 1.$$

In terms of the axial unit vectors $a, b,$ and c spanning the hypersurface Θ_3 , we may write

$$N_\mu = \frac{\varepsilon_{\mu\nu\rho\sigma} a^\nu b^\rho c^\sigma}{\varepsilon_{\alpha\beta\lambda\eta} N^\alpha a^\beta b^\lambda c^\eta},$$

where $\varepsilon_{\mu\nu\rho\sigma}$ are the components of the completely anti-symmetric four-dimensional Levi-Civita permutation tensor density.

Now, the tetrad satisfies the following projective relations:

$$\omega_\mu^i N^\mu = 0, \quad \omega_\mu^i \omega_k^\mu = \delta_k^i, \quad \omega_i^\mu \omega_\nu^i = \delta_\nu^\mu - N^\mu N_\nu.$$

If we denote the local metric tensor of Θ_3 by h , we obtain the following relations:

$$h_{ik} = \omega_i^\mu \omega_k^\nu g_{\mu\nu},$$

$$g_{\mu\nu} = \omega_\mu^i \omega_\nu^k h_{ik} + N_\mu N_\nu.$$

Furthermore, in the hypersurface Θ_3 , let us set $\nabla_i = \omega_i^\mu \nabla_\mu$ and $\partial_i = \frac{\partial}{\partial X^i} = \omega_i^\mu \partial_\mu$. Then we have the following fundamental expressions:

$$\nabla_\nu \omega_\mu^i = Z^i_k \omega_\nu^k N_\mu = \partial_\nu \omega_\mu^i - \omega_\sigma^i \Gamma_{\mu\nu}^\sigma + \Gamma_{k\nu}^i \omega_\mu^k \omega_\nu^l,$$

$$\nabla_k \omega_i^\mu = Z_{ik} N^\mu = \partial_k \omega_i^\mu - \omega_p^\mu \Gamma_{ik}^p + \Gamma_{\rho\sigma}^\mu \omega_i^\rho \omega_k^\sigma,$$

$$\omega_i^\mu \nabla_\nu \omega_\mu^k = 0,$$

$$\nabla_i N^\mu = -Z^i_k \omega_k^\mu,$$

where Z is the extrinsic curvature tensor of the hypersurface Θ_3 , which is generally asymmetric in our theory.

The connection of the hypersurface Θ_3 is linked to that of the space-time \mathbb{S}_4 via

$$\Gamma_{ik}^p = \omega_\mu^p \partial_k \omega_i^\mu + \omega_\lambda^p \Gamma_{\mu\nu}^\lambda \omega_i^\mu \omega_k^\nu.$$

After some algebra, we obtain

$$\Gamma_{\mu\nu}^\lambda = \omega_i^\lambda \partial_\nu \omega_\mu^i + \omega_p^\lambda \Gamma_{ik}^p \omega_\mu^i \omega_\nu^k + N^\lambda \partial_\nu N_\mu + N^\lambda Z_{ik} \omega_\mu^i \omega_\nu^k - N_\mu Z^i_k \omega_i^\lambda \omega_\nu^k.$$

The fundamental geometric relations describing our embedding theory are then given by the following expressions (see [4] for instance):

$$R_{ijkl} = Z_{ik} Z_{jl} - Z_{il} Z_{jk} + R_{\mu\nu\rho\sigma} \omega_i^\mu \omega_j^\nu \omega_k^\rho \omega_l^\sigma - \omega_i^\mu \Lambda_{\mu jkl},$$

$$\nabla_l Z_{ik} - \nabla_k Z_{il} = -R_{\mu\nu\rho\sigma} N^\mu \omega_i^\nu \omega_k^\rho \omega_l^\sigma - 2\Gamma_{[kl]}^p Z_{ip} + N^\mu \Lambda_{\mu ikl},$$

$$\Lambda_{ijk}^\mu = (\partial_k \partial_j - \partial_j \partial_k) \omega_i^\mu + \omega_i^\sigma \Gamma_{\sigma\rho}^\mu (\partial_k \omega_j^\rho - \partial_j \omega_k^\rho).$$

Actually, these relations are just manifestations of the following single expression:

$$(\nabla_k \nabla_j - \nabla_j \nabla_k) \omega_i^\mu = R_{ijk}^p \omega_p^\mu - R_{\nu\rho\sigma}^\mu \omega_i^\nu \omega_j^\rho \omega_k^\sigma - 2\Gamma_{[jk]}^p Z_{ip} N^\mu + \Lambda_{ijk}^\mu.$$

We may note that $\Gamma_{[ik]}^p$ and

$$R_{jkl}^i = \partial_k \Gamma_{jl}^i - \partial_l \Gamma_{jk}^i + \Gamma_{jl}^p \Gamma_{pk}^i - \Gamma_{jk}^p \Gamma_{pl}^i$$

are the components of the torsion tensor and the intrinsic curvature tensor of the hypersurface Θ_3 , respectively.

Now, let us observe that

$$\partial_\nu \omega_\mu^i - \partial_\mu \omega_\nu^i = 2 \left(\omega_\lambda^i \Gamma_{[\mu\nu]}^\lambda - \Gamma_{[kl]}^i \omega_\mu^k \omega_\nu^l + Z^i_k \omega_{[\nu}^k N_{\mu]} \right).$$

Hence letting

$$F_{\mu\nu}^i = 2\omega_\lambda^i \Gamma_{[\mu\nu]}^\lambda,$$

we arrive at the expression

$$F_{\mu\nu}^i = \partial_\nu \omega_\mu^i - \partial_\mu \omega_\nu^i + 2\Gamma_{[kl]}^i \omega_\mu^k \omega_\nu^l + 2Z^i_k \omega_{[\mu}^k N_{\nu]}.$$

In addition, we also see that

$$\Gamma_{[kl]}^i = \frac{1}{2} \omega_k^\mu \omega_l^\nu F_{\mu\nu}^i - \frac{1}{2} \omega_k^\mu \omega_l^\nu (\partial_\nu \omega_\mu^i - \partial_\mu \omega_\nu^i).$$

Now, with respect to the local coordinate transformation given by $X^i = X^i(\bar{X}^A)$ in Θ_3 , let us invoke the following Cartan-Lie algebra:

$$[e_i, e_k] = e_i \otimes e_k - e_k \otimes e_i = C_{ik}^p e_p,$$

$$C_{ikl} = h_{ip} C_{kl}^p = -2\Gamma_{i[kl]} = -i\hat{g} \in_{ikl},$$

where $e_i = e_i^A \frac{\partial}{\partial \bar{X}^A}$ are the elements of the basis vector spanning Θ_3 , C_{ik}^p are the spin coefficients, $i = \sqrt{-1}$, \hat{g} is a coupling constant, and $\in_{ikl} = \sqrt{\det(h)} \varepsilon_{ikl}$ (where ε_{ikl} are the components of the completely anti-symmetric three-dimensional Levi-Civita permutation tensor density).

Hence we obtain

$$F_{\mu\nu}^i = \partial_\nu \omega_\mu^i - \partial_\mu \omega_\nu^i + i\hat{g} \in_{kl}^i \omega_\mu^k \omega_\nu^l + 2Z^i_k \omega_{[\mu}^k N_{\nu]}.$$

At this point, our key insight is to define the gauge field potential as the tetrad itself, i.e.,

$$B_\mu^i = \omega_\mu^i.$$

Hence, at last, we arrive at the following important expression:

$$F_{\mu\nu}^i = \partial_\nu B_\mu^i - \partial_\mu B_\nu^i + i\hat{g} \in_{kl}^i B_\mu^k B_\nu^l + 2Z^i_k B_{[\mu}^k N_{\nu]}.$$

Clearly, $F_{\mu\nu}^i$ are the components of the generalized Yang-Mills gauge field strength. To show this, consider the hypersurface \mathbb{E}_3 of rigid frames (where the metric tensor is strictly constant) which is a reduction (or, in a way, local infinitesimal representation) of the more general hypersurface Θ_3 . We shall call this an ‘‘isospace’’. In it, we have

$$h_{ik} = \delta_{ik},$$

$$\det(h) = 1,$$

$$\Gamma_{kl}^i = \Gamma_{ikl} = \Gamma_{i[kl]} - \Gamma_{l[ik]} - \Gamma_{k[il]} = \frac{1}{2} i\hat{g} \varepsilon_{ikl},$$

$$Z_{ik} = 0.$$

Hence we arrive at the familiar expression

$$F_{i\mu\nu} = \partial_\nu B_{\mu i} - \partial_\mu B_{\nu i} + i\hat{g} \varepsilon_{ikl} B_{\mu k} B_{\nu l}.$$

In other words, setting $\vec{F}_{\mu\nu} = F_{i\mu\nu} e_i$ and $\vec{B}_\mu = B_{\mu i} e_i$, we obtain

$$\vec{F}_{\mu\nu} = \partial_\nu \vec{B}_\mu - \partial_\mu \vec{B}_\nu - [\vec{B}_\mu, \vec{B}_\nu].$$

Finally, let us define the gauge field potential of the second kind via

$$\omega_{\mu ik} = \varepsilon_{ikp} B_\mu^p,$$

such that

$$B_{\mu}^i = \frac{1}{2} \varepsilon_{ikl} \omega_{\mu kl}.$$

Let us then define the gauge field strength of the second kind via

$$R_{ik\mu\nu} = \varepsilon_{ikp} F_{\mu\nu}^p,$$

such that

$$F_{\mu\nu}^p = \frac{1}{2} \varepsilon^{pik} R_{ik\mu\nu}.$$

Hence we obtain the general expression

$$\begin{aligned} R_{ik\mu\nu} = & i\hat{g} \sqrt{\det(h)} \left\{ \partial_{\nu} \omega_{\mu ik} - \partial_{\mu} \omega_{\nu ik} + \right. \\ & \left. + \frac{1}{\sqrt{\det(h)}} (\omega_{\mu ip} \omega_{\nu kp} - \omega_{\mu kp} \omega_{\nu ip}) \right\} + \\ & + \sqrt{\det(h)} \varepsilon_{ikp} Z_r^p B_{[\mu}^r N_{\nu]}. \end{aligned}$$

We may regard the object given by this expression as the curvature of the local gauge spin connection of the hypersurface Θ_3 .

Again, if we refer this to the isospace \mathbb{E}_3 instead of the more general hypersurface Θ_3 , we arrive at the familiar relation

$$R_{ik\mu\nu} = i\hat{g} (\partial_{\nu} \omega_{\mu ik} - \partial_{\mu} \omega_{\nu ik} + \omega_{\mu ip} \omega_{\nu kp} - \omega_{\mu kp} \omega_{\nu ip}).$$

6 Conclusion

We have just completed our program of building the structure of a unified field theory in which gravity, electromagnetism, material spin, and the non-Abelian Yang-Mills gauge field (which is also capable of describing the weak force in the standard model particle physics) are all geometrized only in four dimensions. As we have seen, we have also generalized the expression for the Yang-Mills gauge field strength.

In our theory, the (generalized) Yang-Mills gauge field strength is linked to the electromagnetic field tensor via the relation

$$F_{\mu\nu} = 2 \frac{m c^2}{e} \Gamma_{[\mu\nu]}^{\lambda} u_{\lambda} = \frac{m c^2}{e} F_{\mu\nu}^i u_i,$$

where $u^i = \omega_{\mu}^i u^{\mu}$. This enables us to express the connection in terms of the Yang-Mills gauge field strength instead of the electromagnetic field tensor as follows:

$$\begin{aligned} \Gamma_{\mu\nu}^{\lambda} = & \frac{1}{2} g^{\lambda\rho} (\partial_{\nu} g_{\rho\mu} - \partial_{\rho} g_{\mu\nu} + \partial_{\mu} g_{\nu\rho}) + \frac{1}{2} u_i (F_{\mu\nu}^i u^{\lambda} - \\ & - F_{\mu}^{i\lambda} u_{\nu} - F_{\nu}^{i\lambda} u_{\mu}) + S_{\mu\nu}^{\lambda} - g^{\lambda\rho} (S_{\mu\rho\nu} + S_{\nu\rho\mu}), \end{aligned}$$

i.e., the Yang-Mills gauge field is nothing but a sub-torsion field in the space-time manifold \mathbb{S}_4 .

The results which we have obtained in this work may subsequently be quantized simply by following the method given in our previous work [1] since, in a sense, the present work is but a further in-depth classical consideration of the fundamental method of geometrization outlined in the previous theory.

Dedication

I dedicate this work to my patron and source of inspiration, Albert Einstein (1879–1955), from whose passion for the search of the ultimate physical truth I have learned something truly fundamental of the meaning of being a true scientist and independent, original thinker, even amidst the adversities often imposed upon him by the world and its act of scientific institutionalization.

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An Exact Mapping from Navier-Stokes Equation to Schrödinger Equation via Riccati Equation

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In the present article we argue that it is possible to write down Schrödinger representation of Navier-Stokes equation via Riccati equation. The proposed approach, while differs appreciably from other method such as what is proposed by R. M. Kiehn, has an advantage, i.e. it enables us extend further to quaternionic and biquaternionic version of Navier-Stokes equation, for instance via Kravchenko's and Gibbon's route. Further observation is of course recommended in order to refute or verify this proposition.

1 Introduction

In recent years there were some attempts in literature to find out Schrödinger-like representation of Navier-Stokes equation using various approaches, for instance by R. M. Kiehn [1, 2]. Deriving exact mapping between Schrödinger equation and Navier-Stokes equation has clear advantage, because Schrödinger equation has known solutions, while exact solution of Navier-Stokes equation completely remains an open problem in mathematical-physics. Considering wide applications of Navier-Stokes equation, including for climatic modelling and prediction (albeit in simplified form called "geostrophic flow" [9]), one can expect that simpler expression of Navier-Stokes equation will be found useful.

In this article we presented an alternative route to derive Schrödinger representation of Navier-Stokes equation via Riccati equation. The proposed approach, while differs appreciably from other method such as what is proposed by R. M. Kiehn [1], has an advantage, i.e. it enables us to extend further to quaternionic and biquaternionic version of Navier-Stokes equation, in particular via Kravchenko's [3] and Gibbon's route [4, 5]. An alternative method to describe quaternionic representation in fluid dynamics has been presented by Sprössig [6]. Nonetheless, further observation is of course recommended in order to refute or verify this proposition.

2 From Navier-Stokes equation to Schrödinger equation via Riccati

Recently, Argentini [8] argues that it is possible to write down ODE form of 2D steady Navier-Stokes equations, and it will lead to second order equation of Riccati type.

Let ρ the density, μ the dynamic viscosity, and f the body force per unit volume of fluid. Then the Navier-Stokes equation for the steady flow is [8]:

$$\rho(v \cdot \nabla v) = -\nabla p + \rho \cdot f + \mu \cdot \Delta v. \quad (1)$$

After some necessary steps, he arrives to an ODE version of 2D Navier-Stokes equations along a streamline [8, p. 5] as

follows:

$$u_1 \cdot \dot{u}_1 = f_1 - \frac{\dot{q}}{\rho} + v \cdot \dot{u}_1, \quad (2)$$

where $v = \frac{\mu}{\rho}$ is the kinematic viscosity. He [8, p. 5] also finds a general exact solution of equation (2) in Riccati form, which can be rewritten as follows:

$$\dot{u}_1 - \alpha \cdot u_1^2 + \beta = 0, \quad (3)$$

where:

$$\alpha = \frac{1}{2v}, \quad \beta = -\frac{1}{v} \left(\frac{\dot{q}}{\rho} - f_1 \right) s - \frac{c}{v}. \quad (4)$$

Interestingly, Kravchenko [3, p. 2] has argued that there is neat link between Schrödinger equation and Riccati equation via simple substitution. Consider a 1-dimensional static Schrödinger equation:

$$\ddot{u} + v \cdot u = 0 \quad (5)$$

and the associated Riccati equation:

$$\dot{y} + y^2 = -v. \quad (6)$$

Then it is clear that equation (6) is related to (7) by the inverted substitution [3]:

$$y = \frac{\dot{u}}{u}. \quad (7)$$

Therefore, one can expect to use the same method (8) to write down the Schrödinger representation of Navier-Stokes equation. First, we rewrite equation (3) in similar form of equation (7):

$$\dot{y}_1 - \alpha \cdot y_1^2 + \beta = 0. \quad (8)$$

By using substitution (8), then we get the Schrödinger equation for this Riccati equation (9):

$$\ddot{u} - \alpha\beta \cdot u = 0, \quad (9)$$

where variable α and β are the same with (4). This Schrödinger representation of Navier-Stokes equation is remarkably simple and it also has advantage that now it is possible to generalize it further to quaternionic (ODE) Navier-Stokes

equation via quaternionic Schrödinger equation, for instance using the method described by Gibbon *et al.* [4, 5].

3 An extension to biquaternionic Navier-Stokes equation via biquaternion differential operator

In our preceding paper [10, 12], we use this definition for biquaternion differential operator:

$$\diamond = \nabla^q + i \nabla^q = \left(-i \frac{\partial}{\partial t} + e_1 \frac{\partial}{\partial x} + e_2 \frac{\partial}{\partial y} + e_3 \frac{\partial}{\partial z} \right) + i \left(-i \frac{\partial}{\partial T} + e_1 \frac{\partial}{\partial X} + e_2 \frac{\partial}{\partial Y} + e_3 \frac{\partial}{\partial Z} \right), \quad (10)$$

where e_1, e_2, e_3 are *quaternion imaginary units* obeying (with ordinary quaternion symbols: $e_1 = i, e_2 = j, e_3 = k$): $i^2 = j^2 = k^2 = -1, ij = -ji = k, jk = -kj = i, ki = -ik = j$ and quaternion *Nabla operator* is defined as [13]:

$$\nabla^q = -i \frac{\partial}{\partial t} + e_1 \frac{\partial}{\partial x} + e_2 \frac{\partial}{\partial y} + e_3 \frac{\partial}{\partial z}. \quad (11)$$

(Note that (11) and (12) include partial time-differentiation.)

Now it is possible to use the same method described above [10, 12] to generalize the Schrödinger representation of Navier-Stokes (10) to the biquaternionic Schrödinger equation, as follows.

In order to generalize equation (10) to quaternion version of Navier-Stokes equations (QNSE), we use first quaternion Nabla operator (12), and by noticing that $\Delta \equiv \nabla \nabla$, we get:

$$\left(\nabla^q \bar{\nabla}^q + \frac{\partial^2}{\partial t^2} \right) u - \alpha \beta \cdot u = 0. \quad (12)$$

We note that the multiplying factor $\alpha \beta$ in (13) plays similar role just like $V(x) - E$ factor in the standard Schrödinger equation [12]:

$$-\frac{\hbar^2}{2m} \left(\nabla^q \bar{\nabla}^q + \frac{\partial^2}{\partial t^2} \right) u + (V(x) - E) u = 0. \quad (13)$$

Note: we shall introduce the second term in order to “neutralize” the partial time-differentiation of $\nabla^q \bar{\nabla}^q$ operator.

To get *biquaternion* form of equation (13) we can use our definition in equation (11) rather than (12), so we get [12]:

$$\left(\diamond \bar{\diamond} + \frac{\partial^2}{\partial t^2} - i \frac{\partial^2}{\partial T^2} \right) u - \alpha \beta \cdot u = 0. \quad (14)$$

This is an alternative version of *biquaternionic* Schrödinger representation of Navier-Stokes equations. Numerical solution of the new Navier-Stokes-Schrödinger equation (15) can be performed in the same way with [12] using Maxima software package [7], therefore it will not be discussed here.

We also note here that the route to *quaternionize* Schrödinger equation here is rather different from what is described by Gibbon *et al.* [4, 5], where the Schrödinger-equivalent to Euler fluid equation is described as [5, p. 4]:

$$\frac{D^2 w}{Dt^2} - (\nabla Q) w = 0 \quad (15)$$

and its quaternion representation is [5, p. 9]:

$$\frac{D^2 w}{Dt^2} - q_b \otimes w = 0 \quad (16)$$

with Riccati relation is given by:

$$\frac{D_a^q}{Dt + q_a \otimes q_a} = q_b \quad (17)$$

Nonetheless, further observation is of course recommended in order to refute or verify this proposition (15).

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Numerical Solution of Radial Biquaternion Klein-Gordon Equation

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In the preceding article we argue that biquaternionic extension of Klein-Gordon equation has solution containing imaginary part, which differs appreciably from known solution of KGE. In the present article we present numerical /computer solution of radial biquaternionic KGE (radialBQKGE); which differs appreciably from conventional Yukawa potential. Further observation is of course recommended in order to refute or verify this proposition.

1 Introduction

In the preceding article [1] we argue that biquaternionic extension of Klein-Gordon equation has solution containing imaginary part, which differs appreciably from known solution of KGE. In the present article we presented here for the first time a numerical/computer solution of radial biquaternionic KGE (radialBQKGE); which differs appreciably from conventional Yukawa potential.

This biquaternionic effect may be useful in particular to explore new effects in the context of low-energy reaction (LENR) [2]. Nonetheless, further observation is of course recommended in order to refute or verify this proposition.

2 Radial biquaternionic KGE (radial BQKGE)

In our preceding paper [1], we argue that it is possible to write biquaternionic extension of Klein-Gordon equation as follows:

$$\left[\left(\frac{\partial^2}{\partial t^2} - \nabla^2 \right) + i \left(\frac{\partial^2}{\partial t^2} - \nabla^2 \right) \right] \varphi(x, t) = -m^2 \varphi(x, t), \quad (1)$$

or this equation can be rewritten as:

$$(\diamond \bar{\diamond} + m^2) \varphi(x, t) = 0, \quad (2)$$

provided we use this definition:

$$\begin{aligned} \diamond = \nabla^q + i \nabla^q = & \left(-i \frac{\partial}{\partial t} + e_1 \frac{\partial}{\partial x} + e_2 \frac{\partial}{\partial y} + e_3 \frac{\partial}{\partial z} \right) + \\ & + i \left(-i \frac{\partial}{\partial T} + e_1 \frac{\partial}{\partial X} + e_2 \frac{\partial}{\partial Y} + e_3 \frac{\partial}{\partial Z} \right), \end{aligned} \quad (3)$$

where e_1, e_2, e_3 are *quaternion imaginary units* obeying (with ordinary quaternion symbols: $e_1 = i, e_2 = j, e_3 = k$):

$$\begin{aligned} i^2 = j^2 = k^2 = & -1, \quad ij = -ji = k, \\ jk = -kj = & i, \quad ki = -ik = j. \end{aligned} \quad (4)$$

and quaternion *Nabla operator* is defined as [1]:

$$\nabla^q = -i \frac{\partial}{\partial t} + e_1 \frac{\partial}{\partial x} + e_2 \frac{\partial}{\partial y} + e_3 \frac{\partial}{\partial z}. \quad (5)$$

(Note that (3) and (4) included partial time-differentiation.)

In the meantime, the standard Klein-Gordon equation usually reads [3, 4]:

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2 \right) \varphi(x, t) = -m^2 \varphi(x, t). \quad (6)$$

Now we can introduce polar coordinates by using the following transformation:

$$\nabla = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) - \frac{\ell^2}{r^2}. \quad (7)$$

Therefore, by substituting (6) into (5), the radial Klein-Gordon equation reads — by neglecting partial-time differentiation — as follows [3, 5]:

$$\left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) - \frac{\ell(\ell+1)}{r^2} + m^2 \right) \varphi(x, t) = 0, \quad (8)$$

and for $\ell = 0$, then we get [5]:

$$\left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + m^2 \right) \varphi(x, t) = 0. \quad (9)$$

The same method can be applied to equation (2) for radial biquaternionic KGE (BQKGE), which for the 1-dimensional situation, one gets instead of (7):

$$\left(\frac{\partial}{\partial r} \left(\frac{\partial}{\partial r} \right) - i \frac{\partial}{\partial r} \left(\frac{\partial}{\partial r} \right) + m^2 \right) \varphi(x, t) = 0. \quad (10)$$

In the next Section we will discuss numerical/computer solution of equation (9) and compare it with standard solution of equation (8) using Maxima software package [6]. It can be shown that equation (9) yields potential which differs appreciably from standard Yukawa potential. For clarity, all solutions were computed in 1-D only.

3 Numerical solution of radial biquaternionic Klein-Gordon equation

Numerical solution of the standard radial Klein-Gordon equation (8) is given by:

$$\begin{aligned}
 & (\%i1) \text{diff}(y,t,2) - \text{diff}(y,r,2) + m^2 * y; \\
 & (\%o1) m^2 \cdot y - \frac{d^2}{dx^2} y \\
 & (\%i2) \text{ode2}(\%o1, y, r); \\
 & (\%o2) y = \%k_1 \cdot \% \exp(mr) + \%k_2 \cdot \% \exp(-mr) \quad (11)
 \end{aligned}$$

In the meantime, numerical solution of equation (9) for radial biquaternionic KGE (BQKGE), is given by:

$$\begin{aligned}
 & (\%i3) \text{diff}(y,t,2) - (\%i+1) * \text{diff}(y,r,2) + m^2 * y; \\
 & (\%o3) m^2 \cdot y - (i + 1) \frac{d^2}{dx^2} y \\
 & (\%i4) \text{ode2}(\%o3, y, r); \\
 & (\%o4) y = \%k_1 \cdot \sin\left(\frac{|m|r}{\sqrt{-\%i-1}}\right) + \%k_2 \cdot \cos\left(\frac{|m|r}{\sqrt{-\%i-1}}\right) \quad (12)
 \end{aligned}$$

Therefore, we conclude that numerical solution of radial biquaternionic extension of Klein-Gordon equation yields different result compared to the solution of standard Klein-Gordon equation; and it *differs appreciably* from the well-known Yukawa potential [3, 7]:

$$u(r) = -\frac{g^2}{r} e^{-mr}. \quad (13)$$

Meanwhile, Comay puts forth argument that the Yukawa lagrangian density has theoretical inconsistency within itself [3].

Interestingly one can find argument that biquaternion Klein-Gordon equation is nothing more than quadratic form of (modified) Dirac equation [8], therefore BQKGE described herein, i.e. equation (12), can be considered as a plausible solution to the problem described in [3]. For other numerical solutions to KGE, see for instance [4].

Nonetheless, we recommend further observation [9] in order to refute or verify this proposition of new type of potential derived from biquaternion Klein-Gordon equation.

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VC would like to dedicate this article for RFF.

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Spin Transport in Mesoscopic Superconducting-Ferromagnetic Hybrid Conductor

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The spin polarization and the corresponding tunneling magnetoresistance (TMR) for a hybrid ferromagnetic/superconductor junction are calculated. The results show that these parameters are strongly depends on the exchange field energy and the bias voltage. The dependence of the polarization on the angle of precession is due to the spin flip through tunneling process. Our results could be interpreted as due to spin imbalance of carriers resulting in suppression of gap energy of the superconductor. The present investigation is valuable for manufacturing magnetic recording devices and nonvolatile memories which imply a very high spin coherent transport for such junction.

1 Introduction

Spintronics and spin-based quantum information processing explore the possibility to add new functionality to today's electronic devices by exploiting the electron spin in addition to its charge [1]. Spin-polarized tunneling plays an important role in the spin dependent transport of magnetic nanostructures [2]. The spin-polarized electrons injected from ferromagnetic materials into nonmagnetic one such as superconductor, semiconductor create a non equilibrium spin polarization in such nonmagnetic materials [3, 4, 5].

Ferromagnetic-superconductor hybrid systems are an attractive subject research because of the competition between the spin asymmetry characteristic of a ferromagnet and the correlations induced by superconductivity [1, 2, 6]. At low energies electronic transport in mesoscopic ferromagnet-superconductor hybrid systems is determined by Andreev-reflection [7]. Superconducting materials are powerful probe for the spin polarization of the current injected from ferromagnetic material [8, 9, 10]. Superconductors are useful for exploring how the injected spin-polarized quasiparticles are transported. In this case the relaxation time can be measured precisely in the superconducting state where thermal noise effects are small.

The present paper, spin-polarized transport through ferromagnetic/superconductor/ferromagnetic double junction is investigated. This investigation will show how Andreev-reflection processes are sensitive to the exchange field energy in the ferromagnetic leads.

2 The model

A mesoscopic device is modeled as superconductor sandwiched between two ferromagnetic leads via double tunnel barriers. The thickness of the superconductor is smaller than the spin diffusion length and the magnetization of the ferromagnetic leads are aligned either parallel or antiparal-

lel. The spin polarization of the conduction electrons due to Andreev reflection at ferromagnetic/superconductor interface could be determined through the following equation as:

$$P = \frac{\Gamma_{\uparrow}(E) - \Gamma_{\downarrow}(E)}{\Gamma_{\uparrow}(E) + \Gamma_{\downarrow}(E)}, \quad (1)$$

where $\Gamma_{\uparrow}(E)$ and $\Gamma_{\downarrow}(E)$ are the tunneling probabilities of conduction electrons with up-spin and down-spin respectively. Since the present device is described by the following Bogoliubov-deGennes (BdG) equation [11]:

$$\begin{pmatrix} H_0 - h_{ex}(z)\sigma & \Delta(z) \\ \Delta^*(z) & -H_0 - \sigma h_{ex}(z) \end{pmatrix} \psi = E \psi, \quad (2)$$

where H_0 is the single particle Hamiltonian and it is expressed as:

$$H_0 = -\frac{\hbar^2}{2m} \nabla^2 - \varepsilon_{nl}, \quad (3)$$

in which the energy, ε_{nl} , is expressed of the Fermi velocity v_F , Fermi-momentum P_F , the magnetic field B as [12]:

$$\begin{aligned} \varepsilon_{nl} = & -(\alpha g + k_F D \sin \theta) \mu_B B \pm \\ & \pm [v_F^2 P_F^2 (1 - \sin \theta)^2 + \Delta^2]^{1/2}. \end{aligned} \quad (4)$$

In Eq. (4), $\alpha = \pm 1/2$ for spin-up and spin down respectively, μ_B is the Bohr magneton, g is the g -factor for electrons and θ is the precession angle.

The interface between left ferromagnetic/superconductor and superconductor/right ferromagnetic leads are located at $z = -L/2$ and $z = L/2$ respectively. The parameter $h_{ex}(z)$ represents the exchange field and is given by [13]:

$$h_{ex} = \begin{cases} h_0 & z < -L/2 \\ 0 & -L/2 < z < L/2 \\ \pm h_0 & z > L/2 \end{cases}, \quad (5)$$

where $+h_0$ and $-h_0$ represents the exchange fields for parallel and anti-parallel alignments respectively, the parameter

$\Delta(z)$ is the superconducting gap:

$$\Delta(z) = \begin{cases} 0 & z < -L/2, L/2 < z \\ \Delta & -L/2 < z < L/2 \end{cases}. \quad (6)$$

The temperature dependence of the superconducting gap is given by [14]:

$$\Delta = \Delta_0 \tanh\left(1.74 \sqrt{\frac{T_c}{T} - 1}\right), \quad (7)$$

where Δ_0 is the superconducting gap at $T = 0$ and T_c is the superconducting critical temperature. Now, in order to get the tunneling probability $\Gamma_{\uparrow\downarrow}(E)$ for both up-spin and down-spin electrons by solving the Bogoliubov-deGennes Eqn. (2) as: The eigenfunction in the left ferromagnetic lead ($z < -L/2$) is given by:

$$\begin{aligned} \psi_{\sigma,nl}^{FM1}(r) = & \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{iP_{\sigma,nl}^+(z+\frac{L}{2})} + \right. \\ & + a_{\sigma,nl} \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{iP_{\sigma,nl}^-(z+\frac{L}{2})} + \\ & \left. + b_{\sigma,nl} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-P_{\sigma,nl}^+(z+\frac{L}{2})} \right] S_{nl}(x, y). \end{aligned} \quad (8)$$

In the superconductor ($-L/2 < z < L/2$), the eigenfunction is given by:

$$\begin{aligned} \psi_{\sigma,nl}^{SC}(r) = & \left[\alpha_{\sigma,nl} \begin{pmatrix} u_0 \\ \nu_0 \end{pmatrix} e^{ik_{nl}^+(z+\frac{L}{2})} + \right. \\ & + \beta_{\sigma,nl} \begin{pmatrix} \nu_0 \\ u_0 \end{pmatrix} e^{-ik_{nl}^-(z+\frac{L}{2})} + \\ & + \xi_{\sigma,nl} \begin{pmatrix} u_0 \\ \nu_0 \end{pmatrix} e^{-ik_{nl}^+(z-\frac{L}{2})} + \\ & \left. + \eta_{\sigma,nl} \begin{pmatrix} u_0 \\ \nu_0 \end{pmatrix} e^{ik_{nl}^-(z-\frac{L}{2})} \right] S_{nl}(x, y). \end{aligned} \quad (9)$$

And the eigenfunction in the right ferromagnetic lead ($L/2 < z$) is given by:

$$\begin{aligned} \psi_{\sigma,nl}^{FM2}(r) = & \left[C_{\sigma,nl} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{iq_{\sigma,nl}^+(z-\frac{L}{2})} + \right. \\ & \left. + d_{\sigma,nl} \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-iq_{\sigma,nl}^-(z-\frac{L}{2})} \right] S_{nl}(x, y). \end{aligned} \quad (10)$$

Since the device is rectangular, the eigenfunction in the transverse (x & y) directions with channels n, l is given by:

$$S_{nl}(x, y) = \sin\left(\frac{n\pi x}{W}\right) \sin\left(\frac{l\pi y}{W}\right), \quad (11)$$

where W is the width of the junction.

The wave numbers in the Eqs. (8), (9), (10) are given by:

$$P_{\sigma,nl}^{\pm} = \sqrt{\frac{2m}{\hbar^2} (\mu_F \pm E \pm \sigma h_{ex})}, \quad (12)$$

$$k_{\sigma,nl}^{\pm} = \sqrt{\frac{2m}{\hbar^2} (\mu_F \pm \zeta) - \varepsilon_{nl}}, \quad (13)$$

$$q_{\sigma,nl}^{\pm} = \sqrt{\frac{2m}{\hbar^2} (\mu_F \pm E \pm \sigma h_{ex} \pm \varepsilon_{nl})}, \quad (14)$$

where $\zeta = \sqrt{E^2 - \Delta^2}$, and the energy ε_{nl} is given by Eq. (4). For the coherence factors of electron and holes u_0 and ν_0 are related as [11]:

$$u_0^2 = 1 - \nu_0^2 = \frac{1}{2} \left[1 + \frac{\sqrt{E^2 - \Delta^2}}{E} \right]. \quad (15)$$

The coefficients in Eqs. (8), (9), (10) are determined by applying the boundary conditions at the interfaces and the matching conditions are:

$$\left. \begin{aligned} \psi_{\sigma,nl}^{FM1}\left(z = -\frac{L}{2}\right) &= \psi_{\sigma,nl}^{SC}\left(z = -\frac{L}{2}\right) \\ \psi_{\sigma,nl}^{SC}\left(z = \frac{L}{2}\right) &= \psi_{\sigma,nl}^{FM2}\left(z = \frac{L}{2}\right) \end{aligned} \right\}, \quad (16)$$

$$\left. \frac{d\psi_{\sigma,nl}^{SC}}{dz} \right|_{z=-\frac{L}{2}} - \left. \frac{d\psi_{\sigma,nl}^{FM1}}{dz} \right|_{z=-\frac{L}{2}} = \frac{2mV}{\hbar^2} \psi_{\sigma,nl}^{FM1}\left(z = -\frac{L}{2}\right), \quad (17)$$

$$\left. \frac{d\psi_{\sigma,nl}^{FM2}}{dz} \right|_{z=\frac{L}{2}} - \left. \frac{d\psi_{\sigma,nl}^{SC}}{dz} \right|_{z=\frac{L}{2}} = \frac{2mV}{\hbar^2} \psi_{\sigma,nl}^{FM2}\left(z = \frac{L}{2}\right). \quad (18)$$

Eqs. (14), (15), (16) are solved numerically [15] for the tunneling probabilities corresponding to up-spin and down-spin for the tunneled electrons. The corresponding polarization, P , Eq. (1) is determined at different parameters V , θ , which will be discussed in the next section.

3 Results and discussion

Numerical calculations are performed for the present device, in which the superconductor is Nb and the ferromagnetic leads are of any one of ferromagnetic materials. The features of the present results are:

Fig. 1 shows the dependence of the polarization, P , on the bias voltage, V , at different parameters B , E , h and T . From the figure, the polarization has a peak at the value of V near the value of the energy gap Δ_0 for the present superconductor (Nb) ($\Delta_0 = 1.5$ meV) [16]. But for higher values of V , the polarization, P , decreases. As shown from Fig. 1a, the polarization does not change with the magnetic field, B , due to the Zeeman-energy. Some authors [17] observed the effect of magnetic field of values greater than 1 T, in this case the superconductivity will be destroyed (for Nb, $B_c = 0.19$ T).

Now in order to observe the effect of the spin precession on the value of the polarization, P , this can be shown from Fig. 2. The dependence of the polarization, P , on the angle of precession, θ , is strongly varies with the variation of the magnetic field, temperature, exchange field and the energy of

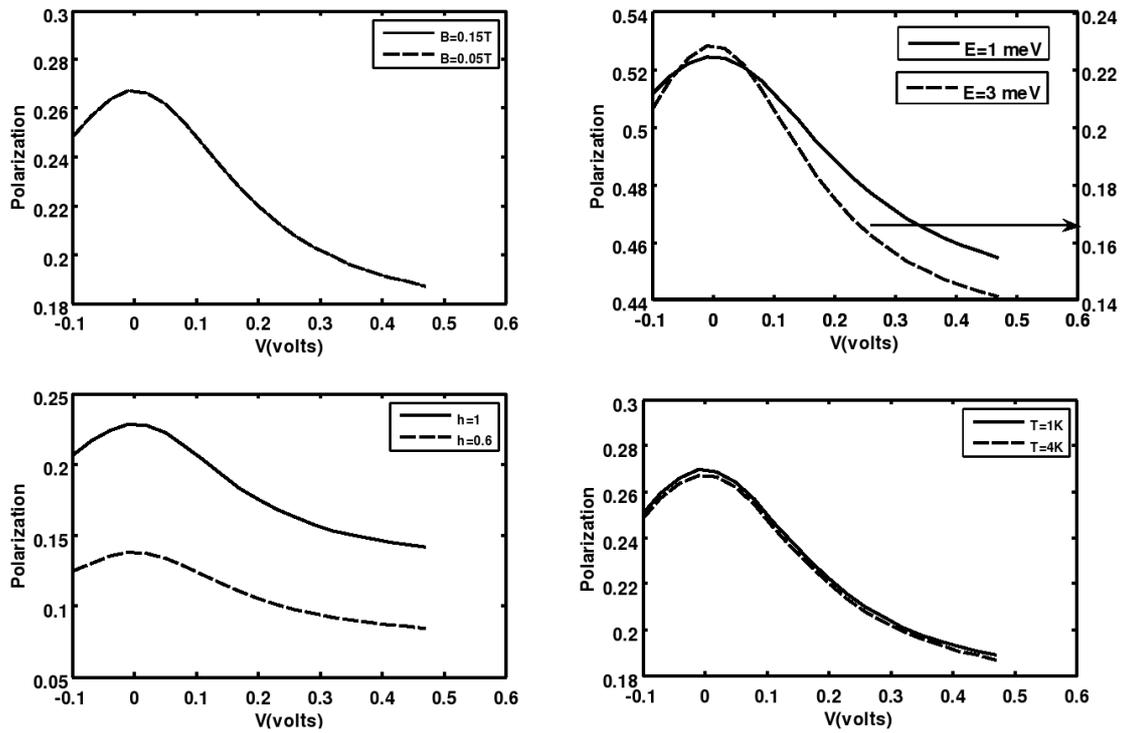


Fig. 1: The dependence of the polarization, P , on the bias voltage, V , at different B , E , h and T .

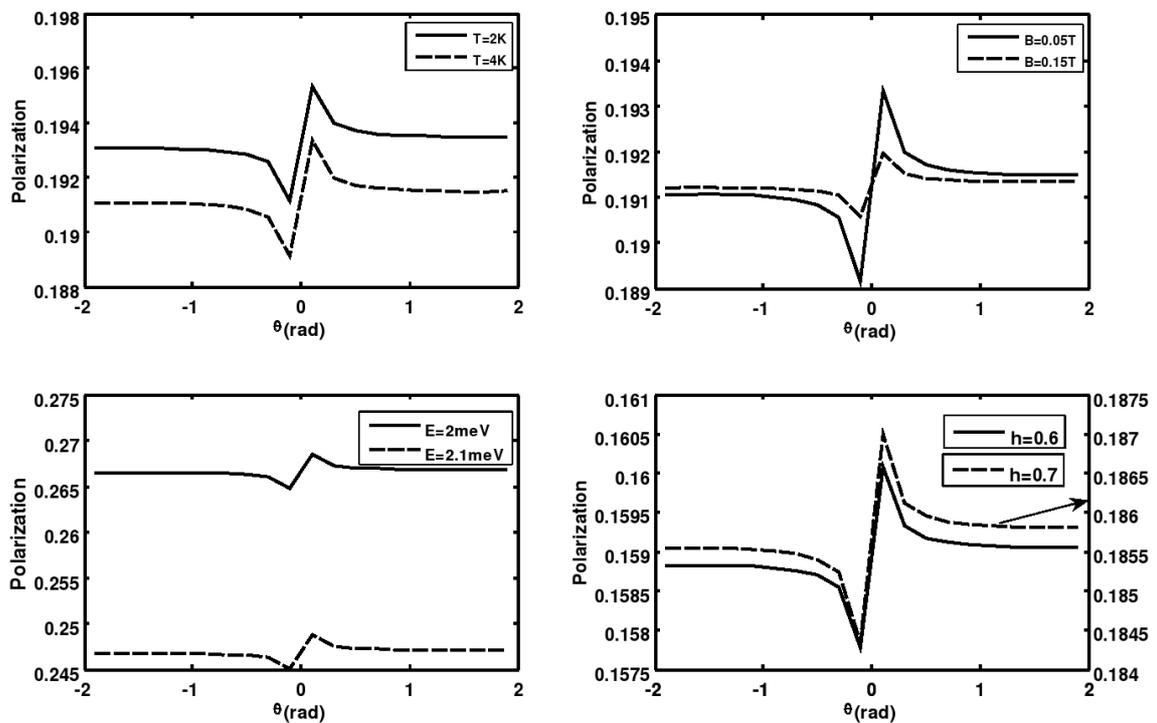


Fig. 2: The dependence of the polarization, P , on the angle of precession at different B , E , h and T .

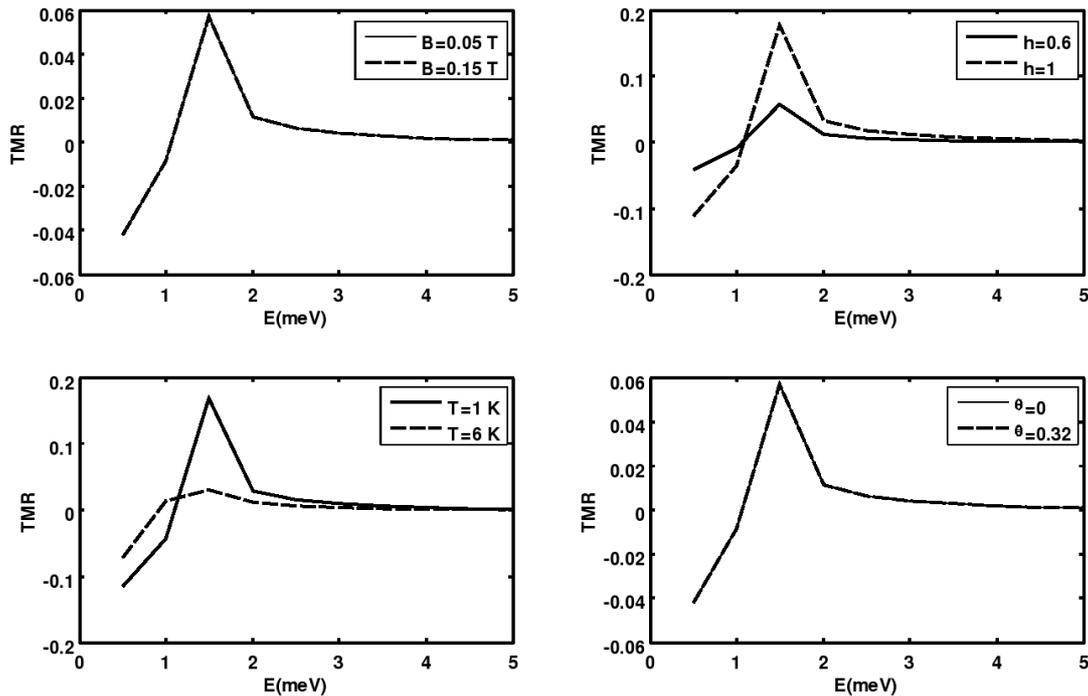


Fig. 3: The variation of the TMR with the energy of the tunneled electrons at different parameters B , T , h and θ .

the tunneled electrons. As shown from Fig. 2, the value of P is minimum at certain values of θ also P is maximum at another values of θ . This trend of the polarization with the angle of the precession is due to the flip of the electron spin when tunneling through the junction.

In order to investigate the spin injection tunneling through such hybrid magnetic system, we calculated the tunnel magnetoresistance (TMR) which is related to the polarization as [18]:

$$TMR = \frac{P^2}{1 - P^2 + \Gamma_s}, \quad (19)$$

where Γ_s is the relaxation parameter and is given by [18]:

$$\Gamma_s = \frac{e^2 N(0) R_T A L}{\tau_s}, \quad (20)$$

where $N(0)$ is the normal-state density of electrons calculated for both up-spin and down-spin distribution function $f_\sigma(E)$, which is expressed as [18]:

$$f_\sigma(E) \cong f_0(E) - \left(\frac{\partial f_0}{\partial E} \right) \sigma \delta\mu, \quad (21)$$

where $\sigma = \pm 1$ for both up and down spin of the electrons, $\delta\mu$ is the shift of the chemical potential, τ_s is the spin relaxation time, A is the junction area and R_T is the resistance at the interface of the tunnel junction.

Fig. 3 shows the variation of the TMR with the energy of the tunneled electrons at different parameters B , T , h and θ . A peak is observed for TMR at a certain value which is in the near value of the gap energy Δ_0 for the superconductor (Nb). These results (Fig. 3) show the interplay between the

spin polarization of electrons and Andreev-reflection process at the ferromagnetic/superconductor interface [19]. From our results; we can conclude that the spin-polarized transport depends on the relative orientation of magnetization in the two ferromagnetic leads. The spin polarization of the tunneled electrons through the junction gives rise to a nonequilibrium spin density in the superconductor. This is due to the imbalance in the tunneling currents carried by the spin-up and spin-down electrons. The trend of the tunneling magnetoresistance (TMR) is due to the spin-orbit scattering in the superconductor. Our results are found concordant with those in literatures [20, 21, 22].

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Human Perception of Physical Experiments and the Simplex Interpretation of Quantum Physics

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In this paper it is argued that knowledge dividing the usual, unusual, transient and transcendental depends on human perception of the world (macro or micro) and depends too on the inclusion of human consciousness in the system. For the analysis of this problem the idea of "Schrödinger's cat" is employed. Transient and transcendental knowledge of the state of Schrödinger's cat corresponds to the case when the observer's consciousness is included in the system. Here it is possible to speak about the latent parameters of the sub quantum world of which Einstein was convinced. Knowledge of the unusual state of Schrödinger's cat, simultaneously alive and dead, corresponds to a case of the open micro world. The usual knowledge of the state of Schrödinger's cat (alive or dead) corresponds to a case of the open macrocosm. Each world separately divides the objective and illusory.

1 Introduction

Scientific cognition frequently avoids the question of interaction of our consciousness with the external world. However, the celebrated known physicist Wigner [1] maintains that separation of our perception from the laws of a nature is no more than simplification and although we are convinced that it has a harmless character, to nevertheless merely forget about it does not follow.

Purposeful perception is sensation and in order to understand more deeply that sensation it is necessary, in the beginning, to be able to distinguish sensation in a macrocosm (spontaneously) from sensation in a microcosm (through the device). Many scientists believe that information recorded with the help of devices can be equally considered with sentient data. Their belief, harmless at first sight would, should not result in the serious misunderstanding. But actually it is not so.

Sensation in a macrocosm, for example, that of a sunrise, and sensation in a microcosm, for example, some number displayed on an ammeter, are not the same. Perception, by definition, is complete subjective reflection: the phenomena are events resulting from direct influence on sense organs, and in a macrocosm it certainly does not depend on the level of our knowledge. Nobody will argue that a sunrise and other such phenomena, events in a macrocosm, are perceived by all people equally. But in a microcosm this is not so. Perception of the invisible world of electrons is not whole or complete and therefore depends on the level of our scientific knowledge. But that knowledge is connected to our consciousness. It becomes clear then why the consciousness of the observer finds itself a place in quantum physics.

The problematic interpretation of quantum mechanics has been a controversial topic of discussion for more than 80 years. The most important upshot of this for physicists is that this problem is related to the problem of consciousness —

an interdisciplinary problem concerning not only physicists, but also philosophers, psychologists, physiologists and biologists. Its solution will result in deeper scientific knowledge. As many scientists have argued, the path to such knowledge should not consider separately the physical phenomena and the phenomena accompanying our thinking. By adhering to this position it is reasonable to conclude that the correct interpretation of the quantum mechanics comprises such knowledge.

Really, the problem of quantum physics, as a choice of one alternative at quantum measurement and a problem of philosophy as to how consciousness functions, is deeply connected with relations between these two. It is quite possible that in solving these two problems, it is likely that experiments in the quantum mechanics will include workings of a brain and consciousness, and it will then be possible to present a new basis for the theory of consciousness

2 Dependence of physical experiment on the state of consciousness

During sensation our brain accepts data and information from an external world. On the basis of these data, during thinking, knowledge is formed. The biological substratum of thinking is the brain. Therefore, knowledge is a product of the brain.

Consciousness, as it is known, is a property of the brain and therefore already concerns the origin of knowledge. Clearly, this relation is either active, i.e. influencing the origin of knowledge, or passive. If active as well as passive, we ask: Does consciousness influence the origin of knowledge? It is possible to answer this because it is known that there are different kinds and levels of consciousness and scientific knowledge which represent various forms and levels of reflection. Considering the definition of knowledge in that it is a reflection of objective characteristics of reality in the consciousness of a person, we are interested with a question:

When and what reflection — passive or active, unequivocal or multiple-valued — takes place?

Passivity or activity of reflection depends on passivity or activity of the consciousness of the observer. Clearly, consciousness is passive if it is not included in the system, being in this case an open system. Consciousness can be active if it is included in the system, being in this case a closed system. Activity or passivity of consciousness is expressed in its ability to influence reflection on reality, i.e. on knowledge. With the contention that active consciousness may influence reflection on reality it is possible to imply that this influence can be directed onto reality as well. Whether or not this is so is however difficult to say. But we know that a closed system should differ from an open one. The difference is expressed in the activity of consciousness, which influences reflection and knowledge.

The unambiguous or the multi-valence nature of reflection does not depend on the activity or passivity of consciousness; it depends on perception, i.e. from integrity of perception. The perception of a macrocosm is complete, but the perception of a microcosm is not complete. Therefore it is clear that reflection on reality in a macrocosm will be unequivocal, but in a microcosm, multiple-valued.

Multiple-valued reflection does not influence knowledge, but, nevertheless, makes knowledge multiple-valued, unclear, and uncertain. It now becomes clear why knowledge of a microcosm results in uncertainties, including the well-known Heisenberg Uncertainties. It is possible that these uncertainties are effects of consciousness, dependent not on the activity of consciousness, but on the impossibility to completely perceive the cognizable world by consciousness.

Thus, in a closed system, reflection is active. In an open system reflection is passive. In a macrocosm it is unequivocal but in a microcosm it is multiple-valued.

For elucidation we shall imagine a mirror; a usual mirror, i.e. a mirror with which we are commonly familiar. Let's assume that this mirror is our consciousness. The mirror is passive, because reflection of objects in it does not depend on itself. Similarly, consciousness is passive, if reflection of reality in it does not depend on itself. Clearly, the passive consciousness appropriate for this mirror is consciousness in an open system, because only in this case is consciousness similar to a mirror that can be counter-posed to a being. If around the mirror there is a bright light, for example, sunlight, the reflection of objects in it will be unequivocal. Perception of these objects will be complete. This case of bright light around of a mirror corresponds to a case of the macrocosm. Really, the macrocosm is our visible world. But now we shall imagine that the mirror is in darkness. Images are absent in the mirror. This case of darkness around the mirror corresponds to a case of the microcosm. The microcosm is our invisible world. Let's now imagine that we want to receive some image from the mirror. For this purpose we artificially illuminate an object. This action corresponds to how we in-

vestigate a microcosm with the help of devices. Artificial illumination is not ideal; therefore reflection of objects in the mirror will be multiple-valued. Clearly, perception will not be complete either. Already, as a result, knowledge cannot be unequivocal. The Heisenberg Uncertainties of a microcosm are the proof. Knowledge from these uncertainties is multiple-valued because it is impossible to determine exactly the localization and speed of a micro-particle. So the usual mirror corresponds to passive consciousness. But what mirror will correspond to active consciousness? In this case the system is closed and the mirror should be unusual; the reflection of objects in it depends on itself. Such a mirror includes a mirror, or more exactly, many mirrors; a mirror in a mirror in a mirror.

So consciousness includes consciousness; it is consciousness in consciousness. One could say that such mirror is a distorting mirror, although a word "distorting" is perhaps not the best description. It is a mirror of unusual reflection. Depending on the mirror, reflection in it varies up to the unrecognisable. To make a distorting mirror a person performs an act — alters a usual mirror. To effect this action he must be included in the system — he cannot simply take a usual mirror in his hands. Similar to this action of the person, consciousness is included in the system, can change consciousness, and reflection of reality will depend on it. Therefore knowledge, being this reflection, will depend on consciousness. In this case, consciousness influences processes in the origin of knowledge. Phenomenologically speaking, reflection of objective reality will already be an actual stream of consciousness.

After we have found out in what case some reflection takes place, we shall be able to answer the aforementioned question: Does consciousness influence the origin of knowledge or not?

Passive consciousness can be excluded from being, from what takes place in an open system. In this case, being is determined according to materialist philosophy. In an open system, passive reflection takes place, and consequently knowledge is defined as passive reflection of reality in the consciousness of a person. As remarked above, passive reflection is unequivocal in a macrocosm, and it is multiple-valued in a microcosm. Therefore, in the case of an open system, in a macrocosm, knowledge is passive and unequivocal. In a microcosm it is passive too, but it is multiple-valued. We shall call this knowledge, accordingly, *usual* and *unusual* knowledge respectively — the unusual because knowledge of the microcosm, including the Heisenberg Uncertainties, is for us, unusual.

Thus, in unusual knowledge there is an affection of consciousness. Hence, it is necessary to consider ontological problems in physics. Many physicists adhere to a definition of being according to materialism. Therefore, constructed by them with the help of theories, physical reality characterizes the world, and excludes the consciousness of the observer

from consideration. We shall call such a concept of physical reality *usual*. Building on it, the physicists do not take into account questions connected with perception and consciousness, so it is possible to act only in the case of a macrocosm.

For a microcosm, physical reality, as constructed by the physicists, should be entirely different; unusual. We shall call physical reality describing a microcosm, as an open system, *ontological*. In this case, effects of consciousness take place, but the effects are connected not with the activity of consciousness, but with reflection or integrity of perception of the cognizable world.

Answering “yes” to the question: Does consciousness influence the origin of knowledge or not? it is evident that consciousness is active and therefore cannot be excluded from the being participating in the closed system. As we have already seen, in the closed system active reflection takes place, so knowledge is active reflection of reality in the consciousness of a person. In this knowledge there is a place for the effects of consciousness, but they are connected not with perception of the cognizable world, as in case of unusual knowledge, but with the activity of the consciousness of the observer.

Can active consciousness of the observer be consciousness of the person? Certainly not! The system, having captured the consciousness of one person, is not closed, because outside it there is the consciousness of another person in which reality can be reflected. Thus, when we speak of consciousness of the observer in the closed system, i.e. about active consciousness, we mean that it cannot be consciousness of the person. The consciousness of the person is a passive consciousness, i.e. this consciousness of the observer in an open system. Knowledge which takes place in this case is a *simple* knowledge of passive consciousness — the person. Accordingly, this knowledge is *usual* (in case of a macrocosm), or *unusual* (in case of a microcosm).

Knowledge, which takes place in the case when the system is closed, is knowledge of active consciousness. This knowledge is absolute knowledge.

Let's consider absolute knowledge in the case when the closed system is a macrocosm. In this case knowledge is active and unequivocal reflection. We shall call such knowledge *transcendental*. Such a name is justified because transcendental knowledge can be understood by passive consciousness. Clearly, such analysis is possible in a macrocosm because in this case we learn of our world, which, in contrast with the microcosm, is visible, audible, and otherwise sentient. *Transcendental* knowledge concerns scientific knowledge.

In the case of a closed system as a microcosm, knowledge is active, but multiple-valued reflection and so gives rise to latent uncertainties which are not Heisenberg Uncertainties. The paradoxes concerning the laws of the quantum world were explained by Albert Einstein as properties of an unobservable, deeper sub-quantum world; hidden variables. With the help of Bell's inequalities it was proved that latent parameters (hidden variables) do not exist. However, if Heisenberg

Uncertainties are open to passive consciousness, i.e. to the consciousness of a person, then the latent parameters are open only to active consciousness. Therefore we also cannot open them. We shall call such knowledge *transient*. Such a name is justified in that it cannot be understood.

Thus, for open systems, knowledge is passive and unequivocal in a macrocosm, passive and multiple-valued in a microcosm. For the closed systems the knowledge is active and unequivocal in a macrocosm, active and multiple-valued in a microcosm. Accordingly, knowledge is divided into the *usual*, *unusual*, *transcendental* and *transient*. Physical reality for these cases are, philosophically speaking, usual, ontological and active.

3 The “Schrödinger cat” experiment

It is known that in a macrocosm a body can be in only one state. Clearly, this knowledge is usual. In a microcosm an elementary particle can be simultaneously in two states. Of course, such knowledge is *unusual*.

However, it has been established that in the result of intensification the superposition of two micro-states turns into superposition of two macro-states. Therefore in a macrocosm there is unusual knowledge. This paradox has been amplified by E. Schrödinger in his mental experiment, known as Schrödinger's cat.

In the paradox of Schrödinger's cat the state of a cat (alive or dead) depends on the act of looking inside the box containing the cat, i.e. depends on the consciousness of the observer. Thus, consciousness becomes an object of quantum physics. We mentioned above that in an open system the consciousness of an observer, being passive, is the consciousness of a person. In an open macrocosm perceived by us unequivocally, the open microcosm is perceived by us as multiple-valued. Frequently it is asked: Where is the border between the macrocosm and the microcosm It is possible to answer that this border is the perception of a person. The state of Schrödinger's cat simultaneously both alive and dead corresponds to an open microcosm. Although we talk about a macro object — a cat — it is connected to a microcosm; it is a microcosm when a person doesn't open the box and look at the cat. As soon as a person looks at the cat in the box, i.e. completely and unequivocally perceive it, the state of the cat is determined, for example, the cat is alive. This state of the cat corresponds to an open macrocosm — to the world which we live.

The state of Schrödinger's cat — simultaneously alive and dead — is the entangled state. In an open system the paradox of Schrödinger's cat is described with the help of the decoherence phenomenon [2]. The open system differs from the closed. In an open system there are some degrees of freedom, including a brain and the consciousness of the observer that by our measurements can give us information. We open the box and find out that the cat is actually alive — it is the deco-

herence. With a statistical ensemble of Schrödinger cats, we can use probability theory and statistical forecast.

What will be Schrödinger's cat in a closed system? The most interesting theory here is the many-world interpretation of quantum mechanics of Everett and Wheeler [3]. The closed system is the whole world, including the observer. Every component of superposition describes the whole world, and none of them has any advantage. The question here is not: What will be the result of measurement? The question here is not: In what world, of many worlds, does the observer appear? In the Everett-Wheeler theory it depends on the consciousness of the observer. In the terminology of Wheeler such consciousness is called active. Knowledge in this case is knowledge of active consciousness and called by us the transcendental (in a macrocosm) and the transient (in a microcosm).

Recall Einstein's objection to Bohr's probabilistic interpretation of the quantum mechanics: "I do not believe that God plays dice". M. B. Menskii [4] writes "Yes, God does not play dice. He equally accepts all possibilities. In dice plays the consciousness of each observer". The author means, that the consciousness of the person, his mind, builds the forecasts, based on concepts of probability theory. Let's agree that the world, about which Einstein speaks, in which God does not play dice, is a real world. The world in which the person plays dice is a sentient world.

Besides these two worlds there exists, according to Max Plank [5], a third — the world of physical science or the physical picture of world. This world is a bridge for us, and with its help we learn of those worlds. It concerns the aforementioned physical reality. Descriptions of the real and sentient worlds in the world of physical science are the quantum and classical worlds, accordingly.

In physics the classical world is very frequently interpreted as the objective world. The quantum world exists as some mathematical image — a state vector, i.e. the wave function. Therefore it is objectively non-existent, an illusion. Such an interpretation, warns Plank, can result in the opinion that there is only a sentient world. Such an outlook cannot be denied logically, because logic itself cannot pluck anyone from his own sentient world. Plank held that besides logic there is also common sense, which tells us that although we may not directly see some world, that world may still exist. From such a point of view, interpretation of the mutual relations between the worlds will be very different — the quantum world is objective, the classical world is an illusion.

It is possible to interpret these worlds from the new point of view. As we saw above for Schrödinger's cat, the border between quantum and classical worlds is erased. Therefore the real world is both the objective quantum world and objective classical world. Furthermore, the sentient world is both an illusion of the quantum world and an illusion of the classical world. Thus, the quantum and classical world each consist of components — objective and illusory components.

Are there an objective classical world and an illusion of the quantum world in our understanding? The classical world is the world of macroscopic objects and our consciousness sees and perceives this world. For us it should be sentient. Illusion of the classical world satisfies this condition. The quantum world is the world of microscopic objects. This world is invisible to us and so cannot be the sentient world. The objective quantum world satisfies this condition. Thus, although there is an objective classical world and an illusion of the quantum world, these worlds are outside the ambit of our consciousness. It becomes clear now why classical and quantum physics essentially and qualitatively differ from each other. Classical physics studies a physical picture of an illusion of the classical world. Quantum physics studies the physical picture of the objective quantum world.

Thus, our consciousness comprehends the objective quantum world. Following Menskii [4], it can be represented symbolically as some complex volumetric figure, and the illusion of the classical world is only one of the projections of this figure. It will be expedient to present this complex volumetric figure, as a simplex.

4 Simplex interpretation of quantum physics

From functional analysis [6] it is known that a point is zero-dimensional, a line is one-dimensional, a triangle is bi-dimensional, a tetrahedron a three-dimensional simplex. The three-dimensional simplex, a tetrahedron has 4 bi-dimensional sides (triangles), 6 one-dimensional sides (lines) and 4 zero-dimensional sides (points), giving a total of 14 sides.

It is impossible to imagine a four-dimensional simplex in our three-dimensional space.

The parallelepiped or cube is not a simplex because for this purpose it is necessary that all 8 points were in six-measured space. Thus, formed from more than four points, is a complex volumetric figure.

Let's assume in experiment with 100 Schrödinger cats, 80 cats are alive and 20 are dead. Points 20 and 80 are two ends of a simplex. At other moment of time or in another experiment let's assume from 100 cats that 60 are alive and 40 are dead. These two points are also ends of a simplex. We can continue our tests, but we shall stop with these two, and thus, we consider a three-dimensional simplex — a tetrahedron. The ribs of our tetrahedron indicate various probabilities. For example, the rib linking the points 80 live cats and 40 dead cats give $80/120 = 2/3$ of probability of the case in which a cat is alive. In the case 60 live and 20 dead cats, the rib of the simplex shows that the probability is $60/80 = 3/4$, etc. The rib linking the points 20 dead and 40 dead cats and the rib linking the points 80 live and 60 live cats each give a probability of 1. Let's consider the faces of the simplex. In the case of a live cat on one of them the probability changes from $2/3$ to 0.8; on another face, from $3/4$ to 0.6; on third face, from $2/3$ to 0.6; on fourth, from $3/4$ to 0.8 etc. As to points of a tetrahedron

they specify determinism of an event. For example, the point of 80 live cats specifies that in fact all 80 cats are alive.

We could construct the simplex with various probabilistic ribs and sides because we are observers from outside. In this case we built a physical picture of the real world. Only in this world is the probabilistic interpretation of the quantum mechanics given by Bohr true.

In a physical picture of the sentient world, we cannot construct a simplex. We can only perceptions as projections, i.e. sides of a simplex. After that, classical probability is applied, but it is applied, we shall repeat, not for a whole simplex, but only for one of its sides. This side, perceived by us as the sentient world, is an illusion because it not unique: there exists a set of worlds alternative to it. With a physical picture of the world, we can even count the number of parallel worlds. As our world is three-dimensional and our consciousness exists in it we can count only sides of a three-dimensional simplex — a tetrahedron, which, as shown above, has only 14 sides.

Returning now to the dispute between Einstein and Bohr, in the real and sentient worlds, of course Einstein was right — really, God does not play dice. However, in the physical picture of the world, Bohr had the right to apply probability and statistics.

Usually in a game of dice we mean only the act of throwing dice. However, dice consists of acts before (we build forecasts) and after (realization of one forecast from possible results). This situation can be likened to a court case; there is a hearing of a case, a verdict and a process after the verdict. In the physical picture of the real world, a game of dice by consciousness is a game up to the act of throwing the dice. Our consciousness can only imagine all sides of a three-dimensional simplex, i.e. all alternative results. But the choice of one of them depends on “active” consciousness. In our sentient world, in the act of throwing the dice, we shall see this choice. In the physical picture of the sentient world, a game of dice by consciousness is a game after the act of throwing the dice. Having these outcomes allow us to statistically forecast.

Thus, uncertainty of the real world qualitatively differs from uncertainty of the sentient world. Thus, uncertainty of the sentient world is not present and, as a matter of fact, the finding of the probability of some casual event has no connexion with uncertainty because this probability exists beforehand, a priori, and by doing a series of tests we simply find it. It becomes clear then why quantum statistics essentially differs from the classical.

This simplex, with various probabilistic ribs and sides, we could construct with the help of epistemological analysis. Knowledge which was analyzed in this case is knowledge of active consciousness. In the case when the simplex from a volumetric figure is converted into one of its projections, we see only one of its sides (a point, a line, a triangle). Knowledge appropriate to this case is knowledge of passive consciousness. In a simplex the lines (80, 20) and (60, 40) where

points 80, 60 are live, and 20, 40 are dead cats, correspond to *usual* knowledge. In this case we use classical statistics (after we have looked in the box, Schrödinger’s cats became simple cats, and we already have data, for example, from 100 cats in one case 80 alive, and in the other case 6, etc.). With the help of this data we find an average and dispersion of a random variable.

But when the ensemble consists not of simple cats, but Schrödinger cats we deal with a microcosm, with a world, the perception of which, is multiple-valued. In this case, for example, the point 80 is already fixed simultaneously and with the point 20, and with the point 40. Therefore the triangle (20, 80, 40) is examined. Similarly, the triangle (40, 60, 20) is also considered. These triangles correspond to *unusual* knowledge. In this case we cannot apply classical statistics. Therefore we use quantum statistics.

There is a question: But what in a simplex will correspond to *transcendental* and *transient* knowledge? We can answer that transcendental knowledge is knowledge of active consciousness in the case of a macrocosm, and corresponds to the entire simplex. Transcendental knowledge can be acquired by us a priori (because we could construct the simplex), but for transient knowledge this is not possible. Knowledge of active consciousness appropriate to transition from a microcosm to macrocosm, i.e. to our world, will be transcendental, and from a microcosm to a microcosm it will be transient. There is no sharp border between macro-world and microcosms, but in fact there is a sharp border between knowledge about them.

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SPECIAL REPORT**On a Geometric Theory of Generalized Chiral Elasticity with Discontinuities**

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In this work we develop, in a somewhat extensive manner, a geometric theory of chiral elasticity which in general is endowed with *geometric discontinuities* (sometimes referred to as *defects*). By itself, the present theory generalizes both Cosserat and void elasticity theories to a certain extent via *geometrization* as well as by taking into account the action of the electromagnetic field, i.e., the incorporation of the electromagnetic field into the description of the so-called *microspin* (*chirality*) also forms the underlying structure of this work. As we know, the description of the electromagnetic field as a unified phenomenon requires four-dimensional space-time rather than three-dimensional space as its background. For this reason we embed the three-dimensional material space in four-dimensional space-time. This way, the electromagnetic spin is coupled to the non-electromagnetic microspin, both being parts of the complete microspin to be added to the macrospin in the full description of vorticity. In short, our objective is to generalize the existing continuum theories by especially describing microspin phenomena in a fully geometric way.

1 Introduction

Although numerous generalizations of the classical theory of elasticity have been constructed (most notably, perhaps, is the so-called Cosserat elasticity theory) in the course of its development, we are somewhat of the opinion that these generalizations simply lack geometric structure. In these existing theories, the introduced quantities supposedly describing microspin and irregularities (such as voids and cracks) seem to have been assumed from *without*, rather than from *within*. By our geometrization of microspin phenomena we mean exactly the description of microspin phenomena in terms of intrinsic geometric quantities of the material body such as its curvature and torsion. In this framework, we produce the microspin tensor and the anti-symmetric part of the stress tensor as intrinsic geometric objects rather than alien additions to the framework of classical elasticity theory. As such, the initial microspin variables are not to be freely chosen to be included in the potential energy functional as is often the case, but rather, at first we identify them with the internal properties of the geometry of the material body. In other words, we can not simply adhere to the simple way of adding external variables that are supposed to describe microspin and defects to those original variables of the classical elasticity theory in the construction of the potential energy functional without first discovering and unfolding their underlying internal geometric existence.

Since in this work we are largely concerned with the behavior of *material points* such as their translational and rotational motion, we need to primarily cast the field equations in a manifestly covariant form of the *Lagrangian* system of material coordinates attached to the material body. Due to

the presence of geometric discontinuities (*geometric singularities*) and the *local non-orientability* of the material points, the full Lagrangian description is necessary. In other words, the compatibility between the spatial (*Eulerian*) and the material coordinate systems can not in general be directly invoked. This is because the smooth transitional transformation from the Lagrangian to the Eulerian descriptions and vice versa breaks down when geometric singularities and the non-orientability of the material points are taken into account. However, for the sake of accommodating the existence of all imaginable systems of coordinates, we shall assume, at least locally, that the material space lies within the three-dimensional space of spatial (*Eulerian*) coordinates, which can be seen as a (flat) hypersurface embedded in four-dimensional space-time. With respect to this embedding situation, we preserve the correspondence between the material and spatial coordinate systems in classical continuum mechanics, although not their equality since the field equations defined in the space of material points are in general not independent of the orientation of that local system of coordinates.

At present, due to the limits of space, we shall concentrate ourselves merely on the construction of the field equations of our geometric theory, from which the equations of motion shall follow. We shall not concern ourselves with the over-determination of the field equations and the extraction of their exact solutions. There is no doubt, however, that in the process of investigating particular solutions to the field equations, we might catch a glimpse into the initial states of the microspin field as well as the evolution of the field equations. We'd also like to comment that we have constructed our theory with a relatively small number of variables only, a

characteristic which is important in order to prevent superfluous variables from encumbering the theory.

2 Geometric structure of the manifold \mathfrak{S}_3 of material coordinates

We shall briefly describe the local geometry of the manifold \mathfrak{S}_3 which serves as the space of material (Lagrangian) coordinates (material points) ξ^i ($i = 1, 2, 3$). In general, in addition to the general non-orientability of its local points, the manifold \mathfrak{S}_3 may contain singularities or geometric defects which give rise to the existence of a local material curvature represented by a generally *non-holonomic* (path-dependent) curvature tensor, a consideration which is normally shunned in the standard continuum mechanics literature. This way, the manifold \mathfrak{S}_3 of material coordinates, may be defined either as a continuum or a discontinuum and can be seen as a three-dimensional hypersurface of non-orientable points, embedded in the physical four-dimensional space-time of spatial-temporal coordinates \mathfrak{R}_4 . Consequently, we need to employ the language of general tensor analysis in which the local metric, the local connection, and the local curvature of the material body \mathfrak{S}_3 form the most fundamental structural objects of our consideration.

First, the material space \mathfrak{S}_3 is spanned by the three curvilinear, covariant (i.e., tangent) basis vectors g_i as \mathfrak{S}_3 is embedded in a four-dimensional space-time of physical events \mathfrak{R}_4 for the sake of general covariance, whose coordinates are represented by y^μ ($\mu = 1, 2, 3, 4$) and whose covariant basis vectors are denoted by ω_μ . In a neighborhood of local coordinate points of \mathfrak{R}_4 we also introduce an enveloping space of spatial (Eulerian) coordinates x^A ($A = 1, 2, 3$) spanned by locally constant orthogonal basis vectors e_A which form a three-dimensional Euclidean space \mathbb{E}_3 . (From now on, it is to be understood that small and capital Latin indices run from 1 to 3, and that Greek indices run from 1 to 4.) As usual, we also define the dual, contravariant (i.e., cotangent) counterparts of the basis vectors g_i , e_A , and ω_μ , denoting them respectively as g^i , e^A , and ω^μ , according to the following relations:

$$\langle g^i, g_k \rangle = \delta_k^i,$$

$$\langle e^A, e_B \rangle = \delta_B^A,$$

$$\langle \omega^\mu, \omega_\nu \rangle = \delta_\nu^\mu,$$

where the brackets $\langle \rangle$ denote the so-called projection, i.e., the inner product and where δ denotes the Kronecker delta. From these basis vectors, we define their tetrad components as

$$\gamma_A^i = \langle g^i, e_A \rangle = \frac{\partial \xi^i}{\partial x^A},$$

$$\zeta_\mu^i = \langle g^i, \omega_\mu \rangle = \frac{\partial \xi^i}{\partial y^\mu},$$

$$e_\mu^A = \langle e^A, \omega_\mu \rangle = \frac{\partial x^A}{\partial y^\mu}.$$

Their duals are given in the following relations:

$$\gamma_A^i \gamma_k^A = \zeta_\mu^i \zeta_k^\mu = \delta_k^i,$$

$$e_\mu^A e_B^A = \delta_B^\mu.$$

(Einstein's summation convention is implied throughout this work.)

The distance between two infinitesimally adjacent points in the (initially undeformed) material body \mathfrak{S}_3 is given by the symmetric bilinear form (with \otimes denoting the tensor product)

$$g = g_{ik} g^i \otimes g^k,$$

called the metric tensor of the material space, as

$$ds^2 = g_{ik} d\xi^i d\xi^k.$$

By means of projection, the components of the metric tensor of \mathfrak{S}_3 are given by

$$g_{ik} = \langle g_i, g_k \rangle.$$

Accordingly, for $a, b = 1, 2, 3$, they are related to the four-dimensional components of the metric tensor of \mathfrak{S}_3 , i.e., $G_{\mu\nu} = \langle \omega_\mu, \omega_\nu \rangle$, by

$$\begin{aligned} g_{ik} &= \zeta_i^\mu \zeta_k^\nu G_{\mu\nu} = \\ &= \zeta_i^\alpha \zeta_k^b G_{ab} + 2k_{(i} b_{k)} + \phi k_i k_k, \end{aligned}$$

where the round brackets indicate symmetrization (in contrast to the square brackets denoting anti-symmetrization which we shall also employ later) and where we have set

$$k_i = \zeta_i^4 = c \frac{\partial t}{\partial \xi^i},$$

$$b_i = G_{4i} = \zeta_i^\alpha G_{4\alpha},$$

$$\phi = G_{44}.$$

Here we have obviously put $y^4 = ct$ with c the speed of light in vacuum and t time.

Inversely, with the help of the following projective relations:

$$g_i = \zeta_i^\mu \omega_\mu,$$

$$\omega_\mu = \zeta_\mu^i g_i + \epsilon n_\mu n,$$

we find that

$$G_{\mu\nu} = \zeta_\mu^i \zeta_\nu^k g_{ik} + \epsilon n_\mu n_\nu,$$

or, calling the dual components of g , as shown in the relations

$$g_{ir} g^{kr} = \delta_i^k,$$

$$G_{\mu\rho} G^{\nu\rho} = \delta_\mu^\nu,$$

we have

$$\gamma_i^\mu \gamma_\nu^i = \delta_\nu^\mu - \in n^\mu n_\nu.$$

Here $\in = \pm 1$ and n^μ and n_ν respectively are the contravariant and covariant components of the unit vector field n normal to the hypersurface of material coordinates \mathfrak{S}_3 , whose canonical form may be given as $\Phi(\xi^i, k) = 0$ where k is a parameter. (Note that the same 16 relations also hold for the inner product represented by $e_A^\mu e_\nu^A$.) We can write

$$n_\mu = \in^{1/2} \frac{\partial \Phi}{\partial y^\mu} \left(G^{\alpha\beta} \frac{\partial \Phi}{\partial y^\alpha} \frac{\partial \Phi}{\partial y^\beta} \right)^{-1/2}.$$

Note that

$$\begin{aligned} n_\mu \gamma_i^\mu &= n_\mu e_i^\mu = 0, \\ n_\mu n^\mu &= \in. \end{aligned}$$

Let now g denote the determinant of the three-dimensional components of the material metric tensor g_{ik} . Then the covariant and contravariant components of the totally anti-symmetric permutation tensor are given by

$$\begin{aligned} \in_{ijk} &= g^{1/2} \varepsilon_{ijk}, \\ \in^{ijk} &= g^{-1/2} \varepsilon^{ijk}, \end{aligned}$$

where ε_{ijk} are the components of the usual *permutation tensor density*. More specifically, we note that

$$g_i \wedge g_j = \in_{ijk} g^k,$$

where the symbol \wedge denotes exterior product, i.e., $g_i \wedge g_j = (\zeta_i^\mu \zeta_j^\nu - \zeta_j^\mu \zeta_i^\nu) \omega_\mu \otimes \omega_\nu$. In the same manner, we define the four-dimensional permutation tensor as one with components

$$\begin{aligned} \in_{\alpha\beta\rho\sigma} &= G^{1/2} \varepsilon_{\alpha\beta\rho\sigma}, \\ \in^{\alpha\beta\rho\sigma} &= G^{-1/2} \varepsilon^{\alpha\beta\rho\sigma}, \end{aligned}$$

where $G = \det G_{\mu\nu}$. Also, we call the following simple transitive rotation group:

$$\omega_\alpha \wedge \omega_\beta = -\in \in_{\alpha\beta\rho\sigma} n^\rho \omega^\sigma,$$

where

$$\begin{aligned} \in_{ijk} n_\sigma &= \zeta_i^\alpha \zeta_j^\beta \zeta_k^\rho \in_{\alpha\beta\rho\sigma}, \\ n_\sigma &= \frac{1}{6} \zeta_i^\alpha \zeta_j^\beta \zeta_k^\rho \in^{ijk} \in_{\alpha\beta\rho\sigma}. \end{aligned}$$

Note the following identities:

$$\begin{aligned} \in_{ijk} \in^{pqr} &= \delta_{ijk}^{pqr} = \delta_i^p (\delta_j^q \delta_k^r - \delta_j^r \delta_k^q) + \delta_i^q (\delta_j^r \delta_k^p - \delta_j^p \delta_k^r) + \\ &+ \delta_i^r (\delta_j^p \delta_k^q - \delta_j^q \delta_k^p), \\ \in_{ijr} \in^{pqr} &= \delta_{ij}^{pq} = \delta_i^p \delta_j^q - \delta_i^q \delta_j^p, \\ \in_{ijs} \in^{ijr} &= \delta_s^r, \end{aligned}$$

where δ_{ijk}^{pqr} and δ_{ij}^{pq} represent generalized Kronecker deltas. In the same manner, the four-dimensional components of the generalized Kronecker delta, i.e.,

$$\delta_{\mu\nu\gamma\lambda}^{\alpha\beta\rho\sigma} = \det \begin{pmatrix} \delta_\mu^\alpha & \delta_\mu^\beta & \delta_\mu^\rho & \delta_\mu^\sigma \\ \delta_\nu^\alpha & \delta_\nu^\beta & \delta_\nu^\rho & \delta_\nu^\sigma \\ \delta_\gamma^\alpha & \delta_\gamma^\beta & \delta_\gamma^\rho & \delta_\gamma^\sigma \\ \delta_\lambda^\alpha & \delta_\lambda^\beta & \delta_\lambda^\rho & \delta_\lambda^\sigma \end{pmatrix}$$

can be used to deduce the following identities:

$$\begin{aligned} \in_{\mu\nu\gamma\lambda} \in^{\alpha\beta\rho\sigma} &= \delta_{\mu\nu\gamma\lambda}^{\alpha\beta\rho\sigma}, \\ \in_{\mu\nu\gamma\sigma} \in^{\alpha\beta\rho\sigma} &= \delta_{\mu\nu\gamma}^{\alpha\beta\rho}, \\ \in_{\mu\nu\rho\sigma} \in^{\alpha\beta\rho\sigma} &= 2 \in \delta_{\mu\nu}^{\alpha\beta}, \\ \in_{\mu\beta\rho\sigma} \in^{\alpha\beta\rho\sigma} &= 6 \in \delta_\mu^\alpha. \end{aligned}$$

Now, for the contravariant components of the material metric tensor, we have

$$\begin{aligned} g^{ik} &= \zeta_a^i \zeta_b^k G^{ab} + 2k^{(i} b^{k)} + \bar{\phi} k^i k^k, \\ G^{\mu\nu} &= \zeta_i^\mu \zeta_k^\nu g^{ik} + \in n^\mu n^\nu, \end{aligned}$$

where

$$\begin{aligned} k^i &= \frac{1}{c} \frac{\partial \xi^i}{\partial t}, \\ b^i &= G^{4i} = \zeta_a^i G^{4a}, \\ \bar{\phi} &= G^{44}. \end{aligned}$$

Obviously, the quantities $\frac{\partial \xi^i}{\partial t}$ in k^i are the contravariant components of the local velocity vector field. If we choose an orthogonal coordinate system for the background space-time \mathfrak{R}_4 , we simply have the following three-dimensional components of the material metric tensor:

$$\begin{aligned} g_{ik} &= \zeta_a^i \zeta_b^k G_{ab} + \phi k_i k_k, \\ g^{ik} &= \zeta_a^i \zeta_b^k G^{ab} + \bar{\phi} k^i k^k. \end{aligned}$$

In a special case, if the space-time \mathfrak{R}_4 is (pseudo-)Euclidean, we may set $\phi = \bar{\phi} = \pm 1$. However, for the sake of generality, we shall not always need to assume the case just mentioned.

Now, the components of the metric tensor of the local Euclidean space of spatial coordinates x^A , $h_{AB} = \langle e_A, e_B \rangle$, are just the components of the *Euclidean Kronecker delta*:

$$h_{AB} = \delta_{AB}.$$

Similarly, we have the following relations:

$$\begin{aligned} g_{ik} &= \gamma_i^A \gamma_k^B h_{AB} = \gamma_i^A \gamma_k^A, \\ h_{AB} &= \gamma_A^i \gamma_B^k g_{ik}. \end{aligned}$$

Now we come to an important fact: from the structure of the material metric tensor alone, we can raise and lower the indices of arbitrary vectors and tensors defined in \mathfrak{S}_3 , and hence in \mathfrak{R}_4 , by means of its components, e.g.,

$$A^i = g^{ik} A_k, A_i = g_{ik} A^k, B^\mu = G^{\mu\nu} B_\nu, \\ B_\mu = G_{\mu\nu} B^\nu, \text{ etc.}$$

Having introduced the metric tensor, let us consider the transformations among the physical objects defined as acting in the material space \mathfrak{S}_3 . An arbitrary tensor field T of rank n in \mathfrak{S}_3 can in general be represented as

$$T = T_{kl\dots}^{ij\dots} g_i \otimes g_j \otimes \dots \otimes g^k \otimes g^l \otimes \dots = \\ = T'^{AB\dots}_{CD\dots} e_A \otimes e_B \otimes \dots \otimes e^C \otimes e^D \otimes \dots = \\ = T''^{\alpha\beta\dots}_{\mu\nu\dots} \omega_\alpha \otimes \omega_\beta \otimes \dots \otimes \omega^\mu \otimes \omega^\nu \otimes \dots$$

In other words,

$$T_{kl\dots}^{ij\dots} = \gamma_A^i \gamma_B^j \dots \gamma_C^k \gamma_D^l \dots T'^{AB\dots}_{CD\dots} = \zeta_\alpha^i \zeta_\beta^j \dots \zeta_\mu^k \zeta_\nu^l \dots T''^{\alpha\beta\dots}_{\mu\nu\dots}, \\ T'^{AB\dots}_{CD\dots} = \gamma_A^i \gamma_B^j \dots \gamma_C^k \gamma_D^l \dots T_{kl\dots}^{ij\dots} = e_A^i e_B^j \dots e_C^k e_D^l \dots T''^{\alpha\beta\dots}_{\mu\nu\dots}, \\ T''^{\alpha\beta\dots}_{\mu\nu\dots} = \zeta_\mu^k \zeta_\nu^l \dots \zeta_i^\alpha \zeta_j^\beta \dots T_{kl\dots}^{ij\dots} = e_\mu^C e_\nu^D \dots e_A^\alpha e_B^\beta T'^{AB\dots}_{CD\dots}.$$

For instance, the material line-element can once again be written as

$$ds^2 = g_{ik} (\xi^p) d\xi^i d\xi^k = \delta_{AB} dx^A dx^B = G_{\mu\nu} (y^\alpha) dy^\mu dy^\nu.$$

We now move on to the notion of a covariant derivative defined in the material space \mathfrak{S}_3 . Again, for an arbitrary tensor field T of \mathfrak{S}_3 , the covariant derivative of the components of T is given as

$$\nabla_p T_{kl\dots}^{ij\dots} = \frac{\partial T_{kl\dots}^{ij\dots}}{\partial \xi^p} + \Gamma_{rp}^i T_{kl\dots}^{rj\dots} + \Gamma_{rp}^j T_{kl\dots}^{ir\dots} + \dots - \\ - \Gamma_{kp}^r T_{rl\dots}^{ij\dots} - \Gamma_{lp}^r T_{kr\dots}^{ij\dots} - \dots,$$

such that

$$\nabla_p T = \frac{\partial T}{\partial \xi^p} = \nabla_p T_{kl\dots}^{ij\dots} g_i \otimes g_j \otimes \dots \otimes g^k \otimes g^l \otimes \dots,$$

where

$$\frac{\partial g_i}{\partial \xi^k} = \Gamma_{ik}^r g_r.$$

Here the $n^3 = 27$ quantities Γ_{jk}^i are the components of the connection field Γ , locally given by

$$\Gamma_{jk}^i = \gamma_A^i \frac{\partial \gamma_j^A}{\partial \xi^k},$$

which, in our work, shall be non-symmetric in the pair of its lower indices (jk) in order to describe both torsion and discontinuities. If $\bar{\xi}^i$ represent another system of coordinates

in the material space \mathfrak{S}_3 , then locally the components of the connection field Γ are seen to transform *inhomogeneously* according to

$$\Gamma_{jk}^i = \frac{\partial \xi^i}{\partial \bar{\xi}^p} \frac{\partial \bar{\xi}^r}{\partial \xi^j} \frac{\partial \bar{\xi}^s}{\partial \xi^k} \bar{\Gamma}_{rs}^p + \frac{\partial \xi^i}{\partial \bar{\xi}^p} \frac{\partial^2 \bar{\xi}^p}{\partial \xi^k \partial \xi^j},$$

i.e., the Γ_{jk}^i do not transform as components of a local tensor field. Before we continue, we shall note a few things regarding some *boundary conditions* of our material geometry. Because we have assumed that the hypersurface \mathfrak{S}_3 is embedded in the four-dimensional space-time \mathfrak{R}_4 , we must in general have instead

$$\frac{\partial g_i}{\partial \xi^k} = \Gamma_{ik}^r g_r + \in K_{ik} n,$$

where $K_{ik} = \langle \nabla_k g_i, n \rangle = n_\mu \nabla_k \zeta_\mu^i$ are the covariant components of the extrinsic curvature of \mathfrak{S}_3 . Then the scalar given by $\bar{K} = \in K_{ik} \frac{d\xi^i}{ds} \frac{d\xi^k}{ds}$, which is the Gaussian curvature of \mathfrak{S}_3 , is arrived at. However our simultaneous embedding situation in which we have also defined an Euclidean space in \mathfrak{R}_4 as the space of spatial coordinates embedding the space of material coordinates \mathfrak{S}_3 , means that the extrinsic curvature tensor, and hence also the Gaussian curvature of \mathfrak{S}_3 , must vanish and we are left simply with $\frac{\partial g_i}{\partial \xi^k} = \Gamma_{ik}^r g_r$. This situation is analogous to the simple situation in which a plane (flat surface) is embedded in a three-dimensional space, where on that plane we define a family of curves which give rise to a system of curvilinear coordinates, however, with discontinuities in the transformation from the plane coordinates to the local curvilinear coordinates and vice versa.

Meanwhile, we have seen that the covariant derivative of the tensor field T is again a tensor field. As such, here we have

$$\nabla_p T_{kl\dots}^{ij\dots} = \gamma_A^i \gamma_B^j \dots \gamma_C^k \gamma_D^l \dots \gamma_p^E \frac{\partial T'^{AB\dots}_{CD\dots}}{\partial x^E}$$

Although a non-tensorial object, the connection field Γ is a fundamental geometric object that establishes comparison of local vectors at different points in \mathfrak{S}_3 , i.e., in the Lagrangian coordinate system. Now, with the help of the material *metrical condition*

$$\nabla_p g_{ik} = 0,$$

i.e.,

$$\frac{\partial g_{ik}}{\partial \xi^p} = \Gamma_{ikp} + \Gamma_{kip},$$

where $\Gamma_{ikp} = g_{ir} \Gamma_{kp}^r$, one solves for Γ_{jk}^i as follows:

$$\Gamma_{jk}^i = \frac{1}{2} g^{ri} \left(\frac{\partial g_{rj}}{\partial \xi^k} - \frac{\partial g_{jk}}{\partial \xi^r} + \frac{\partial g_{kr}}{\partial \xi^j} \right) + \Gamma_{[jk]}^i - \\ - g^{ri} \left(g_{js} \Gamma_{[rk]}^s + g_{ks} \Gamma_{[rj]}^s \right).$$

From here, we define the following geometric objects:

1. The holonomic (path-independent) Christoffel or Levi-Civita connection, sometimes also called the *elastic connection*, whose components are symmetric in the pair of its lower indices (jk) and given by

$$\{^i_{jk}\} = \frac{1}{2} g^{ri} \left(\frac{\partial g_{rj}}{\partial \xi^k} - \frac{\partial g_{jk}}{\partial \xi^r} + \frac{\partial g_{kr}}{\partial \xi^j} \right).$$

2. The non-holonomic (path-dependent) object, a *chirality tensor* called the torsion tensor which describes *local rotation of material points* in \mathfrak{S}_3 and whose components are given by

$$\tau_{jk}^i = \Gamma_{[jk]}^i = \frac{1}{2} \gamma_A^i \left(\frac{\partial \gamma_j^A}{\partial \xi^k} - \frac{\partial \gamma_k^A}{\partial \xi^j} \right).$$

3. The non-holonomic contorsion tensor, a linear combination of the torsion tensor, whose components are given by

$$\begin{aligned} T_{jk}^i &= \Gamma_{[jk]}^i - g^{ri} \left(g_{js} \Gamma_{[rk]}^s + g_{ks} \Gamma_{[rj]}^s \right) = \\ &= \gamma_A^i \check{\nabla}_k \gamma_j^A = \\ &= \gamma_A^i \left(\frac{\partial \gamma_j^A}{\partial \xi^k} - \{^r_{jk}\} \gamma_r^A \right), \end{aligned}$$

which are actually anti-symmetric with respect to the first two indices i and j .

In the above, we have exclusively introduced a covariant derivative with respect to the holonomic connection alone, denoted by $\check{\nabla}_p$. Again, for an arbitrary tensor field T of \mathfrak{S}_3 , we have

$$\begin{aligned} \check{\nabla}_p T_{kl\dots}^{ij\dots} &= \frac{\partial T_{kl\dots}^{ij\dots}}{\partial \xi^p} + \{^i_{rp}\} T_{kl\dots}^{rj\dots} + \{^j_{rp}\} T_{kl\dots}^{ir\dots} + \dots - \\ &- \{^r_{kp}\} T_{rl\dots}^{ij\dots} - \{^r_{lp}\} T_{kr\dots}^{ij\dots} - \dots \end{aligned}$$

Now we can see that the metrical condition $\nabla_p g_{ik} = 0$ also implies that $\check{\nabla}_p g_{ik} = 0$, $\check{\nabla}_k \gamma_i^A = T_{ik}^r \gamma_r^A$, and $\nabla_k \gamma_i^A = 0$.

Finally, with the help of the connection field Γ , we derive the third fundamental geometric objects of \mathfrak{S}_3 , i.e., the local fourth-order curvature tensor of the material space

$$R = R^i_{.jkl} g_i \otimes g^j \otimes g^k \otimes g^l,$$

where

$$R^i_{.jkl} = \frac{\partial \Gamma^i_{.jl}}{\partial \xi^k} - \frac{\partial \Gamma^i_{.jk}}{\partial \xi^l} + \Gamma^r_{.jl} \Gamma^i_{.rk} - \Gamma^r_{.jk} \Gamma^i_{.rl}.$$

These are given in the relations

$$(\nabla_k \nabla_j - \nabla_j \nabla_k) F_i = R^r_{.ijk} F_r - 2\Gamma^r_{[jk]} \nabla_r F_i,$$

where F_i are the covariant components of an arbitrary vector field F of \mathfrak{S}_3 . Correspondingly, for the contravariant components F^i we have

$$(\nabla_k \nabla_j - \nabla_j \nabla_k) F^i = -R^i_{.rjk} F^r - 2\Gamma^r_{[jk]} \nabla_r F^i.$$

The *Riemann-Christoffel curvature tensor* \check{R} here then appears as the part of the curvature tensor R built from the symmetric, holonomic Christoffel connection alone, whose components are given by

$$\check{R}^i_{.jkl} = \frac{\partial}{\partial \xi^k} \{^i_{.jl}\} - \frac{\partial}{\partial \xi^l} \{^i_{.jk}\} + \{^r_{.jl}\} \{^i_{.rk}\} - \{^r_{.jk}\} \{^i_{.rl}\}.$$

Correspondingly, the components of the symmetric Ricci tensor are given by

$$\begin{aligned} \check{R}_{ik} = \check{R}^r_{.irk} &= \frac{\partial}{\partial \xi^r} \{^r_{.ik}\} - \frac{\partial^2 \log(g)}{\partial \xi^k \partial \xi^i} + \\ &+ \{^s_{.ik}\} \frac{\partial^e \log(g)^{1/2}}{\partial \xi^s} - \{^s_{.ir}\} \{^r_{.sk}\}, \end{aligned}$$

where we have used the relations

$$\{^k_{.ik}\} = \frac{\partial^e \log(g)^{1/2}}{\partial \xi^i} = \Gamma^k_{.ki}.$$

Then the Ricci scalar is simply $\check{R} = \check{R}^i_{.i}$, an important geometric object which shall play the role of the *microspin (chirality) potential* in our generalization of classical elasticity theory developed here.

Now, it is easily verified that

$$(\check{\nabla}_k \check{\nabla}_j - \check{\nabla}_j \check{\nabla}_k) F_i = \check{R}^r_{.ijk} F_r$$

and

$$(\check{\nabla}_k \check{\nabla}_j - \check{\nabla}_j \check{\nabla}_k) F^i = -\check{R}^i_{.rjk} F^r.$$

The remaining parts of the curvature tensor R are then the remaining non-holonomic objects J and Q whose components are given as

$$J^i_{.jkl} = \frac{\partial T^i_{.jl}}{\partial \xi^k} - \frac{\partial T^i_{.jk}}{\partial \xi^l} + T^r_{.jl} T^i_{.rk} - T^r_{.jk} T^i_{.rl}$$

and

$$Q^i_{.jkl} = \{^r_{.jl}\} T^i_{.rk} + T^r_{.jl} \{^i_{.rk}\} - \{^r_{.jk}\} T^i_{.rl} - T^r_{.jk} \{^i_{.rl}\}.$$

Hence, we write

$$R^i_{.jkl} = \check{R}^i_{.jkl} + J^i_{.jkl} + Q^i_{.jkl}.$$

More explicitly,

$$R^i_{.jkl} = \check{R}^i_{.jkl} + \check{\nabla}_k T^i_{.jl} - \check{\nabla}_l T^i_{.jk} + T^r_{.jl} T^i_{.rk} - T^r_{.jk} T^i_{.rl}.$$

From here, we define the two important contractions of the components of the curvature tensor above. We have the generalized Ricci tensor whose components are given by

$$R_{ik} = R^r_{.irk} = \check{R}_{ik} + \check{\nabla}_r T^r_{.ik} - T^r_{.is} T^s_{.rk} - \check{\nabla}_k \omega_i + T^r_{.ik} \omega_r,$$

where the $n = 3$ quantities

$$\omega_i = T^k_{.ik} = 2\Gamma^k_{[ik]}$$

define the components of the microspin vector. Furthermore, with the help of the relations $g^{rs}T_{rs}^i = -2g^{ik}\Gamma_{[ks]}^s = -\omega^i$, the generalized Ricci scalar is

$$R = R_{.i}^i = \check{R} - 2\check{\nabla}_i\omega^i - \omega_i\omega^i - T_{ijk}T^{ikj}.$$

It is customary to give the fully covariant components of the Riemann-Christoffel curvature tensor. They can be expressed somewhat more conveniently in the following form (when the g_{ik} are continuous):

$$\check{R}_{ijkl} = \frac{1}{2} \left(\frac{\partial^2 g_{il}}{\partial \xi^k \partial \xi^j} + \frac{\partial^2 g_{jk}}{\partial \xi^l \partial \xi^i} - \frac{\partial^2 g_{ik}}{\partial \xi^l \partial \xi^j} - \frac{\partial^2 g_{jl}}{\partial \xi^k \partial \xi^i} \right) + g_{rs} \left(\{^r_{il}\} \{^s_{jk}\} - \{^r_{ik}\} \{^s_{jl}\} \right).$$

In general, when the g_{ik} are continuous, all the following symmetries are satisfied:

$$\check{R}_{ijkl} = -\check{R}_{jikl} = -\check{R}_{ijlk}, \\ \check{R}_{ijkl} = \check{R}_{klij}.$$

However, for the sake of generality, we may as well drop the condition that the g_{ik} are continuous in their second derivatives, i.e., with respect to the material coordinates ξ^i such that we can define further more non-holonomic, anti-symmetric objects extracted from R such as the tensor field V whose components are given by

$$V_{ik} = R_{.rik}^r = -\gamma_A^l \left(\frac{\partial}{\partial \xi^k} \left(\frac{\partial \gamma_l^A}{\partial \xi^i} \right) - \frac{\partial}{\partial \xi^i} \left(\frac{\partial \gamma_l^A}{\partial \xi^k} \right) \right).$$

The above relations are equivalent to the following $\frac{1}{2}n(n-1) = 3$ equations for the components of the material metric tensor:

$$\frac{\partial}{\partial \xi^l} \left(\frac{\partial g_{ij}}{\partial \xi^k} \right) - \frac{\partial}{\partial \xi^k} \left(\frac{\partial g_{ij}}{\partial \xi^l} \right) = -(R_{ijkl} + R_{jikl}),$$

which we shall denote simply by $\|g_{ij,kl}\|$. When the g_{ik} possess such discontinuities, we may define the *discontinuity potential* by

$$\eta_i = \{^k_{ik}\} = \frac{\partial^e \log(g)^{1/2}}{\partial \xi^i}.$$

Hence we have

$$V_{ik} = \frac{\partial \eta_k}{\partial \xi^i} - \frac{\partial \eta_i}{\partial \xi^k}.$$

From the expression of the determinant of the material metric tensor, i.e.,

$$g = \varepsilon_{ijk} g_{1i} g_{2j} g_{3k}$$

we see, more specifically, that a discontinuum with arbitrary geometric singularities is characterized by the following dis-

continuity equations:

$$\left(\frac{\partial^2}{\partial \xi^s \partial \xi^r} - \frac{\partial^2}{\partial \xi^r \partial \xi^s} \right) g = \\ = \varepsilon_{ijk} g_{2j} g_{3k} \left(\frac{\partial^2}{\partial \xi^s \partial \xi^r} - \frac{\partial^2}{\partial \xi^r \partial \xi^s} \right) g_{1i} + \\ + \varepsilon_{ijk} g_{1i} g_{3k} \left(\frac{\partial^2}{\partial \xi^s \partial \xi^r} - \frac{\partial^2}{\partial \xi^r \partial \xi^s} \right) g_{2j} + \\ + \varepsilon_{ijk} g_{1i} g_{2j} \left(\frac{\partial^2}{\partial \xi^s \partial \xi^r} - \frac{\partial^2}{\partial \xi^r \partial \xi^s} \right) g_{3k}.$$

In other words,

$$\left(\frac{\partial^2}{\partial \xi^s \partial \xi^r} - \frac{\partial^2}{\partial \xi^r \partial \xi^s} \right) g = -\varepsilon_{ijk} (R_{1irs} + R_{i1rs}) g_{2j} g_{3k} - \\ - \varepsilon_{ijk} (R_{2jrs} + R_{j2rs}) g_{1i} g_{3k} - \\ - \varepsilon_{ijk} (R_{3krs} + R_{k3rs}) g_{1i} g_{2j}.$$

It is easy to show that in three dimensions the components of the curvature tensor R obey the following decomposition:

$$R_{ijkl} = W_{ijkl} + g_{ik} R_{jl} + g_{jl} R_{ik} - g_{il} R_{jk} - g_{jk} R_{il} + \\ + \frac{1}{2} (g_{il} g_{jk} - g_{ik} g_{jl}) R,$$

i.e.,

$$R_{.ikl}^{ij} = W_{.ikl}^{ij} + \delta_k^i R_{jl} + \delta_l^j R_{.k}^i - \delta_l^i R_{jk} - \delta_k^j R_{.l}^i + \\ + \frac{1}{2} (\delta_k^i \delta_l^j - \delta_k^j \delta_l^i) R,$$

where W_{ijkl} (and $W_{.ikl}^{ij}$) are the components of the Weyl tensor W satisfying $W_{.irk}^r = 0$, whose symmetry properties follow exactly those of R_{ijkl} . Similarly, for the components of the Riemann-Christoffel curvature tensor \check{R} we have

$$\check{R}_{.ikl}^{ij} = \check{W}_{.ikl}^{ij} + \delta_k^i \check{R}_{jl} + \delta_l^j \check{R}_{.k}^i - \delta_l^i \check{R}_{jk} - \delta_k^j \check{R}_{.l}^i + \\ + \frac{1}{2} (\delta_k^i \delta_l^j - \delta_k^j \delta_l^i) \check{R}.$$

Later, the above equations shall be needed to generalize the components of the elasticity tensor of classical continuum mechanics, i.e., by means of the components

$$\frac{1}{2} (\delta_k^i \delta_l^j - \delta_k^j \delta_l^i) \check{R}.$$

Furthermore with the help of the relations

$$R_{.jkl}^i + R_{.klij}^i + R_{.ljk}^i = -2 \left(\frac{\partial \Gamma_{[jk]}^i}{\partial \xi^l} + \frac{\partial \Gamma_{[kl]}^i}{\partial \xi^j} + \frac{\partial \Gamma_{[lj]}^i}{\partial \xi^k} + \right. \\ \left. + \Gamma_{rj}^i \Gamma_{[kl]}^r + \Gamma_{rk}^i \Gamma_{[lj]}^r + \Gamma_{rl}^i \Gamma_{[jk]}^r \right),$$

we derive the following identities:

$$\nabla_p R_{ijkl} + \nabla_k R_{ijlp} + \nabla_l R_{ijpk} = 2 \left(\Gamma_{[kl]}^r R_{ijrp} + \right. \\ \left. + \Gamma_{[lp]}^r R_{ijrk} + \Gamma_{[pk]}^r R_{ijrl} \right),$$

$$\nabla_i \left(R^{ik} - \frac{1}{2} g^{ik} R \right) = 2g^{ik} \Gamma_{[ri]}^s R_{.s}^r + \Gamma_{[ij]}^r R_{.r}^{ijk}.$$

From these more general identities, we then derive the simpler and more specialized identities:

$$\begin{aligned} \tilde{R}_{ijkl} + \tilde{R}_{iklj} + \tilde{R}_{iljk} &= 0, \\ \tilde{\nabla}_p \tilde{R}_{ijkl} + \tilde{\nabla}_k \tilde{R}_{ijlp} + \tilde{\nabla}_l \tilde{R}_{ijpk} &= 0, \\ \tilde{\nabla}_i \left(\tilde{R}^{ik} - \frac{1}{2} g^{ik} \tilde{R} \right) &= 0, \end{aligned}$$

often referred to as the Bianchi identities.

We are now able to state the following about the sources of the curvature of the material space \mathfrak{S}_3 : there are actually two sources that generate the curvature which can actually be sufficiently represented by the Riemann-Christoffel curvature tensor alone. The first source is the torsion represented by $\Gamma_{[jk]}^i$ which makes the hypersurface \mathfrak{S}_3 *non-orientable* as any field shall in general depend on the twisted path it traces therein. As we have said, this torsion is the source of microspin, i.e., point-rotation. The torsion tensor enters the curvature tensor as an integral part and hence we can equivalently attribute the source of microspin to the Riemann-Christoffel curvature tensor as well. The second source is the possible discontinuities in regions of \mathfrak{S}_3 which, as we have seen, render the components of the material metric tensor $g_{ik} = \gamma_i^A \gamma_k^B \delta_{AB}$ discontinuous at least in their second derivatives with respect to the material coordinates ξ^i . This is explicitly shown in the following relations:

$$\begin{aligned} R_{.jkl}^i &= -\gamma_A^i \left(\frac{\partial}{\partial \xi^l} \left(\frac{\partial \gamma_j^A}{\partial \xi^k} \right) - \frac{\partial}{\partial \xi^k} \left(\frac{\partial \gamma_j^A}{\partial \xi^l} \right) \right) = \\ &= -\gamma_A^i (\nabla_l K_{jk}^A - \nabla_k K_{jl}^A) + \Omega_{.jkl}^i, \end{aligned}$$

where

$$K_{ij}^A = \frac{\partial \gamma_i^A}{\partial \xi^j} = \frac{1}{2} \left(\frac{\partial \gamma_i^A}{\partial \xi^j} + \frac{\partial \gamma_j^A}{\partial \xi^i} \right) + \gamma_k^A \Gamma_{[ij]}^k$$

and

$$\Omega_{.jkl}^i = \gamma_A^i \left(\Gamma_{jk}^r K_{rl}^A - \Gamma_{jl}^r K_{rk}^A - 2\Gamma_{[kl]}^r K_{jr}^A \right).$$

Another way to cognize the existence of the curvature in the material space \mathfrak{S}_3 is as follows: let us inquire into the possibility of “parallelism” in the material space \mathfrak{S}_3 . Take now a “parallel” vector field ${}^p B$ such that

$$\nabla_k {}^p B_i = 0,$$

i.e.,

$$\frac{\partial {}^p B_i}{\partial \xi^k} = \Gamma_{ik}^r {}^p B_r.$$

Then in general we obtain the following non-integrable equations of the form

$$\frac{\partial}{\partial \xi^l} \left(\frac{\partial {}^p B_i}{\partial \xi^k} \right) - \frac{\partial}{\partial \xi^k} \left(\frac{\partial {}^p B_i}{\partial \xi^l} \right) = -R_{.ikl}^r {}^p B_r$$

showing that not even the “parallel” vector field ${}^p B$ is path-independent. Hence even though parallelism may be possibly defined in our geometry, *absolute parallelism* is obtained if and only if the integrability condition $R_{.jkl}^i = 0$ holds, i.e., if the components of the Riemann-Christoffel curvature tensor are given by

$$\tilde{R}_{.jkl}^i = \tilde{\nabla}_l T_{jk}^i - \tilde{\nabla}_k T_{jl}^i + T_{jk}^r T_{rl}^i - T_{jl}^r T_{rk}^i.$$

In other words, in the presence of torsion (microspin) the above situation concerning absolute parallelism is only possible if the material body is free of geometric defects, also known as singularities.

The relations we have been developing so far of course account for arbitrary nonorientability conditions as well as geometric discontinuities of the material space \mathfrak{S}_3 . Consequently, we see that the holonomic field equations of classical continuum mechanics shall be obtained whenever we drop the assumptions of non-orientability of points and geometric discontinuities of the material body. We also emphasize that *geometric non-linearity* of the material body has been fully taken into account. A material body then becomes linear if and only if we neglect any quadratic and higher-order terms involving the connection field Γ of the material space \mathfrak{S}_3 .

3 Elements of the generalized kinematics: deformation analysis

Having described the internal structure of the material space \mathfrak{S}_3 , i.e., the material body, we now move on to the dynamics of the continuum/discontinuum \mathfrak{S}_3 when it is subject to an external displacement field. Our goal in this kinematical section is to generalize the notion of a material derivative with respect to the material motion. We shall deal with the external displacement field in the direction of motion of \mathfrak{S}_3 which brings \mathfrak{S}_3 from its initially undeformed configuration to the deformed configuration ${}^* \mathfrak{S}_3$. We need to generalize the structure of the external displacement (i.e., *external diffeomorphism*) to include two kinds of microspin of material points: the *non-electromagnetic microspin* as well as the *electromagnetic microspin* which is generated, e.g., by electromagnetic polarization.

In this work, in order to geometrically describe the mechanics of the so-called Cosserat continuum as well as other generalized continua, we define the external displacement field ψ as being generally complex according to the decomposition

$$\psi^i = u^i + i\varphi^i,$$

where the diffeomorphism $\mathfrak{S}_3 \xrightarrow{\psi} {}^* \mathfrak{S}_3$ is given by

$${}^* \xi^i = \xi^i + \psi^i.$$

Here u^i are the components of the usual displacement field u in the neighborhood of points in \mathfrak{S}_3 , and φ^i are the

components of the microspin “point” displacement field φ satisfying

$$\nabla_k \varphi_i + \nabla_i \varphi_k = 0,$$

which can be written as an exterior (“Lie”) derivative:

$$L_\varphi g_{ik} = 0.$$

This just says that the components of the material metric remain invariant with respect to the action of the field φ . We shall elaborate on the notion of exterior differentiation in a short while.

The components of the displacement gradient tensor D are then

$$\begin{aligned} D_{ik} &= \nabla_k \psi_i = \\ &= \frac{1}{2} (\nabla_k \psi_i + \nabla_i \psi_k) + \frac{1}{2} (\nabla_k \psi_i - \nabla_i \psi_k) = \\ &= \frac{1}{2} (\nabla_k u_i + \nabla_i u_k) + \frac{1}{2} (\nabla_k u_i - \nabla_i u_k) + \\ &+ \frac{1}{2} i (\nabla_k \varphi_i - \nabla_i \varphi_k) = \\ &= \varepsilon_{ik} + \omega_{ik}. \end{aligned}$$

Accordingly,

$$\varepsilon_{ik} = \frac{1}{2} (\nabla_k u_i + \nabla_i u_k) = \frac{1}{2} L_u g_{ik} \stackrel{lin}{=} \frac{1}{2} (*g_{ik} - g_{ik})$$

are the components of the *linear strain tensor* and

$$\omega_{ik} = \Omega_{ik} + \phi_{ik}$$

are the components of the *generalized spin (vorticity) tensor*, where

$$\Omega_{ik} = \frac{1}{2} (\nabla_k u_i - \nabla_i u_k)$$

are the components of the *ordinary macrospin tensor*, and

$$\phi_{ik} = \frac{1}{2} i (\nabla_k \varphi_i - \nabla_i \varphi_k)$$

are the components of the microspin tensor describing rotation of material points on their own axes due to torsion, or, in the literature, the so-called *distributed moment*. At this point, it may be that the internal rotation of material points is analogous to the spin of electrons if the material point themselves are seen as charged point-particles. However, we know that electrons possess internal spin due to internal structural reasons while the material points also rotate partly due to *externally induced couple stress* giving rise to torsion. For this reason we split the components of the microspin φ into two parts:

$$\varphi_i = \phi_i + e A_i,$$

where ϕ_i describe non-electromagnetic microspin and $e A_i$ describe pure electron spin with e being the electric charge and A_i , up to a constant of proportionality, being the material components of the electromagnetic vector potential A :

$$A_i = q \omega_i^\mu A_\mu,$$

where q is a parametric constant and A_μ are the components of the four-dimensional electromagnetic vector potential in the sense of Maxwellian electrodynamics. Inversely, we have

$$A_\mu = \frac{1}{q} (\omega_\mu^i A_i + \in N n_\mu),$$

where $N = q n_\mu A^\mu$. The correspondence with classical electrodynamics becomes complete if we link the *electromagnetic microspin tensor* f represented by the components

$$f_{ik} = \frac{1}{2} i e (\nabla_k A_i - \nabla_i A_k)$$

to the electromagnetic field tensor $F = F_{\mu\nu} \omega^\mu \otimes \omega^\nu$ through

$$f_{ik} = \frac{\bar{\gamma}}{e} \omega_i^\mu \omega_k^\nu F_{\mu\nu},$$

where $\bar{\gamma} = \frac{1}{2} i q e^2$. The four-dimensional components of the electromagnetic field tensor in canonical form are

$$F_{\mu\nu} = \frac{\partial A_\mu}{\partial y^\nu} - \frac{\partial A_\nu}{\partial y^\mu} = \begin{pmatrix} 0 & -E^1 & -E^2 & -E^3 \\ E^1 & 0 & B^3 & B^2 \\ E^2 & -B^3 & 0 & B^1 \\ E^3 & B^2 & -B^1 & 0 \end{pmatrix},$$

where $E = (E^1, E^2, E^3)$ and $B = (B^1, B^2, B^3)$ are the electric and magnetic fields, respectively. In three-dimensional vector notation, $E = -\frac{1}{c} \frac{\partial \bar{A}}{\partial t} - \vec{\nabla} \phi$ and $B = \text{curl} \bar{A}$, where $\bar{A} = A^\mu \omega_\mu = (\vec{A}, \phi)$. They satisfy Maxwell’s equations in the Lorentz gauge $\text{div} \bar{A} = 0$, i.e.,

$$\frac{1}{c} \frac{\partial E}{\partial t} = \text{curl} B - \frac{4\pi}{c} j,$$

$$\text{div} E = -\nabla^2 \phi = 4\pi \rho_e,$$

$$\frac{1}{c} \frac{\partial B}{\partial t} = -\text{curl} E,$$

$$\text{div} B = 0,$$

where j is the electromagnetic current density vector and ρ_e is the electric charge density. In addition, we can write

$$\nabla_\nu F^{\mu\nu} = \frac{4\pi}{c} j^\mu,$$

i.e., $\nabla_\nu F^{A\nu} = \frac{4\pi}{c} j^A$ and $j^4 = \rho_e$. The inverse transformation relating f_{ik} to F_{ik} is then given by

$$F_{\mu\nu} = \frac{e}{\bar{\gamma}} \omega_\mu^i \omega_\nu^k f_{ik} + \hat{F}_{\mu\nu},$$

where

$$\hat{F}_{\mu\nu} = -\in (n_\mu F_{\nu\sigma} - n_\nu F_{\mu\sigma}) n^\sigma,$$

$$\hat{F}_{\mu\nu} n^\nu = \in^2 F_{\mu\nu} n^\nu = F_{\mu\nu} n^\nu.$$

This way, the components of the generalized vorticity tensor once again are

$$\begin{aligned}\omega_{ik} &= \frac{1}{2} (\nabla_k u_i - \nabla_i u_k) + \frac{1}{2} i (\nabla_k \phi_i - \nabla_i \phi_k) + \\ &+ \frac{1}{2} i e (\nabla_k A_i - \nabla_i A_k) = \\ &= \frac{1}{2} \left(\frac{\partial u_i}{\partial \xi^k} - \frac{\partial u_k}{\partial \xi^i} \right) + \frac{1}{2} i \left(\frac{\partial \phi_i}{\partial \xi^k} - \frac{\partial \phi_k}{\partial \xi^i} \right) + \\ &+ \frac{1}{2} i e \left(\frac{\partial A_i}{\partial \xi^k} - \frac{\partial A_k}{\partial \xi^i} \right) - \Gamma_{[ik]}^r (u_r + i \phi_r + i e A_r) = \\ &= \Omega_{ik} + \varpi_{ik} + \frac{\bar{\gamma}}{e} \omega_i^\mu \omega_k^\nu F_{\mu\nu},\end{aligned}$$

where

$$\varpi_{ik} = \frac{1}{2} i (\nabla_k \phi_i - \nabla_i \phi_k)$$

are the components of the non-electromagnetic microspin tensor. Thus we have now seen, in our generalized deformation analysis, how the microspin field is incorporated into the vorticity tensor.

Finally, we shall now produce some basic framework for equations of motion applicable to arbitrary tensor fields in terms of exterior derivatives. We define the exterior derivative of an arbitrary vector field (i.e., a rank-one tensor field) of \mathfrak{S}_3 , say W , with respect to the so-called *Cartan basis* as the totally anti-symmetric object

$$L_U W = 2U_{[i} W_{k]} g^i \otimes g^k,$$

where U is the velocity vector in the direction of motion of the material body \mathfrak{S}_3 , i.e., $U^i = \frac{\partial \psi^i}{\partial t}$. If we now take the local basis vectors as *directional derivatives*, i.e., the *Cartan coordinate basis vectors* $g_i = \frac{\partial}{\partial \xi^i} = \partial_i$ and $g^i = d\xi^i$, we obtain for instance, in component notation,

$$(L_U W)_i = L_U W_i = U^k \partial_k W_i + W_k \partial_i U^k.$$

Using the exterior product, we actually see that

$$L_U W = U \wedge W = U \otimes W - W \otimes U.$$

Correspondingly, for W^i , we have

$$(L_U W)^i = L_U W^i = U^k \partial_k W^i - W^k \partial_k U^i.$$

The *exterior material derivative* is then a direct generalization of the ordinary material derivative (e.g., as we know, for a scalar field ρ it is given by $\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial \xi^i} U^i$) as follows:

$$\begin{aligned}\frac{DW_i}{Dt} &= \frac{\partial W_i}{\partial t} + L_U W_i = \frac{\partial W_i}{\partial t} + (U \wedge V)_i = \frac{\partial W_i}{\partial t} + \\ &+ U^k \partial_k W_i + W_k \partial_i U^k,\end{aligned}$$

$$\begin{aligned}\frac{DW^i}{Dt} &= \frac{\partial W^i}{\partial t} + L_U W^i = \frac{\partial W^i}{\partial t} + (U \wedge V)^i = \frac{\partial W^i}{\partial t} + \\ &+ U^k \partial_k W^i - W^k \partial_i U^k.\end{aligned}$$

Finally, we obtain the generalized material derivative of the components of an arbitrary tensor field T of \mathfrak{S}_3 as

$$\begin{aligned}\frac{DT_{kl\dots}^{ij\dots}}{Dt} &= \frac{\partial T_{kl\dots}^{ij\dots}}{\partial t} + U^m \partial_m T_{kl\dots}^{ij\dots} + T_{ml\dots}^{ij\dots} \partial_k U^m + \\ &+ T_{km\dots}^{ij\dots} \partial_l U^m + \dots - T_{kl\dots}^{mj\dots} \partial_m U^i - T_{kl\dots}^{im\dots} \partial_m U^j - \dots,\end{aligned}$$

or alternatively as

$$\begin{aligned}\frac{DT_{kl\dots}^{ij\dots}}{Dt} &= \frac{\partial T_{kl\dots}^{ij\dots}}{\partial t} + U^m \nabla_m T_{kl\dots}^{ij\dots} + T_{ml\dots}^{ij\dots} \nabla_k U^m + \\ &+ T_{km\dots}^{ij\dots} \nabla_l U^m + \dots - T_{kl\dots}^{mj\dots} \nabla_m U^i - T_{kl\dots}^{im\dots} \nabla_m U^j - \\ &- \dots + 2\Gamma_{[kp]}^m T_{ml\dots}^{ij\dots} U^p + 2\Gamma_{[lp]}^m T_{kl\dots}^{ij\dots} U^p + \dots - \\ &- 2\Gamma_{[mp]}^i T_{kl\dots}^{mj\dots} U^p - 2\Gamma_{[mp]}^j T_{kl\dots}^{im\dots} U^p - \dots.\end{aligned}$$

Written more simply,

$$\frac{DT_{kl\dots}^{ij\dots}}{Dt} = \frac{\partial T_{kl\dots}^{ij\dots}}{\partial t} + L_U T_{kl\dots}^{ij\dots} = \frac{\partial T_{kl\dots}^{ij\dots}}{\partial t} + (U \wedge T)_{kl\dots}^{ij\dots}.$$

For a scalar field Θ , we have simply

$$\frac{D\Theta}{Dt} = \frac{\partial \Theta}{\partial t} + U^k \partial_k \Theta,$$

which is just the ordinary material derivative.

Now, with the help of the Cartan basis vectors, the torsion tensor can be expressed directly in terms of the permutation tensor as

$$\Gamma_{[jk]}^i = -\frac{1}{2} g^{ip} \epsilon_{pjk}.$$

Hence from the generalized material derivative for the components of the material metric tensor g (defined with respect to the Cartan basis), i.e.,

$$\frac{Dg_{ik}}{Dt} = \frac{\partial g_{ik}}{\partial t} + L_U g_{ik} = \frac{\partial g_{ik}}{\partial t} + (U \wedge g)_{ik},$$

we find especially that

$$\frac{Dg_{ik}}{Dt} = \frac{\partial g_{ik}}{\partial t} + \nabla_k U_i + \nabla_i U_k,$$

with the help of the metrical condition $\nabla_p g_{ik} = 0$. Similarly, we also find

$$\frac{Dg^{ik}}{Dt} = \frac{\partial g^{ik}}{\partial t} - (\nabla^k U^i + \nabla^i U^k).$$

Note also that

$$\frac{D\delta_k^i}{Dt} = 0.$$

The components of the velocity gradient tensor are given by

$$L_{ik} = \nabla_k U_i = \frac{1}{2} (\nabla_k U_i + \nabla_i U_k) + \frac{1}{2} (\nabla_k U_i - \nabla_i U_k),$$

where, following the so-called Helmholtz decomposition theorem, we can write

$$U_i = \nabla_i \alpha + \frac{1}{2} g_{il} \in^{ljk} (\nabla_j \beta_k - \nabla_k \beta_j),$$

for a scalar field α and a vector field β . However, note that in our case we obtain the following generalized identities:

$$\begin{aligned} \operatorname{div} \operatorname{curl} U &= -\frac{1}{2} \in^{ijk} \left(R^l{}_{kij} U_l - 2\Gamma^l{}_{[ij]} \nabla_l A_k \right), \\ \operatorname{curl} \operatorname{grad} \alpha &= \in^{ijk} \Gamma^l{}_{[ij]} \nabla_l \alpha, \end{aligned}$$

which must hold throughout unless a constraint is invoked. We now define the generalized shear scalar by

$$\begin{aligned} \theta &= \nabla_i U^i = \nabla_i \nabla^i \alpha + \frac{1}{2} \in^{ijk} (\nabla_i \nabla_j - \nabla_j \nabla_i) \beta_k = \\ &= \nabla^2 \alpha - \frac{1}{2} \in^{ijk} R^l{}_{kij} \beta_l + \in^{ijk} \Gamma^l{}_{[ij]} \nabla_l \beta_k. \end{aligned}$$

In other words, the shear now depends on the *microspin field* generated by curvature and torsion tensors.

Meanwhile, we see that the “contravariant” components of the *local acceleration vector* will simply be given by

$$\begin{aligned} a^i &= \frac{DU^i}{Dt} = \frac{\partial U^i}{\partial t} + U^k \partial_k U^i - U^k \partial_k U^i = \\ &= \frac{\partial U^i}{\partial t}. \end{aligned}$$

However, we also have

$$a_i = \frac{DU_i}{Dt} = \frac{\partial U_i}{\partial t} + (\nabla_k U_i + \nabla_i U_k) U^k,$$

for the “covariant” components.

Furthermore, we have

$$a_i = \frac{\partial U_i}{\partial t} + \left(\frac{Dg_{ik}}{Dt} - \frac{\partial g_{ik}}{\partial t} \right) U^k.$$

Now, define the *local acceleration covector* through

$$\begin{aligned} \hat{a}^i &= g^{ik} a_i = \\ &= g^{ik} \frac{\partial U_k}{\partial t} + (\nabla_k U^i + \nabla^i U_k) U^k = \\ &= \frac{\partial U^i}{\partial t} - U_k \frac{Dg^{ik}}{Dt}, \end{aligned}$$

such that we have

$$a^i - \hat{a}^i = U_k \frac{Dg^{ik}}{Dt}.$$

Hence we see that the sufficient condition for the two local acceleration vectors to coincide is

$$\frac{Dg_{ik}}{Dt} = 0.$$

In other words, in such a situation we have

$$\frac{\partial g_{ik}}{\partial t} = -(\nabla_k U_i + \nabla_i U_k).$$

In this case, a purely rotational motion is obtained only when the material motion is *rigid*, i.e., when $\frac{\partial g_{ik}}{\partial t} = 0$ or, in other words, when the condition

$$L_U g_{ik} = L_{(ik)} = \nabla_k U_i + \nabla_i U_k = 0$$

is satisfied identically. Similarly, a purely translational motion is obtained when $L_{[ik]} = 0$, which describes a potential motion, where we have $U_i = \nabla_i \alpha$. However, as we have seen, in the presence of torsion even any potential motion of this kind is still obviously path-dependent as the relations $\in^{ijk} \Gamma^l{}_{[ij]} \nabla_l \alpha \neq 0$ hold in general.

We now consider the path-dependent displacement field δ tracing a loop ℓ , say, from point P_1 to point P_2 in \mathfrak{R}_4 with components:

$$\delta^i = \oint_{P_1-P_2} d\psi^i = \oint_{P_1-P_2} (\varepsilon^i{}_{.k} + \omega^i{}_{.k} - \psi^l \Gamma^i{}_{lk}) d\xi^k.$$

Let us observe that

$$\begin{aligned} \psi^k \Gamma^i{}_{kl} &= \psi^k \gamma^i_A \frac{\partial \gamma^A_k}{\partial \xi^l} = -\psi^k \gamma^A_k \frac{\partial \gamma^i_A}{\partial \xi^l} = \\ &= -\psi^A \frac{\partial \gamma^i_A}{\partial \xi^l} = -\left(\frac{\partial}{\partial \xi^l} (\gamma^i_A \psi^A) - \gamma^i_A \frac{\partial \psi^A}{\partial \xi^l} \right) = \\ &= \gamma^i_A \frac{\partial \psi^A}{\partial \xi^l} - \frac{\partial \psi^i}{\partial \xi^l}. \end{aligned}$$

Now since $\psi^i = \delta \xi^i$, and using $\frac{\partial \delta f}{\partial \xi^i} = \delta \frac{\partial f}{\partial \xi^i}$ for an arbitrary function f , we have

$$\psi^k \Gamma^i{}_{kl} = \gamma^i_A \delta \left(\frac{\partial x^A}{\partial x^B} \right) \gamma^B - \delta \left(\frac{\partial \xi^i}{\partial \xi^l} \right) = 0,$$

and we are left with

$$\delta^i = \oint_{P_1-P_2} d\psi^i = \oint_{P_1-P_2} (\varepsilon^i{}_{.k} + \omega^i{}_{.k}) d\xi^k.$$

Assuming that the ε_{ik} are continuous, we can now derive the following relations:

$$D\psi^i = \nabla_k \psi^i d\xi^k = \frac{\partial \psi^i}{\partial \xi^k} d\xi^k,$$

$$\nabla_l \omega^i{}_{.k} d\xi^k = (\nabla_l \varepsilon^i{}_{.k} - \nabla^i \varepsilon_{lk}) d\xi^k = \left(\frac{\partial \varepsilon^i{}_{.k}}{\partial \xi^l} - \frac{\partial \varepsilon_{lk}}{\partial \xi^i} \right) d\xi^k.$$

With the help of the above relations and by direct partial integration, we then have

$$\delta^i = \oint_{P_1-P_2} d\psi^i = \omega^i{}_{.k} \xi^k \Big|_{P_1}^{P_2} - \oint_{P_1-P_2} (\nabla_l \psi^i{}_{.k} - \nabla_k \psi^i{}_{.l}) \delta \xi^k d\xi^l,$$

where

$$\psi^i_{,k} = \varepsilon^i_{,k} - \xi^j (\nabla_j \varepsilon^i_{,k} - \nabla^i \varepsilon_{jk}) .$$

It can be seen that

$$\nabla_i \psi^i_{,k} - \nabla_k \psi^i_{,l} = -Z^i_{,jkl} \xi^j ,$$

where we have defined another non-holonomic tensor Z with the components

$$Z^i_{,jkl} = \nabla_l \nabla_j \varepsilon^i_{,k} + \nabla_k \nabla^i \varepsilon_{jl} - \nabla_k \nabla_j \varepsilon^i_{,l} - \nabla_l \nabla^i \varepsilon_{jk} .$$

Now, the linearized components of the Riemann-Christoffel curvature tensor are given by

$$\tilde{R}_{ijkl} \stackrel{lin}{=} \frac{1}{2} \left(\frac{\partial^2 g_{il}}{\partial \xi^k \partial \xi^j} + \frac{\partial^2 g_{jk}}{\partial \xi^l \partial \xi^i} - \frac{\partial^2 g_{ik}}{\partial \xi^l \partial \xi^j} - \frac{\partial^2 g_{jl}}{\partial \xi^k \partial \xi^i} \right) .$$

Direct calculation gives

$$\delta_\psi \tilde{R}_{ijkl} \stackrel{lin}{=} \frac{1}{2} (\nabla_k \nabla_j \delta_\psi g_{il} + \nabla_l \nabla_i \delta_\psi g_{jk} - \nabla_l \nabla_j \delta_\psi g_{ik} - \nabla_k \nabla_i \delta_\psi g_{jl}) .$$

However, $\delta_\psi g_{ik} = \varepsilon_{ik}$, and hence we obtain

$$\delta_\psi \tilde{R}_{ijkl} \stackrel{lin}{=} \frac{1}{2} (\nabla_k \nabla_j \varepsilon_{il} + \nabla_l \nabla_i \varepsilon_{jk} - \nabla_l \nabla_j \varepsilon_{ik} - \nabla_k \nabla_i \varepsilon_{jl}) .$$

In other words,

$$Z_{ijkl} \stackrel{lin}{=} -2\delta_\psi \tilde{R}_{ijkl} .$$

Obviously the Z_{ijkl} possess almost the same fundamental symmetries as the components of the Riemann-Christoffel curvature tensor, i.e., $Z_{ijkl} = -Z_{jikl} = -Z_{ijlk}$ as well as the general asymmetry $Z_{ijkl} \neq Z_{klij}$ as

$$\begin{aligned} Z_{ijkl} - Z_{klij} &= (R^r_{,ijl} + R^r_{,jli}) \varepsilon_{rk} + (R^r_{,klj} + R^r_{,lkj}) \varepsilon_{ir} + \\ &+ (R^r_{,jik} + R^r_{,ikj}) \varepsilon_{rl} + (R^r_{,ilk} + R^r_{,kli}) \varepsilon_{jr} - \\ &- 2 \left(\Gamma^r_{[jl]} \nabla_r \varepsilon_{ik} + \Gamma^r_{[ik]} \nabla_r \varepsilon_{jl} + \Gamma^r_{[kj]} \nabla_r \varepsilon_{il} + \right. \\ &\left. + \Gamma^r_{[li]} \nabla_r \varepsilon_{jk} \right) . \end{aligned}$$

When the tensor Z vanishes we have, of course, a set of integrable equations giving rise to the integrability condition for the components of the strain tensor, which is equivalent to the vanishing of the field δ . That is, to the first order in the components of the strain tensor, if the condition

$$\delta_\psi \tilde{R}_{ijkl} = 0$$

is satisfied identically.

Finally, we can write (still to the first order in the components of the strain tensor)

$$\begin{aligned} \delta^i &= \oint_{P_1-P_2} d\psi^i = (\omega^i_{,k} \xi^k) \Big|_{P_1}^{P_2} + \frac{1}{2} \oint_{P_1-P_2} Z^i_{,jkl} \xi^j dS^{kl} = \\ &= (\omega^i_{,k} \xi^k) \Big|_{P_1}^{P_2} - \oint_{P_1-P_2} \delta_\psi \tilde{R}^i_{,jkl} \xi^j dS^{kl} , \end{aligned}$$

where

$$dS^{ik} = d\xi^i \delta \xi^k - d\xi^k \delta \xi^i$$

are the components of an infinitesimal closed surface in \mathfrak{S}_3 spanned by the displacements $d\xi$ and $\delta \xi$ in 2 preferred directions.

Ending this section, let us give further in-depth investigation of the local translational-rotational motion of points on the material body. Define the unit velocity vector by

$$\hat{U}^i = \frac{\zeta_4^i}{\sqrt{g_{kl} \zeta_4^k \zeta_4^l}} = \frac{d\xi^i}{ds} ,$$

such that

$$\zeta_4^i = \frac{\partial \xi^i}{\partial t} = (g_{kl} \zeta_4^k \zeta_4^l)^{1/2} \frac{d\xi^i}{ds} ,$$

i.e.,

$$ds = (g_{ik} \zeta_4^i \zeta_4^k)^{1/2} \frac{\partial t}{\partial \xi^l} d\xi^l = (U_i U^i)^{1/2} dt = U dt .$$

Then the local equations of motion along arbitrary curves on the hypersurface of material coordinates $\mathfrak{S}_3 \subset \mathfrak{R}_4$ can be described by the quadruplet of unit space-time vectors $(\hat{U}, \hat{V}, \hat{W}, n) \in n$ orthogonal to each other where the first three unit vectors (i.e., $\hat{U}, \hat{V}, \hat{W}$) are exclusively defined as local tangent vectors in the hypersurface \mathfrak{S}_3 and n is the unit normal vector to the hypersurface \mathfrak{S}_3 . These equations of motion are derived by generalizing the ordinary Frenet equations of orientable points of a curve in three-dimensional Euclidean space to four-dimensions as well as to include effects of microspin generated by geometric torsion. Setting

$$\hat{U} = u^\mu \omega_\mu = \hat{U}^i g_i ,$$

$$\hat{V} = v^\mu \omega_\mu = \hat{V}^i g_i ,$$

$$\hat{W} = w^\mu \omega_\mu = \hat{W}^i g_i ,$$

$$n = n^\mu \omega_\mu ,$$

we obtain, in general, the following set of equations of motion of the material points on the material body:

$$\frac{\delta u^\mu}{\delta s} = k v^\mu ,$$

$$\frac{\delta v^\mu}{\delta s} = \tau w^\mu - k u^\mu ,$$

$$\frac{\delta w^\mu}{\delta s} = \tau v^\mu + \lambda n^\mu ,$$

$$\frac{\delta n^\mu}{\delta s} = \lambda w^\mu,$$

where the operator

$$\frac{\delta}{\delta s} = \hat{U}^i \nabla_i = u^\mu \nabla_\mu$$

represents the *absolute covariant derivative* in $\mathfrak{S}_3 \subset \mathfrak{R}_4$. In the above equations we have defined the following invariants:

$$k = \left(G_{\mu\nu} \frac{\delta u^\mu}{\delta s} \frac{\delta u^\nu}{\delta s} \right)^{1/2} = \left(g_{ik} \frac{\delta \hat{U}^i}{\delta s} \frac{\delta \hat{U}^k}{\delta s} \right)^{1/2},$$

$$\tau = \epsilon_{\mu\nu\rho\sigma} u^\mu v^\nu \frac{\delta v^\rho}{\delta s} n^\sigma = \epsilon_{ijkl} \hat{U}^i \hat{V}^j \frac{\delta \hat{V}^k}{\delta s},$$

$$\lambda = \left(G_{\mu\nu} \frac{\delta n^\mu}{\delta s} \frac{\delta n^\nu}{\delta s} \right)^{1/2}.$$

In our case, however, the vanishing of the extrinsic curvature of the hypersurface \mathfrak{S}_3 means that the direction of the unit normal vector n is fixed. Consequently, we have

$$\lambda = 0,$$

and our equations of motion can be written as

$$\hat{U}^k \check{\nabla}_k \hat{U}^i = k \hat{V}^i - T_{kl}^i \hat{U}^k \hat{U}^l,$$

$$\hat{U}^k \check{\nabla}_k \hat{V}^i = \tau \hat{W}^i - k \hat{U}^i - T_{kl}^i \hat{V}^k \hat{U}^l,$$

$$\hat{U}^k \check{\nabla}_k \hat{W}^i = \tau \hat{V}^i - T_{kl}^i \hat{W}^k \hat{U}^l$$

in three-dimensional notation. In particular, we note that, just as the components of the contorsion tensor T_{jk}^i , the scalar τ measures the twist of any given curve in \mathfrak{S}_3 due to microspin.

Furthermore, it can be shown that the gradient of the unit velocity vector can be decomposed accordingly as

$$\nabla_k \hat{U}_i = \alpha_{ik} + \beta_{ik} + \frac{1}{4} h_{ik} \hat{\theta} + \hat{U}_k \hat{A}_i,$$

where

$$h_{ik} = g_{ik} - \hat{U}_i \hat{U}_k,$$

$$\alpha_{ik} = \frac{1}{4} h_i^r h_k^s (\nabla_r \hat{U}_s + \nabla_s \hat{U}_r) = \frac{1}{4} h_i^r h_k^s (\check{\nabla}_r \hat{U}_s + \check{\nabla}_s \hat{U}_r) - \frac{1}{2} h_i^r h_k^s T_{[rs]}^l \hat{U}_l,$$

$$\beta_{ik} = \frac{1}{4} h_i^r h_k^s (\nabla_r \hat{U}_s - \nabla_s \hat{U}_r) = \frac{1}{4} h_i^r h_k^s (\check{\nabla}_r \hat{U}_s - \check{\nabla}_s \hat{U}_r) - \frac{1}{2} h_i^r h_k^s T_{[rs]}^l \hat{U}_l,$$

$$\hat{\theta} = \nabla_i \hat{U}^i,$$

$$\hat{A}_i = \frac{\delta \hat{U}_i}{\delta s}.$$

Note that

$$h_{ik} \hat{U}^k = \alpha_{ik} \hat{U}^k = \beta_{ik} \hat{U}^k = 0.$$

Setting $\bar{\lambda} = (g_{ik} \zeta_4^i \zeta_4^k)^{-1/2}$ such that $\hat{U}^i = \bar{\lambda} U^i$, we obtain in general

$$\begin{aligned} \bar{\lambda} \nabla_k U_i &= \frac{1}{4} \bar{\lambda} h_i^r h_k^s (\nabla_r U_s + \nabla_s U_r) + \frac{1}{4} \bar{\lambda} h_i^r h_k^s (\nabla_r U_s - \nabla_s U_r) \\ &+ \frac{1}{2} \bar{\lambda} \nabla_i U_k + \frac{1}{4} g_{ik} \frac{\delta \bar{\lambda}}{\delta s} + \frac{1}{4} \bar{\lambda} g_{ik} \nabla_l U^l + \bar{\lambda} U_i U_k \frac{\delta \bar{\lambda}}{\delta s} + \bar{\lambda}^2 U_k \frac{\delta U_i}{\delta s} - \frac{1}{4} \bar{\lambda}^2 U_i U_k \frac{\delta \bar{\lambda}}{\delta s} - \\ &- \frac{1}{2} \bar{\lambda}^3 U_i \frac{\delta U_k}{\delta s} - \frac{1}{4} \bar{\lambda}^3 U_i U_k \nabla_l U^l. \end{aligned}$$

Again, the vanishing of the extrinsic curvature of the hypersurface \mathfrak{S}_3 gives $\frac{\delta \bar{\lambda}}{\delta s} = 0$. Hence we have

$$\begin{aligned} \nabla_k U_i &= \frac{1}{4} h_i^r h_k^s (\nabla_r U_s + \nabla_s U_r) + \frac{1}{4} h_i^r h_k^s (\nabla_r U_s - \nabla_s U_r) + \\ &+ \frac{1}{2} \nabla_i U_k + \frac{1}{4} g_{ik} \nabla_l U^l + \bar{\lambda} U_k \frac{\delta U_i}{\delta s} - \frac{1}{2} \bar{\lambda}^2 U_i \frac{\delta U_k}{\delta s} - \\ &- \frac{1}{4} \bar{\lambda}^2 U_i U_k \nabla_l U^l, \end{aligned}$$

for the components of the velocity gradient tensor.

Meanwhile, with the help of the identities

$$\begin{aligned} \hat{U}^j \nabla_k \nabla_j \hat{U}_i &= \nabla_k (\hat{U}^j \nabla_j \hat{U}_i) - (\nabla_k \hat{U}_j) (\nabla^j \hat{U}_i) = \\ &= \nabla_k \hat{A}_i - (\nabla_k \hat{U}_j) (\nabla^j \hat{U}_i), \end{aligned}$$

$$\hat{U}^j (\nabla_k \nabla_j - \nabla_j \nabla_k) \hat{U}_i = R_{.ijk}^l \hat{U}_l \hat{U}^j - 2\Gamma_{[jk]}^l \hat{U}^j \nabla_l \hat{U}_i,$$

we can derive the following equation:

$$\begin{aligned} \frac{\delta \hat{\theta}}{\delta s} &= \nabla_i \left(\frac{\delta \hat{U}^i}{\delta s} \right) - (\nabla_i \hat{U}^k) (\nabla_k \hat{U}^i) - R_{ik} \hat{U}^i \hat{U}^k + \\ &+ 2\Gamma_{[ik]}^l \hat{U}^i \nabla_l \hat{U}^k. \end{aligned}$$

Hence we obtain

$$\begin{aligned} \frac{\delta \theta}{\delta s} &= \nabla_i \left(\frac{\delta U^i}{\delta s} \right) - \bar{\lambda} (\nabla_i U^k) (\nabla_k U^i) - 2 \frac{\delta U^i}{\delta s} \nabla_i (e \log \lambda) - \\ &- \bar{\lambda} R_{ik} U^i U^k + 2 \bar{\lambda} \Gamma_{[ik]}^l U^i \nabla_l U^k, \end{aligned}$$

for the rate of shear with respect to the local arc length of the material body.

4 Generalized components of the elasticity tensor of the material body \mathfrak{S}_3 in the presence of microspin and geometric discontinuities (defects)

As we know, the most general form of the components of a fourth-rank isotropic tensor is given in terms of spatial coordinates by

$$I_{ABCD} = C_1 \delta_{AB} \delta_{CD} + C_2 \delta_{AC} \delta_{BD} + C_3 \delta_{AD} \delta_{BC},$$

where C_1, C_2 , and C_3 are constants. In the case of anisotropy, C_1, C_2 , and C_3 are no longer constant but still remain invariant with respect to the change of the coordinate system. Transforming these to material coordinates, we have

$$I_{..kl}^{ij} = C_1 g^{ij} g_{kl} + C_2 \delta_k^i \delta_l^j + C_3 \delta_l^i \delta_k^j.$$

On reasonably relaxing the ordinary symmetries, we now generalize the components of the fourth-rank elasticity tensor with the addition of a geometrized part describing microspin and geometric discontinuities as follows:

$$C_{..kl}^{ij} = \alpha g^{ij} g_{kl} + \beta \left(\delta_k^i \delta_l^j + \delta_l^i \delta_k^j \right) + \gamma \left(\delta_k^i \delta_l^j - \delta_l^i \delta_k^j \right),$$

where

$$\begin{aligned} \alpha &= \frac{2}{15} A_{..i.k}^{ik} - \frac{1}{15} A_{..i.k}^{ki}, \\ \beta &= \frac{1}{10} A_{..ik}^{ki} - \frac{1}{10} A_{..i.k}^{ik}, \\ \gamma &= \frac{1}{2} \eta \tilde{R}, \end{aligned}$$

where η is a non-zero constant, and where

$$A_{..kl}^{ij} = \alpha g^{ij} g_{kl} + \beta \left(\delta_k^i \delta_l^j + \delta_l^i \delta_k^j \right)$$

are, of course, the components of the ordinary, non-microspin (non-micropolar) elasticity tensor obeying the symmetries $A_{..kl}^{ij} = A_{..kl}^{ji} = A_{..lk}^{ij} = A_{kl..}^{ij}$. Now if we define the remaining components by

$$B_{..kl}^{ij} = \frac{1}{2} \eta \left(\delta_k^i \delta_l^j - \delta_l^i \delta_k^j \right) \tilde{R},$$

with $B_{..kl}^{ij} = -B_{..kl}^{ji} = -B_{..lk}^{ij} = B_{kl..}^{ij}$, then we have relaxed the ordinary symmetries of the elasticity tensor. Most importantly, we note that our choice of the Ricci curvature scalar \tilde{R} (rather than the more general curvature scalar R of which \tilde{R} is a component) to enter our generalized elasticity tensor is meant to accommodate very general situations such that in the absence of geometric discontinuities the above equations will in general still hold. This corresponds to the fact that the existence of the Ricci curvature tensor \tilde{R} is primarily due to microspin while geometric discontinuities are described by the full curvature tensor R as we have seen in Section 2.

Now with the help of the decomposition of the Riemann-Christoffel curvature tensor, we obtain

$$\begin{aligned} C_{..kl}^{ij} &= \alpha g^{ij} g_{kl} + \beta \left(\delta_k^i \delta_l^j + \delta_l^i \delta_k^j \right) + \eta \left(\tilde{W}_{..kl}^{ij} + \delta_k^i \tilde{R}_{..l}^j + \right. \\ &\quad \left. + \delta_l^j \tilde{R}_{..k}^i - \delta_l^i \tilde{R}_{..k}^j - \delta_k^j \tilde{R}_{..l}^i - \tilde{R}_{..kl}^{ij} \right) \end{aligned}$$

for the components of the generalized elasticity tensor. Hence for linear elastic continua/discontinua, with the help of the potential energy functional \bar{F} , i.e., the one given by

$$\bar{F} = \frac{1}{2} C_{ij..}^{kl} D^{ij} D_{kl},$$

such that

$$\sigma_{ij} = \frac{\partial \bar{F}}{\partial D^{ij}},$$

i.e.,

$$\sigma_{(ij)} = \frac{\partial \bar{F}}{\partial \varepsilon^{ij}},$$

$$\sigma_{[ij]} = \frac{\partial \bar{F}}{\partial \omega^{ij}},$$

we obtain the following constitutive relations:

$$\sigma_{ij} = C_{ij..}^{kl} D_{kl},$$

relating the components of the stress tensor σ to the components of the displacement gradient tensor D . Then it follows, as we have expected, that the stress tensor becomes asymmetric. Since $B_{(ij)..}^{kl} = 0$, we obtain

$$\sigma_{(ij)} = C_{(ij)..}^{kl} D_{kl} = A_{ij..}^{kl} \varepsilon_{kl} = \alpha g_{ij} \varepsilon_{..k}^k + 2\beta \varepsilon_{ij},$$

for the components of the symmetric part of the stress tensor, in terms of the components strain tensor and the dilation scalar $\kappa = \varepsilon_{..i}^i$. Correspondingly, since $A_{[ij]..}^{kl} = 0$, the components of the anti-symmetric part of the stress tensor are then given by

$$\begin{aligned} \sigma_{[ij]} &= C_{[ij]..}^{kl} D_{kl} = B_{ij..}^{kl} \omega_{kl} = \\ &= \eta \left(\tilde{W}_{ij..}^{kl} - \tilde{R}_{ij..}^{kl} \right) + \eta \left(D_{..i}^k \tilde{R}_{jk} + D_{..j}^k \tilde{R}_{ik} - \right. \\ &\quad \left. - D_{..i}^k \tilde{R}_{jk} - D_{..j}^k \tilde{R}_{ik} \right) = \\ &= \eta \left(\tilde{W}_{ij..}^{kl} - \tilde{R}_{ij..}^{kl} \right) \omega_{kl} + 2\eta \left(\omega_{..i}^k \tilde{R}_{jk} - \omega_{..j}^k \tilde{R}_{ik} \right) = \\ &= \eta \omega_{ij} \tilde{R}, \end{aligned}$$

in terms of the components of the generalized vorticity tensor. We can now define the *geometrized microspin potential* by the scalar

$$S = \eta \tilde{R} = \eta \left(R + 2\tilde{\nabla}_i \omega^i + \omega_i \omega^i + T_{ijk} T^{ikj} \right).$$

Then, more specifically, we write

$$\sigma_{[ij]} = S \left(\Omega_{ij} + \varpi_{ij} + \frac{\tilde{\gamma}}{\varepsilon} \omega_i^\mu \omega_j^\nu F_{\mu\nu} \right).$$

From the above relations, we see that when the electromagnetic contribution vanishes, we arrive at a *geometrized Cosserat elasticity theory*. As we know, the standard Cosserat elasticity theory does not consider effects generated by the electromagnetic field. Various continuum theories which can be described as conservative theories often take into consideration electrostatic phenomena since the electric field is simply described by a gradient of a scalar potential which corresponds to their conservative description of force and stress. But that proves to be a limitation especially because magnetic effects are still neglected.

As usual, should we consider thermal effects, then we would define the components of the thermal stress t by

$$t_{ik} = -\mu_T g_{ik} \Delta T,$$

where μ_T is the thermal coefficient and ΔT is the temperature increment. Hence the components of the generalized stress tensor become

$$\sigma_{ik} = C_{ik..}^{rs} D_{rs} - \mu_T g_{ik} \Delta T.$$

Setting now

$$\bar{\mu}_T = -\frac{1}{3} \frac{\mu_T}{(\alpha + 2\beta)},$$

we can alternatively write

$$\sigma_{ik} = C_{ik..}^{rs} (D_{rs} + \bar{\mu}_T g_{rs} \Delta T).$$

Finally, we shall obtain

$$\sigma_{ik} = (\alpha \varepsilon_{,r}^r - \mu_T \Delta T) g_{ik} + 2\beta \varepsilon_{ik} + \eta \omega_{ik} \tilde{R},$$

i.e., as before

$$\sigma_{(ij)} = (\alpha \varepsilon_{,r}^r - \mu_T \Delta T) g_{ij} + 2\beta \varepsilon_{ij},$$

$$\sigma_{[ij]} = \eta \omega_{ij} \tilde{R} = \frac{1}{2} \eta \varepsilon_{ijk} S^k \tilde{R},$$

where S^k are the components of the generalized vorticity vector S .

We note that, as is customary, in order to accord with the standard physical description of continuum mechanics, we need to set

$$\alpha = G = \frac{E}{2(1+\nu)},$$

$$\beta = G \left(\frac{2\nu}{1-2\nu} \right),$$

where G is the shear modulus, E is Young's modulus, and ν is Poisson's ratio.

Extending the above description, we shall have a glimpse into the more general non-linear constitutive relations given by

$$\sigma_{ij} = C_{ij..}^{kl} D_{kl} + K_{ij..mn}^{kl} D_{kl} D^{mn} + \dots,$$

where the dots represent terms of higher order. Or, up to the second order in the displacement gradient tensor, we have

$$\sigma_{ij} = C_{ij..}^{kl} D_{kl} + K_{ij..mn}^{kl} D_{kl} D^{mn}.$$

Here the K_{ijklmn} are the components of the sixth-rank, isotropic, non-linear elasticity tensor whose most general form appears to be given by

$$\begin{aligned} K_{ijklmn} = & A_1 g_{ij} g_{kl} g_{mn} + A_2 g_{ij} g_{km} g_{nl} + A_3 g_{ij} g_{kn} g_{lm} + \\ & + A_4 g_{kl} g_{im} g_{jn} + A_5 g_{kl} g_{in} g_{jm} + A_6 g_{mn} g_{ik} g_{jl} + \\ & + A_7 g_{mn} g_{il} g_{jk} + A_8 g_{im} g_{jk} g_{nl} + A_9 g_{im} g_{jl} g_{kn} + \\ & + A_{10} g_{in} g_{jk} g_{lm} + A_{11} g_{in} g_{jl} g_{km} + A_{12} g_{jm} g_{ik} g_{nl} + \\ & + A_{13} g_{jm} g_{il} g_{kn} + A_{14} g_{jn} g_{ik} g_{lm} + A_{15} g_{jn} g_{il} g_{km}, \end{aligned}$$

where A_1, A_2, \dots, A_{15} are invariants. In a similar manner as in the generalized linear case, we shall call the following symmetries:

$$K_{ijklmn} = K_{klijmn} = K_{klmni j} = K_{mni jkl}.$$

Hence, we can bring the K_{ijklmn} into the form

$$\begin{aligned} K_{ijklmn} = & B_1 g_{ij} g_{kl} g_{mn} + B_2 g_{ij} (g_{km} g_{nl} + g_{kn} g_{lm}) + \\ & + B_3 g_{ij} (g_{km} g_{nl} - g_{kn} g_{lm}) + B_4 g_{kl} (g_{im} g_{jn} + \\ & + g_{in} g_{jm}) + B_5 g_{kl} (g_{im} g_{jn} - g_{in} g_{jm}) + \\ & + B_6 g_{mn} (g_{ik} g_{jl} + g_{il} g_{jk}) + B_7 g_{mn} (g_{ik} g_{jl} - \\ & - g_{il} g_{jk}) + B_8 g_{im} (g_{jk} g_{nl} + g_{jl} g_{kn}) + \\ & + B_9 g_{im} (g_{jk} g_{nl} - g_{jl} g_{kn}) + B_{10} g_{jm} (g_{ik} g_{nl} + \\ & + g_{il} g_{kn}) + B_{11} g_{jm} (g_{ik} g_{nl} - g_{il} g_{kn}), \end{aligned}$$

where, again, B_1, B_2, \dots, B_{11} are invariants. As in the generalized linear case, relating the coefficients B_3, B_5, B_7, B_9 , and B_{11} to the generator of microspin in our theory, i.e., the Riemann-Christoffel curvature tensor, we obtain

$$\begin{aligned} K_{ijklmn} = & \lambda_1 g_{ij} g_{kl} g_{mn} + \lambda_2 g_{ij} (g_{km} g_{nl} + g_{kn} g_{lm}) + \\ & + \lambda_3 g_{kl} (g_{im} g_{jn} + g_{in} g_{jm}) + \lambda_4 g_{mn} (g_{ik} g_{jl} + \\ & + g_{il} g_{jk}) + \lambda_5 g_{im} (g_{jk} g_{nl} + g_{jl} g_{kn}) + \\ & + \lambda_6 g_{jm} (g_{ik} g_{nl} + g_{il} g_{kn}) + \frac{1}{2} \kappa_1 g_{ij} (g_{km} g_{nl} - \\ & - g_{kn} g_{lm}) \tilde{R} + \frac{1}{2} \kappa_2 g_{kl} (g_{im} g_{jn} - g_{in} g_{jm}) \tilde{R} + \\ & + \frac{1}{2} \kappa_3 g_{mn} (g_{ik} g_{jl} - g_{il} g_{jk}) \tilde{R} + \frac{1}{2} \kappa_4 g_{im} (g_{jk} g_{nl} - \\ & - g_{jl} g_{kn}) \tilde{R} + \frac{1}{2} \kappa_5 g_{jm} (g_{ik} g_{nl} - g_{il} g_{kn}) \tilde{R}, \end{aligned}$$

where we have set $B_1 = \lambda_1, B_2 = \lambda_2, B_4 = \lambda_3, B_6 = \lambda_4, B_8 = \lambda_5, B_{10} = \lambda_6$ and where, for constant $\kappa_1, \kappa_2, \dots, \kappa_5$, the five quantities

$$K_1 = B_3 = \frac{1}{2} \kappa_1 \tilde{R},$$

$$K_2 = B_5 = \frac{1}{2} \kappa_2 \tilde{R},$$

$$K_3 = B_7 = \frac{1}{2} \kappa_3 \tilde{R},$$

$$K_4 = B_9 = \frac{1}{2} \kappa_4 \tilde{R},$$

$$K_5 = B_{11} = \frac{1}{2} \kappa_5 \tilde{R}$$

form a set of *additional microspin potentials*. Hence we see that in the non-linear case, at least there are in general six microspin potentials instead of just one as in the linear case.

Then the constitutive equations are readily derivable by means of the third-order potential functional

$$* \bar{F} = \frac{1}{2} C_{ij..}^{kl} D^{ij} D_{kl} + \frac{1}{3} K_{ij..mn}^{kl} D^{ij} D_{kl} D^{mn}$$

through

$$\sigma_{ij} = \frac{\partial * \bar{F}}{\partial D^{ij}} = {}^{(1)}\sigma_{ij} + {}^{(2)}\sigma_{ij},$$

where (1) indicates the linear part and (2) indicates the non-linear part. Note that this is true whenever the K_{ijklmn} in general possess the above mentioned symmetries. Direct, but somewhat lengthy, calculation gives

$$\begin{aligned} {}^{(2)}\sigma_{ij} &= K_{ij..mn}^{kl} D_{kl} D^{mn} = \\ &= \left(\lambda_1 (\varepsilon_{.k}^k)^2 + 2\lambda_2 \varepsilon_{kl} \varepsilon^{kl} + \kappa_1 \omega_{kl} \omega^{kl} \tilde{R} \right) g_{ij} + \\ &+ \varepsilon_{.k}^k \left((2\lambda_3 + 2\lambda_4) \varepsilon_{ij} + (\kappa_2 + \kappa_3) \omega_{ij} \tilde{R} \right) + \\ &+ D_{i.}^k \left(2\lambda_5 \varepsilon_{jk} + \kappa_4 \omega_{jk} \tilde{R} \right) + D_{j.}^k \left(2\lambda_6 \varepsilon_{ik} + \kappa_5 \omega_{ik} \tilde{R} \right). \end{aligned}$$

Overall, we obtain, for the components of the stress tensor, the following:

$$\begin{aligned} \sigma_{ij} &= (\alpha \varepsilon_{.k}^k - \mu_T \Delta T) g_{ij} + 2\beta \varepsilon_{ij} + \eta \omega_{ij} \tilde{R} + \\ &+ \left(\lambda_1 (\varepsilon_{.k}^k)^2 + 2\lambda_2 \varepsilon_{kl} \varepsilon^{kl} + \kappa_1 \omega_{kl} \omega^{kl} \tilde{R} \right) g_{ij} + \\ &+ \varepsilon_{.k}^k \left((2\lambda_3 + 2\lambda_4) \varepsilon_{ij} + (\kappa_2 + \kappa_3) \omega_{ij} \tilde{R} \right) + \\ &+ D_{i.}^k \left(2\lambda_5 \varepsilon_{jk} + \kappa_4 \omega_{jk} \tilde{R} \right) + D_{j.}^k \left(2\lambda_6 \varepsilon_{ik} + \kappa_5 \omega_{ik} \tilde{R} \right). \end{aligned}$$

5 Variational derivation of the field equations. Equations of motion

We shall now see that our theory can best be described, in the linear case, independently by 2 Lagrangian densities. We give the first Lagrangian density as

$$\begin{aligned} \bar{L} &= \sqrt{g} \left(\sigma^{ik} (\nabla_k \psi_i - D_{ik}) + \frac{1}{2} C_{..kl}^{ij} D_{ij} D^{kl} - \right. \\ &\left. - \mu_T D_{.i}^i \Delta T + U^i (\nabla_i \psi_k) (f \psi^k - \rho_m U^k) \right), \end{aligned}$$

where ρ_m is the material density and f is a scalar potential. From here we then arrive at the following invariant integral:

$$\begin{aligned} I &= \int_{vol} \left(\sigma^{ik} (\nabla_{(k} \psi_{i)} - \varepsilon_{ik}) + \sigma^{ik} (\nabla_{[k} \psi_{i]} - \omega_{ik}) + \right. \\ &+ \frac{1}{2} A_{..kl}^{ij} \varepsilon_{ij} \varepsilon^{kl} + \frac{1}{2} B_{..kl}^{ij} \omega_{ij} \omega^{kl} - \mu_T \varepsilon_{.i}^i \Delta T + \\ &\left. + U^i (\nabla_i \psi_k) (f \psi^k - \rho_m U^k) \right) dV, \end{aligned}$$

where $dV = \sqrt{g} d\xi^1 d\xi^2 d\xi^3$.

Writing $\bar{L} = \sqrt{g} L$, we then have

$$\begin{aligned} \delta I &= \int_{vol} \left(\frac{\partial L}{\partial \sigma^{ik}} \delta \sigma^{ik} + \frac{\partial L}{\partial \varepsilon^{ik}} \delta \varepsilon^{ik} + \frac{\partial L}{\partial \omega^{ik}} \delta \omega^{ik} + \right. \\ &\left. + \frac{\partial L}{\partial (\nabla_i \psi_k)} \delta (\nabla_i \psi_k) \right) dV = 0. \end{aligned}$$

Now

$$\begin{aligned} \int_{vol} \frac{\partial L}{\partial (\nabla_i \psi_k)} \delta (\nabla_i \psi_k) dV &= \int_{vol} \nabla_i \left(\frac{\partial L}{\partial (\nabla_i \psi_k)} \delta \psi_k \right) dV - \\ &- \int_{vol} \nabla_i \left(\frac{\partial L}{\partial (\nabla_i \psi_k)} \right) \delta \psi_k dV = \\ &= - \int_{vol} \nabla_i \left(\frac{\partial L}{\partial (\nabla_i \psi_k)} \right) \delta \psi_k dV, \end{aligned}$$

since the first term on the right-hand-side of the first line is an absolute differential that can be transformed away on the boundary of integration by means of the divergence theorem. Hence we have

$$\begin{aligned} \delta I &= \int_{vol} \left(\frac{\partial L}{\partial \sigma^{ik}} \delta \sigma^{ik} + \frac{\partial L}{\partial \varepsilon^{ik}} \delta \varepsilon^{ik} + \frac{\partial L}{\partial \omega^{ik}} \delta \omega^{ik} - \right. \\ &\left. - \nabla_i \left(\frac{\partial L}{\partial (\nabla_i \psi_k)} \right) \delta \psi_k \right) dV = 0, \end{aligned}$$

where each term in the integrand is independent of the others. Note also that the variations $\delta \sigma^{ik}$, $\delta \varepsilon^{ik}$, $\delta \omega^{ik}$, and $\delta \psi_k$ are arbitrary.

From $\frac{\partial L}{\partial \sigma^{ik}} = 0$, we obtain

$$\varepsilon_{ik} = \nabla_{(k} \psi_{i)},$$

$$\omega_{ik} = \nabla_{[k} \psi_{i]},$$

i.e., the components of the strain and vorticity tensors, respectively.

From $\frac{\partial L}{\partial \varepsilon^{ik}} = 0$, we obtain

$$\sigma^{(ik)} = A_{..rs}^{ik} \varepsilon^{rs} - \mu_T g^{ik} \Delta T,$$

i.e., the symmetric components of the stress tensor.

From $\frac{\partial L}{\partial \omega^{ik}} = 0$, we obtain

$$\sigma^{[ik]} = B_{..rs}^{ik} \omega^{rs} = \eta \omega^{ik} \tilde{R},$$

i.e., the anti-symmetric components of the stress tensor.

Finally, from the fourth variation we now show in detail that it yields the equations of motion. We first see that

$$\frac{\partial L}{\partial (\nabla_i \psi_k)} = \sigma^{ik} + U^i (f \psi^k - \rho_m U^k).$$

Hence

$$\begin{aligned} \nabla_i \left(\frac{\partial L}{\partial (\nabla_i \psi_k)} \right) &= \nabla_i \sigma^{ik} + \nabla_i (f U^i) \psi^k + f U^i \nabla_i \psi^k - \\ &- \nabla_i (\rho_m U^i) U^k - \rho_m U^i \nabla_i U^k. \end{aligned}$$

Define the “extended” shear scalar and the mass current density vector, respectively, through

$$l = \nabla_i (f U^i),$$

$$J^i = \rho_m U^i.$$

Now we readily identify the force per unit mass f and the body force per unit mass b , respectively, by

$$f^i = U^k \nabla_k U^i = \frac{\delta U^i}{\delta t},$$

$$b^i = \frac{1}{\rho_m} (l \psi^i + f (1 - \nabla_k J^k) U^i) =$$

$$= \frac{1}{\rho_m} \left(l \psi^i + f \left(1 + \frac{\partial \rho_m}{\partial t} \right) U^i \right),$$

where we have used the relation

$$\frac{D \rho_m}{Dt} = -\rho_m \nabla_i U^i,$$

i.e., $\frac{\partial \rho_m}{\partial t} + \nabla_i (\rho_m U^i) = 0$, derivable from the four-dimensional conservation law $\nabla_\mu (\rho_m * U^\mu) = 0$ where $*U^\mu = (U^i, c)$.

Hence we have (for arbitrary $\delta \psi_k$)

$$\int_{vol} (\nabla_i \sigma^{ik} + \rho_m b^k - \rho_m f^k) \delta \psi_k dV = 0,$$

i.e., the equations of motion

$$\nabla_i \sigma^{ik} = \rho_m (f^k - b^k).$$

Before we move on to the second Lagrangian density, let’s discuss briefly the so-called couple stress, i.e., the couple per unit area also known as the distributed moment. We denote the couple stress tensor by the second-rank tensor field M . In analogy to the linear constitutive relations relating the stress tensor σ to displacement gradient tensor D , we write

$$M_{ik} = D_{ik..}{}^{rs} N_{rs},$$

where

$$D_{ijkl} = E_{ijkl} + F_{ijkl}$$

are assumed to possess the same symmetry properties as C_{ijkl} (i.e., E_{ijkl} have the same symmetry properties as A_{ijkl} while F_{ijkl} , representing the chirality part, have the same symmetry properties as B_{ijkl}).

Likewise,

$$N_{ik} = N_{(ik)} + N_{[ik]} = X_{ik} + Y_{ik}$$

are comparable to $D_{ik} = D_{(ik)} + D_{[ik]} = \varepsilon_{ik} + \omega_{ik}$.

As a boundary condition, let us now define a completely anti-symmetric third-rank spin tensor as follows:

$$J^{ikl} = J^{[ikl]} = \frac{1}{2} \in^{ikl} \psi,$$

where ψ is a scalar function such that the spin tensor of our theory (which contains both the macrospin and microspin tensors) can be written as a gradient, i.e.,

$$S_i = \eta \in_{ijk} \tilde{R} \omega^{jk} = \nabla_i \psi,$$

such that whenever we desire to subject the above to the integrability condition $\in_{ijk} \nabla^j S^k = 0$, we have $\in^{ijk} \Gamma_{[jk]}^l S_l = 0$, resulting in $Y_{ik} = 0$.

In other words,

$$\psi = \psi_0 + \eta \int \in_{ijk} \tilde{R} \omega^{ij} d\xi^k,$$

where ψ_0 is constant, acts as a scalar generator of spin.

As a consequence, we see that

$$\nabla_l J^{ikl} = \frac{1}{2} \in^{ikl} \nabla_l \psi = \frac{1}{2} \in^{ikl} S_l =$$

$$= \frac{1}{2} \eta \tilde{R} \in^{ikl} \in_{pql} \omega^{pq}$$

$$= \frac{1}{2} \eta \tilde{R} (\delta_p^i \delta_q^k - \delta_q^i \delta_p^k) \omega^{pq}$$

$$= \eta \tilde{R} \omega^{ik},$$

i.e.,

$$\nabla_l J^{ikl} = \sigma^{[ik]}.$$

Taking the divergence of the above equations and using the relations $2 \nabla_{[k} \nabla_{i]} \psi = -2 \Gamma_{[ik]}^l \nabla_l \psi$, we obtain the following divergence equations:

$$\nabla_k \sigma^{[ik]} = \frac{1}{2} \in^{ikl} \Gamma_{[kl]}^r S_r,$$

coupling the components of the spin vector to the components of the torsion tensor. Furthermore, we obtain

$$\nabla_k \omega^{ik} = \frac{1}{2} (\eta \tilde{R}) \in^{ikl} \Gamma_{[kl]}^r S_r - \omega^{ik} \frac{\partial^e \log(\eta \tilde{R})}{\partial \xi^k}.$$

We now form the second Lagrangian density of our theory as

$$\bar{H} = \sqrt{g} \left(M^{ik} (\nabla_k S_i - N_{ik}) + \frac{1}{2} D_{..kl}^{ij} N_{ij} N^{kl} - \right.$$

$$\left. - \in_{..rs}^k (\nabla_i S_k) J^{rsi} + U^i \nabla_i S_k (h S^k - I \rho_m V^k) \right),$$

where h is a scalar function (not to be confused with the scalar function f), I is the moment of inertia, and V^i are the components of the angular velocity field.

Hence the action integral corresponding to this is

$$J = \int_{vol} \left(M^{ik} (\nabla_{(k} S_{i)} - X_{ik}) + M^{ik} (\nabla_{[k} S_{i]} - Y_{ik}) + \right.$$

$$+ \frac{1}{2} E_{..kl}^{ij} X_{ij} X^{kl} + \frac{1}{2} F_{..kl}^{ij} Y_{ij} Y^{kl} - \in_{..rs}^k (\nabla_i S_k) J^{rsi} +$$

$$\left. + U^i (\nabla_i S_k) (h S^k - I \rho_m V^k) \right) dV.$$

As before, writing $\bar{H} = \sqrt{g}H$ and performing the variation $\delta J = 0$, we have

$$\delta J = \int_{vol} \left(\frac{\partial H}{\partial M^{ik}} \delta M^{ik} + \frac{\partial H}{\partial X^{ik}} \delta X^{ik} + \frac{\partial H}{\partial Y^{ik}} \delta Y^{ik} - \nabla_i \left(\frac{\partial H}{\partial (\nabla_i S_k)} \right) \delta S_k \right) dV = 0,$$

with arbitrary variations δM^{ik} , δX^{ik} , δY^{ik} , and δS_k .

From $\frac{\partial H}{\partial M^{ik}} = 0$, we obtain

$$X_{ik} = \nabla_{(k} S_{i)},$$

$$Y_{ik} = \nabla_{[k} S_{i]}.$$

From $\frac{\partial H}{\partial X^{ik}} = 0$, we obtain

$$M^{(ik)} = E_{..rs}^{ik} X^{rs}.$$

From $\frac{\partial H}{\partial Y^{ik}} = 0$, we obtain

$$M^{[ik]} = F_{..rs}^{ik} Y^{rs}.$$

Again, we shall investigate the last variation

$$- \int_{vol} \nabla_i \left(\frac{\partial H}{\partial (\nabla_i S_k)} \right) \delta S_k dV = 0$$

in detail.

Firstly,

$$\frac{\partial H}{\partial (\nabla_i S_k)} = M^{ik} - \epsilon_{..rs}^k J^{rsi} + U^i (hS^k - I\rho V^k).$$

Then we see that

$$\begin{aligned} \nabla_i \left(\frac{\partial H}{\partial (\nabla_i S_k)} \right) &= \nabla_i M^{ik} - \epsilon_{..rs}^k \sigma^{[rs]} + \nabla_i (hU^i) S^k + \\ &+ hU^i \nabla_i S^k - I \nabla_i (\rho_m U^i) V^k - \\ &- I \rho_m U^i \nabla_i V^k. \end{aligned}$$

We now define the angular force per unit mass α by

$$\alpha^i = U^k \nabla_k V^i = \frac{\delta V^i}{\delta t},$$

and the angular body force per unit mass β by

$$\beta^i = \frac{1}{\rho_m} \left(\bar{l} S^i + h \frac{\delta S^i}{\delta t} - I (\nabla_k J^k) V^i \right),$$

where $\bar{l} = \nabla_i (h U^i)$.

We have

$$\int_{vol} \left(\nabla_i M^{ik} - \epsilon_{..rs}^k \sigma^{[rs]} + \rho_m \beta^k - I \rho_m \alpha^k \right) \delta S_k dV = 0.$$

Hence we obtain the equations of motion

$$\nabla_i M^{ik} = \epsilon_{..rs}^k \sigma^{[rs]} + \rho_m (I \alpha^k - \beta^k).$$

6 Concluding remarks

At this point we see that we have reproduced the field equations and the equations of motion of Cosserat elasticity theory by our variational method, and hence we have succeeded in showing parallels between the fundamental equations of Cosserat elasticity theory and those of our present theory. However we must again emphasize that our field equations as well as our equations of motion involving chirality are *fully geometrized*. In other words, we have succeeded in generalizing various extensions of the classical elasticity theory, especially the Cosserat theory and the so-called void elasticity theory by ascribing both microspin phenomena and geometric defects to the action of geometric torsion and to the source of local curvature of the material space. As we have seen, it is precisely this curvature that plays the role of a fundamental, intrinsic differential invariant which explains microspin and defects throughout the course of our work.

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Geodetic Precession of the Spin in a Non-Singular Gravitational Potential

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Using a non-singular gravitational potential which appears in the literature we analytically derived and investigated the equations describing the precession of a body's spin orbiting around a main spherical body of mass M . The calculation has been performed using a non-exact *Schwarzschild* solution, and further assuming that the gravitational field of the Earth is more than that of a rotating mass. General theory of relativity predicts that the direction of the gyroscope will change at a rate of 6.6 arcsec/year for a gyroscope in a 650 km high polar orbit. In our case a precession rate of the spin of a very similar magnitude to that predicted by general relativity was calculated resulting to a $\frac{\Delta S_{geo}}{S_{geo}} = -5.570 \times 10^{-2}$.

1 Introduction

A new non-singular gravitational potential appears in the literature that has the following form (Williams [1])

$$V(r) = -\frac{GM}{r} e^{-\frac{\lambda}{r}}, \quad (1)$$

where the constant λ appearing in the potential above is defined as follows:

$$\lambda = \frac{GM}{c^2} = \frac{R_{grav}}{2}, \quad (2)$$

and G is the Newtonian gravitational constant, M is the mass of the main body that produces the potential, and c is the speed of light. In this paper we wish to investigate the differences that might exist in the results

2 Geodetic precession

One of the characteristics of curved space is that parallel transport of a vector alters its direction, which suggests that we can probably detect the curvature of the space-time near the Earth by actually examining parallel transport. From non gravitational physics we know that if a gyroscope is suspended in frictionless gimbals the result is a parallel transport of its spin direction, which does not help draw any valuable conclusion immediately. Similarly in gravitational physics the transport of such gyroscope will also result in parallel transport of the spin. To find the conditions under which parallel transport of gyroscope can happen, we start with Newton's equation of motion for the spin of a rigid body. A rigid body in a gravitational field is subject to a tidal torque that results to a spin rate of change given by [2]:

$$\frac{dS^n}{dt} = \epsilon^{kln} R_{k0s0} \left(-I_t^s + \frac{1}{3} \delta_t^s I_r^r \right), \quad (3)$$

where $n, k, l, s, r = 1, 2, 3$. Here R_{k0s0} is the Riemann tensor evaluated in the rest frame of the gyroscope, the presence of which signifies that this particular equation of motion does not obey the principle of minimal coupling, and that the gyroscope spin transport does not imitate parallel transport [1] and the quantity ϵ^{kqp} is defined as follows $\epsilon^{123} = \epsilon^{231} = \epsilon^{312} = 1$ and $\epsilon^{321} = \epsilon^{213} = \epsilon^{132} = -1$. For a spherical gyroscope we have that $I_t^s \propto \delta_t^s$, then the tidal torque in the equation (3) becomes zero and the equation reads $\frac{dS^n}{dt} = 0$. I_t^s is the moment of inertia tensor defined in the equation below:

$$I_t^s = \int (r^2 \delta_t^s - x^s x_t) dM, \quad (4)$$

where δ_t^s is the Kronecker delta. This Newtonian equation remains in tact when we are in curved spacetime, and in a reference frame that freely falls along a geodesic line. Thus the Newtonian time t must now interpreted as the proper time τ measured along the geodesic. In the freely falling reference frame the spin of the gyroscope remains constant in magnitude and direction, which means that it moves by parallel transport.

If now an extra non gravitational force acts on the gyroscope and as a result the gyroscope moves into a world line that is different from a geodesic, then we can not simply introduce local geodesic coordinates at every point on of this world line which makes the equation of motion for the spin $\frac{dS^n}{dt} \neq 0$. In flat space-time the precession of an accelerated gyroscope is called *Thomas Precession*. In a general coordinate system the spin vector in parallel obeys the equation:

$$\frac{dS^\mu}{d\tau} = -\Gamma_{\nu\lambda}^\mu S^\nu \frac{dx^\lambda}{d\tau} = -\Gamma_{\nu\lambda}^\mu S^\nu \dot{x}^\lambda = -\Gamma_{\nu\lambda}^\mu S^\nu v^\lambda, \quad (5)$$

where $\Gamma_{\nu\lambda}^\mu$ are the Christoffel symbols of the second kind, and S^μ are the spin vector components (here $\mu, \nu, \lambda = 0, 1, 2, 3$).

Alternative theories of gravitation have also been proposed that predict different magnitudes for this effect [3, 4].

3 Gyroscope in orbit

In order to examine the effect of the new non singular gravitational potential has on the gyroscope let us assume a gyroscope in a circular orbit of radius r around the Earth. In real life somebody measures the change of the gyroscope spin relative to the fixed stars, which is also equivalent of finding this change with respect to a fixed coordinate system at infinity. We can use Cartesian coordinates since they are more convenient in calculating this change of spin direction than polar coordinates. The reason for this is that in Cartesian coordinates any change of the spin can be directly related to the curvature of the space-time, where in polar coordinates there is a contribution from both coordinate curvature and curvature of the space-time [2].

Next let us in a similar way to that of *linear theory* and following Ohanian and Ruffini [2] we write the line element ds^2 in the following way:

$$ds^2 \cong c^2 \left(1 - \frac{2GM}{rc^2} e^{-\lambda/r} \right) dt^2 - \left(1 - \frac{2GM}{rc^2} e^{-\lambda/r} \right)^{-1} (dx^2 + dy^2 + dz^2) \quad (6)$$

further assume that our gyroscope is in orbit around the Earth and let the orbit be located in the $x - y$ plane as shown in Figure 1.

In a circular orbit all points are equivalent and if we know the rate of the spin change at one point we can calculate the rate of change of the spin at any point. For that let us write the line interval in the following way:

$$ds^2 \cong c^2 \left(1 - \frac{2GM}{rc^2} e^{-\lambda/r} \right) dt^2 - \left(1 + \frac{2GM}{rc^2} e^{-\lambda/r} \right) (dx^2 + dy^2 + dz^2), \quad (7)$$

which implies that:

$$g_{00} = \left(1 - \frac{2GM}{rc^2} e^{-\lambda/r} \right), \quad (8)$$

$$g_{11} = g_{22} = g_{33} = - \left(1 + \frac{2GM}{rc^2} e^{-\lambda/r} \right).$$

4 The spin components

To evaluate the spatial components of the spin we will use equation (5), and the right hand symbols must be calculated. For that we need the four-velocity $v^\beta \approx (v_t, v_x, v_y, v_z) = (1, 0, v, 0)$. We also need the S^0 component of the spin, and for that we note that in the rest frame of the gyroscope

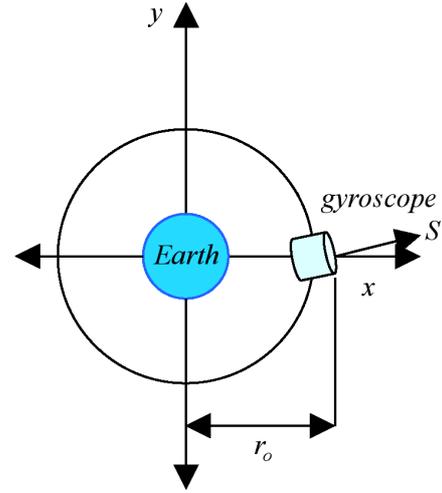


Fig. 1: A gyroscope above a satellite orbiting the Earth, and where its orbital plane coincides with the $x - y$ plane, having at an instant coordinates $x = r_0, y = z = 0$, and where S is the spin vector of the gyroscope.

$S'^0 = 0$ and $v'^\beta = (1, 0, 0, 0)$ and therefore $g'_{\mu\nu} S'^\mu v'^\nu = 0$, and also in our coordinate system we will also have that $g_{\mu\nu} S^\mu v^\nu = 0$, using the latter we have that:

$$S^0 = -\frac{1}{g_{00}} \left[S^1 g_{11} \frac{dx^1}{d\tau} + S^2 g_{22} \frac{dx^2}{d\tau} + S^3 g_{33} \frac{dx^3}{d\tau} \right], \quad (9)$$

$$S^0 = -\frac{1}{g_{00}} \left[S_x g_{11} \frac{dx}{d\tau} + S_y g_{22} \frac{dy}{d\tau} + S_z g_{33} \frac{dz}{d\tau} \right],$$

substituting for the metric coefficients we obtain:

$$S^0 = \frac{\left(1 + \frac{2GM}{rc^2} e^{-\lambda/r} \right)}{\left(1 - \frac{2GM}{rc^2} e^{-\lambda/r} \right)} v S_y \cong \left(1 + \frac{2GM}{rc^2} e^{-\lambda/r} \right)^2 v S_y. \quad (10)$$

Next letting $\mu = 1$ and summing over $\nu = 0, 1, 2, 3$ the component of the spin equation becomes:

$$\frac{dS^1}{d\tau} = -\Gamma_{0\lambda}^1 S^0 v^\lambda - \Gamma_{1\lambda}^1 S^1 v^\lambda - \Gamma_{2\lambda}^1 S^2 v^\lambda - \Gamma_{3\lambda}^1 S^3 v^\lambda, \quad (11)$$

summing over $\lambda = 0, 1, 2, 3$ again we obtain:

$$\frac{dS^1}{d\tau} = -\Gamma_{00}^1 S^0 v^0 - \Gamma_{01}^1 S^0 v^1 - \Gamma_{02}^1 S^0 v^2 - \Gamma_{03}^1 S^0 v^3 - \Gamma_{10}^1 S^1 v^0 - \Gamma_{11}^1 S^1 v^1 - \Gamma_{12}^1 S^1 v^2 - \Gamma_{13}^1 S^1 v^3 - \Gamma_{20}^1 S^2 v^0 - \Gamma_{21}^1 S^2 v^1 - \Gamma_{22}^1 S^2 v^2 - \Gamma_{23}^1 S^2 v^3 - \Gamma_{30}^1 S^3 v^0 - \Gamma_{31}^1 S^3 v^1 - \Gamma_{32}^1 S^3 v^2 - \Gamma_{33}^1 S^3 v^3. \quad (12)$$

Next we will calculate the Cristoffel symbols of the second kind for that we use:

$$\Gamma_{\mu\nu}^\sigma = \frac{1}{2} g^{\sigma\lambda} \left(\frac{\partial g_{\mu\sigma}}{\partial x^\nu} + \frac{\partial g_{\nu\sigma}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\sigma} \right). \quad (13)$$

Since $\Gamma_{\mu\nu}^\sigma = 0$ if $\mu \neq \nu \neq \sigma$ equation (12) further simplifies to:

$$\begin{aligned} \frac{dS^1}{d\tau} = & -\Gamma_{00}^1 S^0 v^0 - \Gamma_{01}^1 S^0 v^1 - \Gamma_{10}^1 S^1 v^0 - \\ & -\Gamma_{11}^1 S^1 v^1 - \Gamma_{12}^1 S^1 v^2 - \Gamma_{13}^1 S^1 v^3 - \\ & -\Gamma_{21}^1 S^2 v^1 - \Gamma_{22}^1 S^2 v^2 - \Gamma_{31}^1 S^3 v^1 - \Gamma_{33}^1 S^3 v^3. \end{aligned} \quad (14)$$

The only non-zero Christoffel symbols calculated at $r = r_0$ are:

$$\Gamma_{01}^0 = \Gamma_{10}^0 = \frac{GM}{r_0^2 c^2} \frac{\left(1 - \frac{\lambda}{r_0}\right) e^{-\lambda/r_0}}{\left(1 - \frac{2GM}{r_0 c^2} e^{-\lambda/r_0}\right)}, \quad (15)$$

$$\Gamma_{00}^1 = \frac{GM}{c^2 r_0^2} \frac{\left(1 - \frac{\lambda}{r_0}\right) e^{-\lambda/r_0}}{\left(1 - \frac{2GM}{r_0 c^2} e^{-\lambda/r_0}\right)}, \quad (16)$$

$$\Gamma_{11}^1 = -\frac{GM}{c^2 r_0^2} \frac{\left(1 - \frac{\lambda}{r_0}\right) e^{-\lambda/r_0}}{\left(1 - \frac{2GM}{r_0 c^2} e^{-\lambda/r_0}\right)}, \quad (17)$$

$$\Gamma_{22}^1 = \frac{GM}{r_0^2 c^2} \frac{\left(1 - \frac{\lambda}{r_0}\right) e^{-\frac{\lambda}{r_0}}}{\left(1 + \frac{2GM}{r_0 c^2} e^{-\frac{\lambda}{r_0}}\right)}, \quad (18)$$

$$\Gamma_{21}^1 = \Gamma_{12}^1 = \frac{GM}{r_0^2 c^2} \frac{\left(1 - \frac{\lambda}{r_0}\right) e^{-\frac{\lambda}{r_0}}}{\left(1 + \frac{2GM}{r_0 c^2} e^{-\frac{\lambda}{r_0}}\right)}, \quad (19)$$

$$\Gamma_{33}^1 = \frac{GM}{r_0^2 c^2} \frac{\left(1 - \frac{\lambda}{r_0}\right) e^{-\frac{\lambda}{r_0}}}{\left(1 + \frac{2GM}{r_0 c^2} e^{-\frac{\lambda}{r_0}}\right)}, \quad (20)$$

$$\Gamma_{13}^3 = \Gamma_{31}^3 = -\frac{GM}{r_0^2 c^2} \frac{\left(1 - \frac{\lambda}{r_0}\right) e^{-\frac{\lambda}{r_0}}}{\left(1 + \frac{2GM}{r_0 c^2} e^{-\frac{\lambda}{r_0}}\right)}. \quad (21)$$

Thus equation (14) further becomes:

$$\frac{dS_x}{d\tau} = -\Gamma_{00}^1 S^0 - \Gamma_{22}^1 S^2 v^2, \quad (22)$$

substituting we obtain:

$$\begin{aligned} \frac{dS_x}{d\tau} = & -\frac{GM}{r_0^2 c^2} \frac{\left(1 - \frac{\lambda}{r_0}\right) e^{-\frac{\lambda}{r_0}}}{\left(1 + \frac{2GM}{r_0 c^2} e^{-\frac{\lambda}{r_0}}\right)} \times \\ & \times \left[1 + \left(1 + \frac{2GM}{r_0 c^2} e^{-\frac{\lambda}{r_0}}\right)^2 \right] v S_y. \end{aligned} \quad (23)$$

Expanding in powers of $\frac{\lambda}{r}$ to first order we can rewrite (23) as follows:

$$\begin{aligned} \frac{dS_x}{d\tau} = & -\frac{GM}{r_0^2 c^2} \frac{\left(1 - \frac{\lambda}{r_0}\right)^2}{\left(1 + \frac{2GM}{r_0 c^2} \left(1 - \frac{\lambda}{r_0}\right)\right)} \times \\ & \times \left[1 + \left(1 + \frac{2GM}{r_0 c^2} \left(1 - \frac{\lambda}{r_0}\right)\right)^2 \right] v S_y, \end{aligned} \quad (24)$$

$$\begin{aligned} \frac{dS_x}{d\tau} \cong & -\frac{GM}{r_0^2 c^2} \frac{\left(1 - \frac{2\lambda}{r_0}\right)}{\left(1 + \frac{2GM}{r_0 c^2} \left(1 - \frac{\lambda}{r_0}\right)\right)} \times \\ & \times \left[1 + \left(1 + \frac{2GM}{r_0 c^2} \left(1 - \frac{\lambda}{r_0}\right)\right)^2 \right] v S_y, \end{aligned} \quad (25)$$

keeping only $\frac{1}{c^2}$ terms and omitting the rest higher powers $\frac{G^2 M^2}{c^4}$ equation (25) can be simplified to:

$$\frac{dS_x}{d\tau} \cong -\frac{2GM}{r_0^2 c^2} \left(1 - \frac{2\lambda}{r_0}\right) v S_y. \quad (26)$$

Similarly the equation for the S_y component of the spin becomes:

$$\frac{dS_y}{d\tau} = -\Gamma_{12}^2 v S_x - \Gamma_{20}^2 S_y - \Gamma_{22}^2 v S_y - \Gamma_{32}^2 v S_y, \quad (27)$$

which becomes:

$$\frac{dS_y}{d\tau} = -\frac{GM}{r_0^2 c^2} \frac{\left(1 - \frac{\lambda}{r}\right) e^{-\frac{\lambda}{r_0}}}{\left(1 + \frac{2GM}{r_0 c^2} e^{-\frac{\lambda}{r_0}}\right)}, \quad (28)$$

can be approximated to:

$$\begin{aligned} \frac{dS_y}{d\tau} = & -\frac{GM}{r_0^2 c^2} \frac{\left(1 - \frac{2\lambda}{r_0}\right)}{\left(1 + \frac{2GM}{r_0 c^2} \left(1 - \frac{\lambda}{r_0}\right)\right)} = \\ = & -\frac{GM}{r_0^2 c^2} \frac{\left(1 - \frac{2\lambda}{r_0}\right)}{\left(1 + \frac{2GM}{r_0 c^2} - \frac{2GM\lambda}{r_0^2 c^2}\right)} v S_x. \end{aligned} \quad (29)$$

Finally the equation for the S_z component becomes:

$$\begin{aligned} \frac{dS^3}{d\tau} = & -\Gamma_{00}^3 S^0 v^0 - \Gamma_{01}^3 S^0 v^1 - \Gamma_{02}^3 S^0 v^2 - \\ & -\Gamma_{03}^3 S^0 v^3 - \Gamma_{10}^3 S^1 v^0 - \Gamma_{11}^3 S^1 v^1 - \\ & -\Gamma_{12}^3 S^1 v^2 - \Gamma_{13}^3 S^1 v^3 - \Gamma_{20}^3 S^2 v^0 - \\ & -\Gamma_{21}^3 S^2 v^1 - \Gamma_{22}^3 S^2 v^2 - \Gamma_{23}^3 S^2 v^3 - \\ & -\Gamma_{30}^3 S^3 v^0 - \Gamma_{31}^3 S^3 v^1 - \Gamma_{32}^3 S^3 v^2 - \Gamma_{33}^3 S^3 v^3, \end{aligned} \quad (30)$$

which finally becomes:

$$\frac{dS_z}{d\tau} = 0. \quad (31)$$

Equations (23), (28), (31) are valid at the chosen $x = r_0$, $y = z = 0$ point. These equations can also be written in a form that is valid at any point of the orbit, if we just recognize that all of them can be combined in the following single 3-D equation:

$$S_x(t) = S_0 \left\{ \cosh \left(\frac{\sqrt{2} GM (2\lambda - r_0)}{c^2 r_0^3 \sqrt{1 + \frac{2GM}{r_0 c^2} - \frac{2GM\lambda}{r_0^2 c^2}}} vt \right) - \sqrt{2 \left(1 + \frac{2GM}{r_0 c^2} - \frac{2GM\lambda}{r_0^2 c^2} \right)} \sinh \left(\frac{\sqrt{2} GM (2\lambda - r_0)}{c^2 r_0^3 \sqrt{1 + \frac{2GM}{r_0 c^2} - \frac{2GM\lambda}{r_0^2 c^2}}} vt \right) \right\} \quad (43)$$

$$S_y(t) \cong S_0 \left\{ \cosh \left(\frac{\sqrt{2} GM (2\lambda - r_0)}{c^2 r_0^3 \sqrt{1 + \frac{2GM}{r_0 c^2} - \frac{2GM\lambda}{r_0^2 c^2}}} vt \right) - \frac{1}{\sqrt{2}} \sinh \left(\frac{\sqrt{2} GM (2\lambda - r_0)}{c^2 r_0^3 \sqrt{1 + \frac{2GM}{r_0 c^2} - \frac{2GM\lambda}{r_0^2 c^2}}} vt \right) \right\} \quad (44)$$

tion in the following way [2]:

$$\frac{dS}{d\tau} = -2v \cdot S \nabla V + vS \cdot \nabla V, \quad (32)$$

where $V = -\frac{GM}{r_0} e^{-\lambda/r_0}$ is the non singular potential used. Below in order to compare we can write down the same equations for the spin components in the case of the Newtonian potential.

$$\begin{aligned} \frac{dS_x}{d\tau} &= -\frac{GM}{r_0^2 c^2} v S_y \left[\left(1 + \frac{2GM}{r_0 c^2} \right) + \frac{1}{\left(1 + \frac{2GM}{r_0 c^2} \right)} \right] \cong \\ &\cong -\frac{2GM}{r_0^2 c^2} v S_y, \end{aligned} \quad (33)$$

$$\frac{dS_y}{d\tau} = \frac{GM}{r_0^2 c^2 \left(1 + \frac{2GM}{r_0 c^2} \right)} v S_x \cong \frac{GM}{r_0^2 c^2} v S_x, \quad (34)$$

$$\frac{dS_z}{d\tau} = 0. \quad (35)$$

5 Non-singular potential solutions

To find the components of the precessing spin let us now solve the system of equations (26) (29) (31) solving we obtain:

$$S_x(t) = C_1 \cosh \left\{ \frac{\sqrt{2} GM (2\lambda - r_0)}{c^2 r_0^3 \sqrt{1 + \frac{2GM}{r_0 c^2} - \frac{2GM\lambda}{r_0^2 c^2}}} vt \right\} + C_2 \sqrt{2} \sinh \left\{ \frac{\sqrt{2} GM (2\lambda - r_0)}{c^2 r_0^3 \sqrt{1 + \frac{2GM}{r_0 c^2} - \frac{2GM\lambda}{r_0^2 c^2}}} vt \right\}, \quad (36)$$

$$S_y(t) = C_2 \cosh \left\{ \frac{\sqrt{2} GM (2\lambda - r_0)}{c^2 r_0^3 \sqrt{1 + \frac{2GM}{r_0 c^2} - \frac{2GM\lambda}{r_0^2 c^2}}} vt \right\} + \frac{C_1}{\sqrt{2}} \sinh \left\{ \frac{\sqrt{2} GM (2\lambda - r_0)}{c^2 r_0^3 \sqrt{1 + \frac{2GM}{r_0 c^2} - \frac{2GM\lambda}{r_0^2 c^2}}} vt \right\}, \quad (37)$$

$$S_z(t) = \text{const} = D_0, \quad (38)$$

since the motion is not relativistic we have that $dt = d\tau$, and the orbital velocity of the gyroscope is $v = \sqrt{\frac{GM}{r_0}}$.

6 Newtonian gravity solutions

Next we can compare the solutions in (36), (37), (38) with those of the system (33), (34), (35) which are:

$$S_x(t) = C_1 \cos \left(\frac{\sqrt{2} GM}{c^2 r_0^2} vt \right) - C_2 \sqrt{2} \sin \left(\frac{\sqrt{2} GM}{c^2 r_0^2} vt \right), \quad (39)$$

$$S_y(t) = C_2 \cos \left(\frac{\sqrt{2} GM}{c^2 r_0^2} vt \right) + \frac{C_1}{\sqrt{2}} \sin \left(\frac{\sqrt{2} GM}{c^2 r_0^2} vt \right), \quad (40)$$

$$S_z(t) = \text{const} = D_0.$$

If we now assume the initial conditions $t = 0$, $S_x(0) = S_y(0) = S_0$ we obtain the final solution:

$$S_x(t) = S_0 \left\{ \cos \left(\frac{\sqrt{2} GM}{c^2 r_0^2} vt \right) - \sqrt{2} \sin \left(\frac{\sqrt{2} GM}{c^2 r_0^2} vt \right) \right\}, \quad (41)$$

$$S_y(t) = S_0 \left\{ \cos \left(\frac{\sqrt{2} GM}{c^2 r_0^2} \sqrt{\frac{GM}{r_0}} t \right) - \frac{1}{\sqrt{2}} \sin \left(\frac{\sqrt{2} GM}{c^2 r_0^2} \sqrt{\frac{GM}{r_0}} t \right) \right\}. \quad (42)$$

Similarly from the solutions of the non-singular Newtonian potential we obtain (43) and (44).

Since numerically $c^2 r_0^3 \gg 2GM r_0^2 - 2GM\lambda r_0$ the above equations take the form (45) and (46).

7 Numerical results for an Earth satellite

Let us now assume a satellite in a circular orbit around the Earth, at an orbital height $h = 650$ km or an orbital radius

$$S_x(t) = S_0 \left\{ \cosh \left(\frac{2\lambda GM}{c^2 r_0^3} \left(1 - \frac{r_0}{2\lambda}\right) \sqrt{\frac{2GM}{r_0 \left(1 + \frac{2GM}{r_0 c^2} - \frac{2GM\lambda}{r_0^2 c^2}\right)}} t \right) - \sqrt{2 \left(1 + \frac{2GM}{r_0 c^2} - \frac{2\lambda GM}{r_0^2 c^2}\right)} \sinh \left(\frac{2\lambda GM}{c^2 r_0^3} \left(1 - \frac{r_0}{2\lambda}\right) \sqrt{\frac{2GM}{r_0 \left(1 + \frac{2GM}{r_0 c^2} - \frac{2GM\lambda}{r_0^2 c^2}\right)}} t \right) \right\} \quad (45)$$

$$S_y(t) = S_0 \left\{ \cosh \left(\frac{2\lambda GM}{c^2 r_0^3} \left(1 - \frac{r_0}{2\lambda}\right) \sqrt{\frac{2GM}{r_0 \left(1 + \frac{2GM}{r_0 c^2} - \frac{2GM\lambda}{r_0^2 c^2}\right)}} t \right) - \frac{1}{\sqrt{2}} \sinh \left(\frac{2\lambda GM}{c^2 r_0^3} \left(1 - \frac{r_0}{2\lambda}\right) \sqrt{\frac{2GM}{r_0 \left(1 + \frac{2GM}{r_0 c^2} - \frac{2GM\lambda}{r_0^2 c^2}\right)}} t \right) \right\} \quad (46)$$

$V(r)$	$\Delta S_x/S_x$	$\Delta S_y/S_y$	Geodetic precession S (arcsec/year)	$\Delta S_{geo}/S_g$
Newtonian	-4.30×10^{-5}	2.00×10^{-5}	-6.6	
Non-Singular	-4.80×10^{-5}	2.40×10^{-5}	-6.289	-5.570×10^{-2}

Table 1: Changes of the spin components and final geodetic precession of an orbiting the Earth satellite t an altitude $h = 650$ km.

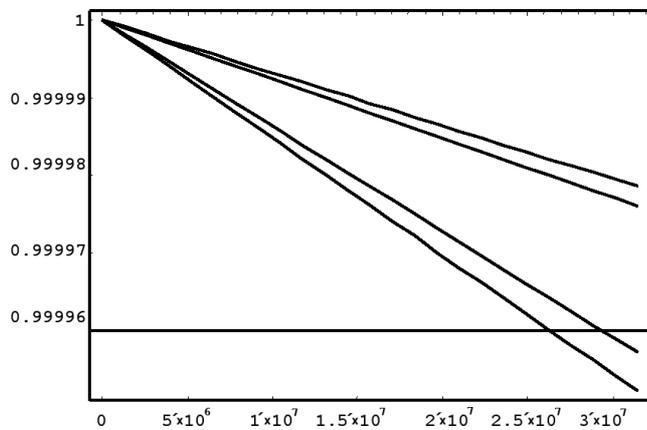


Fig. 2: Gyroscope spin components (S_x, S_y). Newtonian and non-singular potential change in the gyro pin components for a satellite orbiting the earth for a year. Abscissa axis means time.

$r_0 = 7.028 \times 10^6$ m, then $\lambda = 4.372 \times 10^{-3}$ m, $v = 7.676$ km/s, $t = 1$ year $= 3.153 \times 10^7$ s using (41) and (42) we obtain:

Newtonian potential

$$\begin{aligned} S_x &= 0.999957 S_0, \\ S_y &= 1.000020 S_0, \end{aligned} \quad (47)$$

and from (45) and (46) we obtain:

Non-Singular Potential

$$\begin{aligned} S_x &= 0.999952 S_0, \\ S_y &= 0.999976 S_0. \end{aligned} \quad (48)$$

For a gyroscope in orbit around the Earth we can write an expression for the geodetic precession in such a non-singular potential to be equal to:

$$S_{geo} = \frac{3}{2} \nabla \Phi \times v = \frac{3GM}{2c^2 r_0^2} \sqrt{\frac{GM}{r_0}} \left(1 - \frac{\lambda}{r_0}\right) e^{-\frac{\lambda}{r_0}}, \quad (49)$$

substituting values for the parameters above we obtain that:

$$S_{geo} = 1.01099 \times 10^{-12} \text{ rad/s}, \quad (50)$$

$$S_{geo} = 6.289 \text{ arcsec/year}. \quad (51)$$

8 Conclusions

We have derived the equations for the precession of the spin in a case of a non-singular potential and we have compared them with those of the Newtonian potential. In the case of the non-singular gravitational potential both components of the spin are very slow varying functions of time. In a hypothetically large amount of time of the order of $\sim 10^5$ years or more spin components S_x and S_y of the non-singular potential appear to diverge in opposite directions, where those of the Newtonian potential exhibit a week periodic motion in time.

In the case of the non-singular potential we found that $\frac{\Delta S_x}{S_x} = -4.80 \times 10^{-5}$ and $\frac{\Delta S_y}{S_y} = -2.40 \times 10^{-5}$ where in the case of the Newtonian potential we have that $\frac{\Delta S_x}{S_x} = -4.30 \times 10^{-5}$ and $\frac{\Delta S_y}{S_y} = 2.00 \times 10^{-5}$. The calculation has been performed using a non-exact *Schwarzschild* solution. On the other hand the gravitational field of the Earth is not an exact *Schwarzschild* field, but rather the field of a rotating mass. Compared to the Newtonian result, the non-singular potential modifies the original equation of the geodetic precession by the term $(1 - \frac{\lambda}{r_0}) e^{-\frac{\lambda}{r_0}}$ which at the orbital altitude of $h = 650$ km contributes to a spin reduction effect of the order of 9.99×10^{-1} . If such a type of potential exists its effect onto a gyroscope of a satellite orbiting at $h = 650$ km could probably be easily detected.

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A Note on Computer Solution of Wireless Energy Transmit via Magnetic Resonance

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In the present article we argue that it is possible to find numerical solution of coupled magnetic resonance equation for describing wireless energy transmit, as discussed recently by Karalis (2006) and Kurs *et al.* (2007). The proposed approach may be found useful in order to understand the phenomena of magnetic resonance. Further observation is of course recommended in order to refute or verify this proposition.

1 Introduction

In recent years there were some new interests in methods to transmit energy without wire. While it has been known for quite a long-time that this method is possible theoretically (since Maxwell and Hertz), until recently only a few researchers consider this method seriously.

For instance, Karalis *et al* [1] and also Kurs *et al.* [2] have presented these experiments and reported that efficiency of this method remains low. A plausible way to solve this problem is by better understanding of the mechanism of magnetic resonance [3].

In the present article we argue that it is possible to find numerical solution of coupled magnetic resonance equation for describing wireless energy transmit, as discussed recently by Karalis (2006) and Kurs *et al.* (2007). The proposed approach may be found useful in order to understand the phenomena of magnetic resonance.

Nonetheless, further observation is of course recommended in order to refute or verify this proposition.

2 Numerical solution of coupled-magnetic resonance equation

Recently, Kurs *et al.* [2] argue that it is possible to represent the physical system behind wireless energy transmit using coupled-mode theory, as follows:

$$a_m(t) = (i\omega_m - \Gamma_m) a_m(t) + \sum_{n \neq m} i\kappa_{nm} a_n(t) - F_m(t). \quad (1)$$

The simplified version of equation (1) for the system of two resonant objects is given by Karalis *et al.* [1, p. 2]:

$$\frac{da_1}{dt} = -i(\omega_1 - i\Gamma_1) a_1 + i\kappa a_2, \quad (2)$$

and

$$\frac{da_2}{dt} = -i(\omega_2 - i\Gamma_2) a_2 + i\kappa a_1. \quad (3)$$

These equations can be expressed as linear 1st order ODE as follows:

$$f'(t) = -i\alpha f(t) + i\kappa g(t) \quad (4)$$

and

$$g'(t) = -i\beta g(t) + i\kappa f(t), \quad (5)$$

where

$$\alpha = (\omega_1 - i\Gamma_1) \quad (6)$$

and

$$\beta = (\omega_2 - i\Gamma_2) \quad (7)$$

Numerical solution of these coupled-ODE equations can be found using Maxima [4] as follows. First we find test when parameters (6) and (7) are set up to be 1. The solution is:

```
(%i5) 'diff(f(x),x)+%i*f=%i*b*g(x);
(%o5) 'diff(f(x),x,1)+%i*f=%i*b*g(x)
(%i6) 'diff(g(x),x)+%i*g=%i*b*f(x);
(%o6) 'diff(g(x),x,1)+%i*g=%i*b*f(x)
(%i7) desolve([%o5,%o6],[f(x),g(x)]);
```

The solutions for $f(x)$ and $g(x)$ are:

$$f(x) = \frac{[ig(0)b - if(x)] \sin(bx)}{b} - \frac{[g(x) - f(0)b] \cos(bx)}{b} + \frac{g(x)}{b}, \quad (8)$$

$$g(x) = \frac{[if(0)b - ig(x)] \sin(bx)}{b} - \frac{[f(x) - g(0)b] \cos(bx)}{b} + \frac{f(x)}{b}. \quad (9)$$

Translated back to our equations (2) and (3), the solutions for $\alpha = \beta = 1$ are given by:

$$a_1(t) = \frac{[ia_2(0)\kappa - ia_1] \sin(\kappa t)}{\kappa} - \frac{[a_2 - a_1(0)\kappa] \cos(\kappa t)}{\kappa} + \frac{a_2}{\kappa} \quad (10)$$

$$f(x) = e^{-(ic-ia)t/2} \left[\frac{[2if(0)c + 2ig(0)b - f(0)(ic - ia)] \sin\left(\frac{\sqrt{c^2 - 2ac + 4b^2 + a^2}}{2} t\right)}{\sqrt{c^2 - 2ac + 4b^2 + a^2}} + \frac{f(0) \cos\left(\frac{\sqrt{c^2 - 2ac + 4b^2 + a^2}}{2} t\right)}{\sqrt{c^2 - 2ac + 4b^2 + a^2}} \right] \quad (13)$$

$$g(x) = e^{-(ic-ia)t/2} \left[\frac{[2if(0)c + 2ig(0)a - g(0)(ic - ia)] \sin\left(\frac{\sqrt{c^2 - 2ac + 4b^2 + a^2}}{2} t\right)}{\sqrt{c^2 - 2ac + 4b^2 + a^2}} + \frac{g(0) \cos\left(\frac{\sqrt{c^2 - 2ac + 4b^2 + a^2}}{2} t\right)}{\sqrt{c^2 - 2ac + 4b^2 + a^2}} \right] \quad (14)$$

$$a_1(t) = e^{-(i\beta - i\alpha)t/2} \left(\frac{[2ia_1(0)\beta + 2ia_2(0)\kappa - (i\beta - i\alpha)a_1] \sin\left(\frac{\xi}{2} t\right)}{\xi} - \frac{a_1(0) \cos\left(\frac{\xi}{2} t\right)}{\xi} \right) \quad (15)$$

$$a_2(t) = e^{-(i\beta - i\alpha)t/2} \left(\frac{[2ia_2(0)\beta + 2ia_1(0)\kappa - (i\beta - i\alpha)a_2] \sin\left(\frac{\xi}{2} t\right)}{\xi} - \frac{a_2(0) \cos\left(\frac{\xi}{2} t\right)}{\xi} \right) \quad (16)$$

and

$$a_2(t) = \frac{[ia_1(0)\kappa - ia_2] \sin(\kappa t)}{\kappa} - \frac{[a_1 - a_2(0)\kappa] \cos(\kappa t)}{\kappa} + \frac{a_1}{\kappa}. \quad (11)$$

Now we will find numerical solution of equations (4) and (5) when $\alpha \neq \beta \neq 1$. Using Maxima [4], we find:

```
(%i12) 'diff(f(t),t)+%i*a*f(t)=%i*b*g(t);
(%o12) 'diff(f(t),t,1)+%i*a*f(t)=%i*b*g(t)
(%i13) 'diff(g(t),t)+%i*c*g(t)=%i*b*f(t);
(%o13) 'diff(g(t),t,1)+%i*c*g(t)=%i*b*f(t)
(%i14) desolve([%o12,%o13],[f(t),g(t)]);
```

and the solution is found to be quite complicated: these are formulae (13) and (14).

Translated back these results into our equations (2) and (3), the solutions are given by (15) and (16), where we can define a new "ratio":

$$\xi = \sqrt{\beta^2 - 2\alpha\beta + 4\kappa^2 + \alpha^2}. \quad (12)$$

It is perhaps quite interesting to remark here that there is no "distance" effect in these equations.

Nonetheless, further observation is of course recommended in order to refute or verify this proposition.

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VC would like to dedicate this article to R.F.F.

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Structure of Even-Even $^{218-230}\text{Ra}$ Isotopes within the Interacting Boson Approximation Model

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A good description of the excited positive and negative parity states of radium nuclei ($Z = 88$, $N = 130-142$) is achieved using the interacting boson approximation model (IBA-1). The potential energy surfaces, energy levels, parity shift, electromagnetic transition rates $B(E1)$, $B(E2)$ and electric monopole strength $X(E0/E2)$ are calculated for each nucleus. The analysis of the eigenvalues of the model Hamiltonian reveals the presence of an interaction between the positive and negative parity bands. Due to this interaction the $\Delta I = 1$ staggering effect, between the energies of the ground state band and the negative parity state band, is produced including beat patterns.

1 Introduction

The existence of stable octupole deformation in actinide nuclei has encouraged many authors to investigate these nuclei experimentally and theoretically but until now no definitive signatures have been established. Different models have been considered, but none has provided a complete picture of octupole deformation.

Cluster model has been applied to $^{221-226}\text{Ra}$ by many authors [1–7]. The intrinsic multipole transition moment and parity splitting were calculated. Also, the half-lives of cluster emission are predicted. In general, cluster model succeeded in reproducing satisfactory the properties of normal deformed ground state and super deformed excited bands in a wide range of even-even nuclei.

A proposed formalism of the collective model [8, 9, 10] have been used in describing the strong parity shift observed in low-lying spectra of $^{224,226}\text{Ra}$ and $^{224,226}\text{Th}$ with octupole deformations together with the fine rotational band structure developed at higher angular momenta. Beat staggering patterns are obtained also for $^{218-226}\text{Ra}$ and $^{224,226}\text{Th}$.

The mean field model [11] and the analytic quadrupole octupole axially symmetric (AQOA) model [12] have been applied to $^{224,226}\text{Ra}$ and ^{226}Ra nuclei respectively, and found useful for the predictions of the decay properties where the experimental data are scarce.

Spdf interacting boson model [13] has been applied to the even-even $^{218-228}\text{Ra}$ isotopes and an explanation of how the octupole deformation can arise in the rotational limit. The discussion of the properties of the fractional symmetric rigid rotor spectrum [14] and the results of its application to the low excitation energy of the ground state band of $^{214-224}\text{Ra}$ show an agreement with the experimental data.

The aim of the present paper is to calculate and analyze the complete spectroscopic properties of the low-lying positive and negative parity excited states in $^{218-230}\text{Ra}$ isotopes using IBA-1 Hamiltonian. The potential energy surfaces, lev-

els energy, parity shift, electromagnetic transition rates and electric monopole strength $X(E0/E2)$ are calculated.

2 (IBA-1) model

2.1 Level energies

The IBA-1 model describes the low-lying energy states of the even-even radium nuclei as a system of interacting *s*-bosons and *d*-bosons. The π and ν bosons are treated as one boson. Introducing creation ($s^\dagger d^\dagger$) and annihilation ($s\tilde{d}$) operators for *s* and *d* bosons, the most general Hamiltonian [15] which includes one-boson term in boson-boson interaction has been used in calculating the levels energy is:

$$H = EPS \cdot n_d + PAIR \cdot (P \cdot P) + \frac{1}{2} ELL \cdot (L \cdot L) + \frac{1}{2} QQ \cdot (Q \cdot Q) + 5OCT \cdot (T_3 \cdot T_3) + 5HEX \cdot (T_4 \cdot T_4), \quad (1)$$

where

$$P \cdot p = \frac{1}{2} \left[\begin{array}{c} \left\{ (s^\dagger s^\dagger)_0^{(0)} - \sqrt{5} (d^\dagger d^\dagger)_0^{(0)} \right\} x \\ \left\{ (ss)_0^{(0)} - \sqrt{5} (\tilde{d}\tilde{d})_0^{(0)} \right\} \end{array} \right]_0^{(0)}, \quad (2)$$

$$L \cdot L = -10 \sqrt{3} \left[(d^\dagger \tilde{d})^{(1)} x (d^\dagger \tilde{d})^{(1)} \right]_0^{(0)}, \quad (3)$$

$$Q \cdot Q = \sqrt{5} \left[\begin{array}{c} \left\{ (S^\dagger \tilde{d} + d^\dagger s)^{(2)} - \frac{\sqrt{7}}{2} (d^\dagger \tilde{d})^{(2)} \right\} x \\ \left\{ (s^\dagger \tilde{d} + d^\dagger s)^{(2)} - \frac{\sqrt{7}}{2} (d^\dagger \tilde{d})^{(2)} \right\} \end{array} \right]_0^{(0)}, \quad (4)$$

$$T_3 \cdot T_3 = -\sqrt{7} \left[(d^\dagger \tilde{d})^{(2)} x (d^\dagger \tilde{d})^{(2)} \right]_0^{(0)}, \quad (5)$$

$$T_4 \cdot T_4 = 3 \left[(d^\dagger \tilde{d})^{(4)} x (d^\dagger \tilde{d})^{(4)} \right]_0^{(0)}. \quad (6)$$

nucleus	<i>EPS</i>	<i>PAIR</i>	<i>ELL</i>	<i>QQ</i>	<i>OCT</i>	<i>HEX</i>	<i>E2SD</i> (eb)	<i>E2DD</i> (eb)
²¹⁸ Ra	0.3900	0.000	0.0005	-0.0090	0.0000	0.0000	0.2020	-0.5957
²²⁰ Ra	0.3900	0.000	0.0005	-0.0420	0.0000	0.0000	0.1960	-0.5798
²²² Ra	0.0650	0.0000	0.0100	-0.0650	0.0000	0.0000	0.1960	-0.5798
²²⁴ Ra	0.2000	0.0000	0.0060	-0.0450	0.0000	0.0000	0.1640	-0.4851
²²⁶ Ra	0.0700	0.0000	0.0060	-0.0450	0.0000	0.0000	0.1660	-0.4910
²²⁸ Ra	0.0600	0.0000	0.0060	-0.0380	0.0000	0.0000	0.1616	-0.4780
²³⁰ Ra	0.0580	0.0000	0.0060	-0.0502	0.0000	0.0000	0.1560	-0.4615

Table 1: Table 1. Parameters used in IBA-1 Hamiltonian (all in MeV).

In the previous formulas, n_d is the number of boson; $P \cdot P$, $L \cdot L$, $Q \cdot Q$, $T_3 \cdot T_3$ and $T_4 \cdot T_4$ represent pairing, angular momentum, quadrupole, octupole and hexadecupole interactions between the bosons; *EPS* is the boson energy; and *PAIR*, *ELL*, *QQ*, *OCT*, *HEX* is the strengths of the pairing, angular momentum, quadrupole, octupole and hexadecupole interactions.

2.2 Transition rates

The electric quadrupole transition operator [15] employed in this study is given by:

$$T^{(E2)} = E2SD \cdot (s^\dagger \tilde{d} + d^\dagger s)^{(2)} + \frac{1}{\sqrt{5}} E2DD \cdot (d^\dagger \tilde{d})^{(2)}. \quad (7)$$

The reduced electric quadrupole transition rates between $I_i \rightarrow I_f$ states are given by

$$B(E2, I_i - I_f) = \frac{[\langle I_f || T^{(E2)} || I_i \rangle]^2}{2I_i + 1}. \quad (8)$$

3 Results and discussion

3.1 The potential energy surface

The potential energy surfaces [16], $V(\beta, \gamma)$, for radium isotopes as a function of the deformation parameters β and γ have been calculated using:

$$\begin{aligned} E_{N_\pi N_\nu}(\beta, \gamma) &= \langle N_\pi N_\nu; \beta \gamma | H_{\pi\nu} | N_\pi N_\nu; \beta \gamma \rangle = \\ &= \zeta_d(N_\nu N_\pi) \beta^2 (1 + \beta^2) + \beta^2 (1 + \beta^2)^{-2} \times \\ &\times \{ k N_\nu N_\pi [4 - (\bar{X}_\pi \bar{X}_\nu) \beta \cos 3\gamma] \} + \\ &+ \left\{ [\bar{X}_\pi \bar{X}_\nu \beta^2] + N_\nu (N_\nu - 1) \left(\frac{1}{10} c_0 + \frac{1}{7} c_2 \right) \beta^2 \right\}, \end{aligned} \quad (9)$$

where

$$\bar{X}_\rho = \left(\frac{2}{7} \right)^{0.5} X_\rho \quad \rho = \pi \text{ or } \nu. \quad (10)$$

The calculated potential energy surfaces for radium series of isotopes presented in Fig. 1 show that ²¹⁸Ra is a

vibrational-like nucleus where the deformation β is zero. ²²⁰Ra nucleus started to deviate from vibrational-like and a slight prolate deformation appeared. ²²²⁻²³⁰Ra nuclei show more deformation on the prolate and oblate sides, but the deformation on the prolate side is deeper.

3.2 Energy spectra

IBA-1 model has been used in calculating the energy of the positive and negative parity low -lying levels of radium series of isotopes. A comparison between the experimental spectra [17–23] and our calculations, using the values of the model parameters given in Table 1 for the ground and octupole bands, is illustrated in Fig. 2. The agreement between the theoretical and their correspondence experimental values for all the nuclei are slightly higher but reasonable. The most striking is the minimum observed in the negative parity states, Fig. 3, at $N = 136$ which interpreted as ²²⁴Ra is the most deformed nucleus in this chain of isotopes.

3.3 Electromagnetic transitions rates

Unfortunately there is no enough measurements of $B(E1)$ or $B(E2)$ rates for these series of nuclei. The only measured $B(E2, 0_1^+ \rightarrow 2_1^+)$'s are presented, in Table 2a, for comparison with the calculated values. The parameters *E2SD* and *E2DD* used in the present calculations are determined by normalizing the calculated values to the experimentally known ones and displayed in Tables 2a and 2b.

For calculating $B(E1)$ and $B(E2)$ transition rates of intraband and interaband we did not introduce any new parameters. The calculated values some of it are presented in Fig. 4 and Fig. 5 which show bending in the two figures at $N = 136$ which support what we have seen in Fig. 3 as ²²⁴Ra is the most octupole deformed nucleus.

3.4 Electric monopole transitions

The electric monopole transitions, $E0$, are normally occurring between two states of the same spin and parity by transferring energy and zero unit of angular momentum. The strength of the electric monopole transition, $X_{if'f}(E0/E2)$,

$I_i^+ I_f^+$	^{218}Ra	^{220}Ra	^{222}Ra	^{224}Ra	^{226}Ra	^{228}Ra	^{230}Ra
$0_1 \text{ Exp. } 2_1$	1.10(20)	——	4.54(39)	3.99(15)	5.15(14)	5.99(28)	——
$0_1 \text{ Theor. } 2_1$	1.1222	2.4356	4.5630	3.9633	5.1943	5.9933	6.6861
$2_1 0_1$	0.224	0.4871	0.9126	0.7927	1.0389	1.1987	1.3372
$2_2 0_1$	0.0005	0.0028	0.0001	0.0014	0.0002	0.0001	0.0001
$2_2 0_2$	0.086	0.2509	0.5978	0.5287	0.7444	0.8878	1.0183
$2_3 0_1$	0.000	0.0058	0.0001	0.0015	0.0001	0.0001	0.0001
$2_3 0_2$	0.173	0.0854	0.0141	0.0075	0.0122	0.0122	0.0118
$2_3 0_3$	0.022	0.0476	0.0001	0.0011	0.0001	0.0000	0.0000
$2_4 0_3$	0.010	0.1322	0.3662	0.3013	0.5326	0.6481	0.7627
$2_4 0_4$	0.152	0.0707	0.0006	0.0041	0.0001	0.0000	0.0000
$2_2 2_1$	0.300	0.0819	0.0003	0.0034	0.0003	0.0003	0.0001
$2_3 2_1$	0.0001	0.0023	0.0002	0.0022	0.0002	0.0002	0.0001
$2_3 2_2$	0.088	0.4224	0.0690	0.0730	0.0398	0.0348	0.0302
$4_1 2_1$	0.368	0.7474	1.2490	1.0973	1.4449	1.6752	1.8756
$4_1 2_2$	0.0318	0.0337	0.0004	0.0051	0.0004	0.0004	0.0002
$4_1 2_3$	0.0715	0.0331	0.0000	0.0002	0.0000	0.0000	0.0000
$6_1 4_1$	0.4194	0.7924	1.2673	1.1380	1.5138	1.7714	1.9970
$6_1 4_2$	0.0463	0.0270	0.0004	0.0057	0.0005	0.0004	0.0002
$6_1 4_3$	0.0514	0.0249	0.0001	0.0009	0.0000	0.0000	0.0000
$8_1 6_1$	0.3749	0.7217	1.626	1.0830	1.4672	1.7430	1.9864
$8_1 6_2$	0.0529	0.0205	0.0004	0.0051	0.0006	0.0005	0.0002
$8_1 6_3$	0.0261	0.0170	0.0001	0.0018	0.0001	0.0001	0.0000
$10_1 8_1$	0.2346	0.5600	0.9737	0.9649	1.3492	1.6406	1.9005
$10_1 8_2$	0.0553	0.0161	0.0003	0.0041	0.0005	0.0005	0.0002

Table 2: Table 2a. Values of the theoretical reduced transition probability, $B(E2)$ (in $e^2 b^2$).

$I_i^- I_f^+$	^{218}Ra	^{220}Ra	^{222}Ra	^{224}Ra	^{226}Ra	^{228}Ra	^{230}Ra
$1_1 0_1$	0.0008	0.0605	0.1942	0.1886	0.2289	0.2612	0.3033
$1_1 0_2$	0.1203	0.0979	0.0222	0.0293	0.0195	0.0190	0.0183
$3_1 2_1$	0.1117	0.1921	0.3352	0.3301	0.3927	0.4378	0.4937
$3_1 2_2$	0.0451	0.0325	0.0245	0.0330	0.0243	0.0238	0.0228
$3_1 2_3$	0.0025	0.0095	0.0001	0.0001	0.0000	0.0000	0.0001
$3_1 4_1$	0.0015	0.0094	0.0419	0.0458	0.0791	0.0883	0.0926
$3_1 4_2$	0.0007	0.0040	0.0073	0.0100	0.0099	0.0090	0.0074
$5_1 4_1$	0.2397	0.3169	0.4358	0.4349	0.5032	0.5493	0.6043
$5_1 4_2$	0.0531	0.0267	0.0187	0.0260	0.0214	0.0214	0.0205
$5_1 4_3$	0.0017	0.0027	0.0006	0.0005	0.0002	0.0002	0.0003
$7_1 6_1$	0.3839	0.4454	0.5349	0.5388	0.6033	0.6476	0.6996
$7_1 6_2$	0.0476	0.0204	0.0121	0.0187	0.0168	0.0173	0.0169
$9_1 8_1$	0.5429	0.5785	0.6398	0.6479	0.7041	0.7452	0.7936
$9_1 8_2$	0.0295	0.0139	0.0070	0.0129	0.0122	0.0131	0.0132

Table 3: Table 2b. Values of the theoretical reduced transition probability, $B(E1)$ (in $\mu e^2 b$).

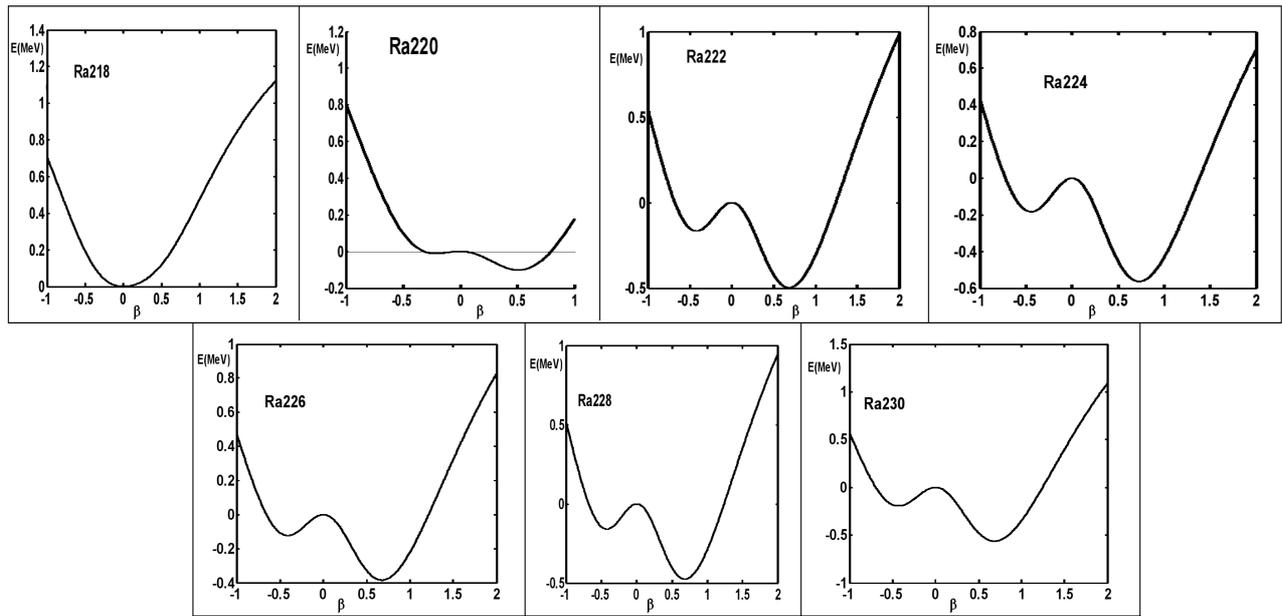


Fig. 1: The potential energy surfaces for $^{218-230}\text{Ra}$ nuclei.

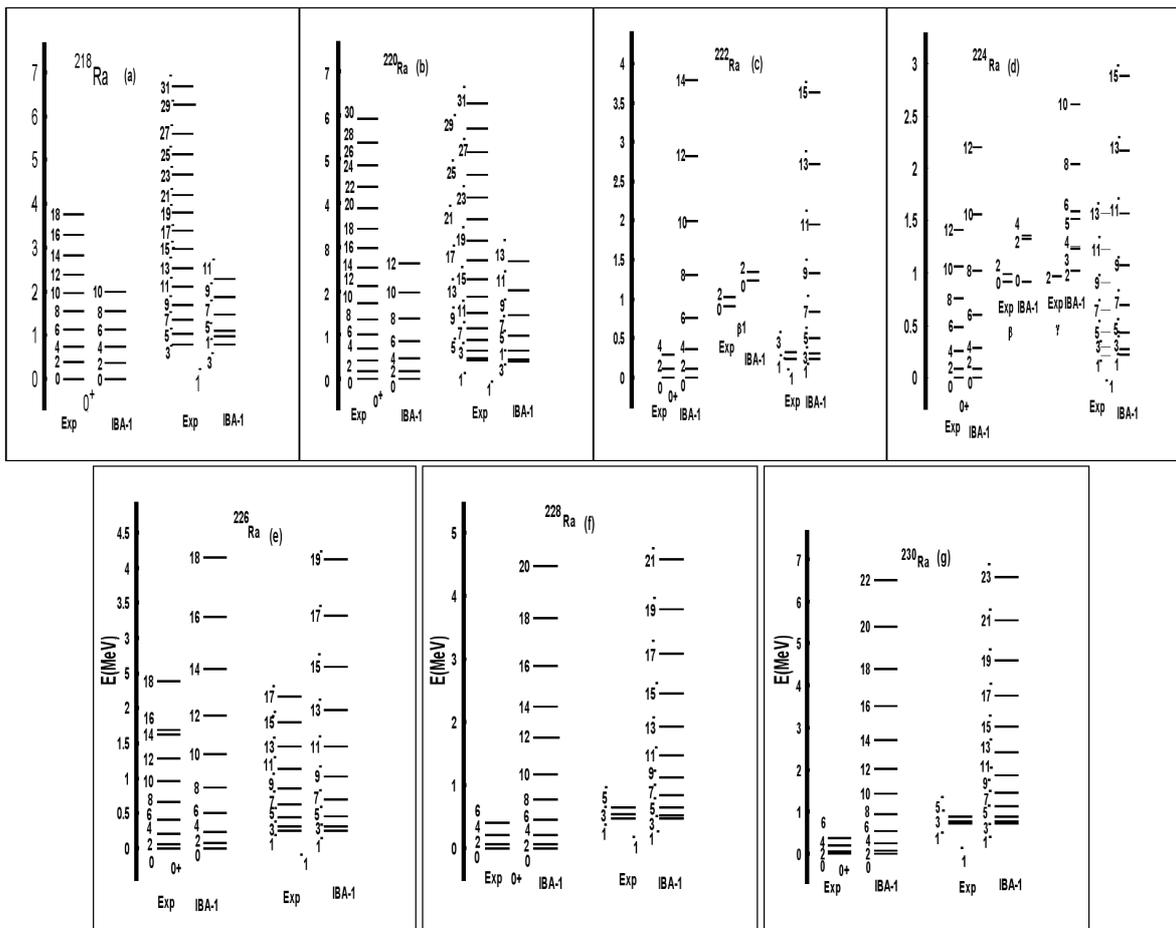


Fig. 2: Comparison between experimental (Exp.) and theoretical (IBA-1) energy levels in $^{218-230}\text{Ra}$, (a-g).

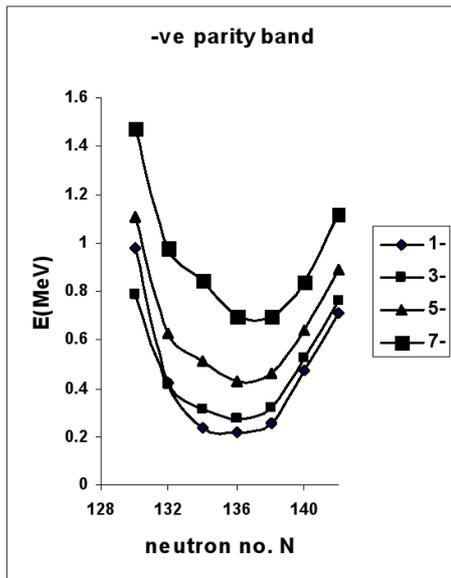


Fig. 3: Energy versus neutron numbers N for the $-ve$ parity band in $^{218-230}\text{Ra}$.

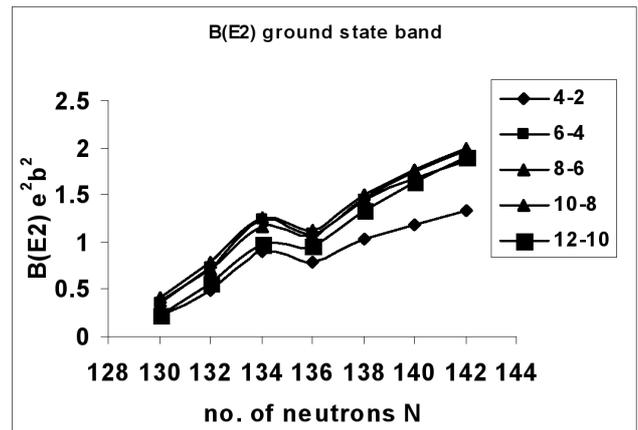


Fig. 4: The calculated $B(E2)$'s for the ground state band of Ra isotopes.

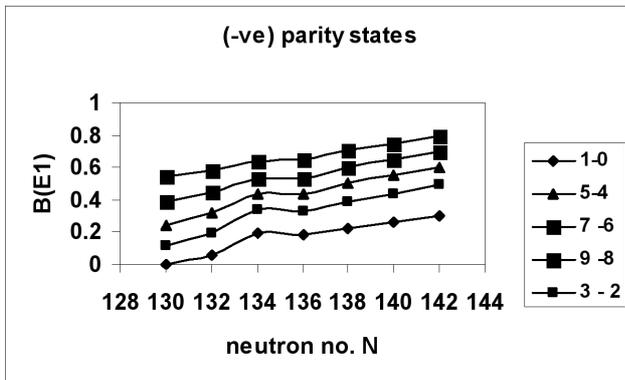


Fig. 5: The calculated $B(E1)$'s for the $(-ve)$ parity band.

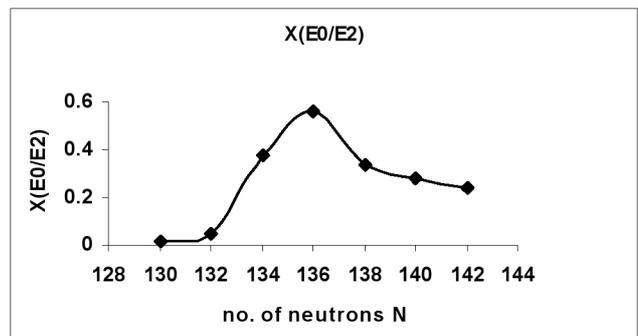


Fig. 6: The calculated $X(E0/E2, 2_2^+ \rightarrow 0_1^+)$ versus N for $^{218-230}\text{Ra}$ isotopes.

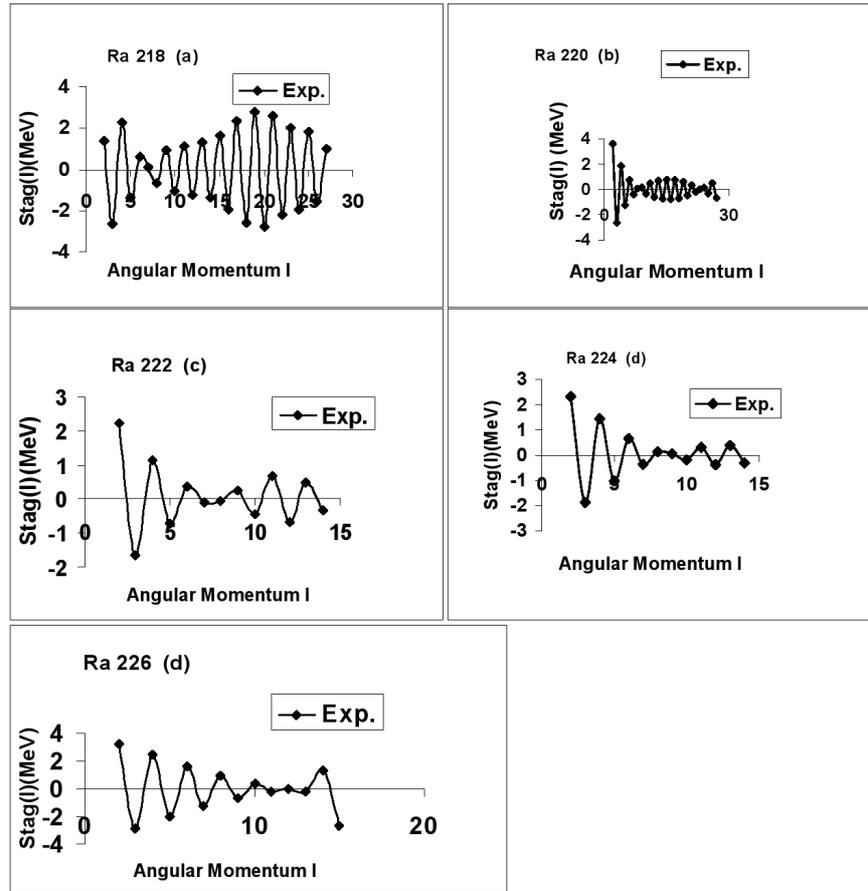


Fig. 7: $\Delta I = 1$, staggering patterns for the ground state and octupole bands of $^{218-230}\text{Ra}$ isotope.

I_i^+	I_f^+	I_{if}^+	^{218}Ra	^{220}Ra	^{222}Ra	^{224}Ra	^{226}Ra	^{228}Ra	^{230}Ra
0 ₂	0 ₁	2 ₁	0.016	0.046	0.376	0.562	0.335	0.279	0.243
0 ₃	0 ₁	2 ₁	0.125	—	—	0.058	0.081	—	0.500
0 ₃	0 ₁	2 ₂	0.007	0.058	—	0.009	0.003	—	0.230
0 ₃	0 ₁	2 ₃	0.015	0.002	—	0.333	0.005	—	0.0005
0 ₃	0 ₂	2 ₁	—	—	10.00	1.705	0.702	2.000	9.000
0 ₃	0 ₂	2 ₂	—	0.029	0.008	0.027	0.027	0.189	4.153
0 ₃	0 ₂	2 ₃	—	0.001	—	9.666	0.054	0.049	0.103
0 ₄	0 ₁	2 ₂	0.009	0.008	1.200	—	1.700	0.391	0.094
0 ₄	0 ₁	2 ₃	0.009	1.000	—	1.230	0.459	1.636	—
0 ₄	0 ₁	2 ₄	0.005	0.018	0.027	4.000	1.307	0.129	5.000
0 ₄	0 ₂	2 ₂	0.018	0.042	1.400	—	0.1000	—	0.037
0 ₄	0 ₂	2 ₃	0.019	5.000	—	0.769	0.027	—	—
0 ₄	0 ₂	2 ₄	0.011	0.093	0.031	2.500	0.076	—	2.000
0 ₄	0 ₃	2 ₁	—	—	0.250	—	0.333	—	—
0 ₄	0 ₃	2 ₂	—	—	0.066	—	0.001	—	—
0 ₄	0 ₃	2 ₃	—	—	—	—	0.027	—	—
0 ₄	0 ₃	2 ₄	—	—	0.001	—	0.076	—	—

Table 3. Theoretical $X_{if'f}$ ($E0/E2$) ratios for $E0$ transitions in Ra isotopes.

[24] can be calculated using equations (11, 12) and presented in Table 3. Fig. 6 shows also that ^{224}Ra has strong electric monopole strength than the other radium isotopes which is in agreement with the previous explanations.

$$X_{if'f}(E0/E2) = \frac{B(E0, I_i - I_f)}{B(E2, I_i - I_f)}, \quad (11)$$

$$X_{if'f}(E0/E2) = (2.54 \times 10^9) A^{3/4} \times \frac{E_\gamma^5(\text{MeV})}{\Omega_{KL}} \alpha(E2) \frac{T_e(E0, I_i - I_f)}{T_e(E2, I_i - I_f)}. \quad (12)$$

3.5 The staggering

A presence of an odd-even staggering effect has been observed for $^{218-230}\text{Ra}$ series of isotopes [8, 9, 10, 25]. Odd-even staggering patterns between the energies of the ground state band and the ($-ve$) parity octupole band have been calculated, $\Delta I = 1$, using staggering function as in equations (13, 14) using the available experimental data [17–23].

$$\text{Stag}(I) = 6\Delta E(I) - 4\Delta E(I-1) - 4\Delta E(I+1) + \Delta E(I+2) + \Delta E(I-2), \quad (13)$$

with

$$\Delta E(I) = E(I+1) - E(I). \quad (14)$$

The calculated staggering patterns are illustrated in Fig. 7, where we can see the beat patterns of the staggering behavior which show an interaction between the ground state and the octupole bands.

3.6 Conclusions

The IBA-1 model has been applied successfully to $^{218-230}\text{Ra}$ isotopes and we have got:

1. The ground state and octupole bands are successfully reproduced;
2. The potential energy surfaces are calculated and show vibrational characters to $^{218,220}\text{Ra}$ and rotational behavior to $^{222-230}\text{Ra}$ isotopes where they are prolate deformed nuclei;
3. Electromagnetic transition rates $B(E1)$ and $B(E2)$ are calculated;
4. The strength of the electric monopole transitions are calculated and show with the other calculated data that ^{224}Ra is the most octupole deformed nucleus;
5. Staggering effect have been observed and beat patterns obtained which show an interaction between the ground state and octupole bands;

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Covariance, Curved Space, Motion and Quantization

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Weak external forces and non-inertial motion are equivalent with the free motion in a curved space. The Hamilton-Jacobi equation is derived for such motion and the effects of the curvature upon the quantization are analyzed, starting from a generalization of the Klein-Gordon equation in curved spaces. It is shown that the quantization is actually destroyed, in general, by a non-inertial motion in the presence of external forces, in the sense that such a motion may produce quantum transitions. Examples are given for a massive scalar field and for photons.

Newton's law. We start with Newton's law

$$m \frac{dv_\alpha}{dt} = f_\alpha, \quad (1)$$

for a particle of mass m , with usual notations. I wish to show here that it is equivalent with the motion of a free particle of mass m in a curved space, *i.e.* it is equivalent with

$$\frac{Du^i}{ds} = \frac{du^i}{ds} + \Gamma_{jk}^i u^j u^k = 0, \quad (2)$$

again with usual notations.*

Obviously, the spatial coordinates of equation (1) are euclidean, and equation (1) is a non-relativistic limit. It follows that the metric we should look for may read

$$ds^2 = (1 + h) c^2 dt^2 + 2cdt g_{0\alpha} dx^\alpha + g_{\alpha\beta} dx^\alpha dx^\beta, \quad (3)$$

where $g_{\alpha\beta} = -\delta_\alpha^\beta (= \delta_{\alpha\beta})$, while functions $h, g_{0\alpha} \ll 1$ are determined such that equation (2) goes into equation (1) in the non-relativistic limit $\frac{v_\alpha}{c} \ll 1$ and for a correspondingly weak force f_α . Such a metric, which recovers Newton's law in the non-relativistic limit, is not unique. The metric given by

*The geometry of the curved spaces originates probably with Gauss (1830). It was given a sense by Riemann (*Ueber die Hypothesen welche der Geometrie zugrunde liegen*, 1854), Grassman (1862), Christoffel (1869), thereafter Klein (*Erlanger Programm, Programm zum Eintritt in die philosophische Fakultät in Erlangen*, 1872), Ricci and Levi-Civita (1901). It was Einstein (1905,1916), Poincare (1905), Minkowski (1907), Sommerfeld (1910), (Kottler, 1912), Weyl (*Raum, Zeit und Materie*, 1918), Hilbert (1917) who made the connection with the physical theories. It is based on point (local) coordinate transforms, cogredient (contravariant) and contragredient (covariant) tensors and the distance element. It is an absolute calculus, as it does not depend on the point, *i.e.* the reference frame. It may be divided into the motion of a particle, the motion of the fields, the motion of the gravitational field, and their applications, especially in cosmology and cosmogony. As the curved space is universal for gravitation, so it is for the non-inertial motion, which we focus upon here. The body which creates the gravitation and the corresponding curved space is here the moving observer for the non-inertial motion, beside forces. It could be very well that the world and the motion are absolute, but they depend on subjectivity, though it could be an universal subjectivity (inter-subjectivity). See W. Pauli, *Theory of Relativity*, Teubner, Leipzig, (1921).

equation (3) can be written as

$$g_{ij} = \begin{pmatrix} 1 + h & g_{10} & g_{20} & g_{30} \\ g_{01} & -1 & 0 & 0 \\ g_{02} & 0 & -1 & 0 \\ g_{03} & 0 & 0 & -1 \end{pmatrix}, \quad (4)$$

(where $g_{0\alpha} = g_{\alpha 0} = g_\alpha$). We perform the calculations up to the first order in h, g_α and $\frac{v_\alpha}{c}$. The distance given by (3) becomes then $ds = cdt (1 + \frac{h}{2})$ and the velocities read

$$u^0 = \frac{dx^0}{ds} = 1 - \frac{h}{2}, \quad u^\alpha = \frac{dx^\alpha}{ds} = \frac{v_\alpha}{c}. \quad (5)$$

It is the Christoffel's symbols (affine connections)

$$\Gamma_{jk}^i = \frac{1}{2} g^{im} \left(\frac{\partial g_{mj}}{\partial x^k} + \frac{\partial g_{mk}}{\partial x^j} - \frac{\partial g_{jk}}{\partial x^m} \right) \quad (6)$$

which require more calculations. First, the contravariant metric is $g^{00} = 1 - h, g^{0\alpha} = g^{\alpha 0}, g_{0\alpha} = g_{\alpha 0}, g_\beta^\alpha = -\delta_\beta^\alpha$, such that $g_{im} g^{mj} = g^{jm} g_{mi} = \delta_i^j$. By making use of (6) we get

$$\left. \begin{aligned} \Gamma_{00}^0 &= \frac{1}{2c} \frac{\partial h}{\partial t}, & \Gamma_{0\alpha}^0 &= \Gamma_{\alpha 0}^0 = \frac{1}{2} \frac{\partial h}{\partial x^\alpha} \\ \Gamma_{\alpha\beta}^0 &= \Gamma_{\beta\alpha}^0 = \frac{1}{2} \left(\frac{\partial g_{0\alpha}}{\partial x^\beta} + \frac{\partial g_{0\beta}}{\partial x^\alpha} \right) \\ \Gamma_{\beta 0}^\alpha &= \Gamma_{0\beta}^\alpha = \frac{1}{2} \left(\frac{\partial g_{0\beta}}{\partial x^\alpha} - \frac{\partial g_{0\alpha}}{\partial x^\beta} \right) \\ \Gamma_{00}^\alpha &= \frac{1}{2} \frac{\partial h}{\partial x^\alpha} - \frac{1}{c} \frac{\partial g_{0\alpha}}{\partial t}, & \Gamma_{\beta\gamma}^\alpha &= 0 \end{aligned} \right\}. \quad (7)$$

Now, the first equation in (2) has $\frac{du^0}{ds} = -\frac{1}{2c} \frac{\partial h}{\partial t}$ and $\Gamma_{jk}^0 u^j u^k = \frac{1}{2c} \frac{\partial h}{\partial t}$, so it is satisfied identically in this approximation. The remaining equations in (2) read

$$\frac{dv_\alpha}{dt} = c^2 \left(\frac{\partial g_{0\alpha}}{c \partial t} - \frac{1}{2} \frac{\partial h}{\partial x^\alpha} \right). \quad (8)$$

By comparing this with Newton's equation (1) we get the functions h and $g_{0\alpha}$ as given by

$$\frac{\partial g_{0\alpha}}{c \partial t} - \frac{1}{2} \frac{\partial h}{\partial x^\alpha} = \frac{f_\alpha}{mc^2}. \quad (9)$$

As it is well-known for a static gravitational potential Φ , the force is given by $f_\alpha = -m \frac{\partial \Phi}{\partial x^\alpha}$, so that $h = \frac{2\Phi}{c^2}$ and also $g_{0\alpha} = \text{const.}$ *

Translations. Suppose that the force \mathbf{f} is given by a static potential φ , such that $\mathbf{f} = -\frac{\partial \varphi}{\partial \mathbf{r}}$. Then $h = \frac{2\varphi}{mc^2}$ and $\mathbf{g} = \text{const.}$

Let us perform a translation

$$\mathbf{r} = \mathbf{r}' + \mathbf{R}(t'), \quad t = t'. \quad (10)$$

Then, Newton's equation $m \frac{d\mathbf{v}}{dt} = \mathbf{f}$ given by (1) becomes

$$m \frac{d\mathbf{v}'}{dt'} = \mathbf{f}' - m \frac{d\mathbf{V}}{dt'}, \quad (11)$$

where \mathbf{f}' is the force in the new coordinates and $\mathbf{V} = \frac{d\mathbf{R}}{dt'}$ is the translation velocity. The inertial force $-m \frac{d\mathbf{V}}{dt'}$ appearing in (11) is accounted by the \mathbf{g} in the metric of the curved space. Indeed, equation (9) gives

$$\mathbf{g} = -\frac{\mathbf{V}}{c}, \quad (12)$$

up to a constant. The constant reflects the principle of inertia. We may put it equal to zero. The time-dependent \mathbf{g} and \mathbf{V} represent a non-inertial motion. Such a non-inertial motion is therefore equivalent with a free motion in a curved space. Of course, this statement is nothing else but the principle of equivalence, or the general principle of relativity. It is however noteworthy that the non-inertial curved space depends on the observer, through the velocity \mathbf{V} , by virtue of the reciprocity of the motion.

Rotations. A rotation of angular frequency Ω about some axis is an orthogonal transformation of coordinates defined locally by

$$d\mathbf{r}' = d\mathbf{r} + (\Omega \times \mathbf{r}) dt, \quad (13)$$

such that the velocity is $\mathbf{v}' = \mathbf{v} + \Omega \times \mathbf{r}$ and

$$\begin{aligned} d\mathbf{v}' &= d\mathbf{v} + (\dot{\Omega} \times \mathbf{r}) dt + (\Omega \times \mathbf{v}) dt + \\ &+ [\Omega \times (\mathbf{v} + \Omega \times \mathbf{r})] dt = \\ &= d\mathbf{v} + (\dot{\Omega} \times \mathbf{r}) dt + 2(\Omega \times \mathbf{v}) dt + [\Omega \times (\Omega \times \mathbf{r})] dt. \end{aligned}$$

It is easy to see that in Newton's law for a particle of mass m there appears a force related to the non-uniform rotation ($\dot{\Omega}$), the Coriolis force $\sim \Omega \times \mathbf{v}$ and the centrifugal force $\sim \Omega^2$. The lagrangian $L = \frac{1}{2} m v'^2 - \varphi$, where φ is a potential, leads to the hamiltonian

*With regard to equation (3), this was for the first time when Einstein "suspected the time" (1905).

$$\begin{aligned} H &= \frac{mv^2}{2} - \frac{m}{2} (\Omega \times \mathbf{r})^2 + \varphi = \\ &= \frac{1}{2m} p^2 - \Omega (\mathbf{r} \times \mathbf{p}) + \varphi = \frac{1}{2m} p^2 - \Omega \mathbf{L} + \varphi, \end{aligned} \quad (14)$$

where $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ is the angular momentum. We can see that neither the Coriolis force nor the centrifugal potential appear anymore in the hamiltonian. Instead, it contains the angular momentum.

The local coordinate transformation (13) leads to a distance given by

$$\begin{aligned} ds^2 &= \left[1 + h - \frac{(\Omega \times \mathbf{r})^2}{c^2} \right] (dx^0)^2 - \\ &- \frac{2}{c} (\Omega \times \mathbf{r}) d\mathbf{r} dx^0 - d\mathbf{r}^2, \end{aligned} \quad (15)$$

where a static potential $\sim h$ is introduced as before, related to the potential φ in (14). It can be checked, through more laborious calculations, that the free motion in the curved space given by (15) is equivalent with the non-relativistic equations of motion given by (14).

As it is well-known, a difficulty appears however in the above metric, related to the unbounded increase with \mathbf{r} of the $\Omega \times \mathbf{r}$. Therefore, we drop out the square of this term in the g_{00} -term above, and keep only the first-order contributions in $\Omega \times \mathbf{r}$ in the subsequent calculations. As one can see, this approximation does not affect the hamiltonian (14). With this approximation, the metric given by (15) is identical with the metric given by equation (4), with the identification

$$\mathbf{g} = -\frac{1}{c} (\Omega \times \mathbf{r}). \quad (16)$$

Coordinate transformations. The translation given by (10) or the rotations given by (13) correspond to local coordinate transformations. As it is well-known, we can define such transformations in general, through suitable matrices (vierbeins). They take locally the infinitesimal coordinates in a flat space into infinitesimal coordinates in a curved space. For instance, the coordinate transformation corresponding to our metric given by equation (3) is given by

$$\left. \begin{aligned} dt &= \frac{(1+h)dt' + (g + \beta\Delta) \frac{dx'}{c}}{\sqrt{(1+h)(1-\beta^2)}} \\ dx &= \frac{c\beta(1+h)dt' + (\beta g + \Delta)dx'}{\sqrt{(1+h)(1-\beta^2)}} \end{aligned} \right\}, \quad (17)$$

$dy = dy'$, $dz = dz'$, where $\Delta = \sqrt{1+h+g^2}$, while \mathbf{g} is along $d\mathbf{x} = dx^1$, $\beta = \frac{V}{c}$ and the velocity V is $V = \frac{dx}{dt}$ for $dx' = 0$ ($dy = dx^2$, $dz = dx^3$). The inverse of this transformation is

$$\left. \begin{aligned} dt' &= \frac{g(\beta dt - \frac{dx}{c}) + \Delta(dt - \frac{\beta dx}{c})}{\Delta \sqrt{(1+h)(1-\beta^2)}} \\ dx' &= \sqrt{1+h} \frac{dx - c\beta dt}{\Delta \sqrt{1-\beta^2}} \end{aligned} \right\}. \quad (18)$$

All the square roots in these equations must exist, which imposes certain restrictions upon h and β (reality conditions; in particular, $1+h > 0$ and $1-\beta^2 > 0$).

In the local transformations given above it is assumed that there exist global transformations $x^i(x')$ and $x'^i(x)$, where x, x' stand for all x^i and, respectively, x'^i , because the coefficients in these transformations are functions of x or, respectively, x' . This restricts appreciably the derivation of metrics by means of (global) coordinate transformations, because in general, as it is well-known, the 10 elements of a metric cannot be obtained by 4 functions $x^i(x')$. Conversely, we can diagonalize the curved metric at any point, such as to reduce it to a locally flat metric (tangent space), but the flat coordinates (axes) will not, in general, be the same for all the points; they depend, in general, on the point.

One can see from (17) that in the flat limit $h, g \rightarrow 0$ the above transformations become the Lorentz transformations, as expected. Therefore, we may have corrections to the flat relativistic motion by first-order contributions of the parameters h and g . Indeed, in this limit, the transformation (18) becomes

$$\left. \begin{aligned} dt &= \frac{(1 + \frac{h}{2}) dt' + (g + \beta) \frac{dx'}{c}}{\sqrt{1 - \beta^2}} \\ dx &= \frac{c\beta(1 + \frac{h}{2}) dt' + (g\beta + 1) dx'}{\sqrt{1 - \beta^2}} \end{aligned} \right\}. \quad (19)$$

which include corrections to the Lorentz transformations, due to the curved space.

The metric given by (3) provides the proper time

$$d\tau = \sqrt{1+h} dt, \quad (20)$$

corresponding to $dx^\alpha = 0$. The metric given by (3) can also be written as

$$ds^2 = c^2(1+h) \left[dt + \frac{1}{c(1+h)} \mathbf{g} d\mathbf{r} \right]^2 - \left[d\mathbf{r}^2 + \frac{1}{1+h} (\mathbf{g} d\mathbf{r})^2 \right], \quad (21)$$

hence the length given by

$$dl^2 = d\mathbf{r}^2 + \frac{1}{1+h} (\mathbf{g} d\mathbf{r})^2 \quad (22)$$

and the time

$$dt' = \sqrt{1+h} \cdot \left[dt + \frac{1}{c(1+h)} \mathbf{g} d\mathbf{r} \right], \quad (23)$$

corresponding to the length dl . The difference $\Delta t = \frac{\mathbf{g} d\mathbf{r}}{c(1+h)}$ between the two times, $dt_1 = \frac{d\tau}{\sqrt{g_{00}}} = dt$ in the proper time (20) and $dt_2 = \frac{dt'}{\sqrt{g_{00}}} = dt + \frac{\mathbf{g} d\mathbf{r}}{c(1+h)}$ in the time given by (23), gives the difference in the synchronization of two simultaneous events, infinitesimally separated. The difference in time

depends on the path followed to reach a point starting from another point.

We limit ourselves to the first order in h, \mathbf{g} , and put $\mathbf{g} = -\frac{\mathbf{V}}{c}$, in order to investigate corrections to the motion under the action of a weak force in a flat space moving with a non-uniform velocity \mathbf{V} with respect to the observer. We will do the calculations basically for translations but a similar analysis can be made for rotations, using equation (16). For the observer, such a motion is then a free motion in a curved space with metric (3). The proper time is then $d\tau = (1 + \frac{h}{2}) dt$, the time given by (23) becomes $dt' = (1 + \frac{h}{2}) dt + \frac{\mathbf{g} d\mathbf{r}}{c}$ and the length is given by $dl^2 = d\mathbf{r}^2$, as for a three-dimensional euclidean space.

Hamilton-Jacobi equation. Let us assume that we have a particle moving freely in a flat space. We denote its contravariant momentum by $(P_0 = \frac{E_0}{c}, \mathbf{P})$ and the corresponding covariant momentum by $(P_0 - \mathbf{P})$, such that $P_0^2 - P^2 = m^2 c^2$, where E_0 is the energy of the particle, and P_0, \mathbf{P} are constant.

We can use the coordinate transformation given by (18) to get the momentum of the particle in the curved space. We prefer to write it down in its covariant form, using the metric (4). We get

$$\left. \begin{aligned} p_0 &= (1+h)p^0 + gp^1 = \sqrt{1+h} \frac{P_0 - \beta P_1}{\sqrt{1-\beta^2}} \\ p_1 &= gp^0 - p^1 = \frac{(g + \beta\Delta)P_0 - (g\beta + \Delta)P_1}{\sqrt{(1+h)(1-\beta^2)}} \end{aligned} \right\}. \quad (24)$$

Then, it seems that we would have already an integral of motion for the motion in the curved space, by using the definition $p_i = mc \frac{dx_i}{ds}$. However, this is not true, because the p_i are at point x' in the curved space, while the coefficients in the transformation (18) are at point x in the flat space. To know the global coordinate transformations $x(x')$ and $x'(x)$ would amount to solve in fact the equations of motion.

We can revert the above transformations for P_0 and P_1 , and make use of $P_0^2 - P^2 = m^2 c^2$, with $p_2 = -P_2, p_3 = -P_3$ for $g = -\beta$. We get

$$(p_0 + gp_1)^2 - \Delta^2(p^2 + m^2 c^2) = 0, \quad (25)$$

or

$$(E - c\mathbf{g}\mathbf{p})^2 - c^2(1+h+g^2)(p^2 + m^2 c^2) = 0, \quad (26)$$

where E is the energy of the particle and \mathbf{p} denotes its three-dimensional momentum. This is the relation between energy and momentum for the motion in the curved space. It gives the Hamilton-Jacobi equation.

Indeed, $p_i = -\frac{\partial S}{\partial x^i}$ and, obviously, for a free particle, $p_i p^i$ is a constant; we put $p_i p^i = m^2 c^2$ and get $g^{ij} p_i p_j = m^2 c^2$ or

$$\left(\frac{\partial S}{\partial t} + c\mathbf{g} \frac{\partial S}{\partial \mathbf{r}} \right)^2 - c^2(1+h+g^2) \left[\left(\frac{\partial S}{\partial \mathbf{r}} \right)^2 + m^2 c^2 \right] = 0. \quad (27)$$

In the limit $\hbar = \frac{2\varphi}{mc^2} \rightarrow 0$ and $\mathbf{g} = -\frac{\mathbf{V}}{c} \rightarrow 0$ it describes the relativistic motion of a particle under the action of the (weak) force $\mathbf{f} = -\frac{\partial\varphi}{\partial\mathbf{r}}$ and for an observer moving with a (small) velocity \mathbf{V} . One can check directly that the coordinate transformations given by equation (19) takes the free Hamilton-Jacobi equation $(\frac{\partial S}{\partial t})^2 - c^2[(\frac{\partial S}{\partial\mathbf{r}})^2 + m^2c^2] = 0$ into the “interacting” Hamilton-Jacobi equation (27), as expected.

The eikonal equation. Waves move through $k_i dx^i = -d\Phi$, where $k_i = -\frac{\partial\Phi}{\partial x^i} = (\frac{\omega}{c}, \mathbf{k})$, ω is the frequency, \mathbf{k} is the wavevector and Φ is called the eikonal. In a flat space k_i are constant, and the wave propagates along a straight line, such that $k_i k^i = 0$, i.e. $\frac{\omega^2}{c^2} - k^2 = 0$ and $\Phi = -\omega t + \mathbf{k}\mathbf{r}$. This is a light ray. In a curved space $k_i k^i = 0$ reads $g^{ij} k_i k_j = 0$, and for g^{ij} slightly departing from the flat metric we have the geometric approximation to the wave propagation. It is governed by the eikonal equation $g^{ij}(\frac{\partial\Phi}{\partial x^i})(\frac{\partial\Phi}{\partial x^j}) = 0$, or

$$\left(\frac{1}{c}\frac{\partial\Phi}{\partial t} + \mathbf{g}\frac{\partial\Phi}{\partial\mathbf{r}}\right)^2 - (1+h+g^2)\left(\frac{\partial\Phi}{\partial\mathbf{r}}\right)^2 = 0, \quad (28)$$

which is the Hamilton-Jacobi equation (27) for $m = 0$.

We neglect the g^2 -contributions to this equation and notice that the first term may not depend on the time (h is a function of the coordinates only). It follows then that the first term in the above equation can be put equal to $\frac{\omega_0}{c}$,

$$\frac{1}{c}\frac{\partial\Phi}{\partial t} + \mathbf{g}\frac{\partial\Phi}{\partial\mathbf{r}} = -\frac{\omega_0}{c}, \quad (29)$$

where ω_0 is the frequency of the wave in the flat space, and

$$\left(\frac{\partial\Phi}{\partial\mathbf{r}}\right)^2 = k^2 = \frac{1}{1+h}\left(\frac{\omega_0}{c}\right)^2 = \frac{1}{1+h}k_0^2, \quad (30)$$

where \mathbf{k}_0 is the wavevector in the flat space. Within our approximation equation (29) becomes

$$\frac{\partial\Phi}{c\partial t} = -\frac{\omega_0}{c} - \mathbf{g}\mathbf{k}_0. \quad (31)$$

We measure the frequency ω corresponding to the proper time, i.e. $\frac{\omega}{c} = -\frac{\partial\Phi}{c\partial\tau}$, where $d\tau = \sqrt{1+h}dt$ for our metric, so the measured frequency of the wave is given by

$$\frac{\omega}{c} = -\frac{\partial\Phi}{c\partial\tau} = -\frac{1}{\sqrt{1+h}}\frac{\partial\Phi}{c\partial t} = \frac{1}{\sqrt{1+h}}\frac{\omega_0}{c} + \mathbf{g}\mathbf{k}_0. \quad (32)$$

There exists, therefore, a shift in frequency

$$\frac{\Delta\omega}{\omega_0} = -\frac{h}{2} + \frac{c\mathbf{g}\mathbf{k}_0}{\omega_0}. \quad (33)$$

The first term in equation (33) is due to the static forces (like the gravitational potential, for instance), while the second term is analogous to the (longitudinal) Doppler effect, for $\mathbf{g} = -\frac{\mathbf{V}}{c}$.

By (30) we have

$$\left(\frac{\partial\Phi}{\partial\mathbf{r}}\right)^2 = (1-h)k_0^2. \quad (34)$$

We assume that h depends only on the radius r , and write the above equation in spherical coordinates; Φ does not depend on θ , and we put $\theta = \frac{\pi}{2}$;

$$\left(\frac{\partial\Phi}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial\Phi}{\partial\varphi}\right)^2 = (1-h)k_0^2; \quad (35)$$

the solution is of the form

$$\Phi = \Phi_r(r) + M\varphi, \quad (36)$$

where M is a constant and

$$\Phi_r(r) = \int_{\infty}^r dr \cdot \sqrt{(1-h)k_0^2 - \frac{M^2}{r^2}}; \quad (37)$$

the trajectory is given by $\frac{\partial\Phi}{\partial M} = \text{const.}^*$ hence

$$\varphi = -\int_{\infty}^r dr \cdot \frac{M}{r^2 \sqrt{(1-h)k_0^2 - \frac{M^2}{r^2}}}. \quad (38)$$

For $h = 0$ we get $r \sin\varphi = \frac{M}{k_0}$, which is a straight line passing at distance $\frac{M}{k_0}$ from the centre. The deviation angle is

$$\Delta\varphi = -\frac{k_0^2}{2} \int_{\infty}^r dr \frac{hM}{r^2 (k_0^2 - \frac{M^2}{r^2})^{3/2}}. \quad (39)$$

Therefore, the light ray is bent by the static forces in a curved space.[†] One can also define the refractive index \mathbf{n} of the curved space, by $\mathbf{k} = \mathbf{n}\frac{\omega}{c}$. Its magnitude is related to $\mathbf{g}\mathbf{k}_0$, while its direction is associated to the inhomogeneity h of the space.

It is worth noting, by (31), that the time-dependent part of the eikonal is given by

$$\Phi_t(t) = -\omega_0 t + \mathbf{k}_0\mathbf{R}(t), \quad (40)$$

for $\mathbf{g} = -\frac{\mathbf{V}}{c}$, i.e. the eikonal corresponding to a translation, as expected. A similar solution of the Hamilton-Jacobi equation can be obtained for massive particles.

Quantization. Suppose that we have a free motion. Then we know its solution, i.e. the dependence of the coordinates, say some x , on some parameter, which may be called some

*Constant M is a generalized moving freely coordinate; therefore, the force acting upon it vanishes, $\frac{\partial L}{\partial M} = 0$, or $\frac{d(\partial S/\partial M)}{dt} = 0$, i.e. $\frac{\partial S}{\partial M} = \text{const.}$

[†]The metric given by (3) for $h = \frac{2\varphi}{c^2}$ differs from the metric created by a gravitational point mass m with $\varphi = \frac{Gm}{r}$; they coincide only in the non-relativistic limit. The deviation angle given by (39) for a gravitational potential is smaller by a factor of 4 than the deviation angle in the gravitational potential of a point mass.

time t . Suppose further that we have a motion under the action of some forces. Then, we know the dependence of its coordinates, say some x' , on some parameter, which may be the same t as in the former case. Then, we may establish a correspondence between x and x' , *i.e.* a global coordinate transformation. It follows that the motion under the action of the forces is a global coordinate transformation applied to the free motion. Similarly, two distinct motions are put in relation to each other by such global coordinate transformations.

This line of thought, due to Einstein, lies at the basis of both the special theory of relativity and the general theory of relativity.

Indeed, it has been noticed that the equations of the electromagnetic field are invariant under Lorentz transformations of the coordinates, which leave the distance given by $s^2 = c^2 t^2 - \mathbf{r}^2$ invariant. These transformations are an expression of the principle of inertia, and this invariance is the principle of relativity. As such, the Lorentz transformations are applicable to the motion of particles, starting, for instance, from a particle at rest. Let $x = \frac{c\beta\tau}{\sqrt{1-\beta^2}}$, $t = \frac{\tau}{\sqrt{1-\beta^2}}$ be these Lorentz transformations where τ is the time of the particle at rest. We may apply these transformations to the momentum $\mathbf{p} = \frac{\partial S}{\partial \mathbf{r}}$ and $p_0 = -\frac{\partial S}{c\partial t} = \frac{E}{c}$, where E is the energy of the particle. Then, we get immediately $\mathbf{p} = \frac{vE}{c^2}$ and $E = \frac{E_0}{\sqrt{1-\beta^2}}$. The non-relativistic limit is recovered for $E_0 = mc^2$, the “inertia of the energy”. The equations of motion are $\frac{d\mathbf{p}}{dt} = \mathbf{f}$, and we can see that indeed, there appear additional, “dynamic forces”, depending on relativistic $\frac{v^2}{c^2}$ -terms, in comparison with Newton’s law. In addition, we get the Hamilton-Jacobi equation $E^2 - c^2(p^2 + m^2 c^2) = 0$. This is the whole theory of special relativity.

The situation is similar in the general theory of relativity, except for the fact that in a curved space we have not the global coordinate transformations, in general, as in a flat space. However, the Hamilton-Jacobi equation gives access to the action function, which may provide a relationship between some integrals of motion. Action S depends on some constants of integration, say M . Then, these constants can be viewed as freely-moving generalized coordinates, so $\frac{\partial S}{\partial M} = \text{const}$, because the force $\frac{\partial L}{\partial M} = \frac{d(\partial S/\partial M)}{dt}$ vanishes. Equation $\frac{\partial S}{\partial M} = \text{const}$ provides the equation of the trajectory. Of course, this is based upon the assumption that the motion is classical, *i.e.* non-quantum, in the sense that there exists a trajectory. For instance, the solution of the Hamilton-Jacobi equation for a free particle is $S = -Et + \mathbf{p}\mathbf{r}$, where E and \mathbf{p} are constants such that $E = \sqrt{m^2 c^4 + c^2 p^2}$. By $\frac{\partial S}{\partial E} = \text{const}$ we get $-t + \frac{E}{c^2 p^2} \mathbf{p}\mathbf{r} = \text{const}$, which is the trajectory of a free particle.

For a classical motion it is useless to attempt to solve the motion in a curved space produced by a non-inertial motion, like non-uniform translations, because it is much simpler to solve the motion in the absence of the non-inertial motion and

then get the solution by a coordinate transformation, like a non-uniform translation for instance. For a quantum motion, however, the things change appreciably.

The Hamilton-Jacobi equation admits another kind of motion too, the quantum motion. Obviously, for a free particle, the classical action given above is the phase of a wave. Then, it is natural to introduce a wavefunction ψ through $S = -i\hbar \ln \psi$, where \hbar turns out to be Planck’s constant. The classical motion is recovered in the limit $\hbar \rightarrow 0$, $\text{Re}\psi = \text{finite}$ and $\text{Im}\psi \rightarrow \infty$, such that $S = \text{finite}$. With this transformation we have $\mathbf{p} = -i\hbar \frac{\partial \psi / \partial \mathbf{r}}{\psi}$ and $E = i\hbar \frac{\partial \psi / \partial t}{\psi}$, which means that momentum and energy are eigenvalues of their corresponding operators, $-i\hbar \frac{\partial}{\partial \mathbf{r}}$ and $i\hbar \frac{\partial}{\partial t}$, respectively.* It follows that the physical quantities have not well-defined values anymore, in contrast to the classical motion. In particular, there is no trajectory of the motion. Instead, they have mean values and deviations, *i.e.* they have a statistical meaning, and the measurement process has to be defined in such terms. It turns out that the wavefunction squared is just the density of probability for the motion to be in some quantum state, and for a defined motion this probability must be conserved.

Klein-Gordon equation. With the substitution $E \rightarrow i\hbar \frac{\partial}{\partial t}$ and $\mathbf{p} \rightarrow -i\hbar \frac{\partial}{\partial \mathbf{r}}$ in the Hamilton-Jacobi equation in the flat space we get the Klein-Gordon equation

$$\frac{\partial^2 \psi}{\partial t^2} - c^2 \frac{\partial^2 \psi}{\partial \mathbf{r}^2} + \frac{m^2 c^4}{\hbar^2} \psi = 0. \quad (41)$$

A similar quantization for the Hamilton-Jacobi equation given by (27) encounters difficulties, since the operators $1 + \hbar + g^2$ and $p^2 + m^2 c^2$ do not commute with each other, nor with the operator $E - c\mathbf{g}\mathbf{p}$.† We may neglect the g^2 -term in $1 + \hbar + g^2$, and write the Hamilton-Jacobi equation (27) as

$$\frac{1}{1 + \hbar} (E - c\mathbf{g}\mathbf{p})^2 = c^2 (p^2 + m^2 c^2), \quad (42)$$

where the two operators in the left side of this equation commute now, up to quantities of the order of $\hbar g$ (or higher), which we neglect. With these approximations, the quantization rules can now be applied, and we get an equation which can be written as

$$\left(\frac{\partial}{\partial t} + c\mathbf{g} \frac{\partial}{\partial \mathbf{r}} \right)^2 \psi - c^2 (1 + \hbar) \left[\frac{\partial^2 \psi}{\partial \mathbf{r}^2} - \frac{m^2 c^2}{\hbar^2} \psi \right] = 0. \quad (43)$$

It can be viewed as describing the quantum motion of a particle under the action of a weak force $-\frac{mc^2}{2} \frac{\partial h(\mathbf{r})}{\partial \mathbf{r}}$, as seen by an observer moving with the small velocity $-c\mathbf{g}(t)$. It can

*Einstein’s (1905) quantization of energy and de Broglie’s (1923) quantization of momentum follow immediately by this assumption, which gives a meaning to the Bohr-Sommerfeld quantization rules (Bohr, 1913, Sommerfeld, 1915). The quantum operators was first seen as matrices by Heisenberg, Born, Jordan, Pauli (1925-1926).

†We recall that \hbar is a function of the coordinates only, $\hbar(\mathbf{r})$, and \mathbf{g} is a function of the time only, $\mathbf{g}(t)$.

be derived directly from (41) by the coordinate transformations (19), in the limit $\hbar, \mathbf{g} \rightarrow 0$.^{*} It is worth noting, however, that there is still a slight inaccuracy in deriving this equation, arising from the fact that the operator $(1 + \hbar)(p^2 + m^2 c^2)$ is not hermitean. It reflects the indefiniteness in writing $(1 + \hbar)(p^2 + m^2 c^2)$ or $(p^2 + m^2 c^2)(1 + \hbar)$ when passing from (42) to (43). This indicates the ambiguities in quantizing the relativistic motion, and they are remedied by the theory of the quantal fields, as it is shown below.

The above equation can be written more conveniently as

$$\left(i\hbar \frac{\partial}{\partial t} - c\mathbf{g}\mathbf{p} \right)^2 \psi - c^2(1 + \hbar)(p^2 + m^2 c^2)\psi = 0, \quad (44)$$

where $\mathbf{p} = -i\hbar \frac{\partial}{\partial \mathbf{r}}$ and $i\hbar \frac{\partial}{\partial t}$ stands for the energy E .

We introduce the operator

$$\begin{aligned} H^2 &= c^2(1 + \hbar)(p^2 + m^2 c^2) = \\ &= c^2(p^2 + m^2 c^2) + c^2 \hbar(p^2 + m^2 c^2), \end{aligned} \quad (45)$$

which is time-independent, and treat the \hbar -term as a small perturbation. It is easy to see, in the first-order of the perturbation theory, that the wavefunctions are labelled by momentum \mathbf{p} , and are plane waves with a weak admixture of plane waves of the order of \hbar ; we denote them by $\varphi(\mathbf{p})$. Similarly, in the first-order of the perturbation theory, the eigenvalues of H^2 can be written as $E^2(\mathbf{p}) = c^2(1 + \hbar)(p^2 + m^2 c^2)$, where $\hbar = \frac{1}{V} \int d\mathbf{r} \cdot \hbar$, V being the volume of normalization. We have, therefore, $H^2 \varphi(\mathbf{p}) = E^2(\mathbf{p}) \varphi(\mathbf{p})$. Now, we look for a time-dependent solution of equation (44) $(i\hbar \frac{\partial}{\partial t} - c\mathbf{g}\mathbf{p})^2 \psi = H^2 \psi$, which can also be written as $(i\hbar \frac{\partial}{\partial t} - c\mathbf{g}\mathbf{p}) \psi = H \psi$, where ψ is a superposition of eigenfunctions

$$\psi = \sum_{\mathbf{p}} c_{\mathbf{p}}(t) e^{-iE(\mathbf{p})t/\hbar} \varphi(\mathbf{p}). \quad (46)$$

We get

$$\dot{c}_{\mathbf{p}'} = -\frac{i}{\hbar} \sum_{\mathbf{p}} c_{\mathbf{p}} e^{-i[E(\mathbf{p}) - E(\mathbf{p}')]/\hbar} c\mathbf{g}\mathbf{p}_{\mathbf{p}'\mathbf{p}}, \quad (47)$$

where $\mathbf{p}_{\mathbf{p}'\mathbf{p}}$ is the matrix element of the momentum \mathbf{p} between the states $\varphi(\mathbf{p}')$ and $\varphi(\mathbf{p})$. We assume $c_{\mathbf{p}} = c_{\mathbf{p}}^0 + c_{\mathbf{p}}^1$, such as $c_{\mathbf{p}'}^0 = 0$ for all $\mathbf{p}' \neq \mathbf{p}$ and $c_{\mathbf{p}}^0 = 1$, and get

$$\dot{c}_{\mathbf{p}'}^1 = -\frac{i}{\hbar} e^{-i[E(\mathbf{p}) - E(\mathbf{p}')]/\hbar} c\mathbf{g}\mathbf{p}_{\mathbf{p}'\mathbf{p}}, \quad (48)$$

which can be integrated straightforwardly. The square $|c_{\mathbf{p}'}^1|^2$ gives the transition probability from state $\varphi(\mathbf{p})$ in state $\varphi(\mathbf{p}')$.

It follows that an observer in a non-uniform translation might see quantum transitions between the states of a relativistic

particle, providing the frequencies in the Fourier expansion of $\mathbf{g}(t)$ match the difference in the energy levels. In the zeroth-order of the perturbation theory the eigenfunctions $\varphi(\mathbf{p})$ are plane waves, and the matrix elements $\mathbf{p}_{\mathbf{p}'\mathbf{p}}$ of the momentum vanish, so there are no such transitions to this order. In general, if the total momentum is conserved, as for free or interacting particles, these transitions do not occur. In the first order of the perturbation theory for the external force represented by \hbar the matrix elements of the momentum do not vanish, in general, and we may have transitions, as an effect of a non-uniform translation. Within this order of the perturbation theory the matrix elements of the momentum are of the order of \hbar , and the transition amplitudes given by (48) are of the order of $g\hbar$. We can see that the time-dependent term of the order of $g\hbar$ neglected in deriving equation (44) produces corrections to the transition amplitudes of the order of $g\hbar^2$, so its neglect is justified.

In general, the solution of the second-order differential equation (43) can be approached by using the Fourier transform. Then, it reduces to a homogeneous matrixial equation, where labels are the frequency and the wavevector (ω, \mathbf{k}) , conveniently ordered. The condition of a non-trivial solution is the vanishing of the determinant of such an equation. This gives a set of conditions for the ordered points (ω, \mathbf{k}) in the (ω, \mathbf{k}) -space, but these conditions do not provide anymore an algebraic connection between the frequency ω and the wavevector \mathbf{k} . This amounts to saying that for a given ω the wavevectors are not determined, and, conversely, for a given wavevector \mathbf{k} the frequencies are not determined, *i.e.* the quantum states do not exist in fact, anymore. The particle exhibits quantum transitions, which make its quantum state undetermined. The same conclusion can also be seen by introducing a non-uniform translation in the phase of a plane wave, expanding the plane wave with respect to this translation, under certain restrictions, and then using the time Fourier expansion of the translation. The frequency of the original plane wave changes correspondingly, which indicates indeed that there are quantum transitions. One may say that for a curved space as the one represented by the metric given here, the quantization question has no meaning anymore, or it has the meaning given here.

In the non-relativistic limit, the above Klein-Gordon equation becomes

$$i\hbar \frac{\partial \psi}{\partial t} = H \psi = \left(mc^2 + \frac{p^2}{2m} + \varphi \right) \psi + c\mathbf{g}\mathbf{p} \psi, \quad (49)$$

which is Schrödinger's equation up to the rest energy mc^2 , and one can see more directly the perturbation $c\mathbf{g}\mathbf{p} = -\mathbf{V}\mathbf{p}$. It is worth noting that the derivation of Schrödinger's equation holds irrespectively of the ambiguities related to the quantization of the Hamilton-Jacobi equation. It follows, that under the conditions mentioned above, *i.e.* in the presence of a (non-trivial) external field φ , an observer in a non-uniform translation may observe quantum transitions in the non-

^{*}It has to be compared with the Klein-Gordon equation written as $(i\hbar \frac{\partial}{\partial t} - e\varphi)^2 \psi - c^2 \left[\left(i\hbar \frac{\partial}{\partial \mathbf{r}} + \frac{e\mathbf{A}}{c} \right)^2 + m^2 c^2 \right] \psi = 0$ for a particle with charge e in the electromagnetic field (φ, \mathbf{A}) , which, historically, was first considered for the Hydrogen atom (Schrödinger, Klein, Gordon, 1926). There, the forces come by the electromagnetic gauge field.

relativistic limit, due to the non-inertial motion.* Obviously, the frequency of this motion must match the quantum energy gaps, for such transitions to be observed.

Similar considerations hold for the metric corresponding to rotations. It is the hamiltonian (14) which is subjected to quantization in that case, so we may have quantum transitions between the states of the particle, providing these states do not conserve the angular momentum. This requires a force, as the one given by a potential φ . The $\Omega \times \mathbf{r}$ is exactly the rotation velocity \mathbf{V} , so we can apply directly the formalism developed above for a non-uniform translation to a non-uniform rotation. The only difference is that the \mathbf{g} for rotations depends on the spatial coordinates too, beside its time dependence. The \mathbf{g} -interaction gives rise to terms of the type $\Omega \mathbf{L}$, and the evaluation of the matrix elements in the interacting terms becomes more cumbersome. It is worth keeping in mind the condition $\Omega r \ll c$ in such evaluations.

The difficulties encountered above with the quantization of the Klein-Gordon equation in curved spaces remain for a corresponding Dirac equation. It is impossible, in general, to get a Dirac equation for equation (43), because the operators $(1 - \frac{\hbar}{2})(E - c\mathbf{g}\mathbf{p})$ and $\alpha c\mathbf{p} + \beta mc^2$ (with α and β the Dirac matrices), which represent the square roots of the two sides of equation (42), do not commute anymore. Nevertheless, if we limit ourselves to the first order of the perturbation theory, we can see that the operator H^2 defined above reduces to $c^2(p^2 + m^2)$ providing we redefine the energy levels such as to include the factor $1 + \hbar$. Within this approximation, we get the Dirac equation

$$\left(i\hbar \frac{\partial}{\partial t} - c\mathbf{g}\mathbf{p} \right) \psi = (\alpha c\mathbf{p} + \beta mc^2) \psi, \quad (50)$$

where ψ contains now a weak admixture of plane waves, of the order of \hbar . It is worth noting that this equation is the Dirac equation corresponding to (41), subjected to the translation $\mathbf{r} = \mathbf{r}' + \mathbf{R}$, and $t = t'$. The non-uniform translation in the left side of equation (50) gives now quantum transitions.

As it is well-known, there remain problems with the quantization of the Klein-Gordon equation, which are not solved by the Dirac equation. These problems find for themselves a natural solution with the quantum fields.

A scalar field in a curved space. Let

$$S = \int dx^0 d\mathbf{r} \sqrt{-g} \left[(\partial_i \psi)(\partial^i \psi) + \frac{m^2 c^2}{\hbar^2} \psi^2 \right] \quad (51)$$

be the lagrangian for the (real) scalar field ψ , where $g = -\Delta^2 = -(1 + h + g^2)$ is the determinant of the metric given

*A suitable unitary transformation of the wavefunction — for instance, $\exp(-i \frac{\mathbf{R}\mathbf{p}}{\hbar})$ — can produce such an interaction in the time-dependent left side of the Schrödinger equation, but, at the same time, it produces an equivalent interaction in the hamiltonian, such that the Schrödinger equation is left unchanged. Such unitary transformations are related to symmetries (Wigner's theorem, 1931) and they are different from a change of coordinates.

by (4).[†] It is easy to see that the principle of least action for ψ in a flat space leads to the Klein-Gordon equation (41). For the metric given by (4), and neglecting g^2 -terms, we get a generalized Klein-Gordon equation

$$\left(\frac{\partial}{\partial t} + c\mathbf{g} \frac{\partial}{\partial \mathbf{r}} \right) \frac{1}{\sqrt{1+h}} \left(\frac{\partial}{\partial t} + c\mathbf{g} \frac{\partial}{\partial \mathbf{r}} \right) \psi - c^2 \frac{\partial}{\partial \mathbf{r}} \sqrt{1+h} \frac{\partial}{\partial \mathbf{r}} \psi + \sqrt{1+h} \frac{m^2 c^4}{\hbar^2} \psi = 0. \quad (52)$$

We can apply the same perturbation approach to this equation as we did for equation (42). Doing so, we get equation (44) and an additional term $i \frac{c^2 \hbar}{2} \frac{\partial h}{\partial \mathbf{r}} \mathbf{p}$, which yields no difficulties in the perturbation approach. The resulting equation reads

$$\left(i\hbar \frac{\partial}{\partial t} - c\mathbf{g}\mathbf{p} \right)^2 \psi - c^2(1+h)(p^2 + m^2 c^2) \psi + \frac{i c^2 \hbar}{2} \left(\frac{\partial h}{\partial \mathbf{r}} \right) \mathbf{p} \psi = 0. \quad (53)$$

It is worth noting that in the limit $\mathbf{g} \rightarrow 0$ this is an exact equation. The qualitative conclusions derived above for equation (44), as regards the quantum transitions produced by the non-uniform translation, remain valid, though, we have now a language of fields. It follows that a quantum particle, either relativistic or non-relativistic, in a curved space of the form analyzed herein becomes a wave packet from a plane wave (or even forms a bound state), as a consequence of the forces, and, at the same time, it may suffer quantum transitions, due to the time-dependent metric (as if in a non-inertial translation for instance). This gives no meaning to the problem of the quantization in curved spaces, or it gives the meaning discussed here.

The density L of lagrangian in the action $S = \int dt d\mathbf{r} \cdot L$ given by (51) gives the momentum $\Pi = \frac{\partial L}{\partial(\partial\psi/\partial t)}$ and the hamiltonian density $\Pi \frac{\partial\psi}{\partial t} - L$. The quantized field reads

$$\psi = \sum_{\mathbf{p}} \frac{c\hbar}{2\sqrt{\varepsilon}} (a_{\mathbf{p}} e^{-i\mathbf{p}\mathbf{r}/\hbar + i\mathbf{p}\mathbf{r}/\hbar} + a_{\mathbf{p}}^{\dagger} e^{i\mathbf{p}\mathbf{r}/\hbar - i\mathbf{p}\mathbf{r}/\hbar}), \quad (54)$$

and

$$\Pi = -i \sum_{\mathbf{p}} \frac{\sqrt{\varepsilon}}{c} (a_{\mathbf{p}} e^{-i\mathbf{p}\mathbf{r}/\hbar + i\mathbf{p}\mathbf{r}/\hbar} - a_{\mathbf{p}}^{\dagger} e^{i\mathbf{p}\mathbf{r}/\hbar - i\mathbf{p}\mathbf{r}/\hbar}), \quad (55)$$

where $\varepsilon = c \sqrt{m^2 c^2 + p^2}$ and $[\psi(t, \mathbf{r}), \Pi(t, \mathbf{r}')] = i\hbar \delta(\mathbf{r} - \mathbf{r}')$ with usual commutation relations for the bosonic operators $a_{\mathbf{p}}$, $a_{\mathbf{p}}^{\dagger}$ and a normalization of one \mathbf{p} -state in a unit volume. The hamiltonian is obtained by integrating its density given

[†]In general, the action for fields must be written by replacing the flat metric η_{ij} by the curved metric g_{ij} (including $\sqrt{-g}$ in the elementary volume of integration) and replacing the derivatives ∂_i by covariant derivatives D_i . The latter requirement can produce technical difficulties, in general. However, for a scalar field or for the electromagnetic field the D_i has the same effect as ∂_i , so the former are superfluous.

above over the whole space. It can be written as $H = H_0 + H_{1h} + H_{1g}$, where

$$H_0 = \int d\mathbf{r} \cdot \left[\frac{1}{4} c^2 \Pi^2 + \left(\frac{\partial \psi}{\partial \mathbf{r}} \right)^2 + \frac{m^2 c^2}{\hbar^2} \psi^2 \right] = \sum_{\mathbf{p}} \frac{\varepsilon}{2} (a_{\mathbf{p}} a_{\mathbf{p}}^+ + a_{\mathbf{p}}^+ a_{\mathbf{p}}) \quad (56)$$

is the free hamiltonian,

$$H_{1h} = \int d\mathbf{r} \times (\sqrt{1+h} - 1) \left[\frac{1}{4} c^2 \Pi^2 + \left(\frac{\partial \psi}{\partial \mathbf{r}} \right)^2 + \frac{m^2 c^2}{\hbar^2} \psi^2 \right] \quad (57)$$

is the interacting part due to the external field h and

$$H_{1g} = -\frac{c}{2} \int d\mathbf{r} \cdot \left[\Pi \left(\mathbf{g} \frac{\partial \psi}{\partial \mathbf{r}} \right) + \left(\mathbf{g} \frac{\partial \psi}{\partial \mathbf{r}} \right) \Pi \right] = -\frac{c}{2} \sum_{\mathbf{p}} (\mathbf{g}\mathbf{p})(a_{\mathbf{p}} a_{\mathbf{p}}^+ + a_{\mathbf{p}}^+ a_{\mathbf{p}}) \quad (58)$$

is the time-dependent interaction. Perturbation theory can now be applied systematically to the first-order of \mathbf{g} and all the orders of h , with the same results as those described above: the quanta will scatter both their wavevectors and their energy. Similar field theories can be set up for charged particles, or for particles with spin $\frac{1}{2}$ and for photons, moving in a curved space given by the metric (4).

Electromagnetic field in curved spaces. Photons. The action for the electromagnetic field is

$$S = -\frac{1}{16\pi c} \int dx^0 d\mathbf{x} \cdot \sqrt{-g} F_{ij} F^{ij}, \quad (59)$$

where the electromagnetic fields F_{ij} are given by the potentials A_i through $F_{ij} = \partial_i A_j - \partial_j A_i$. This leads immediately to the first pair of Maxwell equations (the free equations) $\partial_i F_{jk} + \partial_j F_{ki} + \partial_k F_{ij} = 0$ and the principle of least action gives the second pair of Maxwell equations

$$\partial_j (\sqrt{-g} F^{ij}) = 0. \quad (60)$$

In the presence of charges and currents the right side of equation (60) contains the current, conveniently defined. The antisymmetric tensor F_{ij} consists of a vector and a three-tensor in spatial components, the latter being representable by another vector, its dual. Let these vectors be denoted by \mathbf{E} and \mathbf{B} . Similarly, by raising or lowering the suffixes we can define other two vectors, related to the former pair of vectors, and denoted by \mathbf{D} and \mathbf{H} . Then, the Maxwell equations obtained above take the usual form of Maxwell equations in matter, namely $\text{curl } \mathbf{E} = -\frac{1}{c\sqrt{\gamma}} \frac{\partial(\sqrt{\gamma}\mathbf{B})}{\partial t}$, $\text{div } \mathbf{B} = 0$ (the free equations) and $\text{div } \mathbf{D} = 4\pi\rho$, $\text{curl } \mathbf{H} = \frac{1}{c\sqrt{\gamma}} \frac{\partial(\sqrt{\gamma}\mathbf{D})}{\partial t} + \frac{4\pi}{c} \rho \mathbf{v}$,

where ρ is the density of charge divided by $\sqrt{\gamma}$ and $\gamma_{\alpha\beta} = -g_{\alpha\beta} + \frac{g_{0\alpha}g_{0\beta}}{g_{00}}$ is the spatial metric (div and curl are conveniently defined in the curved space). For our metric, and neglecting g^2 , the matrix γ reduces to the euclidean metric of the space ($\gamma = 1$).

We use $A_0 = 0$, $F_{0\alpha} = \partial_0 A_\alpha$ and $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$. We define an electric field $\mathbf{E} = \text{grad } \mathbf{A}$ and a magnetization field $\mathbf{B} = -\text{curl } \mathbf{A}$. Then, neglecting g^2 , equation (60) can be written as

$$\text{div} \left[\frac{1}{\Delta} (\mathbf{E} + \mathbf{g} \times \mathbf{B}) \right] = 0 \quad (61)$$

and

$$\frac{\partial}{c\partial t} \left[\frac{1}{\Delta} (\mathbf{E} + \mathbf{g} \times \mathbf{B}) \right] = \text{curl} \left[\Delta \mathbf{B} + \frac{1}{\Delta} \mathbf{g} \times \mathbf{E} \right], \quad (62)$$

where $\Delta = \sqrt{1+h}$. One can see that we may have a displacement field $\mathbf{D} = \frac{\mathbf{E} + \mathbf{g} \times \mathbf{B}}{\Delta}$ and a magnetic field $\mathbf{H} = \Delta \mathbf{B} + \frac{\mathbf{g} \times \mathbf{E}}{\Delta}$, and the Maxwell equations $\text{div } \mathbf{D} = 0$, $\frac{\partial \mathbf{D}}{c\partial t} = \text{curl } \mathbf{H}$ without charges.

Equations (61) and (62) can be solved by the perturbation theory, for small values of h and \mathbf{g} , starting with free electromagnetic waves as the unperturbed solution. Doing so, we arrive immediately at the result that the solution must be a wave packet, and the frequencies are not determined anymore, in the sense that either for a given wavevector we have many frequencies or for a given frequency we have many wavevectors. This can be most conveniently expressed in terms of photons which suffer quantum transitions.

The quantization of the electromagnetic field in a curved space proceeds in the usual way. The action given by (59) can be written as

$$S = \frac{1}{8\pi} \int dt d\mathbf{r} \cdot \Delta (\mathbf{D}^2 - \mathbf{B}^2) = \frac{1}{8\pi} \int dt d\mathbf{r} \cdot \frac{1}{\Delta} [\mathbf{E}^2 + 2\mathbf{E}(\mathbf{g} \times \mathbf{B}) - \Delta^2 \mathbf{B}^2], \quad (63)$$

which exhibits the well-known density of lagrangian in the limit $h, \mathbf{g} \rightarrow 0$. We change now to the covariant vector potential $\mathbf{A} \rightarrow -\mathbf{A}$, such that $\mathbf{E} = -\frac{\partial \mathbf{A}}{c\partial t}$ and $\mathbf{B} = \text{curl } \mathbf{A}$. Leaving aside the factor $\frac{1}{8\pi}$, the momentum is given by $\Pi = \frac{\partial L}{\partial(\partial \mathbf{A}/\partial t)} = \frac{2}{\Delta c^2} \left(\frac{\partial \mathbf{A}}{\partial t} - \mathbf{g} \times \mathbf{B} \right)$. The vector potential is represented as

$$\mathbf{A}_\alpha = \sum_{\alpha\mathbf{p}} \frac{c\hbar}{2\sqrt{\varepsilon}} \left[a_{\alpha\mathbf{p}} \mathbf{e}^\alpha e^{-i\mathbf{p}\mathbf{r}/\hbar + i\mathbf{p}\mathbf{r}/\hbar} + hc \right] \quad (64)$$

and the momentum by

$$\Pi_\alpha = -i \sum_{\alpha\mathbf{p}} \frac{\sqrt{\varepsilon}}{c} \left[a_{\alpha\mathbf{p}} \mathbf{e}^\alpha e^{-i\mathbf{p}\mathbf{r}/\hbar + i\mathbf{p}\mathbf{r}/\hbar} - hc \right], \quad (65)$$

where \mathbf{e}^α is the polarization vector along the direction α , perpendicular to $\mathbf{p} = \hbar \mathbf{k}$ (we assume the transversality condition $\text{div } \mathbf{A} = 0$), $\varepsilon = \hbar\omega = cp$, while ω is the frequency and \mathbf{k} is the wavevector. The commutation relations are the usual bosonic

ones, and we get the hamiltonian $H = H_0 + H_{1h} + H_{1g}$, given by

$$\left. \begin{aligned} H_0 &= \int d\mathbf{r} \cdot \left(\frac{1}{4} c^2 \Pi^2 + B^2 \right) = \\ &= \sum_{\alpha\mathbf{p}} \frac{\varepsilon}{2} (a_{\alpha\mathbf{p}}^+ a_{\alpha\mathbf{p}} + a_{\alpha\mathbf{p}} a_{\alpha\mathbf{p}}^+) \\ H_{1h} &= \int d\mathbf{r} \cdot (\sqrt{1 + \hbar} - 1) \left(\frac{1}{4} c^2 \Pi^2 + \mathbf{B}^2 \right) \\ H_{1g} &= -\frac{1}{2} \sum_{\alpha\mathbf{p}} \mathbf{g}\mathbf{p} (a_{\alpha\mathbf{p}}^+ a_{\alpha\mathbf{p}} + a_{\alpha\mathbf{p}} a_{\alpha\mathbf{p}}^+) \end{aligned} \right\} \quad (66)$$

Systematic calculations can now be performed within the perturbation theory, and we can see that quantum transitions between the photonic states may appear, starting with the hg -order of the perturbation theory. Therefore, an observer moving with a non-uniform velocity is able to see a “blue shift” in the frequency of the photons “acted” by a force like the gravitational one.* The shift occurs obviously at the expense of the energy of the observer’s motion.†

Other fields. A similar approach can be used for other fields in a curved space. In particular, it can be applied to spin-1/2 Dirac fields, with similar conclusions, though, technically, it is more cumbersome to write down the action for spinors in curved spaces. It can be speculated upon the question of quantizing the gravitational field in a similar manner. Indeed, weak perturbations of the flat metric can be represented as gravitational waves, which can be quantized by using the gravitational action $\int dx^0 d\mathbf{r} \cdot \sqrt{-g} R$, where R is the curvature of the space.‡ Now, we may suppose that these gravitons move in a curved space with the metric g . We may use the same gravitational action as before, where g is now the metric of the space and R contains the graviton field. Or, alternately, we expand $g = g_0 + \delta g$, where g_0 is the background part and δg is the graviton part. We get a field theory of gravitons interacting with the underlying curved space, and we get quantum transitions of the gravitons, which gives a meaning to the quantization of the gravity, in the sense that either it is not possible or the gravitons suffer quantum transitions. The space and time (the gravitons) are then scattered statistically

*This is similar with the Unruh effect (1976).

†It is worth investigating the change in the equilibrium distribution of the black-body radiation as a consequence of the non-uniform translation in a gravitational field. The frequency shift amounts to a change of temperature, which increases, most likely, by $\frac{\Delta T}{T} \sim (g\hbar)^2$, with temporal and spatial averages (for the quantization of the black-body radiation see Fermi, 1932). In this respect, the effect discussed here, though related to the Unruh effect, is different. The Unruh effect assumes rather that the external non-uniform translation, as a macroscopic motion, consists of a coherent vacuum, so equilibrium photons can be created; the related increase in temperature is rather the measurement made by the observer of its own motion.

‡Though there are difficulties in establishing a relativistically-invariant quantum theory for particles with helicity 2, like the gravitons. Another related difficulty is the general non-localizability of the gravitational energy.

by matter (which in turn suffers a similar process) or by the non-inertial motion.

Conclusion. The quantum motion implies, basically, delocalized waves, like plane wave, both in space and time. The general theory of relativity, gravitation or curved space as the one discussed here, arising from weak static forces and non-inertial motion, imply localized field, both in space and time. Consequently, the quantization is destroyed in those situations involved by the latter case, in the sense that quanta are scattered both in energy and the wavevector, and we have to deal there with transition amplitudes and probabilities, *i.e.* with a statistical perspective. The basic equations for the classical motion in these cases become meaningful only with scattered quanta. This shows indeed that the quantization is both necessary and illusory. The basic aspect of the natural world is its statistical character in terms of quanta.

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LETTERS TO PROGRESS IN PHYSICS**Where is the Science?**

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The important rôle played in society today by scientific research is highlighted, and the related various social, economical and political conditionings of science are discussed. It is suggested that the exclusive emphasis upon the multiple technological applications of science, the use and abuse of scientific research, may lead to the very disappearance of science, transforming scientific research into a routine and almost ritualistic activity, empty of any real content. This may already be seen in the inadequate way present day society tackles the fundamental problems we are confronted with, issues such as the environment, conflict, life and the thinking process.

Science is used and misused today in a great variety of ways, in all of the utmost relevance to human life and activity. Worldwide policy has found it useful for science to be employed by the military, and developed nations spend generously on this application of science. New, sophisticated, powerful weaponry is produced today, by an application of scientific achievements. It has also been found beneficial to put science to work for a more comfortable life; highly-developed technologies, industry, manufacturing, farming, agriculture, commerce, services, transport and communications are science-based today. Education, culture, civilization, a highly-qualified work force are produced on the basis of science. Everything that matters to humans, namely wealth, fame and pleasure, is achieved on an ever larger scale today by using science. Modern science is viewed as an immensely beneficial resource, whose rôle in society is to be tapped more and more for the greatest of profit. In this respect, everybody talks now only of “technology transfer”, “competitiveness”, “innovation”, “leadership”, and last but not least, of “intellectual leadership”, through science. Science is everywhere “oriented” on our epoch towards the military, warfare, technology, industry, economy, education, etc, etc. There is no more “simply science”; it is everywhere determined, oriented, conditioned.

Scientists should feel well and flattered by the great interest shown by society in their art and trade. The fact is that science has provided much for society, through mechanical constructions, thermal machines, electricity, nuclear energy, materials, electronics, and it is natural for society to try to control, accelerate and harness all this in the process of profiting by the use and abuse of science.

Yet nobody is satisfied with such a policy, all around the world. Taxpayers want more and more from science, and the scientists are more and more incapable of responding to their high demands. In addition, politicians stir up heavily this

conflictual issue. The reason for such a failure resides in the inadequacy of this type of science policy.

Indeed, science is not funded, according to this policy, unless it produces something immediately relevant to society, i.e. something useful for the military, for industry, the economy, education, etc. Scientific research, which is the way science advances, is only desired for its applications. Yet all these outlets for science, in various areas of activity and interest, are not science; they are only its applications. Science policy today greatly confuses science with its applications. By laying emphasis exclusively on applications we will end up having no science at all.

Science is a resource, like any other, and yet a bit special. Of course, scientific knowledge does not fade, or degrade, by repeated use, it is not wasted or dissipated by using it. Newton's laws do not vanish by being repeatedly used. But people who have scientific knowledge, and who at least endeavour to maintain it, if not advancing it, i.e. those we call scientists, disappear, if not properly cultivated. We have a lot of applications of science, a serious endeavour for technology transfer, great expectations from using this science, but where is the science? We have no science anymore by such policy which provides exclusively for scientific applications, irrespective of how desirable and beneficial they might be.

A very deeply-rooted fallacy is to think that scientists are in universities. This is profoundly wrong. In universities we have professors who teach science to young people. They need to acquire scientific fuel for this teaching process, from elsewhere. We cannot say reasonably that teachers in universities do both science and teaching contemporaneously, because they then do either half of each or half of neither. It is more appropriate to emphasize the exclusive educational task of the universities, and provide separately for scientists, in distinct laboratories, institutes, etc. The great advances in science and in its applications made by the former Soviet

Union and the USA in the last half of the past century were achieved precisely because these States cultivated distinctly science and scientists, and did not mix up science with teaching or production.

Of course, these things are related, and it is desirable and profitable to cultivate such naturally beneficial relations. How are we going to strengthen the relations between universities, scientists and high-tech entrepreneurs? Simply by doing precisely what we need to: by providing for close relationships between such people, encouraging their meetings, discussions, talks, cooperation, etc. The main cause of the difficulties and dissatisfaction today with the “failure” of science in society is due precisely to the vanishing relationship between scientists, technologists, entrepreneurs, and teachers. We need to urgently provide for such close contacts, but we have to be very careful not to mix things up: to keep the distinction between these socio-professional categories. It is a scientific fact that distinctiveness and variety produce force and motion, whilst admixture increases only the potential of ineffectiveness, resulting in only a restful peace.

If we are going to cultivate, by our policies, the distinction between scientists, teachers, professors, technologists, entrepreneurs, to provide for close collaborative relationships between all them, keeping at the same time the distinction, and not to mistake science and scientific research for teaching or production, then we will be more scientific in our endeavours, and will be more fortunate in our expectations.

We are yet pretty unscientific with respect to basic issues. For instance, nowadays we set for science the mission of reducing, or circumventing, the degradation of the environment, without noticing that every human activity degrades the environment. Indeed, even the mental processes degrade their environment; brains in this case. Life is an organized process whereby entropy is diminished, and therefore it is a great fluctuation, but at the same time we increase also the environmental entropy, including that of our own body, just by living, and the increase is greater than the decrease, and the process goes to equilibrium. We will end with a more balanced world, where life will become extinct, because the fluctuations diminish near equilibrium. We would think of finding a solution for preserving life by creating artificially another similar fluctuation, then with a greater spending of energy. The inherent limitations of such an artificial process will then pose serious issues regarding how, who and how many would be going to live that artificial life. This may present a serious problem for science and technology, and for the future of our society. Another is the process of thinking, for many believe that we should think the thinking process in order to understand what we are tinking. First, they assume erroneously that there exists a conscience, or a consciousness, i.e. a state or process of thinking the thinking process, which is false. Anyone who thinks is not conscious of what he or she is doing, there is no double thinking; consciousness is identical to thinking itself. Thinking is a natural process, associated with

the complexity of the human brain, and so we do not think of thinking, because it is impossible, we just do it. To think is just to be. Such sorts of things we only learn through science, so, providing in our policies for properly cultivating science will greatly enhance our chances of responding to truly relevant questions. Besides, life and the thinking process may be manipulated and controlled by others, but never in those who are doing that. But full power is illusory. We may destroy science in others but never in ourselves. The need for scientific knowledge is essential for survival.

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*LETTERS TO PROGRESS IN PHYSICS***On the Current Situation Concerning the Black Hole Problem**

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This paper reviews a new solution which concerns the black hole problem. The new solution, by S. J. Crothers, doesn't eliminate the line-element of the classical "black hole solution" produced by the founders of the problem, but represents the gravitational collapse condition in terms of physical observable quantities accessible to a real observer whose location is in the real Schwarzschild space itself, not with the quantities in an abstract flat space tangential to it at the point of observation (as it was in the classical solution). Besides, Schwarzschild space is only a very particular case of Einstein spaces of type I. There are minor studies on the physical conditions of gravitational collapse in other spaces of type I, and nothing on Einstein spaces of type II and type III (of which there are hundreds). Einstein spaces (empty spaces, without distributed matter, wherein Ricci's tensor is proportional to the fundamental metric tensor), are spaces filled by an electromagnetic field, dust, or other substances, of which there are many. As a result, studies on the physical conditions of gravitational collapse are only in their infancy.

In a series of pioneering papers, starting in 1979, Leonard S. Abrams (1924–2001) discussed [1] the physical sense of the black hole solution. Abrams claimed that the correct solution for the gravitational field in a Schwarzschild space (an empty space filled by a spherically symmetric gravitational field produced by a spherical source mass) shouldn't lead to a black hole as a physical object. Such a statement has profound consequences for astrophysics.

It is certain that if there is a formal error in the black hole solution, committed by the founders of this theory, in the period from 1915–1920's, a long list of research produced during the subsequent decades would be brought into question. Consequently, Abrams' conclusion has attracted the attention of many physicists. Since millions of dollars have been invested by governments and private organizations into astronomical research connected with black holes, this discussion ignited the scientific community.

Leonard S. Abrams' professional reputation is beyond doubt. As a result, it is particularly noteworthy to observe that Stephen J. Crothers [2], building upon the work of Abrams, was able to deduce solutions for the gravitational field in a Schwarzschild metric space produced in terms of a physical observable (proper) radius. Crothers' solutions fully verify the initial arguments of Abrams. Therefore, the claim that the correct solution for the gravitational field in a Schwarzschild space does not lead to a black hole as a physical object requires serious attention.

Herein, it is important to give a clarification of Crothers' solution from the viewpoint of a theoretical physicist whose professional field is the General Theory of Relativity. The historical aspect of the black hole problem will not be discussed as this has been sufficiently addressed in the scientific literature and, especially, in a historical review [3]. The technical details of Crothers' solution will also not be reana-

lyzed: his calculations were reviewed by many professional relativists prior to publication in *Progress in Physics*. These reviewers had a combined forty years of professional employment in this field and it is thus extremely unlikely that a formal error exists within Crothers' work. Rather, our attention will be focused only upon clarification of the new result in comparison to the classical solution in Schwarzschild space. In other words, the main objective is to answer the question: what have Abrams and Crothers achieved?

In this letter, two important items must be highlighted:

1. The new solution, by Crothers, doesn't eliminate the classical "black hole solution" (i.e. the line-element thereof) produced by the founders of the black hole problem, but represents the perspective of a real observer whose location is in the real Schwarzschild space itself (inhomogeneous and curved), not by quantities in an abstract flat space tangential to it at the point of observation (as it was previously, in the classical solution). Consequently, the new solution opens a doorway to new research on the specific physical conditions accompanying gravitational collapse in Schwarzschild space. This can now be studied in a reasonable manner both through a purely theoretical approach and with the methods of numerical relativity (computers);
2. Schwarzschild space is only a very particular case related to Einstein spaces of type I. There are minor studies on the physical conditions of gravitational collapse in other spaces of type I, but nothing on it in relation to Einstein spaces of type II and type III (of which there are hundreds). Besides Einstein spaces (empty spaces, without distributed matter, wherein Ricci's tensor is proportional to the fundamental metric tensor $R_{\alpha\beta} \sim k g_{\alpha\beta}$), there are spaces filled by an electromagnetic field, dust, or other substances, of which there are

many. As a result, studies on the physical conditions of gravitational collapse are only in their infancy.

First, the corner-stone of Crothers' solution is that it was produced in terms of the physical observed (proper) radius which is dependent on the properties of the space itself, while the classical solution was produced in terms of the coordinate radius determined in the tangentially flat space (it can be chosen at any point of the inhomogeneous, curved space). For instance, when one makes a calculation at such a proper radius where the gravitational collapse condition $g_{00} = 0$ occurs, the calculation result manifests in what might be really measurable on the surface of collapse from the perspective of a real observer who has a real reference body which is located in this space, and is bearing not on the ideal, but on real physical standards whereto this observer compares his measurements. This is in contrast to the classic procedure of calculation oriented to the coordinate quantities measurable by an "abstract" observer who has an "ideal" reference body which, in common with its ideal physical standards, is located in the flat space tangential to the real space at the point of observation, not the real space which is inhomogeneous and curved.

In the years 1910–1920's people had no clear understanding of physical observable quantities in General Relativity. Later, in the years 1930–1940's, many researchers such as Einstein, Lichnerowicz, Cattaneo and others, were working on methods for determination of physical observable quantities in the inhomogeneous curved space of General Relativity. For instance, Landau and Lifshitz, in §84 of their famous book, *The Classical Theory of Fields*, first published in 1939, introduced observable time and the observable three-dimensional interval. But they all limited themselves to only a few particular cases and did not arrive at general mathematical methods to define physical observable quantities in pseudo-Riemannian spaces. The complete mathematical apparatus for calculating physical observable quantities in four-dimensional pseudo-Riemannian space, that is a strict solution to the problem of physical observable quantities in General Relativity, was only constructed in the 1940's, by Abraham Zelmanov (1913–1987), and first published in 1944 in his doctoral dissertation [4].

Therefore David Hilbert and the other founders of the black hole problem*, who did their work during the period 1916–1920's, worked in the circumstances of the gravitational collapse condition $g_{00} = 0$ in Schwarzschild space in terms of the coordinate radius (which isn't the same as the real distance in this space). As a result, they concluded that the spherical mass which produces the gravitational field in Schwarzschild space, with the increase of its density, becomes a "self-closed" object surrounded by the gravitational collapse

*Karl Schwarzschild died in 1916, and had no relation to the black hole solution. He only deduced the metric of a space filled by the spherically symmetric gravitational field produced by a spherical mass therein (such a space is known as a space with a Schwarzschild metric or, alternatively, a Schwarzschild space).

surface of the condition $g_{00} = 0$ so that all events can occur only inside it (this means a singular break in the surface of collapse).

By the new solution, which was obtained by Crothers in terms of the proper radius, there is no observable singular break under any physical conditions: so a real spherical body of a Schwarzschild metric cannot become a "self-closed" object observable as a "black hole" in the space.

This new solution, in common with the classical solution, means that we have two actual pictures of gravitational collapse, drawn by two observers who are respectively located in different spaces: (1) a real observer located in the same Schwarzschild space where the gravitational collapse occurs; (2) an "abstract" observer whose location is in the flat space tangential to the Schwarzschild space at the point of observation.

So, the new solution doesn't eliminate the classical "black hole solution" (i.e. the line-element thereof), but represents the same phenomenon of gravitational collapse in a Schwarzschild space from another perspective, related to real observation and experiment.

Second, Schwarzschild space is only a very particular case of Einstein spaces of Type I. Einstein spaces [5] are empty spaces without distributed matter, wherein Ricci's tensor is proportional to the fundamental metric tensor ($R_{\alpha\beta} \sim k g_{\alpha\beta}$). There are three known kinds of Einstein spaces, and there are many spaces related to each kind (hundreds, as expected, and nobody knows exactly how many). There are almost no studies of the gravitational collapse condition $g_{00} = 0$ in most other Einstein spaces of Type I. There are no studies at all of the collapse condition in Einstein spaces of Type II and Type III. Besides that, General Relativity has many spaces beyond Einstein spaces: spaces filled by distributed matter such as an electromagnetic field, dust, or other substances, of which there are many. Such spaces are closer to real observation and experiment than Schwarzschild space, so it would be very interesting to study the collapse condition in spaces beyond Einstein spaces.

This is why Schwarzschild (empty) space is good for basic considerations, where there are no sharp boundaries for the physical conditions therein. However, such a space becomes unusable under some ultimate physical conditions, which are smooth in the real Universe due to the influences of many other space bodies and fields. Gravitational collapse as the ultimate condition in Schwarzschild space leads to black holes outside a real physical space, with the consequence that the black hole solution in Schwarzschild space has no real meaning (despite the fact that it can be formally obtained). Mathematical curiosities are always interesting, but if these things have no real meaning, then one must make it clear in the end. Consequently, the current mathematical treatment of black holes in Schwarzschild space does not have physical validity in nature, as Crothers explains.

These results are not amazing: many solutions to Ein-

stein's equation have no validity in the physical world. Therefore the collapse condition in a real case, which could be met in the real Universe filled by fields and substance, should be a subject of numerical relativity which produces approximate solutions to Einstein's equations with the use of computers, not an exact theory of the phenomenon.

As a result we see that studies on the physical conditions of gravitational collapse are only beginning. New solutions, given in terms of physical observable quantities, do not close the gravitational collapse problem, but open new horizons for studies by both exact theory and numerical methods of General Relativity.

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SPECIAL REPORT**PLANCK, the Satellite: a New Experimental Test of General Relativity**

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If the origin of a microwave background (EMB) is the Earth, what would be its density and associated dipole anisotropy measured at different altitudes from the surface of the Earth? The mathematical methods of the General Theory of Relativity are applied herein to answer these questions. The density of the EMB is answered by means of Einstein's equations for the electromagnetic field of the Earth. The dipole anisotropy, which is due to the rapid motion of the source (the Earth) in the weak intergalactic field, is analysed by using the geodesic equations for light-like particles (photons), which are mediators for electromagnetic radiation. It is shown that the EMB decreases with altitude so that the density of its energy at the altitude of the COBE orbit (900km) is 0.68 times less than that at the altitude of a U2 aeroplane (25 km). Furthermore, the density at the 2nd Lagrange point (1.5 million km, the position of the WMAP and PLANCK satellites) should be only $\sim 10^{-7}$ of the value detected by a U2 aeroplane or at the COBE orbit. The dipole anisotropy of the EMB doesn't depend on altitude from the surface of the Earth, it should be the same irrespective of the altitude at which measurements are taken. This result is in support to the experimental and observational analysis conducted by P.-M. Robitaille, according to which the 2.7 K microwave background, first observed by Penzias and Wilson, is not of cosmic origin, but of the Earth, and is generated by the oceans. WMAP indicated the same anisotropy of the microwave background at the 2nd Lagrange point that near the Earth. Therefore when PLANCK, which is planned on July, 2008, will manifest the 2.7 K monopole microwave signal decreased at the 2nd Lagrange point, it will be a new experimental verification of Einstein's theory.

1 Introduction

Our recent publication [1] was focused on the mathematical proof in support to the claim made by P.-M. Robitaille: according to the experimental and observational analysis conducted by him [3–10], the 2.7 K monopole microwave background, first detected by Penzias and Wilson [2], is not of cosmic origin, but of the Earth, and is generated by oceanic water*. As shown in the framework of Robitaille's concept, the anisotropy of the background, observed on the 3.35 mK dipole component of it[†], is due to the rapid motion of the whole field in common with its source, the Earth, in a weak intergalactic field so that the anisotropy of the observed microwave background has a purely relativistic origin [21].[‡]

*Robitaille reported the result first in 1999 and 2001 in the short communications [1, 2], then detailed explanation of the problem was given by him in the journal publications [3–6] and also in the reports [9, 10].

[†]The 3.35 mK dipole component of the background was first observed in 1969 by Conklin [11] in a ground-based observation. Then it was studied by Henry [12], Corey [13], and also Smoot, Gorenstein, and Muller (the latest team organized a stratosphere observation on board of a U2 aeroplane [14]). The history of the discovery and all the observations is given in detail in Lineweaver's paper [15]. The anisotropy of the dipole component was found later, in the COBE space mission then verified by the WMAP space mission [16–20].

[‡]This conclusion is based on that fact that, according to the General Theory of Relativity, photons exceeded from a source at radial directions should be carried out with the space wherein this source moves so that the spherical distribution of the signals should experience an anisotropy in the direction of the motion of this source in the space [22, 23].

If the microwave background is of the earthy origin, the density of the field should obviously decrease with altitude from the surface of the Earth. The ground-bound measurements and those made on board of the COBE satellite, at the altitude 900 km, were processed very near the oceans which aren't point-like sources, so the observations were unable to manifest the change of the field density with altitude. Another case — the 2nd Lagrange point, which is located as far as 1.5 mln km from the Earth, the position of the WMAP satellite and the planned PLANCK satellite.

A problem is that WMAP has only differential instruments on board: such an instrument, having a few channels for incoming photons, registers only the difference between the number of photons in the channels. WMAP therefore targeted measurements of the anisotropy of the field, but was unable to measure the field density. PLANCK, which is planned on July, 2008, is equipped by absolute instruments (with just one channel for incoming photons, an absolute instrument gets the integral density of the monopole and all the multipole components of the field). Hence PLANCK will be able to measure the field density at the 2nd Lagrange point.

We therefore were looking for a theory which would be able to represent the density and anisotropy of the Earth's microwave background as the functions of altitude from the Earth's surface.

In our recent publication [1], we created such a theory with use of the mathematical methods of the General The-

ory of Relativity where the physical characteristics of fields are expressed through the geometrical characteristics of the space itself. We have split our tasks into two particular problems: if a microwave background originates from the Earth, what would be the dependency of its density and relativistic anisotropy with altitude? The first problem was solved via Einstein's equations for the electromagnetic field of the Earth. The second problem was solved using the geodesic equations for light-like particles (photons) which are mediators for electromagnetic radiation.

We have determined, according to our solutions [1], that a microwave background that originates at the Earth decreases with altitude so that the density of the energy of such a background in the COBE orbit (the altitude 900 km) is 0.68 times less than that at the altitude of a U2 aeroplane. The density of the energy of the background at the L2 point is only $\sim 10^{-7}$ of the value detected by a U2 aeroplane or at the COBE orbit. The dipole anisotropy of such an earthy microwave background, due to the rapid motion of the Earth relative to the source of a weak intergalactic field which is located in depths of the cosmos, doesn't depend on altitude from the surface of the Earth. Such a dipole will be the same irrespective of the position at which measurements are taken.

In principle, the first problem — how the density of an earthy-origin microwave background decreases with altitude — may be resolved by the methods of classical physics. But this is possible only in a particular case where the space is free of rotation. In real, the Earth experiences daily rotation. We therefore should take into account that fact that the rotation makes the observer's local space non-holonomic: in such a space the time lines are non-orthogonal to the spatial section, so the Riemannian curvature of the space is non-zero. A satellite's motion around the Earth should be also taken into account for the local space of an observer which is located on board of the satellite. Therefore in concern of a real experiment, in both cases of ground-based and satellite-based observations, the first problem can be resolved only in the framework of the General Theory of Relativity.

The second problem can never be resolved in the framework of classical physics due to the purely relativistic origin of the field anisotropy we are considering.

WMAP registered the same parameters of the microwave background anisotropy that the registered by COBE near the Earth. This is according to our theory.

Therefore when PLANCK will manifest the 2.7 K monopole microwave signal decreased at the 2nd Lagrange point, with the same anisotropy of the background that the measured near the Earth (according to WMAP which is as well located at the 2nd Lagrange point), this will be a new experimental verification of the General Theory of Relativity.

A drawback of our theory was only that complicate way in which it was initially constructed. As a result, our recently published calculation [1] is hard to reproduce by the others who have no mathematical skills in the very specific

areas of General Relativity, which are known to only a close circle of the specialists who are no many in the world. We therefore were requested for many additional explanations by those readers who tried to repeat the calculation.

Due to that discussion, we found another way to give representation of our result with much unused stuff removed. We also gave an additional explanation to those parts of our calculation, which were asked by the readers. As a result a new representation of our calculation, with the same result, became as simple as easy to reproduce by everyone who is free in tensor algebra. This representation is given here.

2 The local space metric of a satellite-bound observer

A result of real measurement processed by an observer depends on the properties of his local space. These properties are completely determined by the metric of this space. We therefore are looking for the metric of the local space of an observer, who is located on board of a satellite moved in the Earth's gravitational field.

As one regularly does in construction for a metric, we take a simplest metric which is close to the case we are considering, then modify the metric by introduction of those additional factors which are working in our particular case.

Here is how we do it.

As was proven in the 1940's by Abraham Zelmanov, on the basis of the theory of non-holonomic manifolds [24] constructed in the 1930's by Schouten then applied by Zelmanov to the four-dimensional pseudo-Riemannian space of General Relativity, the non-holonomy of such a space (i.e. the non-orthogonality of the time lines to the spatial section, that is expressed as $g_{0i} \neq 0$ in the fundamental metric tensor $g_{\alpha\beta}$) is manifest as the three-dimensional rotation of this space. Moreover, Zelmanov proven that any non-holonomic space has nonzero Riemannian curvature (nonzero Riemann-Christoffel tensor) due to $g_{0i} \neq 0$. All these was first reported in 1944 by him in his dissertation thesis [25], then also in the latter publications [26–28].

In practice this means that the physical space of the Earth, the planet, is non-holonomic and curved due to the daily rotation of it. This is in addition to that fact that the Earth's space is curved due to the gravitational field of the Earth, described in an approximation by Schwarzschild metric of a centrally symmetric gravitational field, created by a spherical mass in emptiness. The space metric of a satellite-bound observer should also take into account that fact that the satellite moves along its orbit in the Earth's space around the terrestrial globe (the central mass that produces the field). In addition to it the Earth, in common with the satellite and the observer located in it, rapidly moves in the physical space of the Universe associated to the weak intergalactic microwave field. This fact should also be taken into account in the metric.

First, we consider a simplest non-holonomic space — a space wherein all $g_{0i} \neq 0$, and they have the same numerical

$$ds^2 = \left(1 - \frac{2GM}{c^2 r} - \frac{\omega^2 r^2}{c^2}\right) c^2 dt^2 + \frac{2v(\cos\varphi + \sin\varphi)}{c} cdt dr + \frac{2r[v(\cos\varphi - \sin\varphi) - \omega r]}{c} cdt d\varphi + \frac{2v}{c} cdt dz - \left(1 + \frac{2GM}{c^2 r}\right) dr^2 - r^2 d\varphi^2 - dz^2 \quad (7)$$

$$ds^2 = \left(1 - \frac{2GM}{c^2 r} - \frac{\omega^2 r^2}{c^2} + \frac{2v\mathbf{v}}{c^2}\right) c^2 dt^2 + \frac{2v(\cos\varphi + \sin\varphi)}{c} cdt dr + \frac{2r[v(\cos\varphi - \sin\varphi) - \omega r]}{c} cdt d\varphi + \frac{2v}{c} cdt dz - \left(1 + \frac{2GM}{c^2 r}\right) dr^2 + \frac{2v\mathbf{v}(\cos\varphi + \sin\varphi)}{c^2} dr dz - r^2 d\varphi^2 + \frac{2r\mathbf{v}[v(\cos\varphi - \sin\varphi) - \omega r]}{c^2} d\varphi dz - \left(1 - \frac{2v\mathbf{v}}{c^2}\right) dz^2 \quad (8)$$

values. According to Zel'manov [25–28], such a space experiences rotation around all three axes with the same linear velocity $v = v_1 = v_2 = v_3$, where $v_i = -\frac{c g_{0i}}{\sqrt{g_{00}}}$. As obvious, the metric of such a non-holonomic space is

$$ds^2 = c^2 dt^2 + \frac{2v}{c} cdt(dx + dy + dz) - dx^2 - dy^2 - dz^2. \quad (1)$$

For easy taking the Earth's field into account, we change to the cylindrical coordinates r, φ, z , where the r -axis is directed from the centre of gravity of the Earth along its radius. The corresponding transformations of the coordinates are $x = r \cos \varphi, y = r \sin \varphi, z = z$ so that the metric (1) represented in the new coordinates is

$$ds^2 = c^2 dt^2 + \frac{2v}{c} (\cos\varphi + \sin\varphi) cdt dr + \frac{2vr}{c} (\cos\varphi - \sin\varphi) cdt d\varphi + \frac{2v}{c} cdt dz - dr^2 - r^2 d\varphi^2 - dz^2. \quad (2)$$

Next we introduce the factor of the Earth's gravitational field in the same way as it is made in Schwarzschild metric (see §100 in *The Classical Theory of Fields* [29]) — the metric of a spherically symmetric gravitational field, produced by a spherical mass M in emptiness, which in the cylindrical coordinates is

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{2GM}{c^2 r}} - r^2 d\varphi^2 - dz^2, \quad (3)$$

where we should take into account that fact that $\frac{2GM}{c^2 r}$ is small value, so we have

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \left(1 + \frac{2GM}{c^2 r}\right) dr^2 - r^2 d\varphi^2 - dz^2. \quad (4)$$

Besides, we should take into account the factor of rotational motion of the observer, in common with the satellite, along its orbit around the Earth. We see how to do it in the example of a plane metric in the cylindrical coordinates

$$ds^2 = c^2 dt^2 - dr^2 - r^2 d\varphi^2 - dz^2, \quad (5)$$

where we change the reference frame to another one, which rotates relative to the initially reference frame with a constant angular velocity ω . By the applying the transformation of the coordinates $r' = r, \varphi' = \varphi + \omega t, z' = z$, we obtain ds^2 in the rotating reference frame*

$$ds^2 = \left(1 - \frac{\omega^2 r^2}{c^2}\right) c^2 dt^2 - \frac{2\omega r^2}{c} cdt d\varphi - dr^2 - r^2 d\varphi^2 - dz^2. \quad (6)$$

Following with the aforementioned steps[†], we obtain the metric of the *local physical space* of a satellite-bound observer which takes all properties of such a space into account. This resulting metric is represented in formula (7).

This metric will be used by us in calculation for the density of the Earth microwave background, measured by an observer on board of a satellite of the Earth.

This metric is definitely curved due to two factors: non-zero gravitational potential $w = c^2(1 - \sqrt{g_{00}}) \neq 0$ and the space non-holonomy $g_{0i} \neq 0$. Hence we are able to consider Einstein equations in such a space.

On the other hand this metric doesn't take into account that fact that the Earth microwave background, in common with the Earth, moves in a weak intergalactic field with a velocity of $\mathbf{v} = 365 \pm 18$ km/sec (as observational analysis indicates it). To calculate the associated dipole anisotropy of the Earth microwave background, which is due to the motion, we should use such a space metric which takes this motion into account. To do it we take the metric (7) then apply Lorentz' transformations to the z -coordinate (we direct the z -axis with the motion of the Earth in the weak intergalactic field) and time with an obvious approximation of $\mathbf{v} \ll c$ and high order terms omitted: $z' = z + \mathbf{v}t, t' = t + \frac{\mathbf{v}z}{c^2}$. In other word, we "move" the whole local physical space of an earthy satellite-bound observer relative to the source of the weak intergalactic field. As a result the local physical space of such an observer and all physical fields connected to the Earth should experience a drift in the z -direction and a corresponding change the

*See §10.3 in [27], or §3.6 in [28] for detail.

†As known in Riemannian geometry, which is particular to metric geometries, a common metric can be deduced as a superposition of all the particular metrics each of whom takes a particular property of the common space into account.

local physically observed time that should has a sequel on the observed characteristics of the Earth's microwave field.

The resulting metric we have obtained after the transformation is (8). We will use this metric in calculation for the anisotropy of the Earth microwave background measured by a satellite-bound observer.

3 The density of the Earth's microwave background at the 2nd Lagrange point

To calculate the density of a field (distributed matter) dependent from the properties of the space wherein this field is situated we should operate with Einstein's equations

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = -\kappa T_{\alpha\beta} + \lambda g_{\alpha\beta}, \quad (9)$$

the left side of which is for the space geometry, while the right side describes distributed matter (it is with the energy-momentum tensor of distributed matter and the λ -term which describes the distribution of physical vacuum).

Projection of the energy-momentum tensor $T_{\alpha\beta}$ onto the time line and spatial section of an observer's local physical space gives the properties of distributed matter observed by him [25–28]: the density of the energy of distributed matter $\rho = \frac{T_{00}}{g_{00}}$, the density of the momentum $J^i = \frac{cT_{0i}}{\sqrt{g_{00}}}$, and the stress-tensor $U^{ik} = c^2 T^{ik}$. To express the first of these observable quantities through the observable properties of the local physical space is a task in our calculation.

To reach this task we should project the whole Einstein equations onto the ime line and spatial section of the metric space (7) with taking into account that fact that the energy-momentum tensor is of an electromagnetic field. The left side of the projected equations will be containing the observable properties of the local space of such an observer, while the right side will be containing the aforementioned observable properties of distributed matter (the Earth microwave background, in our case). Then we can express the density of the Earth microwave background ρ as a function of the observable properties of the local space.

Einstein's equations projected onto the time line and spatial section of a common case were obtained in the 1940's by Zelmanov [25–28], and are quite complicate in the left side (the observable properties of the local space). We therefore first should obtain the observable properties of the given space (7), then decide what propeties can be omitted from consideration in the framework of our problem.

According to the theory of physical observable quantities of the General Theory of Relativity [25–28], the observable properties of a space are the three-dimensional quantities which are invariant within the fixed spatial section of an observer (so-called *chronometrically invariant quantities*). Those are the three-dimensional metric tensor h_{ik} , the gravitational inertial force F_i , the angular velocity of the space rotation A_{ik} (known as the non-holonomy tensor), the space

deformation tensor D_{ik} , the three-dimensional Christoffel symbols Δ_{kn}^i , and the three-dimensional curvature C_{iklj} , expressed through the gravitational potential $w = c^2(1 - \sqrt{g_{00}})$ and the linear velocity of the space rotation $v_i = -\frac{cg_{0i}}{\sqrt{g_{00}}}$ (whose components are $v^i = -cg^{0i}\sqrt{g_{00}}$ and $v_i = h_{ik}v^k$):

$$h_{ik} = -g_{ik} + \frac{g_{0i}g_{0k}}{g_{00}} = -g_{ik} + \frac{1}{c^2} v_i v_k, \quad (10)$$

$$h^{ik} = -g^{ik}, \quad h_k^i = -g_k^i = \delta_k^i, \quad (11)$$

$$F_i = \frac{1}{\sqrt{g_{00}}} \left(\frac{\partial w}{\partial x^i} - \frac{\partial v_i}{\partial t} \right), \quad (12)$$

$$A_{ik} = \frac{1}{2} \left(\frac{\partial v_k}{\partial x^i} - \frac{\partial v_i}{\partial x^k} \right) + \frac{1}{2c^2} (F_i v_k - F_k v_i), \quad (13)$$

$$D_{ik} = \frac{1}{2} \frac{\partial h_{ik}}{\partial t}, \quad D^{ik} = -\frac{1}{2} \frac{\partial h^{ik}}{\partial t}, \quad (14)$$

$$D = h^{ik} D_{ik} = D_k^k = \frac{\partial \ln \sqrt{h}}{\partial t}, \quad h = \det \| h_{ik} \|, \quad (15)$$

$$\Delta_{jk}^i = \frac{1}{2} h^{im} \left(\frac{\partial h_{jm}}{\partial x^k} + \frac{\partial h_{km}}{\partial x^j} - \frac{\partial h_{jk}}{\partial x^m} \right), \quad (16)$$

$$\Delta_{jk}^j = \frac{\partial \ln \sqrt{h}}{\partial x^k}, \quad (17)$$

$$C_{lkij} = \frac{1}{4} (H_{lkij} - H_{jkil} + H_{klji} - H_{iljk}), \quad (18)$$

$$C_{kj} = C_{kij}^{\dots i} = h^{im} C_{kimj}, \quad C = C_j^j = h^{lj} C_{lj}. \quad (19)$$

Here $\frac{\partial}{\partial t} = \frac{1}{\sqrt{g_{00}}} \frac{\partial}{\partial t}$ and $\frac{\partial}{\partial x^i} = \frac{\partial}{\partial x^i} + \frac{1}{c^2} v_i \frac{\partial}{\partial t}$ are the chronometrically invariant differential operators, while

$$H_{lki}^{\dots j} = \frac{\partial \Delta_{il}^j}{\partial x^k} - \frac{\partial \Delta_{kl}^j}{\partial x^i} + \Delta_{il}^m \Delta_{km}^j - \Delta_{kl}^m \Delta_{im}^j \quad (20)$$

is Zelmanov's tensor constructed by him on the basis of the non-commutativity of the second chronometrically invariant derivatives of an arbitrary spatial vector taken in a given three-dimensional spatial section

$$*\nabla_i * \nabla_k Q_l - *\nabla_k * \nabla_i Q_l = \frac{2A_{ik}}{c^2} \frac{\partial Q_l}{\partial t} + H_{lki}^{\dots j} Q_j, \quad (21)$$

where $*\nabla_i Q_l = \frac{\partial Q_l}{\partial x^i} - \Delta_{ji}^l Q_l$ is the chronometrically invariant derivative of the vector ($*\nabla_i Q^l = \frac{\partial Q^l}{\partial x^i} + \Delta_{ji}^l Q^l$ respectively). The tensor $H_{lki}^{\dots j}$ was introduced by Zelmanov similarly to Schouten's tensor of the theory of non-holonomic manifolds [24] so that the three-dimensional curvature tensor C_{lkij} possesses all the algebraic properties of the Riemann-Christoffel curvature tensor in the spatial section.

We take the components of the fundamental metric tensor $g_{\alpha\beta}$ from the metric of the local physical space of a satellite-bound observer (7), then calculate the aforementioned observable quantities. In this calculation we take into account that fact that $\frac{2GM}{c^2 r}$ and $\frac{\omega^2 r^2}{c^2}$ are in order of 10^{-9} near the sur-

face of the Earth, and the values decrease with altitude. We therefore operate these terms according to the rules of small values. We also neglect all high order terms. We however cannot neglect $\frac{2GM}{c^2 r}$ and $\frac{\omega^2 r^2}{c^2}$ in $g_{00} = 1 - \frac{2GM}{c^2 r} - \frac{\omega^2 r^2}{c^2}$ when calculating the gravitational potential $w = c^2(1 - \sqrt{g_{00}})$ according to the rule of small values

$$\begin{aligned} w &= c^2 \left(1 - \sqrt{1 - \frac{2GM}{c^2 r} - \frac{\omega^2 r^2}{c^2}} \right) = \\ &= c^2 \left\{ 1 - \left(1 + \frac{GM}{c^2 r} + \frac{\omega^2 r^2}{2c^2} \right) \right\} \end{aligned} \quad (22)$$

because these terms are multiplied by c^2 . We also assume the linear velocity of the space rotation v to be small to the velocity of light c . We assume that v doesn't depend from the z -coordinate. This assumption is due to the fact that the Earth, in common with its space, moves relative to a weak intergalactic microwave background that causes the anisotropy of the Earth's microwave field.

As a result we obtain the substantially non-zero components of the characteristics of the space

$$w = \frac{GM}{r} + \frac{\omega^2 r^2}{2}, \quad (23)$$

$$\left. \begin{aligned} v_1 &= -v(\cos \varphi + \sin \varphi) \\ v_2 &= -r[v(\cos \varphi - \sin \varphi) - \omega r] \\ v_3 &= -v \end{aligned} \right\}, \quad (24)$$

$$\left. \begin{aligned} F_1 &= (\cos \varphi + \sin \varphi) v_t + \omega^2 r - \frac{GM}{r^2} \\ F_2 &= r(\cos \varphi - \sin \varphi) v_t, \quad F_3 = v_t \end{aligned} \right\}, \quad (25)$$

$$\left. \begin{aligned} A_{12} &= \omega r + \frac{1}{2} [(\cos \varphi + \sin \varphi) v_\varphi - \\ &\quad - r(\cos \varphi - \sin \varphi) v_r] \\ A_{23} &= -\frac{v_\varphi}{2}, \quad A_{13} = -\frac{v_r}{2} \end{aligned} \right\}, \quad (26)$$

$$\left. \begin{aligned} h_{11} &= h_{33} = 1, \quad h_{22} = r^2, \quad h^{11} = h^{33} = 1 \\ h^{22} &= \frac{1}{r^2}, \quad h = r^2, \quad \frac{\partial \ln \sqrt{h}}{\partial r} = \frac{1}{r} \end{aligned} \right\}, \quad (27)$$

$$\Delta_{22}^1 = -r, \quad \Delta_{12}^2 = \frac{1}{r} \quad (28)$$

while all components of the tensor of the space deformation D_{ik} and the three-dimensional curvature C_{iklj} are negligible in the framework of the first order approximation (the four-dimensional Riemannian curvature isn't negligible).

The quantities v_r , v_φ , and v_t denote the partial derivatives of the linear velocity of the space rotation v by the respective coordinate and time. (Here $v_z = 0$ according to the initially assumptions in the framework of our problem.)

We consider the projected Einstein equations in complete form, published in [25–28]

$$\left. \begin{aligned} \frac{* \partial D}{\partial t} + D_{jl} D^{jl} + A_{jl} A^{lj} + \left(* \nabla_j - \frac{1}{c^2} F_j \right) F^j &= \\ = -\frac{\kappa}{2} (\rho c^2 + U) + \lambda c^2 \\ * \nabla_j (h^{ij} D - D^{ij} - A^{ij}) + \frac{2}{c^2} F_j A^{ij} &= \kappa J^i \\ \frac{* \partial D_{ik}}{\partial t} - (D_{ij} + A_{ij})(D_k^j + A_k^j) + D D_{ik} + \\ + 3A_{ij} A_k^j + \frac{1}{2} (* \nabla_i F_k + * \nabla_k F_i) - \frac{1}{c^2} F_i F_k - \\ - c^2 C_{ik} &= \frac{\kappa}{2} (\rho c^2 h_{ik} + 2U_{ik} - U h_{ik}) + \lambda c^2 h_{ik} \end{aligned} \right\}. \quad (29)$$

We withdraw the λ -term, the space deformation D_{ik} , and the three-dimensional curvature C_{iklj} from consideration. We also use the aforementioned assumptions on small values and high order terms that reduce the chronometrically invariant differential operators to the regular differential operators: $\frac{* \partial}{\partial t} = \frac{\partial}{\partial t}$, $\frac{* \partial}{\partial x^i} = \frac{\partial}{\partial x^i}$. As a result of all these, the projected Einstein equations take the simplified form

$$\left. \begin{aligned} \frac{\partial F^i}{\partial x^i} + \frac{\partial \ln \sqrt{h}}{\partial x^i} F^i - A_{ik} A^{ik} &= -\frac{\kappa}{2} (\rho c^2 + U) \\ \frac{\partial A^{ik}}{\partial x^k} + \frac{\partial \ln \sqrt{h}}{\partial x^k} A^{ik} &= -\kappa J^i \\ 2A_{ij} A_k^j + \frac{1}{2} \left(\frac{\partial F_i}{\partial x^k} + \frac{\partial F_k}{\partial x^i} - 2\Delta_{ik}^m F_m \right) &= \\ = \frac{\kappa}{2} (\rho c^2 h_{ik} + 2U_{ik} - U h_{ik}) \end{aligned} \right\}. \quad (30)$$

We substitute hereto the obtained observable characteristics of the local physical space of a satellite-bound observer. Because the value v is assumed to be small, we neglect not only the square of it, but also the square of its derivative and the products of the derivatives.

The Einstein equations (30) have been written for a space filled with an arbitrary matter, which is described by the energy-momentum tensor written in the common form $T_{\alpha\beta}$. In other word, the distributed matter can be the superposition of an electromagnetic field, dust, liquid or other matter. Concerning our problem, we consider only an electromagnetic field. As known [29], the energy-momentum tensor $T_{\alpha\beta}$ of any electromagnetic field should satisfy the condition $T = \rho c^2 - U$. We therefore assume that the right side of the Einstein equations contains the energy-momentum tensor of only an electromagnetic field (no dust, liquid, or other matter distributed near the Earth). In other word we should mean, in the right side,

$$\rho c^2 = U. \quad (32)$$

Besides, because all measurement in the framework of our problem are processed by an observer on board of a satel-

$$\left. \begin{aligned}
& -2\omega^2 - 2\omega (\cos \varphi + \sin \varphi) \frac{v_\varphi}{r} + 2\omega (\cos \varphi - \sin \varphi) v_r + (\cos \varphi + \sin \varphi) v_{tr} + (\cos \varphi - \sin \varphi) \frac{v_{t\varphi}}{r} = -\kappa \rho c^2 \\
& \frac{1}{2} \left[(\cos \varphi + \sin \varphi) \left(\frac{v_r}{r} + \frac{v_{\varphi\varphi}}{r^2} \right) + (\cos \varphi - \sin \varphi) \left(\frac{v_\varphi}{r^2} - \frac{v_{r\varphi}}{r} \right) \right] = -\kappa J^1 \\
& \frac{1}{2} \left[(\cos \varphi + \sin \varphi) \left(\frac{v_\varphi}{r^3} - \frac{v_{r\varphi}}{r^2} \right) + (\cos \varphi - \sin \varphi) \frac{v_{rr}}{r} \right] = -\kappa J^2 \\
& \frac{1}{2} \left(v_{rr} + \frac{v_r}{r} + \frac{v_{\varphi\varphi}}{r^2} \right) = -\kappa J^3 \\
& 2\omega^2 + 2\omega (\cos \varphi + \sin \varphi) \frac{v_\varphi}{r} - 2\omega (\cos \varphi - \sin \varphi) v_r + (\cos \varphi + \sin \varphi) v_{tr} = \kappa U_{11} \\
& \frac{r^2}{2} \left[(\cos \varphi + \sin \varphi) \frac{v_{t\varphi}}{r^2} + (\cos \varphi - \sin \varphi) \frac{v_{tr}}{r} \right] = \kappa U_{12} \\
& \omega \frac{v_\varphi}{r} + \frac{1}{2} v_{tr} = \kappa U_{13} \\
& 2\omega^2 + 2\omega (\cos \varphi + \sin \varphi) \frac{v_\varphi}{r} - 2\omega (\cos \varphi - \sin \varphi) v_r + (\cos \varphi - \sin \varphi) \frac{v_{t\varphi}}{r} = \kappa \frac{U_{22}}{r^2} \\
& \frac{r^2}{2} \left(\frac{v_{t\varphi}}{r^2} - 2\omega \frac{v_r}{r} \right) = \kappa U_{23} \\
& \kappa U_{33} = 0
\end{aligned} \right\} \quad (31)$$

lite, we should also take into account the weightlessness condition

$$\frac{GM}{r^2} = \omega^2 r. \quad (33)$$

As a result, we obtain the system of the projected Einstein equations (30) in the form (31) which is specific to the real physical space of a satellite-bound observer.

In other word, that fact that we used the conditions (32) and (33) means that our theoretical calculation targets measurement of an electromagnetic field in the weightlessness state in an orbit of the Earth.

We are looking for the quantity ρ as a function of the properties of the space from the first (scalar) equation of the Einstein equations (31). This isn't a trivial task, because the aforementioned scalar Einstein equation

$$\begin{aligned}
\kappa \rho c^2 &= 2\omega^2 + 2\omega (\cos \varphi + \sin \varphi) \frac{v_\varphi}{r} - \\
& - 2\omega (\cos \varphi - \sin \varphi) v_r - (\cos \varphi + \sin \varphi) v_{tr} - \\
& - (\cos \varphi - \sin \varphi) \frac{v_{t\varphi}}{r}
\end{aligned} \quad (34)$$

contains the distribution functions of the linear velocity of the space rotation (the functions v_r , v_φ , and v_t), which are unknown. We therefore should first find the functions.

According to our assumption, $\rho c^2 = U$. Therefore $\kappa \rho c^2$ and κU are the same in the framework of our problem. We calculate the quantity

$$\kappa U = \kappa h^{ik} U_{ik} = \kappa \left(U_{11} + \frac{U_{22}}{r^2} + U_{33} \right) \quad (35)$$

as the sum of the 5th and the 8th equations of the system of the Einstein equations (31) with taking into account that fact that, in our case, $U_{33} = 0$ (as seen from the 10th equation, with $\rho c^2 = U$). We obtain

$$\begin{aligned}
\kappa U &= 4\omega^2 + 4\omega (\cos \varphi + \sin \varphi) \frac{v_\varphi}{r} - \\
& - 4\omega (\cos \varphi - \sin \varphi) v_r + (\cos \varphi + \sin \varphi) v_{tr} + \\
& + (\cos \varphi - \sin \varphi) \frac{v_{t\varphi}}{r}.
\end{aligned} \quad (36)$$

Subtracting $\kappa \rho c^2$ (34) from κU (36) then equalizing the result to zero, according to the electromagnetic field condition $\rho c^2 = U$, we obtain the geometrization condition for the electromagnetic field

$$\begin{aligned}
\omega^2 + \omega (\cos \varphi + \sin \varphi) \frac{v_\varphi}{r} - \omega (\cos \varphi - \sin \varphi) v_r + \\
+ (\cos \varphi + \sin \varphi) v_{tr} + (\cos \varphi - \sin \varphi) \frac{v_{t\varphi}}{r} = 0.
\end{aligned} \quad (37)$$

With this condition, all the components of the energy-momentum tensor of the field $T_{\alpha\beta}$ (the right side of the Einstein equations) are expressed in only the properties of the space (the left side of the Einstein equations). Hence we have geometrized the electromagnetic field. This is an important result: earlier only isotropic electromagnetic fields (they are satisfying Rainich's condition and Nordtvedt-Pagels condition) were geometrized.

To find the distribution functions of v , we consider the conservation law $\nabla_\sigma T^{\alpha\sigma} = 0$, expressed in terms of the phys-

$$\left. \begin{aligned} &(\cos \varphi - \sin \varphi) \left(\frac{v_{tr\varphi}}{r} - \frac{v_{t\varphi}}{r^2} \right) + \omega (\cos \varphi + \sin \varphi) \left(\frac{v_r\varphi}{r} - \frac{v_\varphi}{r^2} \right) - \\ & - \omega (\cos \varphi - \sin \varphi) v_{rr} + (\cos \varphi + \sin \varphi) v_{trr} = 0 \\ &(\cos \varphi + \sin \varphi) \left(\frac{v_{tr\varphi}}{r^2} - \frac{v_{t\varphi}}{r^3} \right) + (\cos \varphi - \sin \varphi) \left(\frac{v_{t\varphi\varphi}}{r^3} + \frac{v_{tr}}{r^2} \right) + \\ & + \omega (\cos \varphi + \sin \varphi) \left(\frac{v_{\varphi\varphi}}{r^3} + \frac{v_r}{r^2} \right) + \omega (\cos \varphi - \sin \varphi) \left(\frac{v_\varphi}{r^3} - \frac{v_r\varphi}{r^2} \right) = 0 \end{aligned} \right\} \quad (41)$$

ical observed quantities [25–28]

$$\left. \begin{aligned} &\frac{* \partial \rho}{\partial t} + D\rho + \frac{1}{c^2} D_{ij} U^{ij} + \\ & + \left(* \nabla_i - \frac{1}{c^2} F_i \right) J^i - \frac{1}{c^2} F_i J^i = 0 \\ &\frac{* \partial J^k}{\partial t} + 2 \left(D_i^k + A_i^k \right) J^i + \\ & + \left(* \nabla_i - \frac{1}{c^2} F_i \right) U^{ik} - \rho F^k = 0 \end{aligned} \right\} \quad (38)$$

which, under the assumptions specific in our problem, is

$$\left. \begin{aligned} &\frac{\partial J^i}{\partial x^i} + \frac{\partial \ln \sqrt{h}}{\partial x^i} J^i = 0 \\ &\frac{\partial J^k}{\partial t} + 2 A_i^k J^i + \frac{\partial U^{ik}}{\partial x^i} + \Delta_{im}^k U^{im} + \\ & + \frac{\partial \ln \sqrt{h}}{\partial x^i} U^{ik} - \rho F^k = 0 \end{aligned} \right\} \quad (39)$$

The first (scalar) equation of the system of the conservation equations (39) means actually that the chronometrically invariant derivative of the vector J^i is zero

$$* \nabla_i J^i = \frac{\partial J^i}{\partial x^i} + \frac{\partial \ln \sqrt{h}}{\partial x^i} J^i = 0, \quad (40)$$

i.e. the flow of the vector J^i (the flow of the density of the field momentum) is constant. So, the first equation of (39) satisfies identically as $* \nabla_i J^i = 0$.

The rest three (vectorial) equations of the system (39), with the properties of the local space of a satellite-bound observer and the components of the energy-momentum tensor substituted (the latest should be taken from the Einstein equations), take the form (41). As seen, only first two equations still remaining meaningful, while the third of the vectorial equations of conservation vanishes becoming the identity zero equals zero.

In other word, we have obtained the equations of the conservation law specific to the real physical space of a satellite-bound observer.

Let's suppose that the function v has the form

$$v = T(t) r e^{i\varphi}, \quad (42)$$

hence the partial derivatives of this function are

$$\left. \begin{aligned} v_r &= T e^{i\varphi} & v_\varphi &= i T r e^{i\varphi} \\ v_{tr} &= \dot{T} e^{i\varphi} & v_{t\varphi} &= i \dot{T} r e^{i\varphi} \\ v_{rr} &= 0 & v_{trr} &= 0 \\ v_{tr\varphi} &= i \dot{T} e^{i\varphi} & v_{t\varphi\varphi} &= -\dot{T} r e^{i\varphi} \\ v_{\varphi\varphi} &= -T r e^{i\varphi} & v_{r\varphi} &= i T e^{i\varphi} \end{aligned} \right\} \quad (43)$$

After the functions substituted into the equations of the conservation law (41), we see that the equations satisfy identically. Hence $v = T(t) r e^{i\varphi}$ is exact solution of the conservation equations with respect to v .

Now we need to find only the unknown function $T(t)$. This function can be found from the electromagnetic field condition $\rho c^2 = U$ expressed by us through the properties of the space itself as the formula (37).

We assume that the satellite, on board of which the observer is located, displaces at small angle along its orbit during the process of his observation. This is obvious assumption, because the very fast registration process for a single photon. Therefore φ is small value in the framework of our problem. Hence in concern of the formula (37), we should mean $\cos \varphi \simeq 1 + \varphi$ and $\sin \varphi \simeq \varphi$. We also take into account only real parts of the function v and its derivatives. (This is due to that fact that a real instrument processes measurement with only real quantities.) Concerning those functions which are contained in the formula (37), all these means that

$$\left. \begin{aligned} v &= T r (1 + \varphi) \\ v_r &= T (1 + \varphi) & \frac{v_\varphi}{r} &= -T \varphi \\ v_{tr} &= \dot{T} (1 + \varphi) & \frac{v_{t\varphi}}{r} &= -\dot{T} \varphi \end{aligned} \right\} \quad (44)$$

Substituting these into (37), we obtain

$$(1 + 2\varphi) \dot{T} - (1 + 2\varphi) \omega T + \omega^2 = 0, \quad (45)$$

or, because $\varphi = \omega t$ and ω is small value (we also neglect the terms which order is higher than ω^2),

$$\dot{T} - \omega T = -\frac{\omega^2}{1 + 2\omega t} = -\omega^2 (1 - 2\omega t) = -\omega^2. \quad (46)$$

This is a linear differential equation of the first order

$$\dot{y} + f(t) y = g(t) \quad (47)$$

whose exact solution is (see Part I, Chapter I, §4.3 in Erich Kamke's reference book [30])

$$y = e^{-F} \left(y_0 + \int_{t_0=0}^t g(t) e^F dt \right), \quad (48)$$

where

$$F(t) = \int f(t) dt. \quad (49)$$

We substitute $f = -\omega$ and $g = -\omega^2$. So we obtain, for small values of ω ,

$$e^F = e^{\int -\omega dt} = e^{-\omega t}, \quad e^{-F} = e^{\omega t}, \quad (50)$$

hence the function y is

$$y = e^{\omega t} \left(y_0 - \omega^2 \int_{t_0=0}^t e^{-\omega t} dt \right) = e^{\omega t} [y_0 + \omega (e^{-\omega t} - 1)]. \quad (51)$$

We assume the numerical value of the function $y = T(t)$ to be zero at the initial moment of observation: $y_0 = T_0 = 0$. As a result we obtain the solution for the function $T(t)$:

$$T = \omega (1 - e^{\omega t}). \quad (52)$$

Applying this solution, we can find a final formula for the density of the energy of the Earth's microwave background $W = \rho c^2$ observed by a satellite-bound observer.

First, we substitute the distribution functions of v (44) into the initially formula for ρc^2 (34) which is originated from the scalar Einstein equation. Assuming $\cos \varphi \simeq 1 + \varphi$ and $\sin \varphi \simeq \varphi$, we obtain

$$\kappa \rho c^2 = -2\omega^2 - 2\omega T (1 + 2\varphi) - (1 + 2\varphi) \dot{T}. \quad (53)$$

Then we do the same substitution into the geometrization condition (37) which is originated from the Einstein equations, and is necessary to be applied to our case due to that fact that we have only an electromagnetic field distributed in the space ($\rho c^2 = U$ in the right side of the Einstein equations, as for any electromagnetic field). After algebra the geometrization condition (37) takes the form

$$\omega^2 - \omega T (1 + 2\varphi) + (1 + 2\varphi) \dot{T} = 0. \quad (54)$$

We express $(1 + 2\varphi) \dot{T} = \omega T (1 + 2\varphi) - 4\omega^2$ from this formula, then substitute it into the previous expression (53) with taking into account that fact that the angle φ is a small value. As a result, we obtain

$$\rho c^2 = \frac{3\omega}{\kappa} (\omega - T) = \frac{3\omega^2}{\kappa} [1 - (1 - e^{\omega t})]. \quad (55)$$

Expanding the exponent into the series $e^{\omega t} = 1 + \omega t + \frac{1}{2} \omega^2 t^2 + \dots \simeq 1 + \omega t$ and taking into account that fact

that ω is small value*, we arrive to the final formula for calculation the density of the energy of the Earth's microwave background observed on board of a satellite

$$\rho c^2 = \frac{3\omega^2}{\kappa}, \quad (56)$$

which is obviously dependent on altitude from the surface of the Earth due to that fact that $\omega = \sqrt{GM_{\oplus}/R^3}$.

With this final formula (55), we calculate the ratio between the density of the Earth's microwave background expected to be measured at different altitudes from the surface of the Earth. According to this formula, the ratio between the density at the altitude of the COBE orbit ($R_{\text{COBE}} = 6,370 + 900 = 7,270$ km) and that at the altitude of a U2 aeroplane ($R_{\text{U2}} = 6,370 + 25 = 6,395$ km) should be

$$\frac{\rho_{\text{COBE}}}{\rho_{\text{U2}}} = \frac{R_{\text{U2}}^3}{R_{\text{COBE}}^3} \simeq 0.68, \quad (57)$$

the ratio between the density at the 2nd Lagrange point ($R_{\text{L2}} = 1.5$ million km) and that at the COBE orbit should be

$$\frac{\rho_{\text{L2}}}{\rho_{\text{COBE}}} = \frac{R_{\text{COBE}}^3}{R_{\text{L2}}^3} \simeq 1.1 \times 10^{-7}, \quad (58)$$

and the ratio between the density at the 2nd Lagrange point and that at the altitude of a U2 aeroplane should be

$$\frac{\rho_{\text{L2}}}{\rho_{\text{U2}}} = \frac{R_{\text{U2}}^3}{R_{\text{L2}}^3} \simeq 7.8 \times 10^{-8}. \quad (59)$$

As a result of our calculation, processed on the basis of the General Theory of Relativity, we see that a microwave background field which originates in the Earth (the Earth microwave background) should have almost the same density at the position of a U2 aeroplane and the COBE satellite. However, at the 2nd Lagrange point (1.5 million km from the Earth, the point of location of the WMAP satellite and the planned PLANCK satellite), the density of the background should be only $\sim 10^{-7}$ of that registered either by the U2 aeroplane or by the COBE satellite.

4 The anisotropy of the Earth's microwave background at the 2nd Lagrange point

We consider the anisotropy of the Earth's microwave background which is due to the rapid motion of the source of this field (the Earth) in a weak intergalactic microwave field[†]. From views of physics this means that photons, the mediators for electromagnetic radiation, being radiated by the source of the field (the Earth) should experience a carrying in the direc-

*The quantity $\omega = \sqrt{GM_{\oplus}/R^3}$, the frequency of the rotation of the Earth space for an observer existing in the weightless state, takes its maximum numerical value at the equator of the Earth's surface, where $\omega = 1.24 \times 10^{-3} \text{ sec}^{-1}$, and decreases with altitude above the surface.

[†]As observational analysis indicates it, the Earth moves in the weak intergalactic field with a velocity of $v = 365 \pm 18$ km/sec in the direction of the anisotropy.

tion whereto the Earth flies in the weak intergalactic field. From mathematical viewpoint this problem can be formulated as a shift of the trajectories experienced by photons of the Earth's microwave field in the direction of this motion.

A light-like free particle, e.g. a free photon, moves along isotropic geodesic trajectories whose four-dimensional (general covariant) equations are [25–28]

$$\frac{dK^\alpha}{d\sigma} + \Gamma_{\mu\nu}^\alpha K^\mu \frac{dx^\nu}{d\sigma} = 0, \quad (60)$$

where $K^\alpha = \frac{\Omega}{c} \frac{dx^\alpha}{d\sigma}$ is the four-dimensional wave vector of the photon (the vector satisfies the condition $K_\alpha K^\alpha = 0$ which is specific to any isotropic vector), Ω is the proper cyclic frequency of the photon, while $d\sigma$ is the three-dimensional chronometrically invariant (observable) spatial interval determined as $d\sigma^2 = (-g_{ik} + \frac{g_{0i}g_{0k}}{g_{00}}) dx^i dx^k = h_{ik} dx^i dx^k$. The quantity $d\sigma$ is chosen as a parameter of differentiation along isotropic geodesics, because along them the four-dimensional interval is zero $ds^2 = c^2 d\tau^2 - d\sigma^2 = 0$ while $d\sigma = cd\tau \neq 0$ (where $d\tau$ is the interval of the physical observable time determined as $d\tau = \sqrt{g_{00}} dt + \frac{g_{0i}}{c\sqrt{g_{00}}} dx^i$).

In terms of the physical observables, the isotropic geodesic equations are represented by their projections on the time line and spatial section of an observer [25–28]

$$\left. \begin{aligned} \frac{d\Omega}{d\tau} - \frac{\Omega}{c^2} F_i c^i + \frac{\Omega}{c^2} D_{ik} c^i c^k &= 0 \\ \frac{d}{d\tau} (\Omega c^i) + 2\Omega (D_k^i + A_k^i) c^k - \\ - \Omega F^i + \Omega \Delta_{kn}^i c^k c^n &= 0 \end{aligned} \right\} \quad (61)$$

where $c^i = \frac{dx^i}{d\tau}$ is the three-dimensional vector of the observable velocity of light (the square of the vector satisfies $c_k c^k = h_{ik} c^i c^k = c^2$ in the spatial section of the observer). The first of the equations (the scalar equation) represents the law of energy for the particle, while the vectorial equation is the three-dimensional equation of its motion.

The terms $\frac{F_i}{c^2}$ and $\frac{D_{ik}}{c^2}$ are negligible in the framework of our assumption. We obtain, from the scalar equation of (61), that the proper frequency of the photons, registered by the observer, is constant. In such a case the vectorial equations of isotropic geodesics (61), written in component notation, are

$$\left. \begin{aligned} \frac{dc^1}{d\tau} + 2(D_k^1 + A_k^1) c^k - F^1 + \Delta_{22}^1 c^2 c^2 + \\ + 2\Delta_{23}^1 c^2 c^3 + \Delta_{33}^1 c^3 c^3 &= 0 \\ \frac{dc^2}{d\tau} + 2(D_k^2 + A_k^2) c^k - F^2 + 2\Delta_{12}^2 c^1 c^2 + \\ + 2\Delta_{13}^2 c^1 c^3 + \Delta_{33}^2 c^3 c^3 &= 0 \\ \frac{dc^3}{d\tau} + 2(D_k^3 + A_k^3) c^k - F^3 + \Delta_{11}^3 c^1 c^1 + \\ + 2\Delta_{12}^3 c^1 c^2 + 2\Delta_{13}^3 c^1 c^3 + \\ + \Delta_{22}^3 c^2 c^2 + 2\Delta_{23}^3 c^2 c^3 &= 0 \end{aligned} \right\}, \quad (62)$$

where $c^1 = \frac{dr}{d\tau}$, $c^2 = \frac{d\varphi}{d\tau}$, and $c^3 = \frac{dz}{d\tau}$, while $\frac{d}{d\tau} = \frac{*}{\partial t} + v^i \frac{*}{\partial x^i}$.

We direct the z -axis of our cylindrical coordinates along the motion of the Earth in the weak intergalactic field. In such a case the local physical space of a satellite-bound observer is described by the metric (8). We therefore will solve the isotropic geodesic equations in the metric (8).

The metric (7) we used in the first part of the problem is a particular to the metric (8) in a case, where $v = 0$. Therefore the solution $v = T(t) r e^{i\varphi}$ (42) we have obtained for the metric (7) is also lawful for the generatized metric (8). We therefore calculate the observable characteristics of the space with taking this function into account. As earlier, we take into account only real part of the function $e^{i\varphi} = \cos \varphi + i \sin \varphi \simeq (1 + \varphi) + i \varphi$. We also take into account the derivatives of this function (43) and the function $T = \omega(1 - e^{\omega t})$ we have found earlier (52).

As well as in the first part of the problem, we assume φ to be small value: $\cos \varphi \simeq 1 + \varphi$ and $\sin \varphi \simeq \varphi$. Because ω is small value too, we neglect $\omega^2 \varphi$ terms. We also take the weightlessness condition $\frac{GM}{r^2} = \omega^2 r$ into account in calculation for the gravitational inertial force. It should be noted that the weightlessness condition is derived from the derivative of the gravitational potential $w = c^2(1 - \sqrt{g_{00}})$. We therefore cannot mere substitute the weightlessness condition into $g_{00} = 1 - \frac{2GM}{c^2 r} - \frac{\omega^2 r^2}{c^2} + \frac{2v v}{c^2}$ taken from the metric (8). We first calculate $w = c^2(1 - \sqrt{g_{00}})$, then take derivative of it by the respective coordinate that is required in the formula for the gravitational inertial force F_i (12). Only then the weightlessness condition $\frac{GM}{r^2} = \omega^2 r$ is lawful to be substituted.

Besides these, we should take into account that fact that the anisotropy of a field is a second order effect. We therefore cannot neglect the terms divided by c^2 . This is in contrast to the first part of the problem, where we concerned only a first order effect. As a result the space deformation and the three-dimensional curvature, neglected in the first part, now cannot be neglected. We however take into account only the space deformation D_{ik} . The three-dimensional curvature C_{iklj} isn't considered here due to the fact that this quantity isn't contained in the equations of motion.

In the same time, in the framework of our assumption for a weak gravitational field and a low speed of the space rotation, $\frac{*}{\partial t} = \frac{1}{\sqrt{g_{00}}} \frac{\partial}{\partial t} \simeq \frac{\partial}{\partial t}$ and $\frac{*}{\partial x^i} = \frac{\partial}{\partial x^i} + \frac{1}{c^2} v_i \frac{*}{\partial t} \simeq \frac{\partial}{\partial x^i}$.

Applying all these conditions to the definitions of v_i , h_{ik} , F_i , A_{ik} , D_{ik} , and Δ_{km}^i , given in Page 7, we obtain substantially non-zero components of the characteristics of the space whose metric is (8):

$$w = \frac{GM}{r} + \frac{\omega^2 r^2}{2} - v v, \quad (63)$$

$$\left. \begin{aligned} v_1 &= \omega^2 t r \\ v_2 &= \omega r^2 (\omega t + 1) \\ v_3 &= \omega^2 t r \end{aligned} \right\}, \quad (64)$$

$$\left. \begin{aligned} \ddot{r} - \omega^2 \left(t - \frac{r\mathbf{v}}{c^2} \right) \dot{z} + \omega^2 (r - \mathbf{v}t) + \frac{\omega^2 \mathbf{v}t}{c^2} \dot{z}^2 &= 0 \\ \ddot{\varphi} + 2\omega \left(1 + \frac{\omega t}{2} \right) \frac{\dot{r}}{r} + \frac{\omega^2 \mathbf{v}}{c^2} \dot{z} + \omega^2 + \frac{2\omega \mathbf{v} \left(1 + \frac{\omega t}{2} \right)}{c^2 r} \dot{r} \dot{z} &= 0 \\ \ddot{z} + \omega^2 \left(t + \frac{r\mathbf{v}}{c^2} \right) \dot{r} + \frac{2\omega^2 \mathbf{v} r}{c^2} \dot{z} + \omega^2 r + \frac{\omega^2 \mathbf{v} t}{c^2} \dot{r}^2 + \frac{2\omega^2 \mathbf{v} t}{c^2} \dot{r} \dot{z} &= 0 \end{aligned} \right\} \quad (70)$$

$$\dot{r}^2 + \frac{2\omega^2 r \mathbf{v} t}{c^2} \dot{r} \dot{z} + \left(1 - \frac{2\omega^2 r \mathbf{v} t}{c^2} \right) \dot{z}^2 = c^2 \quad (71)$$

$$\left. \begin{aligned} F_1 &= -\omega^2 (r - \mathbf{v}t) \\ F_2 &= -\omega^2 r^2 \\ F_3 &= -\omega^2 r \end{aligned} \right\}, \quad (65)$$

$$\left. \begin{aligned} A_{12} &= \omega r \left(1 + \frac{\omega t}{2} \right) \\ A_{23} &= 0 \\ A_{13} &= \frac{\omega^2 t}{2} \end{aligned} \right\}, \quad (66)$$

$$\left. \begin{aligned} h_{11} &= 1, & h_{13} &= \frac{\omega^2 \mathbf{v} t r}{c^2} \\ h_{22} &= r^2, & h_{23} &= \frac{\omega r^2 \mathbf{v} (1 + \omega t)}{c^2} \\ h_{33} &= 1 - \frac{2\omega^2 \mathbf{v} t r}{c^2} \\ h^{11} &= 1, & h^{13} &= -\frac{\omega^2 \mathbf{v} t r}{c^2} \\ h^{22} &= \frac{1}{r^2}, & h^{23} &= -\frac{\omega \mathbf{v} (1 + \omega t)}{c^2} \\ h^{33} &= 1 + \frac{2\omega^2 \mathbf{v} t r}{c^2} \\ h &= r^2 \left(1 + \frac{2\omega^2 \mathbf{v} t r}{c^2} \right) \end{aligned} \right\}, \quad (67)$$

$$\left. \begin{aligned} D_{13} &= \frac{\omega^2 r \mathbf{v}}{2c^2}, & D_{23} &= \frac{\omega^2 r^2 \mathbf{v}}{2c^2} \\ D_{33} &= \frac{\omega^2 r \mathbf{v}}{c^2}, & D &= \frac{\omega^2 r \mathbf{v}}{c^2} \end{aligned} \right\}, \quad (68)$$

$$\left. \begin{aligned} \Delta_{22}^1 &= -r, & \Delta_{23}^1 &= -\frac{\omega r \mathbf{v}}{c^2} \left(1 + \frac{\omega t}{2} \right) \\ \Delta_{33}^1 &= \frac{\omega^2 \mathbf{v} t}{c^2}, & \Delta_{12}^2 &= \frac{1}{r} \\ \Delta_{13}^2 &= \frac{\omega \mathbf{v}}{c^2 r} \left(1 + \frac{\omega t}{2} \right), & \Delta_{11}^3 &= \frac{\omega^2 \mathbf{v} t}{c^2} \\ \Delta_{12}^3 &= \frac{\omega^2 r \mathbf{v} t}{2c^2}, & \Delta_{13}^3 &= -\frac{\omega^2 \mathbf{v} t}{c^2} \end{aligned} \right\}, \quad (69)$$

where we present only those components of Christoffel's symbols which will be used in the geodesic equations (equations of motion).

After substitution of the components, the vectorial equations of isotropic geodesic (62) take the form (70). The condition $h_{ik} c^i c^k = c^2$ — a chronometrically invariant expression of the condition $ds^2 = c^2 d\tau^2 - d\sigma^2 = 0$, which is specific to isotropic trajectories — takes the form (71).

We consider a light beam (a couple of photons) travelling from the Earth along the radial direction r . Therefore, looking for anisotropy in the distribution of the photons' trajectories in the field, we are interested to solve only the third isotropic geodesic equation of (70), which is the equation of motion of a photon along the z -axis orthogonal to the light beam's direction r .

Before to solve the equation, a few notes on our assumptions should be made.

First, because the Earth moves relative to the weak microwave background with a velocity \mathbf{v}^1 along the z -direction, only $\mathbf{v}^3 = \dot{z}$ of the components \mathbf{v}^i is non-zero. Besides that, as easy to see from our previous considerations, we should mean $\frac{\partial}{\partial t} = \frac{1}{\sqrt{g_{00}}} \frac{\partial}{\partial t} \simeq \frac{\partial}{\partial t}$ and $\frac{\partial}{\partial z} = \frac{\partial}{\partial x^3} + \frac{1}{c^2} v_3 \frac{\partial}{\partial t} \simeq \frac{\partial}{\partial z} = 0$. Hence, we apply $\frac{d}{d\tau} = \frac{\partial}{\partial t} + \mathbf{v}^3 \frac{\partial}{\partial z} = \frac{d}{dt}$ to our calculation.

Second, the orbital velocity of a satellite of the Earth, ~ 8 km/sec, is much lesser than the velocity of light. We therefore assume that a light beam doesn't sense the orbital motion of such a satellite. The coordinate φ in the equations of isotropic geodesics is related to the light beam (a couple of single photons), not the rotation of the reference space of a satellite bound observer. Hence, we assume $\varphi = \text{const}$ in our calculation, i.e. $c^2 = \frac{d\varphi}{dt} = \dot{\varphi} = 0$.

Third, we are talking about the counting for single photons in a detector which is located on board of a satellite. The process of the measurement is actually instant. In other word, the measurement is processed very close the moment $t_0 = 0$. Hence we assume $\dot{z} = 0$ in our calculation, while the acceleration \ddot{z} can be non-zero in the z -direction orthogonally to the initially r -direction of such a photon.

Fourth, we apply the relations $\dot{r} = c$ and $r = ct$ which are obvious for such a photon. If such a photon, travelling ini-

tially in the r -direction, experiences a shift to the z -direction (the direction of the motion of the Earth relative to the weak intergalactic field), the distribution of photons of the Earth's microwave field has an anisotropy to the z -direction.

After taking all the factors into account, the third equation of the system (70), which is the equation of motion of a single photon in the z -direction, takes the simple form

$$\ddot{z} + \omega^2 \left(ct + \frac{r\mathbf{v}}{c} \right) + \omega^2 (r + \mathbf{v}t) = 0 \quad (72)$$

which, due to the weightlessness condition $\frac{GM}{r^2} = \omega^2 r$ and the condition $r = ct$, is

$$\ddot{z} + \frac{2GM_{\oplus}}{c^2 t^2} \left(1 + \frac{\mathbf{v}}{c} \right) = 0. \quad (73)$$

Integration of this equation gives

$$\dot{z} = \frac{2GM_{\oplus}}{cr} \left(1 + \frac{\mathbf{v}}{c} \right) = \dot{z}' + \Delta z'. \quad (74)$$

The first term of the solution (74) manifests that fact that such a photon, initially launched in the r -direction (radial direction) in the gravitational field of the Earth, is carried into the z -direction by the rotation of the space of the Earth. The second term, $\Delta z'$, manifests the carriage of the photon into the z -direction due to the motion of the Earth in this direction through the weak intergalactic field.

As a result we obtain the carriage of the three-dimensional vector of the observable velocity of light from the initially r -direction to the z -direction, due to the common motion of the space of the Earth in the point of observation:

$$\frac{\Delta \dot{z}'}{\dot{z}'} = \frac{\mathbf{v}}{c}. \quad (75)$$

Such a carriage of a photon radiated from the Earth's surface, being applied to a microwave background generated by oceanic water, reveals the anisotropy associated with the dipole component of the microwave background.

As seen from the obtained formula (75), such a carriage of a photon into the z -direction, doesn't depend on the path travelled by such a photon in the radial direction r from the Earth. In other word, the anisotropy associated with the dipole component of the Earth microwave background shouldn't be dependent on altitude from the surface of the Earth: the anisotropy of the Earth microwave background should be the same if measured on board a U2 aeroplane (25 km), at the orbit of the COBE satellite (900 km), and at the 2nd Lagrange point (the WMAP satellite and PLANCK satellite, 1.5 million km from the Earth).

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SPECIAL REPORT**New Approach to Quantum Electrodynamics**

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It is shown that a photon with a specific frequency can be identified with the Dirac magnetic monopole. When a Dirac-Wilson line forms a Dirac-Wilson loop, it is a photon. This loop model of photon is exactly solvable. From the winding numbers of this loop-form of photon, we derive the quantization properties of energy and electric charge. A new QED theory is presented that is free of ultraviolet divergences. The Dirac-Wilson line is as the quantum photon propagator of the new QED theory from which we can derive known QED effects such as the anomalous magnetic moment and the Lamb shift. The one-loop computation of these effects is simpler and is more accurate than that in the conventional QED theory. Furthermore, from the new QED theory, we have derived a new QED effect. A new formulation of the Bethe-Salpeter (BS) equation solves the difficulties of the BS equation and gives a modified ground state of the positronium. By the mentioned new QED effect and by the new formulation of the BS equation, a term in the orthopositronium decay rate that is missing in the conventional QED is found, resolving the orthopositronium lifetime puzzle completely. It is also shown that the graviton can be constructed from the photon, yielding a theory of quantum gravity that unifies gravitation and electromagnetism.

1 Introduction

It is well known that the quantum era of physics began with the quantization of energy of electromagnetic field, from which Planck derived the radiation formula. Einstein then introduced the light-quantum to explain the photoelectric effects. This light-quantum was regarded as a particle called photon [1–3]. Quantum mechanics was then developed, ushering in the modern quantum physics era. Subsequently, the quantization of the electromagnetic field and the theory of Quantum Electrodynamics (QED) were established.

In this development of quantum theory of physics, the photon plays a special role. While it is the beginning of quantum physics, it is not easy to understand as is the quantum mechanics of other particles described by the Schrödinger equation. In fact, Einstein was careful in regarding the light-quantum as a particle, and the acceptance of the light-quantum as a particle called photon did not come about until much later [1]. The quantum field theory of electromagnetic field was developed for the photon. However, such difficulties of the quantum field theory as the ultraviolet divergences are well known. Because of the difficulty of understanding the photon, Einstein once asked: “What is the photon?” [1].

On the other hand, based on the symmetry of the electric and magnetic field described by the Maxwell equation and on the complex wave function of quantum mechanics, Dirac derived the concept of the magnetic monopole, which is hypothetically considered as a particle with magnetic charge, in analogy to the electron with electric charge. An important feature of this magnetic monopole is that it gives the quanti-

zation of electric charge. Thus it is interesting and important to find such particles. However, in spite of much effort, no such particles have been found [4, 5].

In this paper we shall establish a mathematical model of photon to show that the magnetic monopole can be identified as a photon. Before giving the detailed model, let us discuss some thoughts for this identification in the following.

First, if the photon and the magnetic monopole are different types of elementary quantum particles in the electromagnetic field, it is odd that one type can be derived from the other. A natural resolution of this oddity is the identification of the magnetic monopole as a photon.

The quantum field theory of the free Maxwell equation is the basic quantum theory of photon [6]. This free field theory is a linear theory and the models of the quantum particles obtained from this theory are linear. However, a stable particle should be a soliton, which is of the nonlinear nature. Secondly, the quantum particles of the quantum theory of Maxwell equation are collective quantum effects in the same way the phonons which are elementary excitations in a statistical model. These phonons are usually considered as quasi-particles and are not regarded as real particles. Regarding the Maxwell equation as a statistical wave equation of electromagnetic field, we have that the quantum particles in the quantum theory of Maxwell equation are analogous to the phonons. Thus they should be regarded as quasi-photons and have properties of photons but not a complete description of photons.

In this paper, a nonlinear model of photon is established. In the model, we show that the Dirac magnetic monopole

can be identified with the photon with some frequencies. We provide a $U(1)$ gauge theory of Quantum Electrodynamics (QED), from which we derive photon as a quantum Dirac-Wilson loop $W(z, z)$ of this model. This nonlinear loop model of the photon is exactly solvable and thus may be regarded as a quantum soliton. From the winding numbers of this loop model of the photon, we derive the quantization property of energy in Planck's formula of radiation and the quantization property of charge. We show that the quantization property of charge is derived from the quantization property of energy (in Planck's formula of radiation), when the magnetic monopole is identified with photon with certain frequencies. This explains why we cannot physically find a magnetic monopole. It is simply a photon with a specific frequency.

From this nonlinear model of the photon, we also construct a model of the electron which has a mass mechanism for generating mass of the electron. This mechanism of generating mass supersedes the conventional mechanism of generating mass (through the Higgs particles) and makes hypothesizing the existence of the Higgs particles unnecessary. This explains why we cannot physically find such Higgs particles.

The new quantum gauge theory is similar to the conventional QED theory except that the former is not based on the four dimensional space-time (t, \mathbf{x}) but is based on the proper time s in the theory of relativity. Only in a later stage in the new quantum gauge theory, the space-time variable (t, \mathbf{x}) is derived from the proper time s through the Lorentz metric $ds^2 = dt^2 - d\mathbf{x}^2$ to obtain space-time statistics and explain the observable QED effects.

The derived space variable \mathbf{x} is a random variable in this quantum gauge theory. Recall that the conventional quantum mechanics is based on the space-time. Since the space variable \mathbf{x} is actually a random variable as shown in the new quantum gauge theory, the conventional quantum mechanics needs probabilistic interpretation and thus has a most mysterious measurement problem, on which Albert Einstein once remarked: "God does not play dice with the universe." In contrast, the new quantum gauge theory does not involve the mentioned measurement problem because it is not based on the space-time and is deterministic. Thus this quantum gauge theory resolves the mysterious measurement problem of quantum mechanics.

Using the space-time statistics, we employ Feynman diagrams and Feynman rules to compute the basic QED effects such as the vertex correction, the photon self-energy and the electron self-energy. In this computation of the Feynman integrals, the dimensional regularization method in the conventional QED theory is also used. Nevertheless, while the conventional QED theory uses it to reduce the dimension 4 of space-time to a (fractional) number n to avoid the ultraviolet divergences in the Feynman integrals, the new QED theory uses it to increase the dimension 1 of the proper time to a number n less than 4, which is the dimension of the space-

time, to derive the space-time statistics. In the new QED theory, there are no ultraviolet divergences, and the dimensional regularization method is not used for *regularization*.

After this increase of dimension, the renormalization method is used to derive the well-known QED effects. Unlike the conventional QED theory, the renormalization method is used in the new QED theory to compute the space-time statistics, but not to remove the ultraviolet divergences, since the ultraviolet divergences do not occur in the new QED theory. From these QED effects, we compute the anomalous magnetic moment and the Lamb shift [6]. The computation does not involve numerical approximation as does that of the conventional QED and is simpler and more accurate.

For getting these QED effects, the quantum photon propagator $W(z, z')$, which is like a line segment connecting two electrons, is used to derive the electrodynamic interaction. (When the quantum photon propagator $W(z, z')$ forms a closed circle with $z = z'$, it then becomes a photon $W(z, z)$.) From this quantum photon propagator, a photon propagator is derived that is similar to the Feynman photon propagator in the conventional QED theory.

The photon-loop $W(z, z)$ leads to the renormalized electric charge e and the mass m of electron. In the conventional QED theory, the bare charge e_0 is of less importance than the renormalized charge e , in the sense that it is unobservable. In contrast, in this new theory of QED, the bare charge e_0 and the renormalized charge e are of equal importance. While the renormalized charge e leads to the physical results of QED, the bare charge e_0 leads to the universal gravitation constant G . It is shown that $e = n_e e_0$, where n_e is a very large winding number and thus e_0 is a very small number. It is further shown that the gravitational constant $G = 2e_0^2$ which is thus an extremely small number. This agrees with the fact that the experimental gravitational constant G is a very small number. The relationships, $e = n_e e_0$ and $G = 2e_0^2$, are a part of a theory unifying gravitation and electromagnetism. In this unified theory, the graviton propagator and the graviton are constructed from the quantum photon propagator. This construction leads to a theory of quantum gravity. In short, a new theory of quantum gravity is developed from the new QED theory in this paper, and unification of gravitation and electromagnetism is achieved.

In this paper, we also derive a new QED effect from the seagull vertex of the new QED theory. The conventional Bethe-Salpeter (BS) equation is reformulated to resolve its difficulties (such as the existence of abnormal solutions [7–32]) and to give a modified ground state wave function of the positronium. By the new QED effect and the reformulated BS equation, another new QED effect, a term in the orthopositronium decay rate that is missing in the conventional QED is discovered. Including the discovered term, the computed orthopositronium decay rate now agrees with the experimental rate, resolving the *orthopositronium lifetime puzzle* completely [33–52]. We note that the recent resolution of

this orthopositronium lifetime puzzle resolves the puzzle only partially due to a special statistical nature of this new term in the orthopositronium decay rate.

This paper is organized as follows. In Section 2 we give a brief description of a new QED theory. With this theory, we introduce the classical Dirac-Wilson loop in Section 3. We show that the quantum version of this loop is a nonlinear exactly solvable model and thus can be regarded as a soliton. We identify this quantum Dirac-Wilson loop as a photon with the $U(1)$ group as the gauge group. To investigate the properties of this Dirac-Wilson loop, we derive a chiral symmetry from the gauge symmetry of this quantum model. From this chiral symmetry, we derive, in Section 4, a conformal field theory, which includes an affine Kac-Moody algebra and a quantum Knizhnik-Zamolodchikov (KZ) equation. A main point of our model on the quantum KZ equation is that we can derive two KZ equations which are dual to each other. This duality is the main point for the Dirac-Wilson loop to be exactly solvable and to have a winding property which explains properties of photon. This quantum KZ equation can be regarded as a quantum Yang-Mills equation.

In Sections 5 to 8, we solve the Dirac-Wilson loop in a form with a winding property, starting with the KZ equations. From the winding property of the Dirac-Wilson loop, we derive, in Section 9 and Section 10, the quantization of energy and the quantization of electric charge which are properties of photon and magnetic monopole. We then show that the quantization property of charge is derived from the quantization property of energy of Planck's formula of radiation, when we identify photon with the magnetic monopole for some frequencies. From this nonlinear model of photon, we also derive a model of the electron in Section 11. In this model of electron, we provide a mass mechanism for generating mass to electron. In Section 12, we show that the photon with a specific frequency can carry electric charge and magnetic charge, since an electron is formed from a photon with a specific frequency for giving the electric charge and magnetic charge. In Section 13, we derive the statistics of photons and electrons from the loop models of photons and electrons.

In Sections 14 to 22, we derive a new theory of QED, wherein we perform the computation of the known basic QED effects such as the photon self-energy, the electron self-energy and the vertex correction. In particular, we provide simpler and more accurate computation of the anomalous magnetic moment and the Lamb shift. Then in Section 23, we compute a new QED effect. Then from Section 24 to Section 25, we reformulate the Bethe-Salpeter (BS) equation. With this new version of the BS equation and the new QED effect, a modified ground state wave function of the positronium is derived. Then by this modified ground state of the positronium, we derive in Section 26 another new QED effect, a term missing in the theoretic orthopositronium decay rate of the conventional QED theory, and show that this new theoretical orthopositronium decay rate agrees with the experimental de-

cay rate, completely resolving the orthopositronium life time puzzle [33–52].

In Section 27, the graviton is derived from the photon. This leads to a new theory of quantum gravity and a new unification of gravitation and electromagnetism. Then in Section 28, we show that the quantized energies of gravitons can be identified as dark energy. Then in a way similar to the construction of electrons by photons, we use gravitons to construct particles which can be regarded as dark matter. We show that the force among gravitons can be repulsive. This gives the diffusion phenomenon of dark energy and the accelerating expansion of the universe [53–57].

2 New gauge model of QED

Let us construct a quantum gauge model, as follows. In probability theory we have the Wiener measure ν which is a measure on the space $C[t_0, t_1]$ of continuous functions [58]. This measure is a well defined mathematical theory for the Brownian motion and it may be symbolically written in the following form:

$$d\nu = e^{-L_0} dx, \quad (1)$$

where $L_0 := \frac{1}{2} \int_{t_0}^{t_1} \left(\frac{dx}{dt}\right)^2 dt$ is the energy integral of the Brownian particle and $dx = \frac{1}{N} \prod_t dx(t)$ is symbolically a product of Lebesgue measures $dx(t)$ and N is a normalized constant.

Once the Wiener measure is defined we may then define other measures on $C[t_0, t_1]$, as follows [58]. Let a potential term $\frac{1}{2} \int_{t_0}^{t_1} V dt$ be added to L_0 . Then we have a measure ν_1 on $C[t_0, t_1]$ defined by:

$$d\nu_1 = e^{-\frac{1}{2} \int_{t_0}^{t_1} V dt} d\nu. \quad (2)$$

Under some condition on V we have that ν_1 is well defined on $C[t_0, t_1]$. Let us call (2) as the Feynman-Kac formula [58].

Let us then follow this formula to construct a quantum model of electrodynamics, as follows. Then similar to the formula (2) we construct a quantum model of electrodynamics from the following energy integral:

$$\begin{aligned} - \int_{s_0}^{s_1} Dds := & - \int_{s_0}^{s_1} \left[\frac{1}{2} \left(\frac{\partial A_1}{\partial x^2} - \frac{\partial A_2}{\partial x^1} \right)^* \left(\frac{\partial A_1}{\partial x^2} - \frac{\partial A_2}{\partial x^1} \right) + \right. \\ & + \left(\frac{dZ^*}{ds} + ie_0 \left(\sum_{j=1}^2 A_j \frac{dx^j}{ds} \right) Z^* \right) \times \\ & \left. \times \left(\frac{dZ}{ds} - ie_0 \left(\sum_{j=1}^2 A_j \frac{dx^j}{ds} \right) Z \right) \right] ds, \end{aligned} \quad (3)$$

where the complex variable $Z = Z(z(s))$ and the real variables $A_1 = A_1(z(s))$ and $A_2 = A_2(z(s))$ are continuous functions in a form that they are in terms of a (continuously differentiable) curve $z(s) = C(s) = (x^1(s), x^2(s))$, $s_0 \leq s \leq s_1$, $z(s_0) = z(s_1)$ in the complex plane where s is a parameter representing the proper time in relativity. (We shall also write

$z(s)$ in the complex variable form $C(s) = z(s) = x^1(s) + ix^2(s)$, $s_0 \leq s \leq s_1$.) The complex variable $Z = Z(z(s))$ represents a field of matter, such as the electron (Z^* denotes its complex conjugate), and the real variables $A_1 = A_1(z(s))$ and $A_2 = A_2(z(s))$ represent a connection (or the gauge field of the photon) and e_0 denotes the (bare) electric charge.

The integral (1.1) has the following gauge symmetry:

$$\begin{aligned} Z'(z(s)) &:= Z(z(s)) e^{ie_0 a(z(s))} \\ A'_j(z(s)) &:= A_j(z(s)) + \frac{\partial a}{\partial x^j}, \quad j = 1, 2 \end{aligned} \quad (4)$$

where $a = a(z)$ is a continuously differentiable real-valued function of z .

We remark that this QED theory is similar to the conventional Yang-Mills gauge theories. A feature of (1.1) is that it is not formulated with the four-dimensional space-time but is formulated with the one dimensional proper time. This one dimensional nature let this QED theory avoid the usual ultraviolet divergence difficulty of quantum fields. As most of the theories in physics are formulated with the space-time let us give reasons of this formulation. We know that with the concept of space-time we have a convenient way to understand physical phenomena and to formulate theories such as the Newton equation, the Schrödinger equation, e.t.c. to describe these physical phenomena. However we also know that there are fundamental difficulties related to space-time such as the ultraviolet divergence difficulty of quantum field theory. To resolve these difficulties let us reexamine the concept of space-time. We propose that the space-time is a statistical concept which is not as basic as the proper time in relativity. Because a statistical theory is usually a convenient but incomplete description of a more basic theory this means that some difficulties may appear if we formulate a physical theory with the space-time. This also means that a way to formulate a basic theory of physics is to formulate it not with the space-time but with the proper time only as the parameter for evolution. This is a reason that we use (1.1) to formulate a QED theory. In this formulation we regard the proper time as an independent parameter for evolution. From (1.1) we may obtain the conventional results in terms of space-time by introducing the space-time as a statistical method.

Let us explain in more detail how the space-time comes out as a statistics. For statistical purpose when many electrons (or many photons) present we introduce space-time (t, \mathbf{x}) as a statistical method to write ds^2 in the form

$$ds^2 = dt^2 - d\mathbf{x}^2. \quad (5)$$

We notice that for a given ds there may have many dt and $d\mathbf{x}$ which correspond to many electrons (or photons) such that (5) holds. In this way the space-time is introduced as a statistics. By (5) we shall derive statistical formulas for many electrons (or photons) from formulas obtained from (1.1). In this way we obtain the Dirac equation as a statistical equation for electrons and the Maxwell equation as a statistical

equation for photons. In this way we may regard the conventional QED theory as a statistical theory extended from the proper-time formulation of this QED theory (From the proper-time formulation of this QED theory we also have a theory of space-time statistics which give the results of the conventional QED theory). This statistical interpretation of the conventional QED theory is thus an explanation of the mystery that the conventional QED theory is successful in the computation of quantum effects of electromagnetic interaction while it has the difficulty of ultraviolet divergence.

We notice that the relation (5) is the famous Lorentz metric. (We may generalize it to other metric in General Relativity.) Here our understanding of the Lorentz metric is that it is a statistical formula where the proper time s is more fundamental than the space-time (t, \mathbf{x}) in the sense that we first have the proper time and the space-time is introduced via the Lorentz metric only for the purpose of statistics. This reverses the order of appearance of the proper time and the space-time in the history of relativity in which we first have the concept of space-time and then we have the concept of proper time which is introduced via the Lorentz metric. Once we understand that the space-time is a statistical concept from (1.1) we can give a solution to the quantum measurement problem in the debate about quantum mechanics between Bohr and Einstein. In this debate Bohr insisted that with the probability interpretation quantum mechanics is very successful. On the other hand Einstein insisted that quantum mechanics is incomplete because of probability interpretation. Here we resolve this debate by constructing the above QED theory which is a quantum theory as the quantum mechanics and unlike quantum mechanics which needs probability interpretation we have that this QED theory is deterministic since it is not formulated with the space-time.

Similar to the usual Yang-Mills gauge theory we can generalize this gauge theory with $U(1)$ gauge symmetry to non-abelian gauge theories. As an illustration let us consider $SU(2) \otimes U(1)$ gauge symmetry where $SU(2) \otimes U(1)$ denotes the direct product of the groups $SU(2)$ and $U(1)$.

Similar to (1.1) we consider the following energy integral:

$$\begin{aligned} L := \int_{s_0}^{s_1} & \left[\frac{1}{2} \text{Tr} (D_1 A_2 - D_2 A_1)^* (D_1 A_2 - D_2 A_1) + \right. \\ & \left. + (D_0^* Z^*) (D_0 Z) \right] ds, \end{aligned} \quad (6)$$

where $Z = (z_1, z_2)^T$ is a two dimensional complex vector; $A_j = \sum_{k=0}^3 A_j^k t^k$ ($j = 1, 2$) where A_j^k denotes a component of a gauge field A^k ; $t^k = iT^k$ denotes a generator of $SU(2) \otimes U(1)$ where T^k denotes a self-adjoint generator of $SU(2) \otimes U(1)$ (here for simplicity we choose a convention that the complex i is absorbed by t^k and t^k is absorbed by A_j ; and the notation A_j is with a little confusion with the notation A_j in the above formulation of (1.1) where A_j , $j = 1, 2$ are real valued); and $D_l = \frac{\partial}{\partial x^l} - e_0 (\sum_{j=1}^2 A_j \frac{dx^j}{ds})$ for $l = 1, 2$; and $D_0 = \frac{d}{ds} - e_0 (\sum_{j=1}^2 A_j \frac{dx^j}{ds})$ where e_0 is the bare electric

charge for general interactions including the strong and weak interactions.

From (6) we can develop a nonabelian gauge theory as similar to that for the above abelian gauge theory. We have that (6) is invariant under the following gauge transformation:

$$\begin{aligned} Z'(z(s)) &:= U(a(z(s))) Z(z(s)) \\ A'_j(z(s)) &:= U(a(z(s))) A_j(z(s)) U^{-1}(a(z(s))) + \\ &+ U(a(z(s))) \frac{\partial U^{-1}}{\partial x^j}(a(z(s))), \quad j = 1, 2 \end{aligned} \quad (7)$$

where $U(a(z(s))) = e^{a(z(s))}$; $a(z(s)) = \sum_k e_0 a^k(z(s)) t^k$ for some functions a^k . We shall mainly consider the case that a is a function of the form $a(z(s)) = \sum_k \text{Re } \omega^k(z(s)) t^k$ where ω^k are analytic functions of z . (We let the function $\omega(z(s)) := \sum_k \omega^k(z(s)) t^k$ and we write $a(z) = \text{Re } \omega(z)$.)

The above gauge theory is based on the Banach space X of continuous functions $Z(z(s)), A_j(z(s)), j = 1, 2, s_0 \leq s \leq s_1$ on the one dimensional interval $[s_0, s_1]$.

Since L is positive and the theory is one dimensional (and thus is simpler than the usual two dimensional Yang-Mills gauge theory) we have that this gauge theory is similar to the Wiener measure except that this gauge theory has a gauge symmetry. This gauge symmetry gives a degenerate degree of freedom. In the physics literature the usual way to treat the degenerate degree of freedom of gauge symmetry is to introduce a gauge fixing condition to eliminate the degenerate degree of freedom where each gauge fixing will give equivalent physical results [59]. There are various gauge fixing conditions such as the Lorentz gauge condition, the Feynman gauge condition, etc. We shall later in the Section on the Kac-Moody algebra adopt a gauge fixing condition for the above gauge theory. This gauge fixing condition will also be used to derive the quantum KZ equation in dual form which will be regarded as a quantum Yang-Mill equation since its role will be similar to the classical Yang-Mill equation derived from the classical Yang-Mill gauge theory.

3 Classical Dirac-Wilson loop

Similar to the Wilson loop in quantum field theory [60] from our quantum theory we introduce an analogue of Wilson loop, as follows. (We shall also call a Wilson loop as a Dirac-Wilson loop.)

Definition A classical Wilson loop $W_R(C)$ is defined by:

$$W_R(C) := W(z_0, z_1) := P e^{e_0 \int_C A_j dx^j}, \quad (8)$$

where R denotes a representation of $SU(2)$; $C(\cdot) = z(\cdot)$ is a fixed closed curve where the quantum gauge theories are based on it as specific in the above Section. As usual the notation P in the definition of $W_R(C)$ denotes a path-ordered product [60–62].

Let us give some remarks on the above definition of Wilson loop, as follows.

1. We use the notation $W(z_0, z_1)$ to mean the Wilson loop $W_R(C)$ which is based on the whole closed curve $z(\cdot)$. Here for convenience we use only the end points z_0 and z_1 of the curve $z(\cdot)$ to denote this Wilson loop (We keep in mind that the definition of $W(z_0, z_1)$ depends on the whole curve $z(\cdot)$ connecting z_0 and z_1).

Then we extend the definition of $W_R(C)$ to the case that $z(\cdot)$ is not a closed curve with $z_0 \neq z_1$. When $z(\cdot)$ is not a closed curve we shall call $W(z_0, z_1)$ as a Wilson line.

2. In constructing the Wilson loop we need to choose a representation R of the $SU(2)$ group. We shall see that because a Wilson line $W(z_0, z_1)$ is with two variables z_0 and z_1 a natural representation of a Wilson line or a Wilson loop is the tensor product of the usual two dimensional representation of the $SU(2)$ for constructing the Wilson loop. \diamond

A basic property of a Wilson line $W(z_0, z_1)$ is that for a given continuous path $A_j, j = 1, 2$ on $[s_0, s_1]$ the Wilson line $W(z_0, z_1)$ exists on this path and has the following transition property:

$$W(z_0, z_1) = W(z_0, z) W(z, z_1) \quad (9)$$

where $W(z_0, z_1)$ denotes the Wilson line of a curve $z(\cdot)$ which is with z_0 as the starting point and z_1 as the ending point and z is a point on $z(\cdot)$ between z_0 and z_1 .

This property can be proved as follows. We have that $W(z_0, z_1)$ is a limit (whenever exists) of ordered product of $e^{A_j \Delta x^j}$ and thus can be written in the following form:

$$\begin{aligned} W(z_0, z_1) &= I + \int_{s'}^{s''} e_0 A_j(z(s)) \frac{dx^j(s)}{ds} ds + \\ &+ \int_{s'}^{s''} e_0 A_j(z(s_2)) \frac{dx^j(s_2)}{ds} \times \\ &\times \left[\int_{s'}^{s_2} e_0 A_j(z(s_3)) \frac{dx^j(s_3)}{ds} ds_3 \right] ds_2 + \dots \end{aligned} \quad (10)$$

where $z(s') = z_0$ and $z(s'') = z_1$. Then since A_i are continuous on $[s', s'']$ and $x^i(z(\cdot))$ are continuously differentiable on $[s', s'']$ we have that the series in (10) is absolutely convergent. Thus the Wilson line $W(z_0, z_1)$ exists. Then since $W(z_0, z_1)$ is the limit of ordered product we can write $W(z_0, z_1)$ in the form $W(z_0, z) W(z, z_1)$ by dividing $z(\cdot)$ into two parts at z . This proves the basic property of Wilson line. \diamond

Remark (classical and quantum versions of Wilson loop)

From the above property we have that the Wilson line $W(z_0, z_1)$ exists in the classical pathwise sense where A_i are as classical paths on $[s_0, s_1]$. This pathwise version of the Wilson line $W(z_0, z_1)$; from the Feynman path integral point of view; is as a partial description of the quantum version of the Wilson line $W(z_0, z_1)$ which is as an operator when A_i are as operators. We shall in the next Section derive and define a quantum generator J of $W(z_0, z_1)$ from the quantum gauge theory. Then by using this generator J we shall compute the quantum version of the Wilson line $W(z_0, z_1)$.

We shall denote both the classical version and quantum version of Wilson line by the same notation $W(z_0, z_1)$ when

there is no confusion. \diamond

By following the usual approach of deriving a chiral symmetry from a gauge transformation of a gauge field we have the following chiral symmetry which is derived by applying an analytic gauge transformation with an analytic function ω for the transformation:

$$\begin{aligned} W(z_0, z_1) &\mapsto W'(z_0, z_1) = \\ &= U(\omega(z_1))W(z_0, z_1)U^{-1}(\omega(z_0)), \end{aligned} \quad (11)$$

where $W'(z_0, z_1)$ is a Wilson line with gauge field:

$$A'_\mu = \frac{\partial U(z)}{\partial x^\mu} U^{-1}(z) + U(z) A_\mu U^{-1}(z). \quad (12)$$

This chiral symmetry is analogous to the chiral symmetry of the usual gauge theory where U denotes an element of the gauge group [61]. Let us derive (11) as follows. Let $U(z) := U(\omega(z(s)))$ and $U(z + dz) \approx U(z) + \frac{\partial U(z)}{\partial x^\mu} dx^\mu$ where $dz = (dx^1, dx^2)$. Following [61] we have

$$\begin{aligned} U(z + dz)(1 + e_0 dx^\mu A_\mu)U^{-1}(z) &= \\ &= U(z + dz)U^{-1}(z) + e_0 dx^\mu U(z + dz)A_\mu U^{-1}(z) \\ &\approx 1 + \frac{\partial U(z)}{\partial x^\mu} U^{-1}(z)dx^\mu + e_0 dx^\mu U(z + dz)A_\mu U^{-1}(z) \\ &\approx 1 + \frac{\partial U(z)}{\partial x^\mu} U^{-1}(z)dx^\mu + e_0 dx^\mu U(z)A_\mu U^{-1}(z) \\ &=: 1 + \frac{\partial U(z)}{\partial x^\mu} U^{-1}(z)dx^\mu + e_0 dx^\mu U(z)A_\mu U^{-1}(z) \\ &=: 1 + e_0 dx^\mu A'_\mu. \end{aligned} \quad (13)$$

From (13) we have that (11) holds.

As analogous to the WZW model in conformal field theory [65, 66] from the above symmetry we have the following formulas for the variations $\delta_\omega W$ and $\delta_{\omega'} W$ with respect to this symmetry (see [65] p.621):

$$\delta_\omega W(z, z') = W(z, z')\omega(z) \quad (14)$$

and

$$\delta_{\omega'} W(z, z') = -\omega'(z')W(z, z'), \quad (15)$$

where z and z' are independent variables and $\omega'(z') = \omega(z)$ when $z' = z$. In (14) the variation is with respect to the z variable while in (15) the variation is with respect to the z' variable. This two-side-variations when $z \neq z'$ can be derived as follows. For the left variation we may let ω be analytic in a neighborhood of z and extended as a continuously differentiable function to a neighborhood of z' such that $\omega(z') = 0$ in this neighborhood of z' . Then from (11) we have that (14) holds. Similarly we may let ω' be analytic in a neighborhood of z' and extended as a continuously differentiable function to a neighborhood of z such that $\omega'(z) = 0$ in this neighborhood of z . Then we have that (15) holds.

4 Gauge fixing and affine Kac-Moody algebra

This Section has two related purposes. One purpose is to find a gauge fixing condition for eliminating the degenerate degree of freedom from the gauge invariance of the above quantum gauge theory in Section 2. Then another purpose is to find an equation for defining a generator J of the Wilson line $W(z, z')$. This defining equation of J can then be used as a gauge fixing condition. Thus with this defining equation of J the construction of the quantum gauge theory in Section 2 is then completed.

We shall derive a quantum loop algebra (or the affine Kac-Moody algebra) structure from the Wilson line $W(z, z')$ for the generator J of $W(z, z')$. To this end let us first consider the classical case. Since $W(z, z')$ is constructed from $SU(2)$ we have that the mapping $z \rightarrow W(z, z')$ (We consider $W(z, z')$ as a function of z with z' being fixed) has a loop group structure [63, 64]. For a loop group we have the following generators:

$$J_n^a = t^a z^n \quad n = 0, \pm 1, \pm 2, \dots \quad (16)$$

These generators satisfy the following algebra:

$$[J_m^a, J_n^b] = if_{abc} J_{m+n}^c. \quad (17)$$

This is the so called loop algebra [63, 64]. Let us then introduce the following generating function J :

$$J(w) = \sum_a J^a(w) = \sum_a j^a(w) t^a, \quad (18)$$

where we define

$$J^a(w) = j^a(w)t^a := \sum_{n=-\infty}^{\infty} J_n^a(z)(w-z)^{-n-1}. \quad (19)$$

From J we have

$$J_n^a = \frac{1}{2\pi i} \oint_z dw (w-z)^n J^a(w), \quad (20)$$

where \oint_z denotes a closed contour integral with center z . This formula can be interpreted as that J is the generator of the loop group and that J_n^a is the directional generator in the direction $\omega^a(w) = (w-z)^n$. We may generalize (20) to the following directional generator:

$$\frac{1}{2\pi i} \oint_z dw \omega(w) J(w), \quad (21)$$

where the analytic function $\omega(w) = \sum_a \omega^a(w)t^a$ is regarded as a direction and we define

$$\omega(w)J(w) := \sum_a \omega^a(w)J^a. \quad (22)$$

Then since $W(z, z') \in SU(2)$, from the variational formula (21) for the loop algebra of the loop group of $SU(2)$ we

have that the variation of $W(z, z')$ in the direction $\omega(w)$ is given by

$$W(z, z') \frac{1}{2\pi i} \oint_z dw \omega(w) J(w). \quad (23)$$

Now let us consider the quantum case which is based on the quantum gauge theory in Section 2. For this quantum case we shall define a quantum generator J which is analogous to the J in (18). We shall choose the equations (34) and (35) as the equations for defining the quantum generator J . Let us first give a formal derivation of the equation (34), as follows. Let us consider the following formal functional integration:

$$\begin{aligned} \langle W(z, z') A(z) \rangle &:= \\ &:= \int dA_1 dA_2 dZ^* dZ e^{-L} W(z, z') A(z), \end{aligned} \quad (24)$$

where $A(z)$ denotes a field from the quantum gauge theory. (We first let z' be fixed as a parameter.)

Let us do a calculus of variation on this integral to derive a variational equation by applying a gauge transformation on (24) as follows. (We remark that such variational equations are usually called the Ward identity in the physics literature.)

Let (A_1, A_2, Z) be regarded as a coordinate system of the integral (24). Under a gauge transformation (regarded as a change of coordinate) with gauge function $a(z(s))$ this coordinate is changed to another coordinate (A'_1, A'_2, Z') . As similar to the usual change of variable for integration we have that the integral (24) is unchanged under a change of variable and we have the following equality:

$$\begin{aligned} \int dA'_1 dA'_2 dZ'^* dZ' e^{-L'} W'(z, z') A'(z) &= \\ = \int dA_1 dA_2 dZ^* dZ e^{-L} W(z, z') A(z), \end{aligned} \quad (25)$$

where $W'(z, z')$ denotes the Wilson line based on A'_1 and A'_2 and similarly $A'(z)$ denotes the field obtained from $A(z)$ with (A_1, A_2, Z) replaced by (A'_1, A'_2, Z') .

Then it can be shown that the differential is unchanged under a gauge transformation [59]:

$$dA'_1 dA'_2 dZ'^* dZ' = dA_1 dA_2 dZ^* dZ. \quad (26)$$

Also by the gauge invariance property the factor e^{-L} is unchanged under a gauge transformation. Thus from (25) we have

$$0 = \langle W'(z, z') A'(z) \rangle - \langle W(z, z') A(z) \rangle, \quad (27)$$

where the correlation notation $\langle \cdot \rangle$ denotes the integral with respect to the differential

$$e^{-L} dA_1 dA_2 dZ^* dZ \quad (28)$$

We can now carry out the calculus of variation. From the gauge transformation we have the formula:

$$W'(z, z') = U(a(z)) W(z, z') U^{-1}(a(z')), \quad (29)$$

where $a(z) = \text{Re } \omega(z)$. This gauge transformation gives a variation of $W(z, z')$ with the gauge function $a(z)$ as the variational direction a in the variational formulas (21) and (23). Thus analogous to the variational formula (23) we have that the variation of $W(z, z')$ under this gauge transformation is given by

$$W(z, z') \frac{1}{2\pi i} \oint_z dw a(w) J(w), \quad (30)$$

where the generator J for this variation is to be specific. This J will be a quantum generator which generalizes the classical generator J in (23).

Thus under a gauge transformation with gauge function $a(z)$ from (27) we have the following variational equation:

$$\begin{aligned} 0 = \left\langle W(z, z') \left[\delta_a A(z) + \right. \right. \\ \left. \left. + \frac{1}{2\pi i} \oint_z dw a(w) J(w) A(z) \right] \right\rangle, \end{aligned} \quad (31)$$

where $\delta_a A(z)$ denotes the variation of the field $A(z)$ in the direction $a(z)$. From this equation an ansatz of J is that J satisfies the following equation:

$$W(z, z') \left[\delta_a A(z) + \frac{1}{2\pi i} \oint_z dw a(w) J(w) A(z) \right] = 0. \quad (32)$$

From this equation we have the following variational equation:

$$\delta_a A(z) = \frac{-1}{2\pi i} \oint_z dw a(w) J(w) A(z). \quad (33)$$

This completes the formal calculus of variation. Now (with the above derivation as a guide) we choose the following equation (34) as one of the equation for defining the generator J :

$$\delta_\omega A(z) = \frac{-1}{2\pi i} \oint_z dw \omega(w) J(w) A(z), \quad (34)$$

where we generalize the direction $a(z) = \text{Re } \omega(z)$ to the analytic direction $\omega(z)$. (This generalization has the effect of extending the real measure of the pure gauge part of the gauge theory to include the complex Feynman path integral since it gives the transformation $ds \rightarrow -ids$ for the integral of the Wilson line $W(z, z')$.)

Let us now choose one more equation for determine the generator J in (34). This choice will be as a gauge fixing condition. As analogous to the WZW model in conformal field theory [65–67] let us consider a J given by

$$J(z) := -k_0 W^{-1}(z, z') \partial_z W(z, z'), \quad (35)$$

where we define $\partial_z = \partial_{x_1} + i\partial_{x_2}$ and we set $z' = z$ after the differentiation with respect to z ; $k_0 > 0$ is a constant which is fixed when the J is determined to be of the form (35) and the minus sign is chosen by convention. In the WZW model [65, 67] the J of the form (35) is the generator of the chiral

symmetry of the WZW model. We can write the J in (35) in the following form:

$$J(w) = \sum_{\alpha} J^{\alpha}(w) = \sum_{\alpha} j^{\alpha}(w) t^{\alpha}. \quad (36)$$

We see that the generators t^{α} of $SU(2)$ appear in this form of J and this form is analogous to the classical J in (18). This shows that this J is a possible candidate for the generator J in (34).

Since $W(z, z')$ is constructed by gauge field we need to have a gauge fixing for the computations related to $W(z, z')$. Then since the J in (34) and (35) is constructed by $W(z, z')$ we have that in defining this J as the generator J of $W(z, z')$ we have chosen a condition for the gauge fixing. In this paper we shall always choose this defining equations (34) and (35) for J as the gauge fixing condition.

In summary we introduce the following definition.

Definition The generator J of the quantum Wilson line $W(z, z')$ whose classical version is defined by (8), is an operator defined by the two conditions (34) and (35). \diamond

Remark We remark that the condition (35) first defines J classically. Then the condition (34) raises this classical J to the quantum generator J . \diamond

Now we want to show that this generator J in (34) and (35) can be uniquely solved. (This means that the gauge fixing condition has already fixed the gauge that the degenerate degree of freedom of gauge invariance has been eliminated so that we can carry out computation.)

Let us now solve J . From (11) and (35) the variation $\delta_{\omega} J$ of the generator J in (35) is given by [65, p. 622] and [67]:

$$\delta_{\omega} J = [J, \omega] - k_0 \partial_z \omega. \quad (37)$$

From (34) and (37) we have that J satisfies the following relation of current algebra [65–67]:

$$J^a(w) J^b(z) = \frac{k_0 \delta_{ab}}{(w-z)^2} + \sum_c i f_{abc} \frac{J^c(z)}{(w-z)}, \quad (38)$$

where as a convention the regular term of $J^a(w) J^b(z)$ is omitted. Then by following [65–67] from (38) and (36) we can show that the J_n^{α} in (18) for the corresponding Laurent series of the quantum generator J satisfy the following Kac-Moody algebra:

$$[J_m^a, J_n^b] = i f_{abc} J_{m+n}^c + k_0 m \delta_{ab} \delta_{m+n,0}, \quad (39)$$

where k_0 is usually called the central extension or the level of the Kac-Moody algebra.

Remark Let us also consider the other side of the chiral symmetry. Similar to the J in (35) we define a generator J' by:

$$J'(z') = k_0 \partial_{z'} W(z, z') W^{-1}(z, z'), \quad (40)$$

where after differentiation with respect to z' we set $z = z'$.

Let us then consider the following formal correlation:

$$\begin{aligned} \langle A(z') W(z, z') \rangle &:= \\ &:= \int dA_1 dA_2 dZ^* dZ A(z') W(z, z') e^{-L}, \end{aligned} \quad (41)$$

where z is fixed. By an approach similar to the above derivation of (34) we have the following variational equation:

$$\delta_{\omega'} A(z') = \frac{-1}{2\pi i} \oint_{z'} d\omega A(z') J'(w) \omega'(w), \quad (42)$$

where as a gauge fixing we choose the J' in (42) be the J' in (40). Then similar to (37) we also have

$$\delta_{\omega'} J' = [J', \omega'] - k_0 \partial_{z'} \omega'. \quad (43)$$

Then from (42) and (43) we can derive the current algebra and the Kac-Moody algebra for J' which are of the same form of (38) and (39). From this we have $J' = J$. \diamond

Now with the above current algebra J and the formula (34) we can follow the usual approach in conformal field theory to derive a quantum Knizhnik-Zamolodchikov (KZ) equation for the product of primary fields in a conformal field theory [65–67]. We derive the KZ equation for the product of n Wilson lines $W(z, z')$. Here an important point is that from the two sides of $W(z, z')$ we can derive two quantum KZ equations which are dual to each other. These two quantum KZ equations are different from the usual KZ equation in that they are equations for the quantum operators $W(z, z')$ while the usual KZ equation is for the correlations of quantum operators. With this difference we can follow the usual approach in conformal field theory to derive the following quantum Knizhnik-Zamolodchikov equation [65, 66, 68]:

$$\begin{aligned} \partial_{z_i} W(z_1, z'_1) \cdots W(z_n, z'_n) &= \\ &= \frac{-e_0^2}{k_0 + g_0} \sum_{j \neq i}^n \frac{\sum_a t_i^a \otimes t_j^a}{z_i - z_j} W(z_1, z'_1) \cdots W(z_n, z'_n), \end{aligned} \quad (44)$$

for $i = 1, \dots, n$ where g_0 denotes the dual Coxeter number of a group multiplying with e_0^2 and we have $g_0 = 2e_0^2$ for the group $SU(2)$ (When the gauge group is $U(1)$ we have $g_0 = 0$). We remark that in (44) we have defined $t_i^a := t^a$ and:

$$\begin{aligned} t_i^a \otimes t_j^a W(z_1, z'_1) \cdots W(z_n, z'_n) &:= W(z_1, z'_1) \cdots \\ \cdots [t^a W(z_i, z'_i)] \cdots [t^a W(z_j, z'_j)] \cdots W(z_n, z'_n). \end{aligned} \quad (45)$$

It is interesting and important that we also have the following quantum Knizhnik-Zamolodchikov equation with respect to the z'_i variables which is dual to (44):

$$\begin{aligned} \partial_{z'_i} W(z_1, z'_1) \cdots W(z_n, z'_n) &= \\ &= \frac{-e_0^2}{k_0 + g_0} \sum_{j \neq i}^n W(z_1, z'_1) \cdots W(z_n, z'_n) \frac{\sum_a t_i^a \otimes t_j^a}{z'_j - z'_i} \end{aligned} \quad (46)$$

for $i = 1, \dots, n$ where we have defined:

$$W(z_1, z'_1) \cdots W(z_n, z'_n) t_i^\alpha \otimes t_j^\alpha := W(z_1, z'_1) \cdots \cdots [W(z_i, z'_i) t_i^\alpha] \cdots [W(z_j, z'_j) t_j^\alpha] \cdots W(z_n, z'_n). \quad (47)$$

Remark From the quantum gauge theory we derive the above quantum KZ equation in dual form by calculus of variation. This quantum KZ equation in dual form may be considered as a quantum Euler-Lagrange equation or as a quantum Yang-Mills equation since it is analogous to the classical Yang-Mills equation which is derived from the classical Yang-Mills gauge theory by calculus of variation. \diamond

5 Solving quantum KZ equation in dual form

Let us consider the following product of two quantum Wilson lines:

$$G(z_1, z_2, z_3, z_4) := W(z_1, z_2)W(z_3, z_4), \quad (48)$$

where the quantum Wilson lines $W(z_1, z_2)$ and $W(z_3, z_4)$ represent two pieces of curves starting at z_1 and z_3 and ending at z_2 and z_4 respectively.

We have that this product $G(z_1, z_2, z_3, z_4)$ satisfies the KZ equation for the variables z_1, z_3 and satisfies the dual KZ equation for the variables z_2 and z_4 . Then by solving the two-variables-KZ equation in (44) we have that a form of $G(z_1, z_2, z_3, z_4)$ is given by [69–71]:

$$e^{-\hat{t} \log[\pm(z_1-z_3)]} C_1, \quad (49)$$

where $\hat{t} := \frac{e_0^2}{k_0+g_0} \sum_a t^a \otimes t^a$ and C_1 denotes a constant matrix which is independent of the variable $z_1 - z_3$.

We see that $G(z_1, z_2, z_3, z_4)$ is a multi-valued analytic function where the determination of the \pm sign depended on the choice of the branch.

Similarly by solving the dual two-variable-KZ equation in (46) we have that G is of the form

$$C_2 e^{\hat{t} \log[\pm(z_4-z_2)]}, \quad (50)$$

where C_2 denotes a constant matrix which is independent of the variable $z_4 - z_2$.

From (49), (50) and letting:

$$C_1 = A e^{\hat{t} \log[\pm(z_4-z_2)]}, \quad C_2 = e^{-\hat{t} \log[\pm(z_1-z_3)]} A, \quad (51)$$

where A is a constant matrix we have that $G(z_1, z_2, z_3, z_4)$ is given by:

$$G(z_1, z_2, z_3, z_4) = e^{-\hat{t} \log[\pm(z_1-z_3)]} A e^{\hat{t} \log[\pm(z_4-z_2)]}, \quad (52)$$

where at the singular case that $z_1 = z_3$ we define $\log[\pm(z_1 - z_3)] = 0$. Similarly for $z_2 = z_4$.

Let us find a form of the initial operator A . We notice that there are two operators $\Phi_\pm(z_1 - z_3) := e^{-\hat{t} \log[\pm(z_1-z_3)]}$ and $\Psi_\pm(z_4 - z_2) = e^{\hat{t} \log[\pm(z_4-z_2)]}$ acting on the two sides of

A respectively where the two independent variables z_1, z_3 of Φ_\pm are mixed from the two quantum Wilson lines $W(z_1, z_2)$ and $W(z_3, z_4)$ respectively and the two independent variables z_2, z_4 of Ψ_\pm are mixed from the two quantum Wilson lines $W(z_1, z_2)$ and $W(z_3, z_4)$ respectively. From this we determine the form of A as follows.

Let D denote a representation of $SU(2)$. Let $D(g)$ represent an element g of $SU(2)$ and let $D(g) \otimes D(g)$ denote the tensor product representation of $SU(2)$. Then in the KZ equation we define

$$[t^\alpha \otimes t^\alpha][D(g_1) \otimes D(g_1)] \otimes [D(g_2) \otimes D(g_2)] := [t^\alpha D(g_1) \otimes D(g_1)] \otimes [t^\alpha D(g_2) \otimes D(g_2)] \quad (53)$$

and

$$[D(g_1) \otimes D(g_1)] \otimes [D(g_2) \otimes D(g_2)][t^\alpha \otimes t^\alpha] := [D(g_1) \otimes D(g_1)t^\alpha] \otimes [D(g_2) \otimes D(g_2)t^\alpha]. \quad (54)$$

Then we let $U(\mathfrak{a})$ denote the universal enveloping algebra where \mathfrak{a} denotes an algebra which is formed by the Lie algebra $su(2)$ and the identity matrix.

Now let the initial operator A be of the form $A_1 \otimes A_2 \otimes A_3 \otimes A_4$ with $A_i, i = 1, \dots, 4$ taking values in $U(\mathfrak{a})$. In this case we have that in (52) the operator $\Phi_\pm(z_1 - z_3)$ acts on A from the left via the following formula:

$$t^\alpha \otimes t^\alpha A = [t^\alpha A_1] \otimes A_2 \otimes [t^\alpha A_3] \otimes A_4. \quad (55)$$

Similarly the operator $\Psi_\pm(z_4 - z_2)$ in (52) acts on A from the right via the following formula:

$$A t^\alpha \otimes t^\alpha = A_1 \otimes [A_2 t^\alpha] \otimes A_3 \otimes [A_4 t^\alpha]. \quad (56)$$

We may generalize the above tensor product of two quantum Wilson lines as follows. Let us consider a tensor product of n quantum Wilson lines: $W(z_1, z'_1) \cdots W(z_n, z'_n)$ where the variables z_i, z'_i are all independent. By solving the two KZ equations we have that this tensor product is given by:

$$W(z_1, z'_1) \cdots W(z_n, z'_n) = \prod_{ij} \Phi_\pm(z_i - z_j) A \prod_{ij} \Psi_\pm(z'_i - z'_j), \quad (57)$$

where \prod_{ij} denotes a product of $\Phi_\pm(z_i - z_j)$ or $\Psi_\pm(z'_i - z'_j)$ for $i, j = 1, \dots, n$ where $i \neq j$. In (57) the initial operator A is represented as a tensor product of operators $A_{ij'i'j'}$, $i, j, i', j' = 1, \dots, n$ where each $A_{ij'i'j'}$ is of the form of the initial operator A in the above tensor product of two-Wilson-lines case and is acted by $\Phi_\pm(z_i - z_j)$ or $\Psi_\pm(z'_i - z'_j)$ on its two sides respectively.

6 Computation of quantum Wilson lines

Let us consider the following product of two quantum Wilson lines:

$$G(z_1, z_2, z_3, z_4) := W(z_1, z_2)W(z_3, z_4), \quad (58)$$

where the quantum Wilson lines $W(z_1, z_2)$ and $W(z_3, z_4)$ represent two pieces of curves starting at z_1 and z_3 and ending at z_2 and z_4 respectively. As shown in the above Section we have that $G(z_1, z_2, z_3, z_4)$ is given by the following formula:

$$G(z_1, z_2, z_3, z_4) = e^{-\hat{t} \log[\pm(z_1-z_3)]} A e^{\hat{t} \log[\pm(z_4-z_2)]}, \quad (59)$$

where the product is a 4-tensor.

Let us set $z_2 = z_3$. Then the 4-tensor $W(z_1, z_2)W(z_3, z_4)$ is reduced to the 2-tensor $W(z_1, z_2)W(z_2, z_4)$. By using (59) the 2-tensor $W(z_1, z_2)W(z_2, z_4)$ is given by:

$$\begin{aligned} W(z_1, z_2)W(z_2, z_4) &= \\ &= e^{-\hat{t} \log[\pm(z_1-z_2)]} A_{14} e^{\hat{t} \log[\pm(z_4-z_2)]}, \end{aligned} \quad (60)$$

where $A_{14} = A_1 \otimes A_4$ is a 2-tensor reduced from the 4-tensor $A = A_1 \otimes A_2 \otimes A_3 \otimes A_4$ in (59). In this reduction the \hat{t} operator of $\Phi = e^{-\hat{t} \log[\pm(z_1-z_2)]}$ acting on the left side of A_1 and A_3 in A is reduced to acting on the left side of A_1 and A_4 in A_{14} . Similarly the \hat{t} operator of $\Psi = e^{\hat{t} \log[\pm(z_4-z_2)]}$ acting on the right side of A_2 and A_4 in A is reduced to acting on the right side of A_1 and A_4 in A_{14} .

Then since \hat{t} is a 2-tensor operator we have that \hat{t} is as a matrix acting on the two sides of the 2-tensor A_{14} which is also as a matrix with the same dimension as \hat{t} . Thus Φ and Ψ are as matrices of the same dimension as the matrix A_{14} acting on A_{14} by the usual matrix operation. Then since \hat{t} is a Casimir operator for the 2-tensor group representation of $SU(2)$ we have that Φ and Ψ commute with A_{14} since Φ and Ψ are exponentials of \hat{t} . (We remark that Φ and Ψ are in general not commute with the 4-tensor initial operator A .) Thus we have

$$\begin{aligned} e^{-\hat{t} \log[\pm(z_1-z_2)]} A_{14} e^{\hat{t} \log[\pm(z_4-z_2)]} &= \\ &= e^{-\hat{t} \log[\pm(z_1-z_2)]} e^{\hat{t} \log[\pm(z_4-z_2)]} A_{14}. \end{aligned} \quad (61)$$

We let $W(z_1, z_2)W(z_2, z_4)$ be as a representation of the quantum Wilson line $W(z_1, z_4)$:

$$\begin{aligned} W(z_1, z_4) &:= W(z_1, w_1)W(w_1, z_4) = \\ &= e^{-\hat{t} \log[\pm(z_1-w_1)]} e^{\hat{t} \log[\pm(z_4-w_1)]} A_{14}. \end{aligned} \quad (62)$$

This representation of the quantum Wilson line $W(z_1, z_4)$ means that the line (or path) with end points z_1 and z_4 is specific that it passes the intermediate point $w_1 = z_2$. This representation shows the quantum nature that the path is not specific at other intermediate points except the intermediate point $w_1 = z_2$. This unspecification of the path is of the same quantum nature of the Feynman path description of quantum mechanics.

Then let us consider another representation of the quantum Wilson line $W(z_1, z_4)$. We consider the three-product $W(z_1, w_1)W(w_1, w_2)W(w_2, z_4)$ which is obtained from the

three-tensor $W(z_1, w_1)W(w_1, w_2)W(w_2, z_4)$ by two reductions where $z_j, w_j, u_j, j = 1, 2$ are independent variables. For this representation we have:

$$\begin{aligned} W(z_1, w_1)W(w_1, w_2)W(w_2, z_4) &= e^{-\hat{t} \log[\pm(z_1-w_1)]} \times \\ &\times e^{-\hat{t} \log[\pm(z_1-w_2)]} e^{\hat{t} \log[\pm(z_4-w_1)]} e^{\hat{t} \log[\pm(z_4-w_2)]} A_{14}. \end{aligned} \quad (63)$$

This representation of the quantum Wilson line $W(z_1, z_4)$ means that the line (or path) with end points z_1 and z_4 is specific that it passes the intermediate points w_1 and w_2 . This representation shows the quantum nature that the path is not specific at other intermediate points except the intermediate points w_1 and w_2 . This unspecification of the path is of the same quantum nature of the Feynman path description of quantum mechanics.

Similarly we may represent $W(z_1, z_4)$ by path with end points z_1 and z_4 and is specific only to pass at finitely many intermediate points. Then we let the quantum Wilson line $W(z_1, z_4)$ as an equivalent class of all these representations. Thus we may write:

$$\begin{aligned} W(z_1, z_4) &= W(z_1, w_1)W(w_1, z_4) = \\ &= W(z_1, w_1)W(w_1, w_2)W(w_2, z_4) = \dots \end{aligned} \quad (64)$$

Remark Since A_{14} is a 2-tensor we have that a natural group representation for the Wilson line $W(z_1, z_4)$ is the 2-tensor group representation of the group $SU(2)$.

7 Representing braiding of curves by quantum Wilson lines

Consider again the $G(z_1, z_2, z_3, z_4)$ in (58). We have that $G(z_1, z_2, z_3, z_4)$ is a multi-valued analytic function where the determination of the \pm sign depended on the choice of the branch.

Let the two pieces of curves represented by $W(z_1, z_2)$ and $W(z_3, z_4)$ be crossing at w . In this case we write $W(z_i, z_j)$ as $W(z_i, w)W(w, z_j)$ where $i = 1, 3, j = 2, 4$. Thus we have

$$\begin{aligned} W(z_1, z_2)W(z_3, z_4) &= \\ &= W(z_1, w)W(w, z_2)W(z_3, w)W(w, z_4). \end{aligned} \quad (65)$$

If we interchange z_1 and z_3 , then from (65) we have the following ordering:

$$W(z_3, w)W(w, z_2)W(z_1, w)W(w, z_4). \quad (66)$$

Now let us choose a branch. Suppose these two curves are cut from a knot and that following the orientation of a knot the curve represented by $W(z_1, z_2)$ is before the curve represented by $W(z_3, z_4)$. Then we fix a branch such that the product in (59) is with two positive signs:

$$W(z_1, z_2)W(z_3, z_4) = e^{-\hat{t} \log(z_1-z_3)} A e^{\hat{t} \log(z_4-z_2)}. \quad (67)$$

Then if we interchange z_1 and z_3 we have

$$W(z_3, w)W(w, z_2)W(z_1, w)W(w, z_4) = e^{-\hat{t} \log[-(z_1-z_3)]} A e^{\hat{t} \log(z_4-z_2)}. \quad (68)$$

From (67) and (68) as a choice of branch we have

$$W(z_3, w)W(w, z_2)W(z_1, w)W(w, z_4) = RW(z_1, w)W(w, z_2)W(z_3, w)W(w, z_4), \quad (69)$$

where $R = e^{-i\pi\hat{t}}$ is the monodromy of the KZ equation. In (69) z_1 and z_3 denote two points on a closed curve such that along the direction of the curve the point z_1 is before the point z_3 and in this case we choose a branch such that the angle of $z_3 - z_1$ minus the angle of $z_1 - z_3$ is equal to π .

Remark We may use other representations of the product $W(z_1, z_2)W(z_3, z_4)$. For example we may use the following representation:

$$W(z_1, w)W(w, z_2)W(z_3, w)W(w, z_4) = e^{-\hat{t} \log(z_1-z_3)} e^{-2\hat{t} \log(z_1-w)} e^{-2\hat{t} \log(z_3-w)} \times A e^{\hat{t} \log(z_4-z_2)} e^{2\hat{t} \log(z_4-w)} e^{2\hat{t} \log(z_2-w)}. \quad (70)$$

Then the interchange of z_1 and z_3 changes only $z_1 - z_3$ to $z_3 - z_1$. Thus the formula (69) holds. Similarly all other representations of $W(z_1, z_2)W(z_3, z_4)$ will give the same result. \diamond

Now from (69) we can take a convention that the ordering (66) represents that the curve represented by $W(z_1, z_2)$ is up-crossing the curve represented by $W(z_3, z_4)$ while (65) represents zero crossing of these two curves.

Similarly from the dual KZ equation as a choice of branch which is consistent with the above formula we have

$$W(z_1, w)W(w, z_4)W(z_3, w)W(w, z_2) = W(z_1, w)W(w, z_2)W(z_3, w)W(w, z_4)R^{-1}, \quad (71)$$

where z_2 is before z_4 . We take a convention that the ordering in (71) represents that the curve represented by $W(z_1, z_2)$ is under-crossing the curve represented by $W(z_3, z_4)$. Here along the orientation of a closed curve the piece of curve represented by $W(z_1, z_2)$ is before the piece of curve represented by $W(z_3, z_4)$. In this case since the angle of $z_3 - z_1$ minus the angle of $z_1 - z_3$ is equal to π we have that the angle of $z_4 - z_2$ minus the angle of $z_2 - z_4$ is also equal to π and this gives the R^{-1} in this formula (71).

From (69) and (71) we have

$$W(z_3, z_4)W(z_1, z_2) = RW(z_1, z_2)W(z_3, z_4)R^{-1}, \quad (72)$$

where z_1 and z_2 denote the end points of a curve which is before a curve with end points z_3 and z_4 . From (72) we see that the algebraic structure of these quantum Wilson lines $W(z, z')$ is analogous to the quasi-triangular quantum group [66, 69].

8 Computation of quantum Dirac-Wilson loop

Consider again the quantum Wilson line $W(z_1, z_4)$ given by $W(z_1, z_4) = W(z_1, z_2)W(z_2, z_4)$. Let us set $z_1 = z_4$. In this case the quantum Wilson line forms a closed loop. Now in (61) with $z_1 = z_4$ we have that the quantities $e^{-\hat{t} \log \pm(z_1-z_2)}$ and $e^{\hat{t} \log \pm(z_1-z_2)}$ which come from the two-side KZ equations cancel each other and from the multi-valued property of the log function we have:

$$W(z_1, z_1) = R^N A_{14}, \quad N = 0, \pm 1, \pm 2, \dots \quad (73)$$

where $R = e^{-i\pi\hat{t}}$ is the monodromy of the KZ equation [69].

Remark It is clear that if we use other representation of the quantum Wilson loop $W(z_1, z_1)$ (such as the representation $W(z_1, z_1) = W(z_1, w_1)W(w_1, w_2)W(w_2, z_1)$) then we will get the same result as (73).

Remark For simplicity we shall drop the subscript of A_{14} in (73) and simply write $A_{14} = A$.

9 Winding number of Dirac-Wilson loop as quantization

We have the equation (73) where the integer N is as a winding number. Then when the gauge group is $U(1)$ we have

$$W(z_1, z_1) = R_{U(1)}^N A, \quad (74)$$

where $R_{U(1)}$ denotes the monodromy of the KZ equation for $U(1)$. We have

$$R_{U(1)}^N = e^{iN \frac{\pi e_0^2}{k_0 + g_0}}, \quad N = 0, \pm 1, \pm 2, \dots \quad (75)$$

where the constant e_0 denotes the bare electric charge (and $g_0 = 0$ for $U(1)$ group). The winding number N is as the quantization property of photon. We show in the following Section that the Dirac-Wilson loop $W(z_1, z_1)$ with the abelian $U(1)$ group is a model of the photon.

10 Magnetic monopole is a photon with a specific frequency

We see that the Dirac-Wilson loop is an exactly solvable non-linear observable. Thus we may regard it as a quantum soliton of the above gauge theory. In particular for the abelian gauge theory with $U(1)$ as gauge group we regard the Dirac-Wilson loop as a quantum soliton of the electromagnetic field. We now want to show that this soliton has all the properties of photon and thus we may identify it with the photon.

First we see that from (75) it has discrete energy levels of light-quantum given by

$$h\nu := N \frac{\pi e_0^2}{k_0}, \quad N = 0, 1, 2, 3, \dots \quad (76)$$

where h is the Planck's constant; ν denotes a frequency and the constant $k_0 > 0$ is determined from this formula. This formula is from the monodromy $R_{U(1)}$ for the abelian gauge theory. We see that the Planck's constant h comes out from this winding property of the Dirac-Wilson loop. Then since this Dirac-Wilson loop is a loop we have that it has the polarization property of light by the right hand rule along the loop and this polarization can also be regarded as the spin of photon. Now since this loop is a quantum soliton which behaves as a particle we have that this loop is a basic particle of the above abelian gauge theory where the abelian gauge property is considered as the fundamental property of electromagnetic field. This shows that the Dirac-Wilson loop has properties of photon. We shall later show that from this loop model of photon we can describe the absorption and emission of photon by an electron. This property of absorption and emission is considered as a basic principle of the light-quantum hypothesis of Einstein [1]. From these properties of the Dirac-Wilson loop we may identify it with the photon.

On the other hand from Dirac's analysis of the magnetic monopole we have that the property of magnetic monopole comes from a closed line integral of vector potential of the electromagnetic field which is similar to the Dirac-Wilson loop [4]. Now from this Dirac-Wilson loop we can define the magnetic charge q and the minimal magnetic charge q_{min} which are given by:

$$eq := enq_{min} := n_e e_0 n \frac{n_m e_0 \pi}{k_0}, \quad n = 0, 1, 2, 3, \dots \quad (77)$$

where $e := n_e e_0$ is as the observed electric charge for some positive integer n_e ; and $q_{min} := \frac{n_m e_0 \pi}{k_0}$ for some positive integer n_m and we write $N = n n_e n_m$, $n = 0, 1, 2, 3, \dots$ (by absorbing the constant k_0 to e_0^2 we may let $k_0 = 1$).

This shows that the Dirac-Wilson loop gives the property of magnetic monopole for some frequencies. Since this loop is a quantum soliton which behaves as a particle we have that this Dirac-Wilson loop may be identified with the magnetic monopole for some frequencies. Thus we have that photon may be identified with the magnetic monopole for some frequencies. With this identification we have the following interesting conclusion: Both the energy quantization of electromagnetic field and the charge quantization property come from the same property of photon. Indeed we have:

$$n h \nu_1 := n \frac{n_e n_m e_0^2 \pi}{k_0} = eq, \quad n = 0, 1, 2, 3, \dots \quad (78)$$

where ν_1 denotes a frequency. This formula shows that the energy quantization gives the charge quantization and thus these two quantizations are from the same property of the photon when photon is modelled by the Dirac-Wilson loop and identified with the magnetic monopole for some frequencies. We notice that between two energy levels $n e q_{min}$ and $(n + 1) e q_{min}$ there are other energy levels which may be regarded as the excited states of a particle with charge $n e$.

11 Nonlinear loop model of electron

In this Section let us also give a loop model to the electron. This loop model of electron is based on the above loop model of the photon. From the loop model of photon we also construct an observable which gives mass to the electron and is thus a mass mechanism for the electron.

Let $W(z, z)$ denote a Dirac-Wilson loop which represents a photon. Let Z denotes the complex variable for electron in (1.1). We then consider the following observable:

$$W(z, z) Z. \quad (79)$$

Since $W(z, z)$ is solvable we have that this observable is also solvable where in solving $W(z, z)$ the variable Z is fixed. We let this observable be identified with the electron. Then we consider the following observable:

$$Z^* W(z, z) Z. \quad (80)$$

This observable is with a scalar factor $Z^* Z$ where Z^* denotes the complex conjugate of Z and we regard it as the mass mechanism of the electron (79). For this observable we model the energy levels with specific frequencies of $W(z, z)$ as the mass levels of electron and the mass m of electron is the lowest energy level $h\nu_1$ with specific frequency ν_1 of $W(z, z)$ and is given by:

$$mc^2 = h\nu_1, \quad (81)$$

where c denotes the constant of the speed of light and the frequency ν_1 is given by (78). From this model of the mass mechanism of electron we have that electron is with mass m while photon is with zero mass because there does not have such a mass mechanism $Z^* W(z, z) Z$ for the photon. From this definition of mass we have the following formula relating the observed electric charge e of electron, the magnetic charge q_{min} of magnetic monopole and the mass m of electron:

$$mc^2 = eq_{min} = h\nu_1. \quad (82)$$

By using the nonlinear model $W(z, z) Z$ to represent an electron we can then describe the absorption and emission of a photon by an electron where photon is as a parcel of energy described by the loop $W(z, z)$, as follows. Let $W(z, z) Z$ represents an electron and let $W_1(z_1, z_1)$ represents a photon. Then the observable $W_1(z_1, z_1) W(z, z) Z$ represents an electron having absorbed the photon $W_1(z_1, z_1)$. This property of absorption and emission is as a basic principle of the hypothesis of light-quantum stated by Einstein [1]. Let us quote the following paragraph from [1]:

... First, the light-quantum was conceived of as a parcel of energy as far as the properties of pure radiation (no coupling to matter) are concerned. Second, Einstein made the assumption — he call it the heuristic principle — that also in its coupling to matter (that is, in emission and absorption), light is created or annihilated in similar discrete parcels of energy. That, I

believe, was Einstein's one revolutionary contribution to physics. It upset all existing ideas about the interaction between light and matter. . .

12 Photon with specific frequency carries electric and magnetic charges

In this loop model of photon we have that the observed electric charge $e := n_e e_0$ and the magnetic charge q_{min} are carried by the photon with some specific frequencies. Let us here describe the physical effects from this property of photon that photon with some specific frequency carries the electric and magnetic charge. From the nonlinear model of electron we have that an electron $W(z, z)Z$ also carries the electric charge when a photon $W(z, z)$ carrying the electric and magnetic charge is absorbed to form the electron $W(z, z)Z$. This means that the electric charge of an electron is from the electric charge carried by a photon. Then an interaction (as the electric force) is formed between two electrons (with the electric charges).

On the other hand since photon carries the constant e_0^2 of the bare electric charge e_0 we have that between two photons there is an interaction which is similar to the electric force between two electrons (with the electric charges). This interaction however may not be of the same magnitude as the electric force with the magnitude e^2 since the photons may not carry the frequency for giving the electric and magnetic charge. Then for stability such interaction between two photons tends to give repulsive effect to give the diffusion phenomenon among photons.

Similarly an electron $W(z, z)Z$ also carries the magnetic charge when a photon $W(z, z)$ carrying the electric and magnetic charge is absorbed to form the electron $W(z, z)Z$. This means that the magnetic charge of an electron is from the magnetic charge carried by a photon. Then a closed-loop interaction (as the magnetic force) may be formed between two electrons (with the magnetic charges).

On the other hand since photon carries the constant e_0^2 of the bare electric charge e_0 we have that between two photons there is an interaction which is similar to the magnetic force between two electrons (with the magnetic charges). This interaction however may not be of the same magnitude as the magnetic force with the magnetic charge q_{min} since the photons may not carry the frequency for giving the electric and magnetic charge. Then for stability such interaction between two photons tends to give repulsive effect to give the diffusion phenomenon among photons.

13 Statistics of photons and electrons

The nonlinear model $W(z, z)Z$ of an electron gives a relation between photon and electron where the photon is modelled by $W(z, z)$ which is with a specific frequency for $W(z, z)Z$

to be an electron, as described in the above Sections. We want to show that from this nonlinear model we may also derive the required statistics of photons and electrons that photons obey the Bose-Einstein statistics and electrons obey the Fermi-Dirac statistics. We have that $W(z, z)$ is as an operator acting on Z . Let $W_1(z, z)$ be a photon. Then we have that the nonlinear model $W_1(z, z)W(z, z)Z$ represents that the photon $W_1(z, z)$ is absorbed by the electron $W(z, z)Z$ to form an electron $W_1(z, z)W(z, z)Z$. Let $W_2(z, z)$ be another photon. Then we have that the model $W_1(z, z)W_2(z, z)W(z, z)Z$ again represents an electron where we have:

$$\begin{aligned} W_1(z, z)W_2(z, z)W(z, z)Z &= \\ &= W_2(z, z)W_1(z, z)W(z, z)Z. \end{aligned} \tag{83}$$

More generally the model $\prod_{n=1}^N W_n(z, z)W(z, z)Z$ represents that the photons $W_n(z, z), n = 1, 2, \dots, N$ are absorbed by the electron $W(z, z)Z$. This model shows that identical (but different) photons can appear identically and it shows that photons obey the Bose-Einstein statistics. From the polarization of the Dirac-Wilson loop $W(z, z)$ we may assign spin 1 to a photon represented by $W(z, z)$.

Let us then consider statistics of electrons. The observable $Z^*W(z, z)Z$ gives mass to the electron $W(z, z)Z$ and thus this observable is as a scalar and thus is assigned with spin 0. As the observable $W(z, z)Z$ is between $W(z, z)$ and $Z^*W(z, z)Z$ which are with spin 1 and 0 respectively we thus assign spin $\frac{1}{2}$ to the observable $W(z, z)Z$ and thus electron represented by this observable $W(z, z)Z$ is with spin $\frac{1}{2}$.

Then let Z_1 and Z_2 be two independent complex variables for two electrons and let $W_1(z, z)Z_1$ and $W_2(z, z)Z_2$ represent two electrons. Let $W_3(z, z)$ represents a photon. Then the model $W_3(z, z)(W_1(z, z)Z_1 + W_2(z, z)Z_2)$ means that two electrons are in the same state that the operator $W_3(z, z)$ is acted on the two electrons. However this model means that a photon $W(z, z)$ is absorbed by two distinct electrons and this is impossible. Thus the models $W_3(z, z)W_1(z, z)Z_1$ and $W_3(z, z)W_2(z, z)Z_2$ cannot both exist and this means that electrons obey Fermi-Dirac statistics.

Thus this nonlinear loop model of photon and electron gives the required statistics of photons and electrons.

14 Photon propagator and quantum photon propagator

Let us then investigate the quantum Wilson line $W(z_0, z)$ with $U(1)$ group where z_0 is fixed for the photon field. We want to show that this quantum Wilson line $W(z_0, z)$ may be regarded as the quantum photon propagator for a photon propagating from z_0 to z .

As we have shown in the above Section on computation of quantum Wilson line; to compute $W(z_0, z)$ we need to write $W(z_0, z)$ in the form of two (connected) Wilson lines: $W(z_0, z) = W(z_0, z_1)W(z_1, z)$ for some z_1 point. Then we

have:

$$\begin{aligned} W(z_0, z_1)W(z_1, z) &= \\ &= e^{-\hat{t} \log[\pm(z_1 - z_0)]} A e^{\hat{t} \log[\pm(z - z_1)]}, \end{aligned} \quad (84)$$

where $\hat{t} = -\frac{e_0^2}{k_0}$ for the $U(1)$ group (k_0 is a constant and we may for simplicity let $k_0 = 1$) where the term $e^{-\hat{t} \log[\pm(z - z_0)]}$ is obtained by solving the first form of the dual form of the KZ equation and the term $e^{\hat{t} \log[\pm(z_0 - z)]}$ is obtained by solving the second form of the dual form of the KZ equation.

Then we may write $W(z_0, z)$ in the following form:

$$W(z_0, z) = W(z_0, z_1)W(z_1, z) = e^{-\hat{t} \log \frac{(z_1 - z_0)}{(z - z_1)}} A. \quad (85)$$

Let us fix z_1 with z such that:

$$\frac{|z_1 - z_0|}{|z - z_1|} = \frac{r_1}{n_e^2} \quad (86)$$

for some positive integer n_e such that $r_1 \leq n_e^2$; and we let z be a point on a path of connecting z_0 and z_1 and then a closed loop is formed with z as the starting and ending point. (This loop can just be the photon-loop of the electron in this electromagnetic interaction by this photon propagator (85).) Then (85) has a factor $e_0^2 \log \frac{r_1}{n_e^2}$ which is the fundamental solution of the two dimensional Laplace equation and is analogous to the fundamental solution $\frac{e^2}{r}$ (where $e := e_0 n_e$ denotes the observed (renormalized) electric charge and r denotes the three dimensional distance) of the three dimensional Laplace equation for the Coulomb's law. Thus the operator $W(z_0, z) = W(z_0, z_1)W(z_1, z)$ in (85) can be regarded as the quantum photon propagator propagating from z_0 to z .

We remark that when there are many photons we may introduce the space variable \mathbf{x} as a statistical variable via the Lorentz metric $ds^2 = dt^2 - d\mathbf{x}^2$ to obtain the Coulomb's law $\frac{e^2}{r}$ from the fundamental solution $e_0^2 \log \frac{r_1}{n_e^2}$ as a statistical law for electricity (We shall give such a space-time statistics later).

The quantum photon propagator (85) gives a repulsive effect since it is analogous to the Coulomb's law $\frac{e^2}{r}$. On the other hand we can reverse the sign of \hat{t} such that this photon operator can also give an attractive effect:

$$W(z_0, z) = W(z_0, z_1)W(z_1, z) = e^{\hat{t} \log \frac{(z - z_1)}{(z_1 - z_0)}} A, \quad (87)$$

where we fix z_1 with z_0 such that:

$$\frac{|z - z_1|}{|z_1 - z_0|} = \frac{r_1}{n_e^2} \quad (88)$$

for some positive integer n_e such that $r_1 \geq n_e^2$; and we again let z be a point on a path of connecting z_0 and z_1 and then a closed loop is formed with z as the starting and ending point. (This loop again can just be the photon-loop of the electron in this electromagnetic interaction by this photon propagator

(85).) Then (87) has a factor $-e_0^2 \log \frac{r_1}{n_e^2}$ which is the fundamental solution of the two dimensional Laplace equation and is analogous to the attractive fundamental solution $-\frac{e^2}{r}$ of the three dimensional Laplace equation for the Coulomb's law.

Thus the quantum photon propagator in (85), and in (87), can give repulsive or attractive effect between two points z_0 and z for all z in the complex plane. These repulsive or attractive effects of the quantum photon propagator correspond to two charges of the same sign and of different sign respectively.

On the other hand when $z = z_0$ the quantum Wilson line $W(z_0, z_0)$ in (85) which is the quantum photon propagator becomes a quantum Wilson loop $W(z_0, z_0)$ which is identified as a photon, as shown in the above Sections.

Let us then derive a form of photon propagator from the quantum photon propagator $W(z_0, z)$. Let us choose a path connecting z_0 and $z = z(s)$. We consider the following path:

$$\begin{aligned} z(s) &= z_1 + a_0 [\theta(s_1 - s)e^{-i\beta_1(s_1 - s)} + \\ &+ \theta(s - s_1)e^{i\beta_1(s_1 - s)}], \end{aligned} \quad (89)$$

where $\beta_1 > 0$ is a parameter and $z(s_0) = z_0$ for some proper time s_0 ; and a_0 is some complex constant; and θ is a step function given by $\theta(s) = 0$ for $s < 0$, $\theta(s) = 1$ for $s \geq 0$. Then on this path we have:

$$\begin{aligned} W(z_0, z) &= \\ &= W(z_0, z_1)W(z_1, z) = e^{\hat{t} \log \frac{(z - z_1)}{(z_1 - z_0)}} A = \\ &= e^{\hat{t} \log \frac{a_0 [\theta(s - s_1)e^{-i\beta_1(s_1 - s)} + \theta(s_1 - s)e^{i\beta_1(s_1 - s)}]}{(z_1 - z_0)}} A = \\ &= e^{\hat{t} \log b [\theta(s - s_1)e^{-i\beta_1(s_1 - s)} + \theta(s_1 - s)e^{i\beta_1(s_1 - s)}]} A = \\ &= b_0 [\theta(s - s_1)e^{-i\hat{t}\beta_1(s_1 - s)} + \theta(s_1 - s)e^{i\hat{t}\beta_1(s_1 - s)}] A \end{aligned} \quad (90)$$

for some complex constants b and b_0 . From this chosen of the path (89) we have that the quantum photon propagator is proportional to the following expression:

$$\frac{1}{2\lambda_1} [\theta(s - s_1)e^{-i\lambda_1(s - s_1)} + \theta(s_1 - s)e^{i\lambda_1(s - s_1)}] \quad (91)$$

where we define $\lambda_1 = -\hat{t}\beta_1 = e_0^2\beta_1 > 0$. We see that this is the usual propagator of a particle $x(s)$ of harmonic oscillator with mass-energy parameter $\lambda_1 > 0$ where $x(s)$ satisfies the following harmonic oscillator equation:

$$\frac{d^2x}{ds^2} = -\lambda_1^2 x(s). \quad (92)$$

We regard (91) as the propagator of a photon with mass-energy parameter λ_1 . Fourier transforming (91) we have the following form of photon propagator:

$$\frac{i}{k_E^2 - \lambda_1}, \quad (93)$$

where we use the notation k_E (instead of the notation k) to denote the proper energy of photon. We shall show in the next Section that from this photon propagator by space-time statistics we can get a propagator with the k_E replaced by the energy-momentum four-vector k which is similar to the Feynman propagator (with a mass-energy parameter $\lambda_1 > 0$). We thus see that the quantum photon propagator $W(z_0, z)$ gives a classical form of photon propagator in the conventional QED theory.

Then we notice that while $\lambda_1 > 0$ which may be think of as the mass-energy parameter of a photon the original quantum photon propagator $W(z_0, z)$ can give the Coulomb potential and thus give the effect that the photon is massless. Thus the photon mass-energy parameter $\lambda_1 > 0$ is consistent with the property that the photon is massless. Thus in the following Sections when we compute the vertex correction and the Lamb shift we shall then be able to let $\lambda_1 > 0$ without contradicting the property that the photon is massless. This then can solve the infrared-divergence problem of QED.

We remark that if we choose other form of paths for connecting z_0 and z we can get other forms of photon propagator corresponding to a choice of gauge. From the property of gauge invariance the final result should not depend on the form of propagators. We shall see that this is achieved by renormalization. This property of renormalizable is as a property related to the gauge invariance. Indeed we notice that the quantum photon propagator with a photon-loop $W(z, z)$ attached to an electron represented by Z has already given the renormalized charge e (and the renormalized mass m of the electron) for the electromagnetic interaction.

It is clear that this renormalization by the quantum photon propagator with a photon-loop $W(z, z)$ is independent of the chosen photon propagator (because it does not need to choose a photon propagator). Thus the renormalization method as that in the conventional QED theory for the chosen of a photon propagator (corresponding to a choice of gauge) should give the observable result which does not depend on the form of the photon propagators since these two forms of renormalization must give the same effect of renormalization.

In the following Section and the Sections from Section 16 to Section 23 on Quantum Electrodynamics (QED) we shall investigate the renormalization method which is analogous to that of the conventional QED theory and the computation of QED effects by using this renormalization method.

15 Renormalization

In this Section and the following Sections from Section 16 to Section 23 on Quantum Electrodynamics (QED) we shall use the density (1.1) and the notations from this density where $A_j, j = 1, 2$ are real components of the photon field. Following the conventional QED theory let us consider the following

renormalization:

$$\begin{aligned} A_j &= z_A^{\frac{1}{2}} A_{jR}, \quad j = 1, 2; \quad Z = z_Z^{\frac{1}{2}} Z_R; \\ e_0 &= \frac{z_e}{z_Z z_A^{\frac{1}{2}}} e = \frac{1}{n_e} e; \end{aligned} \quad (94)$$

where $z_A, z_Z,$ and z_e are renormalization constants to be determined and $A_{Rj}, j = 1, 2, Z_R$ are renormalized fields. From this renormalization the density D of QED in (1.1) can be written in the following form:

$$\begin{aligned} D &= \frac{1}{2} z_A \left(\frac{\partial A_{1R}}{\partial x^2} - \frac{\partial A_{2R}}{\partial x^1} \right)^* \left(\frac{\partial A_{1R}}{\partial x^2} - \frac{\partial A_{2R}}{\partial x^1} \right) + \\ &+ z_Z \left(\frac{dZ_R^*}{ds} + ie \left(\sum_{j=1}^2 A_{jR} \frac{dx^j}{ds} \right) Z_R^* \right) \times \\ &\times \left(\frac{dZ_R}{ds} - ie \left(\sum_{j=1}^2 A_{jR} \frac{dx^j}{ds} \right) Z_R \right) = \\ &= \left\{ \frac{1}{2} \left(\frac{\partial A_{1R}}{\partial x^2} - \frac{\partial A_{2R}}{\partial x^1} \right)^* \left(\frac{\partial A_{1R}}{\partial x^2} - \frac{\partial A_{2R}}{\partial x^1} \right) + \right. \\ &+ \frac{dZ_R^*}{ds} \frac{dZ_R}{ds} + \mu^2 Z_R^* Z_R - \mu^2 Z_R^* Z_R + \\ &+ ie \left(\sum_{j=1}^2 A_{jR} \frac{dx^j}{ds} \right) Z_R^* \frac{dZ_R}{ds} - \\ &- ie \left(\sum_{j=1}^2 A_{jR} \frac{dx^j}{ds} \right) \frac{dZ_R^*}{ds} Z_R + \\ &\left. + e^2 \left(\sum_{j=1}^2 A_{jR} \frac{dx^j}{ds} \right)^2 Z_R^* Z_R \right\} + \\ &+ \left\{ (z_A - 1) \left[\frac{1}{2} \left(\frac{\partial A_{1R}}{\partial x^2} - \frac{\partial A_{2R}}{\partial x^1} \right)^* \left(\frac{\partial A_{1R}}{\partial x^2} - \frac{\partial A_{2R}}{\partial x^1} \right) \right] + \right. \\ &+ (z_Z - 1) \frac{dZ_R^*}{ds} \frac{dZ_R}{ds} + \\ &+ (z_e - 1) \left[+ie \left(\sum_{j=1}^2 A_{jR} \frac{dx^j}{ds} \right) Z_R^* \frac{dZ_R}{ds} - \right. \\ &- ie \left(\sum_{j=1}^2 A_{jR} \frac{dx^j}{ds} \right) \frac{dZ_R^*}{ds} Z_R \left. \right] + \\ &\left. + \left(\frac{z_e^2}{z_Z} - 1 \right) e^2 \left(\sum_{j=1}^2 A_{jR} \frac{dx^j}{ds} \right)^2 Z_R^* Z_R \right\} := \\ &:= D_{phy} + D_{cnt}, \end{aligned} \quad (95)$$

where D_{phy} is as the physical term and the D_{cnt} is as the counter term; and in D_{phy} the positive parameter μ is introduced for perturbation expansion and for renormalization.

Similar to that the Ward-Takahashi identities in the conventional QED theory are derived by the gauge invariance of the conventional QED theory; by using the gauge invariance of this QED theory we shall also derive the corresponding Ward-Takahashi identities for this QED theory in the Section on electron self-energy. From these Ward-Takahashi identities we then show that there exists a renormalization procedure such that $z_e = z_Z$; as similar to that in the conventional QED theory. From this relation $z_e = z_Z$ we then have:

$$e_0 = \frac{e}{z_A^{\frac{1}{2}}} = \frac{1}{n_e} e \quad (96)$$

and that in (95) we have $\frac{z_e^2}{z_Z} - 1 = z_e - 1$.

16 Feynman diagrams and Feynman rules for QED

Let us then transform ds in (1.1) to $\frac{1}{(\beta+ih)}ds$ where $\beta, h > 0$ are parameters and h is as the Planck constant. The parameter h will give the dynamical effects of QED (as similar to the conventional QED). Here for simplicity we only consider the limiting case that $\beta \rightarrow 0$ and we let $h = 1$. From this transformation we get the Lagrangian \mathcal{L} from $-\int_{s_0}^{s_1} Dds$ changing to $\int_{s_0}^{s_1} \mathcal{L}ds$. Then we write $\mathcal{L} = \mathcal{L}_{phy} + \mathcal{L}_{cnt}$ where \mathcal{L}_{phy} corresponds to D_{phy} and \mathcal{L}_{cnt} corresponds to D_{cnt} . Then from the following term in \mathcal{L}_{phy} :

$$-i \left[\left(\frac{dZ_R}{ds} \right)^* \frac{dZ_R}{ds} - \mu^2 Z_R^* Z_R \right] \quad (97)$$

and by the perturbation expansion of $e^{\int_{s_0}^{s_1} \mathcal{L}ds}$ we have the following propagator:

$$\frac{i}{p_E^2 - \mu^2} \quad (98)$$

which is as the (primitive) electron propagator where p_E denotes the proper energy variable of electron.

Then from the pure gauge part of \mathcal{L}_{phy} we get the photon propagator (93), as done in the above Sections and the Section on photon propagator.

Then from \mathcal{L}_{phy} we have the following seagull vertex term:

$$ie^2 \left(\sum_{j=1}^2 A_{jR} \frac{dx^j}{ds} \right)^2 Z_R^* Z_R. \quad (99)$$

This seagull vertex term gives the vertex factor ie^2 . (We remark that the ds of the paths $\frac{dx^j}{ds}$ are not transformed to $-ids$ since these paths are given paths and thus are independent of the transformation $ds \rightarrow -ids$.)

From this vertex by using the photon propagator (93) in the above Section we get the following term:

$$\frac{ie^2}{2\pi} \int \frac{ik_E}{k_E^2 - \lambda_1^2} = -\frac{ie^2}{2\lambda_1} =: -i\omega^2. \quad (100)$$

The parameter ω is regarded as the mass-energy parameter of electron. Then from the perturbation expansion of $e^{\int_{s_0}^{s_1} \mathcal{L}ds}$ we have the following geometric series (which is similar to the Dyson series in the conventional QED):

$$\begin{aligned} & \frac{i}{p_E^2 - \mu^2} + \frac{i}{p_E^2 - \mu^2} (-i\omega^2 + i\mu^2) \frac{i}{p_E^2 - \mu^2} + \dots = \\ & = \frac{i}{p_E^2 - \mu^2 - \omega^2 + \mu^2} = \frac{i}{p_E^2 - \omega^2}, \end{aligned} \quad (101)$$

where the term $i\mu$ of $-i\omega^2 + i\mu^2$ is the $i\mu$ term in \mathcal{L}_{phy} . (The other term $-i\mu$ in \mathcal{L}_{phy} has been used in deriving (98).) Thus we have the following electron propagator:

$$\frac{i}{p_E^2 - \omega^2}. \quad (102)$$

This is as the electron propagator with mass-energy parameter ω . From ω we shall get the mass m of electron. (We shall later introduce a space-time statistics to get the usual electron propagator of the Dirac equation. This usual electron propagator is as the statistical electron propagator.) As the Feynman diagrams in the conventional QED we represent this electron propagator by a straight line.

In the above Sections and the Section on the photon propagator we see that the photon-loop $W(z, z)$ gives the renormalized charge $e = n_e e_0$ and the renormalized mass m of electron from the bare charge e_0 by the winding numbers of the photon loop such that m is with the winding number factor n_e . Then we see that the above one-loop energy integral of the photon gives the mass-energy parameter ω of electron which gives the mass m of electron. Thus these two types of photon-loops are closely related (from the relation of photon propagator and quantum photon propagator) such that the mass m obtained by the winding numbers of the photon loop $W(z, z)$ reappears in the one-loop energy integral (100) of the photon.

Thus we see that even there is no mass term in the Lagrangian of this gauge theory the mass m of the electron can come out from the gauge theory. This actually resolves the mass problem of particle physics that particle can be with mass even without the mass term. Thus we do not need the Higgs mechanism for generating masses to particles.

On the other hand from the one-loop-electron form of the seagull vertex we have the following term:

$$\frac{ie^2}{2\pi} \int \frac{idp_E}{p_E^2 - \mu^2} = -\frac{ie^2}{2\mu} =: -i\lambda_2^2. \quad (103)$$

So for photon from the perturbation expansion of $e^{\int_{s_0}^{s_1} \mathcal{L}ds}$ we have the following geometric series:

$$\begin{aligned} & \frac{i}{k_E^2 - \lambda_1^2} + \frac{i}{k_E^2 - \lambda_1^2} (-i\lambda_2^2) \frac{i}{k_E^2 - \lambda_1^2} + \dots = \\ & = \frac{i}{k_E^2 - \lambda_1^2 - \lambda_2^2} =: \frac{i}{k_E^2 - \lambda_0^2}, \end{aligned} \quad (104)$$

where we define $\lambda_0^2 = \lambda_1^2 + \lambda_2^2$. Thus we have the following photon propagator:

$$\frac{i}{k_E^2 - \lambda_0^2}, \quad (105)$$

which is of the same form as (93) where we replace λ_1 with λ_0 . As the Feynman diagrams in the conventional QED we represent this photon propagator by a wave line.

Then the following interaction term in \mathcal{L}_{phy} :

$$\begin{aligned} & -ie \frac{dZ_R^*}{ds} \left(\sum_{j=1}^2 A_{jR} \frac{dx^j}{ds} \right) Z_R + \\ & + ie \frac{dZ_R}{ds} \left(\sum_{j=1}^2 A_{jR} \frac{dx^j}{ds} \right) Z_R^* \end{aligned} \quad (106)$$

gives the vertex factor $(-ie)(p_E + q_E)$ which corresponds to the usual vertex of Feynman diagram with two electron straight lines (with energies p_E and q_E) and one photon wave line in the conventional QED.

Then as the Feynman rules in the conventional QED a sign factor $(-1)^n$, where n is the number of the electron loops in a Feynman diagram, is to be included for the Feynman diagram.

17 Statistics with space-time

Let us introduce space-time as a statistical method for a large amount of basic variables Z_R and A_{1R}, A_{2R} . As an illustration let us consider the electron propagator $\frac{i}{p_E^2 - \omega^2}$ and the following Green's function corresponding to it:

$$\frac{i}{2\pi} \int \frac{e^{-ip_E(s-s')}}{p_E^2 - \omega^2} dp_E, \quad (107)$$

where s is the proper time.

We imagine each electron (and photon) occupies a space region (This is the creation of the concept of space which is associated to the electron. Without the electron this space region does not exist). Then we write

$$p_E(s - s') = p_E(t - t') - \mathbf{p}(\mathbf{x} - \mathbf{x}'), \quad (108)$$

where $\mathbf{p}(\mathbf{x} - \mathbf{x}')$ denotes the inner product of the three dimensional vectors \mathbf{p} and $\mathbf{x} - \mathbf{x}'$ and (t, \mathbf{x}) is the time-space coordinate where \mathbf{x} is in the space region occupied by $Z_R(s)$ and that

$$\omega^2 - \mathbf{p}^2 = m^2 > 0, \quad (109)$$

where m is the mass of electron. This mass m is greater than 0 since each Z_R occupies a space region which implies that when $t - t'$ tends to 0 we can have that $|\mathbf{x} - \mathbf{x}'|$ does not tend to 0 (\mathbf{x} and \mathbf{x}' denote two coordinate points in the regions occupied by $Z_R(s)$ and $Z_R(s')$ respectively) and thus (109) holds. Then by linear summing the effects of a large amount of basic variables Z_R and letting ω varies from m to ∞ from (107), (108) and (109) we get the following statistical expression:

$$\frac{i}{(2\pi)^4} \int \frac{e^{-ip(x-x')}}{p^2 - m^2} dp, \quad (110)$$

which is the usual Green's function of a free field with mass m where p is a four vector and $x = (t, \mathbf{x})$.

The result of the above statistics is that (110) is induced from (107) with the scalar product p_E^2 of a scalar p_E changed to an indefinite inner product p^2 of a four vector p and the parameter ω is reduced to m .

Let us then introduce Fermi-Dirac statistics for electrons. As done by Dirac for deriving the Dirac equation we factorize $p^2 - m^2$ into the following form:

$$\begin{aligned} p^2 - m^2 &= (p_E - \omega)(p_E + \omega) = \\ &= (\gamma_\mu p^\mu - m)(\gamma_\mu p^\mu + m), \end{aligned} \quad (111)$$

where γ_μ are the Dirac matrices. Then from (110) we get the

following Green's function:

$$\begin{aligned} \frac{i}{(2\pi)^4} \int e^{-ip(x-x')} \frac{\gamma_\mu p^\mu + m}{p^2 - m^2} dp &= \\ = \frac{i}{(2\pi)^4} \int \frac{e^{-ip(x-x')} dp}{\gamma_\mu p^\mu - m}. \end{aligned} \quad (112)$$

Thus we have the Fermi-Dirac statistics that the statistical electron propagator is of the form $\frac{i}{\gamma_\mu p^\mu - m}$ which is the propagator of the Dirac equation and is the electron propagator of the conventional QED.

Let us then consider statistics of photons. Since the above quantum gauge theory of photons is a gauge theory which is gauge invariant we have that the space-time statistical equation for photons should be gauge invariant. Then since the Maxwell equation is the only gauge invariant equation for electromagnetism which is based on the space-time we have that the Maxwell equation must be a statistical equation for photons.

Then let us consider the vertexes. The tree vertex (106) with three lines (one for photon and two for electron) gives the factor $-ie(p_E + q_E)$ where p_E and q_E are from the factor $\frac{dZ_R}{ds}$ for electron.

We notice that this vertex is with two electron lines (or electron propagator) and one photon line (or photon propagator). In doing a statistics on this photon line when it is as an external electromagnetic field on the electron this photon line is of the statistical form $\gamma_\mu A^\mu$ where A^μ denotes the four electromagnetic potential fields of the Maxwell equation. Thus the vertex $-ie(p_E + q_E)$ after statistics is changed to the form $-ie(p_E + q_E) \frac{\gamma^\mu}{2}$ where for each γ^μ a factor $\frac{1}{2}$ is introduced for statistics.

Let us then introduce the on-mass-shell condition as in the conventional QED theory (see [6]). As similar to the on-mass-shell condition in the conventional QED theory our on-mass-shell condition is that $p_E = m$ where m is the observable mass of the electron. In this case $-ie(p_E + q_E) \frac{\gamma^\mu}{2}$ is changed to $-iem\gamma^\mu$. Then the m is absorbed to the two external spinors $\frac{1}{\sqrt{E}}u$ (where E denotes the energy of the electron satisfied the Dirac equation while the E of p_E is only as a notation) of the two electrons lines attached to this vertex such that the factor $\frac{1}{\sqrt{E}}$ of spin 0 of the Klein-Gordon equation is changed to the factor $\sqrt{\frac{m}{E}}$ of spinors of the Dirac equation. In this case we have the magnitude of p_E and q_E reappears in the two external electron lines with the factor \sqrt{m} . The statistical vertex then becomes $-ie\gamma^\mu$. This is exactly the usual vertex in the conventional QED. Thus after a space-time statistics on the original vertex $-ie(p_E + q_E)$ we get the statistical vertex $-ie\gamma^\mu$ of the conventional QED.

18 Basic effects of Quantum Electrodynamics

To illustrate this new theory of QED let us compute the three basic effects of QED: the one-loop photon and electron self-energies and the one-loop vertex correction.

As similar to the conventional QED we have the Feynman rules such that the one-loop photon self-energy is given by the following Feynman integral:

$$i\Pi_0(k_E) := i^2(-i)^2 \frac{e^2}{2\pi} \times \int \frac{(2p_E+k_E)(2p_E+k_E)dp_E}{(p_E^2-\omega^2)((k_E+p_E)^2-\omega^2)}, \quad (113)$$

where e is the renormalized electric charge.

Then as the Feynman rules in the conventional QED for the space-time statistics a statistical sign factor $(-1)^j$, where j is the number of the electron loops in a Feynman diagram, will be included for the Feynman diagram. Thus for the one-loop photon self-energy (113) a statistical factor $(-1)^j$ will be introduced to this one-loop photon self-energy integral.

Then similarly we have the Feynman rules such that the one-loop electron self-energy is given by the following Feynman integral:

$$-i\Sigma_0(p_E) := i^2(-i)^2 \frac{e^2}{2\pi} \times \int \frac{(2p_E-k_E)(2p_E-k_E)dk_E}{(k_E^2-\lambda_0^2)((p_E-k_E)^2-\omega^2)}. \quad (114)$$

Similarly we have the Feynman rules that the one-loop vertex correction is given by the following Feynman integral:

$$(-ie)\Lambda_0(p_E, q_E) := (i)^3(-i)^3 \frac{e^3}{2\pi} \int \frac{(2p_E-k_E)(2q_E-k_E)(p_E+q_E-2k_E)dk_E}{((p_E-k_E)^2-\omega^2)((q_E-k_E)^2-\omega^2)(k_E^2-\lambda_0^2)}. \quad (115)$$

Let us first compute the one-loop vertex correction and then compute the photon self-energy and the electron self-energy.

As a statistics we extend the one dimensional integral $\int dk_E$ to the n -dimensional integral $\int d^n k$ ($n \rightarrow 4$) where $k = (k_E, \mathbf{k})$. This is similar to the dimensional regularization in the conventional quantum field theories (However here our aim is to increase the dimension for statistics which is different from the dimensional regularization which is to reduce the dimension from 4 to n to avoid the ultraviolet divergence). With this statistics the factor 2π is replaced by the statistical factor $(2\pi)^n$. From this statistics on (115) we have the following statistical one loop vertex correction:

$$\frac{e^3}{(2\pi)^n} \int_0^1 dx \int_0^1 2y dy \int d^n k \times \int \frac{4p_E q_E (p_E+q_E)-2k_E((p_E+q_E)^2+4p_E q_E)+5k_E^2(p_E+q_E)-2k_E^3}{[k^2-2k(px+y+q(1-x)y)-p_E^2xy-q_E^2(1-x)y+m^2y+\lambda^2(1-y)]^3}, \quad (116)$$

where $k^2 = k_E^2 - \mathbf{k}^2$, and k^2 is from the free parameters ω, λ_0 where we let $\omega^2 = m^2 + \mathbf{k}^2$, $\lambda_0^2 = \lambda^2 + \mathbf{k}^2$ for the electron mass m and a mass-energy parameter λ for photon; and:

$$\left. \begin{aligned} k(px+y+q(1-x)y) &:= k_E(p_E xy + q_E(1-x)y) \\ -\mathbf{k} \cdot \mathbf{0} &= k_E(p_E xy + q_E(1-x)y) \end{aligned} \right\}. \quad (117)$$

By using the formulae for computing Feynman integrals

we have that (116) is equal to (see [6, 72]):

$$\begin{aligned} & \frac{ie^3}{(2\pi)^n} \int_0^1 dx \int_0^1 2y dy \times \\ & \times \left[\frac{4p_E q_E (p_E+q_E)\pi^{\frac{n}{2}} \Gamma(3-\frac{n}{2})}{\Gamma(3)(\Delta-r^2)^{3-2}} \frac{1}{(-\Delta+r^2)^{2-\frac{n}{2}}} - \right. \\ & - \frac{2((p_E+q_E)^2+4p_E q_E)\pi^{\frac{n}{2}} \Gamma(3-\frac{n}{2})r}{\Gamma(3)(\Delta-r^2)^{3-2}} \frac{1}{(-\Delta+r^2)^{2-\frac{n}{2}}} + \\ & + \frac{5(p_E+q_E)\pi^{\frac{n}{2}} \Gamma(3-1-\frac{n}{2})\frac{n}{2}}{\Gamma(3)(\Delta-r^2)^{3-2-1}} \frac{1}{(-\Delta+r^2)^{2-\frac{n}{2}}} + \\ & + \frac{5(p_E+q_E)\pi^{\frac{n}{2}} \Gamma(3-\frac{n}{2})r^2}{\Gamma(3)(\Delta-r^2)^{3-2}} \frac{1}{(-\Delta+r^2)^{2-\frac{n}{2}}} - \\ & - \frac{(\frac{n+2}{2})2\pi^{\frac{n}{2}} \Gamma(3-1-\frac{n}{2})r}{\Gamma(3)(\Delta-r^2)^{3-2-1}} \frac{1}{(-\Delta+r^2)^{2-\frac{n}{2}}} - \\ & \left. - \frac{2\pi^{\frac{n}{2}} \Gamma(3-\frac{n}{2})r^3}{\Gamma(3)(\Delta-r^2)^{3-2}} \frac{1}{(-\Delta+r^2)^{2-\frac{n}{2}}} \right] =: \\ & =: (-ie)\Lambda(p_1, p_2), \end{aligned} \quad (118)$$

where we define:

$$\left. \begin{aligned} r &:= p_E xy + q_E(1-x)y \\ \Delta &:= p_E^2 xy + q_E^2(1-x)y - m^2 y - \lambda^2(1-y) \end{aligned} \right\}. \quad (119)$$

We remark that in this statistics the p_E and q_E variables are remained as the proper variables which are derived from the proper time s .

Let us then introduce the Fermi-Dirac statistics on the electron and we consider the on-mass-shell case as in the conventional QED. We shall see this will lead to the theoretical results of the conventional QED on the anomalous magnetic moment and the Lamb shift.

As a Fermi-Dirac statistics we have shown in the above Section that the vertex term $-ie(p_E + q_E)$ is replaced with the vertex term $-ie(p_E + q_E)\frac{\gamma^\mu}{2}$. Then as a Fermi-Dirac statistics in the above Section we have shown that the statistical vertex is $-ie\gamma^\mu$ under the on-mass-shell condition. We notice that this vertex agrees with the vertex term in the conventional QED theory.

Let us then consider the Fermi-Dirac statistics on the one-loop vertex correction (118). Let us first consider the following term in (118):

$$\frac{ie^3}{(2\pi)^n} \int_0^1 dx \int_0^1 2y dy \times \frac{\pi^2(p_E+q_E)4p_E q_E}{\Gamma(3)(\Delta-r^2)^{3-2}} \frac{1}{(-\Delta+r^2)^{2-\frac{n}{2}}}, \quad (120)$$

where we can (as an approximation) let $n = 4$. From Fermi-Dirac statistics we have that this term gives the following statistics:

$$\frac{ie^3}{(2\pi)^4} \int_0^1 dx \int_0^1 2y dy \frac{\pi^2(p_E+q_E)\frac{1}{2}\gamma^\mu 4p_E q_E}{\Gamma(3)(\Delta-r^2)^{3-2}}. \quad (121)$$

Then we consider the case of on-mass-shell. In this case we have $p_E = m$ and $q_E = m$. Thus from (121) we have the

following term:

$$\frac{ie^3}{(2\pi)^4} \int_0^1 dx \int_0^1 2ydy \frac{\pi^2 \gamma^\mu 4p_E q_E}{\Gamma(3)(\Delta - r^2)^{3-2}}, \quad (122)$$

where a mass factor $m = \frac{1}{2}(p_E + q_E)$ has been omitted and put to the external spinor of the external electron as explained in the above Section on space-time statistics. In (122) we still keep the expression $p_E q_E$ even though in this case of on-mass-shell because this factor will be important for giving the observable Lamb shift, as we shall see. In (122) because of on-mass-shell we have (as an approximation we let $n = 4$):

$$(\Delta - r^2)^{3-2} = -\lambda^2(1 - y) - r^2 = -\lambda^2(1 - y) - m^2 y^2. \quad (123)$$

Thus in the on-mass-shell case (122) is of the following form:

$$ie\gamma^\mu \frac{\alpha}{\pi} \int_0^1 dx \int_0^1 ydy \frac{\pi^2 p_E q_E}{-\lambda^2(1 - y) - m^2 y^2}, \quad (124)$$

where $\alpha = \frac{e^2}{4\pi}$ is the fine structure constant. Carrying out the integrations on y and on x we have that as $\lambda \rightarrow 0$ (124) is equal to:

$$(-ie)\gamma^\mu \frac{\alpha}{\pi} \frac{p_E q_E}{m^2} \log \frac{m}{\lambda}, \quad (125)$$

where the proper factor $p_E q_E$ will be for a linear space-time statistics of summation. We remark that (125) corresponds to a term in the vertex correction in the conventional QED theory with the infra-divergence when $\lambda = 0$ (see [6]). Here since the parameter λ has not been determined we shall later find other way to determine the effect of (125) and to solve the infrared-divergence problem.

Let us first rewrite the form of the proper value $p_E q_E$. We write $p_E q_E$ in the following space-time statistical form:

$$p_E q_E = -2p' \cdot p, \quad (126)$$

where p and p' denote two space-time four-vectors of electron such that $p^2 = m^2$ and $p'^2 = m^2$. Then we have

$$\begin{aligned} p_E q_E &= \\ &= \frac{1}{3}(p_E q_E + p_E q_E + p_E q_E) = \frac{1}{3}(m^2 - 2p' \cdot p + m^2) \\ &= \frac{1}{3}(m^2 - 2p' \cdot p + m^2) = \frac{1}{3}(p'^2 - 2p' \cdot p + p^2) \\ &= \frac{1}{3}(p' - p)^2 \\ &=: \frac{1}{3}q^2, \end{aligned} \quad (127)$$

where following the convention of QED we define $q = p' - p$. Thus from (125) we have the following term:

$$(-ie)\gamma^\mu \frac{\alpha}{3\pi} \frac{q^2}{m^2} \log \frac{m}{\lambda}, \quad (128)$$

where the parameter λ are to be determined. Again this term

(128) corresponds to a term in the vertex correction in the conventional QED theory with the infrared-divergence when $\lambda = 0$ (see [6]).

Let us then consider the following term in (118):

$$\begin{aligned} &\frac{-ie^3}{(2\pi)^n} \int_0^1 dx \times \\ &\times \int_0^1 \frac{2((p_E + q_E)^2 + 4p_E q_E)\pi^{\frac{n}{2}} r 2ydy}{\Gamma(3)(\Delta - r^2)^{3-2}(-\Delta + r^2)^{2-\frac{n}{2}}}. \end{aligned} \quad (129)$$

For this term we can (as an approximation) also let $n = 4$ and we have let $\Gamma(3 - \frac{n}{2}) = 1$. As similar to the conventional QED theory we want to show that this term gives the anomalous magnetic moment and thus corresponds to a similar term in the vertex correction of the conventional QED theory (see [6]).

By Fermi-Dirac statistics the factor $(p_E + q_E)$ in (129) of $(p_E + q_E)^2$ gives the statistical term $(p_E + q_E)^{\frac{1}{2}}\gamma^\mu$. Thus with the on-mass-shell condition the factor $(p_E + q_E)$ gives the statistical term $m\gamma^\mu$. Thus with the on-mass-shell condition the term $(p_E + q_E)^2$ gives the term $m\gamma^\mu(p_E + q_E)$. Then the factor $(p_E + q_E)$ in this statistical term also give $2m$ by the on-mass-shell condition. Thus by Fermi-Dirac statistics and the on-mass-shell condition the factor $(p_E + q_E)^2$ in (129) gives the statistical term $\gamma^\mu 2m^2$. Then since this is a (finite) constant term it can be cancelled by the corresponding counter term of the vertex giving the factor $-ie\gamma^\mu$ and having the factor $z_e - 1$ in (95). From this cancellation the renormalization constant z_e is determined. Since the constant term is depended on the $\delta > 0$ which is introduced for space-time statistics we have that the renormalization constant z_e is also depended on the $\delta > 0$. Thus the renormalization constant z_e (and the concept of renormalization) is related to the space-time statistics.

At this point let us give a summary of this renormalization method, as follows.

Renormalization

1. The renormalization method of the conventional QED theory is used to obtain the renormalized physical results. Here unlike the conventional QED theory the renormalization method is not for the removing of ultraviolet divergences since the QED theory in this paper is free of ultraviolet divergences.

2. We have mentioned in the above Section on photon propagator that the property of renormalizable is a property of gauge invariance that it gives the physical results independent of the chosen photon propagator.

3. The procedure of renormalization is as a part of the space-time statistics to get the statistical results which is independent of the chosen photon propagator. \diamond

Let us then consider again the above computation of the one-loop vertex correction. We now have that the (finite) constant term of the one-loop vertex correction is cancelled by the corresponding counter term with the factor $z_e - 1$ in (95).

Thus the nonconstant term (128) is renormalized to be the following renormalized form:

$$(-ie)\gamma^\mu \frac{\alpha}{3\pi} \frac{q^2}{m^2} \log \frac{m}{\lambda}. \quad (130)$$

Let us then consider the following term in (129):

$$\frac{-ie^3}{(2\pi)^n} \int_0^1 dx \int_0^1 2ydy \frac{8p_E q_E \pi^{\frac{n}{2}} \Gamma(3 - \frac{n}{2}) r}{\Gamma(3)(\Delta - r^2)^{3 - \frac{n}{2}}}, \quad (131)$$

where we can (as an approximation) let $n = 4$. With the on-mass-shell condition we have that $\Delta - r^2$ is again given by (123). Then letting $\lambda = 0$ we have that (131) is given by:

$$\frac{-ie\alpha}{4\pi} \int_0^1 dx \int_0^1 ydy \cdot \frac{8p_E q_E}{-r} \quad (132)$$

With the on-mass-shell condition we have $r = my$. Thus this term (132) is equal to:

$$(-ie) \frac{-\alpha}{4\pi m} 8p_E q_E. \quad (133)$$

Again the factor $p_E q_E$ is for the exchange of energies for two electrons with proper energies p_E and q_E respectively and thus it is the vital factor. This factor is then for the space-time statistics and later it will be for a linear statistics of summation for the on-mass-shell condition. Let us introduce a space-time statistics on the factor $p_E q_E$, as follows. With the on-mass-shell condition we write $p_E q_E$ in the following form:

$$p_E q_E = \frac{1}{2} (mp_E + q_E m) = \frac{1}{2} m(p_E + q_E). \quad (134)$$

Then we introduce a space-time statistics on the proper energies p_E and q_E respectively that p_E gives a statistics βp and q_E gives a statistics $\beta p'$ where p and p' are space-time four vectors such that $p^2 = m^2$; $p'^2 = m^2$; and β is a statistical factor to be determined.

Then we have the following Gordan relation on the space-time four vectors p and p' respectively (see [6] [72]):

$$\left. \begin{aligned} p^\mu &= \gamma^\mu (p \cdot \gamma) + i\sigma^{\mu\nu} p_\nu \\ p'^\mu &= (p' \cdot \gamma) \gamma^\mu - i\sigma^{\mu\nu} p'_\nu \end{aligned} \right\}, \quad (135)$$

where p^μ and p'^μ denote the four components of p and p' respectively. Thus from (134) and the Gordan relation (135) we have the following space-time statistics:

$$\begin{aligned} \frac{1}{2} (mp_E + q_E m) &= \\ &= \frac{1}{2} m\beta (\gamma^\mu (p \cdot \gamma) + (p' \cdot \gamma) \gamma^\mu - i\sigma^{\mu\nu} q_\nu), \end{aligned} \quad (136)$$

where following the convention of QED we define $q = p' - p$.

From (136) we see that the space-time statistics on p_E for giving the four vector p needs the product of two Dirac γ -matrices. Then since the introducing of a Dirac γ -matrix

for space-time statistics requires a statistical factor $\frac{1}{2}$ we have that the statistical factor $\beta = \frac{1}{4}$.

Then as in the literature on QED when evaluated between polarization spinors, the $p' \cdot \gamma$ and $\gamma \cdot p$ terms are deduced to the mass m respectively. Thus the term $\frac{1}{2} m\beta (\gamma^\mu p \cdot \gamma + p' \cdot \gamma \gamma^\mu)$ as a constant term can be cancelled by the corresponding counter term with the factor $z_e - 1$ in (95).

Thus by space-time statistics on $p_E q_E$ from (133) we get the following vertex correction:

$$(-ie) \frac{i\alpha}{4\pi m} \sigma^{\mu\nu} q_\nu \quad (137)$$

where $q = p - p'$ and the factor 8 in (133) is cancelled by the statistical factor $\frac{1}{2}\beta = \frac{1}{8}$. We remark that in the way of getting (137) a factor m has been absorbed by the two polarization spinors u to get the form $\sqrt{\frac{m}{E}} u$ of the spinors of external electrons.

Then from (137) we get the following exact second order magnetic moment:

$$\frac{\alpha}{2\pi} \mu_0, \quad (138)$$

where $\mu_0 = \frac{1}{2m}$ is the Dirac magnetic moment as in the literature on QED (see [6]).

We see that this result is just the second order anomalous magnetic moment obtained from the conventional QED (see [6] [72]- [78]). Here we can obtain this anomalous magnetic moment exactly while in the conventional QED this anomalous magnetic moment is obtained only by approximation under the condition that $|q^2| \ll m^2$. The point is that we do not need to carry out a complicate integration as in the literature in QED when the on-mass-shell condition is applied to the proper energies p_E and q_E , and with the on-mass-shell condition applied to the proper energies p_E and q_E the computation is simple and the computed result is the exact result of the anomalous magnetic moment.

Let us then consider the following terms in the one-loop vertex correction (118):

$$\begin{aligned} &\frac{ie^3}{(2\pi)^n} \int_0^1 dx \int_0^1 2ydy \times \\ &\times \left[\frac{5(p_E + q_E) \pi^{\frac{n}{2}} \Gamma(3 - 1 - \frac{n}{2})^{\frac{n}{2}}}{\Gamma(3)(\Delta - r^2)^{3-2-1}} \frac{1}{(-\Delta + r^2)^{2 - \frac{n}{2}}} + \right. \\ &+ \frac{5(p_E + q_E) \pi^{\frac{n}{2}} \Gamma(3 - \frac{n}{2}) r^2}{\Gamma(3)(\Delta - r^2)^{3-2}} \frac{1}{(-\Delta + r^2)^{2 - \frac{n}{2}}} - \\ &- \frac{(\frac{n+2}{2}) 2 \pi^{\frac{n}{2}} \Gamma(3 - 1 - \frac{n}{2}) r}{\Gamma(3)(\Delta - r^2)^{3-2-1}} \frac{1}{(-\Delta + r^2)^{2 - \frac{n}{2}}} - \\ &\left. - \frac{2 \pi^{\frac{n}{2}} \Gamma(3 - \frac{n}{2}) r^3}{\Gamma(3)(\Delta - r^2)^{3-2}} \frac{1}{(-\Delta + r^2)^{2 - \frac{n}{2}}} \right]. \end{aligned} \quad (139)$$

From the on-mass-shell condition we have $\Delta - r^2 = -r^2$ where we have set $\lambda = 0$. The first and the second term are with the factor $(p_E + q_E)$ which by Fermi-Dirac statistics

gives the statistics $(p_E + q_E)^{\frac{1}{2}} \gamma^\mu$. Then from the following integration:

$$\int_0^1 dx \int_0^1 2yr dy = \int_0^1 dx \int_0^1 2y^2(p_E x + (1-x)q_E) dy \quad (140)$$

we get a factor $(p_E + q_E)$ for the third and fourth terms. Thus all these four terms by Fermi-Dirac statistics are with the statistics $(p_E + q_E)^{\frac{1}{2}} \gamma^\mu$. Then by the on-mass-shell condition we have that the statistics $(p_E + q_E)^{\frac{1}{2}} \gamma^\mu$ gives the statistics $m\gamma^\mu$. Thus (139) gives a statistics which is of the form $(\gamma^\mu \cdot \text{constant})$. Thus this constant term can be cancelled by the corresponding counter term with the factor $z_e - 1$ in (95).

Thus under the on-mass-shell condition the renormalized vertex correction $(-ie)\Lambda_R(p', p)$ from the one-loop vertex correction is given by the sum of (128) and (137):

$$(-ie)\Lambda_R(p', p) = (-ie) \left[\gamma^\mu \frac{\alpha}{3\pi} \frac{q^2}{m^2} \log \frac{m}{\lambda} + \frac{i\alpha}{4\pi m} \sigma^{\mu\nu} q_\nu \right] \quad (141)$$

19 Computation of the Lamb shift: Part I

The above computation of the vertex correction has not been completed since the parameter λ has not been determined. This appearance of the nonzero λ is due to the on-mass-shell condition. Let us in this Section complete the above computation of the vertex correction by finding another way to get the on-mass-shell condition. By this completion of the above computation of the vertex correction we are then able to compute the Lamb shift.

As in the literature of QED we let ω_{\min} denote the minimum of the (virtual) photon energy in the scattering of electron. Then as in the literature of QED we have the following relation between ω_{\min} and λ when $\frac{v}{c} \ll 1$ where v denotes the velocity of electron and c denotes the speed of light (see [6, 68–74]):

$$\log 2\omega_{\min} = \log \lambda + \frac{5}{6} \quad (142)$$

Thus from (141) we have the following form of the vertex correction:

$$(-ie)\gamma^\mu \frac{\alpha}{3\pi} \frac{q^2}{m^2} \left[\log \frac{m}{2\omega_{\min}} + \frac{5}{6} \right] + (-ie)\gamma^\mu \frac{ie\alpha\sigma^{\mu\nu}q_\nu}{4\pi m} \quad (143)$$

Let us then find a way to compute the following term in the vertex correction (143):

$$(-ie)\gamma^\mu \frac{\alpha}{3\pi} \frac{q^2}{m^2} \log \frac{m}{2\omega_{\min}} \quad (144)$$

The parameter $2\omega_{\min}$ is for the exchanging (or shifting) of the proper energies p_E and q_E of electrons. Thus the magnitudes of p_E and q_E correspond to the magnitude of ω_{\min} . When the ω_{\min} is chosen the corresponding p_E and q_E are also chosen and vice versa.

Since ω_{\min} is chosen to be very small we have that the corresponding proper energies p_E and q_E are very small that they are no longer equal to the mass m for the on-mass-shell condition and they are for the virtual electrons. Then to get the on-mass-shell condition we use a linear statistics of summation on the vital factor $p_E q_E$. This means that the large amount of the effects $p_E q_E$ of the exchange of the virtual electrons are to be summed up to statistically getting the on-mass-shell condition.

Thus let us consider again the one-loop vertex correction (118) where we choose p_E and q_E such that $p_E \ll m$ and $q_E \ll m$. This chosen corresponds to the chosen of ω_{\min} . We can choose p_E and q_E as small as we want such that $p_E \ll m$ and $q_E \ll m$. Thus we can let $\lambda = 0$ and set $p_E = q_E = 0$ for the p_E and q_E in the denominators $(\Delta - r^2)^{3-2}$ in (118). Thus (118) is approximately equal to:

$$\begin{aligned} & \frac{ie^3}{(2\pi)^n} \int_0^1 dx \int_0^1 dy \left[\frac{4p_E q_E (p_E + q_E) \pi^{\frac{n}{2}} \Gamma(3-2)}{-m^2} - \right. \\ & \left. - \frac{2((p_E + q_E)^2 + 4p_E q_E) \pi^{\frac{n}{2}} \Gamma(3-2)r}{-m^2} + \right. \\ & \left. + \frac{5(p_E + q_E) \pi^{\frac{n}{2}} \Gamma(2 - \frac{n}{2}) \frac{n}{2}}{(-\Delta + r^2)^{2 - \frac{n}{2}}} + \frac{5(p_E + q_E) \pi^{\frac{n}{2}} \Gamma(3-2)r^2}{-m^2} - \right. \\ & \left. - \frac{(\frac{n+2}{2}) 2\pi^{\frac{n}{2}} \Gamma(2 - \frac{n}{2})r}{(\Delta - r^2)^{2 - \frac{n}{2}}} + \frac{2\pi^{\frac{n}{2}} \Gamma(3-2)r^3}{-m^2} \right]. \quad (145) \end{aligned}$$

Let us then first consider the four terms in (145) without the factor $\Gamma(2 - \frac{n}{2})$. For these four terms we can (as an approximation) let $n = 4$. Carry out the integrations $\int_0^1 dx \int_0^1 y dy$ of these four terms we have that the sum of these four terms is given by:

$$\begin{aligned} & (ie) \frac{\alpha\pi^2}{4\pi^3 m^2} [4p_E q_E (p_E + q_E) - \\ & - \frac{1}{2}((p_E + q_E)^2 + 4p_E q_E)(p_E + q_E) + \\ & + \frac{5}{9}(p_E + q_E)(p_E^2 + q_E^2 + p_E q_E) - \\ & - \frac{1}{8}(p_E^3 + q_E^3 + p_E^2 q_E + p_E q_E^2)] = \\ & = (ie) \frac{\alpha\pi^2}{4\pi^3 m^2} (p_E + q_E) \left[\frac{5}{72} p_E^2 + \frac{5}{72} q_E^2 - \frac{14}{9} p_E q_E \right], \quad (146) \end{aligned}$$

where the four terms of the sum are from the corresponding four terms of (145) respectively.

Then we consider the two terms in (145) with the factor $\Gamma(2 - \frac{n}{2})$. Let $\delta := 2 - \frac{n}{2} > 0$. We have:

$$\Gamma(\delta) \cdot (-\Delta + r^2)^{-\delta} = \left(\frac{1}{\delta} + \text{a finite limit term as } \delta \rightarrow 0 \right) \cdot e^{-\delta \log(-\Delta + r^2)} \quad (147)$$

We have:

$$\frac{1}{\delta} \cdot e^{-\delta \log(-\Delta + r^2)} = \frac{1}{\delta} \cdot [1 - \delta \log(-\Delta + r^2) + 0(\delta^2)] \quad (148)$$

Then we have:

$$\begin{aligned}
& -\frac{1}{\delta} \cdot \delta \log(-\Delta + r^2) = \\
& = -\log m^2 y - \log \frac{1}{m^2} \times \\
& \times [m^2 - p_E^2 x - q_E^2 (1-x) + (p_E x + q_E (1-x))^2 y] = \\
& = -\log m^2 y - \log \left[1 - \frac{p_E^2 x(1-x)y + q_E^2 (1-x)(1-(1-x)y)}{m^2} + \right. \\
& \left. + \frac{2p_E q_E x(1-x)y}{m^2} + 0 \left(\frac{p_E^2 + q_E^2}{m^2} \right) \right]. \quad (149)
\end{aligned}$$

Then the constant term $-\log m^2 y$ in (149) can be cancelled by the corresponding counter term with the factor $z_e - 1$ in (95) and thus can be ignored. When $p_E^2 \ll m^2$ and $q_E^2 \ll m^2$ the second term in (149) is approximately equal to:

$$\begin{aligned}
f(x, y) := & \frac{p_E^2 x(1-x)y + q_E^2 (1-x)(1-(1-x)y)}{m^2} - \\
& - \frac{2p_E q_E x(1-x)y}{m^2}. \quad (150)
\end{aligned}$$

Thus by (150) the sum of the two terms in (145) having the factor $\Gamma(2 - \frac{n}{2})$ is approximately equal to:

$$\begin{aligned}
& \frac{ie^3}{(2\pi)^n} \int_0^1 dx \int_0^1 y dy f(x, y) \times \\
& \times \left[5(p_E + q_E) \pi^{\frac{n}{2}} \frac{n}{2} - 2\pi^{\frac{n}{2}} \frac{(n+2)}{2} r \right], \quad (151)
\end{aligned}$$

where we can (as an approximation) let $n = 4$. Carrying out the integration $\int_0^1 dx \int_0^1 y dy$ of the two terms in (151) we have that (151) is equal to the following result:

$$\begin{aligned}
& (ie) \frac{\alpha \pi^2}{4\pi^3 m^2} (p_E + q_E) \times \\
& \times \left[(-5 \cdot \frac{1}{9} \cdot 2p_E q_E) + (-\frac{7}{24} p_E^2 - \frac{7}{24} q_E^2 + \frac{3}{9} p_E q_E) \right], \quad (152)
\end{aligned}$$

where the first term and the second term in the $[\cdot]$ are from the first term and the second term in (151) respectively.

Combining (146) and (151) we have the following result which approximately equal to (145) when $p_E^2 \ll m^2$ and $q_E^2 \ll m^2$:

$$(ie) \frac{\alpha \pi^2}{4\pi^3 m^2} (p_E + q_E) \left[\frac{2}{9} p_E^2 + \frac{2}{9} q_E^2 + \frac{7}{3} p_E q_E \right], \quad (153)$$

where the exchanging term $\frac{7}{3} p_E q_E$ is of vital importance.

Now to have the on-mass-shell condition let us consider a linear statistics of summation on (153). Let there be a large amount of virtual electrons $z_j, j \in J$ indexed by a set J with the proper energies $p_{Ej}^2 \ll m^2$ and $q_{Ej}^2 \ll m^2, j \in J$. Then from (153) we have the following linear statistics of summation on (153):

$$\begin{aligned}
& \frac{(-ie)\alpha\pi^2(p_{Ej_0} + q_{Ej_0})}{4\pi^3 m^2} \times \\
& \times \left[\frac{2}{9} \sum_j (p_{Ej}^2 + q_{Ej}^2) + \frac{7}{3} \sum_j p_{Ej} q_{Ej} \right], \quad (154)
\end{aligned}$$

where for simplicity we let:

$$p_{Ej} + q_{Ej} = p_{Ej'} + q_{Ej'} = p_{Ej_0} + q_{Ej_0} = 2m_0 \quad (155)$$

for all $j, j' \in J$ and for some (bare) mass $m_0 \ll m$ and for some $j_0 \in J$. Then by applying Fermi-Dirac statistics on the factor $p_{Ej_0} + q_{Ej_0}$ in (154) we have the following Fermi-Dirac statistics for (154):

$$\begin{aligned}
& (-ie) \frac{\alpha \pi^2}{4\pi^3 m^2} \frac{1}{2} \gamma^\mu (p_{Ej_0} + q_{Ej_0}) \times \\
& \times \left[\frac{2}{9} \sum_j (p_{Ej}^2 + q_{Ej}^2) + \frac{7}{3} \sum_j p_{Ej} q_{Ej} \right] = \\
& = (-ie) \frac{\alpha \pi^2 \gamma^\mu m_0}{4\pi^3 m^2} \left[\frac{2}{9} \sum_j (p_{Ej}^2 + q_{Ej}^2) + \frac{7}{3} \sum_j p_{Ej} q_{Ej} \right]. \quad (156)
\end{aligned}$$

Then for the on-mass-shell condition we require that the linear statistical sum $m_0 \frac{7}{3} \sum_j p_{Ej} q_{Ej}$ in (156) is of the following form:

$$m_0 \frac{7}{3} \sum_j p_{Ej} q_{Ej} = \beta_0 m \frac{7}{3} q^2, \quad (157)$$

where $q^2 = (p' - p^2)$ and the form $m q^2 = m(p' - p^2)$ is the on-mass-shell condition which gives the electron mass m ; and that β_0 is a statistical factor (to be determined) for this linear statistics of summation and is similar to the statistical factor $(2\pi)^n$ for the space-time statistics.

Then we notice that (156) is for computing (144) and thus its exchanging term corresponding to $\sum_j p_{Ej} q_{Ej}$ must be equal to (144). From (156) we see that there is a statistical factor 4 which does not appear in (144). Since this exchanging term in (156) must be equal to (144) we conclude that the statistical factor β_0 must be equal to 4 so as to cancel the statistical factor 4 in (156). (We also notice that there is a statistical factor π^2 in the numerator of (156) and thus it requires a statistical factor 4 to form the statistical factor $(2\pi)^2$ and thus $\beta_0 = 4$.) Thus we have that for the on-mass-condition we have that (156) is of the following statistical form:

$$(-ie) \frac{\alpha \pi^2}{\pi^3 m^2} m \gamma^\mu \left[\beta_2 \frac{2}{9} m^2 + \beta_2' \frac{2}{9} m^2 + \frac{7}{3} q^2 \right]. \quad (158)$$

Then from (158) we have the following statistical form:

$$(-ie) \frac{\alpha \pi^2}{\pi^3 m^2} \gamma^\mu \left[\beta_2 \frac{2}{9} m^2 + \beta_2' \frac{2}{9} m^2 + \frac{7}{3} q^2 \right], \quad (159)$$

where the factor m of $m\gamma^\mu$ has been absorbed to the two external spinors of electron. Then we notice that the term corresponding to $\beta_2 \frac{2}{9} m^2 + \beta_2' \frac{2}{9} m^2$ in (159) is as a constant term and thus can be cancelled by the corresponding counter term with the factor $z_e - 1$ in (95). Thus from (159) we have the following statistical form of effect which corresponds to (144):

$$(-ie) \gamma^\mu \frac{\alpha}{\pi m^2} \frac{7}{3} q^2. \quad (160)$$

This effect (160) is as the total effect of q^2 computed from the one-loop vertex with the minimal energy ω_{\min} and thus

includes the effect of q^2 from the anomalous magnetic moment. Thus we have that (144) is computed and is given by the following statistical form:

$$\begin{aligned} (-ie)\gamma^\mu \frac{\alpha}{3\pi} \frac{q^2}{m^2} \log \frac{m}{2\omega_{\min}} &= \\ &= (-ie)\gamma^\mu \frac{\alpha}{3\pi} \frac{q^2}{m^2} \left[7 - \frac{3}{8}\right], \end{aligned} \quad (161)$$

where the term corresponding to the factor $\frac{3}{8}$ is from the anomalous magnetic moment (137) as computed in the literature of QED (see [6]). This completes our computation of (144). Thus under the on-mass-shell condition the renormalized one-loop vertex $(-ie)\Lambda_R(p', p)$ is given by:

$$\begin{aligned} (-ie)\Lambda_R(p', p) &= \\ &= (-ie) \left[\gamma^\mu \frac{\alpha q^2}{3\pi m^2} \left(7 + \frac{5}{6} - \frac{3}{8}\right) + \frac{i\alpha}{4\pi m} \sigma^{\mu\nu} q_\nu \right]. \end{aligned} \quad (162)$$

This completes our computation of the one-loop vertex correction.

20 Computation of photon self-energy

To compute the Lamb shift let us then consider the one-loop photon self energy (113). As a statistics we extend the one dimensional integral $\int dp_E$ to the n -dimensional integral $\int d^n p$ ($n \rightarrow 4$) where $p = (p_E, \mathbf{p})$. This is similar to the dimensional regularization in the existing quantum field theories (However here our aim is to increase the dimension for statistics which is different from the dimensional regularization which is to reduce the dimension from 4 to n to avoid the ultraviolet divergence). With this statistics the factor 2π is replaced by the statistical factor $(2\pi)^n$. From this statistics on (113) we have that the following statistical one-loop photon self-energy:

$$\begin{aligned} (-1)i^2(-i)^2 \frac{e^2}{(2\pi)^n} \times \\ \times \int_0^1 dx \int \frac{(4p_E^2 + 4p_E k_E + k_E^2) d^n p}{(p^2 + 2pk_x + k_E^2 x - m^2)^2}, \end{aligned} \quad (163)$$

where $p^2 = p_E^2 - \mathbf{p}^2$, and \mathbf{p}^2 is from $\omega^2 = m^2 + \mathbf{p}^2$; and:

$$pk := p_E k_E - \mathbf{p} \cdot \mathbf{0} = p_E k_E. \quad (164)$$

As a Feynman rule for space-time statistics a statistical factor (-1) has been introduced for this photon self-energy since it has a loop of electron particles.

By using the formulae for computing Feynman integrals we have that (163) is equal to:

$$\begin{aligned} \frac{(-1)ie^2}{(2\pi)^n} \int_0^1 dx \times \\ \times \left[\frac{k_E^2 (4x^2 - 4x + 1) \pi^{\frac{n}{2}} \Gamma(2 - \frac{n}{2})}{\Gamma(2)(m^2 - k_E^2 x(1-x))^2 - \frac{n}{2}} + \frac{\pi^{\frac{n}{2}} \Gamma(2 - 1 - \frac{n}{2})^{\frac{n}{2}}}{\Gamma(2)(m^2 - k_E^2 x(1-x))^2 - 1 - \frac{n}{2}} \right]. \end{aligned} \quad (165)$$

Let us first consider the first term in the $[\cdot]$ in (165). Let $\delta := 2 - \frac{n}{2} > 0$. As for the one-loop vertex we have

$$\begin{aligned} \Gamma(\delta) \cdot (m^2 - k_E^2 x(1-x))^{-\delta} &= \\ &= \left(\frac{1}{\delta} + \text{a finite term as } \delta \rightarrow 0\right) \cdot e^{-\delta \log(m^2 - k_E^2 x(1-x))}. \end{aligned} \quad (166)$$

We have

$$\begin{aligned} \frac{1}{\delta} \cdot e^{-\delta \log(m^2 - k_E^2 x(1-x))} &= \\ &= \frac{1}{\delta} \cdot [1 - \delta \log(m^2 - k_E^2 x(1-x)) + 0(\delta^2)]. \end{aligned} \quad (167)$$

Then we have

$$\begin{aligned} -\frac{1}{\delta} \cdot \delta \log(m^2 - k_E^2 x(1-x)) &= \\ &= -\log m^2 - \log \left[1 - \frac{k_E^2 x(1-x)}{m^2}\right]. \end{aligned} \quad (168)$$

Then the constant term $-\log m^2$ in (168) can be cancelled by the corresponding counter term with the factor $z_A - 1$ in (95) and thus can be ignored. When $k_E^2 \ll m^2$ the second term in (168) is approximately equal to:

$$\frac{k_E^2 x(1-x)}{m^2}. \quad (169)$$

Carrying out the integration $\int_0^1 dx$ in (163) with $-\log \left[1 - \frac{k_E^2 x(1-x)}{m^2}\right]$ replaced by (169), we have the following result:

$$\int_0^1 dx (4x^2 - 4x + 1) \frac{k_E^2 x(1-x)}{m^2} = \frac{k_E^2}{30m^2}. \quad (170)$$

Thus as in the literature in QED from the photon self-energy we have the following term which gives contribution to the Lamb shift:

$$\frac{k_E^2}{30m^2} = \frac{(p_E - q_E)^2}{30m^2}, \quad (171)$$

where $k_E = p_E - q_E$ and p_E, q_E denote the proper energies of virtual electrons. Let us then consider statistics of a large amount of photon self-energy (168). When there is a large amount of photon self-energies we have the following linear statistics of summation:

$$\frac{\sum_i k_{Ei}^2}{30m^2}, \quad (172)$$

where each i represent a photon. Let us write:

$$k_{Ei}^2 = (p_{Ei} - q_{Ei})^2 = p_{Ei}^2 - 2p_{Ei}q_{Ei} + q_{Ei}^2. \quad (173)$$

Thus we have:

$$\begin{aligned} \sum_i k_{Ei}^2 &= \sum_i (p_{Ei} - q_{Ei})^2 = \\ &= \sum_i (p_{Ei}^2 + q_{Ei}^2) - 2 \sum_i p_{Ei}q_{Ei}. \end{aligned} \quad (174)$$

Now as the statistics of the vertex correction we have the following statistics:

$$\sum_i p_{Ei}q_{Ei} = 4(p' - p)^2 = 4q^2, \quad (175)$$

where 4 is a statistical factor which is the same statistical factor of case of the vertex correction and p, p' are on-mass-shell four vectors of electrons. As the the statistics of the vertex correction this statistical factor cancels another statistical fac-

tor 4. On the other hand as the statistics of the vertex correction we have the following statistics:

$$\sum_i p_{Ei}^2 = \beta_3 m^2, \quad \sum_i q_{Ei}^2 = \beta_4 m^2, \quad (176)$$

where β_3 and β_4 are two statistical factors. As the case of the vertex correction these two sums give constant terms and thus can be cancelled by the corresponding counter term with the factor $z_A - 1$ in (95). Thus from (174) we have that the linear statistics of summation $\sum_i k_{Ei}^2$ gives the following statistical renormalized photon self-energies Π_R and Π_M (where we follow the notations in the literature of QED for photon self-energies Π_M):

$$\begin{aligned} i\Pi_R(k_E) &= ik_E^2 \Pi_M(k_E) = \\ &= ik_E^2 \frac{\alpha}{4\pi} \frac{8q^2}{30m^2} = ik_E^2 \frac{\alpha}{3\pi} \frac{q^2}{5m^2}, \end{aligned} \quad (177)$$

where we let $k_{Ei}^2 = k_E^2$ for all i .

Let us then consider the second term in the $[\cdot]$ in (165). This term can be written in the following form:

$$\begin{aligned} &\frac{\pi^{\frac{n}{2}} \Gamma(2 - \frac{n}{2}) \frac{n}{2}}{(1 - \frac{n}{2}) \Gamma(2) (m^2 - k_E^2 x(1-x))^{2-1-\frac{n}{2}}} = \\ &= \frac{\pi^{\frac{n}{2}} \Gamma(2 - \frac{n}{2}) \frac{n}{2}}{(1 - \frac{n}{2}) \Gamma(2)} [(m^2 - k_E^2 x(1-x)) + 0(\delta)] = \\ &= k_E^2 \left[\frac{1}{\delta} \cdot \frac{(-1) \pi^{\frac{n}{2}} \Gamma(2 - \frac{n}{2}) \frac{n}{2}}{(1 - \frac{n}{2}) \Gamma(2)} x(1-x) \right] + \\ &+ \left[\frac{1}{\delta} \cdot \frac{\pi^{\frac{n}{2}} \Gamma(2 - \frac{n}{2}) \frac{n}{2}}{(1 - \frac{n}{2}) \Gamma(2)} m^2 + 0(\delta) \right] \end{aligned} \quad (178)$$

Then the first term in (178) under the integration $\int_0^1 dx$ is of the form $(k_E^2 \cdot \text{constant})$. Thus this term can also be cancelled by the counter-term with the factor $z_A - 1$ in (95). In summary the renormalization constant z_A is given by the following equation:

$$\begin{aligned} (-1)^3 i(z_A - 1) &= (-i) \left\{ \frac{1}{\delta} \cdot \frac{e^2 \pi^{\frac{n}{2}}}{(2\pi)^n} \int_0^1 dx \times \right. \\ &\left. \times [(4x^2 - 4x + 1) - \frac{nx(1-x)}{2-n}] + c_A \right\}, \end{aligned} \quad (179)$$

where c_A is a finite constant when $\delta \rightarrow 0$. From this equation we have that z_A is a very large number when $\delta > 0$ is very small. Thus $e_0 = z_e (z_Z z_A^{1/2})^{-1} e = \frac{1}{n_e} e$ is a very small constant when $\delta > 0$ is very small (and since $\frac{e^2}{4\pi} = \alpha = \frac{1}{137}$ is small) where shall show that we can let $z_e = z_Z$.

Then the second term in (178) under the integration $\int_0^1 dx$ gives a parameter $\lambda_3 > 0$ for the photon self-energy since $\delta > 0$ is as a parameter.

Combing the effects of the two terms in the $[\cdot]$ in (165) we have the following renormalized one-loop photon self-energy:

$$i(\Pi_R(k_E) + \lambda_3). \quad (180)$$

Then we have the following Dyson series for photon propagator:

$$\begin{aligned} &\frac{i}{k_E^2 - \lambda_0} + \frac{i}{k_E^2 - \lambda_0} (i\Pi_R(k_E) + i\lambda_3) \frac{i}{k_E^2 - \lambda_0} + \dots = \\ &= \frac{i}{k_E^2 (1 + \Pi_M) - (\lambda_0 - \lambda_3)} =: \\ &=: \frac{i}{k_E^2 (1 + \Pi_M) - \lambda_R}, \end{aligned} \quad (181)$$

where λ_R is as a renormalized mass-energy parameter. This is as the renormalized photon propagator. We have the following approximation of this renormalized photon propagator:

$$\frac{i}{k_E^2 (1 + \Pi_M) - \lambda_R} \approx \frac{i}{k_E^2 - \lambda_R} (1 - \Pi_M). \quad (182)$$

21 Computation of the Lamb shift: Part II

Combining the effect of vertex correction and photon self-energy we can now compute the Lamb shift. Combining the effect of photon self-energy $(-ie\gamma^\mu)[- \Pi_M]$ and vertex correction we have:

$$\begin{aligned} &(-ie)\Lambda_R(p', p) + (-ie\gamma^\mu)[- \Pi_M] = \\ &= (-ie) \left[\gamma^\mu \frac{\alpha q^2}{3\pi m^2} \left(7 + \frac{5}{6} - \frac{3}{8} - \frac{1}{5} \right) + \frac{i\alpha}{4\pi m} \sigma^{\mu\nu} q_\nu \right]. \end{aligned} \quad (183)$$

As in the literature of QED let us consider the states $2S_{\frac{1}{2}}$ and the $2P_{\frac{1}{2}}$ in the hydrogen atom [6, 72–78]. Following the literature of QED for the state $2S_{\frac{1}{2}}$ an effect of $\frac{\alpha q^2}{3\pi m^2} (\frac{3}{8})$ comes from the anomalous magnetic moment which cancels the same term with negative sign in (183). Thus by using the method in the computation of the Lamb shift in the literature of QED we have the following second order shift for the state $2S_{\frac{1}{2}}$:

$$\Delta E_{2S_{\frac{1}{2}}} = \frac{m\alpha^5}{6\pi} \left(7 + \frac{5}{6} - \frac{1}{5} \right). \quad (184)$$

Similarly by the method of computing the Lamb shift in the literature of QED from the anomalous magnetic moment we have the following second order shift for the state $2P_{\frac{1}{2}}$:

$$\Delta E_{2P_{\frac{1}{2}}} = \frac{m\alpha^5}{6\pi} \left(-\frac{1}{8} \right). \quad (185)$$

Thus the second order Lamb shift for the states $2S_{\frac{1}{2}}$ and $2P_{\frac{1}{2}}$ is given by:

$$\Delta E = \Delta E_{2S_{\frac{1}{2}}} - \Delta E_{2P_{\frac{1}{2}}} = \frac{m\alpha^5}{6\pi} \left(7 + \frac{5}{6} - \frac{1}{5} + \frac{1}{8} \right) \quad (186)$$

or in terms of frequencies for each of the terms in (186) we have:

$$\begin{aligned} \Delta\nu &= 952 + 113.03 - 27.13 + 16.96 = \\ &= 1054.86 \text{ Mc/sec.} \end{aligned} \quad (187)$$

This agrees with the experimental results [6, 72–78]:

$$\begin{aligned} \Delta\nu^{\text{exp}} &= 1057.86 \pm 0.06 \text{ Mc/sec} \\ &\text{and} = 1057.90 \pm 0.06 \text{ Mc/sec.} \end{aligned} \quad (188)$$

22 Computation of the electron self-energy

Let us then consider the one-loop electron self-energy (113). As a statistics we extend the one dimensional integral $\int dk_E$ to the n -dimensional integral $\int d^n k$ ($n \rightarrow 4$) where $k = (k_E, \mathbf{k})$. This is similar to the dimensional regularization in the existing quantum field theories (However here our aim is to increase the dimension for statistics which is different from the dimensional regularization which is to reduce the dimension from 4 to n to avoid the ultraviolet divergence). With this statistics the factor 2π is replaced by the statistical factor $(2\pi)^n$. From this statistics on (114) we have that the following statistical one-loop electron self-energy $-i\Sigma(p_E)$:

$$-i\Sigma(p_E) := i^2(-i)^2 \frac{e^2}{(2\pi)^n} \int_0^1 dx \int d^n k \times \quad (189)$$

$$\times \frac{(k_E^2 - 4p_E k_E + 4p_E^2) d^n k}{(k^2 - 2k p_E + p_E^2 x - x m^2 - (1-x)\lambda^2)^2},$$

where $k^2 = k_E^2 - \mathbf{k}^2$, and \mathbf{k}^2 is from $\omega^2 = m^2 + \mathbf{k}^2$ and $\lambda_0^2 = \lambda^2 + \mathbf{k}^2$; and $k p := k_E p_E - \mathbf{k} \cdot \mathbf{0} = k_E p_E$. By using the formulae for computing Feynman integrals we have that (189) is equal to:

$$\begin{aligned} & \frac{i e^2}{(2\pi)^n} \int_0^1 dx \left[\frac{p_E^2 (x^2 - 4x + 4) \pi^{\frac{n}{2}} \Gamma(2 - \frac{n}{2})}{\Gamma(2) (x m^2 + (1-x)\lambda^2 - p_E^2 x(1-x))^{2 - \frac{n}{2}}} + \right. \\ & \left. + \frac{\pi^{\frac{n}{2}} \Gamma(2 - 1 - \frac{n}{2}) \frac{n}{2}}{\Gamma(2) (x m^2 + (1-x)\lambda^2 - p_E^2 x(1-x))^{2 - 1 - \frac{n}{2}}} \right] = \\ & = \frac{i e^2}{(2\pi)^n} \int_0^1 dx \left\{ p_E^2 (x^2 - 4x + 4) \pi^{\frac{n}{2}} \times \right. \\ & \times \left[\left(\frac{1}{\delta} + O(\delta) \right) \cdot e^{-\delta \log(x m^2 + (1-x)\lambda^2 - p_E^2 x(1-x))} \right] - \\ & \left. - \pi^{\frac{n}{2}} \frac{1}{\delta} \frac{1}{\delta} [x m^2 + (1-x)\lambda^2 - p_E^2 x(1-x) + 0(\delta)] \right\} = \\ & = \frac{i e^2}{(2\pi)^n} \int_0^1 dx \left\{ p_E^2 (x^2 - 4x + 4) \pi^{\frac{n}{2}} \left[\frac{1}{\delta} - \frac{1}{\delta} \times \right. \right. \\ & \times \delta \log(x m^2 + (1-x)\lambda^2 - p_E^2 x(1-x)) + 0(\delta) \left. \right] - \\ & \left. - \pi^{\frac{n}{2}} \frac{1}{\delta} \frac{1}{\delta} [x m^2 + (1-x)\lambda^2 - p_E^2 x(1-x) + 0(\delta)] \right\} = \\ & = \frac{i e^2}{(2\pi)^n} \int_0^1 dx \left\{ p_E^2 (x^2 - 4x + 4) \pi^{\frac{n}{2}} \times \right. \\ & \times \left[\frac{1}{\delta} - \log(x m^2 + (1-x)\lambda^2 - p_E^2 x(1-x)) + 0(\delta) \right] - \\ & \left. - \pi^{\frac{n}{2}} \frac{1}{\delta} \frac{1}{\delta} [x m^2 + (1-x)\lambda^2 - p_E^2 x(1-x) + 0(\delta)] \right\} = \\ & = \frac{i e^2}{(2\pi)^n} \int_0^1 dx \left\{ p_E^2 (x^2 - 4x + 4) \pi^{\frac{n}{2}} \times \right. \\ & \times \left[\frac{1}{\delta} - \log(x m^2 + (1-x)\lambda^2) - \right. \\ & \left. - \log\left(1 - \frac{p_E^2 x(1-x)}{x m^2 + (1-x)\lambda^2}\right) + 0(\delta) \right] - \\ & \left. - \pi^{\frac{n}{2}} \frac{1}{\delta} \frac{1}{\delta} [x m^2 + (1-x)\lambda^2 - p_E^2 x(1-x) + 0(\delta)] \right\} =: \end{aligned}$$

$$=: \frac{i e^2}{(2\pi)^n} \int_0^1 dx \left\{ p_E^2 (x^2 - 4x + 4) \pi^{\frac{n}{2}} \left[\frac{1}{\delta} - \right. \right.$$

$$\left. - \log(x m^2 + (1-x)\lambda^2) - \log\left(1 - \frac{p_E^2 x(1-x)}{x m^2 + (1-x)\lambda^2}\right) + \right.$$

$$\left. + 0(\delta) \right\} + p_E^2 \cdot \frac{1}{\delta} \pi^{\frac{n}{2}} \frac{n}{2} x(1-x) \left. \right\} + i\omega_3, \quad (190)$$

where $\omega_3 > 0$ is a mass-energy parameter.

Then we notice that from the expressions for $\Sigma_0(p_E)$ and $\Lambda_0(p_E, q_E)$ in (114) and (115) we have the following identity:

$$\frac{\partial}{\partial p_E} \Sigma_0(p_E) = -\Lambda_0(p_E, p_E) + \quad (191)$$

$$+ \frac{i 4 e^2}{2\pi} \int dk \frac{-k_E + 2p_E}{(k_E^2 - \lambda_0^2)((p_E - k_E)^2 - \omega^2)}.$$

This is as a Ward-Takahashi identity which is analogous to the corresponding Ward-Takahashi identity in the conventional QED theory [6].

From (114) and (115) we get their statistical forms by changing $\int dk$ to $\int d^n k$. From this summation form of statistics and the identity (191) we then get the following statistical Ward-Takahashi identity:

$$\frac{\partial}{\partial p_E} \Sigma(p_E) = -\Lambda(p_E, p_E) + \quad (192)$$

$$+ \frac{i 4 e^2}{(2\pi)^n} \int_0^1 dx \int d^n k \frac{-k_E + 2p_E}{(k^2 - 2k p_E + p_E^2 x - x m^2 - (1-x)\lambda^2)^2},$$

where $\Sigma(p_E)$ denotes the statistical form of $\Sigma_0(p_E)$ and is given by (189) and $\Lambda(p_E, q_E)$ denotes the statistical form of $\Lambda_0(p_E, q_E)$ as in the above Sections.

After the differentiation of (190) with respect to p_E the remaining factor p_E of the factor p_E^2 of (190) is absorbed to the external spinors as the mass m and a factor $\frac{\gamma^\mu}{2}$ is introduced by space-time statistics, as the case of the statistics of the vertex correction $\Lambda_0(p_E, q_E)$ in the above Sections. From the absorbing of a factor p_E to the external spinors for both sides of this statistical Ward-Takahashi identity we then get a statistical Ward-Takahashi identity where the Taylor expansion (of the variable p_E) of both sides of this statistical Ward-Takahashi identity are with constant term as the beginning term. From this Ward-Takahashi identity we have that these two constant terms must be the same constant. Then the constant term, denoted by $C(\delta)$, of the vertex correction of this Ward-Takahashi identity is cancelled by the counterterm with the factor $z_e - 1$ in (95), as done in the above computation of the renormalized vertex correction $A_R(p', p)$. (At this point we notice that in computing the constant term of the vertex correction some terms with the factor p_E has been changed to constant terms under the on-mass-shell condition $p_E = m$. This then modifies the definition of $C(\delta)$).

On the other hand let us denote the constant term for the electron self-energy by $B(\delta)$. Then from the above statistical Ward-Takahashi identity we have the following equality:

$$B(\delta) + a_1 \cdot \frac{1}{\delta} + b_1 = C(\delta), \quad (193)$$

where a_1, b_1 are finite constants when $\delta \rightarrow 0$ and the term $a_1 \cdot \frac{1}{\delta}$ is from the second term in the right hand side of (192).

Let us then compute the constant term $B(\delta)$ for the electron self-energy, as follows. As explained in the above the constant term for the electron self-energy can be obtained by differentiation of (190) with respect to p_E and the removing of the remaining factor p_E of p_E^2 . We have:

$$\begin{aligned} & \frac{\partial}{\partial p_E} \left\{ \frac{ie^2}{(2\pi)^n} \int_0^1 dx p_E^2 (x^2 - 4x + 4) \pi^{\frac{n}{2}} \times \right. \\ & \times \left[\frac{1}{\delta} - \log(xm^2 + (1-x)\lambda^2 - p_E^2 x(1-x)) \right] + \\ & \left. + p_E^2 \cdot \frac{1}{\delta} \pi^{\frac{n}{2}} \frac{n}{2} \frac{ie^2}{(2\pi)^n} \int_0^1 x(1-x) dx + i\omega_3 \right\} = \\ & = \frac{ie^2}{(2\pi)^n} \int_0^1 dx 2p_E (x^2 - 4x + 4) \pi^{\frac{n}{2}} \times \\ & \times \left[\frac{1}{\delta} - \log(xm^2 + (1-x)\lambda^2 - p_E^2 x(1-x)) \right] + \\ & + \frac{ie^2}{(2\pi)^n} \int_0^1 dx \frac{p_E^2 (x^2 - 4x + 4) \pi^{\frac{n}{2}} \cdot 2p_E x(1-x)}{xm^2 + (1-x)\lambda^2 - p_E^2 x(1-x)} + \\ & + 2p_E \cdot \frac{1}{\delta} \pi^{\frac{n}{2}} \frac{n}{2} \frac{ie^2}{(2\pi)^n} \int_0^1 x(1-x) dx. \end{aligned} \quad (194)$$

Then by Taylor expansion of (194) and by removing a factor $2p_E$ from (194) the constant term for the electron self-energy is given by:

$$\begin{aligned} B(\delta) & := \frac{-e^2}{(2\pi)^n} \int_0^1 dx (x^2 - 4x + 4) \pi^{\frac{n}{2}} \times \\ & \times \left[\frac{1}{\delta} - \log(xm^2 + (1-x)\lambda^2) \right] - \\ & - \frac{1}{\delta} \pi^{\frac{n}{2}} \frac{n}{2} \frac{e^2}{(2\pi)^n} \int_0^1 x(1-x) dx. \end{aligned} \quad (195)$$

Then as a renormalization procedure for the electron self-energy we choose a $\delta_1 > 0$ which is related to the δ for the renormalization of the vertex correction such that:

$$B(\delta_1) = B(\delta) + a_1 \cdot \frac{1}{\delta} + b_1. \quad (196)$$

This is possible since $B(\delta)$ has a term proportional to $\frac{1}{\delta}$. From this renormalization procedure for the electron self-energy we have:

$$B(\delta_1) = C(\delta). \quad (197)$$

This constant term $B(\delta_1)$ for the electron self-energy is to be cancelled by the counter-term with the factor $z_Z - 1$ in (95). We have the following equation to determine the renormalization constant z_Z for this cancellation:

$$(-1)^3 i (z_Z - 1) = (-i) B(\delta_1). \quad (198)$$

Then from the equality (197) we have $z_e = z_Z$ where z_e is determined by the following equation:

$$(-1)^3 i (z_e - 1) = (-i) C(\delta). \quad (199)$$

Cancelling $B(\delta_1)$ from the electron self-energy (190) we

get the following renormalized one-loop electron self-energy:

$$\begin{aligned} & -ip_E^2 \Sigma_R(p_E) + i\omega_3^2 := -ip_E^2 \frac{\alpha}{4\pi} \times \\ & \times \int_0^1 dx (x^2 - 4x + 4) \log \left[1 - \frac{p_E^2 x(1-x)}{xm^2 + (1-x)\lambda^2} \right] + i\omega_3^2. \end{aligned} \quad (200)$$

We notice that in (200) we can let $\lambda = 0$ since there is no infrared divergence when $\lambda = 0$. This is better than the computed electron self-energy in the conventional QED theory where the computed one-loop electron self-energy is with infrared divergence when $\lambda = 0$ [6].

From this renormalized electron self-energy we then have the renormalized electron propagator obtained by the following Dyson series:

$$\begin{aligned} & \frac{i}{p_E^2 - \omega^2} + \frac{i}{p_E^2 - \omega^2} (-ip_E^2 \Sigma_R(p_E) + i\omega_3^2) \frac{i}{p_E^2 - \omega^2} + \dots = \\ & = \frac{i}{p_E^2 (1 - \Sigma_R(p_E)) - (\omega^2 - \omega_3^2)} =: \\ & =: \frac{i}{p_E^2 (1 - \Sigma_R(p_E)) - \omega_R^2}, \end{aligned} \quad (201)$$

where $\omega_R^2 := \omega^2 - \omega_3^2$ is as a renormalized electron mass-energy parameter. Then by space-time statistics from the renormalized electron propagator (201) we can get the renormalized electron propagator in the spin- $\frac{1}{2}$ form, as that the electron propagator $\frac{i}{\gamma_\mu p^\mu - m}$ in the spin- $\frac{1}{2}$ form can be obtained from the electron propagator $\frac{i}{p_E^2 - \omega^2}$.

23 New effect of QED

Let us consider a new effect for electron scattering which is formed by two seagull vertexes with one photon loop and four electron lines. This is a new effect of QED because the conventional spin $\frac{1}{2}$ theory of QED does not have this seagull vertex. The Feynman integral corresponding to the photon loop is given by

$$\begin{aligned} & \frac{i^2 (i)^2 e^4}{2\pi} \int \frac{dk_E}{(k_E^2 - \lambda_0^2) ((p_E - q_E - k_E)^2 - \lambda_0^2)} = \\ & = \frac{e^4}{2\pi} \int_0^1 \int \frac{dk_E}{(k_E^2 - 2k_E(p_E - q_E)x + (p_E - q_E)^2 x - \lambda_0^2)^2} = \\ & = \frac{e^4}{2\pi} \int_0^1 \int \frac{dk_E}{(k_E^2 - 2k_E(p_E - q_E)x + (p_E - q_E)^2 x - \lambda_0^2)^2}. \end{aligned} \quad (202)$$

Let us then introduce a space-time statistics. Since the photon propagator of the (two joined) seagull vertex interactions is of the form of a circle on a plane we have that the appropriate space-time statistics of the photons is with the two dimensional space for the circle of the photon propagator. From this two dimensional space statistics we then get a three dimensional space statistics by multiplying the statistical factor $\frac{1}{(2\pi)^3}$ of the three dimensional space statistics and by concentrating in a two dimensional subspace of the three dimensional space statistics.

Thus as similar to the four dimensional space-time statistics with the three dimensional space statistics in the above

Sections from (202) we have the following space-time statistics with the two dimensional subspace:

$$\begin{aligned} & \frac{e^4}{(2\pi)^4} \int_0^1 \int \frac{d^3 k}{(k_E^2 - 2k_E(p_E - q_E)x + (p_E - q_E)^2 x - k^2 - \lambda_4^2)^2} = \\ & = \frac{e^4}{(2\pi)^4} \int_0^1 dx \int \frac{d^3 k}{(k^2 - 2k \cdot (p_E - q_E, 0)x + (p_E - q_E)^2 x - \lambda_4^2)^2}, \end{aligned} \quad (203)$$

where the statistical factor $\frac{1}{(2\pi)^3}$ of three dimensional space has been introduced to give the factor $\frac{1}{(2\pi)^4}$ of the four dimensional space-time statistics; and we let $k = (k_E, \mathbf{k})$, $k^2 = k_E^2 - \mathbf{k}^2$ and since the photon energy parameter λ_0 is a free parameter we can write $\lambda_0^2 = \mathbf{k}^2 + \lambda_4^2$ for some λ_4 .

Then a delta function concentrating at 0 of a one dimensional momentum variable is multiplied to the integrand in (203) and the three dimensional energy-momentum integral in (203) is changed to a four dimensional energy-momentum integral by taking the corresponding one more momentum integral.

From this we get a four dimensional space-time statistics with the usual four dimensional momentum integral and with the statistical factor $\frac{1}{(2\pi)^4}$. After this additional momentum integral we then get (203) as a four dimensional space-time statistics with the two dimensional momentum variable.

Then to get a four dimensional space-time statistics with the three dimensional momentum variable a delta function concentrating at 0 of another one dimensional momentum variable is multiplied to (203) and the two dimensional momentum variable of (203) is extended to the corresponding three dimensional momentum variable. From this we then get a four dimensional space-time statistics with the three dimensional momentum variable.

Then we have that (203) is equal to:

$$\frac{e^4}{(2\pi)^4} \frac{i\pi^{\frac{3}{2}} \Gamma(2 - \frac{3}{2})}{\Gamma(2)} \int_0^1 \frac{dx}{((p_E - q_E)^2 x(1-x) - \lambda_4^2)^{\frac{1}{2}}}. \quad (204)$$

Then since the photon mass-energy parameter λ_4 is a free parameter for space-time statistics we can write λ_4 in the following form:

$$\lambda_4^2 = (\mathbf{p} - \mathbf{q})^2 x(1-x), \quad (205)$$

where $\mathbf{p} - \mathbf{q}$ denotes a two dimensional momentum vector.

Then we let $p - q = (p_E - q_E, \mathbf{p} - \mathbf{q})$. Then we have:

$$\begin{aligned} & (p_E - q_E)^2 x(1-x) - \lambda_4^2 = \\ & = (p_E - q_E)^2 x(1-x) - (\mathbf{p} - \mathbf{q})^2 x(1-x) = \\ & = (p - q)^2 x(1-x). \end{aligned} \quad (206)$$

Then we have that (204) is equal to:

$$\begin{aligned} & \frac{e^4}{(2\pi)^4} \frac{i\pi^{\frac{3}{2}} \Gamma(2 - \frac{3}{2})}{\Gamma(2)} \int_0^1 \frac{dx}{((p-q)^2 x(1-x))^{\frac{1}{2}}} \\ & = \frac{e^4}{(2\pi)^4} \frac{i\pi^{\frac{3}{2}} \Gamma(2 - \frac{3}{2})}{\Gamma(2)} \frac{1}{((p-q)^2)^{\frac{1}{2}}} = \\ & = \frac{e^4 i}{16\pi((p-q)^2)^{\frac{1}{2}}} = \frac{e^2 \alpha i}{4((p-q)^2)^{\frac{1}{2}}}. \end{aligned} \quad (207)$$

Thus we have the following potential:

$$V_{seagull}(p - q) = \frac{e^2 \alpha i}{4((p - q)^2)^{\frac{1}{2}}}. \quad (208)$$

This potential (208) is as the seagull vertex potential.

We notice that (208) is a new effect for electron-electron or electron-positron scattering. Recent experiments on the decay of positronium show that the experimental orthopositronium decay rate is significantly larger than that computed from the conventional QED theory [33–52]. In the following Section 24 to Section 26 we show that this discrepancy can be remedied with this new effect (208).

24 Reformulating the Bethe-Salpeter equation

To compute the orthopositronium decay rate let us first find out the ground state wave function of the positronium. To this end we shall use the Bethe-Salpeter equation. It is well known that the conventional Bethe-Salpeter equation is with difficulties such as the relative time and relative energy problem which leads to the existence of nonphysical solutions in the conventional Bethe-Salpeter equation [7–32]. From the above QED theory let us reformulate the Bethe-Salpeter equation to get a new form of the Bethe-Salpeter equation. We shall see that this new form of the Bethe-Salpeter equation resolves the basic difficulties of the Bethe-Salpeter equation such as the relative time and relative energy problem.

Let us first consider the propagator of electron. Since electron is a spin- $\frac{1}{2}$ particle its statistical propagator is of the form $\frac{i}{\gamma_\mu p^\mu - m}$. Thus before the space-time statistics the spin- $\frac{1}{2}$ form of electron propagator is of the form $\frac{i}{p_E - \omega}$ which can be obtained from the electron propagator $\frac{i}{p_E^2 - \omega^2}$ by the factorization: $p_E^2 - \omega^2 = (p_E - \omega)(p_E + \omega)$. Then we consider the following product which is from two propagators of two spin- $\frac{1}{2}$ particles:

$$\begin{aligned} & [p_{E1} - \omega_1][p_{E2} - \omega_2] = \\ & = p_{E1}p_{E2} - \omega_1p_{E2} - \omega_2p_{E1} + \omega_1\omega_2 =: \\ & =: p_E^2 - \omega_b^2, \end{aligned} \quad (209)$$

where we define $p_E^2 = p_{E1}p_{E2}$ and $\omega_b^2 := \omega_1p_{E2} + \omega_2p_{E1} - \omega_1\omega_2$. Then since ω_1 and ω_2 are free mass-energy parameters we have that ω_b is also a free mass-energy parameter with the requirement that it is to be a positive parameter.

Then we introduce the following reformulated relativistic equation of Bethe-Salpeter type for two particles with spin- $\frac{1}{2}$:

$$\begin{aligned} \phi_0(p_E, \omega_b) & = \frac{i^2 \lambda'}{[p_{E1} - \omega_1][p_{E2} - \omega_2]} \times \\ & \times \int \frac{ie^2 \phi_0(q_E, \omega_b) dq_E}{((p_E - q_E)^2 - \lambda_0^2)}, \end{aligned} \quad (210)$$

where we use the photon propagator $\frac{i}{k_E^2 - \lambda_0^2}$ (which is of the effect of Coulomb potential) for the interaction of these two

particles and we write the proper energy k_E^2 of this potential in the form $k_E^2 = (p_E - q_E)^2$; and λ' is as the coupling parameter. We shall later also introduce the seagull vertex term for the potential of binding.

Let us then introduce the space-time statistics. Since we have the seagull vertex term for the potential of binding which is of the form of a circle in a two dimensional space from the above Section on the seagull vertex potential we see that the appropriate space-time statistics is with the two dimensional space. Thus with this space-time statistics from (210) we have the following reformulated relativistic Bethe-Salpeter equation:

$$\phi_0(p) = \frac{-\lambda'}{p^2 - \gamma_0^2} \int \frac{id^3q}{(p-q)^2} \phi_0(q), \quad (211)$$

where we let the free parameters ω_b and λ_0 be such that $p^2 = p_E^2 - \mathbf{p}^2$ with $\omega_b^2 = \mathbf{p}^2 + \gamma_0^2$ for some constant $\gamma_0^2 = \frac{1}{a^2} > 0$ where a is as the radius of the binding system; and $(p-q)^2 = (p_E - q_E)^2 - (\mathbf{p} - \mathbf{q})^2$ with $\lambda_0^2 = (\mathbf{p} - \mathbf{q})^2$. We notice that the potential $\frac{i\alpha}{(p-q)^2}$ of binding is now of the usual (relativistic) Coulomb potential type. In (211) the constant e^2 in (210) has been absorbed into the parameter λ' in (211).

We see that in this reformulated Bethe-Salpeter equation the relative time and relative energy problem of the conventional Bethe-Salpeter equations is resolved [7–32]. Thus this reformulated Bethe-Salpeter equation will be free of abnormal solutions.

Let us then solve (211) for the relativistic bound states of particles. We show that the ground state solution $\phi_0(p)$ can be exactly solved and is of the following form:

$$\phi_0(p) = \frac{1}{(p^2 - \gamma_0^2)^2}. \quad (212)$$

We have:

$$\begin{aligned} & \frac{1}{(p-q)^2} \frac{1}{(q^2 - \gamma_0^2)^2} = \\ & = \frac{(2+1-1)!}{(2-1)!(1-1)!} \int_0^1 \frac{(1-x)dx}{[x(p-q)^2 + (1-x)(q^2 - \gamma_0^2)]^3} = \\ & = \frac{(2+1-1)!}{(2-1)!(1-1)!} \int_0^1 \frac{(1-x)dx}{[q^2 + 2xpq + xp^2 - (1-x)\gamma_0^2]^3} = \\ & = 2 \int_0^1 \frac{(1-x)dx}{[q^2 + 2xpq + xp^2 - (1-x)\gamma_0^2]^3}. \end{aligned} \quad (213)$$

Thus we have:

$$\begin{aligned} & i \int \frac{d^3q}{((p-q)^2)(q^2 - \gamma_0^2)^2} = \\ & = i2 \int_0^1 (1-x)dx \int \frac{d^3q}{[q^2 + 2xpq + xp^2 - (1-x)\gamma_0^2]^3} = \\ & = i2 \frac{2\pi^{\frac{3}{2}} \Gamma(3 - \frac{3}{2})}{\Gamma(3)} \int_0^1 \frac{(1-x)dx}{[+x(1-x)p^2 - (1-x)\gamma_0^2]^{\frac{3}{2}}} = \\ & = -\frac{2\pi^{\frac{3}{2}} \Gamma(3 - \frac{3}{2})}{\Gamma(3)} \int_0^1 \frac{dx}{[+xp^2 - \gamma_0^2][(1-x)(xp^2 - \gamma_0^2)]^{\frac{3}{2}}} = \end{aligned}$$

$$\begin{aligned} & = -\frac{2\pi^{\frac{3}{2}} \Gamma(3 - \frac{3}{2})}{\Gamma(3)} \frac{\partial^2}{\partial(\gamma_0^2)^2} \int_0^1 dx \left[\frac{xp^2 - \gamma_0^2}{1-x} \right]^{\frac{3}{2}} = \\ & = -\frac{2\pi^{\frac{3}{2}} \Gamma(3 - \frac{3}{2})}{\Gamma(3)} \frac{\partial^2}{\partial(\gamma_0^2)^2} \int_0^1 dx \left[\frac{p^2 - \gamma_0^2}{1-x} - p^2 \right]^{\frac{3}{2}} = \\ & = -\frac{2\pi^{\frac{3}{2}} \Gamma(3 - \frac{3}{2})}{\Gamma(3)} \frac{\partial^2}{\partial(\gamma_0^2)^2} \int_1^\infty \frac{dt}{t^2} [(p^2 - \gamma_0^2)t - p^2]^{\frac{3}{2}} = \\ & = -\frac{2\pi^{\frac{3}{2}} \Gamma(3 - \frac{3}{2})}{\Gamma(3)} \int_1^\infty \frac{dt}{t^2} [(p^2 - \gamma_0^2)t - p^2]^{\frac{-3}{2}} = \\ & = -\frac{2\pi^{\frac{3}{2}} \Gamma(3 - \frac{3}{2})}{\Gamma(3)} \frac{1}{(p^2 - \gamma_0^2)} \int_{\gamma_0^2}^\infty x^{-\frac{3}{2}} dx = \\ & = -\frac{\pi^2}{2} \frac{1}{\gamma_0(p^2 - \gamma_0^2)}. \end{aligned} \quad (214)$$

Then let us choose λ' such that $\lambda' = \frac{2\gamma_0}{\pi^2}$. From this value of λ' we see that the BS equation (211) holds. Thus the ground state solution is of the form (212). We see that when $p_E = 0$ and $\omega_b^2 = \mathbf{p}^2 + \gamma_0^2$ then this ground state gives the well known nonrelativistic ground state of the form $\frac{1}{(\mathbf{p}^2 + \gamma_0^2)^2}$ of binding system such as the hydrogen atom.

25 Bethe-Salpeter equation with seagull vertex potential

Let us then introduce the following reformulated relativistic Bethe-Salpeter equation which is also with the seagull vertex potential of binding:

$$\begin{aligned} \phi(p) &= \frac{-\lambda'}{p^2 - \gamma_0^2} \times \\ & \times \int \left[\frac{i}{(p-q)^2} + \frac{i\alpha}{4((p-q)^2)^{\frac{1}{2}}} \right] \phi(q) d^3q, \end{aligned} \quad (215)$$

where a factor e^2 of both the Coulomb-type potential and the seagull vertex potential is absorbed to the coupling constant λ' .

Let us solve (215) for the relativistic bound states of particles. We write the ground state solution in the following form:

$$\phi(p) = \phi_0(p) + \alpha\phi_1(p), \quad (216)$$

where $\phi_0(p)$ is the ground state of the BS equation when the interaction potential only consists of the Coulomb-type potential. Let us then determine the $\phi_1(p)$.

From (215) by comparing the coefficients of the α^j , $j = 0, 1$ on both sides of BS equation we have the following equation for $\phi_1(p)$:

$$\begin{aligned} \phi_1(p) &= \frac{-\lambda'}{p^2 - \gamma_0^2} \int \left[\frac{i}{4((p-q)^2)^{\frac{1}{2}}} \right] \phi_0(q) d^3q + \\ & + \frac{-\lambda'}{p^2 - \gamma_0^2} \int \left[\frac{i}{((p-q)^2)} + \frac{i\alpha}{4((p-q)^2)^{\frac{1}{2}}} \right] \phi_1(q) d^3q. \end{aligned} \quad (217)$$

This is a nonhomogeneous linear Fredholm integral equation. We can find its solution by perturbation. As a first order approximation we have the following approximation of $\phi_1(p)$:

$$\begin{aligned}
 \phi_1(p) &\approx \frac{-\lambda'}{p^2-\gamma_0^2} \int \frac{i}{4((p-q)^2)^{\frac{1}{2}}} \phi_0(q) d^3q = \\
 &= \frac{-\lambda'}{p^2-\gamma_0^2} \int \frac{i}{4((p-q)^2)^{\frac{1}{2}} (q^2-\gamma_0^2)^2} d^3q = \\
 &= \frac{-\lambda'}{p^2-\gamma_0^2} \frac{i\Gamma(1+\frac{1}{2}+2-1)}{4\Gamma(1+\frac{1}{2}-1)\Gamma(1+2-1)} \int_0^1 y^{\frac{1}{2}}(1-y)dy \times \\
 &\times \int \frac{d^3q}{[q^2-2qp_y+p^2y-(1-y)\gamma_0^2]^2+\frac{1}{2}} = \\
 &= \frac{-\lambda'}{p^2-\gamma_0^2} \frac{i\Gamma(\frac{1}{2}+2)}{4\Gamma(\frac{1}{2})\Gamma(2)} \int_0^1 \frac{i\pi^{\frac{3}{2}}\Gamma(\frac{5}{2}-\frac{3}{2})y^{\frac{1}{2}}(1-y)dy}{\Gamma(\frac{5}{2})(p^2y(1-y)-(1-y)\gamma_0^2)} = \quad (218) \\
 &= \frac{\lambda'}{p^2-\gamma_0^2} \frac{\pi^{\frac{3}{2}}}{4\Gamma(\frac{1}{2})} \int_0^1 y^{\frac{1}{2}}dy \frac{1}{(p^2y-\gamma_0^2)} = \\
 &= \frac{\lambda'\pi}{p^2-\gamma_0^2} \frac{1}{4|p|\gamma_0} \log\left|\frac{|p|-\gamma_0}{|p|+\gamma_0}\right| = \\
 &= \frac{\pi}{p^2-\gamma_0^2} \frac{2\gamma_0}{\pi^2} \frac{1}{4|p|\gamma_0} \log\left|\frac{|p|-\gamma_0}{|p|+\gamma_0}\right| = \\
 &= \frac{1}{2\pi(p^2-\gamma_0^2)|p|} \log\left|\frac{|p|-\gamma_0}{|p|+\gamma_0}\right|,
 \end{aligned}$$

where $|p| = \sqrt{p^2}$.

Thus we have the ground state $\phi(p) = \phi_0(p) + \alpha\phi_1(p)$ where p denotes an energy-momentum vector with a two dimensional momentum. Thus this ground state is for a two dimensional (momentum) subspace. We may extend it to the ground state of the form $\phi(p) = \phi_0(p) + \alpha\phi_1(p)$ where p denotes a four dimensional energy-momentum vector with a three dimensional momentum; and due to the special nature that $\phi_1(p)$ is obtained by a two dimensional space statistics the extension $\phi_1(p)$ of $\phi_1(p)$ to with a three dimensional momentum is a wave function obtained by multiplying $\phi_1(p)$ with a delta function concentrating at 0 of a one dimensional momentum variable and the variable p of $\phi_1(p)$ is extended to be a four dimensional energy-momentum vector with a three dimensional momentum.

Let us use this form of the ground state $\phi(p) = \phi_0(p) + \alpha\bar{\phi}_1(p)$ to compute new QED effects in the orthopositronium decay rate where there is a discrepancy between theoretical result and the experimental result [33–52].

26 New QED effect of orthopositronium decay rate

From the seagull vertex let us find new QED effect to the orthopositronium decay rate where there is a discrepancy between theory and experimental result [33–52]. Let us compute the new one-loop effect of orthopositronium decay rate which is from the seagull vertex potential.

From the seagull vertex potential the positronium ground state is modified from $\phi(p)=\phi_0(p)$ to $\phi(p)=\phi_0(p)+\alpha\bar{\phi}_1(p)$. Let us apply this form of the ground state of positronium to the computation of the orthopositronium decay rate.

Let us consider the nonrelativistic case. In this case we

have $\phi_0(\mathbf{p}) = \frac{1}{(p^2+\gamma_0^2)^2}$ and:

$$\phi_1(\mathbf{p}) = \frac{-1}{2\pi(p^2+\gamma_0^2)|\mathbf{p}|} \log\left|\frac{|\mathbf{p}|-\gamma_0}{|\mathbf{p}|+\gamma_0}\right|. \quad (219)$$

Let M denotes the decay amplitude. Let M_0 denotes the zero-loop decay amplitude. Then following the approach in the computation of the positronium decay rate [33–52] the first order decay rate Γ is given by:

$$\begin{aligned}
 \int 8\pi^{\frac{1}{2}}\gamma_0^{\frac{5}{2}}[\phi_0(\mathbf{p}) + \alpha\bar{\phi}_1(\mathbf{p})]M_0(\mathbf{p})d^3\mathbf{p} &=: \\
 &=: \Gamma_0 + \alpha\Gamma_{seagull}, \quad (220)
 \end{aligned}$$

where $8\pi^{\frac{1}{2}}\gamma_0^{\frac{5}{2}}$ is the normalized constant for the usual unnormalized ground state wave function ϕ_0 [33–52].

We have that the first order decay rate Γ_0 is given by [33–52]:

$$\begin{aligned}
 \Gamma_0 &:= \frac{1}{(2\pi)^3} \int 8\pi^{\frac{1}{2}}\gamma_0^{\frac{5}{2}}\phi_0(\mathbf{p})M_0(\mathbf{p})d^3\mathbf{p} = \\
 &= \frac{1}{(2\pi)^3} \int \frac{8\pi^{\frac{1}{2}}\gamma_0^{\frac{5}{2}}}{(p^2+\gamma_0^2)^2} M_0(\mathbf{p})d^3\mathbf{p} \approx \\
 &\approx \psi_0(\mathbf{r}=0)M_0(0) = \quad (221) \\
 &= \frac{8\pi^{\frac{1}{2}}\gamma_0^{\frac{5}{2}}}{(2\pi)^3} \int \frac{d^3\mathbf{p}}{(p^2+\gamma_0^2)^2} M_0(0) = \\
 &= \frac{8\pi^{\frac{1}{2}}\gamma_0^{\frac{5}{2}}}{(2\pi)^3} \frac{\pi^2}{\gamma_0} M_0(0) = \\
 &= \frac{1}{(\pi a^3)^{\frac{1}{2}}} M_0(0),
 \end{aligned}$$

where $\psi_0(\mathbf{r})$ denotes the usual nonrelativistic ground state wave function of positronium; and $a = \frac{1}{\gamma_0}$ is as the radius of the positronium. In the above equation the step \approx holds since $\phi_0(\mathbf{p}) \rightarrow 0$ rapidly as $\mathbf{p} \rightarrow \infty$ such that the effect of $M_0(\mathbf{p})$ is small for $\mathbf{p} \neq 0$; as explained in [33]- [52].

Then let us consider the new QED effect of decay rate from $\bar{\phi}_1(\mathbf{p})$. As the three dimensional space statistics in the Section on the seagull vertex potential we have the following statistics of the decay rate from $\bar{\phi}_1(\mathbf{p})$:

$$\begin{aligned}
 \Gamma_{seagull} &= \frac{1}{(2\pi)^3} \int 8\pi^{\frac{1}{2}}\gamma_0^{\frac{5}{2}}\bar{\phi}_1(\mathbf{p})M_0(\mathbf{p})d^3\mathbf{p} = \\
 &= \frac{1}{(2\pi)^3} \int 8\pi^{\frac{1}{2}}\gamma_0^{\frac{5}{2}}\phi_1(\mathbf{p})M_0(\mathbf{p})d^2\mathbf{p} \approx \\
 &\approx \frac{8\pi^{\frac{1}{2}}\gamma_0^{\frac{5}{2}}}{(2\pi)^3} \int \phi_1(\mathbf{p})M_0(0)d^2\mathbf{p} = \quad (222) \\
 &= \frac{-8\pi^{\frac{1}{2}}\gamma_0^{\frac{5}{2}}}{(2\pi)^3} \int \frac{\log\left|\frac{|p|-\gamma_0}{|p|+\gamma_0}\right|}{2\pi(p^2+\gamma_0^2)|p|} d^2\mathbf{p}M_0(0) = \\
 &= \frac{8\pi^{\frac{1}{2}}\gamma_0^{\frac{5}{2}}}{(2\pi)^3} \frac{\pi^3}{2\pi\gamma_0} M_0(0) = \\
 &= \frac{1}{4(\pi a^3)^{\frac{1}{2}}} M_0(0),
 \end{aligned}$$

where the step \approx holds as similar the equation (221) since in the two dimensional integral of $\phi_1(\mathbf{p})$ we have that $\phi_1(\mathbf{p}) \rightarrow 0$ as $\mathbf{p} \rightarrow \infty$ such that it tends to zero as rapidly as the three dimensional case of $\phi_0(\mathbf{p}) \rightarrow 0$.

Thus we have:

$$\alpha\Gamma_{seagull} = \frac{\alpha}{4}\Gamma_0. \quad (223)$$

From the literature of computation of the orthopositronium decay rate we have that the computed orthopositronium decay rate (up to the order α^2) is given by [33–52]:

$$\begin{aligned} \Gamma_{\text{O-Ps}} &= \Gamma_0 \left[1 + A \frac{\alpha}{\pi} + \frac{\alpha^2}{3} \log \alpha + B \left(\frac{\alpha}{\pi} \right)^2 - \frac{\alpha^3}{2\pi} \log^2 \alpha \right] = \\ &= 7.039934(10) \mu\text{s}^{-1}, \end{aligned} \quad (224)$$

where $A = -10.286\,606(10)$, $B = 44.52(26)$ and $\Gamma_0 = \frac{9}{2}(\pi^2 - 9)m\alpha^6 = 7.211\,169 \mu\text{s}^{-1}$.

Then with the additional decay rate from the seagull vertex potential (or from the modified ground state of positronium) we have the following computed orthopositronium decay rate (up to the order α^2):

$$\begin{aligned} \Gamma_{\text{O-Ps}} + \alpha\Gamma_{seagull} &= \\ &= \Gamma_0 \left[1 + \left(A + \frac{\pi}{4} \right) \frac{\alpha}{\pi} + \frac{\alpha^2}{3} \log \alpha + B \left(\frac{\alpha}{\pi} \right)^2 - \frac{\alpha^3}{2\pi} \log^2 \alpha \right] = \\ &= 7.039934(10) + 0.01315874 \mu\text{s}^{-1} = \\ &= 7.052092(84) \mu\text{s}^{-1}. \end{aligned} \quad (225)$$

This agrees with the two Ann Arbor experimental values where the two Ann Arbor experimental values are given by: $\Gamma_{\text{O-Ps}}(\text{Gas}) = 7.0514(14) \mu\text{s}^{-1}$ and $\Gamma_{\text{O-Ps}}(\text{Vacuum}) = 7.0482(16) \mu\text{s}^{-1}$ [33, 34].

We remark that for the decay rate $\alpha\Gamma_{seagull}$ we have only computed it up to the order α . If we consider the decay rate $\alpha\Gamma_{seagull}$ up to the order α^2 then the decay rate (225) will be reduced since the order α of $\Gamma_{seagull}$ is of negative value.

If we consider only the computed orthopositronium decay rate up to the order α with the term $B \left(\frac{\alpha}{\pi} \right)^2$ omitted, then $\Gamma_{\text{O-Ps}} = 7.038202 \mu\text{s}^{-1}$ (see [33–52]) and we have the following computed orthopositronium decay rate:

$$\Gamma_{\text{O-Ps}} + \alpha\Gamma_{seagull} = 7.05136074 \mu\text{s}^{-1}. \quad (226)$$

This also agrees with the above two Ann Arbor experimental values and is closer to these two experimental values.

On the other hand the Tokyo experimental value given by $\Gamma_{\text{O-Ps}}(\text{Powder}) = 7.0398(29) \mu\text{s}^{-1}$ [35] may be interpreted by that in this experiment the QED effect $\Gamma_{seagull}$ of the seagull vertex potential is suppressed due to the special two dimensional statistical form of $\Gamma_{seagull}$ (Thus the additional effect of the modified ground state ϕ of the positronium is suppressed). Thus the value of this experiment agrees with the computational result $\Gamma_{\text{O-Ps}}$. Similarly the experimental result of another Ann Arbor experiment given by $7.0404(8) \mu\text{s}^{-1}$

[36] may also be interpreted by that in this experiment the QED effect $\Gamma_{seagull}$ of the seagull vertex potential is suppressed due to the special two dimensional statistical form of $\Gamma_{seagull}$.

27 Graviton constructed from photon

It is well known that Einstein tried to find a theory to unify gravitation and electromagnetism [1, 79, 80]. The search for such a theory has been one of the major research topics in physics [80–88]. Another major research topic in physics is the search for a theory of quantum gravity [89–120]. In fact, these two topics are closely related. In this Section, we propose a theory of quantum gravity that unifies gravitation and electromagnetism.

In the above Sections the photon is as the quantum Wilson loop with the $U(1)$ gauge group for electrodynamics. In the above Sections we have also shown that the corresponding quantum Wilson line can be regarded as the photon propagator in analogy to the usual concept of propagator. In this section from this quantum photon propagator, the quantum graviton propagator and the graviton are constructed. This construction forms the foundation of a theory of quantum gravity that unifies gravitation and electromagnetism.

It is well known that Weyl introduced the gauge concept to unify gravitation and electromagnetism [80]. However this gauge concept of unifying gravitation and electromagnetism was abandoned because of the criticism of the path dependence of the gauge (it is well known that this gauge concept later is important for quantum physics as phase invariance) [1]. In this paper we shall use again Weyl's gauge concept to develop a theory of quantum gravity which unifies gravitation and electromagnetism. We shall show that the difficulty of path dependence of the gauge can be solved in this quantum theory of unifying gravitation and electromagnetism.

Let us consider a differential of the form $g(s)ds$ where $g(s)$ is a field variable to be determined. Let us consider a symmetry of the following form:

$$g(s)ds = g'(s')ds', \quad (227)$$

where s is transformed to s' and $g'(s)$ is a field variable such that (227) holds. From (227) we have a symmetry of the following form:

$$g(s)^* g(s) ds^2 = g^{*(s')} g'(s') ds'^2, \quad (228)$$

where $g^*(s)$ and $g^{*(s)}$ denote the complex conjugate of $g(s)$ and $g'(s)$ respectively. This symmetry can be considered as the symmetry for deriving the gravity since we can write $g(s)^* g(s) ds^2$ into the following metric form for the four dimensional space-time in General Relativity:

$$g(s)^* g(s) ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \quad (229)$$

where we write $ds^2 = a_{\mu\nu} dx^\mu dx^\nu$ for some functions $a_{\mu\nu}$ by introducing the space-time variable x^μ , $\mu = 0, 1, 2, 3$ with x^0 as the time variable; and $g_{\mu\nu} = g(s)^* g(s) a_{\mu\nu}$. Thus from the symmetry (227) we can derive General Relativity.

Let us now determine the variable $g(s)$. Let us consider $g(s) = W(z_0, z(s))$, a quantum Wilson line with $U(1)$ group where z_0 is fixed. When $W(z_0, z(s))$ is the classical Wilson line then it is of path dependence and thus there is a difficulty to use it to define $g(s) = W(z_0, z(s))$. This is also the difficulty of Weyl's gauge theory of unifying gravitation and electromagnetism. Then when $W(z_0, z(s))$ is the quantum Wilson line because of the quantum nature of unspecification of paths we have that $g(s) = W(z_0, z(s))$ is well defined where the whole path of connecting z_0 and $z(s)$ is unspecified (except the two end points z_0 and $z(s)$).

Thus for a given transformation $s' \rightarrow s$ and for any (continuous and piecewise smooth) path connecting z_0 and $z(s)$ the resulting quantum Wilson line $W'(z_0, z(s(s')))$ is again of the form $W(z_0, z(s)) = W(z_0, z(s(s')))$. Let $g'(s') = W'(z_0, z(s(s')))$ $\frac{ds}{ds'}$. Then we have:

$$\begin{aligned} g'^*(s')g'(s')ds'^2 &= \\ &= W'^*(z_0, z(s(s'))))W'(z_0, z(s(s'))))\left(\frac{ds}{ds'}\right)^2 ds'^2 = \\ &= W^*(z_0, z(s))W(z_0, z(s))\left(\frac{ds}{ds'}\right)^2 ds'^2 = \\ &= g(s)^*g(s)ds^2. \end{aligned} \tag{230}$$

This shows that the quantum Wilson line $W(z_0, z(s))$ can be the field variable for the gravity and thus can be the field variable for quantum gravity since $W(z_0, z(s))$ is a quantum field variable.

Then we consider the operator $W(z_0, z)W(z_0, z)$. From this operator $W(z_0, z)W(z_0, z)$ we can compute the operator $W^*(z_0, z)W(z_0, z)$ which is as the absolute value of this operator. Thus this operator $W(z_0, z)W(z_0, z)$ can be regarded as the quantum graviton propagator while the quantum Wilson line $W(z_0, z)$ is regarded as the quantum photon propagator for the photon field propagating from z_0 to z . Let us then compute this quantum graviton propagator $W(z_0, z)W(z_0, z)$. We have the following formula:

$$\begin{aligned} W(z, z_0)W(z_0, z) &= \\ &= e^{-\hat{t} \log[\pm(z-z_0)]} A e^{\hat{t} \log[\pm(z_0-z)]}, \end{aligned} \tag{231}$$

where $\hat{t} = -\frac{e_0^2}{k_0}$ for the $U(1)$ group ($k_0 > 0$ is a constant and we may let $k_0 = 1$) where the term $e^{-\hat{t} \log[\pm(z-z_0)]}$ is obtained by solving the first form of the dual form of the KZ equation and the term $e^{\hat{t} \log[\pm(z_0-z)]}$ is obtained by solving the second form of the dual form of the KZ equation.

Then we change the $W(z, z_0)$ of $W(z, z_0)W(z_0, z)$ in (231) to the second factor $W(z_0, z)$ of $W(z, z_0)W(z_0, z)$ by reversing the proper time direction of the path of connecting

z and z_0 for $W(z, z_0)$. This gives the graviton propagator $W(z_0, z)W(z_0, z)$. Then the reversing of the proper time direction of the path of connecting z and z_0 for $W(z, z_0)$ also gives the reversing of the first form of the dual form of the KZ equation to the second form of the dual form of the KZ equation. Thus by solving the second form of dual form of the KZ equation we have that $W(z_0, z)W(z_0, z)$ is given by:

$$\begin{aligned} W(z_0, z)W(z_0, z) &= e^{\hat{t} \log[\pm(z-z_0)]} A e^{\hat{t} \log[\pm(z-z_0)]} = \\ &= e^{2\hat{t} \log[\pm(z-z_0)]} A. \end{aligned} \tag{232}$$

In (232) let us define the following constant G :

$$G := -2\hat{t} = 2 \frac{e_0^2}{k_0}. \tag{233}$$

We regard this constant G as the gravitational constant of the law of Newton's gravitation and General Relativity. We notice that from the relation $e_0 = \left(z_A^{\frac{1}{2}}\right)^{-1} e = \frac{1}{n_e} e$ where the renormalization number $n_e = z_A^{\frac{1}{2}}$ is a very large number we have that the bare electric charge e_0 is a very small number. Thus the gravitational constant G given by (233) agrees with the fact that the gravitational constant is a very small constant. This then gives a closed relationship between electromagnetism and gravitation.

We remark that since in (232) the factor $-G \log r_1 = G \log \frac{1}{r_1} < 0$ (where we define $r_1 = |z - z_0|$ and r_1 is restricted such that $r_1 > 1$) is the fundamental solution of the two dimensional Laplace equation we have that this factor (together with the factor $e^{-G \log r_1} = e^{G \log \frac{1}{r_1}}$) is analogous to the fundamental solution $-G \frac{1}{r}$ of the three dimensional Laplace equation for the law of Newton's gravitation. Thus the operator $W(z_0, z)W(z_0, z)$ in (232) can be regarded as the graviton propagator which gives attractive effect when $r_1 > 1$. Thus the graviton propagator (232) gives the same attractive effect of $-G \frac{1}{r}$ for the law of Newton's gravitation.

On the other hand when $r_1 \leq 1$ we have that the factor $-G \log r_1 = G \log \frac{1}{r_1} \geq 0$. In this case we may consider that this graviton propagator gives repulsive effect. This means that when two particles are very close to each other then the gravitational force can be from attractive to become repulsive. This repulsive effect is a modification of $-G \frac{1}{r}$ for the law of Newton's gravitation for which the attractive force between two particles tends to ∞ when the distance between the two particles tends to 0.

Then by multiplying two masses m_1 and m_2 (obtained from the winding numbers of Wilson loops in (73) of two particles to the graviton propagator (232) we have the following formula:

$$G m_1 m_2 \log \frac{1}{r_1}. \tag{234}$$

From this formula (234) by introducing the space variable x as a statistical variable via the Lorentz metric: $ds^2 =$

$= dt^2 - dx^2$ we have the following statistical formula which is the potential law of Newton's gravitation:

$$-GM_1M_2\frac{1}{r}, \quad (235)$$

where M_1 and M_2 denotes the masses of two objects.

We remark that the graviton propagator (232) is for matters. We may by symmetry find a propagator $f(z_0, z)$ of the following form:

$$f(z_0, z) := e^{-2\hat{t}\log[\pm(z-z_0)]}A. \quad (236)$$

When $|z - z_0| > 1$ this propagator $f(z_0, z)$ gives repulsive effect between two particles and thus is for anti-matter particles where by the term anti-matter we mean particles with the repulsive effect (236). Then since $|f(z_0, z)| \rightarrow \infty$ as $|z - z_0| \rightarrow \infty$ we have that two such anti-matter particles can not physically exist. However in the following Section on dark energy and dark matter we shall show the possibility of another repulsive effect among gravitons.

As similar to that the quantum Wilson loop $W(z_0, z_0)$ is as the photon we have that the following double quantum Wilson loop can be regarded as the graviton:

$$W(z_0, z)W(z_0, z)W(z, z_0)W(z, z_0). \quad (237)$$

28 Dark energy and dark matter

By the method of computation of solutions of KZ equations and the computation of the graviton propagator (232) we have that (237) is given by:

$$\begin{aligned} W(z_0, z)W(z_0, z)W(z, z_0)W(z, z_0) &= \\ &= e^{2\hat{t}\log[\pm(z-z_0)]}A_g e^{-2\hat{t}\log[\pm(z-z_0)]} = \\ &= R^{2n}A_g, \quad n = 0, \pm 1, \pm 2, \pm 3, \dots \end{aligned} \quad (238)$$

where A_g denotes the initial operator for the graviton. Thus as similar to the quantization of energy of photons we have the following quantization of energy of gravitons:

$$h\nu = 2\pi\epsilon_0^2 n, \quad n = 0, \pm 1, \pm 2, \pm 3, \dots \quad (239)$$

As similar to that a photon with a specific frequency can be as a magnetic monopole because of its loop nature we have that the graviton (237) with a specific frequency can also be regarded as a magnetic monopole (which is similar to but different from the magnetic monopole of the photon kind) because of its loop nature. (This means that the loop nature gives magnetic property.)

Since we still can not directly observe the graviton in experiments the quantized energies (239) of gravitons can be identified as dark energy. Then as similar to the construction of electrons from photons we construct matter from gravitons

by the following formula:

$$W(z_0, z)W(z_0, z)W(z, z_0)W(z, z_0)Z, \quad (240)$$

where Z is a complex number as a state acted by the graviton.

Similar to the mechanism of generating mass of electron we have that the mechanism of generating the mass m_d of these particles is given by the following formula:

$$m_d c^2 = 2\pi\epsilon_0^2 n_d = \pi G n_d = h\nu_d \quad (241)$$

for some integer n_d and some frequency ν_d .

Since the graviton is not directly observable it is consistent to identify the quantized energies of gravitons as dark energy and to identify the matters (240) constructed by gravitons as dark matter.

It is interesting to consider the quantum gravity effect between two gravitons. When a graviton propagator is connected to a graviton we have that this graviton propagator is extended to contain a closed loop since the graviton is a closed loop. In this case as similar to the quantum photon propagator this extended quantum graviton propagator can give attractive or repulsive effect. Then for stability the extended quantum graviton propagator tends to give the repulsive effect between the two gravitons. Thus the quantum gravity effect among gravitons can be repulsive which gives the diffusion of gravitons and thus gives a diffusion phenomenon of dark energy. Furthermore for stability more and more open-loop graviton propagators in the space form closed loops. Thus more and more gravitons are forming and the repulsive effect of gravitons gives the accelerating expansion of the universe [53–57].

Let us then consider the quantum gravity effect between two particles of dark matter. When a graviton propagator is connected to two particles of dark matter not by connecting to the gravitons acting on the two particles of dark matter we have that the graviton propagator gives only attractive effect between the two particles of dark matter. Thus as similar to the gravitational force among the usual non-dark matters the gravitational force among dark matters are mainly attractive. Then when the graviton propagator is connected to two particles of dark matter by connecting to the gravitons acting on the two particles of dark matter then as the above case of two gravitons we have that the graviton propagator can give attractive or repulsive effect between the two particles of dark matter.

29 Conclusion

In this paper a quantum loop model of photon is established. We show that this loop model is exactly solvable and thus may be considered as a quantum soliton. We show that this nonlinear model of photon has properties of photon and magnetic monopole and thus photon with some specific frequency may be identified with the magnetic monopole. From the discrete winding numbers of this loop model we can derive the

quantization property of energy for the Planck's formula of radiation and the quantization property of electric charge. We show that the charge quantization is derived from the energy quantization. On the other hand from the nonlinear model of photon a nonlinear loop model of electron is established. This model of electron has a mass mechanism which generates mass to the electron where the mass of the electron is from the photon-loop. With this mass mechanism for generating mass the Higgs mechanism of the conventional QED theory for generating mass is not necessary.

We derive a QED theory which is not based on the four dimensional space-time but is based on the one dimensional proper time. This QED theory is free of ultraviolet divergences. From this QED theory the quantum loop model of photon is established. In this QED theory the four dimensional space-time is derived for statistics. Using the space-time statistics, we employ Feynman diagrams and Feynman rules to compute the basic QED effects such as the vertex correction, the photon self-energy and the electron self-energy. From these QED effects we compute the anomalous magnetic moment and the Lamb shift. The computation is of simplicity and accuracy and the computational result is better than that of the conventional QED theory in that the computation is simpler and it does not involve numerical approximation as that in the conventional QED theory where the Lamb shift is approximated by numerical means.

From the QED theory in this paper we can also derive a new QED effect which is from the seagull vertex of this QED theory. By this new QED effect and by a reformulated Bethe-Salpeter (BS) equation which resolves the difficulties of the BS equation (such as the existence of abnormal solutions) and gives a modified ground state wave function of the positronium. Then from this modified ground state wave function of the positronium a new QED effect of the orthopositronium decay rate is derived such that the computed orthopositronium decay rate agrees with the experimental decay rate. Thus the *orthopositronium lifetime puzzle* is completely resolved where we also show that the recent resolution of this orthopositronium lifetime puzzle only partially resolves this puzzle due to the special nature of two dimensional space statistics of this new QED effect.

By this quantum loop model of photon a theory of quantum gravity is also established where the graviton is constructed from the photon. Thus this theory of quantum gravity unifies gravitation and electromagnetism. In this unification of gravitation and electromagnetism we show that the universal gravitation constant G is proportional to e_0^2 where e_0 is the bare electric charge which is a very small constant and is related to the renormalized charge e by the formula $e_0 = \frac{1}{n_e} e$ where the renormalized number n_e is a very large winding number of the photon-loop. This relation of G with e_0 (and thus with e) gives a closed relationship between gravitation and electromagnetism. Then since gravitons are not directly observable the quantized energies of gravitons are as dark en-

ergy and the particles constructed by gravitons are as dark matter. We show that the quantum gravity effect among particles of dark matter is mainly attractive (and it is possible to be repulsive when a graviton loop is formed in the graviton propagator) while the quantum gravity effect among gravitons can be repulsive which gives the diffusion of gravitons and thus gives the diffusion phenomenon of dark energy and the accelerating expansion of the universe.

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SPECIAL REPORT**Reconsideration of the Uncertainty Relations and Quantum Measurements**

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Discussions on uncertainty relations (UR) and quantum measurements (QMS) persisted until nowadays in publications about quantum mechanics (QM). They originate mainly from the conventional interpretation of UR (CIUR). In the most of the QM literature, it is underestimated the fact that, over the years, a lot of deficiencies regarding CIUR were signaled. As a rule the alluded deficiencies were remarked disparately and discussed as punctual and non-essential questions. Here we approach an investigation of the mentioned deficiencies collected in a conclusive ensemble. Subsequently we expose a reconsideration of the major problems referring to UR and QMS. We reveal that all the basic presumption of CIUR are troubled by insurmountable deficiencies which require the indubitable failure of CIUR and its necessary abandonment. Therefore the UR must be deprived of their statute of crucial pieces for physics. So, the aboriginal versions of UR appear as being in postures of either (i) thought-experimental fictions or (ii) simple QM formulae and, any other versions of them, have no connection with the QMS. Then the QMS must be viewed as an additional subject comparatively with the usual questions of QM. For a theoretical description of QMS we propose an information-transmission model, in which the quantum observables are considered as random variables. Our approach directs to natural solutions and simplifications for many problems regarding UR and QMS.

1 Introduction

The uncertainty relations (UR) and quantum measurements (QMS) constitute a couple of considerable popularity, frequently regarded as a crucial pieces of quantum mechanics (QM). The respective crucial character is often glorified by assertions like:

- (i) UR are expression of "*the most important principle of the twentieth century physics*" [1];
- (ii) the description of QMS is "*probably the most important part of the theory (QM)*" [2].

The alluded couple constitute the basis for the so-called Conventional Interpretation of UR (CIUR). Discussions about CIUR are present in a large number of early as well as recent publications (see [1–11] and references therein). Less mentioned is the fact that CIUR ideas are troubled by a number of still unsolved deficiencies. As a rule, in the main stream of CIUR partisan publications, the alluded deficiencies are underestimated (through unnatural solutions or even by omission).

Nevertheless, during the years, in scientific literature were recorded remarks such as:

- (i) UR "*are probably the most controverted formulae in the whole of the theoretical physics*" [12];
- (ii) "*the word ("measurement") has had such a damaging effect on the discussions that... it should be banned altogether in quantum mechanics*" [13];

- (iii) "*the idea that there are defects in the foundations of orthodox quantum theory is unquestionable present in the conscience of many physicists*" [14];
- (iv) "*Many scientists have considered the conceptual framework of quantum theory to be unsatisfactory. The very foundations of Quantum Mechanics is a matter that needs to be resolved in order to achieve and gain a deep physical understanding of the underlying physical procedures that constitute our world*" [15].

The above mentioned status of things require further studies and probably new views. We believe that a promising strategy to satisfy such requirements is to develop an investigation guided by the following objectives (obj.):

- (obj.1) to identify the basic presumptions of CIUR;
- (obj.2) to reunite together all the significant deficiencies of CIUR;
- (obj.3) to examine the verity and importance of the respective deficiencies;
- (obj.4) to see if such an examination defends or incriminate CIUR;
- (obj.5) in the latter case to admit the failure of CIUR and its abandonment;
- (obj.6) to search for a genuine reinterpretation of UR;
- (obj.7) to evaluate the consequences of the UR reinterpretation for QMS;
- (obj.8) to promote new views about QMS;

(obj.9) to note a number of remarks on some adjacent questions.

A such guided investigation we are approaching in the next sections of this paper. The present approach try to complete and to improve somewhat less elaborated ideas from few of our previous writings. But, due to a lot of unfortunate chances, and contrary to my desire, the respective writings were edited in modest publications [16–18] or remained as preprints registred in data bases of LANL and CERN libraries (see [19]).

2 Shortly on CIUR history and its basic presumptions

The story of CIUR began with the Heisenberg's seminal work [20] and it starts [21] from the search of general answers to the primary questions (q.):

- (q.1) Are all measurements affected by measuring uncertainties?
- (q.2) How can the respective uncertainties be described quantitatively?

In connection with the respective questions, in its subsequent extension, CIUR promoted the suppositions (s.):

- (s.1) The measuring uncertainties are due to the perturbations of the measured microparticle (system) by its interactions with the measuring instrument;
- (s.2) In the case of macroscopic systems the mentioned perturbations can be made arbitrarily small and, consequently, always the corresponding uncertainties can be considered as negligible;
- (s.3) On the other hand, in the case of quantum microparticles (of atomic size) the alluded perturbations are essentially unavoidable and consequently for certain measurements (see below) the corresponding uncertainties are non-negligible.

Then CIUR limited its attention only to the quantum cases, for which restored to an amalgamation of the following motivations (m.):

- (m.1) Analysis of some thought (gedanken) measuring experiments;
- (m.2) Appeal to the theoretical version of UR from the existing QM.

NOTIFICATION: In the present paper we will use the term “observable” (introduced by CIUR literature) for denoting a physical quantity referring to a considered microparticle (system).

Now let us return to the begining of CIUR history. Firstly [20, 22], for argumentation of the above noted motivation (m.1) were imagined some thought experiments on a quantum microparticle, destined to simultaneous measurements of two (canonically) conjugated observables A and B (such as coordinate q and momentum p or time t and energy E). The

corresponding “*thought experimental*” (te) uncertainties were noted with $\Delta_{te}A$ and $\Delta_{te}B$. They were found as being interconnected trough the following te -UR

$$\Delta_{te}A \cdot \Delta_{te}B \geq \hbar, \quad (1)$$

where \hbar denotes the reduced Planck constant.

As regard the usage of motivation (m.2) in order to promote CIUR few time later was introduced [23, 24] the so-called Robertson Schrödinger UR (RSUR):

$$\Delta_{\Psi}A \cdot \Delta_{\Psi}B \geq \frac{1}{2} \left| \langle [\hat{A}, \hat{B}] \rangle_{\Psi} \right|. \quad (2)$$

In this relation one finds usual QM notations i.e.: (i) \hat{A} and \hat{B} denote the quantum operators associated with the observables A and B of the same microparticle, (ii) $\Delta_{\Psi}A$ and $\Delta_{\Psi}B$ signify the standard deviation of the respective observables, (iii) $\langle (\dots) \rangle_{\Psi}$ represents the mean value of (\dots) in the state described by the wave function Ψ , (iv) $[\hat{A}, \hat{B}]$ depict the commutator of the operators \hat{A} and \hat{B} (for some other details about the QM notations and validity of RSUR (2) see the next section).

CIUR was built by regarding the relations (1) and (2), as standard (reference) elements. It started through the writings (and public lectures) of the so-called Copenhagen School partisans. Later CIUR was adopted, more or less explicitly, in a large number of publications.

An attentive examination of the alluded publications show that in the main CIUR is builded onthe following five basic presumptions (P):

- P_1 : Quantities $\Delta_{te}A$ and $\Delta_{\Psi}A$ from relations (1) and (2) denoted by a unique symbol ΔA , have similar significance of measuring uncertainty for the observable A referring to the same microparticle. Consequently the respective relations have the same generic interpretation as UR regarding the simultaneous measurements of observables A and B of the alluded microparticle;
- P_2 : In case of a solitary observable A , for a microparticle, the quantity ΔA can have always an unbounded small value. Therefore such an observable can be measured without uncertainty in all cases of microparticles (systems) and states;
- P_3 : When two observables A and B are commutable (i.e. $[\hat{A}, \hat{B}] = 0$) relation (2) allows for the quantities ΔA and ΔB , regarding the same microparticle, to be unlimedly small at the same time. That is why such observables can be measured simultaneously and without uncertainties for any microparticle (system) or state. Therefore they are considered as compatible;
- P_4 : If two observables A and B are non-commutable (i.e. $[\hat{A}, \hat{B}] \neq 0$) relation (2) shows that, for a given microparticle, the quantities ΔA and ΔB can be never reduced concomitantly to null values. For that reason

such observables can be measured simultaneously only with non-null and interconnected uncertainties, irrespective of the microparticle (system) or state. Hence such observables are considered as incompatible;

P₅ : Relations (1) and (2), Planck's constant \hbar as well as the measuring peculiarities noted in **P₄** are typically QM things which have not analogies in classical (non-quantum) macroscopic physics.

Here it must be recorded the fact that, in individual publications from the literature which promote CIUR, the above noted presumptions **P₁**–**P₅** often appear in non-explicit forms and are mentioned separately or only few of them. Also in the same publications the deficiencies of CIUR are omitted or underestimated. On the other hand in writings which tackle the deficiencies of CIUR the respective deficiencies are always discussed as separate pieces not reunited in some elucidative ensembles. So, tacitly, in our days CIUR seems to remain a largely adopted doctrine which dominates the questions regarding the foundation and interpretation of QM.

3 Examination of CIUR deficiencies regarded in an elucidative collection

In order to evaluate the true significance of deficiencies regarding CIUR we think that it must be discussed together many such deficiencies reunited, for a good examination, in an elucidative collection. Such a kind of discussion we try to present below in this section.

Firstly let us examine the deficiencies regarding the relation (1). For such a purpose we note the following remark (**R**):

R₁: On the relation (1)

In reality the respective relation is an improper piece for a reference/standard element of a supposed solid doctrine such as CIUR. This fact is due to the circumstance that such a relation has a transitory/temporary character because it was founded on old resolution criteria (introduced by Abe and Rayleigh — see [22, 25]). But the respective criteria were improved in the so-called super-resolution techniques worked out in modern experimental physics (see [26–31] and references). Then it is possible to imagine some super-resolution-thought-experiments (*srte*). So, for the corresponding *srte*-uncertainties $\Delta_{srte}A$ and $\Delta_{srte}B$ of two observables A and B the following relation can be promoted

$$\Delta_{srte}A \cdot \Delta_{srte}B \leq \hbar. \quad (3)$$

Such a relation is possibly to replace the CIUR basic formula (1). But the alluded possibility invalidates the presumption **P₁** and incriminates CIUR in connection with one of its main points.

End of **R₁**

For an argued examination of CIUR deficiencies regarding the relation (2) it is of main importance the following remark:

R₂: On the aboriginal QM elements

Let us remind briefly some significant elements, selected from the aboriginal framework of usual QM. So we consider a QM microparticle whose state (of orbital nature) is described by the wave function Ψ . Two observables A_j ($j = 1, 2$) of the respective particle will be described by the operators \hat{A}_j . The notation (f, g) will be used for the scalar product of the functions f and g . Correspondingly, the quantities $\langle A_j \rangle_\Psi = (\Psi, \hat{A}_j \Psi)$ and $\delta_\Psi \hat{A}_j = \hat{A}_j - \langle \hat{A}_j \rangle_\Psi$ will depict the mean (expected) value respectively the deviation-operator of the observable A_j regarded as a random variable. Then, by denoting the two observables with $A_1 = A$ and $A_2 = B$, we can write the following Cauchy-Schwarz relation:

$$\begin{aligned} (\delta_\Psi \hat{A} \Psi, \delta_\Psi \hat{A} \Psi) (\delta_\Psi \hat{B} \Psi, \delta_\Psi \hat{B} \Psi) &\geq \\ &\geq \left| (\delta_\Psi \hat{A} \Psi, \delta_\Psi \hat{B} \Psi) \right|^2. \end{aligned} \quad (4)$$

For an observable A_j considered as a random variable the quantity $\Delta_\Psi A_j = (\delta_\Psi \hat{A}_j \Psi, \delta_\Psi \hat{A}_j \Psi)^{\frac{1}{2}}$ signifies its standard deviation. From (4) it results directly that the standard deviations $\Delta_\Psi A$ and $\Delta_\Psi B$ of the mentioned observables satisfy the relation

$$\Delta_\Psi A \cdot \Delta_\Psi B \geq \left| (\delta_\Psi \hat{A} \Psi, \delta_\Psi \hat{B} \Psi) \right|, \quad (5)$$

which can be called *Cauchy-Schwarz formula* (CSF). Note that CSF (5) (as well as the relation (4)) is always valid, i.e. for all observables, particles and states. Here it is important to specify the fact that the CSF (5) is an aboriginal piece which implies the subsequent and restricted RSUR (1) only in the cases when the operators $\hat{A} = \hat{A}_1$ and $\hat{B} = \hat{A}_2$ satisfy the conditions

$$(\hat{A}_j \Psi, \hat{A}_k \Psi) = (\Psi, \hat{A}_j \hat{A}_k \Psi), \quad (j, k = 1, 2). \quad (6)$$

Indeed in such cases one can write the relation

$$\begin{aligned} (\delta_\Psi \hat{A} \Psi, \delta_\Psi \hat{B} \Psi) &= \\ &= \frac{1}{2} (\Psi, (\delta_\Psi \hat{A} \cdot \delta_\Psi \hat{B} \Psi + \delta_\Psi \hat{B} \cdot \delta_\Psi \hat{A} \Psi)) - \\ &- \frac{i}{2} (\Psi, i [\hat{A}, \hat{B}] \Psi), \end{aligned} \quad (7)$$

where the two terms from the right hand side are purely real and imaginary quantities respectively. Therefore in the mentioned cases from (5) one finds

$$\Delta_\Psi A \cdot \Delta_\Psi B \geq \frac{1}{2} \left| \langle [\hat{A}, \hat{B}] \rangle_\Psi \right| \quad (8)$$

i.e. the well known RSUR (2).

The above reminded aboriginal QM elements prove the following fact. In reality for a role of standard (reference) piece regarding the interpretation of QM aspects must be considered the CSF (5) but not the RSUR (2). But such a reality

incriminate in an indubitable manner all the basic presumptions P_1 – P_5 of CIUR.

End of R_2

The same QM elements reminded in R_2 , motivate the next remark:

R_3 : On a denomination used by CIUR

The denomination “*uncertainty*” used by CIUR for quantities like $\Delta_\Psi A$ from (2) is groundless because of the following considerations. As it was noted previously in the aboriginal QM framework, $\Delta_\Psi A$ signifies the standard deviation of the observable A regarded as a random variable. The mentioned framework deals with theoretical concepts and models about the intrinsic (inner) properties of the considered particle but not with aspects of the measurements performed on the respective particle. Consequently, for a quantum microparticle, the quantity $\Delta_\Psi A$ refers to the intrinsic characteristics (reflected in fluctuations) of the observable A . Moreover it must noted the following realities:

- (i) For a particle in a given state the quantity $\Delta_\Psi A$ has a well defined value connected with the corresponding wave function Ψ ;
- (ii) The value of $\Delta_\Psi A$ is not related with the possible modifications of the accuracy regarding the measurement of the observable A .

The alluded realities are attested by the fact that for the same state of the measured particle (i.e. for the same value of $\Delta_\Psi A$) the measuring uncertainties regarding the observable A can be changed through the improving or worsening of experimental devices/procedures. Note that the above mentioned realities imply and justify the observation [32] that, for two variables x and p of the same particle, the usual CIUR statement “*as Δx approaches zero, Δp becomes infinite and vice versa*” is a doubtful speculation. Finally we can conclude that the ensemble of the things revealed in the present remark contradict the presumptions P_2 – P_4 of CIUR. But such a conclusion must be reported as a serious deficiency of CIUR.

End of R_3

A class of CIUR conceptual deficiencies regards the following pairs of canonically conjugated observables: L_z - φ , N - ϕ and E - t ($L_z = z$ component of angular momentum, $\varphi = az$ -imuthal angle, $N =$ number, $\phi =$ phase, $E =$ energy, $t =$ time). The respective pairs were and still are considered as being unconformable with the accepted mathematical rules of QM. Such a fact roused many debates and motivated various approaches planned to elucidate in an acceptable manner the missing conformity (for significant references see below within the remarks R_4 – R_6). But so far such an elucidation was not ratified (or admitted unanimously) in the scientific literature. In reality one can prove that, for all the three mentioned pairs of observables, the alluded unconformity refers not to conflicts with aboriginal QM rules but to serious disagreements with RSUR (2). Such proofs and their conse-

quences for CIUR we will discuss below in the following remarks:

R_4 : On the pair L_z - φ

The parts of above alluded problems regarding of the pair L_z - φ were examined in all of their details in our recent paper [33]. There we have revealed the following indubitable facts:

- (i) In reality the pair L_z - φ is unconformable only in respect with the secondary and limited piece which is RSUR (2);
- (ii) In a deep analysis, the same pair proves to be in a natural conformity with the true QM rules presented in R_2 ;
- (iii) The mentioned conformity regards mainly the CSF (5) which can degenerate in the trivial equality $0 = 0$ in some cases regarding the pair L_z - φ .

But such facts points out an indubitable deficiency of CIUR’s basic presumption P_4 .

End of R_4

R_5 : On the pair N - ϕ

The involvement of pair N - ϕ in debates regarding CIUR started [35] subsequently of the Dirac’s idea [36] to transcribe the ladder (lowering and raising) operators \hat{a} and \hat{a}^+ in the forms

$$\hat{a} = e^{i\hat{\phi}} \sqrt{\hat{N}}, \quad \hat{a}^+ = \sqrt{\hat{N}} e^{-i\hat{\phi}}. \quad (9)$$

By adopting the relation $[\hat{a}, \hat{a}^+] = \hat{a}\hat{a}^+ - \hat{a}^+\hat{a} = 1$ from (9) it follows that the operators \hat{N} and $\hat{\phi}$ satisfy the commutation formula

$$[\hat{N}, \hat{\phi}] = i. \quad (10)$$

This relation was associated directly with the RSUR (2) respectively with the presumption P_4 of CIUR. The mentioned association guided to the rash impression that the N - ϕ pair satisfy the relation

$$\Delta_\Psi N \cdot \Delta_\Psi \phi \geq \frac{1}{2}. \quad (11)$$

But, lately, it was found that relation (11) is false — at least in some well-specified situations. Such a situation appears in the case of a quantum oscillator (QO). The mentioned falsity can be pointed out as follows. The Schrödinger equation for a QO stationary state has the form:

$$E\Psi = \frac{1}{2m_0} \hat{p}^2 \Psi + \frac{1}{2} m_0 \omega^2 \hat{x}^2 \Psi, \quad (12)$$

where m_0 and ω represent the mass and (angular) frequency of QO while $\hat{p} = -i\hbar \frac{\partial}{\partial x}$ and $x = x \cdot$ denote the operators of the Cartesian moment p and coordinate x . Then the operators \hat{a} , \hat{a}^+ and \hat{N} have [34] the expressions

$$\hat{a} = \frac{m_0 \omega \hat{x} + i\hat{p}}{\sqrt{2m_0 \omega \hbar}}, \quad \hat{a}^+ = \frac{m_0 \omega \hat{x} - i\hat{p}}{\sqrt{2m_0 \omega \hbar}}, \quad \hat{N} = \hat{a}^+ \hat{a}. \quad (13)$$

The solution of the equation (12) is an eigenstate wave

function of the form

$$\Psi_n(x) = \Psi_n(\xi) \propto \exp\left(-\frac{\xi^2}{2}\right) \mathcal{H}_n(\xi), \quad (14)$$

where $\xi = x \sqrt{\frac{m_0\omega}{\hbar}}$, while $n = 0, 1, 2, 3, \dots$ signifies the oscillation quantum number and $\mathcal{H}_n(\xi)$ stand for Hermite polynomials of ξ . The noted solution correspond to the energy eigenvalue $E = E_n = \hbar\omega(n + \frac{1}{2})$ and satisfy the relation $\hat{N}\Psi_n(x) = n \cdot \Psi_n(x)$.

It is easy to see that in a state described by a wave function like (14) one find the results

$$\Delta_\Psi N = 0, \quad \Delta_\Psi \phi \leq 2\pi. \quad (15)$$

The here noted restriction $\Delta\phi \leq 2\pi$ (more exactly $\Delta\phi = \pi/\sqrt{3}$ — see below in (19)) is due to the natural fact that the definition range for ϕ is the interval $[0, 2\pi)$. Through the results (15) one finds a true falsity of the presumed relation (11). Then the harmonization of N - ϕ pair with the CIUR doctrine reaches to a deadlock. For avoiding the mentioned deadlock in many publications were promoted various adjustments regarding the pair N - ϕ (see [35, 37–43] and references therein). But it is easy to observe that all the alluded adjustments are subsequent (and dependent) in respect with the RSUR (2) in the following sense. The respective adjustments consider the alluded RSUR as an absolutely valid formula and try to adjust accordingly the description of the pair N - ϕ for QO. So the operators \hat{N} and $\hat{\phi}$, defined in (9) were replaced by some substitute (*sbs*) operators $\hat{N}_{sbs} = f(\hat{N})$ and $\hat{\phi}_{sbs} = g(\hat{\phi})$, where the functions f and g are introduced through various ad hoc procedures. The so introduced substitute operators \hat{N}_{sbs} and $\hat{\phi}_{sbs}$ pursue to be associated with corresponding standard deviations $\Delta_\Psi N_{sbs}$ and $\Delta_\Psi \phi_{sbs}$ able to satisfy relations resembling more or less with RSUR (2) or with (11). But we appreciate as very doubtful the fact that the afferent “*substitute observables*” N_{sbs} and ϕ_{sbs} can have natural (or even useful) physical significances. Probably that this fact as well as the ad hoc character of the functions f and g constitute the reasons for which until now, in scientific publications, it does not exist a unanimous agreement able to guarantee a genuine elucidation of true status of the N - ϕ pair comparatively with CIUR concepts.

Our opinion is that an elucidation of the mentioned kind can be obtained only through a discussion founded on the aboriginal QM elements presented above in the remark **R**₂. For approaching such a discussion here we add the following supplementary details. For the alluded QO the Schrödinger equation (12) as well as its solution (14) are depicted in a “*coordinate x-representation*”. But the same equation and solution can be described in a “*phase ϕ -representation*”. By taking into account the relation (10) it results directly that in the ϕ -representation the operators \hat{N} and $\hat{\phi}$ have the expressions $\hat{N} = i\left(\frac{\partial}{\partial\phi}\right)$ and $\hat{\phi} = \phi$. In the same representation the

Schrödinger equation (12) takes the form

$$E\Psi_n(\phi) = \hbar\omega\left(i\frac{\partial}{\partial\phi} + \frac{1}{2}\right)\Psi_n(\phi) \quad (16)$$

where $\phi \in [0, 2\pi)$. Then the solution of the above equation is given by the relation

$$\Psi_n(\phi) = \frac{1}{\sqrt{2\pi}} \exp(in\phi) \quad (17)$$

with $n = \frac{E}{\hbar\omega} - \frac{1}{2}$. If, similarly with the case of a classical oscillator, for a QO the energy E is considered to have non-negative values one finds $n = 0, 1, 2, 3, \dots$.

Now, for the case of a QO, by taking into account the wave function (17), the operators \hat{N} and $\hat{\phi}$ in the ϕ -representation, as well as the aboriginal QM elements presented in **R**₂, we can note the following things. In the respective case it is verified the relation

$$(\hat{N}\Psi_n, \hat{\phi}\Psi_n) = (\Psi_n, \hat{N}\hat{\phi}\Psi_n) + i. \quad (18)$$

This relation shows directly the circumstance that in the mentioned case the conditions (6) are not fulfilled by the operators \hat{N} and $\hat{\phi}$ in connection with the wave function (17). But such a circumstance point out the observation that in the case under discussion the RSUR (2)/(8) is not valid. On the other hand one can see that CSF (5) remains true. In fact it take the form of the trivial equality $0 = 0$ because in the due case one obtains

$$\Delta_\Psi N = 0, \quad \Delta_\Psi \phi = \frac{\pi}{\sqrt{3}}, \quad (\delta\hat{N}\Psi_n, \delta\hat{\phi}\Psi_n) = 0. \quad (19)$$

The above revealed facts allow us to note the following conclusions. In case of QO states (described by the wave functions (14) or (17)) the N - ϕ pair is in a complete disagreement with the RSUR (2)/(8) and with the associated basic presumption **P**₄ of CIUR. But, in the alluded case, the same pair is in a full concordance with the aboriginal QM element by the CSF (5). Then it is completely clear that the here noted conclusions reveal an authentic deficiency of CIUR.

OBSERVATION: Often in CIUR literature the N - ϕ pair is discussed in connection with the situations regarding ensembles of particles (e.g. fluxes of photons). But, in our opinion, such situations are completely different comparatively with the above presented problem about the N - ϕ pair and QO wave functions (states). In the alluded situations the Dirac’s notations/formulas (9) can be also used but they must be utilized strictly in connection with the wave functions describing the respective ensembles. Such utilization can offer examples in which the N - ϕ pair satisfy relations which are semblable with RSUR (2) or with the relation (11). But it is less probable that the alluded examples are able to consolidate the CIUR concepts. This because in its primary form CIUR regards on the first place the individual quantum particles but not ensembles of such particles.

End of R₅

R₆: On the E - t pair

Another pair of (canonically) conjugated observables which are unconformable in relation with the CIUR ideas is given by energy E and time t . That is why the respective pair was the subject of a large number of (old as well as recent) controversial discussions (see [2, 44–48] and references therein). The alluded discussions were generated by the following observations. On one hand, in conformity with the CIUR tradition, in terms of QM, E and t regarded as conjugated observables, ought to be described by the operators

$$\hat{E} = i\hbar \frac{\partial}{\partial t}, \quad \hat{t} = t. \quad (20)$$

respectively by the commutation relation

$$[\hat{E}, \hat{t}] = i\hbar. \quad (21)$$

In accordance with the RSUR (2) such a description require the formula

$$\Delta_{\Psi} E \cdot \Delta_{\Psi} t \geq \frac{\hbar}{2}. \quad (22)$$

On the other hand because in usual QM the time t is a deterministic but not a random variable for any quantum situation (particle/system and state) one finds the expressions

$$\Delta_{\Psi} E = \text{a finite quantity}, \quad \Delta_{\Psi} t \equiv 0. \quad (23)$$

But these expressions invalidate the relation (22) and consequently show an anomaly in respect with the CIUR ideas (especially with the presumption P_4). For avoiding the alluded anomaly CIUR partisans invented a lot of adjusted $\Delta_{\Psi} E - \Delta_{\Psi} t$ formulae destined to substitute the questionable relation (22) (see [2, 44–48] and references). The mentioned formulae can be written in the generic form

$$\Delta_v E \cdot \Delta_v t \geq \frac{\hbar}{2}. \quad (24)$$

Here $\Delta_v E$ and $\Delta_v t$ have various (v) significances such as:

- (i) $\Delta_1 E$ = line-breadth of the spectrum characterizing the decay of an excited state and $\Delta_1 t$ = half-life of the respective state;
- (ii) $\Delta_2 E$ = $\hbar\Delta\omega$ = spectral width (in terms of frequency ω) of a wave packet and $\Delta_2 t$ = temporal width of the respective packet;
- (iii) $\Delta_3 E = \Delta_{\Psi} E$ and $\Delta_3 t = \Delta_{\Psi} A \cdot (d\langle A \rangle_{\Psi} / dt)^{-1}$, with A = an arbitrary observable.

Note that in spite of the efforts and imagination implied in the disputes connected with the formulae (24) the following observations remain of topical interest.

- (i) The diverse formulae from the family (24) are not mutually equivalent from a mathematical viewpoint. Moreover they have no natural justification in the framework of usual QM (that however give a huge number of good results in applications);

- (ii) In the specific literature (see [2, 44–48] and references therein) none of the formulas (24) is agreed unanimously as a correct substitute for relation (22).

Here it must be added also another observation regarding the E - t pair. Even if the respective pair is considered to be described by the operators (20), in the true QM terms, one finds the relation

$$(\hat{E}\Psi, \hat{t}\Psi) = (\Psi, \hat{E}\hat{t}\Psi) - i\hbar. \quad (25)$$

This relation shows clearly that for the E - t pair the condition (6) is never satisfied. That is why for the respective pair the RSUR (2)/(8) is not applicable at all. Nevertheless for the same pair, described by the operators (20), the CSF (5) is always true. But because in QM the time t is a deterministic (i.e. non-random) variable in all cases the mentioned CSF degenerates into the trivial equality $0 = 0$.

Due to the above noted observations we can conclude that the applicability of the CIUR ideas to the E - t pair persists in our days as a still unsolved question. Moreover it seems to be most probable the fact that the respective question can not be solved naturally in accordance with the authentic and aboriginal QM procedures. But such a fact must be reported as a true and serious deficiency of CIUR.

End of R₆

In the above remarks R_1 – R_6 we have approached few facts which through detailed examinations reveal indubitable deficiencies of CIUR. The respective facts are somewhat known due to their relative presence in the published debates. But there are a number of other less known things which point out also deficiencies of CIUR. As a rule, in publications, the respective things are either ignored or mentioned with very rare occasions. Now we attempt to re-examine the mentioned things in a spirit similar with the one promoted in the remarks R_1 – R_6 from the upper part of this section. The announced re-examination is given below in the next remarks.

R₇: On the commutable observables

For commutable observables CIUR adopt the presumption P_3 because the right hand side term from RSUR (2) is a null quantity. But as we have shown in remark R_2 the respective RSUR is only a limited by-product of the general relation which is the CSF (5). However by means of the alluded CSF one can find examples where two commutable observable A and B can have simultaneously non-null values for their standard deviations ΔA and ΔB .

An example of the mentioned kind is given by the cartesian momenta p_x and p_y for a particle in a 2D potential well. The observables p_x and p_y are commutable because $[\hat{p}_x, \hat{p}_y] = 0$. The well is delimited as follows: the potential energy V is null for $0 < x_1 < a$ and $0 < y_1 < b$ respectively $V = \infty$ otherwise, where $0 < a < b$, $x_1 = \frac{(x+y)}{\sqrt{2}}$ and $y_1 = \frac{(y-x)}{\sqrt{2}}$. Then for the particle in the lowest energetic state

one finds

$$\Delta_{\Psi} p_x = \Delta_{\Psi} p_y = \hbar \frac{\pi}{ab} \sqrt{\frac{a^2 + b^2}{2}}, \quad (26)$$

$$|\langle (\delta_{\Psi} \hat{p}_x \Psi, \delta_{\Psi} \hat{p}_y \Psi) \rangle| = \left(\frac{\hbar \pi}{ab} \right)^2 \cdot \left(\frac{b^2 - a^2}{2} \right). \quad (27)$$

With these expressions it results directly that for the considered example the momenta p_x and p_y satisfy the CSF (5) in a non-trivial form (i.e. as an inequality with a non-null value for the right hand side term).

The above noted observations about commutable observables constitute a fact that conflicts with the basic presumption P_3 of CIUR. Consequently such a fact must be reported as an element which incriminates the CIUR doctrine.

End of R_7

R_8 : On the eigenstates

The RSUR (2) fails in the case when the wave function Ψ describes an eigenstate of one of the operators \hat{A} or \hat{B} . The fact was mentioned in [49] but it seems to remain unremarked in the subsequent publications. In terms of the here developed investigations the alluded failure can be discussed as follows. For two non-commutable observables A and B in an eigenstate of A one obtains the set of values: $\Delta_{\Psi} A = 0$, $0 < \Delta_{\Psi} B < \infty$ and $\langle [\hat{A}, \hat{B}]_{\Psi} \rangle \neq 0$. But, evidently, the respective values infringe the RSUR(2). Such situations one finds particularly with the pairs $L_z - \phi$ in some cases detailed in [33] and $N - \phi$ in situations presented above in R_5 .

Now one can see that the question of eigenstates does not engender any problem if the quantities $\Delta_{\Psi} A$ and $\Delta_{\Psi} B$ are regarded as QM standard deviations (i.e. characteristics of quantum fluctuations) (see the next Section). Then the mentioned set of values show that in the respective eigenstate A has no fluctuations (i.e. A behaves as a deterministic variable) while B is endowed with fluctuations (i.e. B appears as a random variable). Note also that in the cases of specified eigenstates the RSUR (2) are not valid. This happens because of the fact that in such cases the conditions (6) are not satisfied. The respective fact is proved by the observation that its opposite imply the absurd result

$$a \cdot \langle B \rangle_{\Psi} = \langle [\hat{A}, \hat{B}]_{\Psi} \rangle + a \cdot \langle B \rangle_{\Psi} \quad (28)$$

with $\langle [\hat{A}, \hat{B}]_{\Psi} \rangle \neq 0$ and $a = \text{eigenvalue of } \hat{A}$ (i.e. $\hat{A}\Psi = a\Psi$). But in the cases of the alluded eigenstates the CSF (5) remain valid. It degenerates into the trivial equality $0 = 0$ (because $\delta_{\Psi} \hat{A} \Psi = 0$).

So one finds a contradiction with the basic presumption P_4 — i.e. an additional and distinct deficiency of CIUR.

End of R_8

R_9 : On the multi-temporal relations

Now let us note the fact RSUR (2)/(8) as well as its precursor CSF (5) are one-temporal formulas. This because all the

quantities implied in the respective formulas refer to the same instant of time. But the mentioned formulas can be generalized into multi-temporal versions, in which the corresponding quantities refer to different instants of time. So CSF (5) is generalizable in the form

$$\Delta_{\Psi_1} A \cdot \Delta_{\Psi_2} B \geq \left| \left(\delta_{\Psi_1} \hat{A} \Psi_1, \delta_{\Psi_2} \hat{B} \Psi_2 \right) \right| \quad (29)$$

where Ψ_1 and Ψ_2 represent the wave function for two different instants of time t_1 and t_2 . If in (29) one takes $|t_2 - t_1| \rightarrow \infty$ in the CIUR vision the quantities $\Delta_{\Psi_1} A$ and $\Delta_{\Psi_2} B$ have to refer to A and B regarded as independent solitary observables. But in such a regard if $(\delta_{\Psi_1} \hat{A} \Psi_1, \delta_{\Psi_2} \hat{B} \Psi_2) \neq 0$ the relation (29) refute the presumption P_2 and so it reveals another additional deficiency of CIUR. Note here our opinion that the various attempts [50, 51], of extrapolating the CIUR vision onto the relations of type (29) are nothing but artifacts without any real (physical) justification. We think that the relation (29) does not engender any problem if it is regarded as fluctuations formula (in the sense which will be discussed in the next Section). In such a regard the cases when $(\delta_{\Psi_1} \hat{A} \Psi_1, \delta_{\Psi_2} \hat{B} \Psi_2) \neq 0$ refer to the situations in which, for the time moments t_1 and t_2 , the corresponding fluctuations of A and B are correlated (i.e. statistically dependent).

Now we can say that, the previously presented discussion on the multi-temporal relations, disclose in fact a new deficiency of CIUR.

End of R_9

R_{10} : On the many-observable relations

Mathematically the RSUR (2)/(8) is only a restricted by-product of CSF (5) which follows directly from the two-observable true relation (4). But further one the alluded relation (4) appear to be merely a simple two-observable version of a more general many-observable formula. Such a general formula has the the form

$$\det \left[\left(\delta_{\Psi} \hat{A}_j \Psi, \delta_{\Psi} \hat{A}_k \Psi \right) \right] \geq 0. \quad (30)$$

Here $\det [\alpha_{jk}]$ denotes the determinant with elements α_{jk} and $j = 1, 2, \dots, r$; $k = 1, 2, \dots, r$ with $r \geq 2$. The formula (30) results from the mathematical fact that the quantities $(\delta_{\Psi} \hat{A}_j \Psi, \delta_{\Psi} \hat{A}_k \Psi)$ constitute the elements of a Hermitian and non-negatively defined matrix (an abstract presentation of the mentioned fact can be found in [52]).

Then, within a consistent judgment of the things, for the many-observable relations (30), CIUR must to give an interpretation concordant with its own doctrine (summarized in its basic presumptions $P_1 - P_5$). Such an interpretation was proposed in [53] but it remained as an unconvincing thing (because of the lack of real physical justifications). Other discussions about the relations of type (30) as in [38] elude any interpretation of the mentioned kind. A recent attempt [54] meant to promote an interpretation of relations like (30), for three or more observables. But the respective attempt has not

a helping value for CIUR doctrine. This is because instead of consolidating the CIUR basic presumptions P_1 – P_5) it seems rather to support the idea that the considered relations are fluctuations formulas (in the sense which will be discussed bellow in the next Section). We opine that to find a CIUR-concordant interpretation for the many-observable relations (30) is a difficult (even impossible) task on natural ways (i.e. without esoteric and/or non-physical considerations). An exemplification of the respective difficulty can be appreciated by investigating the case of observables $A_1 = p$, $A_2 = x$ and $A_3 = H = \text{energy}$ in the situations described by the wave functions (14) of a QO.

Based on the above noted appreciations we conclude that the impossibility of a natural extension of CIUR doctrine to a interpretation regarding the many-observable relations (30) reveal another deficiency of the respective doctrine.

End of R_{10}

R_{11} : On the quantum-classical probabilistic similarity

Now let us call attention on a quantum-classical similarity which directly contradicts the presumption P_5 of CIUR. The respective similarity is of probabilistic essence and regards directly the RSUR (2)/(8) as descendant from the CSF (5). Indeed the mentioned CSF is completely analogous with certain two-observable formula from classical (phenomenological) theory of fluctuations for thermodynamic quantities. The alluded classical formula can be written [55, 56] as follows

$$\Delta_w \mathbb{A} \cdot \Delta_w \mathbb{B} \geq |\langle \delta_w \mathbb{A} \cdot \delta_w \mathbb{B} \rangle_w|. \quad (31)$$

In this formula \mathbb{A} and \mathbb{B} signify two classical global observables which characterize a thermodynamic system in its wholeness. In the same formula w denotes the phenomenological probability distribution, $\langle (\dots) \rangle_w$ represents the mean (expected value) of the quantity (\dots) evaluated by means of w while $\Delta_w \mathbb{A}$, $\Delta_w \mathbb{B}$ and $\langle \delta_w \mathbb{A} \cdot \delta_w \mathbb{B} \rangle_w$ stand for characteristics (standard deviations respectively correlation) regarding the fluctuations of the mentioned observables. We remind the appreciation that in classical physics the alluded characteristics and, consequently, the relations (31) describe the intrinsic (own) properties of thermodynamic systems but not the aspects of measurements performed on the respective systems. Such an appreciation is legitimated for example by the research regarding the fluctuation spectroscopy [57] where the properties of macroscopic (thermodynamic) systems are evaluated through the (spectral components of) characteristics like $\Delta_w \mathbb{A}$ and $\langle \delta_w \mathbb{A} \cdot \delta_w \mathbb{B} \rangle_w$.

The above discussions disclose the groundlessness of idea [58–60] that the relations like (31) have to be regarded as a sign of a macroscopic/classical complementarity (similar with the quantum complementarity motivated by CIUR presumption P_4). According to the respective idea the quantities $\Delta_w \mathbb{A}$ and $\Delta_w \mathbb{B}$ appear as macroscopic uncertainties. Note that the mentioned idea was criticized partially in [61, 62] but without any explicit specification that the quantities $\Delta_w \mathbb{A}$ and

$\Delta_w \mathbb{B}$ are quantities which characterise the macroscopic fluctuations.

The previously notified quantum-classical similarity together with the reminded significance of the quantities implied in (31) suggests and consolidates the following regard (argued also in R_3). The quantities $\Delta_\Psi A$ and $\Delta_\Psi B$ from RSUR (2)/(8) as well as from CSF (5) must be regarded as describing intrinsic properties (fluctuations) of quantum observables A and B but not as uncertainties of such observables.

Now, in conclusion, one can say that the existence of classical relations (31) contravenes to both presumptions P_1 and P_5 of CIUR. Of course that such a conclusion must be announced as a clear deficiency of CIUR.

End of R_{11}

R_{12} : On the higher order fluctuations moments

In classical physics the fluctuations of thermodynamic observables \mathbb{A} and \mathbb{B} implied in (31) are described not only by the second order probabilistic moments like $\Delta_w \mathbb{A}$, $\Delta_w \mathbb{B}$ or $\langle \delta_w \mathbb{A} \delta_w \mathbb{B} \rangle_w$. For a better evaluation the respective fluctuations are characterized additionally [63] by higher order moments like $\langle (\delta_w \mathbb{A})^r (\delta_w \mathbb{B})^s \rangle_w$ with $r + s \geq 3$. This fact suggests the observation that, in the context considered by CIUR, we also have to use the quantum higher order probabilistic moments like $(\langle \delta_\Psi \hat{A}_j \rangle_\Psi, \langle \delta_\Psi \hat{A}_k \rangle_\Psi)$, $r + s \geq 3$. Then for the respective quantum higher order moments CIUR is obliged to offer an interpretation compatible with its own doctrine. But it seems to be improbable that such an interpretation can be promoted through credible (and natural) arguments resulting from the CIUR own presumptions.

That improbability reveal one more deficiency of CIUR.

End of R_{12}

R_{13} : On the so-called “macroscopic operators”

Another obscure aspect of CIUR was pointed out in connection with the question of the so called “macroscopic operators”. The question was debated many years ago (see [64, 65] and references) and it seems to be ignored in the last decades, although until now it was not elucidated. The question appeared due to a forced transfer of RSUR (2) for the cases of quantum statistical systems. Through such a transfer CIUR partisans promoted the formula

$$\Delta_\rho \mathbb{A} \cdot \Delta_\rho \mathbb{B} \geq \frac{1}{2} \left| \langle [\hat{\mathbb{A}}, \hat{\mathbb{B}}] \rangle_\rho \right|. \quad (32)$$

This formula refers to a quantum statistical system in a state described by the statistical operator (density matrix) $\hat{\rho}$.

With \mathbb{A} and \mathbb{B} are denoted two macroscopic (global) observables associated with the operators $\hat{\mathbb{A}}$ and $\hat{\mathbb{B}}$. The quantity

$$\Delta_\rho \mathbb{A} = \left\{ \text{Tr} \left[(\hat{\mathbb{A}} - \langle \mathbb{A} \rangle_\rho)^2 \right] \right\}^{\frac{1}{2}}$$

denotes the standard deviation of the macroscopic observable \mathbb{A} regarded as a (generalised) random variable. In its expression the respective quantity imply the notation $\langle \mathbb{A} \rangle_\rho = \text{Tr}(\hat{\mathbb{A}} \hat{\rho})$

for the mean (expected) value of the macroscopic observable $\hat{\mathbb{A}}$.

Relation (32) entailed discussions because of the conflict between the following two findings:

- (i) On the one hand (32) is introduced by analogy with RSUR (2) on which CIUR is founded. Then, by extrapolating CIUR, the quantities $\Delta_\rho \hat{\mathbb{A}}$ and $\Delta_\rho \hat{\mathbb{B}}$ from (32) should be interpreted as (global) uncertainties subjected to stipulations as the ones indicated in the basic presumption \mathbf{P}_1 ;
- (ii) On the other hand, in the spirit of the presumption \mathbf{P}_5 , CIUR agrees the possibility that macroscopic observables can be measured without any uncertainty (i.e. with unbounded accuracy). For an observable the mentioned possibility should be independent of the fact that it is measured solitarily or simultaneously with other observables. Thus, for two macroscopic (thermodynamic) observables, it is senselessly to accept CIUR basic presumptions \mathbf{P}_3 and \mathbf{P}_4 .

In order to elude the mentioned conflict it was promoted the idea to abrogate the formula (32) and to replace it with an adjusted macroscopic relation concordant with CIUR vision. For such a purpose the global operators $\hat{\mathbb{A}}$ and $\hat{\mathbb{B}}$ from (32) were substituted [64, 65] by the so-called “macroscopic operators” $\hat{\mathcal{A}}$ and $\hat{\mathcal{B}}$. The respective “macroscopic operators” are considered to be representable as quasi-diagonal matrices (i.e. as matrices with non-null elements only in a “microscopic neighbourhood” of principal diagonal). Then one supposes that $[\hat{\mathcal{A}}, \hat{\mathcal{B}}] = 0$ for any pairs of “macroscopic observables” $\hat{\mathcal{A}}$ and $\hat{\mathcal{B}}$. Consequently instead of (32) it was introduced the formula

$$\Delta_\rho \hat{\mathcal{A}} \cdot \Delta_\rho \hat{\mathcal{B}} \geq 0. \quad (33)$$

In this formula CIUR partisans see the fact that the uncertainties $\Delta_\rho \hat{\mathcal{A}}$ and $\Delta_\rho \hat{\mathcal{B}}$ can be unboundedly small at the same time moment, for any pair of observables $\hat{\mathcal{A}}$ and $\hat{\mathcal{B}}$ and for any system. Such a fact constitute the CIUR vision about macroscopic observables. Today it seems to be accepted the belief that mentioned vision solves all the troubles of CIUR caused by the formula (32).

A first disapproval of the mentioned belief results from the following observations:

- (i) Relation (32) cannot be abrogated if the entire mathematical apparatus of quantum statistical physics is not abrogated too. More exactly, the substitution of operators from the usual global version $\hat{\mathbb{A}}$ into a “macroscopic” variant $\hat{\mathcal{A}}$ is a senseless invention as long as in practical procedures of quantum statistical physics [66, 67] for lucrative operators one uses $\hat{\mathbb{A}}$ but not $\hat{\mathcal{A}}$;
- (ii) The substitution $\hat{\mathbb{A}} \rightarrow \hat{\mathcal{A}}$ does not metamorphose automatically (32) into (33), because if two operators are quasi-diagonal, in sense required by the partisans of CIUR, it is not surely that they commute.

For an illustration of the last observation we quote [68] the Cartesian components of the global magnetization $\hat{\mathbb{M}}$ of a paramagnetic system formed of N independent $\frac{1}{2}$ -spins. The alluded components are described by the global operators

$$\hat{\mathbb{M}}_\alpha = \frac{\gamma \hbar}{2} \hat{\sigma}_\alpha^{(1)} \oplus \frac{\gamma \hbar}{2} \hat{\sigma}_\alpha^{(2)} \oplus \dots \oplus \frac{\gamma \hbar}{2} \hat{\sigma}_\alpha^{(N)}, \quad (34)$$

where $\alpha = x, y, z$; $\gamma =$ magneto-mechanical factor and $\hat{\sigma}_\alpha^{(i)}$ = Pauli matrices associated to the i -th spin (particle). Note that the operators (34) are quasi-diagonal in the sense required by CIUR partisans, i.e. $\hat{\mathbb{M}}_\alpha \equiv \hat{\mathcal{M}}_\alpha$. But, for all that, they do not commute because $[\hat{\mathcal{M}}_\alpha, \hat{\mathcal{M}}_\beta] = i \hbar \gamma \cdot \varepsilon_{\alpha\beta\mu} \cdot \hat{\mathcal{M}}_\mu$ ($\varepsilon_{\alpha\beta\mu}$ denote the Levi-Civita tensor).

A second disapproval of the belief induced by the substitution $\hat{\mathbb{A}} \rightarrow \hat{\mathcal{A}}$ is evidenced if the relation (32) is regarded in an ab original QM approach like the one presented in \mathbf{R}_2 . In such regard it is easy to see that in fact the formula (32) is only a restrictive descendant from the generally valid relation

$$\Delta_\rho \hat{\mathbb{A}} \cdot \Delta_\rho \hat{\mathbb{B}} \geq \left| \langle \delta_\rho \hat{\mathbb{A}} \cdot \delta_\rho \hat{\mathbb{B}} \rangle_\rho \right|, \quad (35)$$

where $\delta_\rho \hat{\mathbb{A}} = \hat{\mathbb{A}} - \langle \hat{\mathbb{A}} \rangle_\rho$. In the same regard for the “macroscopic operators” $\hat{\mathcal{A}}$ and $\hat{\mathcal{B}}$ instead of the restricted relation (33) it must considered the more general formula

$$\Delta_\rho \hat{\mathcal{A}} \cdot \Delta_\rho \hat{\mathcal{B}} \geq \left| \langle \delta_\rho \hat{\mathcal{A}} \cdot \delta_\rho \hat{\mathcal{B}} \rangle_\rho \right|. \quad (36)$$

The above last two relations justify the following affirmations:

- (i) Even in the situations when $[\hat{\mathcal{A}}, \hat{\mathcal{B}}] = 0$ the product $\Delta_\rho \hat{\mathcal{A}} \cdot \Delta_\rho \hat{\mathcal{B}}$ can be lower bounded by a non-null quantity. This happens because it is possible to find cases in which the term from the right hand side of (36) has a non-null value;
- (ii) In fact the substitution $\hat{\mathbb{A}} \rightarrow \hat{\mathcal{A}}$ replace (35) with (36). But for all that the alluded replacement does not guarantee the validity of the relation (33) and of the corresponding speculations.

The just presented facts warrant the conclusion that the relation (32) reveal a real deficiency of CIUR. The respective deficiency cannot be avoided by resorting to the so-called “macroscopic operators”. But note that the same relation does not rise any problem if it is considered together with (35) as formulas which refer to the fluctuations of macroscopic (global) observables regarding thermodynamic systems.

End of \mathbf{R}_{13}

\mathbf{R}_{14} : On the similarities between calassical Boltzmann’s and quantum Planck’s constants k_B and \hbar

The quantum-classical similarity revealed in \mathbf{R}_{11} entails also a proof against the CIUR presumption \mathbf{P}_5 . According to the respective presumptions the Planck constant \hbar has no analog in classical (non-quantum) physics. The announced proof can be pointed out as follows.

The here discussed similarity regards the groups of classical respectively quantum relations (31) and (5) (the last ones including their restricted descendant RSUR (2)/(8)). The respective relations imply the standard deviations $\Delta_w \mathbb{A}$ or $\Delta_\Psi A$ associated with the fluctuations of the corresponding classical and quantum observables. But mathematically the standard deviation indicate the randomness of an observable. This in the sense that the alluded deviation has a positive or null value as the corresponding observable is a random or, alternatively, a deterministic (non-random) variable. Therefore the quantities $\Delta_w \mathbb{A}$ and $\Delta_\Psi A$ can be regarded as similar indicators of randomness for the classical respectively quantum observables.

For diverse cases (of observables, systems and states) the classical standard deviations $\Delta_w \mathbb{A}$ have various expressions in which, apparently, no common element seems to be implied. Nevertheless such an element can be found out [69] as being materialized by the Boltzmann constant k_B . So, in the framework of phenomenological theory of fluctuations (in Gaussian approximation) one obtains [69]

$$(\Delta_w \mathbb{A})^2 = k_B \cdot \sum_{\alpha} \sum_{\beta} \frac{\partial \bar{\mathbb{A}}}{\partial \bar{X}_{\alpha}} \cdot \frac{\partial \bar{\mathbb{A}}}{\partial \bar{X}_{\beta}} \cdot \left(\frac{\partial^2 \bar{S}}{\partial \bar{X}_{\alpha} \partial \bar{X}_{\beta}} \right)^{-1}. \quad (37)$$

In this relation $\bar{\mathbb{A}} = \langle \mathbb{A} \rangle_w$, $\bar{S} = \bar{S}(\bar{X}_{\alpha})$ denotes the entropy of the system written as a function of independent thermodynamic variables \bar{X}_{α} , ($\alpha = 1, 2, \dots, r$) and $(a_{\alpha\beta})^{-1}$ represent the elements for the inverse of matrix $(a_{\alpha\beta})$. Then from (37) it result that the expressions for $(\Delta_w \mathbb{A})^2$ consist of products of k_B with factors which are independent of k_B . The respective independence is evidenced by the fact that the alluded factors must coincide with deterministic (non-random) quantities from usual thermodynamics (where the fluctuations are neglected). Or it is known that such quantities do not imply k_B at all. See [69] for concrete exemplifications of the relations (37) with the above noted properties.

Then, as a first aspect, from (37) it results that the fluctuations characteristics $(\Delta_w \mathbb{A})^2$ (i.e. dispersions = squares of the standard deviations) are directly proportional to k_B and, consequently, they are non-null respectively null quantities as $k_B \neq 0$ or $k_B \rightarrow 0$. (Note that because k_B is a physical constant the limit $k_B \rightarrow 0$ means that the quantities directly proportional with k_B are negligible comparatively with other quantities of same dimensionality but independent of k_B .) On the other hand, the second aspect (mentioned also above) is the fact that $\Delta_w \mathbb{A}$ are particular indicators of classical randomness. Conjointly the two mentioned aspects show that k_B has the qualities of an authentic generic indicator of thermal randomness which is specific for classical macroscopic (thermodynamic) systems. (Add here the observation that the same quality of k_B can be revealed also [69] if the thermal randomness is studied in the framework of classical statistical mechanics).

Now let us discuss about the quantum randomness whose

indicators are the standard deviations $\Delta_\Psi A$. Based on the relations (26) one can say that in many situations the expressions for $(\Delta_\Psi A)^2$ consist in products of Planck constant \hbar with factors which are independent of \hbar . (Note that a similar situation can be discovered [33] for the standard deviations of the observables L_z and φ in the case of quantum torsion pendulum.) Then, by analogy with the above discussed classical situations, \hbar places itself in the posture of generic indicator for quantum randomness.

In the mentioned roles as generic indicators k_B and \hbar , in direct connections with the quantities $\Delta_w \mathbb{A}$ and $\Delta_\Psi A$, regard the onefold (simple) randomness, of classical and quantum nature respectively. But in physics is also known a twofold (double) randomness, of a combined thermal and quantum nature. Such a kind of randomness one encounters in cases of quantum statistical systems and it is evaluated through the standard deviations $\Delta_\rho \mathbb{A}$ implied in relations (32) and (35). The expressions of the mentioned deviations can be obtained by means of the fluctuation-dissipation theorem [70] and have the form

$$(\Delta_\rho \mathbb{A})^2 = \frac{\hbar}{2\pi} \int_{-\infty}^{\infty} \coth \left(\frac{\hbar\omega}{2k_B T} \right) \chi''(\omega) d\omega. \quad (38)$$

Here $\chi''(\omega)$ denote the imaginary parts of the susceptibility associated with the observable \mathbb{A} and T represents the temperature of the considered system. Note that $\chi''(\omega)$ is a deterministic quantity which appear also in non-stochastic framework of macroscopic physics [71]. That is why $\chi''(\omega)$ is independent of both k_B and \hbar . Then from (38) it results that k_B and \hbar considered together appear as a couple of generic indicators for the twofold (double) randomness of thermal and quantum nature. The respective randomness is negligible when $k_B \rightarrow 0$ and $\hbar \rightarrow 0$ and significant when $k_B \neq 0$ and $\hbar \neq 0$ respectively.

The above discussions about the classical and quantum randomness respectively the limits $k_B \rightarrow 0$ and $\hbar \rightarrow 0$ must be supplemented with the following specifications.

- (i) In the case of the classical randomness it must considered the following fact. In the respective case one associates the limits $k_B \rightarrow 0$ respectively “(classical) microscopic approach” \rightarrow “(classical) macroscopic approach”. But in this context $k_B \rightarrow 0$ is concomitant with the condition $N \rightarrow 0$ (N = number of microscopic constituents (molecules) of the considered system). The respective concomitance assures the transformation $k_B N \rightarrow \nu R$, i.e. transition of physical quantities from “microscopic version” into a “macroscopic version” (because R sidnify the macoscopic gass constant and ν denotes the macroscopic amount of substance;
- (ii) On the other hand in connection with the quantum case it must taken into account the following aspect. The corresopnding randomness regards the cases of observables of orbital and spin types respectively;

(iii) In the orbital cases the limit $\hbar \rightarrow 0$ is usually associated with the quantum \rightarrow classical limit. The respective limit implies an unbounded growth of the values of some quantum numbers so as to ensure a correct limit for the associated observables regarding orbital movements. Then one finds [72, 73] that, when $\hbar \rightarrow 0$, the orbital-type randomness is in one of the following two situations:

(a) it converts oneself in a classical-type randomness of the corresponding observables (e.g. in the cases of φ and L_z of a torsional pendulum or of x and p of a rectilinear oscillator), or

(b) it disappears, the corresponding observables becoming deterministic classical variables (e.g. in the case of the distance r of the electron in respect with the nucleus in a hydrogen atom);

(iv) The quantum randomness of spin-type regards the spin observables. In the limit $\hbar \rightarrow 0$ such observables disappear completely (i.e. they lose both their mean values and the affined fluctuations).

In the alluded posture the Planck constant \hbar has an authentic classical analog represented by the Boltzmann constant k_B . But such an analogy contradicts strongly the presumption P_5 and so it reveals a new deficiency of CIUR.

End of R_{14}

Within this section, through the remarks R_1 – R_{14} , we examined a collection of things whose ensemble point out deficiencies which incriminate all the basic presumptions P_1 – P_5 of CIUR, considered as single or grouped pieces. In regard to the truth qualities of the respective deficiencies here is the place to note the following completion remark:

R_{15} : On the validity of the above signallized CIUR deficiencies

The mentioned deficiencies are indubitable and valid facts which can not be surmounted (avoided or rejected) by solid and verisimilar arguments taken from the inner framework of CIUR doctrine.

End of R_{15}

4 Consequences of the previous examination

The discussions belonging to the examination from the previous section impose as direct consequences the following remarks:

R_{16} : On the indubitable failure of CIUR

In the mentioned circumstances CIUR proves oneself to be indubitably in a failure situation which deprives it of necessary qualities of a valid scientific construction. That is why CIUR must be abandoned as a wrong doctrine which, in fact, has no real value.

End of R_{16}

R_{17} : On the true significance of the relations (1) and (2)

The alluded abandonment has to be completed by a natural re-interpretation of the basic CIUR's relations (1) and (2). We opine that the respective re-interpretation have to be done and argued by taking into account the discussions from the previous Section, mainly those from the remarks R_1 , R_2 and R_3 . We appreciate that in the alluded re-interpretation must be included the following viewpoints:

(i) On the one hand the relations (1) remain as provisional fictions destitute of durable physical significance;

(ii) On the other hand the relations (2) are simple fluctuations formulae, from the same family with the microscopic and macroscopic relations from the groups (4), (5), (29), (30) respectively (31), (32), (35);

(iii) None of the relations (1) and (2) or their adjustments have not any connection with the description of QMS.

Consequently in fact the relations (1) and (2) must be regarded as pieces of fiction respectively of mathematics without special or extraordinary status/significance for physics.

End of R_{17}

R_{18} : On the non-influences towards the usual QM

The above noted reconsideration of CIUR does not disturb in some way the framework of usual QM as it is applied concretely in the investigations of quantum microparticles. (Few elements from the respective framework are reminded above in the remark R_2).

End of R_{18}

5 Some considerations on the quantum and classical measurements

The question regarding the QMS description is one of the most debated subject associated with the CIUR history. It generated a large diversity of viewpoints relatively to its importance and/or approach (see [1–9] and references). The respective diversity inserts even some extreme opinions such are the ones noted in the Section 1 of the present paper. As a notable aspect many of the existing approaches regarding the alluded question are grounded on some views which presume and even try to extend the CIUR doctrine. Such views (v.) are:

(v.1) The descriptions of QMS must be developed as confirmations and extensions of CIUR concepts;

(v.2) The peculiarities of QMS incorporated in CIUR presumptions P_2 – P_4 are connected with the corresponding features of the measuring perturbations. So in the cases of observables referred in P_2 – P_3 respectively in P_4 the alluded perturbations are supposed to have an avoidable respectively an unavoidable character;

(v.3) In the case of QMS the mentioned perturbations cause specific jumps in states of the measured quantum mi-

cro-particles (systems). In many modern texts the respective jumps are suggested to be described as follows. For a quantum observable A of a microparticle in the state Ψ a QMS is assumed to give as result a single value say a_n which is one of the eigenvalues of the associated operator \hat{A} . Therefore the description of the respective QMS must include as essential piece a “collapse” (sudden reduction) of the wave function i.e a relation of the form:

$$\Psi \left(\begin{array}{c} \text{before} \\ \text{measurement} \end{array} \right) \rightarrow \Psi_n \left(\begin{array}{c} \text{after} \\ \text{measurement} \end{array} \right), \quad (39)$$

where Ψ_n (after measurement) denotes the eigenfunction of the operator \hat{A} corresponding to the eigenvalue a_n ;

- (v.4) With regard to the observables of quantum and classical type respectively the measuring inconveniences (perturbations and uncertainties) show an essential difference. Namely they are unavoidable respectively avoidable characteristics of measurements. The mentioned difference must be taken into account as a main point in the descriptions of the measurements regarding the two types of observables;
- (v.5) The description of QMS ought to be incorporated as an inseparable part in the framework of QM. Adequately QM must be considered as a unitary theory both of intrinsic properties of quantum microparticles and of measurements regarding the respective properties.

Here is the place to insert piece-by-piece the next remark:

R₁₉: Counter-arguments to the above views

The above mentioned views about QMS must be appreciated in conformity with the discussions detailed in the previous sections. For such an appreciation we think that it must taken into account the following counter-arguments (c-a):

- (c-a.1) According to the remark **R₁₆**, in fact CIUR is nothing but a wrong doctrine which must be abandoned. Consequently CIUR has to be omitted but not extended in any lucrative scientific question, particularly in the description of QMS. That is why the above view (v.1) is totally groundless;
- (c-a.2) The view (v.2) is inspired and argued by the ideas of CIUR about the relations (1) and (2). But, according to the discussions from the previous sections, the respective ideas are completely unfounded. Therefore the alluded view (v.2) is deprived of any necessary and well-grounded justification;
- (c-a.3) The view (v.3) is inferred mainly from the belief that the mentioned jumps have an essential importance for QMS.
But the respective belief appears as entirely unjustified if one takes into account the following natural and indubitable observation [74]: “*it seems essential to the*

notion of measurement that it answers a question about the given situation existing before the measurement. Whether the measurement leaves the measured system unchanged or brings about a new and different state of that system is a second and independent question”.

Also the same belief appears as a fictitious thing if we take into account the quantum-classical probabilistic similarity presented in the remark **R₁₁**. According to the respective similarity, a quantum observable must be regarded mathematically as a random variable. Then a measurement of such a observable must consist not in a single trial (which give a unique value) but in a statistical selection/sampling (which yields a spectrum of values). For more details regarding the measurements of random observables see below in this and in the next sections.

So we can conclude that the view (v.3) is completely unjustified;

- (c-a.4) The essence of the difference between classical and quantum observables supposed in view (v.4) is questionable at least because of the following two reasons:
- (a) In the classical case the mentioned avoidance of the measuring inconveniences have not a significance of principle but only a relative and limited value (depending on the performances of measuring devices and procedures). Such a fact seems to be well known by experimenters.
- (b) In the quantum case until now the alluded unavoidable-ness cannot be justified by valid arguments of experimental nature (see the above remark **R₁₆** and the comments regarding the relation (3));
- (c-a.5) The view (v.5) proves to be totally unjustified if the usual conventions of physics are considered. According to the respective conventions, in all the basic chapters of physics, each observable of a system is regarded as a concept “*per se*” (in its essence) which is denuded of measuring aspects. Or QM is nothing but such a basic chapter, like classical mechanics, thermodynamics, electrodynamics or statistical physics. On the other hand in physics the measurements appear as main purposes for experiments. But note that the study of the experiments has its own problems [75] and is done in frameworks which are additional and distinct in respect with the basic chapters of physics. The above note is consolidated by the observation that [76]: “*the procedures of measurement (comparison with standards) has a part which cannot be described inside the branch of physics where it is used*”.
- Then, in contrast with the view (v.5), it is natural to accept the idea that QM and the description of QMS have to remain distinct scientific branches. However the two branches have to use some common concepts

and symbols. This happens because, in fact, both of them also imply elements regarding the same quantum microparticles (systems).

The here presented counter-arguments contradict all the above presented views (v.1)–(v.5) promoted in many of the existing approaches regarding the QMS description.

End of R₁₉

On the basis of discussions presented in **R₁₁** and reminded in (c-a.3) from **R₁₉** a quantum observables must be considered as random variables having similar characteristics which correspond to the classical random observables. Then it results that, on principle, the description of QMS can be approached in a manner similar with the one regarding the corresponding classical measurements. That is why below we try to resume a model promoted by us in [77, 78] and destined to describe the measurement of classical random observables.

For the announced resume we consider a classical random observable from the family discussed in **R₁₁**. Such an observable and its associated probability distribution will be depicted with the symbols $\tilde{\mathbb{A}}$ respectively $w = w(a)$. The individual values a of $\tilde{\mathbb{A}}$ belong to the spectrum $a \in (-\infty, \infty)$. For the considered situation a measurement preserve the spectrum of $\tilde{\mathbb{A}}$ but change the distribution $w(a)$ from a “in” (input) version $w_{in}(a)$ into an “out” (output) reading $w_{out}(a)$. Note that $w_{in}(a)$ describes the intrinsic properties of the measured system while $w_{out}(a)$ incorporates the information about the same system, but obtained on the recorder of measuring device. Add here the fact that, from a general perspective, the distributions $w_{in}(a)$ and $w_{out}(a)$ incorporate informations referring to the measured system. That is why a measurement appears as an “*informational input → output transmission process*”. Such a process is symbolized by a transformation of the form $w_{in}(a) \rightarrow w_{out}(a)$. When the measurement is done by means of a device with stationary and linear characteristics, the the mentioned transformation can be described as follows:

$$w_{out}(a) = \int_{-\infty}^{\infty} G(a, a') w_{in}(a') da'. \quad (40)$$

Here the kernel $G(a, a')$ represents a transfer probability with the significances:

- (i) $G(a, a') da$ enotes the (infinitesimal) probability that by measurement the *in*-value a' of $\tilde{\mathbb{A}}$ to be recorded in the *out*-interval $(a; a + da)$;
- (ii) $G(a, a') da'$ stands for the probability that the *out*-value a to result from the *in*-values which belong to the interval $(a'; a' + da')$.

Due to the mentioned significances the kernel $G(a, a')$ satisfies the conditions

$$\int_{-\infty}^{\infty} G(a, a') da = \int_{-\infty}^{\infty} G(a, a') da' = 1. \quad (41)$$

Add here the fact that, from a physical perspective, the kernel $G(a, a')$ incorporates the theoretical description of all the characteristics of the measuring device. For an ideal device which ensure $w_{out}(a) = w_{in}(a)$ it must be of the form $G(a, a') = \delta(a - a')$ (with $\delta(a - a')$ denoting the Dirac’s function of argument $a - a'$).

By means of $w_{\eta}(a)$ ($\eta = in, out$) the corresponding global (or numerical) characteristics of $\tilde{\mathbb{A}}$ regarded as random variable can be introduced. In the spirit of usual practice of physics we refer here only to the two lowest order such characteristics. They are the η — mean (expected) value $\langle \mathbb{A} \rangle_{\eta}$ and η — standard deviations $\Delta_{\eta} \mathbb{A}$ defined as follows

$$\left. \begin{aligned} \langle \mathbb{A} \rangle_{\eta} &= \int_{-\infty}^{\infty} a w_{\eta}(a) da \\ (\Delta_{\eta} \mathbb{A})^2 &= \langle (\mathbb{A} - \langle \mathbb{A} \rangle_{\eta})^2 \rangle_{\eta} \end{aligned} \right\}. \quad (42)$$

Now, from the general perspective of the present paper, it is of interest to note some observations about the measuring uncertainties (errors). Firstly it is important to remark that for the discussed observable \mathbb{A} , the standard deviations $\Delta_{in} \mathbb{A}$ and $\Delta_{out} \mathbb{A}$ are not estimators of the mentioned uncertainties. Of course that the above remark contradicts some loyalties induced by CIUR doctrine. Here it must be pointed out that:

- (i) On the one hand $\Delta_{in} \mathbb{A}$ together with $\langle \mathbb{A} \rangle_{in}$ describe only the intrinsic properties of the measured system;
- (ii) On the other hand $\Delta_{out} \mathbb{A}$ and $\langle \mathbb{A} \rangle_{out}$ incorporate composite information about the respective system and the measuring device.

Then, in terms of the above considerations, the measuring uncertainties of \mathbb{A} are described by the following error indicators (characteristics)

$$\left. \begin{aligned} \varepsilon \{ \langle \mathbb{A} \rangle \} &= | \langle \mathbb{A} \rangle_{out} - \langle \mathbb{A} \rangle_{in} | \\ \varepsilon \{ \Delta \mathbb{A} \} &= | \Delta_{out} \mathbb{A} - \Delta_{in} \mathbb{A} | \end{aligned} \right\}. \quad (43)$$

Note that because \mathbb{A} is a random variable for an acceptable evaluation of its measuring uncertainties it is completely insufficient the single indicator $\varepsilon \{ \langle \mathbb{A} \rangle \}$. Such an evaluation requires at least the couple $\varepsilon \{ \langle \mathbb{A} \rangle \}$ and $\varepsilon \{ \Delta \mathbb{A} \}$ or even the differences of the higher order moments like

$$\varepsilon \{ \langle (\delta \mathbb{A})^n \rangle \} = | \langle (\delta_{out} \mathbb{A})^n \rangle_{out} - \langle (\delta_{in} \mathbb{A})^n \rangle_{in} |, \quad (44)$$

where $\delta_{\eta} \mathbb{A} = \tilde{\mathbb{A}} - \langle \mathbb{A} \rangle_{\eta}$; $\eta = in, out$; $n \geq 3$).

Now we wish to specify the fact that the errors (uncertainties) indicators (43) and (44) are *theoretical (predicted) quantities*. This because all the above considerations consist in a theoretical (mathematical) modelling of the discussed measuring process. Or within such a modelling we operate only with theoretical (mathematical) elements presumed to reflect

in a plausible manner all the main characteristics of the respective process. On the other hand, comparatively, in experimental physics, the indicators regarding the measuring errors (uncertainties) are *factual entities* because they are estimated on the basis of factual experimental data. But such entities are discussed in the framework of observational error studies.

6 An informational model for theoretical description of QMS

In the above, (c-a.5) from R_{19} , we argued for the idea that QM and the description of QMS have to remain distinct scientific branches which nevertheless have to use some common concepts and symbols. Here we wish to put in a concrete form the respective idea by recommending a reconsidered model for description of QMS. The announced model will assimilate some elements discussed in the previous section in connection with the measurements of classical random observables.

We restrict our considerations only to the measurements of quantum observables of orbital nature (i.e. coordinates, momenta, angles, angular momenta and energy). The respective observables are described by the following operators \hat{A}_j ($j = 1, 2, \dots, n$) regarded as generalized random variables. As a measured system we consider a spinless microparticle whose state is described by the wave function $\Psi = \Psi(\vec{r})$, taken in the coordinate representation (\vec{r} stand for microparticle position). Add here the fact that, because we consider only a non-relativistic context, the explicit mention of time as an explicit argument in the expression of Ψ is unimportant.

Now note the observation that the wave function $\Psi(\vec{r})$ incorporate information (of probabilistic nature) about the measured system. That is why a QMS can be regarded as a *process of information transmission*: from the measured microparticle (system) to the recorder of the measuring device. Then, on the one hand, the input (*in*) information described by $\Psi_{in}(\vec{r})$ refers to the intrinsic (own) properties of the respective microparticle (regarded as information source). The expression of $\Psi_{in}(\vec{r})$ is deducible within the framework of usual QM (e.g. by solving the adequate Schrödinger equation). On the other hand, the output (*out*) information, described by the wave function $\Psi_{out}(\vec{r})$, refers to the data obtained on the device recorder (regarded as information receiver). So the measuring device plays the role of the transmission channel for the alluded information. Accordingly the measurement appears as a processing information operation. By regarding the things as above the description of the QMS must be associated with the transformation

$$\Psi_{in}(\vec{r}) \rightarrow \Psi_{out}(\vec{r}). \tag{45}$$

As in the classical model (see the previous section), without any loss of generality, here we suppose that the quantum observables have identical spectra of values in both *in*- and *out*-situations. In terms of QM the mentioned supposition

means that the operators \hat{A}_j have the same mathematical expressions in both *in*- and *out*-readings. The respective expressions are the known ones from the usual QM.

In the framework delimited by the above notifications the description of QMS requires to put the transformation (45) in concrete forms by using some of the known QM rules. Additionally the same description have to assume suggestions from the discussions given in the previous section about measurements of classical random observables. That is why, in our opinion, the transformation (45) must be detailed in terms of quantum probabilities carriers. Such carriers are the probabilistic densities ρ_η and currents \vec{J}_η defined by

$$\rho_\eta = |\Psi_\eta|^2, \quad \vec{J}_\eta = \frac{\hbar}{m_0} |\Psi_\eta|^2 \cdot \nabla \Phi_\eta. \tag{46}$$

Here $|\Psi_\eta|$ and Φ_η represents the modulus and the argument of Ψ_η respectively (i.e. $\Psi_\eta = |\Psi_\eta| \exp(i\Phi_\eta)$) and m_0 denotes the mass of microparticle.

The alluded formulation is connected with the observations [79] that the couple $\rho-\vec{J}$ “*encodes the probability distributions of quantum mechanics*” and it “*is in principle measurable by virtue of its effects on other systems*”. To be added here the possibility [80] of taking in QM as primary entity the couple $\rho_{in}-\vec{J}_{in}$ but not the wave function Ψ_{in} (i.e. to start the construction of QM from the continuity equation for the mentioned couple and subsequently to derive the Schrödinger equation for Ψ_{in}).

According to the above observations the transformations (45) have to be formulated in terms of ρ_η and \vec{J}_η . But ρ_η and \vec{J}_η refer to the position and the motion kinds of probabilities respectively. Experimentally the two kinds can be regarded as measurable by distinct devices and procedures. Consequently the mentioned formulation has to combine the following two distinct transformations

$$\rho_{in} \rightarrow \rho_{out}, \quad \vec{J}_{in} \rightarrow \vec{J}_{out}. \tag{47}$$

The considerations about the classical relation (40) suggest that, by completely similar arguments, the transformations (47) admit the following formulations

$$\rho_{out}(\vec{r}) = \iiint \Gamma(\vec{r}, \vec{r}') \rho_{in}(\vec{r}') d^3 \vec{r}' \tag{48}$$

$$J_{out; \alpha} = \sum_{\beta=1}^3 \iiint \Lambda_{\alpha\beta}(\vec{r}, \vec{r}') J_{in; \beta}(\vec{r}') d^3 \vec{r}'. \tag{49}$$

In (49) $J_{\eta; \alpha}$ with $\eta = in, out$ and $\alpha = 1, 2, 3 = x, y, z$ denote Cartesian components of \vec{J}_η .

Note the fact that the kernels Γ and $\Lambda_{\alpha\beta}$ from (48) and (49) have significance of transfer probabilities, completely analogous with the meaning of the classical kernel $G(a, a')$ from (40). This fact entails the following relations

$$\iiint \Gamma(\vec{r}, \vec{r}') d^3 \vec{r}' = \iiint \Gamma(\vec{r}, \vec{r}') d^3 \vec{r} = 1, \tag{50}$$

$$\begin{aligned} \sum_{\alpha=1}^3 \iiint \Lambda_{\alpha\beta}(\vec{r}, \vec{r}') d^3\vec{r}' &= \\ &= \sum_{\beta=1}^3 \iiint \Lambda_{\alpha\beta}(\vec{r}, \vec{r}') d^3\vec{r}' = 1. \end{aligned} \quad (51)$$

The kernels Γ and $\Lambda_{\alpha\beta}$ describe the transformations induced by QMS in the data (information) about the measured microparticle. Therefore they incorporate some extra-QM elements regarding the characteristics of measuring devices and procedures. The respective elements do not belong to the usual QM framework which refers to the intrinsic (own) characteristics of the measured microparticle (system).

The above considerations facilitate an evaluation of the effects induced by QMS on the probabilistic estimators of here considered orbital observables A_j . Such observables are described by the operators \hat{A}_j whose expressions depend on \vec{r} and ∇ . According to the previous discussions the mentioned operators are supposed to remain invariant under the transformations which describe QMS. So one can say that in the situations associated with the wave functions Ψ_η ($\eta = in, out$) the mentioned observables are described by the following probabilistic estimators/characteristics (of lower order): mean values $\langle A_j \rangle_\eta$, correlations $C_\eta(A_j, A_k)$ and standard deviations $\Delta_\eta A_j$. With the usual notation $(f, g) = \int f^* g d^3\vec{r}$ for the scalar product of functions f and g , the mentioned estimators are defined by the relations

$$\left. \begin{aligned} \langle A_j \rangle_\eta &= (\Psi_\eta, \hat{A}_j \Psi_\eta) \\ \delta_\eta \hat{A}_j &= \hat{A}_j - \langle A_j \rangle_\eta \\ C_\eta(A_j, A_k) &= (\delta_\eta \hat{A}_j \Psi_\eta, \delta_\eta \hat{A}_k \Psi_\eta) \\ \Delta_\eta A_j &= \sqrt{C_\eta(A_j, A_j)} \end{aligned} \right\}. \quad (52)$$

Add here the fact that the *in*-version of the estimators (52) are calculated by means of the wave function Ψ_{in} , known from the considerations about the inner properties of the investigated system (e.g. by solving the corresponding Schrödinger equation).

On the other hand the *out*-version of the respective estimators can be evaluated by using the probability density and current ρ_{out} and \vec{J}_{out} . So if \hat{A}_j does not depend on ∇ (i.e. $\hat{A}_j = A_j(\vec{r})$) in evaluating the scalar products from (52) one can use the evident equality $\Psi_\eta \hat{A}_j \Psi_\eta = \hat{A}_j \rho_\eta$. When \hat{A}_j depends on ∇ (i.e. $\hat{A}_j = A_j(\nabla)$) in the same products can be appealed to the substitution

$$\Psi_\eta^* \nabla \Psi_\eta = \frac{1}{2} \nabla \rho_\eta + \frac{im}{\hbar} \vec{J}_\eta, \quad (53)$$

$$\Psi_\eta^* \nabla^2 \Psi_\eta = \rho_\eta^{\frac{1}{2}} \nabla^2 \rho_\eta^{\frac{1}{2}} + \frac{im}{\hbar} \nabla \vec{J}_\eta - \frac{m^2}{\hbar^2} \frac{\vec{J}_\eta^2}{\rho_\eta}. \quad (54)$$

The mentioned usage seems to allow the avoidance of the implications regarding [79] “*a possible nonuniqueness of current*” (i.e. of the couple $\rho_\eta - \vec{J}_\eta$).

Within the above presented model of QMS the errors (uncertainties) associated with the measurements of observables A_j can be evaluated through the following indicators

$$\left. \begin{aligned} \varepsilon \{ \langle A_j \rangle \} &= | \langle A_j \rangle_{out} - \langle A_j \rangle_{in} | \\ \varepsilon \{ C(A_j, A_k) \} &= | C_{out}(A_j, A_k) - C_{in}(A_j, A_k) | \\ \varepsilon \{ \Delta A_j \} &= | \Delta_{out} A_j - \Delta_{in} A_j | \end{aligned} \right\}. \quad (55)$$

These quantum error indicators are entirely similar with the classical ones (43). Of course that, mathematically, they can be completed with error indicators like $\varepsilon \{ ((\delta_\Psi \hat{A}_j)^r \Psi, (\delta_\Psi \hat{A}_k)^s \Psi) \}$, $r + s \geq 3$, which regard the higher order probabilistic moments mentioned in \mathbf{R}_{12} .

The above presented model regarding the description of QMS is exemplified in the end of this paper in Annex.

Now is the place to note that the *out*-version of the estimators (52), as well as the error indicators (55), have a theoretical significance.

In practice the verisimilitude of such estimators and indicators must be tested by comparing them with their *experimental (factual) correspondents* (obtained by sampling and processing of the data collected from the recorder of the measuring device). If the test is confirmative both theoretical descriptions, of QM intrinsic properties of system (microparticle) and of QMS, can be considered as adequate. But if the test gives an invalidation of the results, at least one of the mentioned descriptions must be regarded as inadequate.

In the end of this section we wish to add the following two observations:

- (i) The here proposed description of QMS does not imply some interconnection of principle between the measuring uncertainties of two distinct observables. This means that from the perspective of the respective description there are no reasons to discuss about a measuring compatibility or incompatibility of two observables;
- (ii) The above considerations from the present section refer to the QMS of orbital observables. Similar considerations can be also done in the case of QMS regarding the spin observables. In such a case besides the probabilities of spin-states (well known in QM publications) it is important to take into account the spin current density (e.g. in the version proposed recently [81]).

7 Some conclusions

We started the present paper from the ascertained fact that in reality CIUR is troubled by a number of still unsolved defici-

encies. For a primary purpose of our text, we resumed the CIUR history and identified its basic presumptions. Then, we attempt to examine in details the main aspects as well as the validity of CIUR deficiencies regarded in an elucidative collection.

The mentioned examination, performed in Section 3 reveal the following aspects:

- (i) A group of the CIUR deficiencies appear from the application of usual RSUR (2) in situations where, mathematically, they are incorrect;
- (ii) The rest of the deficiencies result from unnatural associations with things of other nature (e.g. with the thought experimental relations or with the presence/absence of \hbar in some formulas);
- (iii) Moreover one finds that, if the mentioned applications and associations are handled correctly, the alluded deficiencies prove themselves as being veridic and unavoidable facts. The ensemble of the respective facts invalidate all the basic presumptions of CIUR.

In consensus with the above noted findings, in Section 4, we promoted the opinion that CIUR must be abandoned as an incorrect and useless (or even misleading) doctrine. Conjointly with the respective opinion we think that the primitive UR (the so called Heisenberg's relations) must be regarded as:

- (i) fluctuation formulas — in their theoretical RSUR version (2);
- (ii) fictitious things, without any physical significance — in their thought-experimental version (1).

Abandonment of CIUR requires a re-examination of the question regarding QMS theoretical description. To such a requirement we tried to answer in Sections 5 and 6. So, by a detailed investigation, we have shown that the CIUR-connected approaches of QMS are grounded on dubitable (or even incorrect) views.

That is why we consider that the alluded question must be reconsidered by promoting new and more natural models for theoretical description of QMS. Such a model, of somewhat informational concept, is developed in Section 6 and it is exemplified in Annex.

Of course that, as regards the QMS theoretical description, our proposal from Section 6, can be appreciated as only one among other possible models. For example, similarly with the discussions regarding classical errors [77, 78], the QMS errors can be evaluated through the informational (Shannon) entopies.

It is to be expected that, in connection with QMS, other models will be also promoted in the next months/years. But as a general rule all such models have to take into account the indubitable fact that the usual QM and QMS theoretical description must be referred to distinct scientific questions (objectives).

Annex: A simple exemplification for the model presented in Section 6

For the announced exemplification let us refer to a microparticle in a one-dimensional motion along the x -axis. We take $\Psi_{in}(x) = |\Psi_{in}(x)| \cdot \exp\{i\Phi_{in}(x)\}$ with

$$|\Psi_{in}(x)| \propto \exp\left\{-\frac{(x-x_0)^2}{4\sigma^2}\right\}, \quad \Phi_{in}(x) = kx. \quad (56)$$

Note that here as well as in other relations from this Annex we omit an explicit notation of the normalisation constants which can be added easy by the interested readers.

Correspondingly to the Ψ and Φ from (56) we have

$$\rho_{in}(x) = |\Psi_{in}(x)|^2, \quad J_{in}(x) = \frac{\hbar k}{m_0} |\Psi_{in}(x)|^2. \quad (57)$$

So the intrinsic properties of the microparticle are described by the parameters x_0 , σ and k .

If the errors induced by QMS are small the kernels Γ and Λ in (48)–(49) can be considered of Gaussian forms like

$$\Gamma(x, x') \propto \exp\left\{-\frac{(x-x')^2}{2\gamma^2}\right\}, \quad (58)$$

$$\Lambda(x, x') \propto \exp\left\{-\frac{(x-x')^2}{2\lambda^2}\right\}, \quad (59)$$

where γ and λ describe the characteristics of the measuring devices. Then for ρ_{out} and J_{out} one finds

$$\rho_{out}(x) \propto \exp\left\{-\frac{(x-x')^2}{2(\sigma^2 + \gamma^2)}\right\}, \quad (60)$$

$$J_{out}(x) \propto \hbar k \cdot \exp\left\{-\frac{(x-x')^2}{2(\sigma^2 + \lambda^2)}\right\}. \quad (61)$$

It can be seen that in the case when both $\gamma \rightarrow 0$ and $\lambda \rightarrow 0$ the kernels $\Gamma(x, x')$ and $\Lambda(x, x')$ degenerate into the Dirac's function $\delta(x-x')$. Then $\rho_{out} \rightarrow \rho_{in}$ and $J_{out} \rightarrow J_{in}$. Such a case corresponds to an ideal measurement. Alternatively the cases when $\gamma \neq 0$ and/or $\lambda \neq 0$ are associated with non-ideal measurements.

As observables of interest we consider coordinate x and momentum p described by the operators $\hat{x} = x$ and $\hat{p} = -i\hbar \frac{\partial}{\partial x}$. Then, in the measurement modeled by the expressions (56), (58) and (59), for the errors (uncertainties) of the considered observables one finds

$$\varepsilon\{x\} = 0, \quad \varepsilon\{p\} = 0, \quad \varepsilon\{C(x, p)\} = 0, \quad (62)$$

$$\varepsilon\{\Delta x\} = \sqrt{\sigma^2 + \gamma^2} - \sigma, \quad (63)$$

$$\varepsilon \{ \Delta p \} = \hbar \left[\left| \frac{k^2 (\sigma^2 + \gamma^2)}{\sqrt{(\sigma^2 + \lambda^2)(\sigma^2 + 2\gamma^2 - \lambda^2)}} - k^2 + \frac{1}{4(\sigma^2 + \gamma^2)} \right|^{\frac{1}{2}} - k \right]. \quad (64)$$

If in (56) we restrict to the values $x_0 = 0$, $k = 0$ and $\sigma = \sqrt{\frac{\hbar}{2m_0\omega}}$ our system is just a linear oscillator in its ground state ($m_0 =$ mass and $\omega =$ angular frequency). This means that the “in”-wave function (56) has the same expression with the one from (14) for $n = 0$. As observable of interest we consider the energy described by the Hamiltonian

$$\hat{H} = -\frac{\hbar^2}{2m_0} \frac{d^2}{dx^2} + \frac{m_0\omega^2}{2} x^2. \quad (65)$$

Then for the respective observable one finds

$$\langle H \rangle_{in} = \frac{\hbar\omega}{2}, \quad \Delta_{in} H = 0, \quad (66)$$

$$\langle H \rangle_{out} = \frac{\omega \left[\hbar^2 + (\hbar + 2m_0\omega\gamma^2)^2 \right]}{4(\hbar + 2m_0\omega\gamma^2)}, \quad (67)$$

$$\Delta_{out} H = \frac{\sqrt{2} m_0 \omega^2 \gamma^2 (\hbar + m_0 \omega \gamma^2)}{(\hbar + 2m_0 \omega \gamma^2)}. \quad (68)$$

The corresponding errors of mean value respectively of standard deviation of oscillator energy have the expressions

$$\varepsilon \{ \langle H \rangle \} = | \langle H \rangle_{out} - \langle H \rangle_{in} | \neq 0, \quad (69)$$

$$\varepsilon \{ \Delta H \} = | \Delta_{out} H - \Delta_{in} H | \neq 0. \quad (70)$$

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A Model of Discrete-Continuum Time for a Simple Physical System

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Proceeding from the assumption that the time flow of an individual object is a real physical value, in the framework of a physical kinetics approach we propose an analogy between time and temperature. The use of such an analogy makes it possible to work out a discrete-continuum model of time for a simple physical system. The possible physical properties of time for the single object and time for the whole system are discussed.

Commonly, time is considered to be a fundamental property of the Universe, and the origin which is not yet clear enough for natural sciences, although it is widely used in scientific and practical activity. Different hypotheses of temporal influence on physical reality and familiar topics have been discussed in modern scientific literature (see, e.g., [1–3] and references therein). In particular, the conception of discrete time-space was proposed in order to explain a number of physical effects (e.g., the problem of asymmetry of some physical phenomena and divergences in field theory) [2–4]. Following this theme, in the present paper we shall consider some aspects of the pattern of discrete-continuum time for a single object and for the whole system. We will focus on the difference between time taken as a property of a single object and a property of the system. We would also touch upon the question of why the discreteness of time is not obvious in ordinary conditions.

As a “given” property of existence time is assumed to be an absolutely passive physical factor and the flow of time is always uniform in ordinary conditions (here we consider the non-relativistic case) for all objects of our world. Therefore classical mechanics proceeds from the assumption that the properties of space and time do not depend on the properties of moving material objects. However even mechanics suggests that other approaches are possible.

From the point of view of classical mechanics a reference frame is in fact a geometrical reference frame of each material object with an in-built “clock” registering time for each particular object. So in fixing the reference frame we deal with the time of each unique object only and subsequently this time model is extended to all other objects of concrete reality. Thus, we always relate time to some concrete object (see, e.g., [5]). Here we seem to neglect the fact that such an assumption extends the time scale as well as the time flow of only one object onto reality in general. This approach is undoubtedly valid for mechanical systems. In the framework of such an approach there is no difference between the time of an unique object and the time of the system containing a lot of objects.

But is it really so? Will the time scale of the system taken as a whole be the same as the time scale of each of the ele-

ments constituting the system? It is of interest to consider the opposite case, i.e. when time for a single object and time for the system of objects do not coincide. So we set out to try to develop a time model for a physical system characterized by continuum and discrete time properties which arise from the assumption that the time flow of an individual object is a real physical value as, for example, the mass or the charge of the electron. In other words, following Mach, we are going to proceed from the assumption that if there is no matter, there is no time.

In order to show the plausibility of such an approach we shall consider a set of material N objects, for example, structureless particles without any force-field interaction between them. Each object is assumed to have some individual physical characteristics and each object is the carrier of its own local time, i.e. for each i -object we shall define its own time flow with some temporal scale θ_i as

$$dt_i = \theta_i dt, \quad (1)$$

where t is the ordinary Newtonian time. Generally speaking, one can expect dependence of θ_i on the physical characteristics of the object, for example, both kinetic and potential energy of the object. However, here we shall restrict ourselves to consideration of the simple case when $\theta_i = \text{const}$.

Since we associate objects with particles we shall also assume that there are collisions between particles and the value of θ_i remains constant until the object comes into contact with other objects, as θ_i may be changed only during the impact, division or merger of objects. This means that the dynamics of a single object without interaction with other objects is determined only by its own time t_i . If, however, we consider the dynamics of an i -object with another j -object we have to take into account some common time of i - and j -objects which we are to determine.

This consideration suggests that in order to describe the whole system (here we shall use the term “system” to denote a set of N objects which act as a single object) one should use something close or similar to a physical kinetics pattern where macroscopic parameters like density, temperature etc., are defined by averaging the statistically significant ensemble of objects. In particular, for the system containing N particles

the temperature may be written as

$$T = \frac{1}{N} \sum_{i=1}^N v_i^2 - \left(\frac{1}{N} \sum_{i=1}^N v_i \right)^2, \quad (2)$$

where v_i is the velocity of the i -object. For the whole system we introduce the general time τ to replace the local time of the i -object (1) as

$$d\tau = (1 + D) dt, \quad (3)$$

where D is determined by the differential relation

$$D(\tau) = \left[\frac{1}{N} \sum_{i=1}^N (\theta_i)^2 - \left(\frac{1}{N} \sum_{i=1}^N \theta_i \right)^2 \right]^{1/2}. \quad (4)$$

By such a definition the general time of the system is the sum total of its Newtonian times and some nonlinear time $D(t_i)$ which is a function that depends on the dispersion of the individual times dt_i . It is noteworthy that this simplest possible statistical approach is similar to that of [6, 7].

It is quite evident that we have Newtonian-like time even if $D = \text{const} \neq 0$. Indeed, from (3) it follows that

$$\tau = (1 + D)t. \quad (5)$$

The pure Newtonian case in relation (3) is realized when all objects have the same temporal scales $\theta_i = \theta_0$.

At the same time there exist a number of cases in which the violation of the pure Newtonian case may occur. For example, let us assume that we have got a system where some number of objects would perish, disappear, whilst another set of the objects might come into existence. In this case the number N is variable and we have to consider D as an explosive step-like function with respect to N , which we ought to integrate (3) only in some interval $t_0 \leq t \leq t_x$ where D remains constant. Here it is obvious that the value of such an interval $t_x - t_0$ is initially unknown. Instead of the Newtonian continuum time (5) we now get a piecewise linear continuous time which is determined by the following recurrence relation

$$\tau = (1 + D)(t - t_0) + \tau_0, \quad t_0 \leq t \leq t_x, \quad (6)$$

where $\tau_0 = \tau(t_0)$. This relation remains true whilst D is not changed. At the moment of local time $t = t_x$ the value D becomes $D + \Delta D$, so we have to redefine τ_0 and other parameters as

$$\left. \begin{aligned} \tau_0 &:= \tau(t_x) = (1 + D)(t_x - t_0) + \tau_0 \\ t_0 &:= t_x, \quad D := D + \Delta D \end{aligned} \right\}. \quad (7)$$

Thus, instead of the linear Newtonian time for a single object we get the broken linear dependence for the time of the whole system if the number of objects forming this system is continuously changing.

Since in reality the majority of objects, as a rule, form some systems consisting of elementary units, it can be concluded that the number of constituent elements might change,

as was shown above in the example considered. In this case D becomes variable and one deals with the manifestation of a piecewise linear dependence of time.

However, it is clear that the effects of this non-uniform time can be revealed to best advantage in a system with a rather small N , since in the limit $N \rightarrow \infty$ the parameter D becomes little sensitive to the changes in N . That is the basic reason why, in ordinary conditions, we may satisfy ourselves with the Newtonian time conception alone.

In the present paper we have tried to draw an analogy between time and temperature for the simplest possible physical system without collective interaction of the objects constituting the system, in order to show the difference in the definition of time for unique objects and for whole systems. One should consider this case as a basic simplified example of the system where the discrete-continuum properties of time may be observed. Thus one should consider it as a rather artificial case since there are no physical objects without field-like interactions between them.

However, despite the simplified case considered above, the piecewise linear properties of time may in fact be observed in reality (in ordinary, non-simplified conditions), though they are by no means obvious. In order to reveal the of dispersion time $D(\tau)$ it is necessary to create some specific experimental conditions. Temporal effects, in our opinion, are best observed in systems characterized by numerous time scales and a relatively small number of constituent elements.

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Models for Quarks and Elementary Particles — Part I: What is a Quark?

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A quark is not a tiny sphere. The formal model idea is based on a vector group which is constructed like an outer vector product. The vectors perform dynamic movements. Two vectors (vector pair) which rotate in opposite directions in a plane have an increasing and diminishing result vector as consequence. At the same time the vector group rotates about the bisectrix of the vector pair. The two movements matched to each other result in that the tip of the resultant vector draws so-called geometrical locus loops in a plane. The u- and the d-quarks have characteristic loops. Each vector group has its own orthogonal, hyperbolic space. By joining three such spaces each, two groups of spaces, one group with a quasi-Euclidian and one group with a complex space are obtained. Based on the u- and d-quarks characterized with their movements and spaces a first elementary particle order is compiled.

1 Introduction

The models are presented in a comprehensive work* and comprise a large number of aspects. Not all of these can be reflected in the present publication. For this reason, only the prominent aspects are presented in four short Parts I to IV.

It is clear that the answer to the question of the heading cannot originate from experiments. A quark is a part of the confinements, of the interior of the elementary particles, which are not accessible for experiments. For this reason the answer in the present case is based on a model, (*lexically = draft, hypothetical presentation to illustrate certain statements; hypothesis = initially unconfirmed assumption of legitimacy with the objective of making them a guaranteed part of our knowledge through confirmation later on*) which on the one hand is based on secured, e.g. QED, physical theories (*lexically = scientific presentation, system of scientific principles*). The answer is not based on one or several axioms (*lexically = immediately obvious tenet which in itself cannot be justified*).

The model or the models were developed during a journey of thinking taking decades from the galaxies to the quarks, to the elementary particles, back to the stars and again to the confinement, the universe as a puzzle.

2 The vector principle

The photon contains electric and magnetic fields and is described with appropriate vectors. This formal description possibility is utilised. Why does the photon have the electric and magnetic vectors positioned vertically to the direction of flight and vertically with regard to each other, the understanding of this will be developed during the course of the model

*There is a homepage under the Internet address www.universum-un.de where a book with the title "Models for Quarks and Elementary Particles" will be displayed, having a volume of approximately 250 DIN A4 pages.

development. For this reason it is obvious that a long distance over highly formal stretches was covered which is not re-enacted here in detail.

It is highly productive to start from the idea of the outer vector product known from mathematics: two vectors of identical size start in a coordinate origin and open up a plane. The resultant vector (EV) stands vertically on this plane and likewise starts in the coordinate origin. In the next step the three vectors of the outer vector product are given a dynamic characteristic. Two movements are introduced:

Firstly, the two identically sized vectors perform a movement in opposite directions. Since the angle between the two vectors $\mathbf{V1}$ and $\mathbf{V2}$ is called 2φ , this is described as φ -rotation or φ -swivelling; see Fig. 1.

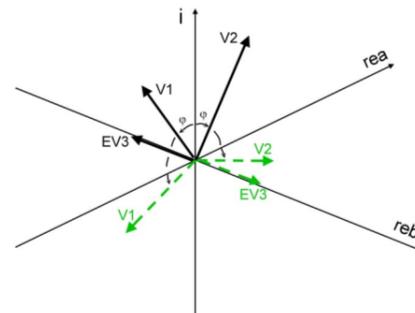


Fig. 1

If the two vectors according to Fig. 1 perform smaller and opposing φ -swivel movements, the resultant vector $\mathbf{EV3}$ becomes greater and smaller in its orientation.

Secondly, the entire vector arrangement measured by Fig. 1 performs a rotation about the bisectrix between the vectors $\mathbf{V1}$ and $\mathbf{V2}$. Since this angle of rotation is referred to as ρ , this rotation is a ρ -rotation or a ρ -swivelling. If the vectors $\mathbf{V1}$ and $\mathbf{V2}$ during the ρ -rotation enclose a fixed angle, the vector tip of the \mathbf{EV} draws an arc of a circle. However,

should the angle 2φ between $\mathbf{V1}$ and $\mathbf{V2}$ change during the ρ -rotation, the tip of the \mathbf{EV} deviates from the arc of the circle; see Fig. 2.

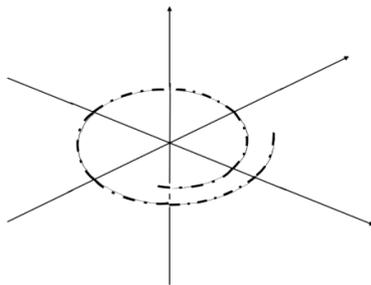


Fig. 2

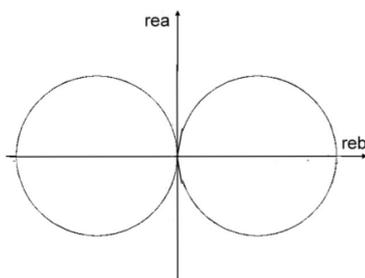


Fig. 3

It is immediately obvious that there are a huge number of possible combinations of the two φ - and ρ -movements in a coordinate system. In developing the models attention was paid to ensure that only φ - and ρ -movements that were matched to one another were considered. If for example each vector $\mathbf{V1}$ and $\mathbf{V2}$ starting from the vertical axis covers an angle $\varphi = 90^\circ$ and the \mathbf{EV} at the same time covers an angle of $\rho = 180^\circ$ in the horizontal plane, the tip of the \mathbf{EV} draws a loop in a plane. Assuming two vector pairs (one drawn black and one green) with the arrangement as in Fig. 1, two loops, see for instance Fig. 3 are obtained. Loops of this type are called geometric loci or geometric locus loops.

3 The three types of space

The limitation to a defined few coordinated φ - and ρ -movements is not yet sufficient to understand quarks. It is necessary to go beyond the Euclidian space with three orthogonal axes. At the same time, the principle of the vectors, especially that of the outer vector product should be maintained. The transition is made from the Euclidian space to the hyperbolic space with right angles between the axes. Here, it must be decided if the hyperbolic space should have one or two imaginary axes. Just as in the case of the vectors only very few models with matched φ - and ρ -movements were found to be carrying further, only few variants carry further with the space as well. (It has not been possible to find a similarly selective way from the amount of the approximately 10^{500} string theories and, in my opinion, will never be found either.)

Just as an outer vector product is productive as idea, it is also productive for the outer vector product, (for the first quark generation) to assume an orthogonal, hyperbolic space with two real axes and one imaginary axis; Fig. 4.

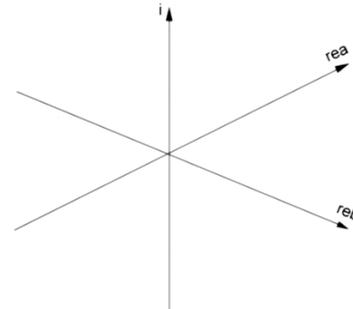


Fig. 4

So as not to create any misunderstanding at this point: it is not that several vector groups (one vector pair, \mathbf{VP} , and one \mathbf{EV} each) are placed in a hyperbolic orthogonal space with two real axes and one imaginary axis but each vector group has its own hyperbolic space. Here, the \mathbf{VP} can be positioned in the real plane or in a Gaussian plane.

Various combinations of the vector groups are possible, as a result of which individual spaces can also be combined differently. As with the φ - and ρ -movements and as with the hyperbolic space, a selection has to be performed also with the combination of individual spaces. Fig. 5 to Fig. 9 show such a selection. The choice of words of the captions to the Figures becomes clear only as this text progresses.

Taking into account quantum chromodynamics, which prescribes three-quark particles, the result of the selected combinations of such individual spaces is the following: only two groups of combined spaces of three vector groups each are obtained: either spaces which in each of the three orientations have at least one real axis (if applicable, superimposed by an imaginary axis) and are therefore called “quasi-Euclidian” (see Fig. 7 and Fig. 8), or spaces which only have imaginary axes in one of the three orientations and are therefore called “complex” (see Fig. 6).

Particles of three quarks have either a quasi-Euclidian or a complex overall space. The Euclidian space from the view of this model is fiction.

Note for Fig. 6 to Fig. 9: Variants of three hyperbolic spaces linked in the coordinate origin consisting of the hyperbolic spaces of a dual-coordination and the hyperbolic space of a singular quark in various arrangements.

4 The four quarks (of the first generation)

Taking into account the construction of a vector group, the matched φ - and ρ -movements, the orientation of \mathbf{VP} and \mathbf{EV} in the hyperbolic space and the electric charge a geometrical locus loop according to Fig. 10 is obtained for the d -quark

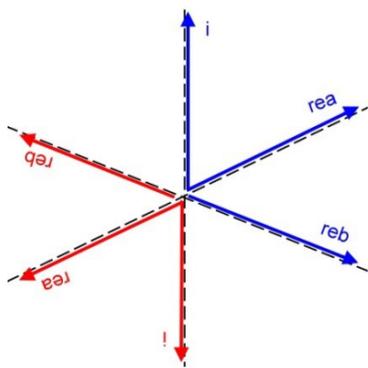


Fig. 5: The two ideal-typically arranged hyperbolic spaces of a dual-coordination as $\underline{dd}, \underline{uu}, \underline{dd}, \underline{uu}$, linked in the coordinate origin.

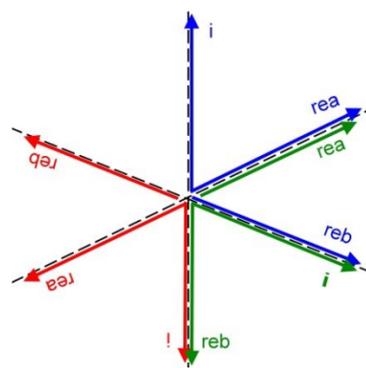


Fig. 9

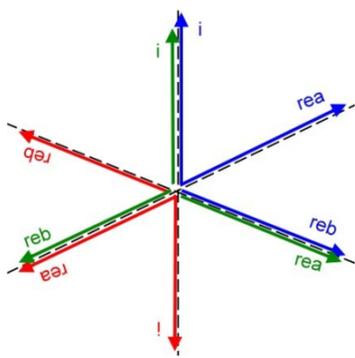


Fig. 6

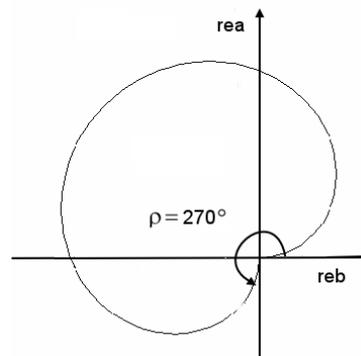


Fig. 10

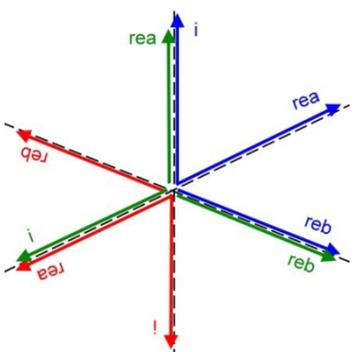


Fig. 7

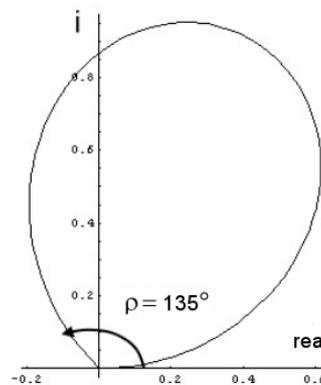


Fig. 11

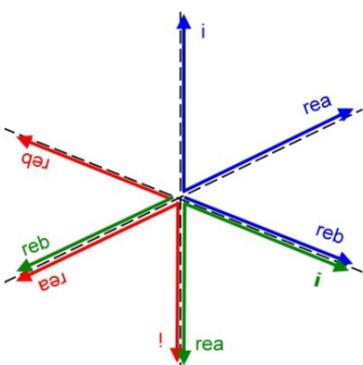


Fig. 8

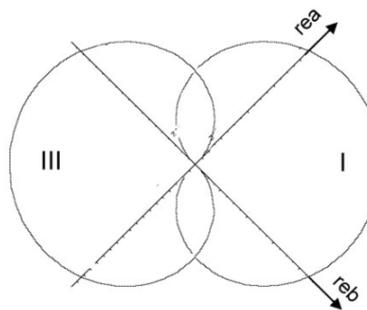


Fig. 12

with negative electric charge and a loop according to Fig. 11 is obtained for the u-quark with positive electric charge.

Antiquarks are characterized by an opposite electric charge so that the geometrical locus loop of the \bar{d} -quarks with positive electric charge is situated in a Gaussian plane and the geometrical locus loop of the \bar{u} -quark with negative electric charge is situated in the real plane.

5 Experiment of an order of elementary particles

Fig. 1 shows two vector groups (black and green) and Fig. 5 shows two hyperbolic spaces (blue and red); the vector groups and the hyperbolic spaces are each inter-linked in the coordinate origin. These presentations stem from the realisation that two quarks of the same type of each three-quark particle assume a particularly close bond. In the text this is called “dual-coordination”, or briefly, “Zk”. The third remaining quark of a three-quark particle is then called a “singular” quark. The different orientations of the quarks (**VP** and **EV**) with their spaces result in that the geometrical locus loops can stand at different angles relative to one another. A $\bar{d}\bar{d}$ -Zk for example has an angle zero between both ρ -rotation planes, see Fig. 12. The same applies to a uu -Zk with angle zero between both ρ -rotation planes. Since the planes are positioned in parallel, the symbol \parallel is used. If the rotation planes of two geometrical loci stand vertically on top of each other, the symbol \perp is used. Table 1 is produced with this system.

Line	ddd	ddu	duu	uuu
A	$\underline{d}\underline{d}\parallel d \equiv e_e$	$\underline{d}\underline{d}\parallel u \equiv n^0$	$d\parallel uu \equiv p^+$	$uu\parallel u \equiv \Delta_\Delta$
B	$\underline{d}\underline{d}\perp d \equiv e^-$	$\underline{d}\underline{d}\perp u \equiv \nu_e$	$d\perp uu \equiv ?^+$	$uu\perp u \equiv (\Delta^{++})$
C	$\underline{d}\underline{d}\underline{d} \equiv \Delta^-$	$\underline{d}\underline{d}u \equiv \Delta^0$	$duu \equiv \Delta^+$	$uuu \equiv \Delta^{++}$

Table 1: The order of particles, sorted by quark flavours and the parallel \parallel and vertical \perp orientation of the geometrical loci.

The esteemed reader will be familiar with four of the spin $\frac{1}{2} \hbar$ -particles (neutron n^0 , proton p^+ , electron e^- and neutrino ν_e) and, if applicable, the Δ -particles with spin $\frac{3}{2} \hbar$ from line C from high-energy physics. Because of the brevity of the present note the individual quark compositions will not be discussed. However, it is immediately evident that highly interesting consequences for the standard model of physics are obtained from the methodology of Table 1. This is evident on the examples of the electron and the neutrino, which, in the standard model, are considered as uniform particles, but here appear to be composed of quarks. In Parts II and IV of the publication the aspect of the electron composed of quarks is deepened. The structural nature of the quarks in the nucleons is another example for statements of these models that clearly go beyond the standard model.

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Models for Quarks and Elementary Particles — Part II: What is Mass?

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It is extremely productive to give the resultant vector (**EV**) from the outer vector product (Part I of this article series) a physical significance. The **EV** is assumed as electric flux ϵ with the dimensions [Vm]. Based on Maxwell’s laws this develops into the idea of the magnetic monopole (MMP) in each quark. The MMP can be brought in relation with the Dirac monopole. The massless MMP is a productive and important idea on the one hand to recognise what mass is and on the other hand to develop the quark structure of massless photon (-likes) from the quark composition of the electron. Based on the experiments by Shapiro it is recognised that the sinusoidal oscillations of the quark can be spiralled in the photons. In an extreme case the spiralling of such a sinusoidal arc produces the geometric locus loop of a quark in a mass-loaded particle.

1 Introduction

Based on some characteristics of the photon mentioned in Part I [1], vectors are introduced to describe the quarks. The formal structures of the quarks (of the first generation) are presented with outer vector product, its angular movements and the corresponding space types. A first order of the elementary particles follows [2].

2 The magnetic monopole (MMP)

It is highly productive to give the vectors from the outer vector product (Part I of this series of papers) a physical meaning. Initially, the **EV** is assumed as electric flux ϵ with the dimensions [Vm].

A very good model for further considerations is given in [3] (see Fig. 7.128, p.398 therein), in which a changing electric field with an enclosing magnetic field is shown. For the present models this should be formulated as follows: a vector pair (**VP**) generates the **EV** issuing from the origin of a coordinate system, which **EV** is now identified with an electric flux ϵ . When this flux is created, almost the entire electric flux ϵ based on Maxwell’s laws creates the magnetic flux Φ located ring-shaped about the ϵ -flux. With this linkage, the models are put on the basis of the QED mentioned in Part I. Feynman [22] calls the QED-theory the best available theory in natural sciences.

The electric source flux ϵ in turn comprises the toroidal magnetic flux Φ , (like the water of a fountain overflowing on all sides), whose maximum radius is designated MAGINPAR, which is illustrated with Fig. 1.

The ϵ -**EV** with toroidal magnetic flux Φ is a substantial part of the description of a quark. With the coverage of the toroidal magnetic flux Φ through the electric source flux ϵ it is also an obvious explanation for the magnetic flux Φ not appearing outside the confinement under normal circumstances. The ϵ -**EV** shown in Fig. 1 does not correspond to a dipole.

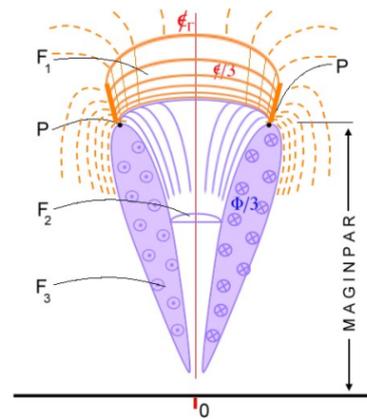


Fig. 1: Schematic section through the Φ - and ϵ -fields of a (d)-quark. In the Φ -tube or funnel the ϵ -field lines created in orange are not indicated. P designates the outer apex line of the Φ -flux which determines the MAGINPAR at the same time. Graphically, the configuration is also called “fountain”. The symbolic ϵ_r -field line lies on the funnel longitudinal axis and is discussed in Part III.

With the latter, the fields shown would be simultaneously visible on two sides of the coordinate origin, while an ϵ -flux trough would also have to appear opposite to a source flux ϵ .

A **Zk** (see Part I) comprises two such ϵ -source fluxes offset by 180° relative to each other which are merely like a dipole. A three-quark particle according to Table 1 (see Part I) comprises three ϵ -source fluxes.

Dirac has stated the charge of the magnetic monopole according to Jackson ([3], p.319), as follows:

$$g^2 = \frac{1}{\epsilon\alpha} \times \frac{n^2}{4} \times 4\pi\mu_0\hbar c \text{ [V}^2\text{s}^2\text{]} \quad \text{or} \quad g = \frac{n}{2} \times \sqrt{\frac{4\pi\mu_0\hbar c}{\epsilon\alpha}}$$

$$g = 4.1357 \times 10^{-15} \text{ [V s]} \quad \text{with } n = 1.$$

If this value is multiplied with double the value of the fine structure constant $2\epsilon\alpha = 1/68.518$, it is identical to the value

of the magnetic flux $\Phi = 6.03593 \times 10^{-17}$ [Vs] of the present models. The dimension of g is likewise identical to the magnetic flux ϕ of the present models, (see [2] Chapter 8.1).

The electron or the electric unit charge of $q = 1.60219 \times 10^{-19}$ [As] according to [1] Table 1 and according to [2] (Chapter 7 therein), consists of three d -quarks. Consequently the natural constant Φ does not stand for a quark either but for a “3QT”, i.e. according to a first assumption for the three d -quarks of the electron. Imagining the electric flux ϵ and the toroidal magnetic flux Φ of a quark according to Fig. 1 the magnetic fluxes of a d -quark or of a u -quark amount to:

$$\Phi_d = \frac{\Phi}{3} = \frac{6.03593 \times 10^{-17}}{3} = 2.01198 \times 10^{-17} \text{ [Vs]},$$

$$\Phi_u = \frac{2\Phi}{3} = 4.02396 \times 10^{-17} \text{ [Vs]}.$$

According to the present models these magnetic fluxes are the values of the magnetic monopoles (MMPs).

Obviously this means that we, and our entire world, also consist of the much sought-after MMPs.

The intensity of the interaction of the Dirac monopole is estimated extremely high. Since the MMP according to the present models is approximately $2^e \alpha$ smaller, the intensity of the interaction of the MMPs is substantially smaller as well. The force between two charged particles corresponds to the product of both charges:

$$\frac{g^2}{\Phi^2} = \frac{(4.1356 \times 10^{-15})^2}{(6.03593 \times 10^{-17})^2} = \frac{(68.518 \times 6.03593 \times 10^{-17})^2}{(6.03593 \times 10^{-17})^2} = \frac{68.518^2}{1} = \frac{4695}{1}.$$

The charge quantity g determined by Dirac thus results in 4695 times greater a force between the charges g than between the fluxes Φ . A further reduction of the interaction obviously results through the $\frac{\Phi}{3}$ and $\frac{2\Phi}{3}$ fragments of the d - or u -quarks. These reduction factors are not the sole cause for the quite obviously much lower intensity of the interaction of the MMPs than assumed by Dirac. The probably decisive reduction factor is the construction of the quark sketched in Fig. 1, where the magnetic flux Φ of a quark is shielded to the outside by the electric flux ϵ .

The literature sketches an MMP as follows:

- A constant magnetic field oriented to the outside on all sides (hedgehog) not allowing an approximation of additional MMPs;
- If two or more (anti-) MMPs attract one another, they are unable to assume a defined position relative to one another because of their point-symmetrical structure;
- The “literature MMP” is the logical continuation of the current world view of the “spheres” which is moderated through probability densities. Atoms are relatively

“large spheres”, nucleons are “very small spheres” therein, and the quarks would consequently be “even smaller spheres” in the nucleons and the electrons are allegedly point-like. The interactions between the “spheres” are secured by the bosons as photons or gluons.

The aspects of these models are:

- The idea of the “sphere chain” is exploded in these models since the swivelling and simultaneously pulsating MMPs act in all particles. Particles can be seen highly simplified as different constellations of MMPs;
- The idea of the “fountain” according to Fig. 1 contains the toroidal magnetic flux Φ as MMP;
- The structures brought about by the MMP are temporally, spatially and electromagnetically highly anisotropic and asymmetrical. Without this structure our world would not be possible. From this it can be concluded that the highly symmetrical “literature MMP” sketched above must not be seen as an elementary part of our world.

3 Some enigmas of the photon

- Why the photon has the electric and magnetic vectors positioned vertically to the direction of flight and vertically to each other is not answered in Part I;
- If the photon is created through “annihilation” of electron and positron as is well known and if the electron according to Table 1, Part I, has the quark structure $\underline{dd} \perp d$, the question arises if the characterisation of the photon with the simple letter γ according to the standard model is correct;
- If electron and positron have a basic mass $m = 0.511 \text{ MeV}/c^2$ why does the photon have the mass zero?
- Why does the wavelength of the light observed by us not fit to the Compton wavelength of the electrons emitting the light?

To solve the enigma, some courageous jumps have to be performed:

First jump: The photon consists of the same quark type as the electron, namely d according to Table 1 (of Part I);

Second jump: The photon contains its own anti-particle, i.e. consists of the quark types d and \bar{d} according to the models;

Third jump: Both quark groups ($3 d$ and $3 \bar{d}$) oscillate by themselves with very similar basic frequencies. This is explained as follows:

The Compton wavelength of the electron ($3 d$) or that of the positron ($3 \bar{d}$) in each case results in a basic frequency of approximately 10^{20} Hz . Thus the photon has two very similar basic frequencies. The beat resulting

from both frequencies has a wavelength or frequency which is greater and lower respectively by the factor 10^5 and with just under 10^{15} Hz is also in the visible range. The beat is the answer to Question (d) concerning the photon.

Some consequences of the courageous jumps:

- (1) The photon must be seen as a composite **yet uniform** particle;
- (2) Two frequencies in this uniform medium create a **beat**;
- (3) According to Table 1 of Part I, Line B, there are three additional leptons in addition to the electron (or its anti-particle positron). It can be expected that from these leptons and each of their anti-particles composite **yet uniform** particles can be formed according to the same pattern as with the photon. These particles are called “photon-like” in the models.

In Table 1, the quark structure of the electron is introduced with $\underline{d}\underline{d} \perp d$. Using the anti- d -quark the positron has the same structure. If both elementary particles in the photon are connected it should be unsurprisingly expected that both structures can be found again in the photon. In addition to this it should be expected that both particles are closely connected with each other. This is expressed in that the two singular quarks of electron and positron in turn assume a close bond. In the models this is called “bond coordination” or “Bk” in brief and in the case of the photon $\underline{d}\underline{d}$ as structural element. Consequently the overall structure of \bar{e} of the photon appears as $\underline{d}\underline{d} \perp \underline{d}\underline{d} \perp \underline{d}\underline{d}$. The overall photon-like structure of the neutrinos would be $\underline{d}\underline{d} \perp \underline{u}\underline{u} \perp \underline{d}\underline{d}$, etc.

In contrast with the three-quark particles of Table 1 the photon-likes are six-quark particles. It is clear that the six-quark structure of the photon-likes has substantial consequences on the reaction equations of the weak interaction. This is reported in Part IV. The quark structure of the photon is the answer to Question (b) concerning the photon.

4 The “pioneering” experiments of Shapiro

Years after the discovery of the quark structure of the photons and long after the insight, as to what mass actually is, was gained, the experiments by Shapiro [5] were brought into relation with both. Here, the experiments by Shapiro are dealt with first in order to facilitate introduction to the subjects.

Towards the end of the nineteen-sixties, Shapiro observed a reduced speed of light c_M near the Sun. The cause is the “refractive index of the vacuum”. Deviating from the interpretation through the standard model of physics and utilising new insights through these models the following is determined in a first jump:

Under the effect of directed electric fields the flat sinusoidal oscillation of the photon becomes helical (see [2], pages 167 and 179). This results in that at constant frequency

the penetration points of the sine curve through the “ x -axis” are situated closer together and the speed of the photon in direction of flight is no longer c but c_M .

Following this thought pattern it can be determined in a second jump: Under the effect of extremely strong highly directional electric fields the initially flat sinusoidal oscillation of the photon is spiralled to such an extent that the geometrical locus loops used for “stationary” particles appear (see [2], page 165ff and Fig. 2 and 3).

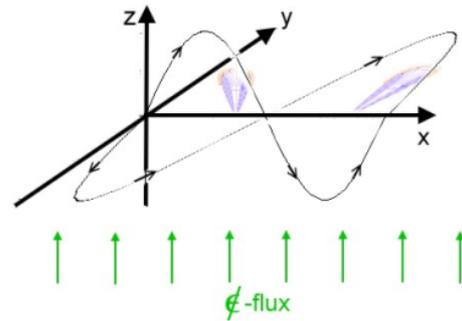


Fig. 2: A photon with initially flat sinusoidal arcs and with schematically sketched “fountain” runs vertically to the direction of an electric field while the arrows on the sinusoidal arcs indicate the sequence of the amplitude.

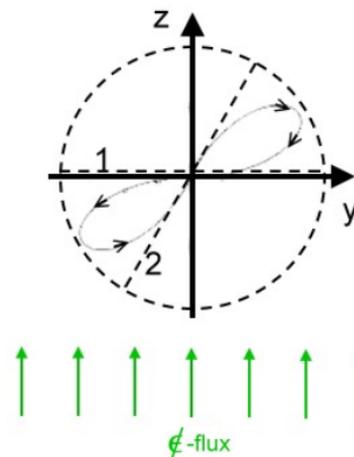


Fig. 3: Projection of the helically deformed initially horizontal and flat sinusoidal arcs of a photon according to Fig. 2 in the $y-z$ plane.

Looking at the helical sinusoidal oscillation in the direction of the x -axis a sinusoidal arc presents itself as a narrower or wider loop. If the loop is very narrow the progressive speed c_M of the photon is only a little smaller than c [5]. However if the loop is very wide the photon is practically unable to move. This means that the photon is then captured in an electron.

The extremely strong directional electric fields can be found in the source fields of the “fountain”, Fig. 1, of the electron quarks. This means that an electron with suitable MAG-INPAR is able to spiral the lateral sinusoidal oscillation of an approaching photon to such an extent that the lateral sinu-

soidal oscillation becomes a central-symmetrical sinusoidal oscillation. If the amplitudes or MAGINPAR of both particles fit to each other the photon is stored in the electron. This also means that an electron charged in this way — and that is every electron from our environment — has central-symmetrical sinusoidal oscillations of 3 d -quarks as well as stored 3 d - and 3 \bar{d} -groups of the photons.

It is now evident: the flat oscillation of the photon is converted to the radial oscillation in the electron or in the fermion through the extremely strong directional electric quark source fields. The geometrical locus loops developed from formal aspects which are shown in Part I for instance with Fig. 3 are sine curves or sinusoidal oscillations which are presented in polar coordinates for a centre each.

5 What is mass?

In Table 1 the neutron and the neutrino are positioned below each other. Both have the same types and quantities of quarks, however with different structural signs! The mass of the neutron almost amounts to 940 MeV, the mass of the neutrino according to the standard model below one eV. The electron and the positron each have a basic mass of 0.511 MeV, while the photon consisting of the same quarks has no mass at all.

Quite obviously, “mass” is not a characteristic of a quark. Mass is a characteristic which arises from the constellation of several quarks. Only certain elementary particles have mass! These include those where the MMPs perform central-symmetrical sinusoidal oscillations, e.g. the three-quark particles of Table 1. The amplitude of the central-symmetrical sinusoidal oscillations is practically identical with the MAGINPAR R . The magnitude of the MAGINPAR R is determined by the frequency ν via $\nu = \frac{c}{\lambda} = \frac{c}{X\pi R}$.

In [2] (page 164), mass is defined as follows:

$$m = \frac{h}{c^2} \nu = \frac{\Phi q}{2^{\epsilon} \alpha c^2} \nu = 7.3726 \times 10^{-51} \left[\frac{\text{V A s}^4}{\text{m}^2} \right] \times \nu \left[\frac{1}{\text{s}} \right]. \quad (1)$$

Conclusion: Mass is nothing other than the very, very frequent occurrence of the MMPs Φ at the coordinate centre of the particle in accordance with the frequency ν multiplied by the electric charge q divided by c^2 and also $2^{\epsilon} \alpha$. The constants jointly have the value $7.3726 \times 10^{-51} [\text{V A s}^4/\text{m}^2]$. These statements satisfy a desire of physics that has remained unanswered for a very long time. The masses of the mass-loaded elementary particles known to us that could only be experimentally measured in the past can be calculated from elementary quantities.

With a photon, the six quarks or MMPs involved describe a lateral movement along a line. The sinusoidal oscillations of the MMPs are not central-symmetrical. According to the

definition such particles have no mass. The lateral movement is the answer to Question (c) regarding the photon.

The well-known relation of mass m [$\text{V A s}^3/\text{m}^2$] and inertia ${}^N\Theta$ [$\text{V A s}^3/\text{m}$] becomes visible by introducing the equations $h = {}^N\hbar/2^{\epsilon} \alpha$ and ${}^N\hbar = {}^N\Theta \times c$. (See [2], Fig. 8.3a of Chapter 8.2.1 therein.) By this equation, equation (1) transforms to

$$m = \frac{{}^N\Theta \nu}{2^{\epsilon} \alpha c} \quad \text{or} \quad m = \frac{{}^N\Theta}{2^{\epsilon} \alpha X \pi R},$$

which is the short version of equation (8-II) on page 156 of [2].

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Electric Charge as a Form of Imaginary Energy

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Electric charge is considered as a form of imaginary energy. With this consideration, the energy of an electrically charged particle is a complex number. The real part is proportional to the mass, while the imaginary part is proportional to the electric charge. The energy of an antiparticle is given by conjugating the energy of its corresponding particle. Newton's law of gravity and Coulomb's law of electric force are classically unified into a single expression of the interaction between the complex energies of two electrically charged particles. Interaction between real energies (or masses) is the gravitational force. Interaction between imaginary energies (or electric charges) is the electromagnetic force. Since radiation is also a form of real energy, there are another two types of interactions between real energies: the mass-radiation interaction and the radiation-radiation interaction. Calculating the work done by the mass-radiation interaction on a photon, we can derive the Einsteinian gravitational redshift. Calculating the work done by the radiation-radiation interaction on a photon, we can obtain a radiation redshift. This study suggests the electric charge as a form of imaginary energy, so that classically unifies the gravitational and electric forces and derives the Einsteinian gravitational redshift.

1 Introduction

It is well known that mass and electric charge are two fundamental properties (inertia and electricity) of matter, which directly determine the gravitational and electromagnetic interactions via Newton's law of gravity [1] and Coulomb's law of electric force [2]. Mass is a quantity of matter [3], and the inertia of motion is solely dependent upon the mass. According to Einstein's energy-mass expression (or Einstein's first law) [4], mass is also understood as a form of real energy. The real energy is always positive. It cannot be destroyed but can be transferred from one form to another. Therefore, the mass is understood not only based on the gravitational interaction but also on the quantity of matter, the inertia of motion, and the energy

Electric charge has two varieties of either positive or negative. It appears always in association with mass to form positive or negative electrically charged particles with different masses. The interaction between electric charges, however, is independent of the mass. Positive and negative charges can annihilate or cancel each other and produce in pair with the total electric charges conserved. So far, the electric charge is understood only based on the electromagnetic interactions. Its own physics meaning of a pure electric charge is still unclear.

In this paper, the pure electric charge is suggested to be a form of imaginary energy. With this suggestion or idea of imaginary energy, we can express an electrically charged particle as a pack of certain amount of complex energy, in which the real part is proportional to the mass and the imaginary part is proportional to the electric charge. We can combine the

gravitational and electromagnetic interactions between two electrically charged particles into the interaction between their complex energies. We can also naturally obtain the energy of an antiparticle by conjugating the energy of its corresponding particle and derive the Einsteinian gravitational redshift from the mass-radiation interaction, a type of interaction between real energies.

2 Electric charge — a form of imaginary energy

With the idea that the electric charge is a form of imaginary energy, total energy of a particle can be generally expressed as a complex number

$$E = E^M + iE^Q, \quad (1)$$

where $i = \sqrt{-1}$ is the imaginary number. The real energy $\text{Re}(E) = E^M$ is proportional to the particle mass

$$E^M = Mc^2, \quad (2)$$

while the imaginary energy $\text{Im}(E) = E^Q$ is proportional to the particle electric charge defined by

$$E^Q = \frac{Q}{\sqrt{G}}c^2 = \alpha E^M, \quad (3)$$

where G is the gravitational constant, c is the light speed, and α is the charge-mass ratio (or the imaginary-real energy ratio) defined by

$$\alpha \equiv \frac{E^Q}{E^M} = \frac{Q}{\sqrt{GM}}, \quad (4)$$

in the cgs unit system. The imaginary energy has the same sign as the electric charge has. Including the electric charge,

we can modify Einstein's first law as

$$E = (1 + i\alpha) Mc^2. \tag{5}$$

The modulus of the complex energy is

$$|E| = \sqrt{1 + \alpha^2} Mc^2. \tag{6}$$

For an electrically charged particle, the absolute value of α is a big number. For instance, proton's α is about 10^{18} and electron's α is about -2×10^{21} . Therefore, an electrically charged particle holds a large amount of imaginary energy in comparison with its real or rest energy. A neutral particle such as a neutron, photon, or neutrino has only a real energy.

3 Unification of Newton's law of gravity and Coulomb's law

Considering two pointlike electrically charged objects with masses M_1, M_2 , electric charges Q_1, Q_2 , and distance r , we can unify Newton's law of gravity and Coulomb's law of electric force by the following single expression of the interaction between complex energies

$$\vec{F} = -G \frac{E_1 E_2}{c^4 r^3} \vec{r}, \tag{7}$$

where E_1 is the energy of object one and E_2 is the energy of object two. Eq. (7) shows that the interaction between two particles is proportional to the product of their energies and inversely proportional to the square of the distance between them.

Replacing E_1 and E_2 by using the complex energy expression (1), we obtain

$$\begin{aligned} \vec{F} &= -G \frac{M_1 M_2}{r^3} \vec{r} + \frac{Q_1 Q_2}{r^3} \vec{r} - i\sqrt{G} \frac{M_1 Q_2 + M_2 Q_1}{r^3} \vec{r} = \\ &= \vec{F}_{MM} + \vec{F}_{QQ} + i\vec{F}_{MQ}. \end{aligned} \tag{8}$$

The first term of Eq. (8) represents Newton's law for the gravitational interaction between two masses \vec{F}_{MM} . The second term represents Coulomb's law for the electromagnetic interaction between two electric charges \vec{F}_{QQ} . The third term is an imaginary force between the mass of one object and the electric charge of the other object $i\vec{F}_{MQ}$. This imaginary force is interesting and may play an essential role in adhering an electric charge on a mass or in combining an imaginary energy with a real energy. A negative imaginary force adheres a positive electric charge on a mass, while a positive imaginary force adheres a negative electric charge on a mass. Figure 1 sketches all of the interactions between two electrically charged particles as included in Eq. (8).

Electric charges have two varieties and thus three types of interactions: (1) repelling between positive electric charges \vec{F}_{++} , (2) repelling between negative electric charges \vec{F}_{--} , and (3) attracting between positive and negative electric charges \vec{F}_{+-} . Figure 2 shows the three types of the Coulomb interactions between two electric charges.

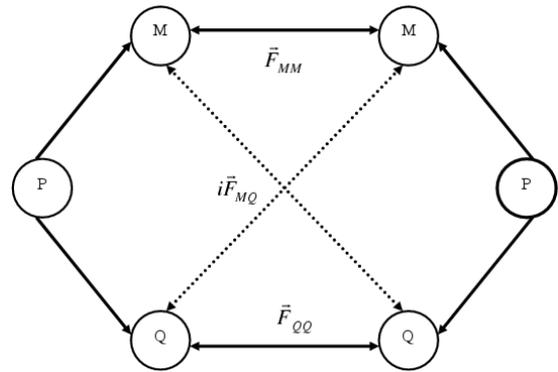


Fig. 1: Interactions between two electrically charged particles. They include (1) the gravitational force between masses, (2) the electric force between charges, and (3) the imaginary force between mass and charge.

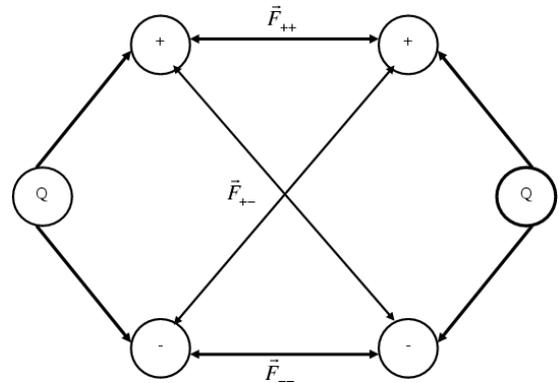


Fig. 2: Interactions between two electric charges. They include (1) repelling between two positive charges, (2) repelling between two negative charges, and (3) attraction between positive and negative charges.

4 Energy of antiparticles

The energy of an antiparticle [5, 6] is naturally obtained by conjugating the energy of the corresponding particle

$$E^* = (E^M + iE^Q)^* = E^M - iE^Q. \tag{9}$$

The only difference between a particle and its corresponding antiparticle is that their imaginary energies (thus their electric charges) have opposite signs. A particle and its antiparticle have the same real energy but have the sign-opposite imaginary energy.

In a particle-antiparticle annihilation process, their real energies completely transfer into radiation photon energies and their imaginary energies annihilate or cancel each other. Since there are no masses to adhere, the electric charges come together due to the electric attraction and cancel each other (or form a positive-negative electric charge pair (+,-)). In a particle-antiparticle pair production process, the radiation photon energies transfer to rest energies with a pair of imaginary energies, which combine with the rest energies to form a particle and an antiparticle.

To describe the energies of all particles and antiparticles, we can introduce a two-dimensional energy space. It is a complex space with two axes denoted by the real energy $\text{Re}(E)$ and the imaginary energy $\text{Im}(E)$. There are two phases in the energy space. In phase I, both real and imaginary energies are positive, while, in phase II, the imaginary energy is negative. Neutral particles including massless radiation photons are located on the real energy axis. Electrically charged particles are distributed between the real and imaginary energy axes. A particle and its antiparticle cannot be located in the same phase of the energy space.

5 Quantization of imaginary energy

The imaginary energy is quantized. Each electric charge quantum e (the electric charge of proton) has the following imaginary energy

$$E^e = \frac{e}{\sqrt{G}} c^2 \sim 1.67 \times 10^{15} \text{ ergs} \sim 10^{27} \text{ eV}, \quad (10)$$

which is about 10^{18} times greater than proton's real energy (or the energy of proton's mass). Dividing the size of proton (10^{-15} cm) by proton's imaginary-real energy ratio (10^{18}), we obtain a scale length $l_Q = 10^{33}$ cm.

On the other hand, Kaluza-Klein theory geometrically unified the four-dimensional Einsteinian general theory of relativity and Maxwellian electromagnetic theory into a five-dimensional unification theory ([7–9] for the original studies, [10] for an extensive review, and [11, 12] for the field solutions). In this unification theory, the fifth dimension is a compact (one-dimensional circle) space with radius 10^{33} cm [13], which is about the order of l_Q obtained above. The reason why the fifth dimensional space is small and compact might be due to that the imaginary energy of an electrically charged particle is many orders of magnitude higher than its real energy. The charge is from the extra (or fifth) dimension [14], a small compact space. A pure electric charge is not measurable and is thus reasonably represented by an imaginary energy.

The imaginary energy of the electric charge quantum is about the thermal energy of the particle at a temperature $T_Q = 2E^e/k_B \sim 2.4 \times 10^{31}$ °K. At this extremely high temperature, an electrically charged particle (e.g. proton) has a real energy in the same order of its imaginary energy. According to the standard big bang cosmology, the temperature at the grand unification era and earlier can be higher than about T_Q [15]. To have a possible explanation for the origin of the universe (or the origin of all the matter and energy), we suggest that a large electric charge such as 10^{46} Coulombs ($\sim 10^{76}$ ergs) was burned out, so that a huge amount of imaginary energies transferred into real energies at the temperature T_Q and above during the big bang of the universe. This suggestion gives a possible explanation for the origin of the universe from nothing to the real world in a process of transferring a

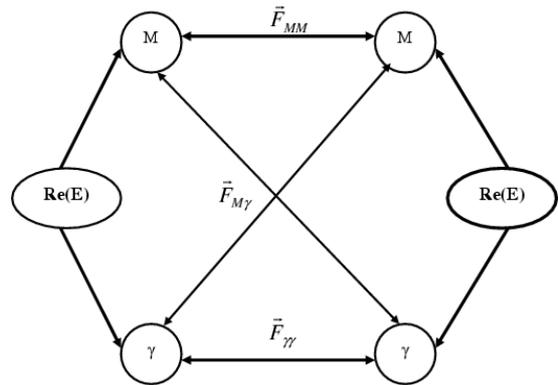


Fig. 3: Three types of gravitational interactions between real energies: (1) the mass-mass interaction, (2) the mass-radiation interaction, and (3) the radiation-radiation interaction.

large amount of imaginary energy (or electric charge) to real energy.

6 Gravitational and radiation redshifts

Real energies actually have two components: matter with mass and matter without mass (i.e. radiation). The interactions between real energies may be referred as the gravitation in general. In this sense, we have three types of gravitations: (1) mass-mass interaction \vec{F}_{MM} , (2) mass-radiation interaction $\vec{F}_{M\gamma}$, and (3) radiation-radiation interaction $\vec{F}_{\gamma\gamma}$. Figure 3 sketches all these interactions between real energies.

The energy of a radiation photon is given by $h\nu$, where h is the Planck's constant and ν is the frequency of the radiation. According to Eq. (7), the mass-radiation interaction between a mass M and a photon γ is given by

$$\vec{F} = -G \frac{M h \nu}{c^2 r^3} \vec{r}, \quad (11)$$

and the radiation-radiation interaction between two photons γ_1 and γ_2 is given by

$$\vec{F}_{\gamma\gamma} = -G \frac{(h\nu_1)(h\nu_2)}{c^4 r^3} \vec{r}. \quad (12)$$

Newton's law of gravity describes the gravitational force between two masses \vec{F}_{MM} . The Einsteinian general theory of relativity has successfully described the effect of matter (or mass) on the space-time and thus the interaction of matter on both matter and radiation (or photon). If we appropriately introduce a radiation energy-momentum tensor into the Einstein field equation, the Einsteinian general theory of relativity can also describe the effect of radiation on the space-time and thus the interaction of radiation on both matter and radiation.

When a photon of light travels relative to an object (e.g. the Sun) from \vec{r} to $\vec{r} + d\vec{r}$, it changes its energy or frequency from ν to $\nu + d\nu$. The work done on the photon by the mass-radiation interaction ($\vec{F}_{M\gamma} \cdot d\vec{r}$) is equal to the photon energy

change ($h d\nu$), i.e.,

$$-G \frac{M h \nu}{c^2 r^2} dr = h d\nu. \quad (13)$$

Eq. (13) can be rewritten as

$$\frac{d\nu}{\nu} = -\frac{GM}{c^2 r^2} dr. \quad (14)$$

Integrating Eq. (14) with respect to r from R to ∞ and ν from ν_e to ν_o , we have

$$\ln \frac{\nu_o}{\nu_e} = -\frac{GM}{c^2 R}, \quad (15)$$

where R is the radius of the object, ν_e is the frequency of the light when it is emitted from the surface of the object, ν_o is the frequency of the light when it is observed by the observer at an infinite distance from the object. Then, the redshift of the light is

$$Z_G = \frac{\lambda_o - \lambda_e}{\lambda_e} = \frac{\nu_e - \nu_o}{\nu_o} = \exp\left(\frac{GM}{c^2 R}\right) - 1. \quad (16)$$

In the weak field approximation, it reduces

$$Z_G \simeq \frac{GM}{c^2 R}. \quad (17)$$

Therefore, calculating the work done by the mass-radiation interaction on a photon, we can derive the Einsteinian gravitational redshift in the weak field approximation.

Similarly, calculating the work done on a photon from an object by the radiation-radiation gravitation $\vec{F}_{\gamma\gamma}$, we obtain a radiation redshift,

$$Z_\gamma = \frac{4GM}{15c^5} \sigma A T_c^4 + \frac{G}{c^5} \sigma A T_s^4, \quad (18)$$

where σ is the Stephan-Boltzmann constant, A is the surface area, T_c is the temperature at the center, T_s is the temperature on the surface. Here we have assumed that the inside temperature linearly decreases from the center to the surface. The radiation redshift contains two parts. The first term is contributed by the inside radiation. The other is contributed by the outside radiation. The redshift contributed by the outside radiation is negligible because $T_s \ll T_c$.

The radiation redshift derived here is significantly small in comparison with the empirical expression of radiation redshift proposed by Finlay-Freundlich [16]. For the Sun with $T_c = 1.5 \times 10^7$ °K and $T_s = 6 \times 10^3$ °K, the radiation redshift is only about $Z_\gamma = 1.3 \times 10^{-13}$, which is much smaller than the gravitational redshift $Z_G = 2.1 \times 10^{-6}$.

7 Discussions and conclusions

A quark has not only the electric charge but also the color charge [17, 18]. The electric charge has two varieties (positive and negative), while the color charge has three values (red, green, and blue). Describing both electric and color charges as imaginary energies, we may unify all of the four fundamental interactions into a single expression of the inter-

action between complex energies. Details of the study including the color charge will be given in the next paper.

Eq. (1) does not include the self-energy — the contribution to the energy of a particle that arises from the interaction between different parts of the particle. In the nuclear physics, the self-energy of a particle has an imaginary part [19, 20]. The mass-mass, mass-charge, and charge-charge interactions between different parts of an electrically charged particle will be studied in future.

As a summary, a pure electric charge (not observable and from the extra dimension) has been suggested as a form of imaginary energy. Total energy of an electrically charged particle is a complex number. The real part is proportional to the mass, while the imaginary part is proportional to the electric charge. The energy of an antiparticle is obtained by conjugating the energy of its corresponding particle. The gravitational and electromagnetic interactions have been classically unified into a single expression of the interaction between complex energies.

The interactions between real energies are gravitational forces, categorized by the mass-mass, mass-radiation, and radiation-radiation interactions. The work done by the mass-radiation interaction on a photon derives the Einsteinian gravitational redshift, and the work done by the radiation-radiation interaction on a photon gives the radiation redshift, which is significantly small in comparison with the gravitational redshift.

The interaction between imaginary energies is electromagnetic force. Since an electrically charged particle contains many order more imaginary energy than real energy, the interaction between imaginary energies are much stronger than that between real energies.

Overall, this study develops a new physics concept for electric charges, so that classically unifies the gravitational and electric forces and derives the Einsteinian gravitational redshift.

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An “Earth-Planet” or “Earth-Star” Couplet as a Gravitational Wave Antenna, wherein the Indicators are Microseismic Peaks in the Earth

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An “Earth-planet” or “Earth-star” couplet can be considered as a gravitational wave antenna. There in such an antenna a gravitational wave should lead to a peak in the microseismic background spectrum on the Earth (one of the ends of the antenna). This paper presents numerous observational results on the Earth’s microseismic background. The microseismic spectrum, being compared to the distribution of the relative location of the nearest stars, found a close peak-to-peak correspondence. Hence such peaks can be a manifestation of an oscillation in the couplet “Earth-star” caused by gravitational waves arriving from the cosmos.

1 Introduction

Use the following simplest model. Focus on two gravitationally-connected objects such as the couplets “Earth-Moon”, “Earth-Jupiter”, “Earth-Saturn”, “Earth-Sun”, or “Earth-star” (a near star is meant). Such a couplet can be considered as a gravitational wave antenna. A gravitational wave, falling down onto such an antenna, should produce an oscillation in the system that leads to a peak in the microseismic background spectrum of the Earth (one of the ends of the antenna).

Gravitational waves radiated on different frequencies may have an origin in gravitationally unstable objects in the Universe. For instance, a gravitationally unstable cosmic cloud wherein a stellar form may be such a source. A mechanism which generates gravitational waves on a wide spectrum can be shown in such an example. There is a theorem: “if a system is in the state of unstable equilibrium, such a system can oscillatorily bounce at low frequencies in the stable area of the states; the frequency decreases while the system approaches the state of equilibrium (threshold of instability) with a finite wavenumber at zero frequency” [1, 2]. This theorem is applicable exactly to the case of the gravitational instability of the cosmic clouds. Such a gravitational instability is known as Jeans’s instability, and leads to the process of the formation of stars [3]. In this process intense gravitational radiation should be produced. Besides the spectrum of the waves should be continuously shifting on low frequency scales as such a cloud approaches to the threshold of instability. Hence, gravitational waves radiated on the wide spectrum of frequencies should be presented in the Universe always as stellar creation process.

Hence, the peaks of the microseismic background on the

Earth (if any observed), if correlated to the parameters of the “Earth-space body” system (such as the distance L between them), should manifest the reaction in the “Earth-space body” couplet of the gravitational waves arriving from the cosmos. The target of this study is the search for such correlation peaks in the microseismic background of the Earth.

2 Observations

Our observations were processed at the Seismic Station of Simpheropol University (Sevastopol, Crimea Peninsula), using a laser interferometer [4]. Six peaks were registered at 2.3 Hz, 1 Hz, 0.9 Hz, 0.6 Hz, 0.4 Hz, 0.2 Hz (see Fig. 1a and Fig. 1b). The graphs were drawn directly on the basis of the records made by the spectrum analyzer SK4-72. The spectrum analyzer SK4-72 accumulates output signals from an interferometer, then enhances periodic components of the signal relative to the chaotic components. 1,024 segregate records, 40-second length each, were averaged.

Many massive gravitating objects are located near the solar system at the distance of 1.3, 2.7, 3.5, 5, 8, and 11 parsecs. All the distances L between the Earth and these objects correspond to all the observed peaks (see Fig. 1a and Fig. 1b). The calculated distribution of the gravitational potential of the nearest stars is shown in Fig. 1c. Comparing Fig. 1a and Fig. 1b to Fig. 1c, we reveal a close similarity between the corresponding curves: each peak of Fig. 1a and Fig. 1b corresponds to a peak in Fig. 1c, and vice versa. Besides there are small deviations, that should be pointed out for clarity. For distances $L > 4$ parsecs the data were taken only for the brightest star, and the curve of the gravitational potential corresponding to this distance is lower than that for the $L < 4$ shown in the theoretical Fig. 1c. Another deviation is the presence of a uniform growth for the low-frequency background component in the experimental Fig. 1a and Fig. 1b, which doesn’t appear in Fig. 1c. Such a uniform component of the microseismic background is usually described by the law $A_\omega \sim 1/\omega^2$ [5, 6].

*Posthumous publication prepared by Prof. Simon E. Shnoll (Institute of Theoretical and Experimental Biophysics, Russian Academy of Sciences, Pushino, Moscow Region, 142290, Russia), who was close to the author. E-mail of the submitter: shnoll@iteb.ru; shnoll@mail.ru. See Afterword for the biography and bibliography of the author, Prof. Vladimir A. Dubrovskiy (1935–2006).

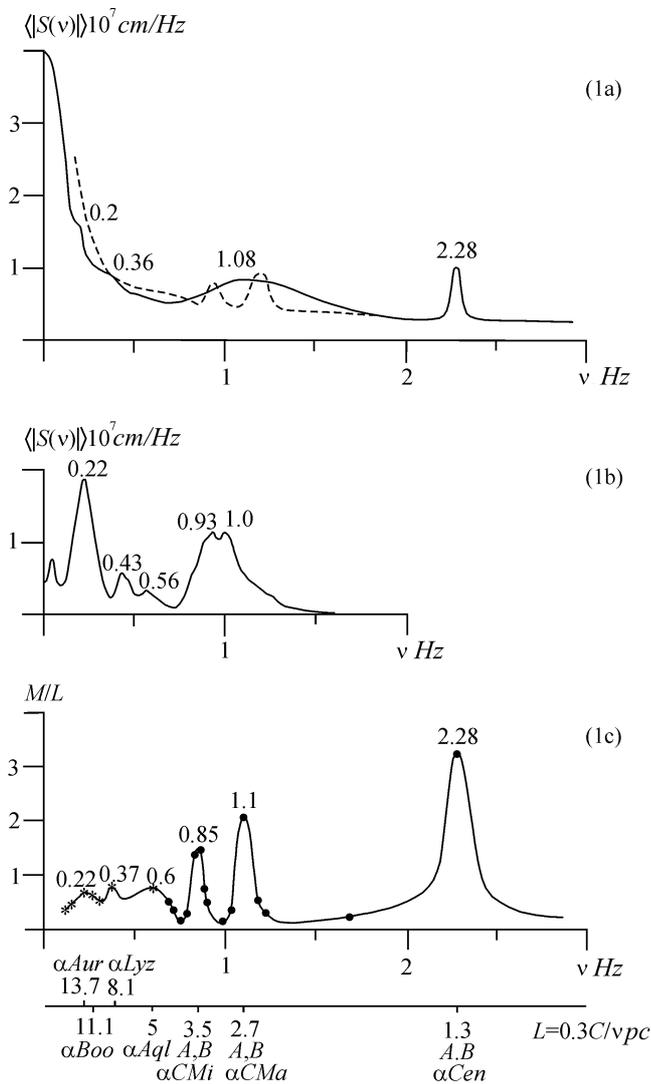


Fig. 1: The observed microseismic background (solid curve) after accumulation of the background signals from the interferometer output: Fig. 1a shows the range 0.1–5 Hz; Fig. 1b shows the range 0.1–2 Hz. The dotted curve of Fig. 1a shows the calculated distribution of the gravitational potential of the stars in common with the uniform part of the microseismic background. This dotted curve is normalized so that it is the same as that of the solid curve at 2.28 Hz. Fig. 1c shows the calculated distribution of the gravitational potential of the stars. The solid points correspond to all the nearest stars, a distance to which is $L < 4$ parsecs, and to all the brightest stars located at $L > 4$ parsecs. Masses M are expressed in the mass of the Sun. α Aur, α Lyz, etc. mean α stars of the constellations according to the astronomical notation [7, 8]. A, B sign for the components of the binaries. The numbers typed at the extrema are frequencies.

Moreover, the quantitative correlation between the frequency peaks and the distribution of the nearest stars is found. Namely, the sharpest peak at 2.28 Hz corresponds to the distance between the Earth and the nearest binary stars A and B , α Centaurus [7, 8]. The broader peak at 1 Hz (see Fig. 1a, and Fig. 1b) corresponds to the distances to the stars which are distributed over the range from 2.4 to 3.8 parsecs [7, 8]. The spectrum analyzer SK4-72 averages all the peaks in the range 2.4–3.8 parsecs into one broad peak near 1 Hz (Fig. 1a). At the same time the broad peak of Fig. 1a, being taken under detailed study, is shown to be split into two peaks (Fig. 1b) if the spectrum analyzer SK4-72 processes the frequency range from 0.1 to 2 Hz (the exaggeration of the frequency scale). This subdivision of the frequency range corresponds to the division of the group of stars located as far as in the range from 2.4 to 3.8 parsecs into two subgroups which are near 2.7 and 3.5 parsecs (Fig. 1c).

The distribution of the gravitational potential over the subgroups, in common with the uniform background spectrum, is shown by the dotted curve in Fig. 1a. We see therein both the quantitative and qualitative correlation between the frequent spectra of the microseismic background and the distribution of the gravitational potential in the subgroups.

The Sevastopol data correlation on the frequency spectra between the microseismic background and the distances between the Earth and the nearest stars are the same as the data registered in Arizona. The Sevastopol and Arizona data are well-overlapping with coincidence in three peaks [9].

It is possible to propose more decisive observations. Namely, it would be reasonable to look for peaks which could be corresponding to the Earth-Moon (~ 240 MHz), “Earth-Sun” (~ 0.6 MHz), “Earth-Venus” (~ 0.3 – 2.2 MHz), “Earth-Jupiter” (~ 100 – 150 kHz), and “Earth-Saturn” (~ 58 – 72 kHz) antennae. Moreover, the peaks corresponding to Venus, Jupiter and Saturn should change their frequency in accordance with the change in the distance between the Earth and these planets during their orbital motion around the Sun. If such a correlation could be registered in an experiment, this would be experimentum crucis in support of the above presented results.

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Afterword by the Editor

In addition to the posthumous paper by Prof. Dubrovskiy, I should provide an explanation why we publish it in a form substantially truncated to the originally version of the manuscript.

The originally Dubrovskiy manuscript, submitted by Prof. Simon E. Shnoll, was based on the preprint uploaded in 2001 into the Cornell arXiv.org, astro-ph/0106350. In that manuscript, aside for the experimental data presented in the current publication, Dubrovskiy tried to use the data as a verification to the Laplace speed of gravitation, which is many orders higher than the velocity of light. His belief in Laplace's theory unfortunately carried him into a few formally errors.

Laplace supposed such a speed as a result of his solution of the gravitational two-body problem, which concerns the motion of two point particles that interact only with each other, due to gravity. In this problem a body A experiences that the force of gravitation which acts at that point where the body A is located in the moment. Because a body B (the source of the force) is distant from the body A and moves with respect to it with a velocity, there is incoincidence of two directions: the line connecting both bodies in the moment and the direction from the body A to that point where the body B was located, due to its motion, some time ago. What line is the location of the centre of gravity in such a system? If it is located in the first line, a force accelerating the body A should appear. If it is the second line, a non-compensated component of the momentum should appear in the body B, that is the breaking of the conservation law. As a result such a system becomes unstable anyway. This is a paradox of the two body problem of the 18th century. Using the mathematical methods accessed in the end of the 18th century, Laplace resolved this problem by introduction of the speed of gravitation, which should be, in the sample of the planets, at least ten orders higher than the velocity of light.

The contemporary Newtonian celestial mechanics resolves the two body problem with use of the methods of higher mathematics. This is a classical example, which shows that two bodies orbiting a common centre of gravity under specific conditions move along stable elliptic orbits so that they cannot leave the system or fall onto each other. This classical problem, known as the Kepler problem, is described in detail in §13 of *Short Course of Theoretical Physics. Mechanics. Electrodynamics* by Landau and Lifshitz (Nauka Publishers, Moscow, 1969).

The same situation takes a place in the General Theory of Relativity in a case where the physical conditions of the motion are close to the non-relativistic Newtonian mechanics. This problem is

discussed in detail in §101 of *The Classical Theory of Fields* by Landau and Lifshitz (Butterworth-Heinemann, 1980). The mechanical energy and the moment of momentum of a two body system remain unchanged with only a small correction for the energy-momentum loss with gravitational radiation. In a system like the solar system the power of gravitational radiation, which is due to the orbiting planets, is nothing but only a few kilowatts. Therefore such a system is stable with the speed of gravitation equal to the velocity of light: the planets cannot leave the solar system or fall onto each other within a duration compared to the age of the Universe.

Due to the aforementioned reason, I substantially corrected the originally Dubrovskiy manuscript. I removed everything on the superluminal Laplace velocity of gravitation. I also corrected minor errors in the description of gravitational wave antennae.

I did it through the prior permission of Dr. Victor N. Sergeev (e-mail: svn@idg.chph.ras.ru), who was a close friend of Prof. Dubrovskiy and a co-author of many his works.

Dr. Sergeev is in contact with Prof. Shnoll. He read the corrected version of the manuscript, and agreed with the edition. Sergeev wrote, in a private letter of January 29, 2008: "... He [Dubrovskiy] considered the manuscript as a verification to his theory of gravitation where gravitational waves travel with a superluminal velocity. However the presence of a correlation of the microseismic spectra to the cosmic bodies, the result itself is important independent from interpretation given to it. Of course, it would be very good to publish this result. Besides, the edited version has nothing of those contradicting to the views of V. A. Dubrovskiy."

In general, an idea about a free-mass gravitational wave antenna whose basis is set up by an "Earth-planet" or "Earth-star" couplet is highly original. No such an idea met in the science before Dubrovskiy. Moreover, the correlation of the microseismic oscillations to the distances found by him gives good chances that such a couplet can be used as a huge free-mass gravitational wave detector in the future. The interstellar distances are extremely larger to 5 mln. km of the basis of LISA — the Laser Interferometer Space Antenna planned by the European Space Agency to launch on the next decade. So the displacement effect in the Dubrovskiy mass-detector due the a falling gravitational wave should be large that could result a microseismic activity in the Earth.

With such a fine result, this paper will leave fond memories of Prof. Dubrovskiy. May his memory live for ever!

*Dmitri Rabounski, Editor-in-Chief
Progress in Physics*

Vladimir A. Dubrovskiy (1935–2006)

Vladimir Anatolievich Dubrovskiy was born on March 20, 1935, in the formerly-known Soviet Union. In 1953–1959 he was a student in the Physics Department of Moscow University. Then he worked on the research staff of the Academy of Sciences of URSS (now the Russian Academy of Sciences, RAS) all his life. During the first period, from 1959 to 1962, he was employed as a research scientist at the Institute of Mathematics in the Siberian Branch of the Academy of Sciences, where he worked on the physics of elementary particles. During the second period, from 1962 to 1965, he completed post-graduate education at the Institute of Applied Mechanics: his theme was a "quasi-classical approximation of the equations of Quantum Mechanics". During two decades, from 1966 to 1998, he worked at

the Laboratory of Seismology of the Institute of the Physics of the Earth, in Moscow, where he advanced from a junior scientist to the Chief of the Laboratory. His main research at the Institute concerned the internal constitution and evolution of the Earth.

From 1972 to 1992 Dubrovskiy was the Executive Secretary of the “Intergovernmental Commission URSS-USA on the Prediction for Earthquakes”. In 1986–1991 he was the Executive Secretary of the “Commission on the Constitution, Composition, and Evaluation of the Earth’s Interior” by the Academy of Science of URSS and the German Research Foundation (Deutsche Forschungsgemeinschaft). In 1997 he was elected a Professor in the Department of Mechanics and Mathematics of Moscow University.

In the end of 1996, Dubrovskiy and all the people working with him at his Laboratory of Seismology were ordered for discharge from the Institute of the Physics of the Earth due to a conflict between Dubrovskiy and the Director of the Institute. Then, in February of 1997, Dubrovskiy accused the Director with repression in science like those against genetics during the Stalin regime, and claimed hungry strike. A month later, in March, his health condition had become so poor, forcing him to be hospitalized. (Despite the urgent medical treatment, his health didn’t come back to him; he was still remaining very ill, and died nine years later.) All the story met a resonance in the scientific community. As a result, Dubrovskiy, in common with two his co-workers, was invited by another Institute of the Academy of Sciences, the Institute of Geospheres Dynamics in Moscow, where he worked from 1998 till death. He died on November 12, 2006, in Moscow.

Dubrovskiy authored 102 research papers published in scientific journals and the proceedings of various scientific conferences. A brief list of his scientific publications attached.

Main scientific legacy of V. A. Dubrovskiy

A five dimensional approach to the quasiclassical approach of the equations of Quantum Mechanics:

- Dubrovskiy V. A. and Skuridin G. A. Asymptotic decomposition in wave mechanics. *Magazine of Computational Mathematics and Mathematical Physics*, 1964, v. 5, no. 4.

The hypothesis on the iron oxides contents of the Earth’s core:

- Dubrovskiy V. A. and Pan’kov V. L. On the composition of the Earth’s core. *Izvestiya of the Academy of Sciences of USSR, Earth Phys.*, 1972, no. 7, 48–54.

Now this hypothesis has been verified by many scientists in their experimental and theoretical studies. A new idea is that the d -electrons of the transition elements (mainly iron), being under high pressure, participate with high activity in the formation of the additional covalent bindings. As a result the substances become dense, so the iron oxide FeO can be seen as the main part of the contents formation of the core of the Earth.

The theory of eigenoscillation of the elastic inhomogeneities:

- Dubrovskiy V. A. Formation of coda waves. In: *The Soviet-American Exchange in Earthquake Prediction. U.S. Geological Survey. Open-File Report*, 81–1150, 1981, 437–456.
- Dubrovskiy V. A. and Morochnik V. S. Natural vibrations of a spherical inhomogeneity in an elastic medium. *Izvestiya of the Academy of Sciences of USSR, Physics of the Solid Earth*, 1981, v. 17, no. 7, 494–504.
- Dubrovskiy V. A. and Morochnik V. S. Nonstationary scattering of elastic waves by spherical inclusion. *Izvestiya of the Academy of*

Sciences of USSR, Physics of the Solid Earth, 1989, v. 25, no. 8, 679–685.

This presents the analytic solution of the boundary problem. The frequent equation is derived for both radial, torsional and spheroidal vibrations. A new method of solution for the diffraction problem is developed for a spherical elastic inclusion into an infinite elastic medium. The obtained analytical solution is checked by numerical computation. Formulae are obtained for the coda waves envelop in two limiting cases: single scattering and diffusion scattering. A frequency dependence on the quality factor is manifest through the corresponding dependence on the scattering cross-section.

The mechanism of the tectonic movements:

- Artemjev M. E., Bune V. J., Dubrovskiy V. A., and Kambarov N. Sh. Seismicity and isostasy. *Phys. Earth Planet. Interiors*, 1972, v. 6, no. 4, 256–262.
- Dubrovskiy V. A. Mechanism of tectonic movements. *Izvestiya of the Academy of Sciences of USSR, Physics of the Solid Earth*, 1986, v. 22, no. 1, 18–27.
- Dubrovskiy V. A., Sergeev V. N., and Fuis G. S. Generalized condition of isostasy. *Doklady of the Russian Academy of Sciences*, 1995, v. 342, no. 1.
- Dubrovskiy V. A. and Sergeev V. N. Physics of tectonic waves. *Izvestiya of the Russian Academy of Sciences, Physics of the Solid Earth*, 1997, v. 33, no. 10, 865–866.

This mechanism is seen to be at work in a “lithosphere-asthenosphere” system which has the density inversion between the lithosphere and asthenosphere. The substance of the elastic lithosphere is denser than that of the liquid asthenosphere. A solution for the model of the elastic layer above the incompressible fluid with the density inversion is found. It is found that there is a nontrivial, unstable equilibrium on nonzero displacement of the elastic layer. The bifurcation point is characterized by a critical wavelength of the periodic disturbance. This wavelength is that of the wave disturbance when the Archimedian force reaches the elastic force of disturbance.

Two-level convection in Earth’s mantle:

- Dubrovskiy V. A. Two-level convection in the Earth’s mantle. *Doklady of the Russian Academy of Sciences*, 1994, v. 334, no. 1.
- Dubrovskiy V. A. Convective instability motions in the Earth’s interiors. *Izvestiya of the Russian Academy of Sciences, Physics of the Solid Earth*, 1995, no. 9.

The mantle convection is considered at two levels: a convection in the lower mantle is the chemical-density convection due to the core-mantle boundary differentiation into the different compositionally light and heavy components, while the other convection is the heat-density convection in the “elastic lithosphere — fluid asthenosphere” system. The last one manifests itself in different tectonic phenomena such as the tectonic waves, the oceanic plate tectonic and continental tectonic as a result of the density inversion in the “lithosphere-asthenosphere” system. The lower mantle chemical convection gives the heat energy flow to the upper mantle heat convection.

Generation for the magnetic, electric and vortex fields in magnetohydrodynamics, electrohydrodynamics and vortex hydrodynamics:

- Dubrovskiy V. A. and Skuridin G. A. The propagation of small disturbances in magnetohydrodynamics. *Geomagnetism and Aeronomy*, 1965, v. 5, no. 2, 234–250.
- Dubrovskiy V. A. The equations of electrohydrodynamics and electroelasticity. *Soviet Physics Doklady*, v. 29(12), December 1984 (transl. from *Doklady Akademii Nauk URSS*, 1984, v. 279, 857–860).
- Dubrovskiy V. A. Conditions for magnetic field Generation. *Doklady Akademii Nauk URSS*, 1986, v. 286, no. 1, 74–77.
- Dubrovskiy V. A. and Rusakov N. N. Mechanism of generation of an elastic field. *Doklady Akademii Nauk URSS*, 1989, v. 306, no. 6, 64–67.

- Dubrovskiy V. A. On a relation between strains and vortices in hydrodynamic flows. *Doklady Physics*, 2000, v. 45, no. 2, 52–54 (transl. from *Doklady Akademii Nauk URSS*, 2000, v. 370, no. 6, 754–756).

A nonlinear system of the equations is obtained, which manifests a mutual influence between the motion of a dielectric medium and an electric field. This theory well-describes the atmospheric electricity, including ball lightning. The theory proves: the motion of a magnetohydrodynamical, electrohydrodynamical or hydrodynamical incompressible fluid is locally unstable everywhere relative to the disturbances of a vortex, magnetic or electric field. A mutual, pendulumlike conversion energy of the fluid flow and energy of a magnetic, electric or vortex field is possible. Two-dimensional motions are stable in a case where they are large enough. The magnetic restraint of plasma is impossible in three-dimensional case.

The elastic model of the physical vacuum:

- Dubrovskiy V. A. Elastic model of the physical vacuum. *Soviet Physics Doklady*, v. 30(5), May 1985 (translated from *Doklady Akademii Nauk URSS*, 1985, v. 283, 83–85).
- Dubrovskiy V. A. Measurements of the gravity waves velocity. arXiv: astro-ph/0106350.
- Dubrovskiy V. A. Relation of the microseismic background with cosmic objects. *Vestnik MGU* (Transactions of the Moscow University), 2004, no. 4.
- Dubrovskiy V. A. and Smirnov N. N. Experimental evaluation of the gravity waves velocity. In: *Proc. of the 54th International Astronautical Congress*, September 29 — October 3, 2003, Bremen, Germany.

New variables in the theory of elasticity are used (e.g. the velocity, vortex, and dilation set up instead the velocity and stress used in the standard theory). This gives a new system of the equations describing the wave motion of the velocity, vortex and dilation. In such a model, transversal waves and longitudinal waves are associated to electromagnetic and gravitational waves respectively. Such an approach realizes the field theory wherein elementary particles are the singularities in the elastic physical vacuum.

A universal precursor for the geomechanical catastrophes:

- Dubrovskiy V. A. Tectonic waves. *Izvestiya of the Academy of Sciences of URSS, Earth Physics*, 1985, v. 21, no. 1, 20–23.
- Dubrovskiy V. A. and Dieterich D. Wave propagation along faults and the onset of slip instability. *EOS*, 1990, v. 71, no. 17, 635–636.
- Dubrovskiy V. A., McEvelly T. V., Belyakov A. S., Kuznetsov V. Y., and Timonov M. V. Borehole seismoacoustical emission study at the Parkfield prognosis range. *Doklady of the Russian Academy of Sciences*, 1992, v. 325, no. 4.
- Dubrovskiy V. A. and Sergeev V. N. The necessary precursor for a catastrophe. In: *Tectonic of Neogeny: General and Regional Aspects*, GEOS, Moscow, 2001, v. 1, 222–226.

Unstable phenomena such as earthquakes can occur in a geomechanical system, if there is an unstable state of equilibrium in a set of critical geophysical parameters. There are two fields of the geophysical parameters, which correspond to the stable and unstable states. According to Dubrovskiy (1985) and also Dubrovskiy and Sergeev (2001), in the stable field of the parameters the geosystem has vibratory eigenmotions, where the frequencies tend to zero if the system approaches unstable equilibrium (during an earthquake occurrence, for instance). However the critical wavelength of the vibrations remains finite at zero frequency, and characterizes the size of the instability. Change in the eigenfrequencies affects the spectrum of seismoacoustic emission in an area surrounding an impending earthquake. Such a change indicates the fact that the geomechanical system is close to an unstable threshold, and the critical wavelength determines the energy and space dimensions of the developing instability source. Such an approach to the study of a systems in the state of unstable equilibrium is applicable to all system, whose behavior is described by hyperbolic equations in partial derivatives, i.e. not only geomechanical systems.

Remarks on Conformal Mass and Quantum Mass

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One shows how in certain model situations conformal general relativity corresponds to a Bohmian-Dirac-Weyl theory with conformal mass and Bohmian quantum mass identified.

The article [12] was designed to show relations between conformal general relativity (CGR) and Dirac-Weyl (DW) theory with identification of conformal mass \hat{m} and quantum mass \mathfrak{M} following [7, 9, 11, 25] and precision was added via [21]. However the exposition became immersed in technicalities and details and we simplify matters here. Explicitly we enhance the treatment of [7] by relating \mathfrak{M} to an improved formula for the quantum potential based on [21] and we provide a specific Bohmian-Dirac-Weyl theory wherein the identification of CGR and DW is realized. Much has been written about these matters and we mention here only [1–7, 9–20, 23–28] and references therein. One has an Einstein form for GR of the form

$$S_{GR} = \int d^4x \sqrt{-g} (R - \alpha |\nabla\psi|^2 + 16\pi L_M) \quad (1.1)$$

(cf. [7, 22]) whose conformal form (conformal GR) is an integrable Weyl geometry based on

$$\begin{aligned} \hat{S}_{GR} &= \int d^4x \sqrt{-\hat{g}} e^{-\psi} \times \\ &\times \left[\hat{R} - \left(\alpha - \frac{3}{2} \right) |\hat{\nabla}\psi|^2 + 16\pi e^{-\psi} L_M \right] = \\ &= \int d^4x \sqrt{-\hat{g}} \left[\hat{\phi} \hat{R} - \left(\alpha - \frac{3}{2} \right) \frac{|\hat{\nabla}\hat{\phi}|^2}{\hat{\phi}} + 16\pi \hat{\phi}^2 L_M \right] \end{aligned} \quad (1.2)$$

where $\Omega^2 = \exp(-\psi) = \hat{\phi}$ with $\hat{g}_{ab} = \Omega^2 g_{ab}$ and $\hat{\phi} = \exp(\psi) = \hat{\phi}^{-1}$ (note $(\hat{\nabla}\psi)^2 = (\hat{\nabla}\hat{\phi})^2 / (\hat{\phi})^2$). One sees also that (1.2) is the same as the Brans-Dicke (BD) action when $L_M = 0$, namely (using \hat{g} as the basic metric)

$$S_{BD} = \int d^4x \sqrt{-\hat{g}} \left[\hat{\phi} \hat{R} - \frac{\omega}{\hat{\phi}} |\hat{\nabla}\hat{\phi}|^2 + 16\pi L_M \right]; \quad (1.3)$$

which corresponds to (1.2) provided $\omega = \alpha - \frac{3}{2}$ and $L_M = 0$. For (1.2) we have a Weyl gauge vector $w_a \sim \partial_a \psi = \partial_a \hat{\phi} / \hat{\phi}$ and a conformal mass $\hat{m} = \hat{\phi}^{-1/2} m$ with $\Omega^2 = \hat{\phi}^{-1}$ as the conformal factor above. Now in (1.2) we identify \hat{m} with the quantum mass \mathfrak{M} of [25] where for certain model situations $\mathfrak{M} \sim \beta$ is a Dirac field in a Bohmian-Dirac-Weyl theory as in (1.8) below with quantum potential Q determined via $\mathfrak{M}^2 = m^2 \exp(Q)$ (cf. [10, 11, 21, 25] and note that $m^2 \propto T$ where $8\pi T^{ab} = (1/\sqrt{-g})(\delta\sqrt{-g} \mathfrak{L}_M / \delta g_{ab})$). Then $\hat{\phi}^{-1} = \hat{m}^2 / m^2 = \mathfrak{M}^2 / m^2 \sim \Omega^2$ for Ω^2 the standard conformal

factor of [25]. Further one can write **(1A)** $\sqrt{-\hat{g}} \hat{\phi} \hat{R} = \hat{\phi}^{-1} \sqrt{-\hat{g}} \hat{\phi}^2 \hat{R} = \hat{\phi}^{-1} \sqrt{-g} \hat{R} = (\beta^2 / m^2) \sqrt{-g} \hat{R}$. Recall here from [11] that for $g_{ab} = \hat{\phi} \hat{g}_{ab}$ one has $\sqrt{-g} = \hat{\phi}^2 \sqrt{-\hat{g}}$ and for the Weyl-Dirac geometry we give a brief survey following [11, 17]:

1. Weyl gauge transformations: $g_{ab} \rightarrow \tilde{g}_{ab} = e^{2\lambda} g_{ab}$; $g^{ab} \rightarrow \tilde{g}^{ab} = e^{-2\lambda} g^{ab}$ — weight e.g. $\Pi(g^{ab}) = -2$. β is a Dirac field of weight -1. Note $\Pi(\sqrt{-g}) = 4$;
2. Γ_{ab}^c is Riemannian connection; Weyl connection is $\hat{\Gamma}_{ab}^c$ and $\hat{\Gamma}_{ab}^c = \Gamma_{ab}^c = g_{ab} w^c - \delta_b^c w_a - \delta_a^c w_b$;
3. $\nabla_a B_b = \partial_a B_b - B_c \Gamma_{ab}^c$; $\nabla_a B^b = \partial_a B^b + B^c \Gamma_{ca}^b$;
4. $\hat{\nabla}_a B_b = \partial_a B_b - B_c \hat{\Gamma}_{ab}^c$; $\hat{\nabla}_a B^b = \partial_a B^b + B^c \hat{\Gamma}_{ca}^b$;
5. $\hat{\nabla}_\lambda g^{ab} = -2g^{ab} w_\lambda$; $\hat{\nabla}_\lambda g_{ab} = 2g_{ab} w_\lambda$ and for $\Omega^2 = \exp(-\psi)$ the requirement $\nabla_c g_{ab} = 0$ is transformed into $\hat{\nabla}_c \hat{g}_{ab} = \partial_c \psi \hat{g}_{ab}$ showing that $w_c = -\partial_c \psi$ (cf. [7]) leading to $w_\mu = \hat{\phi}_\mu / \hat{\phi}$ and hence via $\beta = m \hat{\phi}^{-1/2}$ one has $w_c = 2\beta_c / \beta$ with $\hat{\phi}_c / \hat{\phi} = -2\beta_c / \beta$ and $w^a = -2\beta^a / \beta$.

Consequently, via $\beta^2 \hat{R} = \beta^2 R - 6\beta^2 \nabla_\lambda w^\lambda + 6\beta^2 w^\lambda w_\lambda$ (cf. [11, 12, 16, 17]), one observes that $-\beta^2 \nabla_\lambda w^\lambda = -\nabla_\lambda (\beta^2 w^\lambda) + 2\beta \partial_\lambda \beta w^\lambda$, and the divergence term will vanish upon integration, so the first integral in (1.2) becomes

$$I_1 = \int d^4x \sqrt{-g} \left[\frac{\beta^2}{m^2} R + 12\beta \partial_\lambda \beta w^\lambda + 6\beta^2 w^\lambda w_\lambda \right]. \quad (1.4)$$

Setting now $\alpha - \frac{3}{2} = \gamma$ the second integral in (1.2) is

$$\begin{aligned} I_2 &= -\gamma \int d^4x \sqrt{-\hat{g}} \hat{\phi} \frac{|\hat{\nabla}\hat{\phi}|^2}{|\hat{\phi}|^2} = \\ &= -4\gamma \int d^4x \sqrt{-\hat{g}} \hat{\phi}^{-1} \hat{\phi}^2 \frac{|\hat{\nabla}\beta|^2}{\beta^2} = \\ &= -\frac{4\gamma}{m^2} \int d^4x \sqrt{-g} |\hat{\nabla}\beta|^2, \end{aligned} \quad (1.5)$$

while the third integral in the formula (1.2) becomes **(1B)** $16\pi \int \sqrt{-g} d^4x L_M$. Combining now (1.4), (1.5), and **(1B)** gives then

$$\begin{aligned} \hat{S}_{GR} &= \frac{1}{m^2} \int d^4x \sqrt{-g} \left[\beta^2 R + 6\beta^2 w^\alpha w_\alpha + \right. \\ &\left. + 12\beta \partial_\alpha \beta w^\alpha - 4\gamma |\hat{\nabla}\beta|^2 + 16\pi m^2 L_M \right]. \end{aligned} \quad (1.6)$$

We will think of $\hat{\nabla}\beta$ in the form **(1C)** $\hat{\nabla}_\mu\beta = \partial_\mu\beta - w_\mu\beta = -\partial_\mu\beta$. Putting then $|\hat{\nabla}\beta|^2 = |\partial\beta|^2$ (1.6) becomes (recall $\gamma = \alpha - \frac{3}{2}$)

$$\hat{S}_{GR} = \frac{1}{m^2} \int d^4x \sqrt{-g} \times \quad (1.7)$$

$$\times [\beta^2 R + (3 - 4\alpha)|\partial\beta|^2 + 16\pi m^2 L_M].$$

One then checks this against some Weyl-Dirac actions. Thus, neglecting terms $W^{ab}W_{ab}$ we find integrands involving $d^4x \sqrt{-g}$ times

$$-\beta^2 R + 3(3\sigma + 2)|\partial\beta|^2 + 2\Lambda\beta^4 + \mathfrak{L}_M \quad (1.8)$$

(see e.g. [11, 12, 17, 25]); the term $2\Lambda\beta^4$ of weight -4 is added gratuitously (recall $\Pi(\sqrt{-g}) = 4$). Consequently, omitting the Λ term, (1.8) corresponds to (1.7) times m^2 for $\mathfrak{L}_M \sim \sim 16\pi L_M$ and **(1D)** $9\sigma + 4\alpha + 3 = 0$. Hence one can identify conformal GR (without Λ) with a Bohmian-Weyl-Dirac theory where conformal mass \hat{m} corresponds to quantum mass \mathfrak{M} .

REMARK 1.1. The origin of a β^4 term in (1.8) from \hat{S}_{GR} in (1.2) with a term $2\sqrt{-\hat{g}}\hat{\Lambda}$ in the integrand would seem to involve writing **(1E)** $2\sqrt{-\hat{g}}\hat{\Lambda} = 2\sqrt{-\hat{g}}\hat{\phi}^2\Omega^4\hat{\Lambda} = = 2\sqrt{-g}\beta^4\hat{\Lambda}/m^4$ so that Λ in (1.8) corresponds to $\hat{\Lambda}$. Normally one expects $\Lambda\sqrt{-g} \rightarrow \sqrt{-\hat{g}}\hat{\phi}^2\Lambda$ (cf. [2]) or perhaps $\Lambda \rightarrow \hat{\phi}^2\Lambda = \Omega^{-4}\Lambda = \hat{\Lambda}$. In any case the role and nature of a cosmological constant seems to still be undecided. ■

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Gravitation and Electricity

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The equations of gravitation together with the equations of electromagnetism in terms of the General Theory of Relativity allow to conceive an interdependence between the gravitational field and the electromagnetic field. However the technical difficulties of the relevant problems have precluded from expressing clearly this interdependence. Even the simple problem related to the field generated by a charged spherical mass is not correctly solved. In the present paper we reexamine from the outset this problem and propose a new solution.

1 Introduction

Although gravitation and electromagnetism are distinct entities, the principles of General Relativity imply that they affect each other. In fact, the equations of electromagnetism, considered in the spacetime of General Relativity, depend on the gravitational tensor, so that the electromagnetic field depends necessarily on the gravitational potentials. On the other hand, the electromagnetism is involved in the equations of gravitation by means of the corresponding energy-momentum tensor, so that the gravitational potentials depend necessarily on the electromagnetic field. It follows that, in order to bring out the relationship between gravitation and electromagnetism, we must consider together the equations of electromagnetism, which depend on the gravitational tensor, and the equations of gravitation, which depend on the electromagnetic potentials. So we have to do with a complicated system of equations, which are intractable in general. Consequently it is very difficult to bring out in explicit form the relationship between gravitation and electromagnetism. However the problem can be rigorously solved in the case of the field (gravitational and electric) outside a spherical charged mass. The classical solution of this problem, the so-called Reissner-Nordström metric, involves mathematical errors which distort the relationship between gravitational and electric field. In dealing with the derivation of this metric, H. Weyl notices that “For the electrostatic potential we get the same formula as when the gravitation is disregarded” [5], without remarking that this statement includes an inconsistency: The electrostatic potential without gravitation is conceived in the usual spacetime, whereas the gravitation induces a non-Euclidean structure affecting the metrical relations and, in particular, those involved in the definition of the electrostatic potential. The correct solution shows, in fact, that the electrostatic potential depends on the gravitational tensor.

In the present paper we reexamine from the outset the problem related to the joint action of the gravitation and electromagnetism which are generated by a spherical charged source. We assume that the distribution of matter and charges

is such that the corresponding spacetime metric is $S\Theta(4)$ -invariant (hence also $\Theta(4)$ -invariant), namely a spacetime metric of the following form [3,4]

$$ds^2 = f^2 dx_0^2 + 2f f_1 (x dx) dx_0 - \ell_1^2 dx^2 + \left(\frac{\ell_1^2 - \ell^2}{\rho^2} + f_1^2 \right) (x dx)^2, \tag{1.1}$$

(where $f = f(x_0, \|x\|)$, $f_1 = f_1(x_0, \|x\|)$, $\ell_1 = \ell_1(x_0, \|x\|)$, $\ell = \ell(x_0, \|x\|)$, $\rho = \|x\|$).

It is useful to write down the components of (1.1):

$$g_{00} = f^2, \quad g_{0i} = g_{i0} = x_i f f_1,$$

$$g_{ii} = -\ell_1^2 + \left(\frac{\ell_1^2 - \ell^2}{\rho^2} + f_1^2 \right) x_i^2,$$

$$g_{ij} = \left(\frac{\ell_1^2 - \ell^2}{\rho^2} + f_1^2 \right) x_i x_j, \quad (i, j = 1, 2, 3; i \neq j),$$

the determinant of which equals $-f^2 \ell^2 \ell_1^4$. Then an easy computation gives the corresponding contravariant components:

$$g^{00} = \frac{\ell^2 - \rho^2 f_1^2}{f^2 \ell^2}, \quad g^{0i} = g^{i0} = x_i \frac{f_1}{f \ell^2},$$

$$g^{ii} = -\frac{1}{\ell_1^2} - \frac{1}{\rho^2} \left(\frac{1}{\ell^2} - \frac{1}{\ell_1^2} \right) x_i^2,$$

$$g^{ij} = -\frac{1}{\rho^2} \left(\frac{1}{\ell^2} - \frac{1}{\ell_1^2} \right) x_i x_j, \quad (i, j = 1, 2, 3; i \neq j).$$

Regarding the electromagnetic field, with respect to (1.1), it is defined by a skew-symmetrical $S\Theta(4)$ -invariant tensor field of degree 2 which may be expressed either by its covariant components

$$\sum V_{\alpha\beta} dx_\alpha \otimes dx_\beta, \quad (V_{\alpha\beta} = -V_{\beta\alpha}),$$

or by its contravariant components

$$\sum V^{\alpha\beta} \frac{\partial}{\partial x_\alpha} \otimes \frac{\partial}{\partial x_\beta}, \quad (V^{\alpha\beta} = -V^{\beta\alpha}).$$

2 Electromagnetic field outside a spherical charged source. Vanishing of the magnetic field

According to a known result [2], the skew-symmetrical $S\Theta(4)$ -invariant tensor field $\sum V_{\alpha\beta} dx_\alpha \otimes dx_\beta$ is the direct sum of the following two tensor fields:

(a) A $\Theta(4)$ -invariant skew-symmetrical tensor field

$$q(x_0, \|x\|)(dx_0 \otimes F(x) - F(x) \otimes dx_0),$$

$$\left(F(x) = \sum_{i=1}^3 x_i dx_i \right),$$

which represents the electric field with components

$$\left. \begin{aligned} V_{01} = -V_{10} = qx_1, \quad V_{02} = -V_{20} = qx_2, \\ V_{03} = -V_{30} = qx_3. \end{aligned} \right\}; \quad (2.1)$$

(b) A purely $S\Theta(4)$ -invariant skew-symmetrical tensor field

$$q_1(x_0, \|x\|) [x_1(dx_2 \otimes dx_3 - dx_3 \otimes dx_2) +$$

$$+ x_2(dx_3 \otimes dx_1 - dx_1 \otimes dx_3) +$$

$$+ x_3(dx_1 \otimes dx_2 - dx_2 \otimes dx_1)],$$

which represents the magnetic field with components

$$\left. \begin{aligned} V_{23} = -V_{32} = q_1 x_1, \quad V_{31} = -V_{13} = q_1 x_2, \\ V_{12} = -V_{21} = q_1 x_3. \end{aligned} \right\}. \quad (2.2)$$

Since the metric (1.1) plays the part of a fundamental tensor, we can introduce the contravariant components of the skew-symmetrical tensor field $\sum V_{\alpha\beta} dx_\alpha \otimes dx_\beta$ with respect to (1.1).

Proposition 2.1 *The contravariant components of the $S\Theta(4)$ -invariant skew-symmetrical tensor field $\sum V_{\alpha\beta} dx_\alpha \otimes dx_\beta$ are defined by the following formulae:*

$$V^{01} = -V^{10} = -\frac{q x_1}{f^2 \ell^2}, \quad V^{02} = -V^{20} = -\frac{q x_2}{f^2 \ell^2},$$

$$V^{03} = -V^{30} = -\frac{q x_3}{f^2 \ell^2},$$

$$V^{23} = -V^{32} = \frac{q_1 x_1}{\ell_1^4}, \quad V^{31} = -V^{13} = \frac{q_1 x_2}{\ell_1^4},$$

$$V^{12} = -V^{21} = \frac{q_1 x_3}{\ell_1^4}.$$

Proof. The components V^{01} and V^{23} , for instance, result from the obvious formulae

$$V^{01} = \sum g^{0\alpha} g^{1\beta} V_{\alpha\beta} = (g^{00} g^{11} - g^{01} g^{10}) V_{01} +$$

$$+ (g^{00} g^{12} - g^{02} g^{10}) V_{02} + (g^{00} g^{13} - g^{03} g^{10}) V_{03} +$$

$$+ (g^{02} g^{13} - g^{03} g^{12}) V_{23} + (g^{03} g^{11} - g^{01} g^{13}) V_{31} +$$

$$+ (g^{01} g^{12} - g^{02} g^{11}) V_{12}$$

and

$$V^{23} = \sum g^{2\alpha} g^{3\beta} V_{\alpha\beta} = (g^{20} g^{31} - g^{21} g^{30}) V_{01} +$$

$$+ (g^{20} g^{32} - g^{22} g^{30}) V_{02} + (g^{20} g^{33} - g^{23} g^{30}) V_{03} +$$

$$+ (g^{22} g^{33} - g^{23} g^{32}) V_{23} + (g^{23} g^{31} - g^{21} g^{33}) V_{31} +$$

$$+ (g^{21} g^{32} - g^{22} g^{31}) V_{12}$$

after effectuating the indicated operations.

Proposition 2.2 *The functions $q = q(x_0, \rho)$, $q_1 = q_1(x_0, \rho)$, ($x_0 = ct$, $\rho = \|x\|$), defining the components (2.1) and (2.2) outside the charged spherical source are given by the formulae*

$$q = \frac{\varepsilon f \ell}{\rho^3 \ell_1^2}, \quad q_1 = \frac{\varepsilon_1}{\rho^3},$$

$$(\varepsilon = \text{const}, \varepsilon_1 = \text{const}.)$$

(The equations of the electromagnetic field are to be considered together with the equations of gravitation, and since these last are inconsistent with a punctual source, there exists a length $\alpha > 0$ such that the above formulae are valid only for $\rho \geq \alpha$.)

Proof. Since outside the source there are neither charges nor currents, the components (2.1), (2.2) are defined by the classical equations

$$\frac{\partial V_{\alpha\beta}}{\partial x_\gamma} + \frac{\partial V_{\beta\gamma}}{\partial x_\alpha} + \frac{\partial V_{\gamma\alpha}}{\partial x_\beta} = 0, \quad (2.3)$$

($x_0 = ct$, $(\alpha, \beta, \gamma) \in \{(0, 1, 2), (0, 2, 3), (0, 3, 1), (1, 2, 3)\}$),

$$\sum_{\beta=0}^3 \frac{\partial}{\partial x_\beta} \left(\sqrt{-G} V^{\alpha\beta} \right) = 0, \quad (2.4)$$

$$(\alpha = 0, 1, 2, 3; G = -f^2 \ell^2 \ell_1^4).$$

Taking $(\alpha, \beta, \gamma) = (0, 1, 2)$, we have, on account of (2.3),

$$\frac{\partial(qx_1)}{\partial x_2} + \frac{\partial(q_1 x_3)}{\partial x_0} - \frac{\partial(qx_2)}{\partial x_1} = 0$$

and since

$$\frac{\partial q}{\partial x_i} = \frac{\partial q}{\partial \rho} \frac{x_i}{\rho}, \quad (i = 1, 2, 3),$$

we obtain

$$x_1 x_2 \frac{\partial q}{\partial \rho} - x_2 x_1 \frac{\partial q}{\partial \rho} + x_3 \frac{\partial q_1}{\partial x_0} = 0,$$

whence $\frac{\partial q_1}{\partial x_0} = 0$, so that q_1 depends only on ρ , $q_1 = q_1(\rho)$.

On the other hand, taking $(\alpha, \beta, \gamma) = (1, 2, 3)$, the equation (2.3) is written as

$$\frac{\partial(q_1 x_3)}{\partial x_3} + \frac{\partial(q_1 x_1)}{\partial x_1} + \frac{\partial(q_1 x_2)}{\partial x_2} = 0,$$

whence $3q_1 + \rho q_1' = 0$, so that $3\rho^2 q_1 + \rho^3 q_1' = 0$ or $(\rho^3 q_1)' = 0$ and $\rho^3 q_1 = \varepsilon_1 = \text{const}$ or $q_1 = \frac{\varepsilon_1}{\rho^3}$.

Consider now the equation (2.4) with $\alpha = 1$. Since $G = -f^2 \ell^2 \ell_1^4$,

$$V^{11} = 0, \quad V^{10} = \frac{qx_1}{f^2 \ell^2}, \quad V^{12} = \frac{q_1 x_3}{\ell_1^4}, \quad V^{13} = -\frac{q_1 x_2}{\ell_1^4},$$

we have

$$\frac{\partial}{\partial x_0} \left(\frac{q \ell_1^2}{f \ell} x_1 \right) + \frac{\partial}{\partial x_2} \left(\frac{q_1 f \ell}{\ell_1^2} x_3 \right) - \frac{\partial}{\partial x_3} \left(\frac{q_1 f \ell}{\ell_1^2} x_2 \right) = 0.$$

Because of

$$\frac{\partial}{\partial x_2} \left(\frac{q_1 f \ell}{\ell_1^2} x_3 \right) = \frac{x_3 x_2}{\rho} \frac{\partial}{\partial \rho} \left(\frac{q_1 f \ell}{\ell_1^2} \right) = \frac{\partial}{\partial x_3} \left(\frac{q_1 f \ell}{\ell_1^2} x_2 \right),$$

we obtain

$$x_1 \frac{\partial}{\partial x_0} \left(\frac{q \ell_1^2}{f \ell} \right) = 0$$

so that $\frac{q \ell_1^2}{f \ell}$ depends only on ρ : $\frac{q \ell_1^2}{f \ell} = \varphi(\rho)$.

Now the equation (2.4) with $\alpha = 0$ is written as

$$\frac{\partial}{\partial x_1} (x_1 \varphi(\rho)) + \frac{\partial}{\partial x_2} (x_2 \varphi(\rho)) + \frac{\partial}{\partial x_3} (x_3 \varphi(\rho)) = 0,$$

whence $3\varphi(\rho) + \rho\varphi'(\rho) = 0$ and $3\rho^2\varphi(\rho) + \rho^3\varphi'(\rho) = 0$ or $(\rho^3\varphi(\rho))' = 0$.

Consequently $\rho^3\varphi(\rho) = \varepsilon = \text{const}$ and $q = \frac{\varepsilon f \ell}{\rho^3 \ell_1^2}$.

The meaning of the constants ε and ε_1 :

Since the function q occurs in the definition of the electric field (2.1), it is natural to identify the constant ε with the electric charge of the source. Does a similar reasoning is applicable to the case of the magnetic field (2.2)? In other words, does the constant ε_1 represents a magnetic charge of the source? This question is at first related to the case where $\varepsilon = 0, \varepsilon_1 \neq 0$, namely to the case where the spherical source appears as a magnetic monopole. However, although the existence of magnetic monopoles is envisaged sometimes as a theoretical possibility, it is not yet confirmed experimentally. Accordingly we are led to assume that $\varepsilon_1 = 0$, namely that the purely $S\Theta(4)$ -invariant magnetic field vanishes. So we have to do only with the electric field (2.1), which, on account of $q = \frac{\varepsilon f \ell}{\rho^3 \ell_1^2}$, depends on the gravitational tensor (contrary to Weyl's assertion).

3 Equations of gravitation outside the charged source

We recall that, if an electromagnetic field

$$\sum V_{\alpha\beta} dx_\alpha \otimes dx_\beta, \quad (V_{\alpha\beta} = -V_{\beta\alpha})$$

is associated with a spacetime metric

$$\sum g_{\alpha\beta} dx_\alpha \otimes dx_\beta,$$

then it gives rise to an energy-momentum tensor

$$\sum W_{\alpha\beta} dx_\alpha \otimes dx_\beta$$

defined by the formulae

$$W_{\alpha\beta} = \frac{1}{4\pi} \left(\frac{1}{4} g_{\alpha\beta} \sum V_{\gamma\delta} V^{\gamma\delta} - \sum V_{\alpha\delta} V_{\beta}^{\cdot\delta} \right). \quad (3.1)$$

In the present situation, the covariant and contravariant components $V_{\gamma\delta}$ and $V^{\gamma\delta}$ are already known. So it remains to compute the mixed components

$$V_{\beta}^{\cdot\delta} = \sum g^{\gamma\delta} V_{\beta\gamma} = -\sum g^{\delta\gamma} V_{\gamma\beta} = -V_{\cdot\beta}^{\delta}.$$

Taking into account the vanishing of the magnetic field, an easy computation gives

$$V_0^{\cdot 0} = \frac{\rho^2 q f_1}{f \ell^2}, \quad V_0^{\cdot k} = -\frac{q x_k}{\ell^2}, \quad (k = 1, 2, 3),$$

$$V_k^{\cdot 0} = -\frac{\ell^2 - \rho^2 f_1^2}{f^2 \ell^2} q x_k, \quad (k = 1, 2, 3),$$

$$V_k^{\cdot k} = -\frac{q f_1}{f \ell^2} x_k^2, \quad (k = 1, 2, 3),$$

$$V_2^{\cdot 3} = -\frac{q f_1}{f \ell^2} x_2 x_3 = V_3^{\cdot 2},$$

$$V_3^{\cdot 1} = -\frac{q f_1}{f \ell^2} x_3 x_1 = V_1^{\cdot 3},$$

$$V_1^{\cdot 2} = -\frac{q f_1}{f \ell^2} x_1 x_2 = V_2^{\cdot 1}.$$

It follows that

$$\sum V_{\gamma\delta} V^{\gamma\delta} = -\frac{2\rho^2 q^2}{f^2 \ell^2},$$

$$\sum V_{0\delta} V_0^{\cdot\delta} = -\frac{\rho^2 q^2}{\ell^2},$$

$$\sum V_{0\delta} V_1^{\cdot\delta} = -\frac{\rho^2 f_1 q^2}{f \ell^2} x_1,$$

$$\sum V_{1\delta} V_2^{\cdot\delta} = \frac{\ell^2 - \rho^2 f_1^2}{f^2 \ell^2} q^2 x_1 x_2,$$

$$\sum V_{1\delta} V_1^{\cdot\delta} = \frac{\ell^2 - \rho^2 f_1^2}{f^2 \ell^2} q^2 x_1^2,$$

and then the formula (3.1) gives the components $W_{00}, W_{01}, W_{11}, W_{12}$ of the energy-momentum tensor. The other components are obtained simply by permuting indices.

Proposition 3.1 *The energy-momentum tensor associated with the electric field (2.1) is a $\Theta(4)$ -invariant tensor defined by the following formulae*

$$W_{00} = E_{00}, \quad W_{0i} = W_{i0} = x_i E_{01},$$

$$W_{ii} = E_{11} + x_i^2 E_{22}, \quad W_{ij} = W_{ji} = x_i x_j E_{22},$$

$$(i, j = 1, 2, 3; i \neq j),$$

where

$$E_{00} = \frac{1}{8\pi} \rho^2 f^2 E, \quad E_{01} = \frac{1}{8\pi} \rho^2 f f_1 E,$$

$$E_{11} = \frac{1}{8\pi} \rho^2 \ell_1^2 E, \quad E_{22} = \frac{1}{8\pi} (-\ell_1^2 - \ell^2 + \rho^2 f_1^2) E$$

with

$$E = \frac{q^2}{f^2 \ell^2} = \frac{\varepsilon^2}{\rho^2 g^4}.$$

Regarding the Ricci tensor $R_{\alpha\beta}$, we already know [4] that it is a symmetric $\Theta(4)$ -invariant tensor defined by the functions

$$Q_{00} = Q_{00}(t, \rho), \quad Q_{01} = Q_{01}(t, \rho),$$

$$Q_{11} = Q_{11}(t, \rho), \quad Q_{22} = Q_{22}(t, \rho)$$

as follows

$$R_{00} = Q_{00}, \quad R_{0i} = R_{i0} = Q_{01} x_i, \quad R_{ii} = Q_{11} + x_i^2 Q_{22},$$

$$R_{ij} = R_{ji} = x_i x_j Q_{22}, \quad (i, j = 1, 2, 3; i \neq j).$$

So, assuming that the cosmological constant vanishes, we have to do from the outset with four simple equations of gravitation, namely

$$Q_{00} - \frac{R}{2} f^2 + \frac{8\pi k}{c^4} E_{00} = 0,$$

$$Q_{01} - \frac{R}{2} f f_1 + \frac{8\pi k}{c^4} E_{01} = 0,$$

$$Q_{11} + \frac{R}{2} \ell_1^2 + \frac{8\pi k}{c^4} E_{11} = 0,$$

$$Q_{11} + \rho^2 Q_{22} - \frac{R}{2} (\rho^2 f_1^2 - \ell^2) + \frac{8\pi k}{c^4} (E_{11} + \rho^2 E_{22}) = 0.$$

An additional simplification results from the fact that the mixed components of the electromagnetic energy-momentum tensor satisfy the condition $\Sigma W_\alpha^\alpha = 0$, and then the equations of gravitation imply (by contraction) that the scalar curvature R vanishes. Moreover, introducing as usual the functions $h = \rho f_1$, $g = \rho \ell_1$, and taking into account that $q = \frac{\varepsilon f \ell}{\rho^3 \ell_1^2}$, we obtain

$$E = \frac{\varepsilon^2}{\rho^2 g^4}, \quad E_{00} = \frac{\varepsilon^2 f^2}{8\pi g^4}, \quad E_{01} = \frac{\varepsilon^2 f f_1}{8\pi g^4},$$

$$E_{11} = \frac{\varepsilon^2 \ell_1^2}{8\pi g^4}, \quad E_{11} + \rho^2 E_{22} = \frac{\varepsilon^2 (-\ell^2 + h^2)}{8\pi g^4},$$

so that by setting

$$\nu^2 = \frac{k}{c^4} \varepsilon^2,$$

we get the definitive form of the equations of gravitation

$$Q_{00} + \frac{\nu^2}{g^4} f^2 = 0, \tag{3.1}$$

$$Q_{01} + \frac{\nu^2}{g^4} f f_1 = 0, \tag{3.2}$$

$$Q_{11} + \frac{\nu^2}{g^4} \ell_1^2 = 0, \tag{3.3}$$

$$Q_{11} + \rho^2 Q_{22} + \frac{\nu^2}{g^4} (-\ell^2 + h^2) = 0. \tag{3.4}$$

4 Stationary solutions outside the charged spherical source

In the case of a stationary field, the functions Q_{00} , Q_{01} , Q_{11} , Q_{22} depend only on ρ and their expressions are already known [3, 4]

$$Q_{00} = f \left(-\frac{f''}{\ell^2} + \frac{f' \ell'}{\ell^3} - \frac{2f' g'}{\ell^2 g} \right), \tag{4.1}$$

$$Q_{01} = \frac{h}{\rho f} Q_{00}, \tag{4.2}$$

$$Q_{11} = \frac{1}{\rho^2} \left(-1 + \frac{g'^2}{\ell^2} + \frac{g g''}{\ell^2} - \frac{\ell' g g'}{\ell^3} + \frac{f' g g'}{f \ell^2} \right), \tag{4.3}$$

$$Q_{11} + \rho^2 Q_{22} = \frac{f''}{f} + \frac{2g''}{g} - \frac{f' \ell'}{f \ell} - \frac{2\ell' g'}{\ell g} + \frac{h^2}{f^2} Q_{00}. \tag{4.4}$$

On account of (4.2), the equation (3.2) is written as

$$\left(Q_{00} + \frac{\nu^2}{g^4} f^2 \right) h = 0$$

so that it is verified because of (3.1).

Consequently it only remains to take into account the equations (3.1), (3.3), (3.4).

From (3.1) we obtain

$$\frac{\nu^2}{g^4} = -\frac{Q_{00}}{f^2}$$

and inserting this expression into (3.4) we obtain the relation

$$f^2(Q_{11} + \rho^2 Q_{22}) - (-\ell^2 + h^2)Q_{00} = 0$$

which, on account of (4.1) and (4.4), reduces, after cancellations, to the simple equation

$$\frac{g''}{g'} = \frac{f'}{f} + \frac{\ell'}{\ell}$$

which does not contain the unknown function h and implies

$$f \ell = c g', \quad (c = \text{const}). \tag{4.5}$$

Next, from (3.1) and (3.3) we deduce the equation

$$Q_{11} - \frac{Q_{00}}{f^2} \ell_1^2 = 0 \tag{4.6}$$

which does not contain the function h either.

Now, from (4.5) we find

$$f = \frac{c g'}{\ell}$$

and inserting this expression of f into (4.6), we obtain an equation which can be written as

$$\frac{d}{d\rho} \left(\frac{F'}{2g'} \right) = 0$$

with

$$F' = g^2 - \frac{g^2 g'^2}{\ell^2}.$$

It follows that

$$F' = 2A_1 g - A_2, \quad (A_1 = \text{const}, A_2 = \text{const}),$$

and

$$g'^2 = \ell^2 \left(1 - \frac{2A_1}{g} + \frac{A_2}{g^2} \right). \quad (4.7)$$

On account of (4.5), the derivative g' does not vanish. In fact $g' = 0$ implies either $f = 0$ or $\ell = 0$, which gives rise to a degenerate spacetime metric, namely a spacetime metric meaningless physically. Then, in particular, it follows from (4.7) that

$$1 - \frac{2A_1}{g} + \frac{A_2}{g^2} > 0.$$

The constant A_1 , obtained by means of the Newtonian approximation, is already known:

$$A_1 = \frac{km}{c^2} = \mu.$$

In order to get A_2 , we insert first

$$\frac{f'}{f} = \frac{g''}{g'} - \frac{\ell'}{\ell}$$

into (4.3) thus obtaining

$$\rho^2 Q_{11} = -1 + \frac{g'^2}{\ell^2} + \frac{2gg''}{\ell^2} - \frac{2\ell'gg'}{\ell^3}. \quad (4.8)$$

Next by setting

$$Q(g) = 1 - \frac{2A_1}{g} + \frac{A_2}{g^2}$$

we have

$$g' = \ell \sqrt{Q(g)},$$

$$g'' = \ell' \sqrt{Q(g)} + \ell^2 \left(\frac{A_1}{g^2} - \frac{A_2}{g^3} \right)$$

and inserting these expressions of g' and g'' into (4.8), we find

$$\rho^2 Q_{11} = -\frac{A_2}{g^2}.$$

The equation (3.3) gives finally the value of the constant A_2 :

$$A_2 = \nu^2 = \frac{k\varepsilon^2}{c^4}.$$

It follows that the general stationary solution outside the charged spherical source is defined by two equations, namely

$$f\ell = c \frac{dg}{d\rho}, \quad (4.9)$$

$$\frac{dg}{d\rho} = \ell \sqrt{1 - \frac{2\mu}{g} + \frac{\nu^2}{g^2}}, \quad (4.10)$$

$$\left(\mu = \frac{km}{c^2}, \quad \nu = \frac{\sqrt{k}}{c^2} |\varepsilon|, \quad 1 - \frac{2\mu}{g} + \frac{\nu^2}{g^2} > 0 \right).$$

The interdependence of the two fields, gravitational and electric, is now obvious: The electric charge ε , which defines the electric field, is also involved in the definition of the gravitational field by means of the term

$$\frac{\nu^2}{g^2} = \frac{k}{c^4} \left(\frac{\varepsilon}{g} \right)^2.$$

On the other hand, since

$$q = \frac{\varepsilon f \ell}{\rho^3 \ell_1^2} = \frac{c\varepsilon}{\rho g^2} \frac{dg}{d\rho},$$

the components of the electric field:

$$\begin{aligned} V_{01} = -V_{10} &= q x_1 = \frac{c\varepsilon}{g^2} \frac{dg}{d\rho} \frac{x_1}{\rho} = \\ &= -c\varepsilon \frac{\partial}{\partial x_1} \left(\frac{1}{g} \right) = -c \frac{\partial}{\partial x_1} \left(\frac{\varepsilon}{g} \right), \end{aligned}$$

$$V_{02} = -V_{20} = q x_2 = -c \frac{\partial}{\partial x_2} \left(\frac{\varepsilon}{g} \right),$$

$$V_{03} = -V_{30} = q x_3 = -c \frac{\partial}{\partial x_3} \left(\frac{\varepsilon}{g} \right)$$

result from the electric potential:

$$\frac{\varepsilon}{g} = \frac{\varepsilon}{g(\rho)}$$

which is thus defined by means of the curvature radius $g(\rho)$, namely by the fundamental function involved in the definition of the gravitational field.

Note that, among the functions occurring in the spacetime metric, only the function $h = \rho f_1$ does not appear in the equations (4.9) and (4.10). The problem does not require a uniquely defined h . Every differentiable function h satisfying the condition $|h| \leq \ell$ is allowable. And every allowable h gives rise to a possible conception of the time coordinate. Contrary to the Special Relativity, we have to do, in General Relativity, with an infinity of possible definitions of the time coordinate. In order to elucidate this assertion in the present situation, let us denote by ρ_1 the radius of the spherical stationary source, and consider a photon emitted radially from the sphere $\|x\| = \rho_1$ at an instant τ . The equation of motion

of this photon, namely

$$f(\rho)dt + h(\rho)d\rho = \ell(\rho)d\rho$$

implies

$$\frac{dt}{d\rho} = \frac{-h(\rho) + \ell(\rho)}{f(\rho)}$$

whence $\tau = t - \psi(\rho)$ with

$$\psi(\rho) = \int_{\rho_1}^{\rho} \frac{-h(u) + \ell(u)}{f(u)} du.$$

For every value of $\rho \geq \rho_1$, $\pi(t, \rho) = t - \psi(\rho)$ is the instant of radial emission of a photon reaching the sphere $\|x\| = \rho$ at the instant t . The function $\pi(t, \rho)$ will be called *propagation function*, and we see that to each allowable h there corresponds a uniquely defined propagation function. Moreover each propagation function characterizes uniquely a conception of the notion of time. Regarding the radial velocity of propagation of light, namely

$$\frac{d\rho}{dt} = \frac{f(\rho)}{-h(\rho) + \ell(\rho)},$$

it is not bounded by a barrier as in Special Relativity. In the limit case where the allowable h equals ℓ , this velocity becomes infinite.

This being said, we return to the equations (4.9) and (4.10) which contain the remaining unknown functions f , ℓ , g . Their investigation necessitates a rather lengthy discussion which will be carried out in another paper. At present we confine ourselves to note two significant conclusions of this discussion:

- (a) Pointwise sources do not exist, so that the spherical source cannot be reduced to a point. In particular the notion of black hole is inconceivable;
- (b) Among the solutions defined by (4.9) and (4.10), particularly significant are those obtained by introducing the radial geodesic distance

$$\delta = \int_0^{\rho} \ell(u) du.$$

Then we have to define the curvature radius $G(\delta) = g(\rho(\delta))$ by means of the equation

$$\frac{dG}{d\delta} = \sqrt{1 - \frac{2\mu}{G} + \frac{\nu^2}{G^2}}$$

the solutions of which need specific discussion according as $\nu^2 - \mu^2 > 0$ or $\nu^2 - \mu^2 = 0$ or $\nu^2 - \mu^2 < 0$. The first approach to this problem appeared in the paper [1].

We note finally that the derivation of the Reissner-Nordström metric contains topological errors and moreover identifies erroneously the fundamental function $g(\rho)$ with a ra-

dial coordinate. This is why the Reissner-Nordström metric is devoid of geometrical and physical meaning.

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Low-Lying Collective Levels in $^{224-234}\text{Th}$ Nuclei

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The low-lying collective levels in $^{224-234}\text{Th}$ isotopes are investigated in the frame work of the interacting boson approximation model (IBA-1). The contour plot of the potential energy surfaces, $V(\beta, \gamma)$, shows two wells on the prolate and oblate sides which indicate that all thorium nuclei are deformed and have rotational characters. The levels energy, electromagnetic transition rates $B(E1)$ and $B(E2)$ are calculated. Bending at angular momentum $I^+ = 20$ has been observed for ^{230}Th . Staggering effect has been calculated and beat patterns are obtained which indicate the existence of an interaction between the ground state band, (GSB), and the octupole negative parity band, (NPB). All calculated values are compared with the available experimental data and show reasonable agreement.

1 Introduction

The level schemes of $^{224-234}\text{Th}$ isotopes are characterized by the existence of two bands of opposite parity and lie in the region of octupole deformations. The primary evidence for this octupole deformation comes from the parity-doublet bands, fast electric transition ($E1$) between the negative and positive parity bands and the low-lying 1^- , 0_2^+ and 2_2^+ excitation energy states. This kind of deformation has offered a real challenge for nuclear structure models. Even-even thorium nuclei have been studied within the frame work of the *Spdf* interacting boson model [1] and found the properties of the low-lying states can be understood without stable octupole deformation. High spin states in some of these nuclei suggest that octupole deformation develops with increasing spin.

A good description of the first excited positive and negative parity bands of nuclei in the rare earth and the actinide region has achieved [2–4] using the interacting vector boson model. The analysis of the eigen values of the model Hamiltonian reveals the presence of an interaction between these bands. Due to this interaction staggering effect has reproduced including the beat patterns.

Shanmugam-Kamalahran (SK) model [5] for α -decay has been applied successfully to $^{226-232}\text{Th}$ for studying their shapes, deformations of the parent and daughter nuclei as well as the charge distribution process during the decay. Also, a solution of the Bohr Hamiltonian [6] aiming at the description of the transition from axial octupole deformation to octupole vibrations in light actinides ^{224}Ra and ^{226}Th is worked out. The parameter free predictions of the model are in good agreement with the experimental data of the two nuclei, where they known to lie closest to the transition from octupole deformation to octupole vibrations in this region. A new frame-work for comparing fusion probabilities in reactions [7] forming heavy elements, ^{220}Th , eliminates both theoretical and experimental uncertainties, allowing insights into systematic be-

havior, and revealing previously hidden characteristics in fusion reactions forming heavy elements.

It is found that cluster model [8] succeeded in reproducing satisfactorily the properties of normal deformed ground state and super deformed excited bands [9, 10] in a wide range of even-even nuclei, $222 \leq A \leq 242$ [11]. The calculated spin dependences [12] to the parity splitting and the electric multipole transition moments are in agreement with the experimental data. Also, a new formula between half-lives, decay energies and microscopic density-dependent cluster model [13] has been used and the half-lives of cluster radioactivity are well reproduced.

A new empirical formula [14], with only three parameters, is proposed for cluster decay half-lives. The parameters of the formula are obtained by making least square fit to the available experimental cluster decay data. The calculated half-lives are compared with the results of the earlier proposed models, experimental available data and show excellent agreement. A simple description of the cluster decay by suggesting a folding cluster-core interaction based on a self-consistent mean-field model [15]. Cluster decay in even-even nuclei above magic numbers have investigated.

Until now scarce informations are available about the actinide region in general and this is due to the experimental difficulties associated with this mass region. The aim of the present work is to:

- (1) calculate the potential energy surfaces, $V(\beta, \gamma)$, and know the type of deformation exists;
- (2) calculate levels energy, electromagnetic transition rates $B(E1)$ and $B(E2)$;
- (3) study the relation between the angular momentum I , the rotational angular frequency $\hbar\omega$ and see if there any bending for any of thorium isotopes;
- (4) calculate staggering effect and beat patterns to study the interaction between the (+ve) and (-ve) parity bands.

nucleus	<i>EPS</i>	<i>PAIR</i>	<i>ELL</i>	<i>QQ</i>	<i>OCT</i>	<i>HEX</i>	<i>E2SD(eb)</i>	<i>E2DD(eb)</i>
²²⁴ Th	0.2000	0.000	0.0081	-0.0140	0.0000	0.0000	0.2150	-0.6360
²²⁶ Th	0.2000	0.000	0.0058	-0.0150	0.0000	0.0000	0.2250	-0.6656
²²⁸ Th	0.2000	0.0000	0.0052	-0.0150	0.0000	0.0000	0.1874	-0.5543
²³⁰ Th	0.2000	0.0000	0.0055	-0.0150	0.0000	0.0000	0.1874	-0.5543
²³² Th	0.2000	0.0000	0.0055	-0.0150	0.0000	0.0000	0.1820	-0.5384
²³⁴ Th	0.2000	0.0000	0.0063	-0.0150	0.0000	0.0000	0.1550	-0.4585

Table 1: Parameters used in IBA-1 Hamiltonian (all in MeV).

2 (IBA-1) model

2.1 Level energies

The IBA-1 model was applied to the positive and negative parity low-lying states in even-even ²²⁴⁻²³⁴Th isotopes. The proton, π , and neutron, ν , bosons are treated as one boson and the system is considered as an interaction between s -bosons and d -bosons. Creation ($s^\dagger d^\dagger$) and annihilation ($s\tilde{d}$) operators are for s and d bosons. The Hamiltonian [16] employed for the present calculation is given as:

$$\begin{aligned}
H = & EPS \cdot n_d + PAIR \cdot (P \cdot P) \\
& + \frac{1}{2} ELL \cdot (L \cdot L) + \frac{1}{2} QQ \cdot (Q \cdot Q) \\
& + 5OCT \cdot (T_3 \cdot T_3) + 5HEX \cdot (T_4 \cdot T_4),
\end{aligned} \quad (1)$$

where

$$P \cdot P = \frac{1}{2} \left[\begin{array}{c} \left\{ (s^\dagger s^\dagger)_0^{(0)} - \sqrt{5} (d^\dagger d^\dagger)_0^{(0)} \right\} x \\ \left\{ (s s)_0^{(0)} - \sqrt{5} (\tilde{d} \tilde{d})_0^{(0)} \right\} \end{array} \right]_0^{(0)}, \quad (2)$$

$$L \cdot L = -10 \sqrt{3} \left[(d^\dagger \tilde{d})^{(1)} x (d^\dagger \tilde{d})^{(1)} \right]_0^{(0)}, \quad (3)$$

$$Q \cdot Q = \sqrt{5} \left[\begin{array}{c} \left\{ (S^\dagger \tilde{d} + d^\dagger s)^{(2)} - \frac{\sqrt{7}}{2} (d^\dagger \tilde{d})^{(2)} \right\} x \\ \left\{ (s^\dagger \tilde{d} + \tilde{d} s)^{(2)} - \frac{\sqrt{7}}{2} (d^\dagger \tilde{d})^{(2)} \right\} \end{array} \right]_0^{(0)}, \quad (4)$$

$$T_3 \cdot T_3 = -\sqrt{7} \left[(d^\dagger \tilde{d})^{(2)} x (d^\dagger \tilde{d})^{(2)} \right]_0^{(0)}, \quad (5)$$

$$T_4 \cdot T_4 = 3 \left[(d^\dagger \tilde{d})^{(4)} x (d^\dagger \tilde{d})^{(4)} \right]_0^{(0)}. \quad (6)$$

In the previous formulas, n_d is the number of boson; $P \cdot P$, $L \cdot L$, $Q \cdot Q$, $T_3 \cdot T_3$ and $T_4 \cdot T_4$ represent pairing, angular momentum, quadrupole, octupole and hexadecupole interactions between the bosons; EPS is the boson energy; and $PAIR$, ELL , QQ , OCT , HEX is the strengths of the pairing, angular momentum, quadrupole, octupole and hexadecupole interactions.

2.2 Transition rates

The electric quadrupole transition operator [16] employed in this study is given by:

$$\begin{aligned}
T^{(E2)} = & E2SD \cdot (s^\dagger \tilde{d} + d^\dagger s)^{(2)} + \\
& + \frac{1}{\sqrt{5}} E2DD \cdot (d^\dagger \tilde{d})^{(2)}.
\end{aligned} \quad (7)$$

The reduced electric quadrupole transition rates between $I_i \rightarrow I_f$ states are given by

$$B(E2, I_i \rightarrow I_f) = \frac{[\langle I_f || T^{(E2)} || I_i \rangle]^2}{2I_i + 1}. \quad (8)$$

3 Results and discussion

3.1 The potential energy surface

The potential energy surfaces [17], $V(\beta, \gamma)$, for thorium isotopes as a function of the deformation parameters β and γ have been calculated using:

$$\begin{aligned}
E_{N_\pi N_\nu}(\beta, \gamma) = & \langle N_\pi N_\nu; \beta \gamma | H_{\pi\nu} | N_\pi N_\nu; \beta \gamma \rangle = \\
= & \zeta_d(N_\nu N_\pi) \beta^2 (1 + \beta^2) + \beta^2 (1 + \beta^2)^{-2} \times \\
& \times \{ k N_\nu N_\pi [4 - (\bar{X}_\pi \bar{X}_\nu) \beta \cos 3\gamma] \} + \\
& + \left\{ [\bar{X}_\pi \bar{X}_\nu \beta^2] + N_\nu (N_\nu - 1) \left(\frac{1}{10} c_0 + \frac{1}{7} c_2 \right) \beta^2 \right\},
\end{aligned} \quad (9)$$

where

$$\bar{X}_\rho = \left(\frac{2}{7} \right)^{0.5} X_\rho \quad \rho = \pi \text{ or } \nu. \quad (10)$$

The calculated potential energy surfaces, $V(\beta, \gamma)$, for thorium series of isotopes are presented in Fig. 1. It shows that all nuclei are deformed and have rotational-like characters. The prolate deformation is deeper than oblate in all nuclei except ²³⁰Th. The two wells on both oblate and prolate sides are equals and O(6) characters is expected to the nucleus. The energy and electromagnetic magnetic transition rates ratio are not in favor to that assumption and it is treated as a rotational-like nucleus.

$I_i^+ I_f^+$	^{224}Th	^{226}Th	^{228}Th	^{230}Th	^{232}Th	^{234}Th
$0_1 \text{ Exp. } 2_1$	—	6.85(42)	7.06(24)	8.04(10)	9.28(10)	8.00(70)
$0_1 \text{ Theor. } 2_1$	4.1568	6.8647	7.0403	8.038	9.2881	8.0559
$2_1 0_1$	0.8314	1.3729	1.4081	1.6076	1.8576	1.6112
$2_2 0_1$	0.0062	0.0001	0.0044	0.0088	0.0105	0.0079
$2_2 0_2$	0.4890	0.8357	0.8647	1.0278	1.2683	1.1659
$2_3 0_1$	0.0127	0.0272	0.0211	0.0157	0.0122	0.0075
$2_3 0_2$	0.1552	0.0437	0.0020	0.0023	0.0088	0.0099
$2_3 0_3$	0.1102	0.0964	0.0460	0.0203	0.0079	0.0023
$2_4 0_3$	0.2896	0.4907	0.5147	0.6271	0.8048	0.7786
$2_4 0_4$	0.1023	0.0709	0.0483	0.0420	0.0385	0.0990
$2_2 2_1$	0.1837	0.1153	0.0599	0.0387	0.0286	0.0174
$2_3 2_1$	0.0100	0.0214	0.0211	0.0198	0.0178	0.0118
$2_3 2_2$	0.8461	1.0683	0.5923	0.2989	0.1538	0.0697
$4_1 2_1$	1.3733	2.0662	2.0427	2.2957	2.6375	2.2835
$4_1 2_2$	0.0908	0.1053	0.0764	0.0579	0.0445	0.0266
$4_1 2_3$	0.0704	0.0325	0.0104	0.0038	0.0018	0.0008
$6_1 4_1$	1.5696	2.2921	2.2388	2.4979	2.8606	2.4745
$6_1 4_2$	0.0737	0.0858	0.0685	0.0585	0.0493	0.0312
$6_1 4_3$	0.0584	0.0404	0.0198	0.0106	0.0061	0.0029
$8_1 6_1$	1.5896	2.3199	2.2720	2.5381	2.9105	2.5220
$8_1 6_2$	0.0569	0.0660	0.0554	0.0511	0.0466	0.0314
$8_1 6_3$	0.0483	0.0421	0.0256	0.0166	0.0109	0.0055
$10_1 8_1$	1.4784	2.2062	2.1948	2.4760	2.8586	2.4899
$10_1 8_2$	0.0448	0.0513	0.0438	0.0422	0.0407	0.0290

Table 2: Values of the theoretical reduced transition probability, $B(E2)$ (in $e^2 b^2$).

$I_i^- I_f^+$	^{224}Th	^{226}Th	^{228}Th	^{230}Th	^{232}Th	^{234}Th
$1_1 0_1$	0.0428	0.0792	0.1082	0.1362	0.1612	0.1888
$1_1 0_2$	0.0942	0.0701	0.0583	0.0534	0.0515	0.0495
$3_1 2_1$	0.1607	0.1928	0.2209	0.2531	0.2836	0.3227
$3_1 2_2$	0.0733	0.0829	0.0847	0.0817	0.0768	0.0717
$3_1 2_3$	0.0360	0.0157	0.0054	0.0013	0.0002	0.0000
$3_1 4_1$	0.0233	0.0441	0.0652	0.0884	0.1150	0.1384
$3_1 4_2$	0.0170	0.0285	0.0371	0.0424	0.0460	0.0449
$5_1 4_1$	0.2873	0.3131	0.3363	0.3657	0.3946	—
$5_1 4_2$	0.0787	0.0834	0.0868	0.0865	0.0835	—
$5_1 4_3$	0.0160	0.0101	0.0051	0.0020	0.0006	—
$7_1 6_1$	0.4178	0.4387	0.4581	0.4839	0.5100	—
$7_1 6_2$	0.0732	0.0757	0.0798	0.0817	0.0812	—
$9_1 8_1$	0.5532	0.5690	0.5848	0.6070	0.6301	—
$9_1 8_2$	0.0639	0.0665	0.0707	0.0735	0.0748	—

Table 3: Values of the theoretical reduced transition probability, $B(E1)$ (in $\mu e^2 b$).

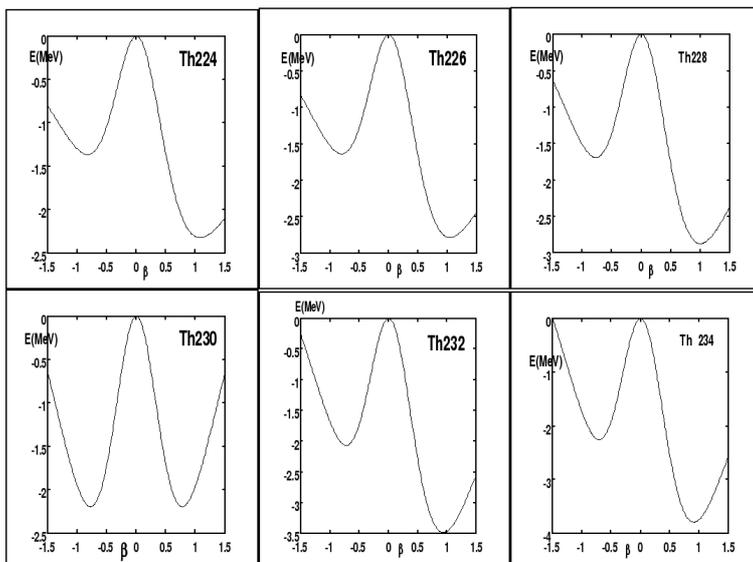


Fig. 1: Potential Energy surfaces for $^{224-234}\text{Th}$ nuclei.

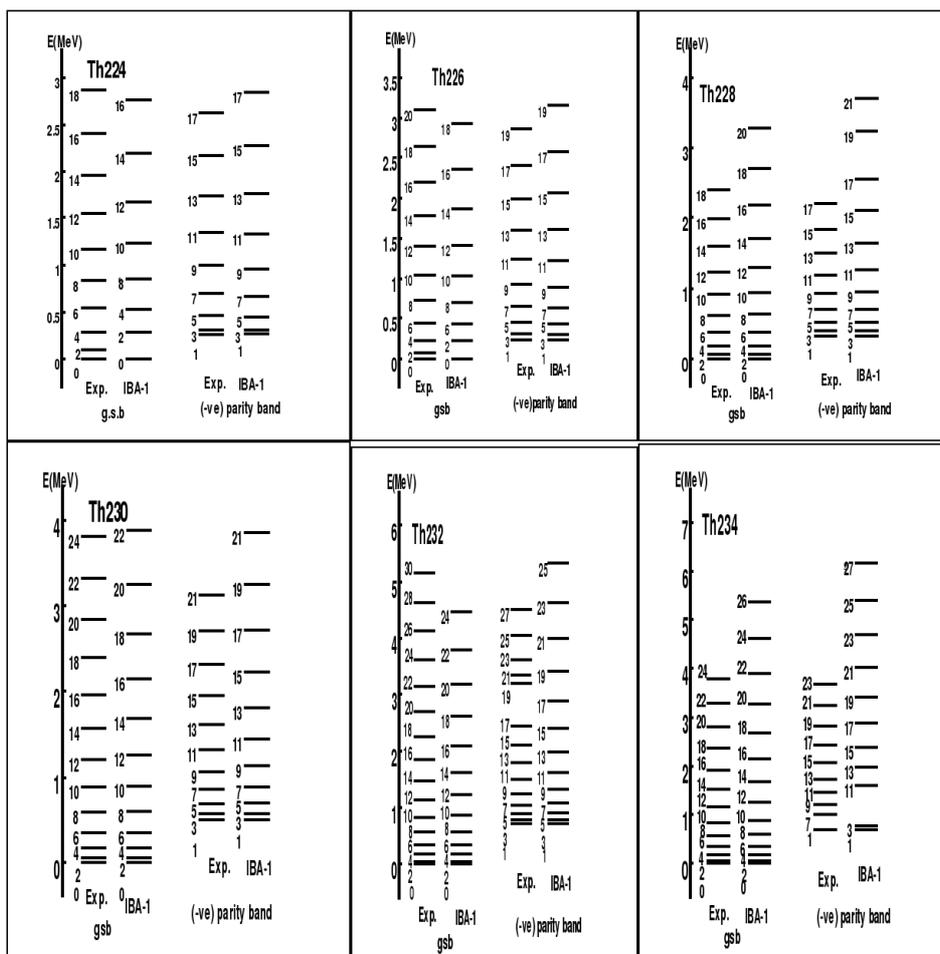


Fig. 2: Comparison between experimental (Exp.) and theoretical (IBA-1) energy levels in $^{224-234}\text{Th}$.

3.2 Energy spectra

IBA-1 model has been used in calculating the energy of the positive and negative parity low -lying levels of thorium series of isotopes. In many deformed actinide nuclei the negative parity bands have been established and these nuclei are considered as an octupole deformed. A simple means to examine the nature of the band is to consider the ratio R which for octupole band, $R > 1$, and defined as [18]:

$$R = \frac{E(I+3) - E(I-1)_{NPB}}{E(I) - E(I-2)_{GSB}}. \quad (11)$$

In the present calculations all values of R for thorium series of isotopes are > 1 , and we treated them as octupole deformed nuclei.

A comparison between the experimental spectra [19–24] and our calculations, using values of the model parameters given in Table 1 for the ground and octupole bands, are illustrated in Fig. 2. The agreement between the calculated levels energy and their correspondence experimental values for all thorium nuclei are slightly higher especially for the higher excited states. We believe this is due to the change of the projection of the angular momentum which is due to band crossing and octupole deformation.

Unfortunately there is no enough measurements of electromagnetic transition rates $B(E2)$ or $B(E1)$ for these series of nuclei. The only measured $B(E2, 0_1^+ \rightarrow 2_1^+)$'s are presented, in Table's 2,3 for comparison with the calculated values. The parameters $E2SD$ and $E2DD$ used in the present calculations are determined by normalizing the calculated values to the experimentally known ones and displayed in Table 1.

For calculating $B(E1)$ and $B(E2)$ electromagnetic transition rates of intraband and interband we did not introduce any new parameters. Some of the calculated values are presented in Fig. 3 and show bending at $N = 136, 142$ which means there is an interaction between the $(+ve)GSB$ and $(-ve)$ parity octupole bands.

The moment of inertia I and energy parameters $\hbar\omega$ are calculated using equations (12, 13):

$$\frac{2I}{\hbar^2} = \frac{4I - 2}{\Delta E(I \rightarrow I - 2)}, \quad (12)$$

$$(\hbar\omega)^2 = (I^2 - I + 1) \left[\frac{\Delta E(I \rightarrow I - 2)}{(2I - 1)} \right]^2. \quad (13)$$

All the plots in Fig. 4 show back bending at angular momentum $I^+ = 20$ for ^{230}Th . It means, there is a band crossing and this is confirmed by calculating staggering effect to these series of thorium nuclei. A disturbance of the regular band structure has observed not only in the moment of inertia but also in the decay properties.

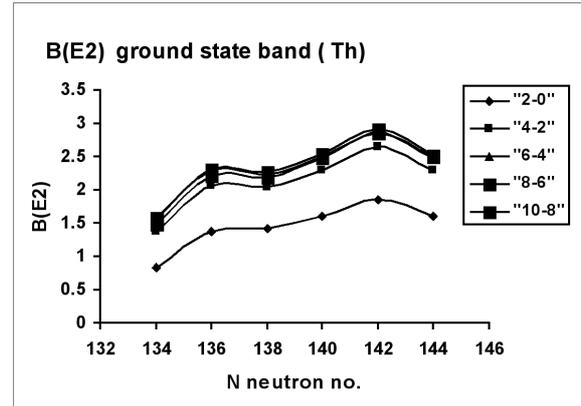


Fig. 3: The calculated $B(E2)$'s for the ground state band of $^{224-234}\text{Th}$ isotopes.

3.3 The staggering

The presence of odd-even parity states has encouraged us to study staggering effect for $^{218-230}\text{Th}$ series of isotopes [10, 12, 25, 26]. Staggering patterns between the energies of the GSB and the $(-ve)$ parity octupole band have been calculated, $\Delta I = 1$, using staggering function equations (14, 15) with the help of the available experimental data [19–24].

$$\text{Stag}(I) = 6\Delta E(I) - 4\Delta E(I-1) - 4\Delta E(I+1) + \Delta E(I+2) + \Delta E(I-2), \quad (14)$$

with

$$\Delta E(I) = E(I+1) - E(I). \quad (15)$$

The calculated staggering patterns are illustrated in Fig. 5, where we can see the beat patterns of the staggering behavior which show an interaction between the ground state and the octupole bands.

3.4 Conclusions

The IBA-1 model has been applied successfully to $^{224-234}\text{Th}$ isotopes and we have got:

1. The ground state and octupole bands are successfully reproduced;
2. The potential energy surfaces are calculated and show rotational behavior to $^{224-234}\text{Th}$ isotopes where they are mainly prolate deformed nuclei;
3. Electromagnetic transition rates $B(E1)$ and $B(E2)$ are calculated;
4. Bending for ^{230}Th has been observed at angular momentum $I^+ = 20$;
5. Staggering effect has been calculated and beat patterns are obtained which show an interaction between the ground state and octupole bands;

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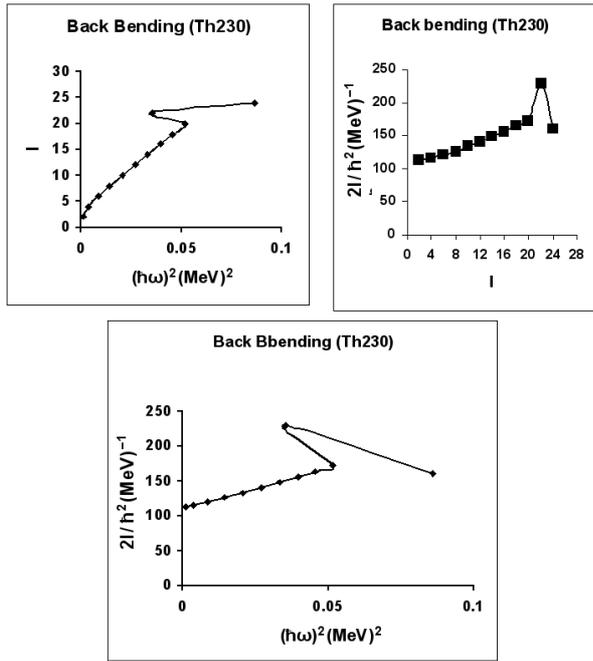


Fig. 4: Angular momentum I as a function of the rotational frequency $(\hbar\omega)^2$ and $2I/\hbar^2$ as a function of $(\hbar\omega)^2$ for the GSB of ^{230}Th .

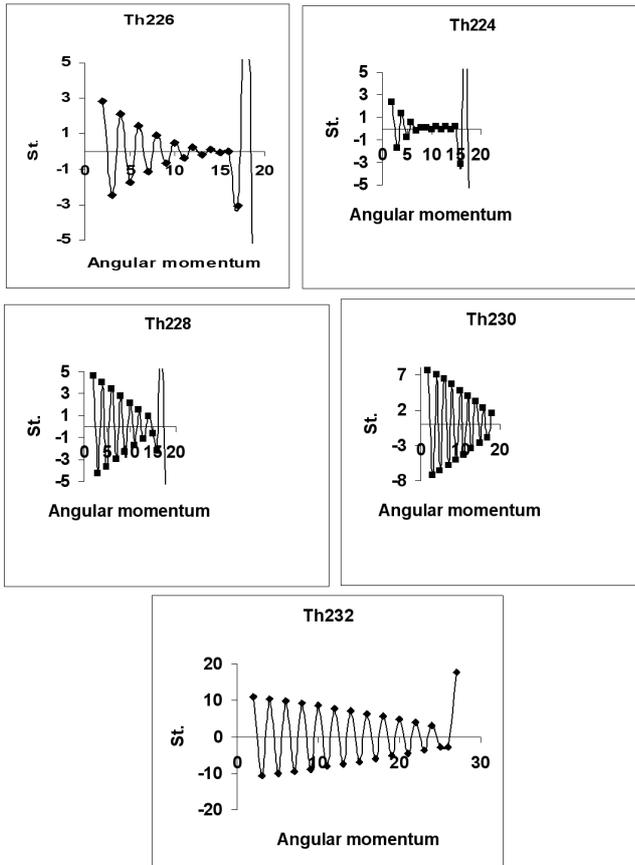


Fig. 5: $\Delta I = 1$, staggering patterns for the ground state and octupole bands of $^{224-232}\text{Th}$ isotope.

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Correlated Detection of sub-mHz Gravitational Waves by Two Optical-Fiber Interferometers

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Results from two optical-fiber gravitational-wave interferometric detectors are reported. The detector design is very small, cheap and simple to build and operate. Using two detectors has permitted various tests of the design principles as well as demonstrating the first simultaneous detection of correlated gravitational waves from detectors spatially separated by 1.1 km. The frequency spectrum of the detected gravitational waves is sub-mHz with a strain spectral index $a = -1.4 \pm 0.1$. As well as characterising the wave effects the detectors also show, from data collected over some 80 days in the latter part of 2007, the dominant earth rotation effect and the earth orbit effect. The detectors operate by exploiting light speed anisotropy in optical-fibers. The data confirms previous observations of light speed anisotropy, earth rotation and orbit effects, and gravitational waves.

1 Introduction

Results from two optical-fiber gravitational-wave interferometric detectors are reported. Using two detectors has permitted various tests of the design principles as well as demonstrating the first simultaneous detection of correlated gravitational waves from detectors spatially separated by 1.1 km. The frequency spectrum of the detected gravitational waves is sub-mHz. As well as characterising the wave effects the detectors also show, from data collected over some 80 days in the latter part of 2007, the dominant earth rotation effect and the earth orbit effect. The detectors operate by exploiting light speed anisotropy in optical-fibers. The data confirms previous observations [1–4, 6–10] of light speed anisotropy, earth rotation and orbit effects, and gravitational waves. These observations and experimental techniques were first understood in 2002 when the Special Relativity effects and the presence of gas were used to calibrate the Michelson interferometer in gas-mode; in vacuum-mode the Michelson interferometer cannot respond to light speed anisotropy [11, 12], as confirmed in vacuum resonant-cavity experiments, a modern version of the vacuum-mode Michelson interferometer [13]. The results herein come from improved versions of the prototype optical-fiber interferometer detector reported in [9], with improved temperature stabilisation and a novel operating technique where one of the interferometer arms is orientated with a small angular offset from the local meridian. The detection of sub-mHz gravitational waves dates back to the pioneering work of Michelson and Morley in 1887 [1], as discussed in [16], and detected again by Miller [2] also using a gas-mode Michelson interferometer, and by Torr and Kolen [6], DeWitte [7] and Cahill [8] using RF waves in coaxial cables, and by Cahill [9] and herein using an optical-fiber interfer-

ometer design, which is very much more sensitive than a gas-mode interferometer, as discussed later.

It is important to note that the repeated detection, over more than 120 years, of the anisotropy of the speed of light is not in conflict with the results and consequences of Special Relativity (SR), although at face value it appears to be in conflict with Einstein's 1905 postulate that the speed of light is an invariant in vacuum. However this contradiction is more apparent than real, for one needs to realise that the space and time coordinates used in the standard SR Einstein formalism are *constructed* to make the speed of light invariant wrt those special coordinates. To achieve that observers in relative motion must then relate their space and time coordinates by a Lorentz transformation that mixes space and time coordinates — but this is only an artifact of this formalism*. Of course in the SR formalism one of the frames of reference could have always been designated as the observable one. Such an ontologically real frame of reference, only in which the speed of light is isotropic, has been detected for over 120 years, yet ignored by mainstream physics. The problem is in not clearly separating a very successful mathematical formalism from its predictions and experimental tests. There has been a long debate over whether the Lorentz 3-space *and* time interpretation or the Einstein spacetime interpretation of observed SR effects is preferable or indeed even experimentally distinguishable.

What has been discovered in recent years is that a dynamical structured 3-space exists, so confirming the Lorentz interpretation of SR, and with fundamental implications for physics — for physics failed to notice the existence of the

*Thus the detected light speed anisotropy does not indicate a breakdown of Lorentz symmetry, contrary to the aims but not the outcomes of [13].

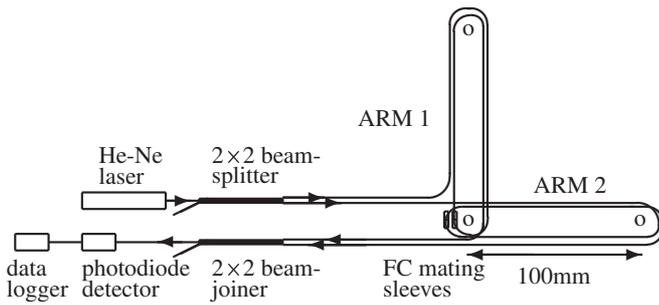


Fig. 1: Schematic layout of the interferometric optical-fiber light-speed anisotropy/gravitational wave detector. Actual detector is shown in Fig. 2. Coherent 633 nm light from the a He-Ne Laser is split into two lengths of single-mode polarisation preserving fibers by the 2×2 beam splitter. The two fibers take different directions, ARM1 and ARM2, after which the light is recombined in the 2×2 beam joiner, in which the phase differences lead to interference effects that are indicated by the outgoing light intensity, which is measured in the photodiode detector/amplifier (Thorlabs PDA36A or PDA36A-EC), and then recorded in the data logger. In the actual layout the fibers make two loops in each arm, but with excess lengths wound around one arm (not shown) — to reduce effective fiber lengths so as to reduce sensitivity. The length of one straight section is 100 mm, which is the center to center spacing of the plastic turners, having diameter = 52 mm, see Fig. 2. The relative travel times, and hence the output light intensity, are affected by the varying speed and direction of the flowing 3-space, by affecting differentially the speed of the light, and hence the net phase difference between the two arms.

main *constituent* defining the universe, namely a dynamical 3-space, with quantum matter and EM radiation playing a minor role. This dynamical 3-space provides an explanation for the success of the SR Einstein formalism. It also provides a new account of gravity, which turns out to be a quantum effect [17], and of cosmology [16, 18–20], doing away with the need for dark matter and dark energy.

2 Dynamical 3-space and gravitational waves

Light-speed anisotropy experiments have revealed that a dynamical 3-space exists, with the speed of light being c , in vacuum, only wrt to this space: observers in motion “through” this 3-space detect that the speed of light is in general different from c , and is different in different directions*. The dynamical equations for this 3-space are now known and involve a velocity field $\mathbf{v}(\mathbf{r}, t)$, but where only relative velocities are observable locally — the coordinates \mathbf{r} are relative to a non-physical mathematical embedding space. These dynamical equations involve Newton’s gravitational constant G and the fine structure constant α . The discovery of this dynamical 3-space then required a generalisation of the Maxwell, Schrödinger and Dirac equations. The wave effects already de-

*Many failed experiments supposedly designed to detect this anisotropy can be shown to have design flaws.



Fig. 2: Photograph of a detector showing the optical fibers forming the two orthogonal arms. See Fig. 1 for the schematic layout. The 2×2 beam splitter and joiner (Thorlabs FC632-50B-FC) are the two small stainless steel cylindrical tubes. The two FC to FC mating sleeves (Thorlabs ADAFC1) are physically adjacent. The overall dimensions of the metal base plate are 160×160 mm. The 2×2 splitter and joiner each have two input and two output fibers, with one not used. Arm 2 is folded over the splitter and joiner, compared to the schematic layout. The interferometer shown costs approximately \$400.

tected correspond to fluctuations in the 3-space velocity field $\mathbf{v}(\mathbf{r}, t)$, so they are really 3-space turbulence or wave effects. However they are better known, if somewhat inappropriately, as “gravitational waves” or “ripples” in “spacetime”. Because the 3-space dynamics gives a deeper understanding of the spacetime formalism we now know that the metric of the induced spacetime, merely a mathematical construct having no ontological significance, is related to $\mathbf{v}(\mathbf{r}, t)$ according to [16, 18, 20]

$$ds^2 = dt^2 - \frac{(\mathbf{dr} - \mathbf{v}(\mathbf{r}, t)dt)^2}{c^2} = g_{\mu\nu} dx^\mu dx^\nu. \quad (1)$$

The gravitational acceleration of matter, and of the structural patterns characterising the 3-space, are given by [16, 17]

$$\mathbf{g} = \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \quad (2)$$

and so fluctuations in $\mathbf{v}(\mathbf{r}, t)$ may or may not manifest as a gravitational force. The general characteristics of $\mathbf{v}(\mathbf{r}, t)$ are now known following the detailed analysis of the experiments noted above, namely its average speed, and removing the earth orbit effect, is some 420 ± 30 km/s, from direction $RA = 5.5 \pm 2^{\text{hr}}$, $Dec = 70 \pm 10^\circ S$ — the center point of the Miller data in Fig. 12b, together with large wave/turbulence effects. The magnitude of this turbulence depends on the timing resolution of each particular experiment, and here we



Fig. 3: (a) Detector 1 (D1) is located inside a sealed air-filled bucket inside an insulated container (blue) containing some 90 kg of water for temperature stabilisation. This detector, in the School of Chemistry, Physics and Earth Sciences, had an orientation of 5° anti-clockwise to the local meridian. Cylindrical He-Ne laser (Melles-Griot 0.5 mW 633 nm 05-LLR-811-230) is located on LHS of bench, while data logger is on RHS. Photodiode detector/pre-amplifier is located atop aluminium plate. (b) Detector 2 (D2) was located 1.1 km North of D1 in the Australian Science and Mathematics School. This detector had an orientation of 11° anti-clockwise to the local meridian. The data was logged on a PC running a PoScope USB DSO (PoLabs <http://www.poscope.com>).

characterise them at sub-mHz frequencies, showing that the fluctuations are very large, as also seen in [8].

3 Gravitational wave detectors

To measure $\mathbf{v}(\mathbf{r}, t)$ has been difficult until now. The early experiments used gas-mode Michelson interferometers, which involved the visual observation of small fringe shifts as the relatively large devices were rotated. The RF coaxial cable experiments had the advantage of permitting electronic recording of the RF travel times, over 500m [6] and 1.5 km [7], by means of two or more atomic clocks, although the experiment reported in [8] used a novel technique that enable the coaxial cable length to be reduced to laboratory size*.

*The calibration of this technique is at present not well understood in view of recent discoveries concerning the Fresnel drag effect in optical fibers.



Fig. 4: (a) Detectors are horizontally located inside an air-filled bucket. The plastic bag reduces even further any air movements, and thus temperature differentials. The blue crystals are silica gel to reduce moisture. (b) Bucket located inside and attached to bottom of the insulated container prior to adding water to the container.

The new optical-fiber detector design herein has the advantage of electronic recording as well as high precision because the travel time differences in the two orthogonal fibers employ light interference effects, but with the interference effects taking place in an optical fiber beam-joiner, and so no optical projection problems arise. The device is very small, very cheap and easily assembled from readily available opto-electronic components. The schematic layout of the detector is given in Fig. 1, with a detailed description in the figure caption. The detector relies on the phenomenon where the 3-space velocity $\mathbf{v}(\mathbf{r}, t)$ affects differently the light travel times in the optical fibers, depending on the projection of $\mathbf{v}(\mathbf{r}, t)$ along the fiber directions. The differences in the light travel times are measured by means of the interference effects in the beam joiner. The difference in travel times is given by

$$\Delta t = k^2 \frac{Lv_P^2}{c^3} \cos(2\theta), \tag{3}$$

where

$$k^2 = \frac{(n^2 - 1)(2 - n^2)}{n}$$

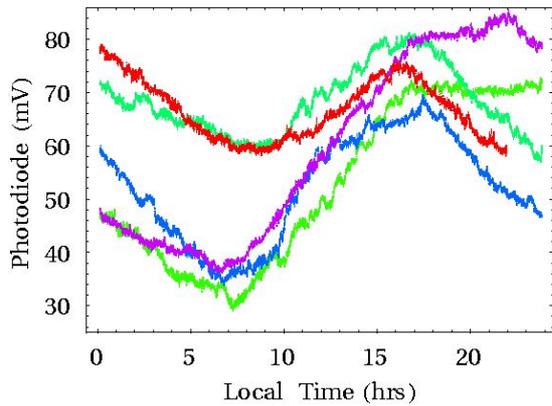


Fig. 5: D1 photodiode output voltage data (mV), recorded every 5 secs, from 5 successive days, starting September 22, 2007, plotted against local Adelaide time (UT = local time + 9.5 hrs). Day sequence is indicated by increasing hue. Dominant minima and maxima is earth rotation effect. Fluctuations from day to day are evident as are fluctuations during each day — these are caused by wave effects in the flowing space. Changes in RA cause changes in timing of min/max, while changes in magnitude are caused by changes in declination and/or speed. Blurring effect is caused by laser noise. Same data is plotted sequentially in Fig. 7a.

is the instrument calibration constant, obtained by taking account of the three key effects: (i) the different light paths, (ii) Lorentz contraction of the fibers, an effect depending on the angle of the fibers to the flow velocity, and (iii) the refractive index effect, including the Fresnel drag effect. Only if $n \neq 1$ is there a net effect, otherwise when $n = 1$ the various effects actually cancel. So in this regard the Michelson interferometer has a serious design flaw. This problem has been overcome by using optical fibers. Here $n = 1.462$ at 633 nm is the effective refractive index of the single-mode optical fibers (Fibercore SM600, temperature coefficient 5×10^{-2} fs/mm/C). Here $L \approx 200$ mm is the average effective length of the two arms, and $v_P(r, t)$ is the projection of $\mathbf{v}(r, t)$ onto the plane of the detector, and the angle θ is that of the projected velocity onto the arm.

The reality of the Lorentz contraction effect is experimentally confirmed by comparing the 2nd order in v/c Michelson gas-mode interferometer data, which requires account be taken of the contraction effect, with that from the 1st order in v/c RF coaxial cable travel time experiments, as in DeWitte [7], which does not require that the contraction effect be taken into account, to give comparable values for v .

For gas-mode Michelson interferometers $k^2 \approx n^2 - 1$, because then $n \approx 1^+$ is the refractive index of a gas. Operating in air, as for Michelson and Morley and for Miller, $n = 1.00029$, so that $k^2 = 0.00058$, which in particular means that the Michelson-Morley interferometer was nearly 2000 times less sensitive than assumed by Michelson, who used Newtonian physics to calibrate the interferometer — that analysis gives $k^2 = n^3 \approx 1$. Consequently the small fringe

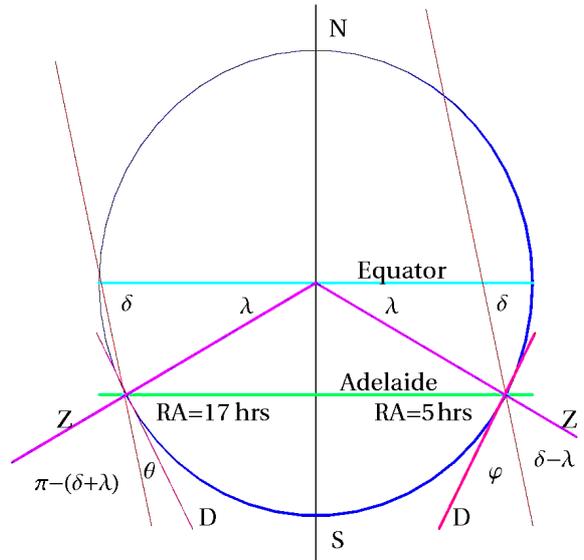


Fig. 6: Schematic of earth and spatial flow at approximate local sidereal times (RA) of 5 hrs and 17 hrs. The detector arms, D, of D1 and D2 are operated at small offset angles from the local meridian. The long straight lines indicate the spatial flow velocity vector, with declination δ . The large earth-rotation induced minima/maxima are caused by the inclination angle varying from a maximum ϕ to a minimum θ , respectively. Wave effects are changes in the velocity vector.

shifts observed by Michelson and Morley actually correspond to a light speed anisotropy of some 400 km/s, that is, the earth has that speed relative to the local dynamical 3-space. The dependence of k on n has been checked [11, 18] by comparing the air gas-mode data against data from the He gas-mode operated interferometers of Illingworth [3] and Joos [4].

The above analysis also has important implications for long-baseline terrestrial vacuum-mode Michelson interferometer gravitational wave detectors — they have a fundamental design flaw and will not be able to detect gravitational waves.

The interferometer operates by detecting changes in the travel time difference between the two arms, as given by (3). The cycle-averaged light intensity emerging from the beam joiner is given by

$$\begin{aligned}
 I(t) &\propto \left(\text{Re}(\mathbf{E}_1 + \mathbf{E}_2 e^{i\omega(\tau + \Delta t)}) \right)^2 = \\
 &= 2|\mathbf{E}|^2 \cos\left(\frac{\omega(\tau + \Delta t)}{2}\right)^2 \approx \\
 &\approx a + b \Delta t.
 \end{aligned}
 \tag{4}$$

Here \mathbf{E}_i are the electric field amplitudes and have the same value as the fiber splitter/joiner are 50%–50% types, and having the same direction because polarisation preserving fibers are used, ω is the light angular frequency and τ is a travel time difference caused by the light travel times not

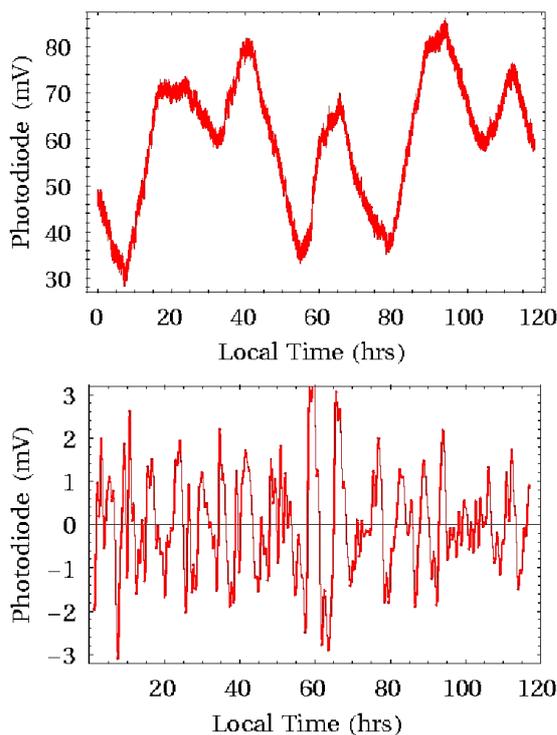


Fig. 7: (a) Plot of 5 days of data from Fig. 5 shown sequentially. The Fourier Transform of this data is shown in Fig. 12a. (b) Plot shows data after filtering out earth-rotation frequencies ($f < 0.025$ mHz) and laser noise frequencies ($f > 0.25$ mHz, $\text{Log}_{10}[0.25] = -0.6$). This shows wave/turbulence effects. Note that the magnitude of the wave component of the signal is some 10% of full signal in this bandwidth.

being identical, even when $\Delta t = 0$, mainly because the various splitter/joiner fibers will not be identical in length. The last expression follows because Δt is small, and so the detector operates, hopefully, in a linear regime, in general, unless τ has a value equal to modulo(T), where T is the light period. The main temperature effect in the detector, so long as a temperature uniformity is maintained, is that τ will be temperature dependent. The temperature coefficient for the optical fibers gives an effective fractional fringe shift error of $\Delta\tau/T = 3 \times 10^{-2}/\text{mm}/\text{C}$, for each mm of length difference. The photodiode detector output voltage $V(t)$ is proportional to $I(t)$, and so finally linearly related to Δt . The detector calibration constants a and b depend on k , τ and the laser intensity and are unknown at present.

4 Data analysis

The data is described in detail in the figure captions.

- Fig. 5 shows 5 typical days of data exhibiting the earth-rotation effect, and also fluctuations during each day and from day to day, revealing dynamical 3-space turbulence — essentially the long-sort-for gravitational waves. It is now known that these gravitational waves

were first detected in the Michelson-Morley 1887 experiment [16], but only because their interferometer was operated in gas-mode. Fig. 12a shows the frequency spectrum for this data;

- Fig. 7b shows the gravitational waves after removing frequencies near the earth-rotation frequency. As discussed later these gravitational waves are predominately sub-mHz;
- Fig. 8 reports one of a number of key experimental tests of the detector principles. These show the two detector responses when (a) operating from the same laser source, and (b) with only D2 operating in interferometer mode. These reveal the noise effects coming from the laser in comparison with the interferometer signal strength. This gives a guide to the S/N ratio of these detectors;
- Fig. 9 shows two further key tests: 1st the time delay effect in the earth-rotation induced minimum caused by the detectors not being aligned NS. The time delay difference has the value expected. The 2nd effect is that wave effects are simultaneous, in contrast to the 1st effect. This is the first coincidence detection of gravitational waves by spatially separated detectors. Soon the separation will be extended to much larger distances;
- Figs. 10 and 11 show the data and calibration curves for the timing of the daily earth-rotation induced minima and maxima over an 80 day period. Because D1 is orientated away from the NS these times permit the determination of the Declination (Dec) and Right Ascension (RA) from the two running averages. That the running averages change over these 80 days reflects three causes (i) the sidereal time effect, namely that the 3-space velocity vector is related to the positioning of the galaxy, and not the Sun, (ii) that a smaller component is related to the orbital motion of the earth about the Sun, and (iii) very low frequency wave effects. This analysis gives the changing Dec and RA shown in Fig. 12b, giving results which are within 13° of the 1925/26 Miller results, and for the RA from the DeWitte RF coaxial cable results. Figs. 10a and 11a also show the turbulence/wave effects, namely deviations from the running averages;
- Fig. 12a shows the frequency analysis of the data. The fourier amplitudes, which can be related to the strain $h = v^2/2c^2$, decrease as f^a where the strain spectral index has the value $a = -1.4 \pm 0.1$, after we allow for the laser noise.

5 Conclusions

Sub-mHz gravitational waves have been detected and partially characterised using the optical-fiber version of a Michelson interferometer. The waves are relatively large and were

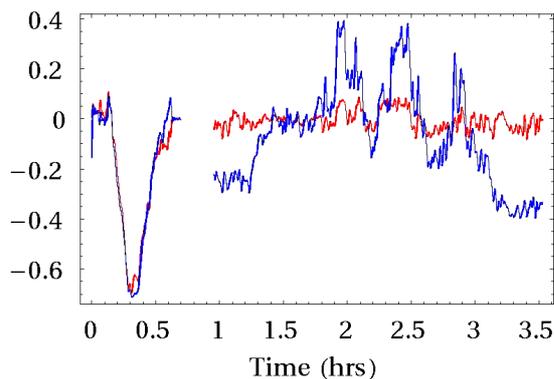


Fig. 8: Two tests of the detectors. (a) The left plot shows data from D1 and D2 when co-located, parallel and operating from the same laser. The data from one has been rescaled to match the data from the other, as they have different calibrations. Both detectors show a simultaneous gravitational wave pulse of duration ≈ 0.5 hrs. (b) The right plot shows data from D2 (blue) and from a direct feed of the common laser source to the photodiode detector of D1 (red), i.e. bypassing the D1 interferometer. This data has been rescaled so that high frequency components have the same magnitude, to compensate for different feed amplitudes. The laser-only signal (red) shows the amplitude and frequency noise level of the laser. The signal from D2 (blue) shows the same noise effects, but with additional larger variations — these are wave effects detected by D2 operating in interferometer mode. This data shows that the laser noise is dominant above approximately 1 mHz.

first detected, though not recognised as such, by Michelson and Morley in 1887. Since then another 6 experiments [2,6–9], including herein, have confirmed the existence of this phenomenon. Significantly three different experimental techniques have been employed, all giving consistent results. In contrast vacuum-mode Michelson interferometers, with mechanical mirror support arms, cannot detect this phenomenon due to a design flaw. A complete characterisation of the waves requires that the optical-fiber detector be calibrated for speed, which means determining the parameter b in (4). Then it will be possible to extract the wave component of $\mathbf{v}(\mathbf{r}, t)$ from the average, and so identify the cause of the turbulence/wave effects. A likely candidate is the in-flow of 3-space into the Milky Way central super-massive black hole — this in-flow is responsible for the high, non-Keplerian, rotation speeds of stars in the galaxy.

The detection of the earth-rotation, earth-orbit and gravitational waves, and over a long period of history, demonstrate that the spacetime formalism of Special Relativity has been very misleading, and that the original Lorentz formalism is the appropriate one; in this the speed of light is not an invariant for all observers, and the Lorentz-Fitzgerald length contraction and the Lamor time dilation are real physical effects on rods and clocks in motion through the dynamical 3-space, whereas in the Einstein formalism they are transferred and attributed to a perspective effect of spacetime, which we now recognise as having no ontological significance — merely a

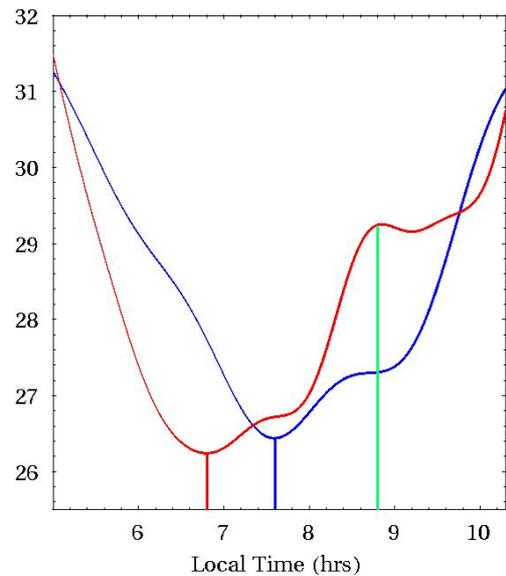


Fig. 9: Photodiode data (mV) on October 4, 2007, from detectors D1 (red plot) and D2 (blue plot) operating simultaneously with D2 located 1.1 km due north of D1. A low-pass FFT filter ($f \leq 0.25$ mHz, $\text{Log}_{10}[f(\text{mHz})] \leq -0.6$) was used to remove laser noise. D1 arm is aligned 5° anti-clockwise from local meridian, while D2 is aligned 11° anti-clockwise from local meridian. The alignment offset between D1 and D2 causes the dominant earth-rotation induced minima to occur at *different* times, with that of D2 at $t = 7.6$ hrs delayed by 0.8 hrs relative to D1 at $t = 6.8$ hrs, as expected from Figs.10b and 11b for $\text{Dec} = 77^\circ$. This is a fundamental test of the detection theory and of the phenomena. As well the data shows a *simultaneous* sub-mHz gravitational wave correlation at $t \approx 8.8$ hrs and of duration ≈ 1 hr. This is the first observed correlation for spatially separated gravitational wave detectors. Two other wave effects (at $t \approx 6.5$ hrs in D2 and $t \approx 7.3$ hrs in D1) seen in one detector are masked by the stronger earth-rotation induced minimum in the other detector.

mathematical construct, and in which the invariance of the speed of light is definitional — not observational.

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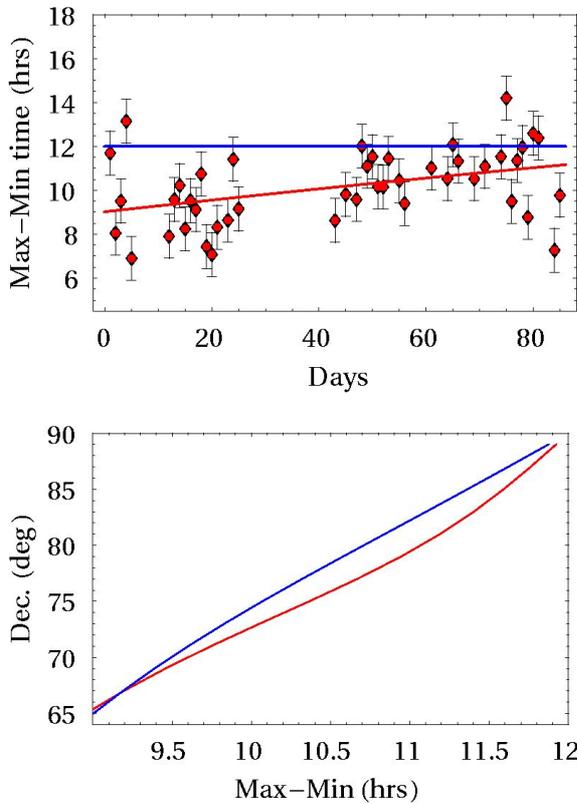


Fig. 10: (a) Time differences between maxima and minima each day from D1, from September 22 to December 16, 2007. Some days are absent due to data logger malfunction. The red curve shows a quadratic best-fit running average. If the detector arm was orientated along the meridian and there were no wave effects then one would see the horizontal line (blue) at 12 hrs. The data shows, however, that the running average has a time varying measure, from 9 hrs to 11 hrs over these days, caused by the orbital motion of the earth about the sun. Wave-effect fluctuations from day to day are also evident. This data in conjunction with the calibration curve in (b) permits a determination of the approximate Declination each day, which is used in the plot shown in Fig. 12b. (b) Declination calibration curve for D1 (red) and D2 (blue). From the orientation of the detector, with an offset angle of 5° for D1 anti-clockwise from the local meridian and 11° for D2 anti-clockwise from the local meridian, and the latitude of Adelaide, the offset angle causes the time duration between a minimum and a maximum to be different from 12 hrs, ignoring wave effects. In conjunction with the running average in Fig. 10a an approximate determination of the Declination on each day may be made without needing to also determine the RA and speed.

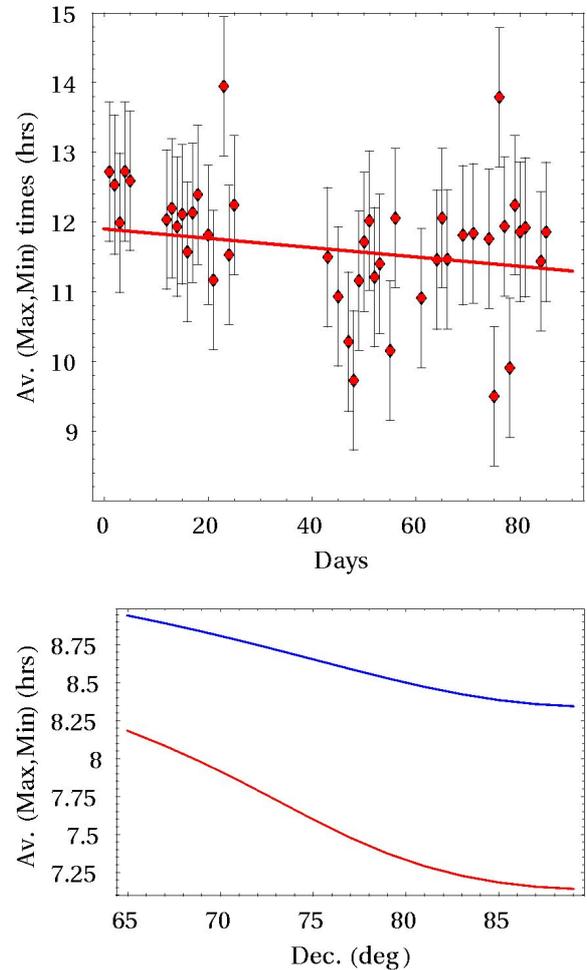


Fig. 11: (a) Time of average of minimum and maximum for each day. The linear best-fit line shows the trend line. Wave effects are again very evident. The decreasing trend line is cause by a combination of the sidereal effect, the earth orbit effect and very low frequency waves. This data may be used to determine the approximate RA for each day. However a correction must be applied as the arm offset angle affects the determination. (b) RA calibration curve for D1 (red) and D2 (blue). The detector offset angles cause the timing of the mid-point between the minima and maxima, ignoring wave effects, to be delayed in time, beyond the 6 hrs if detector were aligned NS. So, for example, if the Declination is found to be 70° , then this calibration curve gives 8 hrs for D1. This 8 hrs is then subtracted from the time in (a) to give the approximate true local time for the minimum to occur, which then permits computation of the RA for that day.

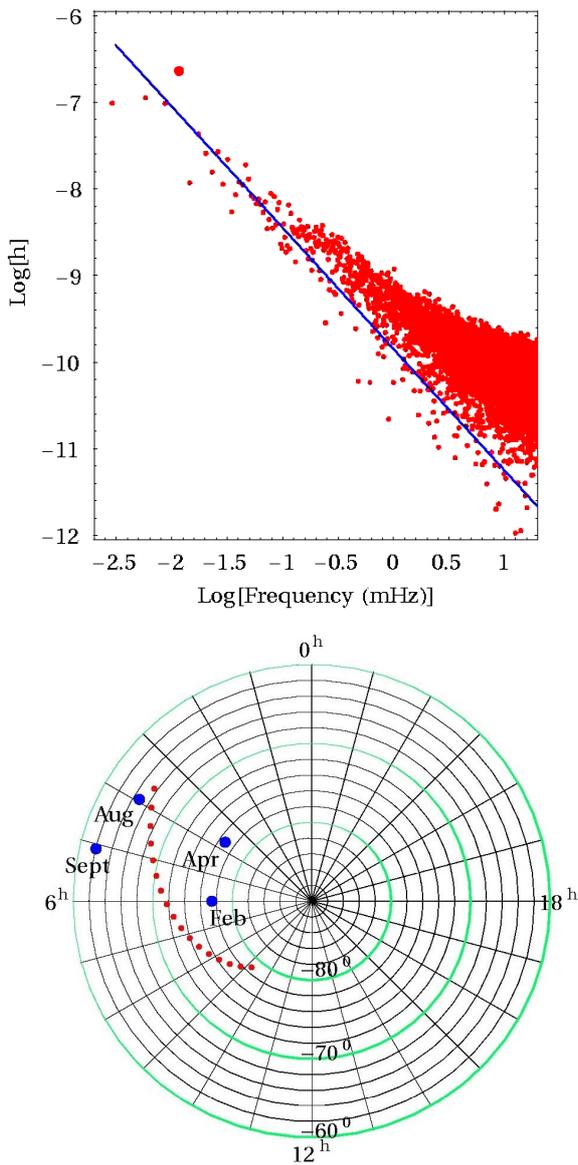


Fig. 12: (a) Log-Log plot of the frequency spectrum $|h(f)|$ of the data from the five days shown in Fig. 7a. $h(f)$ is the strain $v^2/2c^2$ at frequency f , normalised to $v = 400$ km/s at the 24 hr frequency. The largest component (large red point) is the 24 hr earth rotation frequency. The straight line (blue) is a trend line that suggests that the signal has two components — one indicated by the trend line having the form $|h(f)| \propto f^a$ with strain spectral index $a = -1.4 \pm 0.1$, while the second component, evident above 1 mHz, is noise from the laser source, as also indicated by the data in Fig. 8. (b) Southern celestial sphere with RA and Dec shown. The 4 blue points show the results from Miller [2] for four months in 1925/1926. The sequence of red points show the daily averaged RA and Dec as determined from the data herein for every 5 days. The 2007 data shows a direction that moves closer to the south celestial pole in late December than would be indicated by the Miller data. The new results differ by 10° to 13° from the corresponding Miller data points (the plot exaggerates angles). The wave effects cause the actual direction to fluctuate from day to day and during each day.

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Derivation of Maxwell's Equations Based on a Continuum Mechanical Model of Vacuum and a Singularity Model of Electric Charges

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The main purpose of this paper is to seek a mechanical interpretation of electromagnetic phenomena. We suppose that vacuum is filled with a kind of continuously distributed material which may be called $\Omega(1)$ substratum. Further, we speculate that the $\Omega(1)$ substratum might behave like a fluid with respect to translational motion of large bodies through it, but would still possess elasticity to produce small transverse vibrations. Thus, we propose a visco-elastic constitutive relation of the $\Omega(1)$ substratum. Furthermore, we speculate that electric charges are emitting or absorbing the $\Omega(1)$ substratum continuously and establish a fluidic source and sink model of electric charges. Thus, Maxwell's equations in vacuum are derived by methods of continuum mechanics based on this mechanical model of vacuum and the singularity model of electric charges.

1 Introduction

Maxwell's equations in vacuum can be written as [1]

$$\nabla \cdot \mathbf{E} = \frac{\rho_e}{\epsilon_0}, \quad (1)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (2)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (3)$$

$$\frac{1}{\mu_0} \nabla \times \mathbf{B} = \mathbf{j} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}, \quad (4)$$

where \mathbf{E} is the electric field vector, \mathbf{B} is the magnetic induction vector, ρ_e is the density field of electric charges, \mathbf{j} is the electric current density, ϵ_0 is the dielectric constant of vacuum, μ_0 is magnetic permeability of vacuum, t is time, $\nabla = \mathbf{i} \frac{\partial}{\partial x} + \mathbf{j} \frac{\partial}{\partial y} + \mathbf{k} \frac{\partial}{\partial z}$ is the Hamilton operator.

The main purpose of this paper is to derive the aforementioned Maxwell equations in vacuum based on a continuum mechanics model of vacuum and a singularity model of electric charges.

The motivation for this paper was looking for a mechanism of electromagnetic phenomena. The reasons why new mechanical models of electromagnetic fields are interesting may be summarized as follows.

First, there exists various electromagnetic phenomena which could not be interpreted by the present theories of electromagnetic fields, e.g., the spin of an electron [1, 2], the Aharonov-Bohm effect [3, 4], etc. New theories of electromagnetic phenomena may consider these problems from new sides.

Second, there exists some inconsistencies and inner difficulties in Classical Electrodynamics, e.g., the inadequacy of the Liénard-Wiechert potentials [5–7]. New theories of electromagnetic phenomena may overcome such difficulties.

Third, there exists some divergence problems in Quantum Electrodynamics [8]. By Dirac's words, "I must say that I am very dissatisfied with the situation, because this so-called good theory does involve neglecting infinities which appear in its equations, neglecting them in an arbitrary way. This is just not sensible mathematics". New theories of electromagnetic phenomena may open new ways to resolve such problems.

Fourth, since the quantum theory shows that vacuum is not empty and produces physical effects, e.g., the Casimir effect [9–12], it is valuable to reexamine the old concept of electromagnetic aether.

Fifth, from the viewpoint of reductionism, Maxwell's theory of electromagnetic fields can only be regarded as a phenomenological theory. Although Maxwell's theory is a field theory, the field concept is different from that of continuum mechanics [13–16] due to the absence of a medium. Thus, from the viewpoint of reductionism, the mechanism of electromagnetic phenomena is still remaining an unsolved problem of physics [17].

Sixth, one of the puzzles of physics is the problem of dark matter and dark energy (refer to, for instance, [18–26]). New theories of electromagnetic phenomena may provide new ideas to attack this problem.

Finally, one of the tasks of physics is the unification of the four fundamental interactions in the Universe. New theories of electromagnetic phenomena may shed some light on this puzzle.

To conclude, it seems that new considerations for electromagnetic phenomena is needed. It is worthy keeping an open mind with respect to all the theories of electromagnetic phenomena before the above problems been solved.

Now let us briefly review the long history of the mechanical interpretations of electromagnetic phenomena.

According to E. T. Whittaker [17], Descartes was the first person who brought the concept of aether into science by sug-

gested mechanical properties to it. Descartes believed that every physical phenomenon could be interpreted in the framework of a mechanical model of the Universe. William Watson and Benjamin Franklin (independently) constructed the one-fluid theory of electricity in 1746 [17]. H. Cavendish attempted to explain some of the principal phenomena of electricity by means of an elastic fluid in 1771 [17]. Not contented with the above mentioned one-fluid theory of electricity, du Fay, Robert Symmer and C. A. Coulomb developed a two-fluid theory of electricity from 1733 to 1789 [17].

Before the unification of both electromagnetic and light phenomena by Maxwell in 1860's, light phenomena were independent studied on the basis of Descartes' views for the mechanical origin of Nature. John Bernoulli introduced a fluidic aether theory of light in 1752 [17]. Euler believed in an idea that all electrical phenomena are caused by the same aether that moves light. Furthermore, Euler attempted to explain gravity in terms of his single fluidic aether [17].

In 1821, in order to explain polarisation of light, A. J. Frensel proposed an aether model which is able to transmit transverse waves. After the advent of Frensel's successful transverse wave theory of light, the imponderable fluid theories were abandoned. In the 19th century, Frensel's dynamical theory of a luminiferous aether had an important influence on the mechanical theories of Nature [17]. Inspired by Frensel's luminiferous aether theory, numerous dynamical theories of elastic solid aether were established by Stokes, Cauchy, Green, MacCullagh, Boussinesq, Riemann and William Thomson. (See, for instance, [17]).

Thomson's analogies between electrical phenomena and elasticity helped to James Clark Maxwell to establish a mechanical model of electrical phenomena [17]. Strongly impressed by Faraday's theory of lines of forces, Maxwell compared the Faraday lines of forces with the lines of flow of a fluid. In 1861, in order to obtain a mechanical interpretation of electromagnetic phenomena, Maxwell established a mechanical model of a magneto-electric medium. The Maxwell magneto-electric medium is a cellular aether, looks like a honeycomb. Each cell of the aether consists of a molecular vortex surrounded by a layer of idle-wheel particles. In a remarkable paper published in 1864, Maxwell established a group of equations, which were named after his name later, to describe the electromagnetic phenomena.

In 1878, G. F. FitzGerald compared the magnetic force with the velocity in a quasi-elastic solid of the type first suggested by MacCullagh [17]. FitzGerald's mechanical model of such an electromagnetic aether was studied by A. Sommerfeld, by R. Reiff and by Sir J. Larmor later [17].

Because of some dissatisfactions with the mechanical models of an electromagnetic aether and the success of the theory of electromagnetic fields, the mechanical world-view was removed with the electromagnetic world-view gradually. Therefore, the concepts of a luminiferous aether and an elastic solid aether were removed with the concepts of an electro-

magnetic aether or an electromagnetic field. This paradigm shift in scientific research was attributed to many scientists, including Faraday, Maxwell, Sir J. Larmor, H. A. Lorentz, J. J. Thomson, H. R. Hertz, Ludwig Lorenz, Emil Wiechert, Paul Drude, Wilhelm Wien, etc. (See, for instance, [17].)

In a remarkable paper published in 1905, Einstein abandoned the concept of aether [27]. However, Einstein's assertion did not cease the exploration of aether (refer to, for instance, [17,28-37,68,69]). Einstein changed his attitude later and introduced his new concept of aether [38,39]. In 1979, A. A. Golebiewska-Lasta observed the similarity between the electromagnetic field and the linear dislocation field [28]. V. P. Dmitriyev have studied the similarity between the electromagnetism and linear elasticity since 1992 [32,35,37,40]. In 1998, H. Marmanis established a new theory of turbulence based on the analogy between electromagnetism and turbulent hydrodynamics [34]. In 1998, D. J. Larson derived Maxwell's equations from a simple two-component solid-mechanical aether [33]. In 2001, D. Zareski gave an elastic interpretation of electrodynamics [36]. I regret to admit that it is impossible for me to mention all the works related to this field of history.

A. Martin and R. Keys [41-43] proposed a fluidic cosmic gas model of vacuum in order to explain the physical phenomena such as electromagnetism, gravitation, Quantum Mechanics and the structure of elementary particles.

Inspired by the above mentioned works, we show that Maxwell's equations of electromagnetic field can be derived based on a continuum mechanics model of vacuum and a singularity model of electric charges.

2 Clues obtained from dimensional analysis

According to Descartes' scientific research program, which is based on his views about the mechanical origin of Nature, electromagnetic phenomena must be (and can be) interpreted on the basis of the mechanical motions of the particles of aether.

Therefore, all the physical quantities appearing in the theory of electromagnetic field should be mechanical quantities.

Thus, in order to construct a successful mechanical model of electromagnetic fields, we should undertake a careful dimensional analysis (refer to, for instance, [44]) for physical quantities in the theory of electromagnetism (for instance, electric field vector \mathbf{E} , magnetic induction vector \mathbf{B} , the density field of electric charges ρ_e , the dielectric constant of vacuum ϵ_0 , the magnetic permeability of vacuum μ_0 , etc.).

It is known that Maxwell's equations (1-4) in vacuum can also be expressed as [1]

$$\nabla^2 \phi + \frac{\partial}{\partial t}(\nabla \cdot \mathbf{A}) = -\frac{\rho_e}{\epsilon_0}, \quad (5)$$

$$\nabla^2 \mathbf{A} - \nabla(\nabla \cdot \mathbf{A}) - \mu_0 \epsilon_0 \frac{\partial}{\partial t} \left(\nabla \phi - \frac{\partial \mathbf{A}}{\partial t} \right) = -\mathbf{j}, \quad (6)$$

where ϕ is the scalar electromagnetic potential, \mathbf{A} is the vector electromagnetic potential, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the Laplace operator.

In 1846, W. Thomson compared electric phenomena with elasticity. He pointed out that the elastic displacement \mathbf{u} of an incompressible elastic solid is a possible analogy to the vector electromagnetic potential \mathbf{A} [17].

Noticing the similarity between the Eq. (6) and the equation (39) of momentum conservation of elastic solids, it is natural to judge that vacuum is filled with a kind of elastic substratum. Further, we may say that the dimension of the electromagnetic vector potential \mathbf{A} of such an elastic substratum is the same that of the displacement vector \mathbf{u} of an elastic solid. Thus, the dimension of the vector electromagnetic potential \mathbf{A} of the elastic substratum is $[L^0 M^0 T^0]$, where L , M and T stands for the dimensions of length, mass, and time, respectively. Therefore, we can determine the dimensions of the rest physical quantities of the theory of electromagnetism, for instance, the electric field vector \mathbf{E} , the magnetic induction vector \mathbf{B} , the electric charge q_e , the dielectric constant of vacuum ϵ_0 , the magnetic permeability of vacuum μ_0 , etc. For instance, the dimension of an electric charge q_e should be $[L^0 M^1 T^{-1}]$.

Inspired by this clue, we are going to produce, in the next Sections, an investigation in this direction.

3 A visco-elastic continuum model of vacuum

The purpose of this Section is to establish a visco-elastic continuum mechanical model of vacuum.

In 1845–1862, Stokes suggested that aether might behave like a glue-water jelly [45–47]. He believed that such an aether would act like a fluid on the transit motion of large bodies through it, but would still possessing elasticity to produce a small transverse vibration.

Following Stokes, we propose a visco-elastic continuum model of vacuum.

Assumption 1. *Suppose that vacuum is filled with a kind of continuously distributed material.*

In order to distinguish this material with other substratums, we may call this material as $\Omega(1)$ substratum, for convenience. Further, we may call the particles that constitute the $\Omega(1)$ substratum as $\Omega(1)$ particles (for convenience).

In order to construct a continuum mechanical theory of the $\Omega(1)$ substratum, we should take some assumptions based on the experimental data about the macroscopic behavior of vacuum.

Assumption 2. *We suppose that all the mechanical quantities of the $\Omega(1)$ substratum under consideration, such as the density, displacements, strains, stresses, etc., are piecewise continuous functions of space and time. Furthermore, we suppose that the material points of the $\Omega(1)$ substratum remain be in one-to-one correspondence with the material points before a deformation appears.*

Assumption 3. *We suppose that the material of the $\Omega(1)$ substratum under consideration is homogeneous, that is $\frac{\partial \rho}{\partial x} = \frac{\partial \rho}{\partial y} = \frac{\partial \rho}{\partial z} = \frac{\partial \rho}{\partial t} = 0$, where ρ is the density of the $\Omega(1)$ substratum.*

Assumption 4. *Suppose that the deformation processes of the $\Omega(1)$ substratum are isothermal. So we neglect the thermal effects.*

Assumption 5. *Suppose that the deformation processes are not influenced by the gradient of the stress tensor.*

Assumption 6. *We suppose that the material of the $\Omega(1)$ substratum under consideration is isotropic.*

Assumption 7. *We suppose that the deformation of the $\Omega(1)$ substratum under consideration is small.*

Assumption 8. *We suppose that there are no initial stress and strain in the body under consideration.*

When the $\Omega(1)$ substratum is subjected to a set of external forces, the relative positions of the $\Omega(1)$ particles form the body displacement.

In order to describe the deformation of the $\Omega(1)$ substratum, let us introduce a Cartesian coordinate system $\{o, x, y, z\}$ or $\{o, x_1, x_2, x_3\}$ which is static relative to the $\Omega(1)$ substratum. Now we may introduce a definition to the displacement vector \mathbf{u} of every point in the $\Omega(1)$ substratum:

$$\mathbf{u} = \mathbf{r} - \mathbf{r}_0, \tag{7}$$

where \mathbf{r}_0 is the position of the point before the deformation, while \mathbf{r} is the position after the deformation.

The displacement vector may be written as $\mathbf{u} = u_1 \mathbf{i} + u_2 \mathbf{j} + u_3 \mathbf{k}$ or $\mathbf{u} = u \mathbf{i} + v \mathbf{j} + w \mathbf{k}$, where $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are three unit vectors directed along the coordinate axes.

The gradient of the displacement vector \mathbf{u} is the relative displacement tensor $u_{i,j} = \frac{\partial u_i}{\partial x_j}$.

We decompose the tensor $u_{i,j}$ into two parts, the symmetric ϵ_{ij} and the skew-symmetric Ω_{ij} (refer to, for instance, [14,48,49])

$$u_{i,j} = \frac{1}{2}(u_{i,j} + u_{j,i}) + \frac{1}{2}(u_{i,j} - u_{j,i}) = \epsilon_{ij} + \Omega_{ij}, \tag{8}$$

$$\epsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad \Omega_{ij} = \frac{1}{2}(u_{i,j} - u_{j,i}). \tag{9}$$

The symmetric tensor ϵ_{ij} manifests a pure deformation of the body at a point, and is known the strain tensor (refer to, for instance, [14,48,49]). The matrix form and the component notation of the strain tensor ϵ_{ij} are

$$\epsilon_{ij} = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{1}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) & \frac{1}{2} \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \\ \frac{1}{2} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) & \frac{\partial v}{\partial y} & \frac{1}{2} \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \\ \frac{1}{2} \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) & \frac{1}{2} \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right) & \frac{\partial w}{\partial z} \end{pmatrix}, \tag{10}$$

$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{pmatrix}. \quad (11)$$

The strain-displacements equations come from Eq. (10)

$$\left. \begin{aligned} \varepsilon_{11} &= \frac{\partial u}{\partial x}, & \varepsilon_{12} = \varepsilon_{21} &= \frac{1}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \\ \varepsilon_{22} &= \frac{\partial v}{\partial y}, & \varepsilon_{23} = \varepsilon_{32} &= \frac{1}{2} \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \\ \varepsilon_{33} &= \frac{\partial w}{\partial z}, & \varepsilon_{31} = \varepsilon_{13} &= \frac{1}{2} \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) \end{aligned} \right\}. \quad (12)$$

For convenience, we introduce the definitions of the mean strain deviator ε_m and the strain deviator e_{ij} as

$$\varepsilon_m = \frac{1}{3} (\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}), \quad (13)$$

$$e_{ij} = \varepsilon_{ij} - \varepsilon_m = \begin{pmatrix} \varepsilon_{11} - \varepsilon_m & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} - \varepsilon_m & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} - \varepsilon_m \end{pmatrix}. \quad (14)$$

When the $\Omega(1)$ substratum deforms, the internal forces arise due to the deformation. The component notation of the stress tensor σ_{ij} is

$$\sigma_{ij} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix}. \quad (15)$$

For convenience, we introduce the definitions of mean stress σ_m and stress deviator s_{ij} as

$$\sigma_m = \frac{1}{3} (\sigma_{xx} + \sigma_{yy} + \sigma_{zz}), \quad (16)$$

$$s_{ij} = \sigma_{ij} - \sigma_m = \begin{pmatrix} \sigma_{11} - \sigma_m & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} - \sigma_m & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} - \sigma_m \end{pmatrix}. \quad (17)$$

Now let us turn to study the constitutive relation.

An elastic Hooke solid responds instantaneously with respect to an external stress. A Newtonian viscous fluid responds to a shear stress by a steady flow process.

In 19th century, people began to point out that fact that some materials showed a time dependence in their elastic response with respect to external stresses. When a material like pitch, gum rubber, polymeric materials, hardened cement and even glass, is loaded, an instantaneous elastic deformation follows with a slow continuous flow or creep.

Now this time-dependent response is known as viscoelasticity (refer to, for instance, [50–52]). Materials bearing both instantaneous elastic elasticity and creep characteristics are known as viscoelastic materials [51, 52]. Viscoelastic materials were studied long time ago by Maxwell [51–53], Kelvin, Voigt, Boltzmann [51, 52, 54], etc.

Inspired by these contributors, we propose a visco-elastic constitutive relation of the $\Omega(1)$ substratum.

It is natural to say that the constitutive relation of the $\Omega(1)$ substratum may be a combination of the constitutive relations of the Hooke-solid and the Newtonian-fluid.

For the Hooke-solid, we have the generalized Hooke law as follows (refer to, for instance, [14, 48, 49, 55]),

$$\sigma_{ij} = 2G\varepsilon_{ij} + \lambda\theta\delta_{ij}, \quad \varepsilon_{ij} = \frac{\sigma_{ij}}{2G} - \frac{3\nu}{Y}\sigma_m\delta_{ij}, \quad (18)$$

where δ_{ij} is the Kronecker symbol, σ_m is the mean stress, where Y is the Yang modulus, ν is the Poisson ratio, G is the shear modulus, λ is Lamé constant, θ is the volume change coefficient. The definition of θ is $\theta = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$.

The generalized Hooke law Eq. (18) can also be written as [55]

$$s_{ij} = 2G e_{ij}, \quad (19)$$

where s_{ij} is the stress deviator, e_{ij} is the strain deviator.

For the Newtonian-fluid, we have the following constitutive relation

$$\frac{de_{ij}}{dt} = \frac{1}{2\eta} s_{ij}, \quad (20)$$

where s_{ij} is the stress deviator, $\frac{de_{ij}}{dt}$ is the strain rate deviator, η is the dynamic viscosity.

The $\Omega(1)$ substratum behaves like the Hooke-solid during very short duration. We therefore differentiate both sides of Eq. (19), then obtain

$$\frac{de_{ij}}{dt} = \frac{1}{2G} \frac{ds_{ij}}{dt}. \quad (21)$$

A combination of Eq. (21) and Eq. (20) gives

$$\frac{de_{ij}}{dt} = \frac{1}{2\eta} s_{ij} + \frac{1}{2G} \frac{ds_{ij}}{dt}. \quad (22)$$

We call the materials behaving like Eq. (22) “Maxwell-liquid” since Maxwell established such a constitutive relation in 1868 (refer to, for instance, [50–53]).

Eq. (22) is valid only in the case of infinitesimal deformation because the presence of the derivative with respect to time. Oldroyd recognized that we need a special definition for the operation of derivation, in order to satisfy the principle of material frame indifference or objectivity [51, 56]. Unfortunately, there is no unique definition of such a differential operation fulfil the principle of objectivity presently [51].

As an enlightening example, let us recall the description [50] for a simple shear experiment. We suppose

$$\frac{d\sigma_t}{dt} = \frac{\partial\sigma_t}{\partial t}, \quad \frac{de_t}{dt} = \frac{\partial e_t}{\partial t}, \quad (23)$$

where σ_t is the shear stress, e_t is the shear strain.

Therefore, Eq. (22) becomes

$$\frac{\partial e_t}{\partial t} = \frac{1}{2\eta} \sigma_t + \frac{1}{2G} \frac{\partial\sigma_t}{\partial t}. \quad (24)$$

Integration of Eq. (24) gives

$$\sigma_t = e^{-\frac{G}{\eta}t} \left(\sigma_0 + 2G \int_0^t \frac{de_t}{dt} e^{\frac{G}{\eta}t} dt \right). \quad (25)$$

If the shear deformation is kept constant, i.e. $\frac{\partial e_t}{\partial t} = 0$, we have

$$\sigma_t = \sigma_0 e^{-\frac{G}{\eta}t}. \quad (26)$$

Eq. (26) shows that the shear stresses remain in the Maxwell-liquid and are damped in the course of time.

We see that $\frac{\eta}{G}$ must have the dimension of time. Now let us introduce the following definition of Maxwellian relaxation time τ

$$\tau = \frac{\eta}{G}. \quad (27)$$

Therefore, using Eq. (27), Eq. (22) becomes

$$\frac{s_{ij}}{\tau} + \frac{ds_{ij}}{dt} = 2G \frac{de_{ij}}{dt}. \quad (28)$$

Now let us introduce the following hypothesis

Assumption 9. Suppose the constitutive relation of the $\Omega(1)$ substratum satisfies Eq. (22).

Now we can derive the the equation of momentum conservation based on the above hypotheses 9.

Let T be a characteristic time scale of an observer of the $\Omega(1)$ substratum. When the observer's time scale T is of the same order that the period of the wave motion of light, the Maxwellian relaxation time τ is a comparigly large number. Thus, the first term of Eq. (28) may be neglected. Therefore, the observer concludes that the strain and the stress of the $\Omega(1)$ substratum satisfy the generalized Hooke law.

The generalized Hooke law (18) can also be written as [14, 55]

$$\left. \begin{aligned} \sigma_{11} &= \lambda \theta + 2G \varepsilon_{11} \\ \sigma_{22} &= \lambda \theta + 2G \varepsilon_{22} \\ \sigma_{33} &= \lambda \theta + 2G \varepsilon_{33} \\ \sigma_{12} &= \sigma_{21} = 2G \varepsilon_{12} = 2G \varepsilon_{21} \\ \sigma_{23} &= \sigma_{32} = 2G \varepsilon_{23} = 2G \varepsilon_{32} \\ \sigma_{31} &= \sigma_{13} = 2G \varepsilon_{31} = 2G \varepsilon_{13} \end{aligned} \right\}, \quad (29)$$

where $\lambda = \frac{Y\nu}{(1+\nu)(1-2\nu)}$ is Lamé constant, θ is the volume change coefficient. By its definition, $\theta = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$.

The following relationship are useful

$$G = \frac{Y}{2(1+\nu)}, \quad K = \frac{Y}{3(1-2\nu)}, \quad (30)$$

where K is the volume modulus.

It is known that the equations of the momentum conservation are (refer to, for instance, [14, 48, 49, 55, 57, 58]),

$$\frac{\partial \sigma_{11}}{\partial x} + \frac{\partial \sigma_{12}}{\partial y} + \frac{\partial \sigma_{13}}{\partial z} + f_x = \rho \frac{\partial^2 u}{\partial t^2}, \quad (31)$$

$$\frac{\partial \sigma_{21}}{\partial x} + \frac{\partial \sigma_{22}}{\partial y} + \frac{\partial \sigma_{23}}{\partial z} + f_y = \rho \frac{\partial^2 v}{\partial t^2}, \quad (32)$$

$$\frac{\partial \sigma_{31}}{\partial x} + \frac{\partial \sigma_{32}}{\partial y} + \frac{\partial \sigma_{33}}{\partial z} + f_z = \rho \frac{\partial^2 w}{\partial t^2}, \quad (33)$$

where f_x, f_y and f_z are three components of the volume force density \mathbf{f} exerted on the $\Omega(1)$ substratum.

The tensor form of the equations (31-33) of the momentum conservation can be written as

$$\sigma_{ij,j} + f_i = \rho \frac{\partial^2 u_i}{\partial t^2}. \quad (34)$$

Noticing Eq. (29), we write Eqs. (31-33) as

$$2G \left(\frac{\partial \varepsilon_{11}}{\partial x} + \frac{\partial \varepsilon_{12}}{\partial y} + \frac{\partial \varepsilon_{13}}{\partial z} \right) + \lambda \frac{\partial \theta}{\partial x} + f_x = \rho \frac{\partial^2 u}{\partial t^2}, \quad (35)$$

$$2G \left(\frac{\partial \varepsilon_{21}}{\partial x} + \frac{\partial \varepsilon_{22}}{\partial y} + \frac{\partial \varepsilon_{23}}{\partial z} \right) + \lambda \frac{\partial \theta}{\partial y} + f_y = \rho \frac{\partial^2 v}{\partial t^2}, \quad (36)$$

$$2G \left(\frac{\partial \varepsilon_{31}}{\partial x} + \frac{\partial \varepsilon_{32}}{\partial y} + \frac{\partial \varepsilon_{33}}{\partial z} \right) + \lambda \frac{\partial \theta}{\partial z} + f_z = \rho \frac{\partial^2 w}{\partial t^2}. \quad (37)$$

Using Eq. (12), Eqs. (35-37) can also be expressed by means of the displacement \mathbf{u}

$$\left. \begin{aligned} G \nabla^2 u + (G + \lambda) \frac{\partial}{\partial x} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) + f_x &= \rho \frac{\partial^2 u}{\partial t^2} \\ G \nabla^2 v + (G + \lambda) \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) + f_y &= \rho \frac{\partial^2 v}{\partial t^2} \\ G \nabla^2 w + (G + \lambda) \frac{\partial}{\partial z} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) + f_z &= \rho \frac{\partial^2 w}{\partial t^2} \end{aligned} \right\}. \quad (38)$$

The vectorial form of the aforementioned equations (38) can be written as (refer to, for instance, [14, 48, 49, 55, 57, 58]),

$$G \nabla^2 \mathbf{u} + (G + \lambda) \nabla(\nabla \cdot \mathbf{u}) + \mathbf{f} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2}. \quad (39)$$

When no body force in the $\Omega(1)$ substratum, Eqs. (39) reduce to

$$G \nabla^2 \mathbf{u} + (G + \lambda) \nabla(\nabla \cdot \mathbf{u}) = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2}. \quad (40)$$

From Long's theorem [48, 59], there exist a scalar function ψ and a vector function \mathbf{R} such that \mathbf{u} is represented by

$$\mathbf{u} = \nabla \psi + \nabla \times \mathbf{R} \quad (41)$$

and ψ and \mathbf{R} satisfy the following wave equations

$$\nabla^2 \psi - \frac{1}{c_t} \frac{\partial^2 \psi}{\partial t^2} = 0, \quad (42)$$

$$\nabla^2 \mathbf{R} - \frac{1}{c_t} \frac{\partial^2 \mathbf{R}}{\partial t^2} = 0, \quad (43)$$

where c_l is the velocity of longitudinal waves, c_t is the velocity of transverse waves. The definitions of these two elastic wave velocities are (refer to, for instance, [48, 49, 57, 58]),

$$c_l = \sqrt{\frac{\lambda + 2G}{\rho}}, \quad c_t = \sqrt{\frac{G}{\rho}}. \quad (44)$$

ψ and \mathbf{R} is usually known as the scalar displacement potential and the vector displacement potential, respectively.

4 Definition of point source and sink

If there exists a velocity field which is continuous and finite at all points of the space, with the exception of individual isolated points, then, usually, these isolated points are called velocity singularities. Point sources and sinks are examples of such velocity singularities.

Assumption 10. *Suppose there exists a singularity at a point $P_0 = (x_0, y_0, z_0)$ in a continuum. If the velocity field of the singularity at a point $P = (x, y, z)$ is*

$$\mathbf{v}(x, y, z, t) = \frac{Q}{4\pi r^2} \hat{\mathbf{r}}, \quad (45)$$

where $r = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}$, $\hat{\mathbf{r}}$ is the unit vector directed outward along the line from the singularity to this point $P = (x, y, z)$, we call such a singularity a point source in the case of $Q > 0$ or a point sink in the case of $Q < 0$. Here Q is called the strength of the source or sink.

Suppose that a static point source with the strength Q locates at the origin $(0, 0, 0)$. In order to calculate the volume leaving the source per unit of time, we may enclose the source with an arbitrary spherical surface S of the radius a . Calculation shows that

$$\oint_S \mathbf{u} \cdot \mathbf{n} dS = \oint_S \frac{Q}{4\pi a^2} \hat{\mathbf{r}} \cdot \mathbf{n} dS = Q, \quad (46)$$

where \mathbf{n} is the unit vector directed outward along the line from the origin of the coordinates to the field point (x, y, z) . Equation (46) shows that the strength Q of a source or sink evaluates the volume of the fluid leaving or entering a control surface per unit of time.

For the case of continuously distributed point sources or sinks, it is useful to introduce a definition for the volume density ρ_s of point sources or sinks. The definition is

$$\rho_s = \lim_{\Delta V \rightarrow 0} \frac{\Delta Q}{\Delta V}, \quad (47)$$

where ΔV is a small volume, ΔQ is the sum of the strengthes of all the point sources or sinks in the volume ΔV .

5 A point source and sink model of electric charges

The purpose of this Section is to propose a point source and sink model of electric charges.

Let T be the characteristic time of an observer of an electric charge in the $\Omega(1)$ substratum. We may suppose that the observer's time scale T is very large to the Maxwellian relaxation time τ . So the Maxwellian relaxation time τ is a relatively small, and the stress deviator s_{ij} changes very slow. Thus, the second term in the left side of Eq. (28) may be neglected. For such an observer, the constitutive relation of the $\Omega(1)$ substratum may be written as

$$s_{ij} = 2\eta \frac{de_{ij}}{dt}. \quad (48)$$

The observer therefore concludes that the $\Omega(1)$ substratum behaves like a Newtonian-fluid on his time scale.

In order to compare fluid motions with electric fields, Maxwell introduced an analogy between sources or sinks and electric charges [17].

Einstein, Infeld and Hoffmann introduced an idea by which all particles may be looked as singularities in fields [60, 61].

Recently [62], we talked that the universe may be filled with a kind fluid which may be called "tao". Thus, Newton's law of gravitation is derived by methods of hydrodynamics based on a point sink flow model of particles.

R. L. Oldershaw talked that hadrons may be considered as Kerr-Newman black holes if one uses appropriate scaling of units and a revised gravitational coupling factor [63].

Inspired by the aforementioned works, we introduce the following

Assumption 11. *Suppose that all the electric charges in the Universe are the sources or sinks in the $\Omega(1)$ substratum. We define such a source as a negative electric charge. We define such a sink as a positive electric charge. The electric charge quantity q_e of an electric charge is defined as*

$$q_e = -k_Q \rho Q, \quad (49)$$

where ρ is the density of the $\Omega(1)$ substratum, Q is called the strength of the source or sink, k_Q is a positive dimensionless constant.

A calculation shows that the mass m of an electric charge is changing with time as

$$\frac{dm}{dt} = -\rho Q = \frac{q_e}{k_Q}, \quad (50)$$

where q_e is the electric charge quantity of the electric charge.

We may introduce a hypothesis that the masses of electric charges are changing so slowly relative to the time scaler of human beings that they can be treated as constants approximately.

For the case of continuously distributed electric charges, it is useful to introduce the following definition of the volume density ρ_e of electric charges

$$\rho_e = \lim_{\Delta V \rightarrow 0} \frac{\Delta q_e}{\Delta V}, \quad (51)$$

where ΔV is a small volume, Δq_e is the sum of the strengths of all the electric charges in the volume ΔV .

From Eq. (47), Eq. (49) and Eq. (51), we have

$$\rho_e = -k_Q \rho \rho_s. \quad (52)$$

6 Derivation of Maxwell's equations in vacuum

The purpose of this Section is to deduce Maxwell's equations based on the aforementioned visco-elastic continuum model of vacuum and the singularity model of electric charges.

Now, let us deduce the continuity equation of the $\Omega(1)$ substratum from the mass conservation. Consider an arbitrary volume V bounded by a closed surface S fixed in space. Suppose that there are electric charges continuously distributed in the volume V . The total mass in the volume V is

$$M = \iiint_V \rho dV, \quad (53)$$

where ρ is the density of the $\Omega(1)$ substratum.

The rate of the increase of the total mass in the volume V is

$$\frac{\partial M}{\partial t} = \frac{\partial}{\partial t} \iiint_V \rho dV. \quad (54)$$

Using the Ostrogradsky–Gauss theorem (refer to, for instance, [16, 64–67]), the rate of the mass outflow through the surface S is

$$\iint_S \rho(\mathbf{v} \cdot \mathbf{n}) dS = \iiint_V \nabla \cdot (\rho \mathbf{v}) dV, \quad (55)$$

where \mathbf{v} is the velocity field of the $\Omega(1)$ substratum.

The definition of the velocity field \mathbf{v} is

$$v_i = \frac{\partial u_i}{\partial t}, \quad \text{or} \quad \mathbf{v} = \frac{\partial \mathbf{u}}{\partial t}. \quad (56)$$

Using Eq. (52), the rate of the mass created inside the volume V is

$$\iiint_V \rho \rho_s dV = \iiint_V -\frac{\rho_e}{k_Q} dV. \quad (57)$$

Now according to the principle of mass conservation, and making use of Eq. (54), Eq. (55) and Eq. (57), we have

$$\frac{\partial}{\partial t} \iiint_V \rho dV = \iiint_V -\frac{\rho_e}{k_Q} dV - \iiint_V \nabla \cdot (\rho \mathbf{v}) dV. \quad (58)$$

Since the volume V is arbitrary, from Eq. (58) we have

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = -\frac{\rho_e}{k_Q}. \quad (59)$$

According to Assumption 3, the $\Omega(1)$ substratum is homogeneous, that is $\frac{\partial \rho}{\partial x} = \frac{\partial \rho}{\partial y} = \frac{\partial \rho}{\partial z} = \frac{\partial \rho}{\partial t} = 0$. Thus, Eq. (59) becomes

$$\nabla \cdot \mathbf{v} = -\frac{\rho_e}{k_Q \rho}. \quad (60)$$

According to Assumption 11 and Eq. (50), the masses bearing positive electric charges are changing since the strength of a sink evaluates the volume of the $\Omega(1)$ substratum entering the sink per unit of time. Thus, the momentum of a volume element ΔV of the $\Omega(1)$ substratum containing continuously distributed electric charges, and moving with an average speed \mathbf{v}_e , changes. The increased momentum $\Delta \mathbf{P}$ of the volume element ΔV during a time interval Δt is the decreased momentum of the continuously distributed electric charges contained in the volume element ΔV during a time interval Δt , that is,

$$\Delta \mathbf{P} = \rho(\rho_s \Delta V \Delta t) \mathbf{v}_e = -\frac{\rho_e}{k_Q} \Delta V \Delta t \mathbf{v}_e. \quad (61)$$

Therefore, the equation of momentum conservation Eq. (39) of the $\Omega(1)$ substratum should be changed as

$$G \nabla^2 \mathbf{u} + (G + \lambda) \nabla (\nabla \cdot \mathbf{u}) + \mathbf{f} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2} - \frac{\rho_e \mathbf{v}_e}{k_Q}. \quad (62)$$

In order to simplify the Eq. (62), we may introduce an additional assumption as

Assumption 12. We suppose that the $\Omega(1)$ substratum is almost incompressible, or we suppose that θ is a sufficient small quantity and varies very slow in the space so that it can be treated as $\theta = 0$.

From Assumption 12, we have

$$\nabla \cdot \mathbf{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \theta = 0. \quad (63)$$

Therefore, the vectorial form of the equation of momentum conservation Eq. (62) reduces to the following form

$$G \nabla^2 \mathbf{u} + \mathbf{f} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2} - \frac{\rho_e \mathbf{v}_e}{k_Q}. \quad (64)$$

According to the Stokes-Helmholtz resolution theorem (refer to, for instance, [48, 57]), which states that every sufficiently smooth vector field may be decomposed into irrotational and solenoidal parts, there exist a scalar function ψ and a vector function \mathbf{R} such that \mathbf{u} is represented by

$$\mathbf{u} = \nabla \psi + \nabla \times \mathbf{R}. \quad (65)$$

Now let us introduce the definitions

$$\nabla \phi = k_E \frac{\partial}{\partial t} (\nabla \psi), \quad \mathbf{A} = k_E \nabla \times \mathbf{R}, \quad (66)$$

$$\mathbf{E} = -k_E \frac{\partial \mathbf{u}}{\partial t}, \quad \mathbf{B} = k_E \nabla \times \mathbf{u}, \quad (67)$$

where ϕ is the scalar electromagnetic potential, \mathbf{A} is the vector electromagnetic potential, \mathbf{E} is the electric field intensity, \mathbf{B} is the magnetic induction, k_E is a positive dimensionless constant.

From Eq. (65), Eq. (66) and Eq. (67), we have

$$\mathbf{E} = -\nabla \phi - \frac{\partial \mathbf{A}}{\partial t}, \quad \mathbf{B} = \nabla \times \mathbf{A} \quad (68)$$

and

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (69)$$

$$\nabla \cdot \mathbf{B} = 0. \quad (70)$$

Based on Eq. (66) and noticing that

$$\nabla^2 \mathbf{u} = \nabla(\nabla \cdot \mathbf{u}) - \nabla \times (\nabla \times \mathbf{u}), \quad (71)$$

$$\nabla^2 \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla \times (\nabla \times \mathbf{A}), \quad (72)$$

and $\nabla \cdot \mathbf{u} = 0$, $\nabla \cdot \mathbf{A} = 0$, we have

$$k_E \nabla^2 \mathbf{u} = \nabla^2 \mathbf{A}. \quad (73)$$

Therefore, using Eq. (73), Eq. (64) becomes

$$\frac{G}{k_E} \nabla^2 \mathbf{A} + \mathbf{f} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2} - \frac{\rho_e \mathbf{v}_e}{k_Q}. \quad (74)$$

Using Eq. (72), Eq. (74) becomes

$$-\frac{G}{k_E} \nabla \times (\nabla \times \mathbf{A}) + \mathbf{f} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2} - \frac{\rho_e \mathbf{v}_e}{k_Q}. \quad (75)$$

Now using Eq. (68), Eq. (75) becomes

$$-\frac{G}{k_E} \nabla \times \mathbf{B} + \mathbf{f} = -\frac{\rho}{k_E} \frac{\partial \mathbf{E}}{\partial t} - \frac{\rho_e \mathbf{v}_e}{k_Q}. \quad (76)$$

It is natural to say that there are no other body forces or surface forces exerted on the $\Omega(1)$ substratum. Thus, we have $\mathbf{f} = 0$. Therefore, Eq. (76) becomes

$$\frac{k_Q G}{k_E} \nabla \times \mathbf{B} = \frac{k_Q \rho}{k_E} \frac{\partial \mathbf{E}}{\partial t} + \rho_e \mathbf{v}_e. \quad (77)$$

Now let us introduce the following definitions

$$\mathbf{j} = \rho_e \mathbf{v}_e, \quad \epsilon_0 = \frac{k_Q \rho}{k_E}, \quad \frac{1}{\mu_0} = \frac{k_Q G}{k_E}. \quad (78)$$

Therefore, Eq. (77) becomes

$$\frac{1}{\mu_0} \nabla \times \mathbf{B} = \mathbf{j} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}. \quad (79)$$

Noticing Eq. (67) and Eq. (78), Eq. (60) becomes

$$\nabla \cdot \mathbf{E} = \frac{\rho_e}{\epsilon_0}. \quad (80)$$

Now we see that Eq. (69), Eq. (70), Eq. (79) and Eq. (80) coincide with Maxwell's equations (1–4).

7 Mechanical interpretation of electromagnetic waves

It is known that, from Maxwell's equations (1-4), we can obtain the following equations (refer to, for instance, [1])

$$\nabla^2 \mathbf{E} - \frac{1}{\mu_0 \epsilon_0} \frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{1}{\epsilon_0} \nabla \rho_e + \mu_0 \frac{\partial \mathbf{j}}{\partial t}, \quad (81)$$

$$\nabla^2 \mathbf{H} - \frac{1}{\mu_0 \epsilon_0} \frac{\partial^2 \mathbf{H}}{\partial t^2} = -\frac{1}{\mu_0} \nabla \times \mathbf{j}. \quad (82)$$

Eq. (81) and Eq. (82) are the electromagnetic wave equations with sources in the $\Omega(1)$ substratum. In the source free region where $\rho_e = 0$ and $\mathbf{j} = 0$, the equations reduce to the following equations

$$\nabla^2 \mathbf{E} - \frac{1}{\mu_0 \epsilon_0} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0, \quad (83)$$

$$\nabla^2 \mathbf{H} - \frac{1}{\mu_0 \epsilon_0} \frac{\partial^2 \mathbf{H}}{\partial t^2} = 0. \quad (84)$$

Eq. (83) and Eq. (84) are the electromagnetic wave equations without the sources in the $\Omega(1)$ substratum.

From Eq. (83), Eq. (84) and Eq. (78), we see that the velocity c_0 of electromagnetic waves in vacuum is

$$c_0 = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \sqrt{\frac{G}{\rho}}. \quad (85)$$

From Eq. (44) and Eq. (85), we see that the velocity c_0 of electromagnetic waves in the vacuum is the same as the velocity c_t of the transverse elastic waves in the $\Omega(1)$ substratum.

Now we may regard electromagnetic waves in the vacuum as the transverse waves in the $\Omega(1)$ substratum. This idea was first introduced by Frensel in 1821 [17].

8 Conclusion

We suppose that vacuum is not empty and may be filled with a kind continuously distributed material called $\Omega(1)$ substratum. Following Stokes, we propose a visco-elastic constitutive relation of the $\Omega(1)$ substratum. Following Maxwell, we propose a fluidic source and sink model of electric charges. Thus, Maxwell's equations in vacuum are derived by methods of continuum mechanics based on this continuum mechanical model of vacuum and the singularity model of electric charges.

9 Discussion

Many interesting theoretical, experimental and applied problems can be met in continuum mechanics, Classical Electrodynamics, Quantum Electrodynamics and also other related fields of science involving this theory of electromagnetic phenomena. It is an interesting task to generalize this theory of electromagnetic phenomena in the static $\Omega(1)$ substratum to the case of electromagnetic phenomena of moving bodies.

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On the Geometry of Background Currents in General Relativity

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In this preliminary work, we shall reveal the intrinsic geometry of background currents, possibly of electromagnetic origin, in the space-time of General Relativity. Drawing a close analogy between the object of our present study and electromagnetism, we shall show that there exists an inherent, fully non-linear, conservative third-rank radiation current which is responsible for the irregularity in the curvature of the background space(-time), whose potential (generator) is of purely geometric origin.

1 Introduction

Herein we attempt to study, in a way that has never been fully explored before, the nature of background radiation fields from a purely geometric point of view. One may always expect that empty (matter-free) regions in a space(-time) of non-constant sectional curvature are necessarily filled with some kind of pure radiation field that may be associated with a class of null electromagnetic fields. As is common in practice, their description must therefore be attributed to the Weyl tensor alone, as the only remaining geometric object in emptiness (with the cosmological constant neglected). An in-depth detailed elaboration on the nature of the physical vacuum and emptiness, considering space(-time) anisotropy, can be seen in [6, 7].

Our present task is to explore the geometric nature of the radiation fields permeating the background space(-time). As we will see, the thrilling new aspect of this work is that our main stuff of this study (a third-rank background current and its associates) is geometrically non-linear and, as such, it cannot be gleaned in the study of gravitational radiation in weak-field limits alone. Thus, it must be regarded as an essential part of Einstein's theory of gravity.

Due to the intended concise nature of this preliminary work, we shall leave aside the more descriptive aspects of the subject.

2 A third-rank geometric background current in a general metric-compatible manifold

At first, let us consider a general, metric-compatible manifold \mathbb{M}_D of arbitrary dimension D and coordinates x^α . We may extract a third-rank background current from the curvature as follows:

$$J_{\mu\nu\rho} = J_{\mu[\nu\rho]} = \nabla_\lambda R^\lambda_{\mu\nu\rho},$$

where square brackets on a group of indices indicate anti-symmetrization (similarly, round brackets will be used to indicate symmetrization). Of course, ∇ is the covariant derivative, and, with $\partial_\mu = \frac{\partial}{\partial x^\mu}$,

$$R^\lambda_{\mu\nu\rho} = \partial_\nu \Gamma^\lambda_{\mu\rho} - \partial_\rho \Gamma^\lambda_{\mu\nu} + \Gamma^\sigma_{\mu\rho} \Gamma^\lambda_{\sigma\nu} - \Gamma^\sigma_{\mu\nu} \Gamma^\lambda_{\sigma\rho}$$

are the usual components of the curvature tensor R of the metric-compatible connection Γ whose components are given by

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma} (\partial_\sigma g_{\mu\nu} - \partial_\mu g_{\nu\sigma} + \partial_\nu g_{\sigma\mu}) + \Gamma^\lambda_{[\mu\nu]} - g^{\lambda\alpha} (g_{\mu\beta} \Gamma^\beta_{[\alpha\nu]} + g_{\nu\beta} \Gamma^\beta_{[\alpha\mu]}).$$

Here $g_{\mu\nu}$ are the components of the fundamental symmetric metric tensor g and $\Gamma^\lambda_{[\mu\nu]}$ are the components of the torsion tensor. The (generalized) Ricci tensor and scalar are then given, as usual, by the contractions $R_{\mu\nu} = R^\lambda_{\mu\lambda\nu}$ and $R = R^\mu_{\mu}$, respectively.

We may introduce the traceless Weyl curvature tensor W through the decomposition

$$R^\mu_{\alpha\beta\gamma} = K^\mu_{\alpha\beta\gamma} + \frac{1}{D-2} (\delta^\mu_\beta R_{\alpha\gamma} + g_{\alpha\gamma} R^\mu_\beta - \delta^\mu_\gamma R_{\alpha\beta} - g_{\alpha\beta} R^\mu_\gamma),$$

$$K^\mu_{\alpha\beta\gamma} = W^\mu_{\alpha\beta\gamma} + \frac{1}{(D-1)(D-2)} (\delta^\mu_\gamma g_{\alpha\beta} - \delta^\mu_\beta g_{\alpha\gamma}) R,$$

$$K_{\alpha\beta} = K_{(\alpha\beta)} = K^\mu_{\alpha\mu\beta} = -\frac{1}{D-2} g_{\alpha\beta} R,$$

for which $D > 2$. In particular, we shall take into account the following useful relation:

$$\begin{aligned} R^\mu_{\alpha\beta\gamma} R^{\alpha\beta\gamma\nu} &= W^\mu_{\alpha\beta\gamma} W^{\alpha\beta\gamma\nu} + \\ &+ \frac{1}{D-2} (K^\mu_{\alpha\beta} R^{\alpha\beta\gamma\nu} + K^{\alpha\mu\beta\nu} R_{\alpha\beta} - K^{\alpha\beta\mu\nu} R_{[\alpha\beta]}) + \\ &+ \frac{1}{D-2} (2RR^{\mu\nu} + g^{\mu\nu} R_{\alpha\beta} R^{\alpha\beta} - R^\mu_{\alpha} R^{\nu\alpha} - R^\alpha_{\mu} R^{\alpha\nu}) + \\ &+ \frac{2}{D-2} (R^{(\mu\alpha)} R^{[\nu\alpha]} + R^{(\mu\alpha)} R^\nu_{\alpha} - R^\mu_{\alpha} R^{(\nu\alpha)}) - \\ &- \frac{2}{(D-2)^2} RR^{\mu\nu}. \end{aligned}$$

Now, for an arbitrary tensor field T , we have, as usual,

$$\begin{aligned}
 (\nabla_\beta \nabla_\alpha - \nabla_\alpha \nabla_\beta) T_{\rho\sigma\dots}^{\mu\nu\dots} &= \\
 &= R^\lambda_{\rho\alpha\beta} T_{\lambda\sigma\dots}^{\mu\nu\dots} + R^\lambda_{\sigma\alpha\beta} T_{\rho\lambda\dots}^{\mu\nu\dots} + \dots - R^\mu_{\lambda\alpha\beta} T_{\rho\sigma\dots}^{\lambda\nu\dots} - \\
 &- R^\nu_{\lambda\alpha\beta} T_{\rho\sigma\dots}^{\mu\lambda\dots} - 2\Gamma^\lambda_{[\alpha\beta]} \nabla_\lambda T_{\rho\sigma\dots}^{\mu\nu\dots}.
 \end{aligned}$$

For a complete set of general identities involving the curvature tensor R and their relevant physical applications in Unified Field Theory, see [1-5].

At this point, we can define a second-rank background current density (field strength) f through

$$f^{\nu\rho} = f^{[\nu\rho]} = \nabla_\mu J^{\mu\nu\rho} = \nabla_\mu \nabla_\lambda R^{\lambda\mu\nu\rho} = -\nabla_{[\lambda} \nabla_{\mu]} R^{\lambda\mu\nu\rho}.$$

An easy calculation gives, in general,

$$\begin{aligned}
 f^{\mu\nu} &= -\frac{1}{2} (R^\mu_{\alpha\beta\gamma} R^{\alpha\beta\gamma\nu} - R^\nu_{\alpha\beta\gamma} R^{\alpha\beta\gamma\mu}) - \\
 &- R_{[\alpha\beta]} R^{\alpha\beta\mu\nu} - \Gamma^\alpha_{[\rho\sigma]} \nabla_\alpha R^{\rho\sigma\mu\nu}.
 \end{aligned}$$

In analogy to the electromagnetic source, we may define a first-rank current density through

$$j^\mu = \nabla_\nu f^{\mu\nu}.$$

Then, a somewhat lengthy but straightforward calculation shows that

$$\nabla_\mu j^\mu = R_{[\mu\nu]} f^{\mu\nu} + \Gamma^\sigma_{[\mu\nu]} \nabla_\sigma f^{\mu\nu}.$$

We may also define the field strength f through a sixth-rank curvature tensor F whose components are given by

$$\begin{aligned}
 F_{\mu\nu\rho\sigma\lambda\delta} &= F_{[\mu\nu][\rho\sigma]\lambda\delta} = \\
 &= \frac{1}{2} (R_{\mu\delta\lambda\alpha} \bar{R}^\alpha_{\nu\rho\sigma} - R_{\nu\delta\lambda\alpha} \bar{R}^\alpha_{\mu\rho\sigma}) + \\
 &+ \frac{1}{2} (R_{\mu\rho\lambda\alpha} \bar{R}^\alpha_{\nu\delta\sigma} - R_{\nu\rho\lambda\alpha} \bar{R}^\alpha_{\mu\delta\sigma}) + \\
 &+ \frac{1}{2} (R_{\nu\sigma\lambda\alpha} \bar{R}^\alpha_{\mu\delta\rho} - R_{\mu\sigma\lambda\alpha} \bar{R}^\alpha_{\nu\delta\rho}) + \\
 &+ \frac{1}{2} (R_{\mu\lambda\delta\alpha} \bar{R}^\alpha_{\nu\rho\sigma} - R_{\nu\lambda\delta\alpha} \bar{R}^\alpha_{\mu\rho\sigma}) - R_{\mu\nu\delta\alpha} \bar{R}^\alpha_{\lambda\rho\sigma}.
 \end{aligned}$$

where $\bar{R}_{\mu\nu\rho\sigma} = R_{\rho\sigma\mu\nu}$.

If we define a second-rank anti-symmetric tensor B by

$$\begin{aligned}
 B_{\mu\nu} &= F_{\mu\nu\rho\sigma}{}^{\rho\sigma} = \\
 &= \frac{1}{2} (R_{\mu\alpha\beta\gamma} R^{\alpha\beta\gamma\nu} - R_{\nu\alpha\beta\gamma} R^{\alpha\beta\gamma\mu}) - R_{\mu\nu\alpha\beta} R^{[\alpha\beta]},
 \end{aligned}$$

we then obtain

$$f_{\mu\nu} = B_{\mu\nu} + \Gamma^\alpha_{[\rho\sigma]} \nabla_\alpha R^{\rho\sigma}{}_{\mu\nu},$$

such that in the case of vanishing torsion, the quantities f and B are completely equivalent.

3 A third-rank radiation current relevant to General Relativity

Having defined the basic geometric objects of our theory, let us adhere to the standard Riemannian geometry of General Relativity in which the torsion vanishes, that is $\Gamma^\lambda_{[\mu\nu]} = 0$, and so the connection is the symmetric Levi-Civita connection. However, let us also take into account discontinuities in the first derivatives of the components of the metric tensor in order to take into account discontinuity surfaces corresponding to any existing background energy field. As we will see, we shall obtain a physically meaningful background current which is strictly conservative.

Now, in connection with the results of the preceding section, if we employ the simplified relation (which is true in the absence of torsion)

$$\begin{aligned}
 R^\mu_{\alpha\beta\gamma} R^{\alpha\beta\gamma\nu} &= W^\mu_{\alpha\beta\gamma} W^{\alpha\beta\gamma\nu} + \\
 &+ \frac{1}{D-2} (K^\mu{}^\nu{}_\alpha{}_\beta R^{\alpha\beta} + K^{\alpha\mu\beta\nu} R_{\alpha\beta}) + \\
 &+ \frac{1}{D-2} (2RR^{\mu\nu} + g^{\mu\nu} R_{\alpha\beta} R^{\alpha\beta} - 4R^\mu{}_\alpha R^{\nu\alpha}) - \\
 &- \frac{2}{(D-2)^2} RR^{\mu\nu},
 \end{aligned}$$

as well as the relations

$$\begin{aligned}
 K^\mu_{\alpha\beta\gamma} K^{\alpha\beta\gamma\nu} &= W^\mu_{\alpha\beta\gamma} W^{\alpha\beta\gamma\nu} + \frac{1}{(D-1)(D-2)^2} g^{\mu\nu} R^2, \\
 K^{\mu\alpha\nu\beta} R_{\alpha\beta} &= W^{\mu\alpha\nu\beta} R_{\alpha\beta} + \\
 &+ \frac{1}{(D-1)(D-2)} (R^{\mu\nu} - g^{\mu\nu} R) R,
 \end{aligned}$$

we obtain the desired relation

$$\begin{aligned}
 f^{\mu\nu} &= -\frac{1}{2} (W^\mu_{\alpha\beta\gamma} W^{\alpha\beta\gamma\nu} - W^\nu_{\alpha\beta\gamma} W^{\alpha\beta\gamma\mu}) - \\
 &- \frac{1}{D-2} (W^\mu{}^\nu{}_\alpha{}_\beta - W^\nu{}^\mu{}_\alpha{}_\beta) R^{\alpha\beta}.
 \end{aligned}$$

If the metric tensor is perfectly continuous, it is obvious that

$$f^{\mu\nu} = 0.$$

In deriving this relation we have used the symmetry $W_{\mu\nu\rho\sigma} = W_{\rho\sigma\mu\nu}$. This shows that, in the presence of metric discontinuity, the field strength f depends on the Weyl curvature alone which is intrinsic to the background space(-time) only when matter and non-null electromagnetic fields are absent. We see that, in spaces of constant sectional curvature, we will strictly have $J^{\lambda\mu\nu} = 0$ and $f^{\mu\nu} = 0$ since the Weyl curvature vanishes therein. In other words, in the sense of General Relativity, the presence of background currents is responsible for the irregularity (anisotropy) in the curvature of the background space(-time). Matter, if not elementary

particles, in this sense, can indeed be regarded as a form of perturbation with respect to the background space(-time).

Furthermore, it is now apparent that

$$J_{\mu\nu\rho} + J_{\nu\rho\mu} + J_{\rho\mu\nu} = 0.$$

This relation is, of course, reminiscent of the usual Bianchi identity satisfied by the components of the Maxwellian electromagnetic field tensor.

Also, we obtain the conservation law

$$\nabla_\mu j^\mu = 0.$$

which becomes trivial when the metric is perfectly continuous.

Hence, the formal correspondence between our present theory and the ordinary theory of electromagnetism may be completed, in the simplest way, through the relation

$$J_{\mu\nu\rho} = \nabla_\lambda R^\lambda_{\mu\nu\rho} = \nabla_\mu \Phi_{\nu\rho},$$

where the anti-symmetric field tensor Φ given by

$$\Phi_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$$

plays a role similar to that of the electromagnetic field strength. However, it should be emphasized that it exists in General Relativity's fully non-linear regime. In addition, it vanishes identically in the absence of curvature anisotropy. Interestingly, if one is willing to regard electromagnetism as a kind of non-linear gravity, one may alternatively regard Φ as being the complete equivalent of Maxwell's electromagnetic field strength. However, we shall not further pursue this interest here.

Furthermore, we obtain the relation

$$f_{\mu\nu} = \square \Phi_{\mu\nu},$$

where $\square = \nabla_\mu \nabla^\mu$. That is, the wave equation

$$\begin{aligned} \square \Phi_{\mu\nu} = & -\frac{1}{2} (W_{\mu\alpha\beta\gamma} W^{\alpha\beta\gamma}_\nu - W_{\nu\alpha\beta\gamma} W^{\alpha\beta\gamma}_\mu) - \\ & - \frac{1}{D-2} (W_{\mu\alpha\nu\beta} - W_{\nu\alpha\mu\beta}) R^{\alpha\beta}. \end{aligned}$$

In the absence of metric discontinuity, we obtain

$$\square \Phi_{\mu\nu} = 0.$$

Let us now introduce a vector potential ϕ such that the curl of which gives us the field strength f . Instead of writing $f_{\mu\nu} = \partial_\nu \phi_\mu - \partial_\mu \phi_\nu$ and instead of expressing the field strength f in terms of the Weyl tensor, let us write its components in the following equivalent form:

$$\begin{aligned} f_{\mu\nu} = & -\frac{1}{2} (R_{\mu\alpha\beta\gamma} R^{\alpha\beta\gamma}_\nu - R_{\nu\alpha\beta\gamma} R^{\alpha\beta\gamma}_\mu) = \\ = & \nabla_\nu \phi_\mu - \nabla_\mu \phi_\nu. \end{aligned}$$

In order for the potential ϕ to be purely geometric, we shall have

$$\nabla_\nu \phi_\mu = -\frac{1}{2} R_{\mu\alpha\beta\gamma} R^{\alpha\beta\gamma}_\nu,$$

from which an "equation of motion" follows somewhat effortlessly:

$$\frac{D\phi_\mu}{Ds} = -\frac{1}{2} R_{\mu\alpha\beta\gamma} R^{\alpha\beta\gamma}_\nu \frac{dx^\nu}{ds},$$

where $\frac{D\phi_\mu}{Ds} = \frac{dx^\nu}{ds} \nabla_\nu \phi_\mu$.

Note that, in the absence of metric discontinuity, the vector potential ϕ is a mere gradient of a smooth scalar field Θ : $\phi_\mu = \nabla_\mu \Theta$.

Now, it remains to integrate the equation

$$\partial_\nu \phi_\mu = -\frac{1}{2} R_{\mu\alpha\beta\gamma} R^{\alpha\beta\gamma}_\nu + \Gamma^\lambda_{\mu\nu} \phi_\lambda$$

by taking a closed contour P associated with the surface area dS spanned by infinitesimal displacements in two different directions, that is,

$$dS^{\mu\nu} = d_1 x^\mu d_2 x^\nu - d_1 x^\nu d_2 x^\mu.$$

An immediate effect of this closed-loop integration is that, by using the generally covariant version of Stokes' theorem and by explicitly assuming that the integration factor Z given by

$$\begin{aligned} Z^\rho_\mu = & \frac{1}{2} \oint_S (\nabla_\lambda \Gamma^\rho_{\mu\nu} - \nabla_\nu \Gamma^\rho_{\mu\lambda}) dS^{\lambda\nu} = \\ = & \frac{1}{2} \oint_S (R^\rho_{\mu\lambda\nu} + \Gamma^\rho_{\sigma\lambda} \Gamma^\sigma_{\mu\nu} - \Gamma^\rho_{\mu\lambda} \Gamma^\sigma_{\sigma\nu}) dS^{\lambda\nu} = \\ = & \frac{1}{2} \oint_S (R^\rho_{\mu\lambda\nu} + 2\Gamma^\rho_{\sigma\lambda} \Gamma^\sigma_{\mu\nu}) dS^{\lambda\nu} \end{aligned}$$

does not depend on ϕ , the integral $\oint_P \Gamma^\lambda_{\mu\nu} \phi_\lambda dx^\nu$ shall indeed vanish identically.

Hence, we are left with the expression

$$\Delta \phi_\mu = -\frac{1}{2} \oint_P R_{\mu\alpha\beta\gamma} R^{\alpha\beta\gamma}_\nu dx^\nu.$$

By introducing a new integration factor X satisfying $X^{\alpha\beta\gamma} + X^{\beta\gamma\alpha} + X^{\gamma\alpha\beta} = 0$ as follows:

$$\begin{aligned} X^{\alpha\beta\gamma} = & X^{[\alpha\beta]\gamma} = \oint_P R^{\alpha\beta\gamma}_\nu dx^\nu = \\ = & \frac{1}{2} \oint_S (\nabla_\mu R^{\alpha\beta\gamma}_\nu - \nabla_\nu R^{\alpha\beta\gamma}_\mu) dS^{\mu\nu}, \end{aligned}$$

we obtain, through direct partial integration,

$$\Delta \phi_\mu = -\frac{1}{2} \left(R_{\mu\alpha\beta\gamma} X^{\alpha\beta\gamma} - \int X^{\alpha\beta\gamma} dR_{\mu\alpha\beta\gamma} \right).$$

Simplifying, by keeping in mind that $X = X(R, dR)$, we finally obtain

$$\Delta\phi_\mu = \frac{1}{2} \int R_{\mu\alpha\beta\gamma} dX^{\alpha\beta\gamma}.$$

The simplest desired result of this is none other than

$$\Delta\phi_\mu = \frac{1}{2} R_{\mu\alpha\beta\gamma} X^{\alpha\beta\gamma},$$

which, expressed in terms of the Weyl tensor, the Ricci tensor, and the Ricci scalar, is

$$\Delta\phi_\mu = \frac{1}{2} W_{\mu\alpha\beta\gamma} X^{\alpha\beta\gamma} + \frac{1}{D-2} \left(X^\alpha{}_\mu{}^\beta R_{\alpha\beta} - X^\alpha{}_\beta{}^\beta R_{\alpha\mu} \right) + \frac{1}{(D-1)(D-2)} X^\mu{}_\alpha{}^\alpha R.$$

Hence, through Einstein's field equation (i.e. through the energy-momentum tensor T)

$$R_{\mu\nu} = \pm \frac{8\pi G}{c^4} \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right),$$

where G is Newton's gravitational constant and c is the speed of light, we may see how the presence of (distributed) matter affects the potential ϕ .

4 Final remarks

At this point, having outlined our study in brief, it remains to be seen whether our fully geometric background current may be associated with any type of conserved material current which is already known in the literature. It is also tempting to ponder, from a purely physical point of view, on the possibility that the intrinsic curvature of space(-time) owes its existence to null (light-like) electromagnetic fields or simply pure radiation fields.

In this case, let the null electromagnetic (pure radiation) field of the background space(-time) be denoted by φ , for which

$$\varphi_{\mu\nu}\varphi^{\mu\nu} = 0.$$

Then we may express the components of the Weyl tensor as

$$W_{\mu\nu\rho\sigma} = \varphi_{\mu\nu}\varphi_{\rho\sigma} - \varphi_{\mu\rho}\varphi_{\nu\sigma} + \varphi_{\mu\sigma}\varphi_{\nu\rho},$$

such that the relation $W^\rho{}_{\mu\rho\nu} = 0$ is satisfied.

If this indeed is the case, then we shall have a chance to better understand how matter actually originates from such a pure radiation field in General Relativity. This will hopefully also open a new way towards the full geometrization of matter in physics.

Finally, as a pure theory of gravitation, the results in the present work may be compared to those given in [8] and [9], wherein, based on the theory of chronometric invariants [7], a

new geometric formulation of gravity (which is fully equivalent to the standard form of General Relativity) is presented in a way very similar to that of the electromagnetic field, based solely on a second-rank anti-symmetric field tensor.

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The Asymptotic Approach to the Twin Paradox

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The argument of twins' asymmetry, essentially put forward in the common solution of the Twin Paradox, is revealed to be inoperative in some asymptotic situations in which the noninertial effects are insignificant. Consequently the respective solution proves itself as unreliable thing and the Twin Paradox is re-established as an open problem which require further investigations.

1 Introduction

Undoubtely that, in connection with Special Relativity, one of the most disputed subjects was (and still remaining) the so-called the *Twin Paradox*. Essentially this paradox consists in a contradiction between *time-dilatation* (relativistic transformations of time intervals) and the simple belief in symmetry regarding the ageing degrees of two relatively moving twins. The idea of time-dilatation is largely agreed in scientific literature (see [1–5] and references therein) as well as in various (more or less academic) media. However the experimental convincingsness of the respective idea still remains a subject of interest even in the inverstigations of the last decades (see for examples [6–9]).

It is notable the fact that, during the last decades, the disputes regarding the Twin Paradox were diminished and disimulated owing to the *common solution* (CS), which seems to be accredited with a great and unimpeded popularity. In the essence, CS argues that the twins are in completely asymmetric ageing situations due to the difference in the noninertial effects which they feel. Such noninertial effects are connected with the nonuniform motion of only one of the two twins. Starting from the mentioned argumentation, without any other major and credible proof, CS states that the Twin Paradox is nothing but an apparent and fictitious problem.

But even in the situations considered by CS a kind of symmetry between the twins can be restored if the noninertial effects are adequately managed. Such a management is possible if we take into account an asymptotic situation when the motions of the traveling twin is prevalently uniform or, in addition, the nonuniform motions are symmetrically present for both of the twins. Here we will see that the existence of the mentioned asymptotic situations have major consequences/implications for the reliability of CS. Our search is done in the Special Relativity approach (without appeals to General Relativity). This is because we consider such an approach to be sufficiently accurate/adequate for the situations under discussion.

In the end we shall conclude that the existence of the alluded asymptotic situations invalidate the CS and restores the Twin Paradox as a real (non-apparent, non-fictitious) and open problem which requires further investigations.

2 Asymptotic situations in which the noninertial effects are insignificant

In order to follow our project let us reconsider, in a quantitative manner, the twins arrangement used in CS. We consider two twins *A* and *B* whose proper reference frames are K_A and K_B respectively. The situations of the two twins *A* and *B* are reported in comparison with an inertial reference frame *K*.

2.1 Discussions about an asymptotic asymmetric situation

Within the framework of a first approach, we consider the twin *A* remaining at rest in the coordinate origin *O* of the frame *K* while the twin *B* moves forth and back along the positive part of the *x*-axis of *K*. The motion of *B* passes through the points *O*, *M*, *N* and *P* whose *x*-positions are: $x_O = 0$, $x_M = D$, $x_N = D + L$, $x_P = 2D + L$. The motion starts and finishes at *O*, while *P* is a turning point — i.e. the velocity of *B* is zero at *O* and *P*. Along the segments *OM* and *NP* the motion is nonuniform (accelerated or decelerated) with a time *t* dependent velocity $v(t)$. On the other hand, along the segment *MN*, the motion is uniform with the velocity of $v_0 = \text{const}$. In the mentioned situation K_A coincides with *K*, while K_B moves (nonuniformly or uniformly) with respect to *K*. The time variables describing the degrees of ageing of the two twins will be indexed respective to *A* and *B*. Also the mentioned time variables will be denoted respective to τ and t as they refer to the proper (intrinsic) time of the considered twin or, alternatively, to the time measured (estimated) in the reference frame of the other twin. The infinitesimal or finite intervals of τ and t will be denoted by $d\tau$ and dt respectively by $\Delta\tau$ and Δt .

With the menioned specifications, according to the relativity theory, for the time interval from the start to the finish of the motion of the twin *B*, one can write the relations

$$\Delta\tau_A = (\Delta t_B)_n + (\Delta t_B)_u, \quad (1)$$

$$\Delta\tau_B = \int_{(\Delta t_B)_n} \sqrt{1 - \frac{v^2(t_B)}{c^2}} \cdot dt_B + (\Delta t_B)_u \sqrt{1 - \frac{v_0^2}{c^2}}. \quad (2)$$

In these equations the indices n and u refer to the nonuniform respectively uniform motions, while c denotes the light velocity. In (2) it was used the fact (accepted in the relativity theory [10]) that instantaneously, at any moment of time, an arbitrarily moving reference frame can be considered as inertial. Because $v(t_B) \leq v_0 \leq c$, from (1) and (2) a formula follows:

$$\Delta\tau_A > \Delta\tau_B. \tag{3}$$

On the other hand, in the framework of a simple conception (naive belief) these two twins must be in symmetric ageing when B returns at O . This means that, according to the respective conception, the following supposed relation (s.r) have to be taken into account

$$\Delta\tau_A = \Delta\tau_B \quad (\text{s.r.}) \tag{4}$$

Moreover, for the same simple conception, by invoking the relative character of the twins' motion, the roles of A and B in (3) might be (formally) inverted. Then one obtains another supposed relation, namely

$$\Delta\tau_A < \Delta\tau_B \quad (\text{s.r.}) \tag{5}$$

This obvious disagreement between the relativistic formula (3) and the supposed relations (4) and (5) represents just the Twin Paradox.

For resolving of the Twin Paradox, CS invokes [1–3] (as essential and unique argument) the assertion that the twins ageing is completely asymmetric. The respective assertion is argued with an idea that, in the mentioned arrangement of twins, B feels non-null noninertial effects during its nonuniform motions, while A , being at rest, does not feel such effects. Based on the alluded argumentation, without any other major and credible proof, CS rejects the supposed relations (4) and (5) as unfounded and fictitious. Then, according to CS only the relativistic formula (3) must be regarded as a correct relation. Consequently CS infers the conclusion: the Twin Paradox is nothing but a purely and apparent fictitious problem.

But now we have to notify the fact that CS does not approach any discussion on the comparative importance (significance) in the Twin Paradox problem of the respective nonuniform and uniform motions. Particularly, it is not taken into discussions the asymptotic situations where, comparatively, the effects of the noninertial motions become insignificant. Or, it is clear that, as it is considered by CS, the asymmetry of the twins is generated by the nonuniform motions, while the uniform motions have nothing to do on the respective asymmetry. That is why we discuss that the alluded comparative importance is absolutely necessary. Moreover such a discussion should refer (in a quantitative manner) to the comparative value/ratio of L and D . This is because

$$(\Delta t_B)_u = \frac{2L}{v_0}, \tag{6}$$

while, on the other hand, $(\Delta t_B)_n$ depends on D , — e.g. when the nonuniformity of B motions is caused by constant forces, the relativity theory gives

$$(\Delta t_B)_n = \frac{4v_0 D}{c^2 \left(1 - \sqrt{1 - \frac{v_0^2}{c^2}}\right)}. \tag{7}$$

Then with the notation $\eta = \frac{D}{L}$ one obtains

$$\frac{(\Delta t_B)_n}{(\Delta t_B)_u} = \eta \frac{2v_0^2}{c^2 \left(1 - \sqrt{1 - \frac{v_0^2}{c^2}}\right)} \approx 4\eta \quad (\text{for } v_0 \ll c). \tag{8}$$

This means that, in the mentioned circumstances, the ratio $\eta = \frac{D}{L}$ has a property which gives a quantitative description to the comparative importance (significance) of the respective nonuniform and uniform motions. It is natural to consider η as the bearing the mentioned property in the circumstances that are more general than those referred in (7) and (8). That is why we will conduct our discussions in terms of the parameter η .

SPECIFICATION: The quantities D and v_0 are considered as being nonnull and constant, while L is regarded as an adjustable quantity. So we can consider situations where $\eta \ll 1$ or even where $\eta \rightarrow 0$.

Now let us discuss the cases where $\eta \ll 1$. In such a case the twin B moves predominantly uniform, and the noninertial effects on it are prevalently absent. The twins' positions are prevalently symmetric or even become asymptotically symmetric when $\eta \rightarrow 0$. That is why we regard/denote the respective cases as asymptotic situations. In such situations the role of the accelerated motions (and of associated noninertial effects) becomes insignificant (negligible).

These just alluded situations should be appreciated by consideration (prevalently or even asymptotically) of Einstein's postulate of relativity, which states [3] that the inertial frames of references are equivalent to each other, and they cannot be distinguished by means of investigation of physical phenomena. Such an appreciation materializes itself in the relations

$$\left. \begin{aligned} \Delta\tau_A &\approx \Delta\tau_B, & (\eta \ll 1) \\ \lim_{\eta \rightarrow 0} \Delta\tau_A &= \lim_{\eta \rightarrow 0} \Delta\tau_B \end{aligned} \right\}. \tag{9}$$

Also, from (1) and (2) one obtains

$$\Delta\tau_B \approx \Delta\tau_A \sqrt{1 - \frac{v_0^2}{c^2}} < \Delta\tau_A, \quad (\eta \ll 1). \tag{10}$$

By taking into account the mentioned Einstein postulate in (10), the roles of A and B might be inverted and one finds

$$\Delta\tau_A \approx \Delta\tau_B \sqrt{1 - \frac{v_0^2}{c^2}} < \Delta\tau_B, \quad (\eta \ll 1). \tag{11}$$

Note that, in the framework of the discussed case, the relations (9) and (11) are not supposed (or fictitious) pieces as (4) and (5) are, but they are true formulae like (10). This means that for $\eta \ll 1$ the mentioned arrangement of the twins leads to a set of incompatible relations (9)–(11). Within CS the respective incompatibility cannot be avoided by any means.

2.2 Discussions about an asymptotic completely symmetric situation

Now let us consider a new arrangement of the twins as follows. The twin *B* preserves exactly his situation previously presented. On the other hand, the twin *A* moves forward and backward in the negative part of the *x*-axis in *K*, symmetric to as *B* moves with respect to the point *O*. All the mentioned notations remain unchanged as the above. Evidently that, in the framework of the new arrangement, the situations of these two twins *A* and *B*, as well as their proper frames *K_A* and *K_B*, are completely symmetric with respect to *K*. From this fact, for the time intervals between start and finish of the motions, it results directly the relation

$$\Delta\tau_A = \Delta\tau_B. \tag{12}$$

In addition, for asymptotic situations where $\eta \ll 1$, one obtains

$$\Delta\tau_A = \Delta\tau_B \approx \frac{2L}{v_0} \sqrt{1 - \frac{v_0^2}{c^2}}, \quad (\eta \ll 1). \tag{13}$$

On the other hand, by taking into account Einstein’s postulate of relativity, similarly to the relations (10) and (11) for the new arrangement in the asymptotic situations (i.e., where $\eta \ll 1$ and the noninertial effects are insignificant), one finds

$$\Delta\tau_B \approx \Delta\tau_A \sqrt{1 - \frac{w_0^2}{c^2}} < \Delta\tau_A, \quad (\eta \ll 1), \tag{14}$$

$$\Delta\tau_A \approx \Delta\tau_B \sqrt{1 - \frac{w_0^2}{c^2}} < \Delta\tau_B, \quad (\eta \ll 1), \tag{15}$$

with

$$w_0 = \frac{2v_0}{1 + \frac{v_0^2}{c^2}}. \tag{16}$$

It should be noted that fact that, with respect to the relativity theory, the relations (12), (14) and (15) are true formulae: they are not supposed and/or fictitious. On the other hand, one finds that the mentioned relations are incompatible to each other. The respective incompatibility cannot be resolved or avoided in a rational way by CS whose solely major argument is the asymmetry of the twins.

3 Some final comments

The above analysed facts show that, in the mentioned “asymptotic situations”, the noninertial effects are insignificant for the estimation of the time intervals evaluated (felt

by the two twins. Consequently in such situations the inertial-noninertial asymmetry between such two twins cannot play a significant role. Therefore the respective asymmetry cannot be considered a reliable proof in the resolving of the Twin Paradox. This means that the CS loses its essential (and solely) argument. So, the existence of the above mentioned asymptotic situations appears as a true incriminating test for CS.

Regarding to its significance and implications, the mentioned test has to be evaluated/examined concurrently with the “approvingly illustrations” invoked and preached by the supporters of CS. At this point it seems to be of some profit to remind the Feynmann’s remark [11] that, in fact, a conception/theory is invalidated (proved to be wrong) by the real and irrefutable existence of a single incriminating test, indifferently of the number of approving illustrations. Some scientists consider that such a test must be only of experimental but not of theoretical nature. We think that the role of such tests can be played also by theoretical consequences rigorously derived from a given conception. So thinking, it is easy to see that for CS the existence of the above discussed asymptotic situations has all the characteristics of an irrefutable incriminating test. The respective existence invalidate the CS which must be abandoned as a wrong and unreliable approach of the Twin Paradox.

But even if CS is abandoned the incompatibility regarding the relations (9)–(11) or/and the formulae (12), (14) and (15) remains as an unavoidable and intriguing fact. Then what is the significance and importance of the respective fact? We think that it restores the Twin Paradox as an authentic unsolved problem which is still waiting for further investigations. Probably that such investigations will involve a large variety of facts/arguments/opinions.

In connection to the alluded further investigations the following first question seems to be non-trivially interesting: can the investigations on the Twin Paradox be done in a credible manner without troubling the Special Theory of Relativity? If a negative answer, a major importance goes to the second question: can the Twin Paradox, restored as mentioned above, be an incriminating test for the Special Theory of Relativity, in the sense of the previously noted Feynmann’s remark, or not? Can the second question be connected to the “sub-title” of the volume mentioned in the reference [9], or not?

This paper was prepared on the basis of an earlier manuscript of mine [12].

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A New Detector for Perturbations in Gravitational Field

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The paper presents design, principles of operation, and examples of registrations carried out by original device developed and constructed by V. N. Smirnov. The device manifested the possibility to register very weak gravitational perturbations of non-seismic kind both from celestial bodies and from the internal processed in the terrestrial globe.

Given all hypotheses of the possible, do choice for such one which doesn't limit your further thinking on the studied phenomenon.

J. C. Maxwell

1 Introduction

At present day, we have many working properly gravitational wave detectors such as LIGO (USA), GEO-600 (Great Britain and Germany), VIRGO (Italy), TAMA-300 (Japan), miniGRAIL (the Netherlands) and so on. The physical principles of measurement, on a basis of which all the detectors work, lie in the theory of deviation of two particles in the field of a falling gravitational wave meant as a wave of the space metric (so called deformation gravitational waves [1, 2]). The first of such devices was a solid-body (resonant) detector — a 1.500 kg aluminum pig, which is approximated by two particles connected by an elastic force (spring). It was constructed and armed in the end of 1960's by Joseph Weber, the pioneer of these measurements [3–5]. Later there were constructed also free-mass gravitational detectors, built on two mirrors, distantly located from each other and equipped by a laser range-finder to measure the distance between them. Once a gravitational wave falls onto both solid-body or free-mass detector, the detector should have smallest deformation that could be registered as piezo-effect in a solid-body detector or the change of the distance between the mirrors in a free-mass detector [1]. For instance, LIGO (USA) is a free-mass detector, while miniGRAIL (the Netherlands) is a solid-body detector built on a 65-cm metallic sphere, cooled down to liquid Helium. (A spherical solid-body detector is especially good, because it easily registers the direction of the falling gravitational wave that manifests the source of the gravitational radiation.) A device similar to miniGrail will soon be launched at São-Paolo, Brasil. Moreover, it is projected a “big Grail” which mass expects to be 110 tons.

As supposed, the sources of gravitational radiation should be the explosions of super-novae, stellar binaries, pulsars, and the other phenomena in the core of which lies the same process: two masses, which rotate round the common centre of inertia, loose the energy of gravitational interaction with time so shorten the distance between them; the lost energy of grav-

itational interaction exceeds into space with gravitational radiation [1]. In the same time, we may expect the sources of gravitational radiation existing in not only the far cosmos, but also in the solar system and even in the Earth. The nearest cosmic source of gravitational waves should be the system Earth-Moon. Besides, even motion of tectonic masses should generate gravitational radiation. Timely registering gravitational radiation produced by such tectonic masses, we could reach a good possibility for the prediction of earthquakes.

Here we represent a device, which could be considered as a gravitational wave detector of a new kind, which is a resonant-dynamic system. The core of such a detector is a rotating body (made from metal or ceramics) in the state of negative acceleration. Besides the advantage of the whole system is that it gives a possibility for easy registration of the direction of the gravitation wave moved through it.

2 The dynamical scheme of the device

Fig. 1 shows the dynamical scheme of the device, where the rotating body is a 200 g cylindrical pig made from brass and shaped as a cup (it is marked by number 1). The rotor is fixed up on the axis of a micro electrical motor of direct current (number 2). In the continuation of the axis 3 of the motor a thick disc made from aluminum (number 4) is located; the other side of the disc is painted by a light-absorbing black color ink, except of the small reflecting sector 5. There over the disc, an azimuth circle 6 is located, it is for orientation of the device to the azimuth coordinates (they can further be processed into the geographical coordinates of the sources of a registered signal, or the celestial coordinates of it if it is located in the cosmos). The azimuth circle has a optical pair consisting of semiconductor laser as emitter and photodiode as receiver 7. A laser beam, reflected by the sector 5, acts onto the photodiode. The electrical motor 2 is fixed on the rectangular magnetic platform 8, which is suspended by the strong counter-field 0.3 Tesla of the stationary fixed magnet 9. There between the magnetic platforms an inductive detector 10 is located.

We consider the functional dependencies between the elements of this device. The rotor 1 turns into rotation by the

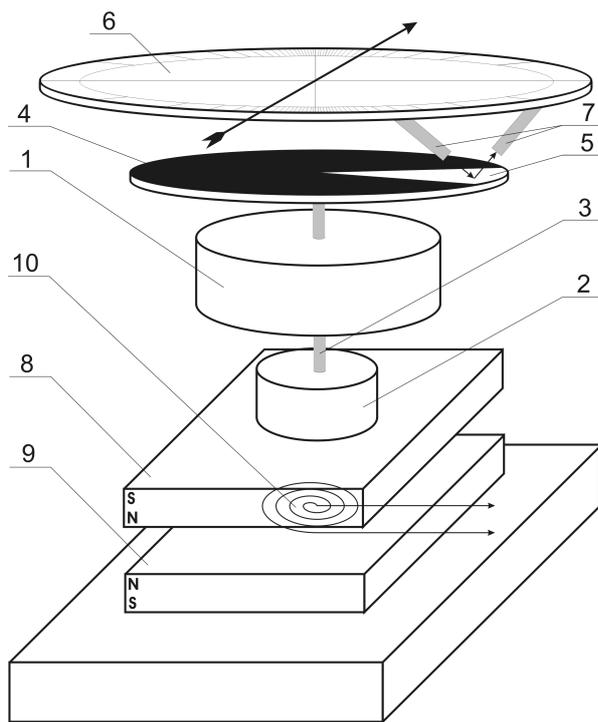


Fig. 1: Dynamical scheme of the device.

electric motor to 4.000 rpm; the disc 4 rotates synchronically with the rotor. Once the reflected laser beam falls onto the photodiode, it produces an electric current. The pulse signal, produced by the photodiode, goes into the control electronic block which produces a rectangular pulse of voltage with the regulated duration in the scale from 1.5 to 4.0 μsec . Next time these impulses go into the input of the motor driver. If the output of the driver had a stable voltage with the polarity (+, -), the inverts to (-, +) in the moment when the electric pulse acts. For this moment the motor's rotation is under action of a negative acceleration: the rotation is braking for a short time. During the braking a reverse pulse current is inducted in the motor circuit, that is a "braking current" appears a form of which is under permanent control on the screen of an control oscilloscope.

Fig. 2 shows the block diagram of the control block. There are: the rectangular magnetic platform 1, in common with the rotor and the motor 3 located on it; the stationary fixed magnetic platform 2; the inductive detector 4; selective amplifier 5 working in the range from 10 Hz to 20 kHz; plotter 6; the source of the power for the electrical motor (number 7); the driver 8; the electronic block for processing of the electric pulse coming out from the photodiode (number 9); the inductive detector of the pulse current (number 10); the indicator of the angular speed of the rotor (number 11); oscilloscope 12.

At the end of braking pulse finishes (if to be absolutely exact — on falling edge of pulse) the electrical motor rotating with inertia re-starts, so a positive acceleration appears in

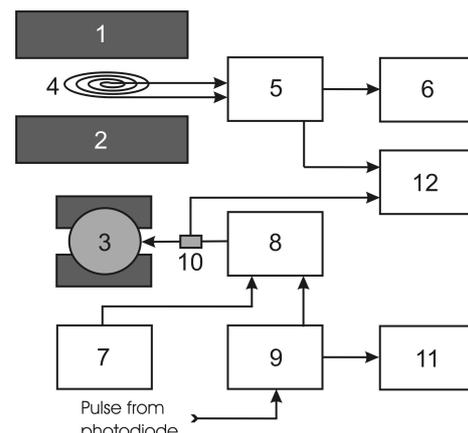


Fig. 2: Diagram of the control block.

the system. The starting pulse is due to the strong starting currents in the power supply circuit. According to Ampere's law, the occurred starting current leads to a mechanical impact experienced by the electrical motor armature (it is the necessary condition for the work of the whole device as a detector of gravitational perturbation). During the rotor's rotation, the whole spectrum of the low frequent oscillations produced by this mechanical impact are transferred to the mechanical platform 1, which induces electromotive force on the detector 4. This signal is transferred to the selective amplifier 5, wherein a corresponding harmonic characterizing the rotor's state is selected. This harmonic, converted into analogous signal, is transferred to the plotter 6.

3 The peculiarities of the experiment

The impulsive mechanical impact experienced by the motor armature is actually applied to the centre of the fixation of the rotor at the axis of electrical motor. The rotor, having a form of cylindrical resonator, reach excitation with low frequency due to this impact. In order to increase the excitation effect, a brass bush seal was set up on the motor's axis: the contact surface between the axis and the rotor became bigger than before that. As a result in the rotor a standing sonar wave occurs which has periodically excited, while all the time between the excitations it dissipates energy. The rotor, as a low frequent resonator, has its own resonant frequency, which was measured with special equipment by the method of the regulated frequent excitation and laser diagnostics. (The necessity to know the resonator frequency of the rotor proceeded from the requirement to choose the frequency of its rotation and also the frequency of its excitation.

Effect produced in the rotor due to a gravitational perturbation consist of the change of the period of its rotation that leads to the change in the initially parameters of the whole system: the shift of the operating point on frequency response function of selective amplifier and also the signal's amplitude changed at the output of the selective amplifier. Besides the

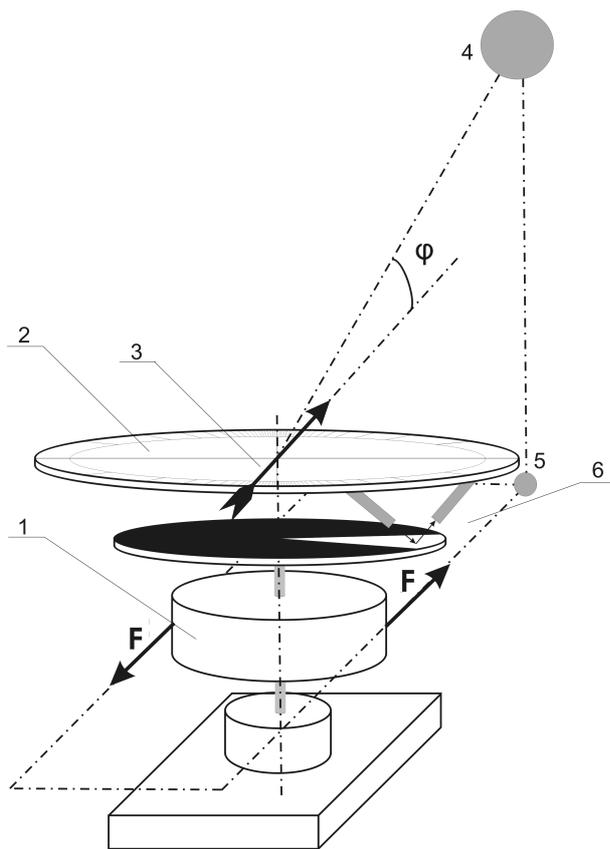


Fig. 3: Diagram of the device orientation at the supposed source of gravitational perturbation.

change of the angular speed of the rotation, due to the momentum conservation law, produces a reaction in the magnetic platform. Because the magnetic has rectangular form, the magnetic field between the platforms 1 and 2 (see Fig. 2) is non-uniform so the derivative of the density of the magnetic flow is substantial. All these lead to the fast change in the level of the signal's amplitude, and are defining the sensitivity of the whole device.

Plotter registered such a summarized change of the signal's amplitude.

Thus the sensitivity of the device is determined by the following parameters: (1) the choice for the required resonant frequency of the rotor; (2) the choice of the angular speed of its rotation; (3) the duration of the braking pulse; (4) the choice for the information sensor which gives information about the rotor; (5) the factor of the orientation of the device at the supposed source of gravitational perturbation.

The vector of the device orientation is the direction of the impulsive braking force F or, that is the same, the negative acceleration vector applied to the rotor. In the moment of braking there a pair of forces F appears, which are applied to the rotor. The plane where the forces act is the antennae parameter of the system. Fig. 3 represent a fragment of the device, where 1 is the rotor, 2 is the azimuth circle, 3 are the

indicators of direction, where the angular scale has the origin of count (zero degree) pre-defined to the Southern pole. If we suppose that the source of gravitational perturbation (it is pictured by gray circle, 4) is a cosmic object, the device should be oriented to the projection of this source onto the horizontal plane (this projection is marked by number 5, and pictured by small gray circle). The plane 6 is that for the acting forces of braking.

4 Experimental results

Here are typical experimental results we got on the device over a years of investigations.

The fact that such a device works as an antenna permits to turn it so that it will be directed in exact at the selected space objects in the sky or the earthy sources located at different geographical coordinates.

First, we were looking for the gravitational field perturbations due to the tectonic processes that could be meant the predecessors of earthquakes. Using the geographic map of the tectonic breaks, we set up an experiment on the orientation of the device to such breaks. Despite the fact that exact measurement of such directions is possible by a system of a few devices (or in that case where the device is located in area of a tectonic brake), the measurement of the azimuth direction by our device was as precise as $\pm 2^\circ$. The azimuthal directions were counted with respect to the South pole. All measurement represented on the experimental diagrams (Fig. 4–9) are given with Moscow time, because the measurement were done at Moscow, Russia. The period of the rotation of the gyro changed in the range from $75 \mu\text{sec}$ to $200 \mu\text{sec}$ during all the measurements produced on: the rises and sets of the planets of the solar system (including the Moon) and also those of the Sun; the moments when the full moon and new moon occurs; the solar and lunar eclipses; the perihelion and aphelion of the Earth, etc. In some experiments (Fig. 6) extremely high gravitational perturbations were registered, during which the period of the rotation of the gyro was changed till $400 \mu\text{sec}$ and even more (the duration of such extremely high perturbations was 5–10 minutes on the average). Further we found a correlation of the registered signals to the earthquakes. The correlation showed: the perturbations of the earthy gravitational field, registered by our device, preceded the earthquakes in the range from 3 to 15 days in the geographic areas whereto the device was directed (Fig. 4–6).

Examples of records in Fig. 7–8 present transit of Venus through the disc of the Sun (Fig. 7) and solar eclipse at Moscow, which occur at November 03, 2005.

Aside for such single signals as presented in Fig. 4–8, our device registered also periodic signals. The periodic signals were registered twice a year, in October and May, that are two points in the chord of the orbit of the Earth which connects the directions to Taurus and Virgo. The time interval between

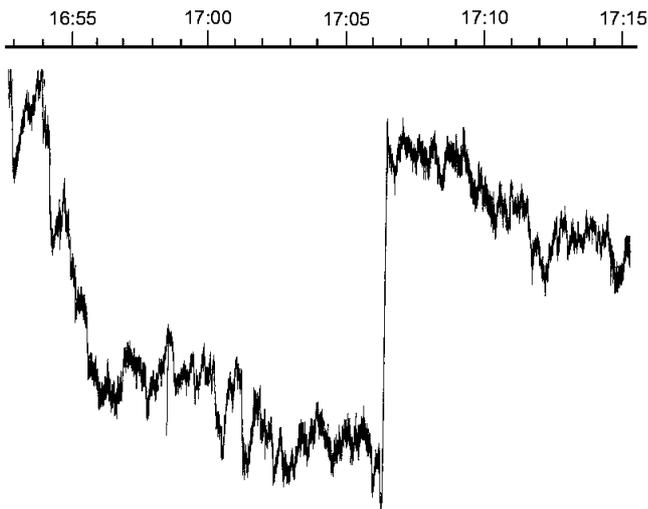


Fig. 4: June 30, 2005. The azimuth of the signal is $\sim 53^\circ$ to East. The predecing signal of the earthquake in the Indian Ocean near Sumatra Island, Indonesia, July 05, 2005.

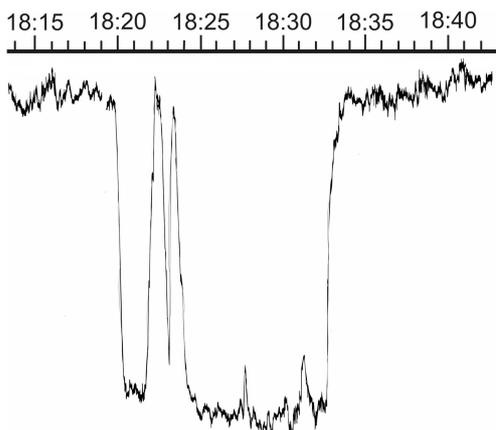


Fig. 5: March 29, 2006. The azimuth of the signal is $\sim 9^\circ$ to East: the predecessor of the earthquake in the Western Iran, which occurred on April 02, 2006.

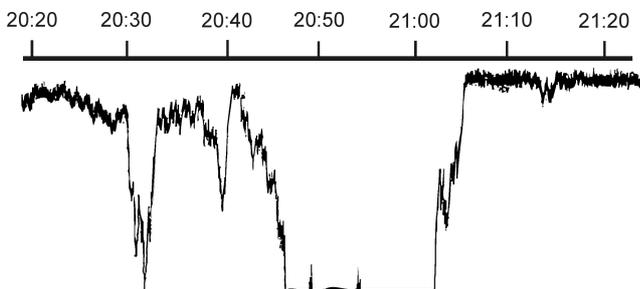


Fig. 6: May 05, 2007. A high altitude gravitational perturbation. The azimuth of the signal is 122° to West. The central states of the USA became under action of 74 destructing tornados two days later, on May 08, 2007.

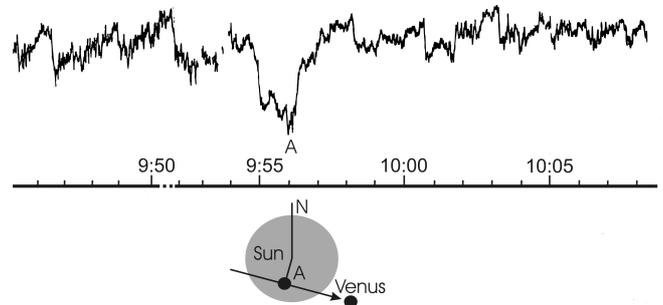


Fig. 7: June 08, 2004. Transit of Venus through the disc of the Sun, $09^h 51^{min}$.



Fig. 8: November 03, 2005. The solar eclipse at Moscow city, Russia. The eclipse phase is ~ 0.18 .

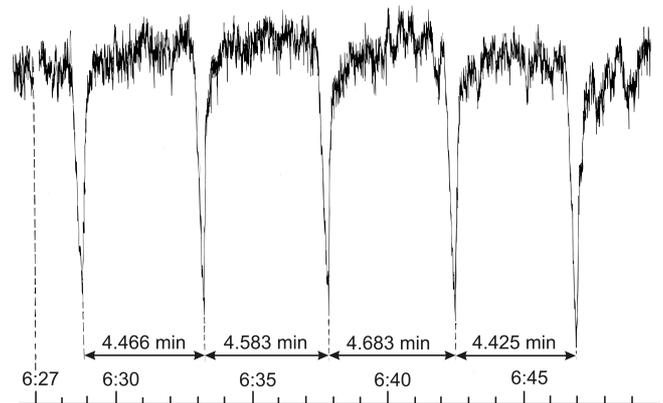


Fig. 9: May 31, 2003. Periodical signals.

the signals growing with the motion of the Earth along its orbit during 5 days then deceased. A fragment of the graph is represented in Fig. 9.

It should be noted that when Joseph Weber claimed about a gravitational wave signal registered with his solid-body detector [3–5], he pointed out that fact that the solely registered signal came from Taurus.

5 Conclusion

The core of the device is a rotating body (in our case it is a rotating brass resonator), which sensitivity to gravitational radiation lies in its excitation expected in the field of a falling gravitational wave. Despite the physical state of the gyro-resonator corresponds, in main part, to the wave-guide solid-body gyros, its internal construction and the principles of work are substantially different from those [6].

The device manifested the possibility to register gravitational perturbations of non-seismic kind from the internal processed in the terrestrial globe, and locate the terrestrial coordinates of the sources of the perturbations.

An auxiliary confirmation of such a principle for the registration of gravitational perturbation is that fact that one of the gyros CMG-3 working on board of the International Space Station “experienced an unusual high vibration” on March 28, 2005 (it was registered by the space station commander Leroy Chiao and the astronaut Salizhan Sharipov [7]), in the same time when a huge earthquake occurred near Nias Island (in the shelf of Indian Ocean, close to Sumatra, Indonesia).

This device is a really working instrument to be used for the aforementioned tasks. In the same time, a lack of attention to it brakes the continuation of the experiments till the stop of the whole research program in the near future.

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Third Quantization in Bergmann-Wagoner Theory

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We present the third quantization of Bergmann-Wagoner scalar-tensor and Brans Dicke solvable toy models. In the first one we used an exponential cosmological term, for the second one we considered vanishing cosmological constant. In both cases, it is found that the number of the universes produced from nothing is very large.

1 Introduction

The Wheeler-DeWitt (WDW) equation is a result of quantization of a geometry and matter (second quantization of gravity), in this paper we consider the third quantization of a solvable inflationary universe model, i.e., by analogy with the quantum field theory, it can be done the second quantization of the universe wavefunction ψ expanding it on the creation and annihilation operators (third quantization) [1]. Because in the recent years there has been a great interest in the study of scalar-tensor theories of gravitation, owing that of the unified theories [2, 3], we choose to work with the most general scalar-tensor theory examined by Bergmann and Wagoner [4, 5], in this theory the Brans-Dicke parameter ω and cosmological function λ depend upon the scalar gravitational field ϕ . The Brans-Dicke theory can be obtained setting $\omega = \text{const}$ and $\lambda = 0$.

The WDW equation is obtained by means of canonical quantization of Hamiltonian H according to the standard canonical rule, this leads to a difficulty known as the problem of time [6]. Also, this equation has problems in its probabilistic interpretation. In the usual formulation of quantum mechanics a conserved positive-definite probability density is required for a consistent interpretation of the physical properties of a given system, and the universe in the quantum cosmology perspective, do not satisfied this requirement, because the WDW equation is a hyperbolic second order differential equation, there is no conserved positive-definite probability density as in the case of the Klein-Gordon equation, an alternative to this, is to regard the wavefunction as a quantum field in minisuperspace rather than a state amplitude [7].

The paper is organized as follows. In Section 2 we obtain the WDW equation. In Section 3 we show third quantization of the universe wavefunction using two complete set of modes for the most easy choice of factor ordering. Finally, Section 4 consists of conclusions.

2 Canonical formalism

Our starting point is the action of Bergmann-Wagoner scalar tensor theory

$$S = \frac{1}{l_p^2} \int_M \sqrt{-g} \left[\phi R^{(4)} - \frac{\omega(\phi)}{\phi} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} + 2\phi \lambda(\phi) \right] d^4x + \frac{2}{l_p^2} \int_{\partial M} \sqrt{h} \phi h_{ij} K^{ij} d^3x, \quad (1)$$

where $g = \det(g_{\mu,\nu})$, $\phi(t)$ is the conventional real scalar gravitational field, while l_p is the Planck length and $\lambda(\phi)$ is the cosmological term. The quantity $R^{(4)}$ is the scalar curvature of the Friedmann-Robertson-Walker theory, which is given, according to the theory, by

$$R^{(4)} = -\frac{6k}{a^2} - 6\frac{\dot{a}^2}{N^2 a^2} - 6\frac{\ddot{a}}{N^2 a} + 6\frac{\dot{a}\dot{N}}{N^3 a}. \quad (2)$$

The second integral in (1) is a surface term involving the induced metric h_{ij} and second fundamental form K^{ij} on the boundary, needed to cancel the second derivatives in $R^{(4)}$ when the action is varied with the metric and scalar field, but not their normal derivatives, fixed on the boundary. Substituting (2) in (1) and integrating with respect to space coordinates, we have

$$S = \frac{1}{2} \int \left[-Nka\phi + \frac{a\phi}{N} \dot{a}^2 + \frac{a^2}{N} \dot{a}\dot{\phi} - \frac{N\omega(\phi)}{6\phi} a^3 \dot{\phi}^2 + \frac{N}{3} a^3 \phi \lambda(\phi) \right] dt, \quad (3)$$

where dot denotes time derivative with respect to the time t , now introducing a new time $d\tau = \phi^{\frac{1}{2}} dt$ and the following independent variables

$$\alpha = a^2 \phi \cosh \int \left(\frac{2\omega(\phi) + 3}{3} \right)^{\frac{1}{2}} \frac{d\phi}{\phi}, \quad (4)$$

$$\beta = a^2 \phi \sinh \int \left(\frac{2\omega(\phi) + 3}{3} \right)^{\frac{1}{2}} \frac{d\phi}{\phi}, \quad (5)$$

$$\lambda(\phi) = 3\phi \left[\Lambda_1 \cosh \int \left(\frac{2\omega(\phi) + 3}{3} \right)^{\frac{1}{2}} \frac{d\phi}{\phi} + \Lambda_2 \sinh \int \left(\frac{2\omega(\phi) + 3}{3} \right)^{\frac{1}{2}} \frac{d\phi}{\phi} \right], \quad (6)$$

where Λ_1 and Λ_2 are constants, with gauge $N = 1$, then action (3) transforms into a symmetric form

$$S = \frac{1}{2} \int \left[\frac{1}{4} (\alpha'^2 - \beta'^2) + \Lambda_1 \alpha + \Lambda_2 \beta - k \right] d\tau, \quad (7)$$

here prime denotes time derivative with respect to τ . The Hamiltonian of the system is

$$H = 2\pi_\alpha^2 - 2\pi_\beta^2 + \frac{1}{2}(k - \Lambda_1 \alpha - \Lambda_2 \beta). \quad (8)$$

After canonical quantization of H , the WDW equation is

$$\left[\partial_\alpha^2 + A\alpha^{-1}\partial_\alpha - \partial_\beta^2 - B\beta^{-1}\partial_\beta + \frac{1}{4}(\Lambda_1 \alpha + \Lambda_2 \beta - k) \right] \psi(\alpha, \beta) = 0, \quad (9)$$

where A and B are ambiguity ordering parameters. The general universe wavefunction for this model can be given in terms of Airy functions.

3 Third quantization

The procedure of the universe wavefunction ψ quantization is called third quantization, in this theory we consider ψ as an operator acting on the state vectors of a system of universes and can be decomposed as

$$\hat{\psi}(\alpha, \beta) = \hat{C}_i \psi_i^+(\alpha, \beta) + \hat{C}_i^\dagger \psi_i^-(\alpha, \beta), \quad (10)$$

where $\psi_i^\pm(\alpha, \beta)$ form complete orthonormal sets of solutions to WDW equation. This is in analogy with the quantum field theory, where \hat{C}_i and \hat{C}_i^\dagger are creation and annihilation operators. Thus, we expect that the vacuum state in a third quantized theory is unstable and creation of universes from the initial vacuum state takes place. In this view, the variable α plays the role of time, and variable β the role of space. $\psi(\alpha, \beta)$ is interpreted as a quantum field in the minisuper-space.

We assume that the creation and annihilation operators of universes obey the standard commutation relations

$$[C(s), C^\dagger(s')] = \delta(s - s'), \quad (11)$$

$$[C(s), C(s')] = [C^\dagger(s), C^\dagger(s')] = 0. \quad (12)$$

The vacuum state $|0\rangle$ is defined by

$$C(s)|0\rangle \text{ for } \forall C, \quad (13)$$

and the Fock space is spanned by $C^\dagger(s_1)C^\dagger(s_2)\dots|0\rangle$. The field $\psi(\alpha, \beta)$ can be expanded in normal modes ψ_s as

$$\psi(\alpha, \beta) = \int_{-\infty}^{+\infty} [C(s)\psi_s(\alpha, \beta) + C^\dagger(s)\psi_s^*(\alpha, \beta)] ds, \quad (14)$$

here, the wave number s is the momentum in Planck units and is very small.

3.1 General model

Let us consider the quantum model (9) for the most easy factor ordering $A = B = 0$, with $\Lambda_2 = 0$ and closed universe $k = 1$. Then, the WDW equation becomes

$$\left[\partial_\alpha^2 - \partial_\beta^2 + \frac{1}{4}(\Lambda_1 \alpha - 1) \right] \psi(\alpha, \beta) = 0, \quad (15)$$

the third-quantized action to yield this equation is

$$S_{3Q} = \frac{1}{2} \int \left[(\partial_\alpha \psi)^2 - (\partial_\beta \psi)^2 - \frac{1}{4}(\Lambda_1 \alpha - 1)\psi^2 \right] d\alpha d\beta, \quad (16)$$

this action can be canonically quantized and we impose the equal time commutation relations

$$[i\partial_\alpha \psi(\alpha, \beta), \psi(\alpha, \beta')] = \delta(\beta - \beta'), \quad (17)$$

$$[i\partial_\alpha \psi(\alpha, \beta), i\partial_\alpha \psi(\alpha, \beta')] = 0, \quad (18)$$

$$[\psi(\alpha, \beta), \psi(\alpha, \beta')] = 0. \quad (19)$$

A suitable complete set of normalized positive frequency solutions to equation (15) are:

$$\psi_s^{out}(\alpha, \beta) = \frac{e^{is\beta}}{(16\Lambda_1)^{\frac{1}{16}}} \left\{ \text{Ai} \left[(2\Lambda_1)^{-\frac{2}{3}}(1 - 4s^2 - \Lambda_1 \alpha) \right] + i \text{Bi} \left[(2\Lambda_1)^{-\frac{2}{3}}(1 - 4s^2 - \Lambda_1 \alpha) \right] \right\}, \quad (20)$$

and

$$\psi_s^{in}(\alpha, \beta) = \frac{\sqrt{2} e^{is\beta}}{(16\Lambda_1)^{\frac{1}{16}}} \times \left\{ e^{\frac{(1-4s^2)^{\frac{3}{2}}}{3\Lambda_1}} \text{Ai} \left[(2\Lambda_1)^{-\frac{2}{3}}(1 - 4s^2 - \Lambda_1 \alpha) \right] + \frac{i}{2} e^{-\frac{(1-4s^2)^{\frac{3}{2}}}{3\Lambda_1}} \text{Bi} \left[(2\Lambda_1)^{-\frac{2}{3}}(1 - 4s^2 - \Lambda_1 \alpha) \right] \right\}, \quad (21)$$

$\psi_s^{out}(\alpha, \beta)$ and $\psi_s^{in}(\alpha, \beta)$ can be seen as a positive frequency out going and in going modes, respectively, and these solutions are orthonormal with respect to the Klein-Gordon scalar product

$$\langle \psi_s, \psi_{s'} \rangle = i \int \psi_s \overleftrightarrow{\partial}_\beta \psi_{s'}^* d\beta = \delta(s - s'). \quad (22)$$

The expansion of $\psi(\alpha, \beta)$ in terms of creation and annihilation operators for the in-mode and out-mode is

$$\psi(\alpha, \beta) = \int [C_{in}(s)\psi_s^{in}(\alpha, \beta) + C_{in}^\dagger(s)\psi_s^{in*}(\alpha, \beta)] ds, \quad (23)$$

and

$$\psi(\alpha, \beta) = \int [C_{out}(s)\psi_s^{out}(\alpha, \beta) + C_{out}^\dagger(s)\psi_s^{out*}(\alpha, \beta)] ds. \quad (24)$$

As both sets (20) and (21) are complete, they are related to each other by the Bogoliubov transformation defined by

$$\psi_s^{out}(\alpha, \beta) = \int [C_1(s, r) \psi_r^{in}(\alpha, \beta) + C_2(s, r) \psi_r^{in*}(\alpha, \beta)] dr, \quad (25)$$

and

$$\psi_s^{in}(\alpha, \beta) = \int [C_1(s, r) \psi_r^{out}(\alpha, \beta) + C_2(s, r) \psi_r^{out*}(\alpha, \beta)] dr. \quad (26)$$

Then, we obtain that the Bogoliubov coefficients $C_1(s, r) = \delta(s - r)C_1(s)$ and $C_2(s, r) = \delta(s + r)C_2(s)$ are

$$C_1(s, r) = \delta(s - r) \frac{1}{\sqrt{2}} \times \left(\frac{1}{2} e^{-\frac{(1-4s^2)^{\frac{3}{2}}}{3\Lambda_1}} + e^{\frac{(1-4s^2)^{\frac{3}{2}}}{3\Lambda_1}} \right), \quad (27)$$

and

$$C_2(s, r) = \delta(s + r) \frac{1}{\sqrt{2}} \times \left(\frac{1}{2} e^{-\frac{(1-4s^2)^{\frac{3}{2}}}{3\Lambda_1}} + e^{\frac{(1-4s^2)^{\frac{3}{2}}}{3\Lambda_1}} \right). \quad (28)$$

The coefficients $C_1(s, r)$ and $C_2(s, r)$ are not equal to zero. Thus, two Fock spaces constructed with the help of the modes (20) and (21) are not equivalent and we have two different third quantized vacuum states (voids): the in-vacuum $|0, in\rangle$ and out-vacuum $|0, out\rangle$ (which are the states with no Friedmann Robertson Walker-like universes) defined by

$$C_{in}(s) |0, in\rangle = 0 \quad \text{and} \quad C_{out}(s) |0, out\rangle = 0, \quad (29)$$

where $s \in \mathbf{R}$. Since the vacuum states $|0, in\rangle$ and $|0, out\rangle$ are not equivalent, the birth of the universes from nothing may have place, where nothing is the vacuum state $|0, in\rangle$. The average number of universes produced from nothing, in the s -tn mode $N(s)$ is

$$N(s) = \langle 0, in | C_{out}^\dagger(s) C_{out}(s) | 0, in \rangle, \quad (30)$$

as follows from equation (25) we get

$$N(s) = \frac{1}{2} \left(\frac{1}{2} e^{-\frac{(1-4s^2)^{\frac{3}{2}}}{3\Lambda_1}} - e^{\frac{(1-4s^2)^{\frac{3}{2}}}{3\Lambda_1}} \right)^2, \quad (31)$$

considering Coleman's wormhole mechanism [8] for the vanishing cosmological constant and the constraint $\Lambda_1 \leq \frac{1}{8} \pi \times 10^{-120} m_p^4$, with $|s| \ll 1$, then the number of state $N(s)$ is

$$N(s) \approx \frac{1}{2} e^{\frac{2}{3\Lambda_1}(1-4s^2)^{\frac{3}{2}}}. \quad (32)$$

This result from third quantization shows that the number of the universes produced from nothing is exponentially large.

3.2 Particular model

An interesting model derived from Bergmann Wagoner action (1) with $\omega(\phi) = \omega_0 = \text{const}$, $\lambda(\phi) = 0$ and $N = 1$ is the Brans-Dicke theory

$$S = \frac{1}{l_p^2} \int \sqrt{-g} \left[\phi R^{(4)} - \frac{\omega(\phi)}{\phi} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} + 2\phi \lambda(\phi) \right] dt. \quad (33)$$

By means of new variables

$$x = \ln(a^2 \phi), \quad y = \ln \phi^{\frac{1}{p}}, \quad dt = a d\tau, \quad (34)$$

where $\rho^2 = \frac{3}{2\omega_0 + 3}$, action (33) transforms into

$$S = \frac{1}{2} \int \left[\frac{x'^2}{4} - \frac{y'^2}{4} - 1 \right] e^x d\tau, \quad (35)$$

the WDW equation for this model is

$$\left[x^{-A} \partial_x (x^A \partial_x) - \partial_y^2 - \frac{e^{2x}}{4} \right] \psi(x, y) = 0, \quad (36)$$

the ambiguity of factor ordering is encoded in the A parameter. The third quantized action to yield the WDW equation (36) is

$$S_{3Q} = \frac{1}{2} \int \left[(\partial_x \psi)^2 - (\partial_y \psi)^2 + \frac{e^{2x}}{4} \psi^2 \right] dx dy. \quad (37)$$

Again, in order to quantize this toy model, we impose equal time commutation relations given by (17–19), and by means of normal mode functions ψ_p we can expand the field $\psi(x, y)$. A suitable normalized out-mode function with positive frequency for large scales, is

$$\psi_p^{out}(x, y) = \frac{1}{2\sqrt{2}} e^{-\frac{\pi}{2}|p|} H_{-q}^{(2)} \frac{ie^x}{2} e^{ipy}, \quad (38)$$

where $H_{-q}^{(2)}$ is a Hankel function and $q = -i|p|$. The normalized in-mode function is

$$\psi_p^{in}(x, y) = \frac{e^{\frac{\pi}{2}|p|}}{2 \sinh^{\frac{1}{2}}(\pi|p|)} J_q \frac{ie^x}{2} e^{ipy}, \quad (39)$$

where J_q is a first class Bessel function. In the classically allowed regions the positive frequency modes correspond to the expanding universe [9]. As both wavefunctions (38) and (39) are complete, they are related to each other by a Bogoliubov transformation. The corresponding coefficients are

$$C_1(p, q) = \delta(p - q) \frac{1}{\sqrt{1 - e^{-2\pi|p|}}}, \quad (40)$$

and

$$C_2(p, q) = \delta(p + q) \frac{1}{\sqrt{e^{2\pi|p|} - 1}}. \quad (41)$$

The coefficients $C_1(p)$ and $C_2(p)$ are not equal to zero and satisfy the probability conservation condition

$$|C_1(p)|^2 - |C_2(p)|^2 = 1.$$

In this way, it can be constructed two not equivalent Fock spaces by means of (38) and (39). These two different third quantized vacuum states, the in-vacuum $|0, in\rangle$ and out-vacuum $|0, out\rangle$ are defined by (29). The average number of universes created from nothing, i.e., the in-vacuum in the p -th $N(p)$, is

$$\begin{aligned} N(p) &= \langle 0, in | C_{out}^\dagger(p) C_{out}(p) | 0, in \rangle = \\ &= |C_2(p)|^2 = \\ &= \frac{1}{e^{2\pi|p|} - 1}. \end{aligned} \quad (42)$$

This expression corresponds to a Planckian distribution of universes.

4 Conclusions

By means of a suitable choice of lapse function and independent variables, we have solved the WDW equation in the Bergmann-Wagoner gravitational theory for a cosmological function of the form $\lambda(\phi) = \Lambda_1 \cosh[2y(\phi)] + \Lambda_2 \sinh[2y(\phi)]$, this kind of cosmological term is important because of new scenario of extended inflation [10]. Also, we have studied on the third quantization of Bergmann-Wagoner and Brans-Dicke models, in which time is related by the scalar factor of universe and the space coordinate is related with the scalar field. The universe is created from stable vacuum obtained by the Bogoliubov-type transformation just as it is in the quantum field theory.

One of the main results of third quantization is that the number of universes produced from nothing is exponentially large. We calculated the number density of the universes creating from nothing and found that the initial state $|0, in\rangle$ is populated by a Planckian distribution of universes.

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Gravity Model for Topological Features on a Cylindrical Manifold

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A model aimed at understanding quantum gravity in terms of Birkhoff's approach is discussed. The geometry of this model is constructed by using a winding map of Minkowski space into a $\mathbb{R}^3 \times S^1$ -cylinder. The basic field of this model is a field of unit vectors defined through the velocity field of a flow wrapping the cylinder. The degeneration of some parts of the flow into circles (topological features) results in inhomogeneities and gives rise to a scalar field, analogous to the gravitational field. The geometry and dynamics of this field are briefly discussed. We treat the intersections between the topological features and the observer's 3-space as matter particles and argue that these entities are likely to possess some quantum properties.

1 Introduction

In this paper we shall discuss a mathematical construction aimed at understanding quantum gravity in terms of Birkhoff's twist Hamiltonian diffeomorphism of a cylinder [1]. We shall also use the idea of compactification of extra dimensions due to Klein [2]. To outline the main idea behind this model in a very simple way, we can reduce the dimensionality and consider the dynamics of a vector field defined on a 2-cylinder $\mathbb{R}^1 \times S^1$. For this purpose we can use the velocity field $u(x, \tau)$ of a two-dimensional flow of ideal incompressible fluid moving through this manifold.

Indeed, the dynamics of the vector field $u(x, \tau)$ with the initial condition $u(x, 0)$ is defined by the evolution equation

$$\delta \int_{\Delta\tau} \int_{\Delta x} dx \wedge u(x, \tau) d\tau \rightarrow 0, \quad (1.1)$$

where we use the restriction of the vector field onto an arbitrary cylinder's element; $\Delta\tau$ is the evolution (time) interval, and Δx is an arbitrary segment of the cylinder's element. In other words, we assume the variation of the integral of the mass carried by the flow through the segment during a finite time interval to be vanishing. That is, as a result of the field evolution, $u(x, 0) \rightarrow u(x, \infty)$, the functional of the flow mass approaches to its maximal value. If, at the initial moment of time, the regular vector field $u(x, 0)$ corresponds to a unit vector forming an angle φ with the cylinder's element, then the evolution of this field is described by the equation

$$\begin{aligned} \delta \int_{\Delta\tau} \int_{\Delta x} dx \wedge u(x, \tau) d\tau &= \\ = \delta \int_{\Delta\tau} \int_{\Delta x} \sin \varphi(\tau) dx d\tau &= \cos \varphi(\tau) \Delta\tau \Delta x \rightarrow 0. \end{aligned} \quad (1.2)$$

Therefore, the case of $\varphi(0) = 0$ corresponds to the absolute instability of the vector field. During its evolution, $u(x, 0) \rightarrow u(x, \infty)$, the field is relatively stable at $0 < \varphi(\tau) < \frac{\pi}{2}$, achieving the absolute stability at the end of this evolution, when $\varphi(\infty) = \frac{\pi}{2}$. If, additionally, we fix the

vector field $u(x, \tau)$ at the endpoints of the segment Δx by imposing some boundary conditions on the evolution equation (1.1), we would get the following dynamical equation:

$$\delta \int_{\Delta\tau} \int_{\Delta x} dx \wedge u(x, \tau) d\tau = 0. \quad (1.3)$$

Let some flow lines of the vector field $u(x, \tau)$ be degenerated into circles (topological features) as a result of the absolute instability of the field and fluctuations during the initial phase of its evolution. Since the dynamics of such topological features is described by (1.3), the features would tend to move towards that side of Δx where the field $u(x, \tau)$ is more stable. Thus, the topological features serve as attraction points for each other and can be used for modelling matter particles (mass points).

We must emphasise that the plane (x, τ) , in which our variational equations are defined, has the Euclidean metric. That is, in the case of the Euclidean plane (x, ϕ) wrapping over a cylinder we can identify the azimuthal parameter ϕ with the evolution parameter τ . By choosing the observer's worldline coinciding with a cylinder's element we can speak of a classical limit, whereas by generalising and involving also the azimuthal (angular) parameter we can speak of the quantisation of our model. So, when the observer's worldline is an arbitrary helix on the cylinder, the variational equation (1.3) reads

$$\delta \int_{\Delta x_0} \int_{\Delta x_1} dx_1 \wedge g(x) dx_0 = 0, \quad (1.4)$$

where the varied is the vector field $g(x)$ defined on the pseudo-Euclidean plane (x_0, x_1) oriented in such a way that one of its isotropic lines covers the cylinder-defining circle and the other corresponds to a cylinder's element. In this case we can speak of a relativistic consideration. If the observer's worldline corresponds to a curved line orthogonal to the flow lines of the vector field $g(x)$, where $g^2(x) > 0$, then we have to use the variational equation defined on a two-dimensional pseudo-Riemann manifold M induced by the vector field

$g(x)$, namely,

$$\delta \int_{\Delta M} g^2(x') \sqrt{-\det g_{ij}} dx'_0 \wedge dx'_1 = 0, \quad (1.5)$$

where $\Delta M = \Delta x'_0 \times \Delta x'_1$ is an arbitrary region of the manifold M ; $x'_0(\phi)$ is the flow line of the vector field $g(x)$ parameterised by the angular coordinate ϕ ; $x'_1(r)$ is the spatial coordinate on the cylinder (orthogonal to the observer worldline) parameterised by the Euclidean length r ; g_{ij} is the Gram matrix corresponding to the pair of tangent vectors $(\frac{dx'_0}{d\phi}, \frac{dx'_1}{dr})$. In this case the dynamics of the vector field is described through the geometry of its flow lines [3–5].

Thus, we can say that our approach to the dynamics of the vector field is based on maximisation of the mass carried by the flow [6, 7], which is not exactly what is typically used in the ergodic theory [8–10]. However, this principle is likely to be related to the the minimum principle for the velocity field [13–15], which is a special case of the more general principle of minimum or maximum entropy production [11, 12].

Before a more detailed discussion of this model we have to make a few preliminary notes. First, throughout this paper we shall use a somewhat unconventional spherical coordinates. Namely, latitude will be measured modulo 2π and longitude – modulo π . In other words, we shall use the following spherical $(\rho, \varphi, \theta_1, \dots, \theta_{n-2})$ to Cartesian (x_1, \dots, x_n) coordinate transformation in \mathbb{R}^n :

$$\begin{aligned} x_1 &= \rho \cos \varphi, \\ x_2 &= \rho \sin \varphi \cos \theta_1, \\ x_3 &= \rho \sin \varphi \sin \theta_1, \\ &\dots\dots\dots \\ x_{n-1} &= \rho \sin \varphi \dots \sin \theta_{n-3} \cos \theta_{n-2}, \\ x_n &= \rho \sin \varphi \dots \sin \theta_{n-3} \sin \theta_{n-2}, \end{aligned}$$

where $0 \leq \rho < \infty$, $0 \leq \varphi < 2\pi$ and $0 \leq \theta_i < \pi$. We shall also be interpreting the projective space RP^n as the space of centrally symmetric lines in \mathbb{R}^{n+1} , that is, as a quotient space $\mathbb{R}^{n+1} \setminus \{0\}$ under the equivalence relation $x \sim r x$, where $r \in \mathbb{R} \setminus \{0\}$.

2 The geometry of the model

We can describe the geometry of our model in terms of the mapping of the Euclidean plane into a 2-sphere, S^2 , by winding the former around the latter. We can also use similar winding maps for the pseudo-Euclidean plane into a cylinder, $\mathbb{R} \times S^1$, or a torus, $S^1 \times S^1$. More formally this could be expressed in the following way [16]. Take the polar coordinates (φ, ρ) defined on the Euclidean plane and the spherical coordinates (θ, ϕ) on a sphere. We can map the Euclidean plane into sphere by using the congruence classes modulo π and 2π . That is,

$$\theta = |\varphi| \bmod \pi, \quad \phi = |\pm \pi \rho| \bmod 2\pi, \quad (2.1)$$

where the positive sign corresponds to the interval $0 \leq \varphi < \pi$ and negative — to the interval $\pi \leq \varphi < 2\pi$. If the projective lines are chosen to be centrally symmetric then the Euclidean plane can be generated as the product $RP^1 \times \mathbb{R}$. Here the components of \mathbb{R} are assumed to be Euclidean, i.e., rigid and with no mirror-reflection operation allowed. Similarly, we can define a space based on unoriented lines in the tangent plane to the sphere. Therefore, the sphere can be generated by the product $RP^1 \times S^1$, the opposite points of the circle being identified with each other. In this representation all centrally symmetric Euclidean lines are mapped as

$$\mathbb{R} \rightarrow S^1 : e^{i\pi x} = e^{\pm i\pi \rho} \quad (2.2)$$

by winding them onto the corresponding circles of the sphere.

The winding mapping of Euclidean space onto a sphere can be extended to any number of dimensions. Here we are focusing mostly on the case of Euclidean space, \mathbb{R}^3 , generated as the product $RP^2 \times \mathbb{R}$ and also on the case of a 3-sphere generated as $RP^2 \times S^1$. In both cases we assume the Euclidean rigidity of straight lines and the identification of the opposite points on a circle. Euclidean space, \mathbb{R}^3 , can be mapped into a sphere, S^3 , by the winding transformation analogous to (2.1). Indeed, for this purpose we only have to establish a relation between the length of the radius-vector in Euclidean space and the spherical coordinate (latitude) measured modulo 2π . The relevant transformations are as follows:

$$\theta_1 = \vartheta, \quad \theta_2 = |\varphi| \bmod \pi, \quad \phi = |\pm \pi \rho| \bmod 2\pi, \quad (2.3)$$

where the sign is determined by the quadrant of φ .

Let (e_0, e_1) be an orthonormal basis on a pseudo-Euclidean plane with coordinates (x_0, x_1) . Let the cylindrical coordinates of $\mathbb{R} \times S^1$ be (ϕ, r) . Then the simplest mapping of this pseudo-Euclidean plane to the cylinder would be

$$\phi = |\pi(x_0 + x_1)| \bmod 2\pi, \quad r = x_0 - x_1. \quad (2.4)$$

That is, the first isotropic line is winded here around the cylinder’s cross-section (circle) and the second line is identified with the cylinder’s element. In this way one can make a correspondence between any non-isotropic (having a non-zero length) vector in the plane and a point on the cylinder. For instance, if a vector x having coordinates (x_0, x_1) forms a hyperbolic angle φ with the e_0 or $-e_0$, then

$$\phi = |\pm \pi e^{-\varphi} \rho| \bmod 2\pi = |\pi(x_0 + x_1)| \bmod 2\pi. \quad (2.5)$$

If this vector forms the hyperbolic angle φ with the e_1 or $-e_1$, then

$$r = \pm e^\varphi \rho = x_0 - x_1, \quad (2.6)$$

where $\varphi = -\ln \left| \frac{x_0 + x_1}{\rho} \right|$; $\rho = |(x_0 + x_1)(x_0 - x_1)|^{1/2}$.

By analogy, one can build a winding map of the pseudo-Euclidean plane into the torus, with the only difference that in the latter case the second isotropic line is winded around the longitudinal (toroidal) direction of the torus.

Now let us consider a 6-dimensional pseudo-Euclidean space \mathbb{R}^6 with the signature $(+, +, +, -, -, -)$. In this case the analogue to the cylinder above is the product $\mathbb{R}^3 \times S^3$, in which the component \mathbb{R}^3 is Euclidean space. In order to wind the space \mathbb{R}^6 over the cylinder $\mathbb{R}^3 \times S^3$ we have to take an arbitrary pseudo-Euclidean plane in \mathbb{R}^6 passing through the (arbitrary) orthogonal lines x_k, x_p that belong to two Euclidean subspaces \mathbb{R}^3 of the space \mathbb{R}^6 . Each plane (x_k, x_p) has to be wound onto a cylinder with the cylindrical coordinates (ϕ_k, r_p) ; the indices k, p correspond to the projective space RP^2 . We can take all the possible planes and wind them over the corresponding cylinders. The mapping transformation of the pseudo-Euclidean space \mathbb{R}^6 into the cylinder $\mathbb{R}^3 \times S^3$ is similar to the expressions (2.5) and (2.6):

$$\begin{aligned} \phi_k &= |\pm \pi e^{-\varphi} \rho| \bmod 2\pi = \\ &= |\pi(x_k + x_p)| \bmod 2\pi, \end{aligned} \quad (2.7)$$

$$r_p = \pm e^\varphi \rho = x_k - x_p. \quad (2.8)$$

By fixing the running index k and replacing it with zero we can get the winding map of the Minkowski space \mathbb{R}^4 into the cylinder $\mathbb{R}^3 \times S^1$, which is a particular case (reduction) of (2.7) and (2.8). Conversely, by winding \mathbb{R}^3 over a 3-sphere, S^3 , we can generalise the case and derive a winding map from \mathbb{R}^6 into $S^3 \times S^3$.

Let us consider the relationship between different orthonormal bases in the pseudo-Euclidean plane, which is wound over a cylinder. It is known that all of the orthonormal bases in a pseudo-Euclidean are equivalent (i.e., none of them can be chosen as privileged). However, by defining a regular field c of unit vectors on the pseudo-Euclidean plane it is, indeed, possible to get such a privileged orthonormal basis (c, c_1) . In turn, a non-uniform unitary vector field $g(x)$, having a hyperbolic angle $\varphi(x)$ with respect to the field c , would induce a non-orthonormal frame $(g'(x), g'_1(x))$. Indeed, if we assume that the following equalities are satisfied:

$$\begin{aligned} \pi &= |\pm \pi e^{-\varphi} \rho(e^\varphi g)| \bmod 2\pi = \\ &= |\pm \pi e^{-\varphi} \rho(g')| \bmod 2\pi, \end{aligned} \quad (2.9)$$

$$\pm 1 = \pm e^\varphi \rho(e^{-\varphi} g_1) = \pm e^\varphi \rho(g'_1), \quad (2.10)$$

we can derive a non-orthonormal frame $(g'(x), g'_1(x))$ by using the following transformation of the orthonormal frame $(g(x), g_1(x))$:

$$g'(x) = e^\varphi g(x), \quad g'_1(x) = e^{-\varphi} g_1(x). \quad (2.11)$$

Then the field $g(x)$ would induce a 2-dimensional pseudo-Riemann manifold with a metric tensor $\{g'_{ij}\}$ ($i, j = 0, 1$), which is the same as the Gram matrix corresponding to the system of vectors $(g'(x), g'_1(x))$. A unitary vector field $g(x)$ defined in the Minkowski space wound onto the cylinder $\mathbb{R}^3 \times S^1$ would induce a 4-dimensional pseudo-Riemann manifold. Indeed, take the orthonormal frame (g, g_1, g_2, g_3) derived by hyperbolically rotating the Minkowski space by

the angle $\varphi(x)$ in the plane $(g(x), c)$. Then the Gram matrix g'_{ij} ($i, j = 0, 1, 2, 3$) corresponding to the set of vectors $\{e^\varphi g, e^{-\varphi} g_1, g_2, g_3\}$ would be related to the metric of the pseudo-Riemann manifold. Note, that, since the determinant of the Gram matrix is unity [17, 18], the induced metric preserves the volume. That is, the differential volume element of our manifold is equal to the corresponding volume element of the Minkowski space.

3 The dynamics of the model

As we have already mentioned in Section 1, the dynamics of the velocity field $u(x, \tau)$ of an ideal incompressible fluid on the surface of a cylinder $\mathbb{R}^3 \times S^1$ can be characterised by using the minimal volume principle, i.e., by assuming that the 4-volume of the flow through an arbitrary 3-surface $\Sigma \subset \mathbb{R}^3$ during the time T is minimal under some initial and boundary conditions, namely:

$$\delta \int_0^T \int_\Sigma dV \wedge u(x, \tau) d\tau = 0, \quad (3.1)$$

where dV is the differential volume element of a 3-surface Σ . This is also equivalent to the minimal mass carried by the flow through the measuring surface during a finite time interval.

In a classical approximation, by using the winding projection of the Minkowski space into a cylinder $\mathbb{R}^3 \times S^1$, we can pass from the dynamics defined on a cylinder to the statics in the Minkowski space. Let the global time t be parameterised by the length of the flow line of the vector field c in the Minkowski space corresponding to some regular vector field on the cylinder and let the length of a single turn around the cylinder be h . Let us take in the Minkowski space a set of orthogonal to c Euclidean spaces \mathbb{R}^3 in the Minkowski space. The distance between these spaces is equal to hz , where $z \in \mathbb{Z}$. The projection of this set of spaces into the cylinder is a three-dimensional manifold, which we shall refer to as a global measuring surface. Then we can make a one-to-one correspondence between the dynamical vector field $u(x, \tau)$ and the static vector field $g(x)$, defined in the Minkowski space. Thus, in a classical approximation there exists a correspondence between the minimisation of the 4-volume of the flow $u(x, \tau)$ on the cylinder and the minimisation of the 4-volume of the static flow defined in the Minkowski space by the vector field $g(x)$, namely:

$$\delta \int_0^{x_0} \int_{\Sigma'} dV \wedge g(x) dx_0 = 0, \quad (3.2)$$

where the first basis vector e_0 coincides with the vector c , and the 3-surfaces, Σ' , lie in the Euclidean sub-spaces orthogonal to the vector c . Let $\{(c_i)\} = (c_0, c_1, c_2, c_3)$ be an orthonormal basis in \mathbb{R}^4 such that $c_0 = c$. Let the reference frame bundle be such that each non-singular point of \mathbb{R}^4 has a corresponding non-orthonormal frame $(g_i(x)) = (g_0, g_1, g_2, g_3)$, where $g_0 = g(x), g_1 = c_1, g_2 = c_2, g_3 = c_3$. Let us form

a matrix $\{g_{ij}\}$ of inner products (c_i, g_j) of the basis vectors $\{c_i\}$ and the frame $\{g_i\}$. The absolute value of its determinant, $\det(g_{ij})$, is equal to the volume of the parallelepiped formed by the vectors (g_0, g_1, g_2, g_3) . It is also equal to the scalar product, $(g(x), c)$. On the other hand, the equation $(g(x), c)^2 = |\det G(x)|$ holds for the Gram matrix, $G(x)$, which corresponds to the set of vectors $\{g_i(x)\}$ [21]. Then, according to the principle (3.2), the vector field $g(x)$ satisfies the variational equation

$$\delta \int_{\Omega} (g(x), c) dx^4 = \delta \int_{\Omega} |\det G(x)|^{\frac{1}{2}} dx^4 = 0, \quad (3.3)$$

where dx^4 is the differential volume element of a cylindrical 4-region Ω of the Minkowski space, having the height T . The cylinder's base is a 3-surface Σ with the boundary condition $g(x) = c$. In order to derive the differential equation satisfying the integral variational equation (3.3), we have to find the elementary region of integration, Ω . Let $\Delta\pi$ be an infinitesimal parallelepiped spanned by the vectors $\Delta x_0, \Delta x_1, \Delta x_2, \Delta x_3$, with ω being a tubular neighbourhood with the base spanned by the vectors $\Delta x_1, \Delta x_2, \Delta x_3$. This (vector) tubular neighbourhood is filled in with the vectors $|\Delta x_0|g(x)$ obtained from the flow lines of the vector field $g(x)$ by increasing the natural parameter (the pseudo-Euclidean length) by the amount $|\Delta x_0|$. Then the localisation expression of the equation (3.3) gives [19]:

$$\delta \int_{\Delta\pi} |\det G(x, t)|^{\frac{1}{2}} dx^4 = \delta \text{Vol}\omega = 0. \quad (3.4)$$

Since the field lines of a nonholonomy vector field $g(x)$ are nonparallel even locally, any variation of such a field (i.e. the increase or decrease of its nonholonomicity) would result in a non-vanishing variation of the volume $\text{Vol}\omega$. Conversely, in the case of a holonomy field its variations do not affect the local parallelism, so that the holonomicity of the field $g(x)$ appears to be the necessary condition for the zero variation of $\text{Vol}\omega$. Given a vector field $g(x)$ with an arbitrary absolute value, the sufficient conditions for the vanishing variation of the volume of the tubular neighbourhood ω are the potentiality of this field and the harmonic character of its potential. In terms of differential forms these conditions correspond to a simple differential equation:

$$d \star g(x) = 0, \quad (3.5)$$

where d is the external differential; \star is the Hodge star operator; $g(x) = d\varphi(x)$; and $\varphi(x)$ is an arbitrary continuous and smooth function defined everywhere in the Minkowski space, except for the singularity points (topological features). Substituting the unitary holonomy field $g(x) = k(x)d\varphi(x)$ in (3.5), where $k(x) = 1/|d\varphi(x)|$, we shall find that the unitary vector field $g(x)$ must satisfy the minimum condition for the integral surfaces of the co-vector field dual to $g(x)$. In this case the magnitude of the scalar quantity $\varphi(x)$ will be equal to the hyperbolic angle between the vectors $g(x)$ and c . We

can also note that the potential vector field $g(x) = d\varphi(x)$ represented by the harmonic functions $\varphi(x)$ is the solution to the following variational equation:

$$\delta \int_0^T \int_{\Sigma} \left[\left(\frac{\partial \varphi(x, t)}{\partial t} \right)^2 - \nabla^2 \varphi(x, t) \right] dx^3 dt = 0, \quad (3.6)$$

in which Σ is a region in Euclidean space of the "global" observer; the function $\varphi(x, t)$ is defined in the Minkowski space. Thus, the stationary scalar field $\varphi(x)$ induced by a topological feature in the global space is identical to the Newtonian gravitational potential of a mass point.

We have to bear in mind that the space of a "real" observer is curved, since the line for measuring time and the surface for measuring the flux is defined by the vector field $g(x)$, and not by the field c as in the case of the global observer. Therefore, if we wish to derive a variational equation corresponding to the real observer, we have to define it on the pseudo-Riemann manifold M induced in the Minkowski space by the holonomy field $g(x)$, whose flux is measured through the surfaces orthogonal to its flow lines and whose flow lines serve for measuring time. The metric on M is given by the Gram matrix of four tangent vectors, one of which corresponds to the flow line $x'_0(\phi)$ parameterised by the angular coordinate of the cylindrical manifold, and the three others are tangent to the coordinate lines of the 3-surface $x'_1(r), x'_2(r), x'_3(r)$ parameterised by the Euclidean length. The following variational equation holds for an arbitrary region ΔM of M :

$$\delta \int_{\Delta M} g^2(x') dV = 0 \quad (3.7)$$

(under the given boundary conditions) where dV is the differential volume element of M . Note that the norm of the vector $g(x)$ coincides with the magnitude of the volume-element deformation of the pseudo-Riemann manifold, which allows making the correspondence between our functional and that of the Hilbert-Einstein action.

Returning to the global space, let us consider some properties of the vector field $g(x)$. Let a point in the Minkowski space has a trajectory $X(\tau)$ and velocity \dot{X} . Its dynamics is determined by the variational equation:

$$\delta \int_0^T (g(x), \dot{X}) d\tau = 0. \quad (3.8)$$

The varied here is the trajectory $X(\tau)$ in the Minkowski space where the vector field $g(x)$ is defined and where the absolute time τ plays the role of the evolution parameter. For small time intervals the integral equation (3.8) can be reduced to

$$\delta (g(x), \dot{X}) = 0, \quad (3.9)$$

which is satisfied by the differential equation

$$\ddot{X} = g(X). \quad (3.10)$$

Taking the orthogonal projection $\xi(\tau) = \text{pr}_{\mathbb{R}^3} X(\tau)$ of the trajectory of a given topological feature in Euclidean space of the global observer, as well as the projection $\nabla\varphi(X) = \text{pr}_{\mathbb{R}^3} g(X)$ of the vector field $g(x)$ at the point $X(\tau)$ gives a simple differential equation

$$\ddot{\xi}(\tau) = \nabla\varphi(x), \quad (3.11)$$

which (as in Newtonian mechanics) expresses the fact that the acceleration of a mass point in an external gravitational field does not depend on the mass.

4 Some implications

Let us consider some implications of our model for a real observer in a classical approximation (by the real observer we mean the reference frame of a topological feature). First, we can note that a real observer moving uniformly along a straight line in the Minkowski space cannot detect the “relative vacuum” determined by the vector c and, hence, cannot measure the global time t . By measuring the velocities of topological features (also uniformly moving along straight lines) our observer would find that for gauging space and time one can use an arbitrary unitary vector field c' defined on the Minkowski space. Therefore, the observer would conclude that the notion of spacetime should be relative. It is seen that the real observer can neither detect the unitary vector field $g(x)$ nor its deviations from the vector c . However, it would be possible to measure the gradient of the scalar (gravitational) field and detect the pseudo-Riemann manifold induced by $g(x)$.

Indeed, in order to gauge time and distances in different points of space (with different magnitudes of the scalar field) one has to use the locally orthonormal basis $\{g'_i\}$ defined on the 4-dimensional pseudo-Riemann manifold with its metric tensor $\{g'_{ij}\}$. Thus, for the real observer, the deformations of the pseudo-Euclidean space could be regarded as if induced by the scalar field. Locally, the deformations could be cancelled by properly accelerating the mass point (topological feature), which implies that its trajectory corresponds to a geodesics of the manifold.

We can see that the dynamics of a topological feature in our model is identical to the dynamics of a mass point in the gravitational field. Indeed, the scalar field around a topological feature is spherically symmetric. At distance r from the origin the metric will be $e^{2\varphi} dt^2 - e^{-2\varphi} dr^2$, which corresponds to the metric tensor of the gravitational field of a point mass, given $e^{2\varphi} \approx 1 + 2\varphi$ for small φ . If $\varphi = H\tau$, i.e., hyperbolic angle φ linearly depends from the evolutionary parameter τ , then we can compare the constant H with the cosmological factor.

Let us now consider some quantum properties of our model. Let the absolute value of the vector field c be a continuous function $|c(x)|$ in the Minkowski space. Then the angular velocity of the flow will be:

$$\dot{\phi}(x) = \frac{d\phi(x)}{dt} = \frac{\pi}{h} |c(x)|, \quad (4.1)$$

where the angular function $\phi(x)$ can be identified with the phase action of the gauge potential in the observer space. On the other hand, it is reasonable to associate the angular velocity $X(\tau)$ of the topological feature with the Lagrangian of a point mass in the Minkowski space:

$$\dot{\phi}(X) = \frac{d\phi(X)}{d\tau} = \frac{\pi}{h} L(x). \quad (4.2)$$

Let us consider the random walk process of the topological feature in the cylinder space $\mathbb{R}^3 \times S^1$. Let a probability density function $\rho(x)$ be defined on a line, such that $\rho(x)$,

$$\int_{-\infty}^{+\infty} \rho(x) dx = 1. \quad (4.3)$$

Let us calculate the expectation value for the random variable $e^{i\pi x}$, which arises when a line is compactified into a circle:

$$\begin{aligned} M(e^{i\pi x}) &= \int_{-\infty}^{+\infty} \rho(e^{i\pi x}) dx = \\ &= \int_{-\infty}^{+\infty} e^{i\pi x} \rho(x) dx = p e^{i\pi\alpha}. \end{aligned} \quad (4.4)$$

Here the quantity $p e^{i\pi\alpha}$ can be called the complex probability amplitude. It characterises two parameters of the random variable distribution, namely, the expectation value itself, $e^{i\pi\alpha}$, and the probability density, p , i.e. the magnitude of the expectation value. If $\rho(x) = \delta(x)$, then $M(e^{i\pi x}) = 1 \cdot e^{i\pi\alpha}$. Conversely, if $\rho(x)$ is uniformly distributed along the line then the expectation value is $M(e^{i\pi x}) = 0$. It follows from these considerations that a distribution in \mathbb{R}^3 of a complex probability amplitude is related to random events in the cylinder space $\mathbb{R}^3 \times S^1$.

In order to specify the trajectories $X(\tau)$ in the Minkowski space with an external angular potential $\phi(x)$ we shall use the procedure proposed by Feynman [22]. Let the probabilistic behaviour of the topological feature be described as a Markov random walk in the cylinder space $\mathbb{R}^3 \times S^1$. An elementary event in this space is a free passage. In the Minkowski space such an event is characterised by two random variables, duration, $\Delta\tau$, and the random path vector, ΔX , whose projection into Euclidean space of the absolute observer is $\Delta\xi$. The ratio $\frac{\Delta\xi}{\Delta\tau}$ is a random velocity vector, ξ . On the other hand, the free passage of a topological feature corresponds to an increment in the phase angle $\Delta\phi(X) = \dot{\phi}(X)\Delta\tau$ (phase action) in the cylinder space $\mathbb{R}^3 \times S^1$.

Let the probability distribution of the phase action has an exponential form, say, $\rho(\Delta\phi) = e^{-\Delta\phi}$ (neglecting the normalisation coefficient). Then, the corresponding probability density for the random variable $e^{i\Delta\phi}$ will be

$$\rho(e^{i\Delta\phi}) = e^{-\Delta\phi} e^{i\Delta\phi}. \quad (4.5)$$

Using the properties of a Markov chain [20], we can derive the probability density for an arbitrary number of random

walks:

$$\rho(e^{i\phi}) = \prod_0^T e^{-\dot{\phi}d\tau} e^{i\dot{\phi}d\tau}. \quad (4.6)$$

To get the expectation value of the random variable $e^{i\phi}$ we have to sum up over the all possible trajectories, that is, to calculate the quantity

$$M(e^{i\phi}) = \sum \prod_0^T e^{-\dot{\phi}d\tau} e^{i\dot{\phi}d\tau}. \quad (4.7)$$

It is known that any non-vanishing variation of the phase action has a vanishing amplitude of the transitional probability and, on the contrary, that the vanishing variation corresponds to a non-vanishing probability amplitude [23–25]. Then it is seen that the integral action corresponding to the topological feature must be minimal. It follows that the “probabilistic trap” of a random walk [26] in the cylinder space $\mathbb{R}^3 \times \mathcal{S}^1$ is determined by the variational principle — the same that determines the dynamics of a mass point in classical mechanics.

5 Conclusions

In conclusion, we have made an attempt to describe the dynamics of spacetime (as well as of matter particles) in terms of the vector field defined on a cylindrical manifold and based on the principle of maximum mass carried by the field flow. The analysis of the observational implications of our model sheds new light on the conceptual problems of quantum gravity.

Still many details of our model are left unexplored. For example, it would be instructive to devise the relationship between the vector field $g(x)$ and the 4-potential of electromagnetic field $A(x)$ and to consider the local perturbations of $g(x)$ as gravitons or/and photons. We also expect that the most important properties of our model would be revealed by extending it to the cylindrical manifold $\mathbb{R}^3 \times \mathcal{S}^3$. In particular, we hope that within such an extended version of our framework it would be possible to find a geometric interpretation of all known gauge fields. It is also expected that studying the dynamics of the minimal unit vector field on a 7-sphere should be interesting for cosmological applications of our approach.

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Kaluza-Klein-Carmeli Metric from Quaternion-Clifford Space, Lorentz' Force, and Some Observables

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It was known for quite long time that a quaternion space can be generalized to a Clifford space, and vice versa; but how to find its neat link with more convenient metric form in the General Relativity theory, has not been explored extensively. We begin with a representation of group with non-zero quaternions to derive closed FLRW metric [1], and from there obtains Carmeli metric, which can be extended further to become 5D and 6D metric (which we propose to call Kaluza-Klein-Carmeli metric). Thereafter we discuss some plausible implications of this metric, beyond describing a galaxy's spiraling motion and redshift data as these have been done by Carmeli and Hartnett [4, 5, 6]. In subsequent section we explain Podkletnov's rotating disc experiment. We also note possible implications to quantum gravity. Further observations are of course recommended in order to refute or verify this proposition.

1 Introduction

It was known for quite long time that a quaternion space can be generalized to a Clifford space, and *vice versa*; but how to find its neat link to more convenient metric form in the General Relativity theory, has not been explored extensively [2].

First it is worth to remark here that it is possible to find a flat space representation of quaternion group, using its algebraic isomorphism with the ring division algebra [3, p.3]:

$$E_i E_j = -\delta_{ij} + f_{ijk} E_k. \quad (1)$$

Working for R^{dim} , we get the following metric [3]:

$$ds^2 = dx_\mu dx^\mu, \quad (2)$$

imposing the condition:

$$x_\mu x^\mu = R^2. \quad (3)$$

This rather elementary definition is noted here because it was based on the choice to use the square of the radius to represent the distance (x_μ), meanwhile as Riemann argued long-time ago it can also been represented otherwise as the square of the square of the radius [3a].

Starting with the complex $n = 1$, then we get [3]:

$$q = x_0 + x_1 E_1 + x_2 E_2 + x_3 E_3. \quad (4)$$

With this special choice of x_μ we can introduce the special metric [3]:

$$ds^2 = R^2 (\delta_{ij} \partial \Phi_i \partial \Phi_j). \quad (5)$$

This is apparently most direct link to describe a flat metric from the ring division algebra. In the meantime, it seems very interesting to note that Trifonov has shown that the geometry of the group of nonzero quaternions belongs to closed FLRW metric. [1] As we will show in the subsequent Section, this

approach is more rigorous than (5) in order to describe neat link between quaternion space and FLRW metric.

We begin with a representation of group with non-zero quaternions to derive closed FLRW metric [1], and from there we argue that one can obtain Carmeli 5D metric [4] from this group with non-zero quaternions. The resulting metric can be extended further to become 5D and 6D metric (which we propose to call Kaluza-Klein-Carmeli metric).

Thereafter we discuss some plausible implications of this metric, beyond describing a galaxy's spiraling motion and redshift data as these have been done by Carmeli and Hartnett [4–7]. Possible implications to the Earth geochronometrics and possible link to coral growth data are discussed. In the subsequent Section we explain Podkletnov's rotating disc experiment. We also note a possible near link between Kaluza-Klein-Carmeli and Yefremov's Q-Relativity, and also possible implications to quantum gravity.

The reasons to consider this Carmeli metric instead of the conventional FLRW are as follows:

- One of the most remarkable discovery from WMAP is that it reveals that our Universe seems to obey Euclidean metric (see Carroll's article in *Nature*, 2003);
- In this regards, to explain this observed fact, most arguments (based on General Relativity) seem to agree that in the edge of Universe, the metric will follow Euclidean, because the matter density tends to approaching zero. But such a proposition is of course in contradiction with the basic "assumption" in GTR itself, i.e. that the Universe is *homogenous isotropic* everywhere, meaning that the matter density should be the same too in the edge of the universe. In other words, we need a new metric to describe the *inhomogeneous isotropic* spacetime.

$$g_{\alpha\beta} = \begin{pmatrix} \tau(\eta)\left(\frac{\dot{R}}{R}\right)^2 & 0 & 0 & 0 \\ 0 & -\tau(\eta) & 0 & 0 \\ 0 & 0 & -\tau(\eta)\sin^2(\chi) & 0 \\ 0 & 0 & 0 & -\tau(\eta)\sin^2(\chi)\sin^2(\vartheta) \end{pmatrix}. \tag{6}$$

- Furthermore, from astrophysics one knows that spiral galaxies do not follow Newtonian potential exactly. Some people have invoked MOND or modified (Post-) Newton potential to describe that deviation from Newtonian potential [8, 9]. Carmeli metric is another possible choice [4], and it agrees with spiral galaxies, and also with the redshift data [5–7].
- Meanwhile it is known, that General Relativity is strictly related to Newtonian potential (Poisson’s equation). All of this seems to indicate that General Relativity is only applicable for some limited conditions, but it may not be able to represent the *rotational aspects of gravitational phenomena*. Of course, there were already extensive research in this area of the generalized gravitation theory, for instance by introducing a torsion term, which vanishes in GTR [10].

Therefore, in order to explain spiral galaxies’ rotation curve and corresponding “dark matter”, one can come up with a different route instead of invoking a kind of strange matter. In this regards, one can consider dark matter as a property of the metric of the spacetime, just like the precession of the first planet is a property of the spacetime in General Relativity.

Of course, there are other methods to describe the *inhomogeneous* spacetime, see [15, 16], for instance in [16] a new differential operator was introduced: $\frac{\delta}{\delta\tau} = \frac{1}{H_0} \frac{1}{c} \frac{\delta}{\delta t}$, which seems at first glance as quite similar to Carmeli method. But to our present knowledge Carmeli metric is the most consistent metric corresponding to generalized FLRW (derived from a quaternion group).

Further observations are of course recommended in order to refute or verify this proposition.

2 FLRW metric associated to the group of non-zero quaternions

The quaternion algebra is one of the most important and well-studied objects in mathematics and physics; and it has natural Hermitian form which induces Euclidean metric [1]. Meanwhile, Hermitian symmetry has been considered as a method to generalize the gravitation theory (GTR), see Einstein paper in *Ann. Math.* (1945).

In this regards, Trifonov has obtained that a natural extension of the structure tensors using nonzero quaternion bases will yield formula (6). (See [1, p.4].)

Interestingly, by *assuming* that [1]:

$$\tau(\eta)\left(\frac{\dot{R}}{R}\right)^2 = 1, \tag{7}$$

then equation (6) reduces to closed FLRW metric [1, p.5]. Therefore one can say that closed FLRW metric is neatly associated to the group of nonzero quaternions.

Now consider equation (7), which can be rewritten as:

$$\tau(\eta)(\dot{R})^2 = R^2. \tag{8}$$

Since we choose (8), then the radial distance can be expressed as:

$$dR^2 = dz^2 + dy^2 + dx^2. \tag{9}$$

Therefore we can rewrite equation (8) in terms of (9):

$$\tau(\eta)(d\dot{R})^2 = (dR)^2 = dz^2 + dy^2 + dx^2, \tag{10}$$

and by defining

$$\tau(\eta) = \tau^2 = \frac{1}{H_0^2(\eta)} = \frac{1}{\alpha(H_0^n)^n}. \tag{11}$$

Then we can rewrite equation (10) in the form:

$$\tau(\eta)(d\dot{R})^2 = \tau^2(dv)^2 = dz^2 + dy^2 + dx^2, \tag{12}$$

or

$$-\tau^2(dv)^2 + dz^2 + dy^2 + dx^2 = 0, \tag{13}$$

which is nothing but an original Carmeli metric [4, p.3, equation (4)] and [6, p.1], where H_0 represents Hubble constant (by setting $\alpha = n = 1$, while in [12] it is supposed that $\alpha = 1.2$, $n = 1$). Further extension is obviously possible, where equation (13) can be generalized to include the *(icdt)* component in the conventional Minkowski metric, to become (Kaluza-Klein)-Carmeli 5D metric [5, p.1]:

$$-\tau^2(dv)^2 + dz^2 + dy^2 + dx^2 + (icdt)^2 = 0. \tag{14}$$

Or if we introduce equation (13) in the general relativistic setting [4, 6], then one obtains:

$$ds^2 = \tau^2(dv)^2 - e^\xi \cdot dr^2 - R^2 \cdot (d\vartheta^2 + \sin^2\vartheta \cdot d\phi^2). \tag{15}$$

The solution for (15) is given by [6, p.3]:

$$\frac{dr}{dv} = \tau \cdot \exp\left(-\frac{\xi}{2}\right), \tag{16}$$

which can be written as:

$$\frac{d\dot{r}}{dr} = \frac{dv}{dr} = \tau^{-1} \cdot \exp\left(\frac{\xi}{2}\right). \tag{17}$$

This result implies that there shall be a metric deformation, which may be associated with astrophysics observation, such as the possible AU differences [11, 12].

Furthermore, this proposition seems to correspond neatly to the Expanding Earth hypothesis, because [13]:

“In order for expansion to occur, the moment of inertia constraints must be overcome. An expanding Earth would necessarily rotate more slowly than a smaller diameter planet so that angular momentum would be conserved.” (Q.1)

We will discuss these effects in the subsequent Sections.

We note however, that in the original Carmeli metric, equation (14) can be generalized to include the potentials to be determined, to become [5, p.1]:

$$ds^2 = \left(1 + \frac{\Psi}{\tau^2}\right) \tau^2 (dv)^2 - dr^2 + \left(1 + \frac{\Phi}{c^2}\right) c^2 dt^2, \quad (18)$$

where

$$dr^2 = dz^2 + dy^2 + dx^2. \quad (19)$$

The line element represents a spherically symmetric *inhomogeneous* isotropic universe, and the expansion is a result of the spacevelocity component. In this regard, metric (18) describes *funfbeen* (“five-legs”) similar to the standard Kaluza-Klein metric, for this reason we propose the name Kaluza-Klein-Carmeli for all possible metrics which can be derived or extended from equations (8) and (10).

To observe the expansion at a definite time, the $(icdt)$ term in equation (14) has been ignored; therefore the metric becomes “*phase-space*” Minkowskian. [5, p.1]. (A similar phase-space Minkowskian has been considered in various places, see for instance [16] and [19].) Therefore the metric in (18) reduces to (by taking into consideration the isotropic condition):

$$dr^2 + \left(1 + \frac{\Psi}{\tau^2}\right) \tau^2 (dv)^2 = 0. \quad (20)$$

Alternatively, one can suppose that in reality this assumption may be reasonable by setting $c \rightarrow 0$, such as by considering the metric for the phonon speed c_s instead of the light speed c ; see Volovik, etc. Therefore (18) can be rewritten as:

$$ds_{phonon}^2 = \left(1 + \frac{\Psi}{\tau^2}\right) \tau^2 (dv)^2 - dr^2 + \left(1 + \frac{\Phi}{c_s^2}\right) c_s^2 dt^2. \quad (21)$$

To summarize, in this Section we find out that not only closed FLRW metric is associated to the group of nonzero quaternions [1], but also the same group yields Carmeli metric. In the following Section we discuss some plausible implications of this proposition.

3 Observable A: the Earth geochronometry

One straightforward implication derived from equation (8) is that the ratio between the velocity and the radius is directly proportional, regardless of the scale of the system in question:

$$\left(\frac{\dot{R}}{R}\right)^2 = \tau(\eta)^{-1}, \quad (22)$$

or

$$\left(\frac{R_1}{R_2}\right) = \left(\frac{\dot{R}_1}{\dot{R}_2}\right) = \sqrt{\tau(\eta)}. \quad (23)$$

Therefore, one can say that there is a direct proportionality between the *spacevelocity* expansion of, let say, Virgo galaxy and the Earth geochronometry. Table 1 displays the calculation of the Earth’s radial expansion using the formula represented above [17]:

Therefore, the Earth’s radius increases at the order of ~ 0.166 cm/year, which may correspond to the decreasing angular velocity (Q.1). This number, albeit very minute, may also correspond to the Continental Drift hypothesis of A. Wegener [13, 17]. Nonetheless the reader may note that our calculation was based on Kaluza-Klein-Carmeli’s phase-space *spacevelocity* metric.

Interestingly, there is a quite extensive literature suggesting that our Earth experiences a continuous deceleration rate. For instance, J. Wells [14] described an increasing day-length of the Earth [14]:

“It thus appears that the length of the day has been increasing throughout geological time and that the number of days in the year has been decreasing. At the beginning of the Cambrian the length of the day would have been 21^h.” (Q.2)

Similar remarks have been made, for instance by G. Smoot [13]:

“In order for this to happen, the lunar tides would have to slow down, which would affect the length of the lunar month. ... an Earth year of 447 days at 1.9 Ga decreasing to an Earth year of 383 days at 290 Ma to 365 days at this time. However, the Devonian coral rings show that the *day is increasing by 24 seconds every million years*, which would allow for an expansion rate of about 0.5% for the past 4.5 Ga, all other factors being equal.” (Q.3)

Therefore, one may compare this result (Table 1) with the increasing day-length reported by J. Wells [13].

4 Observable B: the Receding Moon from the Earth

It is known that the Moon is receding from the Earth at a constant rate of ~ 4 cm/year [17, 18].

Using known values: $G = 6.6724 \times 10^{-8}$ cm²/(g · sec²) and $\rho = 5.5 \times 10^6$ g/m³, and the Moon’s velocity ~ 7.9 km/sec, then one can calculate using known formulas:

$$\text{Vol} = \frac{4}{3} \pi \cdot (R + \Delta R)^3, \quad (24)$$

$$M + \Delta M = \text{Vol} \cdot \rho, \quad (25)$$

$$r + \Delta r = \frac{G \cdot (M + \Delta M)}{v^2}, \quad (26)$$

where r , v , M each represents the distance from the Moon to the Earth, the Moon’s orbital velocity, and the Earth’s mass,

Nebula	Radial velocity (mile/s)	Distance (10 ³ kly)	Ratio (10 ⁻⁵ cm/yr)	the Earth dist. (R, km)	Predicted the Earth exp. (ΔR, cm/year)
Virgo	750	39	2.617	6371	0.16678
Ursa Mayor	9300	485	2.610	6371	0.166299
Hydra	38000	2000	2.586	6371	0.164779
Bootes 2	86000	4500	2.601	6371	0.165742
Average			2.604		0.1659

Table 1: Calculation of the radial expansion from the Galaxy velocity/distance ratio. Source: [17].

respectively. Using this formula we obtain a prediction of the Receding Moon at the rate of 0.00497 m/year. This value is around 10% compared to the observed value 4 cm/year.

Therefore one can say that this calculation shall take into consideration other aspects. While perhaps we can use other reasoning to explain this discrepancy between calculation and prediction, for instance using the “conformal brane” method by Pervushin [20], to our best knowledge this effect has neat link with the known paradox in astrophysics, i.e. the observed matter only contributes around ~1–10% of all matter that is supposed to be “there” in the Universe.

An alternative way to explain this discrepancy is that there is another type of force different from the known Newtonian potential, i.e. by taking into consideration the expansion of the “surrounding medium” too. Such a hypothesis was proposed recently in [21]. But we will use here a simple argument long-time ago discussed in [22], i.e. if there is a force other than the gravitational force acting on a body with mass, then it can be determined by this equation [22, p.1054]:

$$\frac{d(mv_0)}{dt} = F + F_{gr}, \tag{27}$$

where v_0 is the velocity of the particle relative to the absolute space [22a]. The gravitational force can be defined as before:

$$F_{gr} = -m \nabla V, \tag{28}$$

where the function V is solution of Poisson’s equation:

$$\nabla^2 V = 4\pi K \mu, \tag{29}$$

and K represents Newtonian gravitational constant. For system which does not obey Poisson’s equation, see [15].

It can be shown, that the apparent gravitational force that is produced by an aether flow is [22]:

$$F_{gr} = m \frac{\partial v}{\partial t} + m \nabla \left(\frac{v^2}{2} \right) - m v_0 \times \nabla \times v + v \frac{dm}{dt}, \tag{30}$$

which is an extended form of Newton law:

$$\vec{F} = \frac{d}{dt} (\vec{m}\vec{v}) = m \left(\frac{d\vec{v}}{dt} \right) + v \left(\frac{d\vec{m}}{dt} \right). \tag{31}$$

If the surrounding medium be equivalent to Newton’s theory, this expression shall reduce to that given in (27). Supposing the aether be irrotational relative to the particular system

of the coordinates, and $m = \text{const}$, then (29) reduces [22]:

$$F_{gr} = -m \left(-\frac{\partial v}{\partial t} - \nabla \left(\frac{v^2}{2} \right) \right), \tag{32}$$

which will be equivalent to equation (27) only if:

$$\nabla V = \frac{\partial v}{\partial t} + \nabla \left(\frac{v^2}{2} \right). \tag{33}$$

Further analysis of this effect to describe the Receding Moon from the Earth will be discussed elsewhere. In this Section, we discuss how the calculated expanding radius can describe (at least partially) the Receding Moon from the Earth. Another possible effect, in particular the deformation of the surrounding medium, shall also be considered.

5 Observable C: Podkletnov’s rotation disc experiment

It has been discussed how gravitational force shall take into consideration the full description of Newton’s law. In this Section, we put forth the known equivalence between Newton’s law (31) and Lorentz’ force [23], which can be written (supposing m to be constant) as follows:

$$\vec{F} = \frac{d}{dt} (\gamma \vec{m}\vec{v}) = \gamma m \left(\frac{d\vec{v}}{dt} \right) = q \left(\vec{E} + \frac{1}{c} \vec{v} \times \vec{B} \right), \tag{34}$$

where the relativistic factor is defined as:

$$\gamma = \pm \sqrt{\frac{1}{1 - \beta^2}}. \tag{35}$$

while we can expand this equation in the cylindrical coordinates [23], we retain the simplest form in this analysis. In accordance with Spohn, we define [24]:

$$E = -\nabla A. \tag{36}$$

$$B = \nabla \times A. \tag{37}$$

For Podkletnov’s experiment [26–28], it is known that there in a superconductor $E = 0$ [25], and by using the mass m in lieu of the charge ratio $\frac{e}{c}$ in the right hand term of (34) called the “gravitational Lorentz force”, we get:

$$m \left(\frac{d\vec{v}}{dt} \right) = \frac{m}{\gamma} (\vec{v} \times \vec{B}) = \frac{1}{\gamma} (\vec{p} \times \vec{B}). \tag{38}$$

Let us suppose we conduct an experiment with the weight $w = 700$ g, the radius $r = 0.2$ m, and it rotates at $f = 2$ cps (cycle per second), then we get the velocity at the edge of the disc as:

$$v = 2\pi \cdot f r = 2.51 \text{ m/sec}, \quad (39)$$

and with known values for $G = 6.67 \times 10^{-11}$, $c \approx 3 \times 10^8$ m/sec, $M_{earth} = 5.98 \times 10^{24}$ kg, $r_{earth} = 3 \times 10^6$ m, then we get:

$$F_{gr} = \frac{G}{c^2 r} M v \approx 3.71 \times 10^{-9} \text{ newton/kgm sec}. \quad (40)$$

Because $B = F/\text{meter}$, then from (39), the force on the disc is given by:

$$F_{disc} = \vec{B}_{earth} \cdot \vec{p}_{disc} \approx B_{earth} \cdot \left(m \frac{c}{\gamma} \right). \quad (41)$$

High-precision muon experiment suggests that its speed can reach around $\sim 0.99c$. Let us suppose in our disc, the particles inside have the speed $0.982c$, then $\gamma^{-1} = 0.1889$. Now inserting this value into (40), yields:

$$F_{disc} = (3.71 \times 10^{-9}) \cdot (0.7) \cdot (3 \times 10^8) \cdot 0.189 = 0.147 \text{ newton} = 14.7 \text{ gr}. \quad (42)$$

Therefore, from the viewpoint of a static observer, the disc will get a mass reduction as large as $\frac{14.7}{700} = 2.13\%$, which seems quite near with Podkletnov's result, i.e. the disc can obtain a mass reduction up to 2% of the static mass.

We remark here that we use a simplified analysis using Lorentz' force, considering the fact that superconductivity may be considered as a relativistic form of the ordinary electromagnetic field [25].

Interestingly, some authors have used different methods to explain this apparently bizarre result. For instance, using Tajmar and deMatos' [29] equation: $\gamma_0 = \frac{a\Omega}{2} = \frac{0.2 \cdot 2}{2} = 0.2$. In other words, it predicts a mass reduction around $\sim \frac{0.2}{9.8} = 2\%$, which is quite similar to Podkletnov's result.

Another way to describe those rotating disc experiments is by using simple Newton law [33]. From equation (31) one has (by setting $F = 0$ and because $g = \frac{dv}{dt}$):

$$\frac{dm}{dt} = -\frac{m}{v} g = -\frac{m}{\omega R} g, \quad (43)$$

Therefore one can expect a mass reduction given by an angular velocity (but we're not very how Podkletnov's experiment can be explained using this equation).

We end this section by noting that we describe the rotating disc experiment by using Lorentz' force in a rotating system. Further extension of this method in particular in the context of the (extended) Q-relativity theory, will be discussed in the subsequent Section.

6 Possible link with Q-Relativity. Extended 9D metric

In the preceding Section, we have discussed how closed FLRW metric is associated to the group with nonzero quaternions, and that Carmeli metric belongs to the group. The only

problem with this description is that it neglects the directions of the velocity other than against the x line.

Therefore, one can generalize further the metric to become [1, p.5]:

$$-\tau^2 (dv_R)^2 + dz^2 + dy^2 + dx^2 = 0, \quad (44)$$

or by considering each component of the velocity vector [23]:

$$(i\tau dv_X)^2 + (i\tau dv_Y)^2 + (i\tau dv_Z)^2 + dz^2 + dy^2 + dx^2 = 0. \quad (45)$$

From this viewpoint one may consider it as a generalization of Minkowski's metric into biquaternion form, using the modified Q-relativity space [30, 31, 32], to become:

$$ds = (dx_k + i\tau dv_k) q_k. \quad (46)$$

Please note here that we keep using definition of Yefremov's quaternion relativity (Q-relativity) physics [30], albeit we introduce dv instead of dt in the right term. We propose to call this metric *quaternionic Kaluza-Klein-Carmeli metric*.

One possible further step for the generalization this equation, is by keep using the standard Q-relativistic dt term, to become:

$$ds = (dx_k + ic dt_k + i\tau dv_k) q_k, \quad (47)$$

which yields 9-Dimensional extension to the above quaternionic Kaluza-Klein-Carmeli metric. In other words, this generalized 9D KK-Carmeli metric is seemingly capable to bring the most salient features in both the standard Carmeli metric and also Q-relativity metric. Its prediction includes plausible time-evolution of some known celestial motion in the solar system, including but not limited to the Earth-based satellites (albeit very minute). It can be compared for instance using Arbab's calculation, that the Earth accelerates at rate 3.05 arcsec/cy^2 , and Mars at 1.6 arcsec/cy^2 [12]. Detailed calculation will be discussed elsewhere.

We note here that there is quaternionic multiplication rule which acquires the compact form [30–32]:

$$1q_k = q_k 1 = q_k, \quad q_j q_k = -\delta_{jk} + \varepsilon_{jkn} q_n, \quad (48)$$

where δ_{kn} and ε_{jkn} represent 3-dimensional symbols of Kronecker and Levi-Civita, respectively [30]. It may also be worth noting here that in 3D space Q-connectivity has clear geometrical and physical treatment as movable Q-basis with behavior of Cartan 3-frame [30].

In accordance with the standard Q-relativity [30, 31], it is also possible to write the dynamics equations of Classical Mechanics for an inertial observer in the constant Q-basis, as follows:

$$m \frac{d^2}{dt^2} (x_k q_k) = F_k q_k. \quad (49)$$

Because of the antisymmetry of the connection (the generalized angular velocity), the dynamics equations can be written in vector components, by the conventional vector no-

tation [30, 32]:

$$m (\vec{a} + 2\vec{\Omega} \times \vec{v} + \vec{\Omega} \times \vec{r} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r})) = \vec{F}, \quad (50)$$

which represents known types of classical acceleration, i.e. the linear, the Coriolis, the angular, and the centripetal acceleration, respectively.

Interestingly, as before we can use the equivalence between the inertial force and Lorentz' force (34), therefore equation (50) becomes:

$$m \left(\frac{d\vec{v}}{dt} + 2\vec{\Omega} \times \vec{v} + \vec{\Omega} \times \vec{r} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) \right) = q_{\otimes} \left(\vec{E} + \frac{1}{c} \vec{v} \times \vec{B} \right), \quad (51)$$

or

$$\left(\frac{d\vec{v}}{dt} \right) = \frac{q_{\otimes}}{m} \left(\vec{E} + \frac{1}{c} \vec{v} \times \vec{B} \right) - \frac{2\vec{\Omega} \times \vec{v} + \vec{\Omega} \times \vec{r} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r})}{m}. \quad (52)$$

Please note that the variable q here denotes electric charge, not quaternion number.

Therefore, it is likely that one can expect a new effects other than Podkletnov's rotating disc experiment as discussed in the preceding Section.

Further interesting things may be expected, by using (34):

$$\vec{F} = m \left(\frac{d\vec{v}}{dt} \right) = q \left(\vec{E} + \frac{1}{c} \vec{v} \times \vec{B} \right) \Rightarrow \Rightarrow m (d\vec{v}) = q \left(\vec{E} + \frac{1}{c} \vec{v} \times \vec{B} \right) dt. \quad (53)$$

Therefore, by introducing this Lorentz' force instead of the velocity into (44), one gets directly a plausible extension of Q-relativity:

$$ds = \left[dx_k + i\tau \frac{q}{m} \left(\vec{E}_k + \frac{1}{c} \vec{v}_k \times \vec{B}_k \right) dt_k \right] q_k. \quad (54)$$

This equation seems to indicate how a magnetic worm-hole can be induced in 6D Q-relativity setting [16, 19]. The reason to introduce this proposition is because there is known link between magnetic field and rotation [34]. Nonetheless further experiments are recommended in order to refute or verify this proposition.

7 Possible link with quantum gravity

In this Section, we remark that the above procedure to derive the closed FLRW-Carmeli metric from the group with nonzero quaternions has an obvious advantage, i.e. one can find Quantum Mechanics directly from the quaternion framework [35]. In other words, one can expect to put the gravitational metrical (FLRW) setting and the Quantum Mechanics setting in equal footing. After all, this may be just a goal sought in "quantum gravity" theories. See [4a] for discussion

on the plausible quantization of a gravitational field, which may have observable effects for instance in the search of extrasolar planets [35a].

Furthermore, considering the "phonon metric" described in (20), provided that it corresponds to the observed facts, in particular with regards to the "surrounding medium" vortices described by (26–29), one can say that the "surrounding medium" is comprised of the phonon medium. This proposition may also be related to the superfluid-interior of the Sun, which may affect the Earth climatic changes [35b]. Therefore one can hypothesize that the signatures of quantum gravity, in the sense of the quantization in gravitational large-scale phenomena, are possible because the presence of the phonon medium. Nonetheless, further theoretical works and observations are recommended to explore this new proposition.

8 Concluding remarks

In the present paper we begun with a representation of a group with non-zero quaternions to derive closed FLRW metric [1], and we obtained Carmeli 5D metric [4] from this group. The resulting metric can be extended further to become 5D and 6D metric (called by us *Kaluza-Klein-Carmeli metric*).

Thereafter we discussed some plausible implications of this metric. Possible implications to the Earth geochronometrics and possible link to the coral growth data were discussed. In subsequent Section we explained Podkletnov's rotating disc experiment. We also noted possible neat link between Kaluza-Klein-Carmeli metric and Yefremov's Q-Relativity, in particular we proposed a further extension of Q-relativity to become 9D metric. Possible implications to quantum gravity, i.e. possible observation of the quantization effects in gravitation phenomena was also noted.

Nonetheless we do not pretend to have the last word on some issues, including quantum gravity, the structure of the aether (phonon) medium, and other calculations which remain open. There are also different methods to describe the Receding Moon or Podkletnov's experiments. What this paper attempts to do is to derive some known gravitational phenomena, including Hubble's constant, in a simplest way as possible, without invoking a strange form of matter. Furthermore, the Earth geochronometry data may enable us to verify the cosmological theories with unprecedented precision.

Therefore, it is recommended to conduct further observations in order to verify and also to explore the implications of our propositions as described herein.

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The Palindrome Effect

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As initially experimental material of this paper serves sets of histograms built on the base of short samples which provided the daily time series of the α -decay rate fluctuations and the p-n junction current fluctuations. Investigations of the histograms similarity revealed the palindrome effect, which is: two sets of histograms built on the base of two consecutive 12-hours time series are most similar if one set of the histograms is rearranged in inverse order, and the start time of the series is exact six hours later the local noon.

1 Introduction

As was shown in our previous works, the similarity of histograms built on the base of short samples of the time series of fluctuations measured on the processes of different nature, changes the regularly with time. These changes can be characterized by different periods equal the solar (1440 min) and sidereal (1436 min) days, several near 27-day periods, and yearly periods [1–5]. At different geographical locations the shapes of the histograms are similar to each other with high probability for the coincident moments of the local time [6]. Also it was found the dependence of the histogram patterns on the spatial directions of outgoing α -particles [5] and the motion specific to the measurement system [7]. Aforementioned phenomena led us to an idea that the histogram patterns can be dependent on also the sign of the projection obtained from the velocity vector of the measurement system projected onto the Earth's orbital velocity vector. As was found, this supposition is true.

2 The method

A raw experimental data we used for this paper were sets of the histograms built on the base of short samples which provided the daily time series of ^{239}Pu α -decay rate fluctuations and the p-n junction current fluctuations. The experimental data processing and histogram sets analyzing are given in details in [1, 2].

We use the daily time series of fluctuations in the study. Every time series started six hours later the local noon. After the data acquisition, we divided the 24-hours record into two 12-hours ones. On the base of these two consecutive 12-hours time series two sets of histograms (so-called “direct sets”) were obtained for further analysis. The sign of the measurement system's velocity projected onto the Earth's orbital velocity is positive for one set, while the sign is negative for the other. Proceeding from the direct sets, by rearranging in inverse order, we obtained two “inverse” sets of histograms.

The histograms themselves were built on the base of the 60 of 1-sec measurements. So, one histogram durations was 1 min, while the 12-hours time series we used in the present work formed the sets consisting of 720 such histograms. The similarity of the histogram was studied for couplets (“direct-direct” and “direct-inverse”) along the 720-histogram sets. Here we present the results in the form of interval distribution: the number of similar pairs of the histograms is present as a function on the time interval between them.

3 Experimental results

Fig. 1 shows the interval distributions for two couplets of the sets built on the base of the daily time series of ^{239}Pu α -decay rate fluctuations, obtained on April 23, 2004. The left diagram, Fig. 1a, shows the interval distribution for the “direct-inverse” histogram sets. From the right side of the diagram, we get the “direct-direct” histogram sets.

A peak shown in Fig. 1a means that the histograms with the coincident numbers in the “direct-inverse” sets are similar with very high probability. These sets of similar histograms constitute about 20% from the total number (720) of the pairs. In contrast to the “direct-inverse” sets, the interval distributions in the “direct-direct” histogram sets (Fig. 1b) achieve only 5% of the total number of the pairs for the same zero interval.

We call the *palindrome effect*^{*} such a phenomenon, where two sets of the histograms built on the base of two consecutive 12-hours time series are most similar in the case where one of the sets is rearranged in inverse order, while the daily record starts six hours later the local noon.

The palindrome effect doesn't depend from the annual motion of the Earth. This effect is actually the same for all the seasons. This statement is illustrated by Fig. 2, where the palindrome effect is displayed for the measurements carried out on the autumnal equinox, September 22–23, 2005.

^{*}This comes from the Greek word $\pi\alpha\lambda\iota\nu\delta\rho\mu\omicron\varsigma$, which means *there and back*.

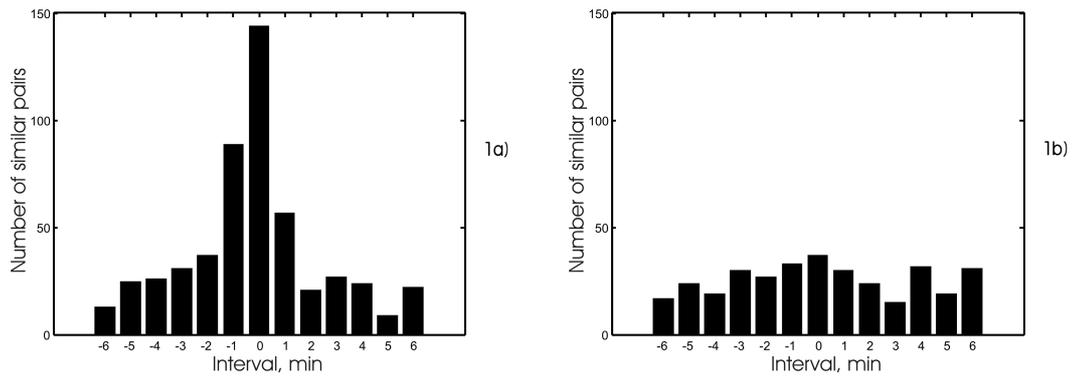


Fig. 1: The palindrome effect in the daily time series of the ^{239}Pu α -decay rate fluctuations, registered on April 23, 2004. The interval distribution for the “direct-inverse” histogram sets are shown in Fig. 1a, while those for the “direct-direct” histograms sets are shown in Fig. 1b.

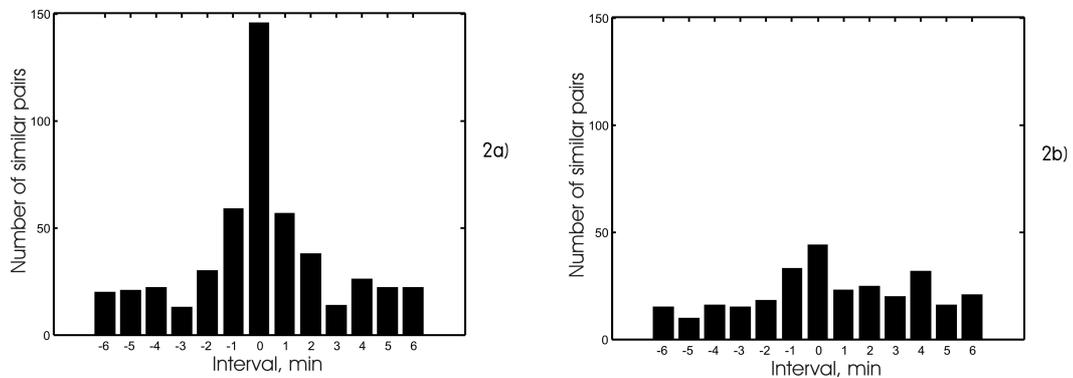


Fig. 2: The palindrome effect in the daily time series of the ^{239}Pu α -decay rate fluctuations, registered on the autumnal equinox, September 22-23, 2005. The interval distribution for the “direct-inverse” histogram sets are shown in Fig. 1a, while those for the “direct-direct” histogram sets are shown in Fig. 2b.

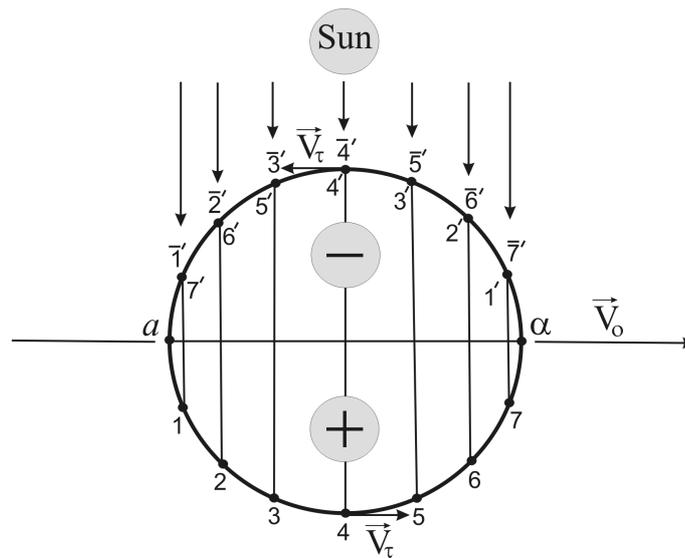


Fig. 3: The palindrome effect.

As easy to see, Fig. 2a and Fig. 2b are similar to Fig. 1a and Fig. 1b respectively. Similarly to Fig. 1 and Fig. 2, the interval distribution was obtained also for the winter and summer solstice.

The aforementioned results mean that, for different locations of the Earth in its circumsolar orbit, we have the same appearance of the palindrome effect.

4 Discussions

It is important to note that the 12-hours time series used in the present work were measured in such a way that the projection of the tangential velocity vector V_τ (Fig. 3) of the measurement system (which is due to the rotatory motion of the Earth) onto the vector of the orbital velocity of the Earth V_o has the same sign. So, two moments of time or, in another word, two singular points a and α exist in the 24-hours daily circle where the sign of the projection changes. The sign of the projection is showed in Fig. 3 by gray circles. The palindrome effect can be observed, if the 12-hours time series start exact at the moments a and α . For the aforementioned results, these moments are determined within a 1-min accuracy by zero peaks shown in Fig. 1–2.

A special investigation on the time series measured within the 20-min neighborhood of the a and α moments was carry out with use of a semiconductor source of fluctuations (fluctuations of p-n junction current). The interval distribution obtained on the base of two sets of the 2-sec histograms constructed from this time series showed these moments to within the 2-sec accuracy. If we get a symmetric shift of the start-point of the time series relative to the a and α points, we find that the peak on the interval distribution (like those shown in Fig. 1–2) has the same time shift relative to zero interval.

The importance of two singular points a and α for the palindrome effect leads us to an idea about the significance of the tangential velocity vector V_τ and its projection onto the vector V_o . If consider the numerical value of the projection, we see that the set $1'-7'$ is symmetric to the $7-1$. In such a case the interval distributions (a) and (b) in Fig. 1–2 should be the same. Because they are different in real, just given supposition is incorrect. We also can consider our measurement system as oriented. In this case the 1 and $1'$ histograms should be the same. This means that zero peaks should be located in the “direct-direct” interval distributions, and be absent on the “direct-inverse” one. As seen in Fig. 1–2, this is not true.

On the other hand, it is possible to formulate a supposition which is qualitatively agreed with the obtained experimental results. This supposition is as follows. There is an external influence unshielded by the Earth, and this influence is orthogonal to V_o . In such a case the inversion of one set of the histograms is understood, and leads to the interval distributions like those of Fig. 1-2. As easy to see, in such an inversion rearrange order of the histograms, the histograms whose location is the same orthogonal line have the same numbers.

This is because we have zero-peak in the “direct-inverse” interval distribution.

The origin of such lines can be the Sun. The only problem in this case is the orbital motion of the Earth. We cannot be located in the same line after 24-hours. As probable, we should suppose that this structure of the lines, which are orthogonal to V_o , moves together with the Earth.

Now we continue this bulky research on the palindrome effect. Detailed description of new results and the verifications to the aforementioned suppositions will be subjected in forthcoming publications.

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On the Second-Order Splitting of the Local-Time Peak

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The paper presents experimental investigations of a local-time peak splitting right up to a second-order splitting. The splitting pattern found in the experiments has a fractal structure. A hypothesis about the possibility of high order splitting is proposed. The obtained experimental result leads to a supposition that the real space possesses a fractal structure.

1 Introduction

The main subject of this paper is a local-time effect, which is one of manifestations of the phenomenon of macroscopic fluctuations. The essence of this phenomenon is that the pattern (shape) of histograms, which are built on the base of short samples of the time series of the fluctuations measured in the processes of different nature, are non-random. Many-years of investigations of such histograms carried out by the method of macroscopic fluctuations [1] revealed a variety of phenomena [2–4]. The most important among the phenomena is the local-time effect [5–8].

The local-time effect consists of the high probability of the similarity of the histogram pairs, which are divided by a time interval equal to the local-time difference between the points of measurement. This effect was registered in the scale of distances from the maximal distance between the locations of measurement which are possible on the Earth’s surface (about 15,000 km) to the distances short as 1 meter. Besides, this effect can be observed on the processes of very different nature [2–4].

The idea of a typical local-time experiment is illustrated by Fig. 1. There in the picture we have two spaced sources of fluctuations 1 and 2, which are fixed on the distance L between them, and synchronously moved with a velocity V in such a way that the line which connects 1 and 2 is parallel to the vector of the measurement system’s velocity V . In this case, after a time duration Δt_0

$$\Delta t_0 = \frac{L}{V}, \tag{1}$$

the source of fluctuations 1 appear in the same position that the source 2 was before. In Fig. 1 these new places are presented as $1'$ and $2'$. According to the local-time effect, coincident spatial positions cause similar histograms patterns. In the interval distribution built on the base of the measurements carried out by the system displayed in Fig. 1 (the number of similar pairs of the histograms as a function of the time interval between them), a single peak in the interval Δt_0 is observed.

In our previous works [6-8], we showed that there within

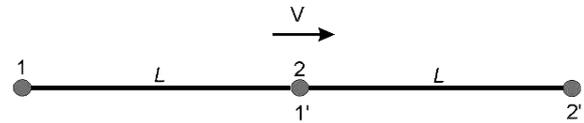


Fig. 1: This diagram illustrates the appearance of the local-time effect.

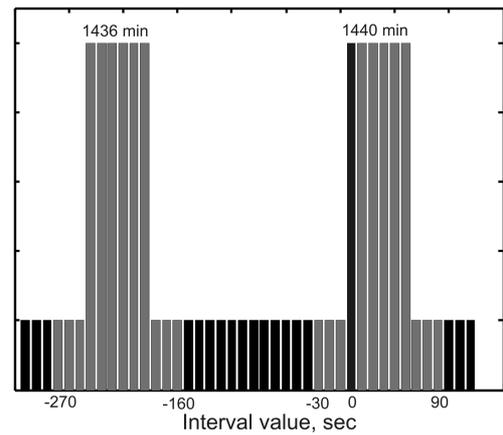


Fig. 2: The sketch of the solar (1440 min) and stellar (1436 min) splitting of the daily period.

the time resolution enhancement (with use histograms, shortest in time) the local-time peak splits onto two sub-peaks. It was found that the ratio between the splitting Δt_1 and the local-time value Δt_0 is $k = 2.78 \times 10^{-3}$. This numerical value is equal, with high accuracy, to the ratio between the daily period splitting 240 sec and the daily period value $T = 86400$ sec [7, 8]. This equality means that the local-time effect and the daily period are originated in the same phenomenon. From this viewpoint, the daily period can be considered as the maximum value of the local-time effect, which can be observed on the Earth.

In our recent work [8], we suggested that the sub-peaks of the local-time peak can also be split with resolution enhancement, and, in general, we can expect an n -order splitting with the value Δt_n

$$\Delta t_n = k^n \Delta t_0, \quad n = 1, 2, 3, \dots \tag{2}$$

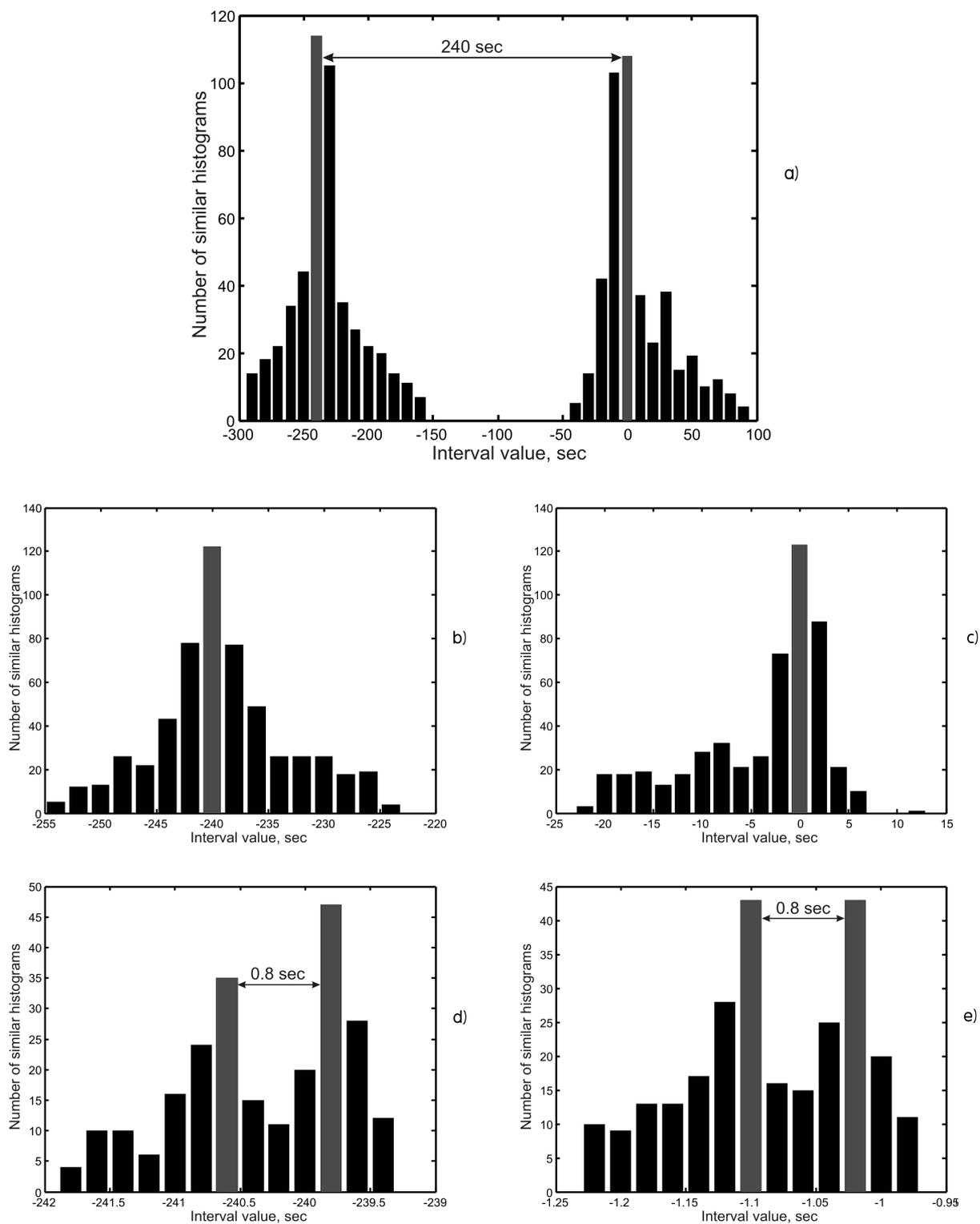


Fig. 3: The interval distributions for the 10-sec histograms (a), 2-sec histograms (b)–(c), and 0.2-sec histograms (d)–(e).

Preliminary results obtained in [8] verified this suggestion in part. The present work provides further investigation on the second-order splitting of the local-time peak.

As easy to see, from (2), every subsequent value of the local-time peak splitting Δt_n needs more than two orders of resolution enhancement. Therefore, most easy way to study Δt_n is to use the maximum value of Δt_0 . Such a value, as stated above, is the daily period $\Delta t_0 = 86400$ sec.

2 Experimental results

To study the second-order splitting of the daily period, we use the known positions of the “solar peak” (1440 min) and the “sidereal peak” (1436 min), which are the first-order splittings of the daily period. The peaks are schematically displayed in Fig. 2. To find the position of the second-order splitting peaks, we used the method of consecutive refinements of the positions of the solar and sidereal peaks. The peaks displayed in Fig. 2 are determined with one-minute accuracy. Since the positions of the solar and stellar peaks are well-known, we can study its closest neighborhood by shortest (to one minute) histograms. In Fig. 2, such a neighborhood is displayed by gray bars (they mean 10-sec histograms). After obtaining the intervals distribution for the 10-sec histograms, the procedure was repeated, while the rôle of the 1-min histograms was played by the 10-sec histograms, and those were substituted for the 2-sec histograms. After this, the procedure was on the 2-sec and 0.2-sec histograms.

Zero interval (Fig. 2) marked by black colour corresponds to the start-point of the records. We used two records, started in the neighboring days at the same moments of the local time. So, the same numbers of histograms were divided by the time interval equal to the duration of solar day: 86400 sec. The interval values shown in Fig. 2 are given relative to zero interval minus 86400 sec.

The time series of the fluctuations in a semiconductor diode were registered on November 2–4, 2007. Each of the measurement consisted of two records with a length of 50000 and 19200000 points measured with the sampling rate 5 Hz and 8 kHz. On the base of these time series, we built the sets of the 10-sec, 2-sec, and 0.2-sec histograms. We used these sets in our further analysis.

The 10-second set of histograms was built on the base of the records, obtained with the sampling rate 5 Hz. Each 10-sec histogram was built from 50 points samples of the time series of fluctuation. The 2-sec and 0.2-sec sets were built on the base of the 50-point samples of the 25 Hz and 250 Hz time series (they were recount from the 8 kHz series).

It is important to note, that the solar day duration is not equal exactly to 86400 sec, but oscillates along the year. Such oscillations are described by the time equation [8]. To provide our measurements, we choose the dates when the time equation has extrema. Due to this fact, the day duration for all the measurements can be considered as the same, and we can

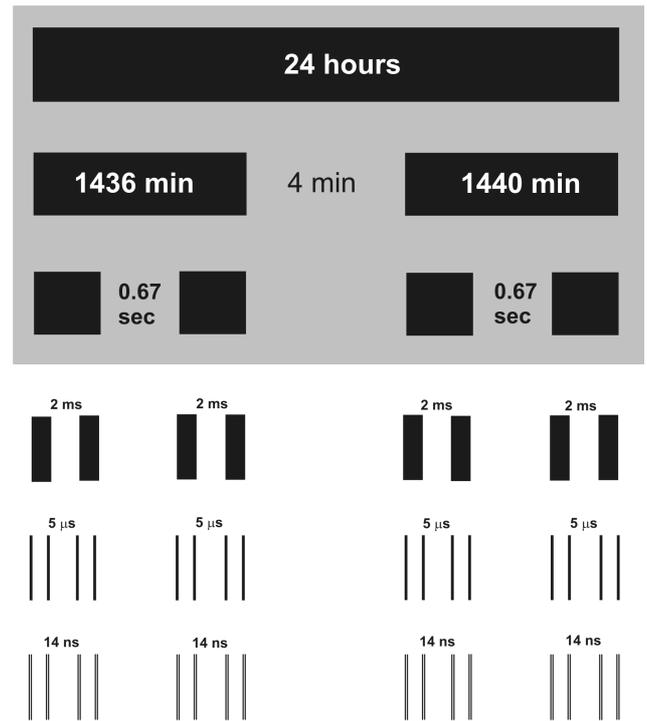


Fig. 4: The daily period splitting. Gray color marks the experimentally found splittings. The splitting displayed below were calculated on the base of the formula (2) for $n = 3 \dots 5$.

average the interval distributions obtained on the base of the time series measured on November 2–4, 2007.

The interval distributions obtained after the comparison of the histograms are given in Fig. 3. The upper graph, Fig. 3a, displays the interval distribution for the 10-sec histograms. As follows from Fig. 3a, the interval distribution in the neighborhood of the 1-min peaks consists of two sharp peaks (displayed by gray bars) which are separated by a time interval of 240 ± 10 sec giving the positions of the solar and sidereal peaks with 10-sec accuracy.

The interval distributions in the neighborhood of the 10-sec peaks (Fig. 3a), for the 2-sec histograms, are displayed in Fig. 3b–3c. Gray bars in Fig. 3b–3c correspond to the new positions of the solar and sidereal peaks with 2-sec accuracy. Considering the neighborhood of the 2-sec peaks (Fig. 3b–3c) to the 0.2-sec histograms, we obtain the interval distributions displayed in Fig. 3d–3e. As easy to see, instead of the more precise position of the 2-sec solar and sidereal peaks, we obtain the splitting of the aforementioned peaks onto two couplets of new distinct peaks. So, from Fig. 3d–3e, we state the second-order splitting of the daily period.

3 Discussion

On the base of the formula (2), for $n = 2$ with use of $\Delta t_0 = 86400$ sec and $k = 2.78 \times 10^{-3}$ for the second-order splitting Δt_2 , we get the value $\Delta t_2 = 0.67$ sec. From the ex-

perimentally obtained interval distribution (Fig. 3d–3e), we have $\Delta t_2 = 0.8 \pm 0.2$ sec. So, the experimental value agrees with the theoretical estimations made on the base of the formula (2).

Such an agreement leads us to a supposition that there is a high-order splitting, which can be obtained from the formula (2). In Fig. 4, we marked by gray colour the experimentally found splitting. The splitting displayed below was calculated on the base of (2) for $n = 3 \dots 5$, $\Delta t_0 = 86400$ sec, and $k = 2.78 \times 10^{-3}$. This splitting will be a subject of our further studies, and only this splitting is accessed to be studied now. For $n > 5$, we will need to get measurements with a sampling rate of about 7.5 THz. Such a sampling rate is out of the technical possibilities for now.

As was stated in Introduction, the local-time effect exists in the scale from the maximal distances, which are possible on the Earth's surface, to the distances close to one meter. Besides, the local-time effect doesn't depend on the origin (nature) of the fluctuating process. In the case, where the spatial basis of the measurements is about one meter, the time required for obtaining of the long-length time series (that is sufficient for further analysis) is about 0.5 sec. Any external influences of geophysical origin, which affect the sources of the fluctuations synchronously, cannot be meant a source of the experimentally obtained results. Only the change we have is the changing of the spatial position due to the motion of whole system with a velocity V originated in the rotatory motion of the Earth (see formula 1). From this, we can conclude that the local-time effect originates in the heterogeneity of the space itself. The results presented in Fig. 3 lead us to a supposition that such an heterogeneity has a fractal structure.

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LETTERS TO PROGRESS IN PHYSICS

On the Explanation of the Physical Cause of the Shnoll Characteristic Histograms and Observed Fluctuations

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Interpretations are given herein regarding the very visionary and important Pu-239 histogram work of Shnoll, and calling attention to background research which was not fully described in that paper. In particular, this Letter gives results of our theoretical and experimental research of gravitational anomalies during total solar eclipses and planetary line-up, and compares interpretations of the data with the work of Shnoll.

I am writing this Letter-to-the-Editor in reference to the very luminary paper authored by S. Shnoll [1], published in this journal, because of the far-reaching impact of the implications of this paper in describing nature, and because I have corresponded scientifically with the author on the subject of his repeat-pattern histogram work [2, 3] since 2001 when I first conveyed to Shnoll that his very meritorious radioactive decay findings of periodicities was an element of a larger and more ubiquitous external-particle net-transfer-of-momenta model and theory in which the origin of gravity due to collision-induced phenomena, was the initial cornerstone [4]. At that time Shnoll reported that the cause of the periodicities in his radioactive decay histograms was unknown but must be due to “profound cosmophysical phenomena” [2, 3]. The cited references within [1] do not convey the full background of the work leading to that paper [1] which Shnoll refers to as a survey, but in my opinion is far beyond a simple review-of-the-literature paper, and is instead a very significant archival work. Additionally, within the text itself [1] there are no references to the private communications References 40 and 41 cited in the list of references in [1]. For these above reasons, I wish to clarify various elements of the paper [1].

I also advised Shnoll in 2003 and 2004 to search out his earlier Pu-239 alpha decay data that were taken at the time of a total solar eclipse [5], doing so because I was impressed with his work during 2003 on characteristic histograms during the New Moon, observed simultaneously independent of location and latitude [6]. As stated, although my work, and that of colleague, Frank Lucatelli, is referenced as private communications in Shnoll’s paper [1], as Refs. 40 and 41, those references are not cited in the text, but instead only in the bibliography, and thus most readers would be unaware of our input into Shnoll’s paper of [1]. I also conveyed to Dr. Shnoll our own work whereby at my request, colleagues had measured a dip in the radioactive decay of Co-60 in southeastern Kansas, and in Po-210 in the Boston area at the time of the total solar eclipse of 4 December, 2002, when the “umbra” passed closest to the isotope sources [7]. We predicted that

this effect would be observed based on the data of Allais [8], and of Saxl and Allen [9] showing decreases in gravity associated with the eclipses of 1954 and 1959, and the eclipse of 1970, respectively, and also based on the dip in gravity which I observed using a dual Newton-cradle pendula system during the planetary line-up of Earth-Sun-Jupiter’s/magnetosphere-Saturn on 18 May 2001 [10]. This prediction was based on my postulate that if gravity were a result of external particle impingement on mass particles, then the other three axiomatic “forces” should also depend upon, or be influenced by the external particle flux.

In this Letter-to-the-Editor, I wish to address points regarding Dr. Shnoll’s interpretation of his decades of data, and of the data of others.

Shnoll has conducted very excellent collimator studies, which showed that when the collimator was pointed north toward the pole star, the near-daily-periods in the repeat histogram patterns of Pu-239 decay were *not* observed, contrasting the data showing repeat histograms when the collimators were oriented east, and when they were oriented west. Shnoll interprets these data stating that . . . “Such a dependence, in its turn, implies a sharp anisotropy of space.” I suggest that a better and more correct manner to interpret these data is in terms of the Earth-Moon-Sun system, spinning and orbiting in the east-west ecliptic plane interrupting, through capture and/or scattering, elementary particles (probably neutrinos) that would otherwise impinge upon the radioactive source and perturb the weak interaction in unstable nuclei. This is not a proof of heterogeneity and anisotropy of space time in a general sense, but indication of celestial body orbits that exist in the general plane of the ecliptic — the external particles being omnidirectional, and the heterogeneity arising generally from supernovae explosions and their consequences. Shnoll earlier in the paper rightfully states, referring to daily, monthly, and yearly periods in repeat forms of the histograms, that “All these periods imply the dependence of the obtained histogram pattern on two factors of rotation — (1) rotation of the Earth around its axis, and (2) move-

ment of the Earth along its circumsolar orbit”, thus supporting the above explanation. Shnoll alludes to this explanation by stating that a heterogeneity in the gravitational field results from the existence of “mass thicknesses” of celestial bodies, and this then must relate to the capture cross-section between nucleons of the mass bodies. Stanley [11] has described in detail the properties of mass that relate to gravity, and treated mathematically the flux of externally impinging neutrinos [11] as related to gravitational interactions. Shnoll invokes a “wave interference” and relates it to a gravitational effect (which associates with our use of interruption and capture, but in our case the phenomenon is particle-based rather than wave-based).

In Section 10 of [1], the author describes the observations of characteristic histogram patterns for the occurrence of the New Moon, and the total solar eclipse. The author writes that the specific patterns do not “depend on position on the Earth’s surface where the Moon’s shadow falls during the eclipse or the New Moon.” We have found, however, that the decrease in gravity signature during a total solar eclipse *does* depend upon the latitude of the location of totality and of the measurements [12], and this is clearly proven in comparing the different data signatures during eclipses in different locations, most notably the work of Wang et al. [13, 14] during the eclipse of March 1997 in China. The work of Stanley and Vezzoli [12] has been able to mathematically describe from first principles the detailed gravimeter data of Wang et al. [13, 14] for the above eclipse, including the parabolic dips in gravity at first contact, and at last contact. The dependence upon latitude of the location of the measurements and of totality is due to the elastic scattering properties of the three-body problem. Shnoll then interprets the overall data in association with the fractality of space-time — a conclusion that we have also reached in our gravity research [11, 15] and that is also described very recently by Loll [16]. Shnoll notes that he also observes a chirality in histograms, which we have shown is fundamental in the nature of materials and the aggregation of mass to form compounds [17].

It is interesting to note that in [1], Shnoll concludes that there is a spatial heterogeneity on the scale of 10^{-13} cm. This is the value that we calculate for the inter-neutrino spacing of the neutrino flux, corresponding to a collision cross-section with nucleons of $\sim 10^{-38}$ cm², and a particle density $3.7 \times 10^{28} - 10^{34}$ particles per cm³.

Our work, and our interpretation of the Shnoll work [1–3], and many other works by Shnoll, correlates very well with the positron annihilation work of Vikin [19] showing that the production of positronium from Na-22 undergoes a maximum near the time of the New Moon, and a minimum near the time of the Full Moon. At the time of the New Moon, the Earth laboratory (whether measurements are of gravitational interactions or of radioactive decay phenomena) faces in the general direction of the line of the Moon and the Sun for a short period of the day, and then rotates such that the labo-

ratory faces free and open space and distant stars during the duration of the day, so that a large complement of neutrinos falls uninterrupted onto the measuring device; also neutrinos that are emitted by the Sun may be scattered by the Moon to affect the data. During the Full Moon, however, the Earth laboratory is always between the Moon and the Sun, and hence the overall collision physics is considerably different.

Shnoll sums the interpretation of the work that he describes within [1] by stating “Taken together, all these facts can mean that we deal with narrowly directed wave fluxes”, which he refers to as beams that are more narrow than the aperture of the collimators of the apparatus (0.9 mm). Our model and theory of gravity [11] is based on a flux of particles, and the “narrow beam” is interpreted due to very low-angle elastic scattering of external particles by the nucleons of the celestial bodies [11, 12], particularly the Moon (near body in [12]) and Sun (far body), such that some particles never reach the detecting apparatus such as pendula, gravimeter, or radioactive source-detector system.

Fundamental to Shnoll’s work is his assertion that these periodic characteristic histograms relate to a wide variety of phenomena ranging from bio-chemical phenomena, to the noise in a gravitational antenna, to alpha decay. This is in agreement with my own work and that of others, and I have found that anomalies in gravity, radioactive decay of Po-210 (and Co-60), and changes in plant growth, orientation, and physiology, as well as embryonic centriole-centriole separation phenomena, and even DNA and its sheathing H₂O, are affected by the Earth-Moon-Sun relationship [10–12, 14, 17, 19, 20]. It has been shown by Gershteyn et al. [21] that the value of G varies at least 0.054% with the orientation of the torsion pendula masses with the stars, and that G is periodic over the sidereal year [21] — this periodicity arguing for a strong link between the Shnoll radioactive decay data and gravity. Furthermore the Shnoll work [1] cites the possibility of a space-time anisotropy in a preferential direction, and refers to the drift of the solar system toward the constellation Hercules. Our theoretical work in collision-induced gravity shows that G is a function of collision cross-section of the neutrino-nucleon interaction [11], and experimental work indicates that G is a function also of at least temperature, phase, and shape [10, 22]. Our very recent experimental work determined that $G = 6.692 \times 10^{-11}$ cubic meters per kg sec² [15] which compares very favorably with the slightly earlier work of Fixler et al [22] using precision an interferometric method in conjunction with cold Cs atoms and a known Pb mass, yielding $G = 6.693 \times 10^{-11}$ cubic meters per kg sec² — these values being considerably larger than the normally utilized value of 6.67×10^{-11} . These data are in accord with an increasing trend in G that could possibly be related to other trends such as that cited by Shnoll [1].

Shnoll reports [1] that the subject histograms have a fine structure that shows what he refers to as “macroscopic fluctuations”. We have reported gravitational fluctuations [10]

that appear at random, and are associated with time intervals of ~ 0.13 sec, indicating another correlation between gravity data and radioactive decay data. The gravitational fluctuations that we detected were observed in the form of two Newton cradle pendula dwelling near each other for prolonged periods of time, but occurring in an unpredictable manner. We tentatively correlated these events with signals arriving from supernovae events that had occurred somewhere in the vastness of the universe. We also had detected on 27 August 2001 a peak in the radioactive decay of our Po-210 source, far in excess of two-sigma Poisson statistics, and later correlated with the arrival of radiation from supernovae explosion SN 2001 dz in UGC 47, emitting energy in all neutrinos of the order of 10^{46} joules.

All of the above points to the ubiquity of a model of nature based on elementary impinging momentum-transferring external particles that can be interrupted by mass particles, rather than nature being based on the conventional four axiomatic forces and their respective field theory. Furthermore, in an external particle based model for gravity, there is no need to invoke a purely mathematical “fabric” to space-time curvilinearity according to geodesics or warping, nor is it necessary to invoke Riemannian space, nor Minkowski space, but instead space-time is considered to be of a fractal geometry, and the trajectory of mass particles and photons through space is curved because of collisions with neutrinos (WIMPS). Although the collision cross-section of the neutrino with the photon is extremely low, the flux density of the neutrino in our region of the universe is extremely high, and we postulate that the bending of light is due to that interaction. It seems that astrophysics is now poised to affirm modifications to Einstein’s theory of General Relativity, and this is not unexpected in that many recent findings have indicated that gravity is quantized [15, 16, 24–26]. Understanding the nature and details of this quantization is one of the very major challenges and objectives in physics of this new century.

See also [27] for corroboration by private communication of periodic behavior of radioactive decay data during New Moon.

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27. In private communications, Professor N. Goleminov (Moscow) had corresponded data to me in the summer of 2006, showing that the standard deviation in his radioactive decay data was oscillatory, giving a minimum at the time of the New Moon, and a maxima in standard deviation occurring prior the New Moon, and another maxima subsequent to the New Moon. In the light of what I have conveyed in this Letter, Goleminov's work can be interpreted to a scattering by the moon of particles emanating from the Sun (during the New Moon), that would otherwise affect the radioactive decay data and cause a higher standard deviation at times other than the New Moon period. This interpretation would also be allied to Shnoll [1] using the term "interference" of flux. Goleminov's data also correlates indirectly with the positron annihilation periodicity work cited in this Letter.

LETTERS TO PROGRESS IN PHYSICS**Reply to the Letter by Gary C. Vezzoli**

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In a letter published by Dr. Vezzoli in the current issue of your journal, he claims priority back to 2001 for an explanation to certain gravitational phenomena, which were first recorded by me and my co-workers at my laboratory. He claims priority to me on the basis of the fact that he shared his results and plans with me in 2001 in private communication. However, I and my co-workers understood the phenomena in the same terms as much as 20 years before that, in the 1980's, and discussed by us in numerous publications during the 1980's, in the Soviet (now Russian) scientific journals. I provide a list of my early publications, refuting Dr. Vezzoli's claim to priority.

Dear Sir,

I refer a letter published by Dr. Vezzoli in the current issue of your journal he claims priority back to 2001 for an explanation to certain gravitational phenomena, which were first recorded by me and my co-workers at my laboratory. Clearly, Dr. Vezzoli is mistaken to think that he was the first person to propose, in 2001, an explanation of the gravitational phenomena recorded by me and my co-workers, at my laboratory. We in fact understood the phenomena in the same terms as much as 20 years before that, in the 1980's, as numerous publications [1–17] testify. For instance, an explanation of the experiments was given by me in 1989 at the International Congress on Geo-Cosmic Relations, in Amsterdam [4, 5]. This explanation was repeated in the other papers, published by us in 1989, 1995, and 2001. Our data, obtained during solar eclipses, began with the eclipse of July 31, 1981, when a large series of measurements was processed by 30 experimentalists connected to my laboratory, located at 10 geographical points stretching from the Atlantic to the Pacific (Sakhalin Island) along the corridor of the eclipse. We got more than 100,000 single measurements of the speed of chemical reactions during that eclipse. Our results were published in 1985 and 1987 [2, 3]. Since 1981 we processed measurements obtained during many solar and lunar eclipses, and also Full Moon and New Moon phases. The results were published in part only because a detailed analysis was required. In 1989 I published a paper wherein I claimed an observed change in the form of histograms obtained from a radioactive decay which was dependent upon the position of the Moon over the horizon [6]. This effect was observed at different geographical points. In the same paper [6] I suggested a gravitational origin of the observed effects.

I was pleased by the fact that a suggestion similar to that of mine was given by our American colleagues (Dr. Vezzoli, Dr. Lucatelli, and others), 20 years subsequent to me. This is despite that fact that their conclusions were made on the basis of scanty experimental data, in contrast to our own.

Dr. Vezzoli's claim to priority in this research, and hence his intellectual property, is I feel due to the following circumstance: the absence of information in the West about most publications made by us during the 1980's in the Soviet (now Russian) scientific journals.

My belief is that I, being a purely experimental physicist, should represent neither theoretical interpretations of the observed phenomena nor hypotheses on the subject given by the other authors. They may do that in their own papers; such a policy would be most reasonable from any standpoint.

Unfortunately, no definite theoretical explanation of the phenomenon we observed [1–16] was published in the scientific press until now. The authors of a series of papers, published in 2001 in *Biophysics*, v. 46, no. 5, presented different hypotheses on the subject. Not one of those hypotheses resulted in a calculation which could be verified by experiment.

I am responsible for a huge volume of experimental data, resulting from decades of continuous experimental research carried out by myself and dozens of my co-workers. I wouldn't like to dilute the data with a survey on hypotheses and theoretical propositions given by the theoretical physicists. Frankly speaking, I have no obligation to give such a survey. I am prepared to provide references to published papers on the subject, if it is suitable according to contents. However I feel that it is wrong to refer any information obtained in private communications before they publish their views on their own account.

I give below a list of my early publications, which refute the claim made by Dr. Vezzoli. Even a cursory inspection of the publications reveals the fact that the information provided to me by Dr. Vezzoli and Dr. Lucatelli wasn't news to me. I do not wish to be embroiled in any quarrel with them. However, having the list of my early publications, it would be strange to raise the issue of priority.

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*LETTERS TO PROGRESS IN PHYSICS***The Earth Microwave Background (EMB), Atmospheric Scattering and the Generation of Isotropy**

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In this work, the presence of substantial microwave power in the atmosphere of the Earth is discussed. It is advanced that this atmospheric microwave power constitutes pools of scattered photons initially produced, at least in substantial part, by the ~ 3 K microwave background. The existence of these microwave pools of photons can serve to explain how the Earth, as an anisotropic source, is able to produce an Earth Microwave Background (EMB) at ~ 3 K which is isotropic.

The ~ 3 K microwave background [1] has always been associated with the primordial universe [2]. Conversely, I have advanced an oceanic origin for this signal [3–7], a scenario supported by Rabounski and Borissova [8–10]. The Earth has an anisotropic surface comprised of water and solid matter. However, the microwave background is isotropic. As a result, if the Earth is the emitter of the ~ 3 K signal [1], isotropy must be achieved by scattering oceanic photons in the atmosphere.

Initially, I invoked a Compton process in the atmosphere in order to generate isotropy from an anisotropic oceanic source [3]. Yet, given the nature of the scattering required and the energies involved, such a mechanism is not likely. I therefore proposed that Mie scattering should be present [4]. Finally, I discussed both Rayleigh and Mie scattering [6]. Rayleigh scattering should be more important at the lower frequencies, while directional Mie scattering would prevail at the higher frequencies [6].

Currently [2], the microwave background is believed to be continuously striking the Earth from all spatial directions. Under steady state, any photon initially absorbed by the atmosphere must eventually be re-emitted, given elastic interactions. Since the incoming microwave background is isotropic [1, 2], then even scattering effects associated with absorption/emission should not reduce the signal intensity on the ground, because of steady state [6]. Thus, there should be no basis for signal attenuation by the atmosphere, as I previously stated [6]. Nonetheless, current astrophysical models of the atmosphere assume that such attenuations of the microwave background occur [i.e. 11, 12]. These models also appear to neglect atmospheric scattering [i.e. 11, 12].

I have mentioned that scattering processes are a central aspect of the behavior of our atmosphere at microwave frequencies [6]. In addition, since steady state assumptions should hold, any scattering of radiation, should build up some kind of reservoir or pool of scattered photons in the atmosphere.

Scattering is known to become more pronounced with increasing frequencies. Consequently, larger photon reservoirs might be seen at the shorter wavelengths.

It is known that the atmosphere interferes with the measurements of the microwave background [11]. However, this interference has been attributed to atmospheric emissions [i.e. 11, 12], not to scattering. Experimental measurements have demonstrated that atmospheric emissions increase substantially with frequency [11, 12]. For instance, emissions attributed to the water continuum tend to increase with the square of the frequency [12]. Atmospheric contributions are so pronounced at the elevated frequencies, that they can contribute in excess of 15 K to the microwave background temperature measurements at wavelengths below 1 cm [see table 4.2 in ref 11]. At a wavelength of 23.2 cm, Penzias and Wilson [1] obtained a 2.3 K contribution to their measurement just from the atmosphere [see table 4.2 in ref 11]. Even at a wavelength of 75 cm, an atmospheric contribution of 1K can be expected [see table 4.2 in ref 11]. Atmospheric modeling used in microwave background studies confirms the increase in interference with frequency and its decrease with altitude [i.e. 11, 12].

A pronounced increase in emission with frequency is expected if scattering is present. As such, it is reasonable to postulate that astrophysics is dealing with scattering in this instance [6], not with simple emission [11, 12]. Microwave background measurements at the elevated frequencies are therefore primarily complicated not by a lack of absolute signal, as I previously believed [6], but rather, by the tremendous interference from the scattered signal reservoirs in the atmosphere. In order to eliminate this effect, we are therefore forced to study the elevated frequencies from mountains top or at higher altitudes using balloons, rockets and satellites [11].

Should the microwave background arise from the universe [2], the atmosphere of the Earth would still generate the same

reservoirs of scattered radiation. The atmosphere cannot distinguish whether a photon approaches from space [2] or from the oceanic surface [6]. Thus, establishing the presence of the scattered pool of photons in the atmosphere cannot reconcile, by itself, whether the microwave background originates from the cosmos, or from the oceans. Nonetheless, since a steady state process is involved, if a ~ 3 K signal is indeed produced by the oceans, then a ~ 3 K signal will be detected, either on Earth [1] or above the atmosphere [13]. The Planckian nature of this signal will remain unaltered precisely because of steady state. This is a key feature of the steady state regimen. Importantly, experimental measures of emission [11, 12] do confirm that substantial microwave power appears to be stored in the scattering reservoirs of the atmosphere. Consequently, a mechanism for creating isotropy from an anisotropic oceanic signal [5] is indeed present for the oceanic ~ 3 K Earth Microwave Background.

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Dedication: This work is dedicated to my three sons, Jacob, Christophe and Luc.

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LETTERS TO PROGRESS IN PHYSICS

Reply to the “Certain Conceptual Anomalies in Einstein’s Theory of Relativity” and Related Questions

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This paper answers twelve most common questions on the basics of Einstein’s theory of relativity. The answers remove most key problems with a real, solid understanding of the theory.

Since its inception, *Progress in Physics*, has maintained the importance of freedom of expression in science [1]. As a result, the journal has sometimes published works even though the editorial staff differed either with the premise or with the conclusions of a paper. The editorial board maintains that it is best to disseminate works, rather than to unknowingly suppress seminal ideas. The validity of all scientific arguments will eventually be discovered. For this reason, the journal strongly upholds the rights of individual scientists relative to publication. At the same time, many questions focusing on fundamental aspects of Einstein’s theory of relativity have been submitted to the journal. Most of these letters were not published as they were conceived by authors who did not properly grasp the concepts outlined within the classic textbooks on this subject, such as *The Classical Theory of Fields* by Landau and Lifshitz [2] and others [3].

Recently, the editorial board made the decision to publish a work by Stephen J. Crothers [4] even though some questions remained relative to its basic premise. We chose to move to publication for two reasons. First, Crothers is a capable scientist who has already demonstrated substantial insight into General Relativity [5]. Indeed, the editorial board has written in support of these ideas [6]. Second, the journal has received substantial correspondence from both amateurs and established scientists. These letters have focused on perceived problems with Einstein’s theory of relativity. The editors therefore feels compelled to address these concerns, both relative to Crothers [4] and to other serious scientists who had previously worked, with success, on numerous applications of the theory of relativity.

In general, the correspondence we have received has expressed doubt concerning the validity of some key points in Einstein’s theory. We found that these questions originated in the fact that the scientists asking the questions were educated as physicists, while the base of Einstein’s theory is Riemannian geometry. It is therefore not surprising that some confusion might arise. The meaning of Einstein’s theory is the geometrization of physics, the expression of all physics through the geometrical properties of the four-dimensional pseudo-Riemannian space (the basic space-time of the theory of relativity) or its extensions. Many physicists came to the the-

ory of relativity from the other fields of physics; they learned Einstein’s theory through brief courses which gave the theory in its historical sense, often with artificially introduced principles and postulates. When the meaning of Einstein’s theory, the geometrization of physics, was finally understood through the joint intellectual powers of Albert Einstein and Marcel Grossmann, all the physical principles came out from the consideration; they all became covered by the particular properties of the geometry within four-dimensional pseudo-Riemannian space. Such a “historical” approach, which is very common in most brief courses on the theory of relativity for physicists, often carries a student away with speculations on the principles and postulates, instead of studying Riemannian geometry itself. As a result, serious physicists erred relative to simple questions which remained open after their brief education. Only a small minority of physicists, who devoted their life to understanding the theory of relativity, were lucky enough to be able to study the special (more advanced) courses on this subject.

Here we collected twelve of the most common questions on the basics of Einstein’s theory, asked by the readers and some of our colleagues. We hope the answers will remove most key problems with a real, solid understanding of the theory.

First. Naturally, each term in Einstein’s equations in emptiness (i.e. with zero right-hand-side) vanishes. This is due to that fact that, in such a case, the scalar curvature is zero $R=0$, so Einstein’s equations become the vanishing condition for Ricci’s tensor: $R_{\alpha\beta}=0$. In the same time, Ricci’s tensor $R_{\alpha\beta}$ isn’t a number, but a 2nd-rank tensor whose components are 16 (only 10 of whom are independent). The formula $R_{\alpha\beta}=0$, i.e. Einstein’s equations in emptiness, means 10 different differential equations with zero elements on the right-hand-side. These are differential equations with respect to the components of the fundamental metric tensor $g_{\alpha\beta}$: each of 10 equations $R_{\alpha\beta}=0$ is expressed in the terms containing the components of $g_{\alpha\beta}$ and their derivatives according to the definition of Ricci’s tensor $R_{\alpha\beta}$. Nothing more. (With non-zero elements on the right-hand-side, these would be Einstein’s equations in a space filled with distributed matter, e.g. electromagnetic field, dust, liquid, etc. In such a case these

would be 10 differential equations with a free term.)

Therefore the vanishing of each term of Einstein's equations in emptiness doesn't matter with respect to the validity of the equations in both general and particular cases.

Second. A common mistake is that a gravitational field is described by Einstein's equations. In fact, a gravitational field is described not by Einstein's equations, but the components of the fundamental metric tensor $g_{\alpha\beta}$ of which only 10 are substantial (out of 16). To find the components, we should solve a system of 10 Einstein's equations, consisting of $g_{\alpha\beta}$ and their derivatives: the differential equations with zero right-hand-side (in emptiness) or non-zero right-hand-side (with distributed matter).

Third. The condition $R_{\alpha\beta} = 0$ doesn't mean flatness, the pseudo-Euclidean space ($g_{00} = 1$, i.e. the absence of gravitational fields), but only emptiness (see the first point that above). Only a trivial case means flatness when $R_{\alpha\beta} = 0$.

Fourth. A mass, the source of a gravitational field, is contained in the time-time component g_{00} of the fundamental metric tensor $g_{\alpha\beta}$: the gravitational potential expresses as $w = c^2 \sqrt{1 - g_{00}}$. Therefore Einstein's equations in emptiness, $R_{\alpha\beta} = 0$, satisfy a gravitational field produced by a mass ($g_{00} \neq 1$). The right-hand-side terms (the energy-momentum tensor $T_{\alpha\beta}$ of matter and the λ -term which describes physical vacuum) describe distributed matter. There is no contradiction between Einstein's equations in emptiness and the equivalence principle.

Fifth. In the case of geometrized matter, the most known of which are isotropic electromagnetic fields (such fields are geometrized due to Rainich's condition and Nortvedt-Pagels' condition), the energy-momentum tensor of the field expresses itself through the components of the fundamental metric tensor. In such a case, we can also construct Einstein's equations containing only the "geometrical" left-hand-side by moving all the right side terms (they consist of only $g_{\alpha\beta}$ and their functions) to the left-hand-side so the right-hand-side becomes zero. But such equations aren't Einstein's equations in emptiness because $R_{\alpha\beta} \neq 0$ therein.

Sixth. Minkowski's space, the basic space-time of Special Relativity, permits test-masses, not point-masses. A test-mass is one which is so small that the gravitational field produced by it is so negligible that it doesn't have any effect on the space metric. A test-mass is a continuous body, which is approximated by its geometrical centre; it has nothing in common with a point-mass whose density should obviously be infinite.

The four-dimensional pseudo-Riemannian space with Minkowski's signature (+---) or (-+++), the space-time of General Relativity, permits continuously gravitating masses (such a mass can be approximated by the centre of its gravity) and test-masses which move in the gravitational field. No point-masses are present in the space-time of both Special Relativity and General Relativity.

Seventh. Einstein's theory of relativity doesn't work on infinite high density. According to Einstein, the theory works on densities up to the nuclear density. When one talks about a singular state of a cosmological solution, one means a so-called singular object. This is not a point, but a compact object with a finite radius and high density close to the nuclear density. Infinite high density may occur on the specific conditions within a finite radius (this is described in the modern relativistic cosmology [7]), but Einstein's theory does consider only the states before and after that transit, when the density lowers to that in atomic nuclei. Such a transit itself is out of consideration in the framework of Einstein's theory.

Eighth. Einstein's pseudotensor isn't the best solution for elucidating the energy of a gravitational field, of course. On the other hand, the other solutions proposed to solve this problem aren't excellent as well. Einstein's pseudotensor of the energy of a gravitational field permits calculation of real physical problems; the calculation results meet experiment nicely. See, for instance, Chapter XI of the famous *The Classical Theory of Fields* by Landau and Lifshitz [2]. This manifests the obvious fact that Einstein's pseudotensor, despite many drawbacks and problems connected to it, is a good approximation which lies in the right path.

Bel's tensor of superenergy, which is constructed in analogy to the tensor of the electromagnetic field, is currently the best of the attempts to solve the problem of the energy of the gravitational field in a way different from that of Einstein. See the original publications by Louis Bel [8]. More can be found on Bel's tensor in Debever's paper [9] and also in Chapter 5 of *Gravitational Waves in Einstein's Theory* by Zakharov [10].

Besides Bel's tensor, a few other solutions were proposed to the problem of the energy of the gravitational field, with less success. Einstein's theory of relativity isn't fossilized, rather it is under active development at the moment.

Ninth. Another very common mistake is the belief that Einstein's equations have no dynamical solution. There are different dynamical solutions, Peres' metric for instance [11]. Peres's metric, one of the empty space metrics, being applied to Einstein's equations in emptiness (which are $R_{\mu\nu} = 0$), leads to a solely harmonic condition along the x^1 and x^2 directions. One can read all these in detail, for instance, in Chapter 9 of the well-known book *Gravitational Waves in Einstein's Theory* by Zakharov [10].

Tenth. The main myths about Einstein's theory proceed in a popular misconception claiming the principal impossibility of an exceptional (absolute) reference frame in the theory of relativity. This is naturally impossible in the space-time of Special Relativity (Minkowski's space, which is the four-dimensional pseudo-Euclidean space with Minkowski's signature) due to that fact that, in such a space, all space-time (mixed) components g_{0i} of the fundamental metric tensor are zero (the space is free of rotation), and also all non-zero components of the metric are independent from time (the space

deformation is zero). This however isn't true in the space-time of General Relativity which is pseudo-Riemannian, so any components of the metric can be non-zero therein. It was shown already in the 1940's, by Abraham Zelmanov, a prominent scientist in the theory of relativity and cosmology, that the space-time of General Relativity permits absolute reference frames connected to the anisotropy of the fields of the space rotation or deformation of the whole Universe, i. e. connected to globally polarized (dipole-fit) fields which are as a global background gyro. See Chapter 4 in his book of 1944, *Chronometric Invariants* [7], for detail.

Eleventh. Another popular myth claims that an experiment, which manifests the anisotropy of the distribution of the velocity of light, is in contradiction to the basics of the theory of relativity due to the world-invariance of the velocity of light. This myth was also completely shattered [12]. According to the theory of physical observables in General Relativity [7], the observable velocity of light lowers from the world-invariance of the velocity by the gravitational potential and the linear velocity of the space rotation at the point of observation. The vector of the observable velocity of light directed towards an attracting body is carried into the direction of our motion in the space. As a result, the distribution of the vectors of the velocity of light beams has a preferred direction in space, depending on the motion, despite the fact that the world-invariance of the velocity of light remains unchanged. In such a case the field of the observable velocities of light is distributed anisotropically. If the space is free from rotation and gravitation (for instance, Minkowski's space of Special Relativity), the anisotropic effect vanishes: the spatial vectors of the observable velocity of light are distributed equally in all directions in the three-dimensional space. The anisotropic effect hence is due to only General Relativity. Here is nothing contradictory to the basics of Einstein's theory.

Twelfth. About Friedmann's models of a homogeneous universe, including the Big Bang scenario. It was already shown in the 1930's [7] that Friedmann's models have substantial drawbacks both in its principal and mathematical approaches. Friedmann's models are empty (free of distributed matter), homogeneous, and isotropic. They were only the first, historical step made by the scientists in the attempt to create physically and mathematically valid models of relativistic cosmology. There are hundreds of thousands of solutions to Einstein's equations. True relativistic cosmology should be stated by models of an inhomogeneous, anisotropic universe, which meet the real physical conditions of the cosmos, and can be applied to only a local volume, not the whole Universe [7]. A classification of the cosmological models, which are theoretically thinkable on the basis of Einstein's equations, was given in the 1940's. See Chapter 4 of *Chronometric Invariants* by Zelmanov [7], for detail. Many different cosmological scenarios are listed there, including such exotics as the transits through the states of infinite rarefaction and infinite

density on a finite volume (that is possible under special physical conditions). The Big Bang model, the model of expansion of a compact object of a finite radius and nuclear density, where the space is free of gravitating bodies, rotation, and deformation, is just one of many. Aside for this model, many other models of an expanding universe can be conceived on the basis of the solutions of Einstein's equations.

Relativistic cosmology is based on the time functions of the density, volume and others obtained from solutions to Einstein's equations. Therefore, only those states are under consideration, which are specific to Einstein's equations (they work up to only the nuclear density). Relativistic cosmology points out only the possibility of the state of infinite density as a theoretically extrem of the density function, while the equations of the theory are valid up to only the nuclear density. It is a very common mistake that Einstein's theory studies the state of infinite density, including a singular point-state.

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LETTERS TO PROGRESS IN PHYSICS**Rational Thinking and Reasonable Thinking in Physics**

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The usual concept of space and time, based on Aristotle's principle of contemplation of the world and of the absoluteness of time, is a product of rational thinking. At the same time, in philosophy, rational thinking differs from reasonable thinking; the aim of logic is to distinguish finite forms from infinite forms. Agreeing that space and time are things of infinity in this work, we shall show that, with regard to these two things, it is necessary to apply reasonable thinking. Spaces with non-Euclidean geometry, for example Riemannian and Finslerian spaces, in particular, the space of the General Theory of the Relativity (four-dimensional pseudo-Riemannian geometry) and also the concept of multi-dimensional space-time are products of reasonable thinking. Consequently, modern physical experiment not dealing with daily occurrences (greater speeds than a low speed to the velocity of light, strong fields, singularities, etc.) can be covered only by reasonable thinking.

In studying the microcosm, the microcosm or any extreme conditions in physics, we deal with neo-classical, unusual physics. For example, the uncertainty principle in quantum physics and the relativity principle in relativistic physics are really unusual to our logic. We may or may not desire such things, but we shall agree with physical experiments in which there is no exact localization of micro-particles or in which, in all inertial systems, light has the same speed and, hence, time is not absolute. Our consent with such experiments, the results of which are illogical from the view-point of ordinary consciousness, means that we accept to start to operate at another level of consciousness which is distinct from the level of consciousness necessary for the acceptance of experimental results of classical physics. The fundamental difference consists of the human consciousness at such a new level which operates with other categories — forms of infinity.

The world is a thing of infinity. Hence, a logic which includes forms of infinity is necessary for its cognition. The logic in itself considers the thinking in its activity and in its product. This product shall then be used by all sciences. The one and only philosophy, underlining that problem of logic is to distinguish finite forms from infinite forms, and to show some necessity to consider thinking in its activity. This activity is supra-sensory activity; though it may look like sensual perception, such as contemplation. Therefore the content of logic is the supra-sensory world and in studying it we will stay (i.e., remain) in this world. Staying in this world, we find the universal. For instance, the general laws of the motion of planets, are invisible (they are not "written in the sky") and inaudible; they exist only as a process of activity of our thinking. Hence, we arrive at Hegel's slogan "what is reasonable, is real" [1] by which the status of thinking is raised to the status of truth. As a result, it is possible not only to assume that our real world has a tie with unusual geometries,

but, in fact, it is true.

From this point of view, it is possible to agree with many mathematicians [2–6], that Euclid could direct natural sciences. In another way, at the same time, he could have taken not space as primary concept, but time.

Aristotle, having proclaimed the general principle of a world-contemplation of motions occurring simultaneously [7], has come to a conclusion (which is only natural to that epoch) that the duration of any phenomenon does not depend on a condition of rest or motion of a body in which this motion is observed, i.e. time is absolute and does not depend on the observer. This principle satisfied requirements of the person for the cognition of the world for such a very time. Why? Because, what is reasonable, is necessarily real. In reasoning itself, there is everything that it is possible to find in experience. Aristotle said, "There is nothing existent in (man's) experience that would not be in reason". Hence, in reasoning, there exist many constructions which can be adjusted to the experience.

Prior to the beginning of the 20th century, the Aristotle's principle of contemplation of world was sufficient for understanding our experiencing the world. The experiment of Michelson-Morley on measuring the velocity of light had not yet surfaced. This experiment appeared only later when there also appeared other experiments confirming relativity theory and quantum mechanics. The new principle of the contemplation of the world, explaining these experiments, has proclaimed things, which are "monstrous" from the point of view of rational thinking. Instead of time, it is the velocity of light which turns out to be the absolute magnitude. The observed duration of events (the perception of time) depends on the rest and motion of the observer. The understanding of this fact hasn't come from rational thinking, but from reasonable thinking. Rational thinking, which can ex-

plain only finite things, has become insufficient for a crucial explanation of new experimental data. Only reasonable thinking can realize such infinite things as, for example, the world, time, space. And only reasonable thinking can understand Aristotle's question whether time (related to that which divides the past and the future) is uniform or not, whether time remains always identical and invariable, or whether it constantly changes. Strict rational thinking protests against such a question, but reasonable thinking answers it. Furthermore, it depends on the level of our thinking (the level of consciousness of the observer). One may object: it depends not on one's level of consciousness, but from one's level of physical experiment. But experiment itself depends on the level of our knowledge and therefore depends on the level of our consciousness. Any principle of contemplation of the world exists in our reasoning. Our reasoning chooses necessary principle for a concrete case. Really, our reasoning is infinite.

As it is known, after the experiments confirming relativity theory our relation to the real world has changed. Riemannian geometry has played a huge role in understanding the structure of physical reality. It was a victory of "reason over mind". Relativity theory and Riemannian geometry (and its special case — pseudo-Euclidian geometry of Minkowski's space which is the basis of the Special Theory of Relativity) are products of reasoning.

We ask ourselves, why is there no unusual geometry related to the ordinary representation of the observer? This results from the fact that in life, in usual experiment, we deal with small speeds and weak fields. In such conditions, the differences among geometries are insignificant. As a simple example, in seeing that bodies are in motion as a result of some action-force, our mind has decided, that it will be carried out in any case. That is, motion is force. It is an example of naive thinking. Newton's first law has finished with this kind of knowledge because, as it became known at some later stage in the history of physics, bodies can move with constant velocity without influence of any force. There are many such examples. Perhaps, among various possible representations, one may further revise the geometries of Lobachevsky, Riemann, and Finsler.

In receiving abnormal results, the mind will treat them somehow, but not in the direction of revision of "obvious" geometrical properties. Thus, if we can overcome the resistance of the mind and reconsider "obvious" things, then our thinking can reproduce from itself new sensations and contemplations.

For example, let's consider multi-dimensional time. Within the limits of existing models that assume multi-dimensional time, there is a set of the parallel worlds (various spatial sections intersecting each other at the same point of a given space-time). It is like a set of possible states of a body in Euclidean space. Let's notice, that our reason at all does not resist to this new sensation in order to construct a new principle of the contemplation of the world.

Even if concepts of multi-dimensional space and time, constructed via reasonable thinking, demand confirmation by physical experiment (which at present seems far-fetched), it is still possible to confirm it in other ways. As Hegel has spoken, experience is done for the cognition of phenomena but not for the cognition of truth itself. One experience is not enough for the cognition of truth. Empirical supervision gives us numerous identical perceptions. However, generality is something different from a simple set. This generality is found only by means of reasoning.

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*LETTERS TO PROGRESS IN PHYSICS***A Blind Pilot: Who is a Super-Luminal Observer?**

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This paper discusses the nature of a hypothetical super-luminal observer who, as well as a real (sub-light speed) observer, perceives the world by light waves. This consideration is due to that fact that the theory of relativity permits different frames of reference, including light-like and super-luminal reference frames. In analogy with a blind pilot on board a supersonic jet aeroplane (or missile), perceived by blind people, it is concluded that the light barrier is observed in the framework of only the light signal exchange experiment.

We outline a few types of the frames of reference which may exist in the space-time of General Relativity — the four-dimensional pseudo-Riemannian space with Minkowski's signature (+---) or (-+++). Particles, including the observer himself, that travel at sub-luminal speed ("inside" the light cone), bear real relativistic mass. In other words, the particles, the body of reference and the observer are in the state of matter commonly referred to as "substance". Therefore any observer whose frame of reference is one of this kind is referred to as a *sub-luminal speed observer*, or as a *substantial observer*.

Particles and the observer that travel at the speed of light (i. e. over the surface of a light hypercone) bear zero rest-mass $m_0 = 0$ but their relativistic mass (mass of motion) is nonzero $m \neq 0$. They are in the light-like state of matter. In other words, such an observer accompanies the light. We therefore call such an observer a *light-like observer*.

Accordingly, we will call particles and the observer that travel at a super-luminal speed *super-luminal* particles and observer respectively. They are in the state of matter for which rest-mass is definitely zero $m_0 = 0$ but the relativistic mass is imaginary.

It is intuitively clear who a sub-luminal speed observer is: this term requires no further explanation. The same more or less applies to a light-like observer. From the point of view of a light-like observer the world around looks like a colourful system of light waves. But who is a super-light observer? To understand this let us give an example.

Imagine a new supersonic jet aeroplane (or missile) to be commissioned into operation. All members of the ground crew are blind, and so is the pilot. Thus we may assume that all information about the surrounding world the pilot and the members of the ground crew gain is from sound, that is, from transverse waves traveling in air. It is sound waves that build a picture that those people will perceive as their "real world".

The aeroplane takes off and begins to accelerate. As long as its speed is less than the speed of sound in air, the blind members of the ground crew will match its "heard" position in the sky to the one we can see. But once the sound barrier is overcome, everything changes. The blind members

of the ground crew will still perceive the speed of the plane equal to the speed of sound regardless of its real speed. The speed of propagation of sound waves in air will be the *maximum speed of propagation of information*, while the real supersonic jet plane will be beyond their "real world", in the world of "imaginary objects", and all its properties will be imaginary too. The blind pilot will hear nothing as well. Not a single sound will reach him from his past reality and only local sounds from the cockpit (which also travels at the supersonic speed) will break his silence. Once the speed of sound is overcome, the blind pilot leaves the subsonic world for a new supersonic one. From his new viewpoint (the supersonic frame of reference) the old subsonic fixed world that contains the airport and the members of the ground crew will simply disappear to become a realm of "imaginary quantities".

What is light? — Transverse waves that run across a certain medium at a constant speed. We perceive the world around through eyesight, receiving light waves from other objects. It is waves of light that build our picture of the "truly real world".

Now imagine a spaceship that accelerates faster and faster to eventually overcome the light barrier at still growing speed. From the purely mathematical viewpoint this is quite possible in the space-time of General Relativity. For us the speed of the spaceship will be still equal to the speed of light whatever is its real speed. For us the speed of light will be the maximum speed of propagation of information, and the real spaceship for us will stay in another "unreal" world of super-light speeds where all properties are imaginary. The same is true for the spaceship's pilot. From his viewpoint, overcoming the light barrier brings him into a new super-light world that becomes his "true reality". And the old world of sub-light speeds is banished to the realm of "imaginary reality".

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Detection of the Relativistic Corrections to the Gravitational Potential using a Sagnac Interferometer

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General Relativity predicts the existence of relativistic corrections to the static Newtonian potential which can be calculated and verified experimentally. The idea leading to quantum corrections at large distances is that of the interactions of massless particles which only involve their coupling energies at low energies. In this short paper we attempt to propose the Sagnac intrerferometric technique as a way of detecting the relativistic correction suggested for the Newtonian potential, and thus obtaining an estimate for phase difference using a satellite orbiting at an altitude of 250 km above the surface of the Earth.

1 Introduction

The potential acting between masses M and m that separated from their centers by a distance r is:

$$V(r) = -\frac{GMm}{r}, \quad (1)$$

where s the Newton's constant of gravitation. This potential is of course only approximately valid [1]. For large masses and or large velocities the theory of General Relativity predicts that there exist relativistic corrections which can be calculated and also verified experimentally [2]. In the microscopic distance domain, we could expect that quantum mechanics, would predict a modification in the gravitational potential in the same way that the radiative corrections of quantum electrodynamics leads to a similar modification of the Coulombic interaction [3].

Even though the theory of General Relativity constitutes a very well defined classical theory, it is still not possible to combine it with quantum mechanics in order to create a satisfied theory of quantum gravity. One of the basic obstacles that prevent this from happening is that General Relativity does not actually fit the present paradigm for a fundamental theory that of a renormalizable quantum field theory. Gravitational fields can be successfully quantized on smooth-enough space-times [4], but the form of gravitational interactions is such that they induce unwanted divergences which can not be absorbed by the renormalization of the parameters of the minimal General Relativity [5]. Somebody can introduce new coupling constants and absorb the divergences then, one is unfortunately led to an infinite number of free parameters. In spite the difficulty above quantum gravity calculations can predict long distance quantum corrections.

The main idea leading to quantum corrections at large distances is due to the interactions of massless particles which

only involve their coupling energies at low energies, something that it is known from the theory of General Relativity, even though at short distances the theory of quantum gravity differs resulting to finite correction of the order, $O\left(\frac{G\hbar}{c^3 r^3}\right)$. The existence of a universal long distance quantum correction to the Newtonian potential should be relevant for a wide class of gravity theories. It is well known that the ultraviolet behaviour of Einstein's pure gravity can be improved, if higher derivative contributions to the action are added, which in four dimensions take the form:

$$\alpha R^{\kappa\lambda} R_{\kappa\lambda} + \beta R^2, \quad (2)$$

where α and β are dimensionless coupling constants. What makes the difference is that the resulting classical and quantum corrections to gravity are expected to significantly alter the gravitational potential at short distances comparable to that of Planck length $\ell_P = \sqrt{\frac{G\hbar}{c^3}} = 10^{-35}$ m, but it should not really affect its behaviour at long distances. At long distances is the structure of the Einstein-Hilbert action that actually determines that. At this point we should mentioned that some of the calculation to the corrections of the Newtonian gravitational potential result in the absence of a cosmological constant Λ which usually complicates the perturbative treatment to a significant degree due to the need to expand about a non-flat background.

In one loop amplitude computation one needs to calculate all first order corrections in G , which will include both the relativistic $O\left(\frac{G^2 m^2}{c^2}\right)$ and the quantum mechanical $O\left(\frac{G\hbar}{c^3}\right)$ corrections to the classical Newtonian potential [6].

2 The corrections to the potential

Our goal is not to present the details of the one loop treatment that leads to the corrections of the Newtonian gravita-

tional potential but rather state the result and then use it in our calculations. Valid in order of G^2 we have that the corrected potential now becomes [6]:

$$V(r) = -\frac{GMm}{r} \left[1 - \frac{G(M+m)}{2c^2 r} - \frac{122 G \hbar}{15\pi c^3 r^2} \right]. \quad (3)$$

Observing (3) we see that in the correction of the static Newtonian potential two different length scales are involved. First, the Planck length $\ell_P = \sqrt{\frac{G\hbar}{c^3}} = 10^{-35}$ m and second the Schwarzschild radii of the heavy sources $r_{sch} = \frac{2GM_n}{c^2}$. Furthermore there are two independent dimensionless parameters which appear in the correction term, and involve the ratio of these two scales with respect to the distance r . Presumably for meaningful results the two length scales are much smaller than r .

3 Perturbations due to oblateness J_2

Because the Earth's gravitational potential is not that of a perfect spherical body, we can approximate its potential as a spherical harmonic expansion of the following form:

$$V(r, \phi) = -\frac{GMm}{r} \left[1 - \sum_{n=2}^{\infty} J_n \left(\frac{R_e}{r} \right)^n P_n(\sin \phi) \right] = \frac{GMm}{r} [V_0 + V_{J_2} + V_{J_3} + \dots], \quad (4)$$

where:

r = geocentric distance,

ϕ = geocentric latitude.

R_e = means equatorial radius of the Earth,

P_n = Legendre polynomial of degree n and order zero,

$J_n = J_{n_0}$ zonal harmonics of order zero, that depend on the latitude ϕ only,

and the first term GMm/r now describes the potential of a homogeneous sphere and thus refers to Keplerian motion, the remaining part represents the Earth's oblateness via the zonal harmonic coefficients and [7]

$$\left. \begin{aligned} V_0 &= -1 \\ V_{J_2} &= \frac{J_2}{2} \left(\frac{R_e}{r} \right)^2 (3 \sin^2 \phi - 1) \\ V_{J_3} &= \frac{J_3}{2} \left(\frac{R_e}{r} \right)^3 (5 \sin^3 \phi - 3 \sin \phi) \end{aligned} \right\} \quad (5)$$

similarly [8]

$$\left. \begin{aligned} J_2 &= 1,082.6 \times 10^{-6} \\ J_3 &= -2.53 \times 10^{-6} \end{aligned} \right\}. \quad (6)$$

Therefore equation (4) can be further written:

$$V(r, \phi) = -\frac{GMm}{r} \times \left[1 - \sum_{n=2}^{\infty} J_n \left(\frac{R_e}{r} \right)^n P_n(\sin \phi) - \frac{G(M+m)}{2rc^2} \right] = \frac{GMm}{r} [V_0 + V_{J_2} + V_{J_3} + \dots - V_{Relativistic}]. \quad (6a)$$

Since J_2 is 400 larger than any other J_n coefficients, we can disregard them and write the following expression for the Earth's potential function including only the relativistic correction and omitting the quantum corrections as being very small we have:

$$V(r, \phi) = -\frac{GM_e m}{r} + \frac{GM_e m R_e^2 J_2}{r^3} \left(\frac{3}{2} \sin^2 \phi - \frac{1}{2} \right) + \frac{G^2 M_e m (M_e + m)}{c^2 r^2}. \quad (7)$$

Since we propose a satellite in orbit that carries the Sagnac instrument it will be of help to express equation (7) for the potential in terms of the orbital elements. We know that $\sin \phi = \sin i \sin(f + \omega)$ where i is the inclination of the orbit, f is the true anomaly and ω is the argument of the perigee. Ignoring long and short periodic terms (those containing ω and f) we write (7) in terms of the inclination as follows:

$$V(r, \phi) = -\frac{GM_e m}{r} + \frac{3GM_e m R_e^2 J_2}{2r^3} \left(\frac{\sin^2 i}{2} - \frac{1}{3} \right) + \frac{G^2 M_e m (M_e + m)}{c^2 r^2}. \quad (8)$$

therefore the corresponding total acceleration that a mass m at $r > R_e$ has becomes:

$$g_{tot} = -\frac{1}{m} \frac{\partial}{\partial r} \left[-\frac{GM_e m}{r} + \frac{3GM_e m R_e^2 J_2}{2r^3} \times \left(\frac{\sin^2 i}{2} - \frac{1}{3} \right) + \frac{G^2 M_e m (M_e + m)}{r^2 c^2} \right] \quad (9)$$

so that:

$$g_{tot} = -\frac{GM_e}{r^2} + \frac{9GM_e R_e^2 J_2}{2r^4} \left(\frac{\sin^2 i}{2} - \frac{1}{3} \right) + \frac{G^2 M_e (M_e + m)}{c^2 r^3}. \quad (10)$$

4 Basic Sagnac interferometric theory

The Sagnac interferometer is based on the *Sagnac effect*, reported by G. Sagnac in 1913 [8]. Two beams are sent in opposite directions around the interferometer until they meet

$$\Delta\phi_{rs} = \frac{8\pi^2 R_s^2 N \Omega_s \nu (R_s \Omega_s + \pi^{-2} v_{orb})}{(c^2 - R_s^2 \Omega_s^2) \left[1 - \frac{R_s}{c^2} \left(-\frac{GM_e}{r^2} + \frac{9GM_e R_s^2 J_2}{2r^4} \left(\frac{\sin^2 i}{2} - \frac{1}{3} \right) + \frac{G^2 M_e^2}{r^3 c^2} \right) \left[1 - \cos \left[\frac{2\pi R_s \Omega_s}{c} \left(1 + \frac{R_s \Omega_s}{c} \right)^{-1} \right] \right] \right]} \quad (15)$$

$$\Delta\phi_{rs} = \frac{8\pi^2 R_s^2 N \Omega_s \nu \left(R_s \Omega_s + \pi^{-2} \sqrt{\frac{GM_e}{(R_e + z_{orb})}} \right)}{(c^2 - R_s^2 \Omega_s^2) \left[1 + \frac{R_s}{c^2} \left(\frac{GM_e}{r^2} - \frac{3GM_e R_s^2 J_2}{4r^4} - \frac{G^2 M_e^2}{r^3 c^2} \right) \left[1 - \cos \left[\frac{2\pi R_s \Omega_s}{c} \left(1 + \frac{R_s \Omega_s}{c} \right)^{-1} \right] \right] \right]} \quad (16)$$

$$\Delta\phi_{rs} = \frac{8\pi^2 R_s^2 N \Omega_s \nu \left(1 + \frac{R_s^2 \Omega_s^2}{c^2} \right) \left(R_s \Omega_s + \pi^{-2} \sqrt{\frac{GM_e}{R_e} \left(1 - \frac{z_{orb}}{R_e} \right)} \right)}{c^2 \left[1 + R_s \left(\frac{GM_e}{c^2 R_s^2} \left(1 - \frac{2z_{orb}}{R_e} \right) - \frac{3GM_e R_s^2 J_2}{4c^2 R_s^4} \left(1 - \frac{4z_{orb}}{R_e} \right) - \frac{G^2 M_e^2}{R_s^3 c^4} \left(1 - \frac{3z_{orb}}{R_e} \right) \right) \left[1 - \cos \left[\frac{2\pi R_s \Omega_s}{c} \left(1 + \frac{R_s \Omega_s}{c} \right)^{-1} \right] \right] \right]} \quad (17)$$

again to create a phase pattern. By rotating the interferometer in the direction of either the clockwise (CW) or counter-clockwise (CCW) beam, a phase difference results between the two beams that its given by:

$$\Delta\Phi_{rs} = \frac{8\pi^2 R_{sag}^2 N \Omega \nu}{(c^2 - a^2 \Omega^2)}, \quad (11)$$

where Ω is the angular velocity of the interferometer, R_{sag} is the radius of the interferometer, N is the number of turns of fiber around the radius and ν is the frequency of light in the fiber.

Let us now assume that the Sagnac interferometer and its light laser beams are in the region of space around the Earth where the gravitational potential is given by equation (3) and let us further assume that the quantum correction to the potential is really negligible. If the Sagnac light loop area has a unit vector that is perpendicular to the acceleration of gravity vector, then the motion of the interferometer will exhibit a red-shift that will be given by:

$$f_{rs} = \frac{f}{1 - \frac{\Delta V}{c^2}} = \frac{f}{1 - \frac{g_{cor} z}{c^2}}, \quad (12)$$

where ΔV is the difference in the potential between to different points P_1 and P_2 , and g_{cor} is the corrected or total acceleration of gravity and z is the difference in vertical distance between the two beams as the interferometer coil rotates. This distance z that the laser beams see is given by:

$$z = R_{sag} \left\{ 1 - \cos \left[\frac{2\pi \Omega R_{sag}}{c \left(1 + \frac{R_{sag} \Omega}{c} \right)} \right] \right\}. \quad (13)$$

This Sagnac effect can also be amplified by an interferometer that is in orbit, where the orbital velocity of the interferometer with respect to the Earth's surface produces an increased phase shift. Both terms involved in the acceleration of gravity in the first one:

$$\Delta\Phi_{rs} = \frac{8\pi^2 R_{sag}^2 N \Omega \nu \left(R_{sag} \Omega + \frac{v_{orb}}{\pi^2} \right)}{(c^2 - R_{sag}^2 \Omega^2) \left[1 - \frac{g_{tot} z}{c^2} \right]} \quad (14)$$

using (14) and taking into account that $M \gg m$ we further obtain (15), where M is the source of the gravitational field = the mass of the Earth in our case M_e , and R is the radius of the massive body = R_e , and $r = R_e + z_{orb}$ it's orbital height plus Earth radius for an Earth-based satellite.

This Sagnac effect can also be amplified by an interferometer that is in orbit, where the orbital velocity of the interferometer with respect to the Earth's surface produces an increased phase shift. Both terms involved in the acceleration of gravity in the first one:

5 Sagnac in circular orbit of known inclination

Let now a Sagnac interferometer be aboard a satellite in a circular polar orbit of inclination $i = 90$ degrees. If the inclination is 90 degrees the term $\sin^2 \frac{i}{2} - \frac{1}{3} = \frac{1}{6}$ and the orbital velocity at some height z above the surface of the Earth is $v_{orb} = \sqrt{\frac{GM_e}{(R_e + z_{orb})}}$ and (6) takes the form (16) can be finally written as (17).

6 Sagnac in elliptical orbit of known inclination

If now a satellite is carrying a Sagnac device is in an elliptical orbit of eccentricity e and semi-major axis a we have that the radial orbital vector and the orbital velocity are given by:

$$r(f) = \frac{a(1 - e^2)}{1 + e \cos f}, \quad (18)$$

$$v^2 = GM_e \left(\frac{2}{r} - \frac{1}{a} \right) = \frac{GM_e}{a} \left[\frac{2(1 + e \cos f)}{(1 - e^2)} - 1 \right], \quad (19)$$

where f is the true anomaly of the orbit. Substituting now in (8) we obtain (20).

If we use the fact that $GM_e = n^2 a^3$ where n is the mean motion of the satellite, equation (20) can be further written as (21).

When the satellite approaches perigee its orbital velocity will increase, so we will expect to see a higher phase difference than any other point of the orbit, and similarly the effect

$$\Delta\phi_{rs} = \frac{8\pi^2 R_s^2 N \Omega_s \nu \left(1 + \frac{R_s^2 \Omega_s^2}{c^2}\right) \left(R_s \Omega_s + \pi^{-2} \sqrt{\frac{GM_e}{a} \left(\frac{1+e^2+2e \cos f}{1-e^2}\right)}\right)}{c^2 \left[1 + R_s \left(\frac{GM_e(1+e \cos f)^2}{c^2 a^2 (1-e^2)^2} - \frac{3GM_e R_e^2 J_2 (1+e \cos f)^4}{4c^2 a^4 (1-e^2)^4} - \frac{G^2 M_e^2 (1+e \cos f)^3}{c^4 a^3 (1-e^2)^3}\right) \left[1 - \cos \left[\frac{2\pi R_s \Omega_s}{c} \left(1 + \frac{R_s \Omega_s}{c}\right)^{-1}\right]\right]\right]} \quad (20)$$

$$\Delta\phi_{rs} = \frac{8\pi^2 R_s^2 N \Omega_s \nu \left(1 + \frac{R_s^2 \Omega_s^2}{c^2}\right) \left(R_s \Omega_s + \pi^{-2} n a \sqrt{\left(\frac{1+e^2+2e \cos f}{1-e^2}\right)}\right)}{c^2 \left[1 + R_s \left(\frac{n^2 a (1+e \cos f)^2}{c^2 (1-e^2)^2} - \frac{3n^2 R_e^2 J_2 (1+e \cos f)^4}{4c^2 a (1-e^2)^4} - \frac{n^4 a^3 (1+e \cos f)^3}{c^4 (1-e^2)^3}\right) \left[1 - \cos \left[\frac{2\pi R_s \Omega_s}{c} \left(1 + \frac{R_s \Omega_s}{c}\right)^{-1}\right]\right]\right]} \quad (21)$$

$$\Delta\phi_{rs}(\text{perigee}) = \frac{8\pi^2 R_s^2 N \Omega_s \nu \left(1 + \frac{R_s^2 \Omega_s^2}{c^2}\right) \left(R_s \Omega_s + \pi^{-2} \sqrt{\frac{GM_e}{a} \left(\frac{1+e}{1-e}\right)}\right)}{c^2 \left[1 + R_s \left(\frac{GM_e}{c^2 a^2 (1-e)^2} - \frac{3GM_e R_e^2 J_2}{4c^2 a^4 (1-e)^4} - \frac{G^2 M_e^2}{c^4 a^3 (1-e)^3}\right) \left[1 - \cos \left[\frac{2\pi R_s \Omega_s}{c} \left(1 + \frac{R_s \Omega_s}{c}\right)^{-1}\right]\right]\right]} \quad (24)$$

$$\Delta\phi_{rs}(\text{perigee}) = \frac{8\pi^2 R_s^2 N \Omega_s \nu \left(1 + \frac{R_s^2 \Omega_s^2}{c^2}\right) \left(R_s \Omega_s + \pi^{-2} n a \sqrt{\frac{1+e}{1-e}}\right)}{c^2 \left[1 + R_s \left(\frac{n^2 a}{c^2 (1-e)^2} - \frac{3n^2 R_e^2 J_2}{4c^2 a (1-e)^4} - \frac{n^4 a^3}{c^4 (1-e)^3}\right) \left[1 - \cos \left[\frac{2\pi R_s \Omega_s}{c} \left(1 + \frac{R_s \Omega_s}{c}\right)^{-1}\right]\right]\right]} \quad (25)$$

$$\Delta\phi_{rs}(\text{apogee}) = \frac{8\pi^2 R_s^2 N \Omega_s \nu \left(1 + \frac{R_s^2 \Omega_s^2}{c^2}\right) \left(R_s \Omega_s + \pi^{-2} \sqrt{\frac{GM_e}{a} \left(\frac{1-e}{1+e}\right)}\right)}{c^2 \left[1 + R_s \left(\frac{GM_e}{c^2 a^2 (1+e)^2} - \frac{3GM_e R_e^2 J_2}{4c^2 a^4 (1+e)^4} - \frac{G^2 M_e^2}{c^4 a^3 (1+e)^3}\right) \left[1 - \cos \left[\frac{2\pi R_s \Omega_s}{c} \left(1 + \frac{R_s \Omega_s}{c}\right)^{-1}\right]\right]\right]} \quad (26)$$

$$\Delta\phi_{rs}(\text{apogee}) = \frac{8\pi^2 R_s^2 N \Omega_s \nu \left(1 + \frac{R_s^2 \Omega_s^2}{c^2}\right) \left(R_s \Omega_s + \pi^{-2} n a \sqrt{\frac{1+e}{1-e}}\right)}{c^2 \left[1 + R_s \left(\frac{n^2 a}{c^2 (1+e)^2} - \frac{3n^2 R_e^2 J_2}{4c^2 a (1+e)^4} - \frac{n^4 a^3}{c^4 (1+e)^3}\right) \left[1 - \cos \left[\frac{2\pi R_s \Omega_s}{c} \left(1 + \frac{R_s \Omega_s}{c}\right)^{-1}\right]\right]\right]} \quad (27)$$

will be minimum at the point of apogee because the satellite's velocity is minimal. The distance at perigee and apogee are given by the equations below:

$$\left. \begin{aligned} r_{pg} &= a(1-e) \\ r_{apg} &= a(1+e) \end{aligned} \right\} \quad (22)$$

also the corresponding velocities are:

$$\left. \begin{aligned} v_{pg}^2 &= \frac{GM}{a} \left(\frac{1+e}{1-e}\right) \\ v_{apg}^2 &= \frac{GM_e}{a} \left(\frac{1-e}{1+e}\right) \end{aligned} \right\}, \quad (23)$$

therefore the phase difference detected by the Sagnac due to the contribution of the Earth's oblateness plus relativistic correction to the potential at perigee and apogee can be written as (24) or again (25).

Similarly the phase difference at apogee can be written as (26) or again (27).

For this last case of the elliptical orbit in (25) and (26) where the Sagnac interferometer is on the satellite and we assume $R_s = 1$ m, $\nu = 2 \times 10^{14}$ Hz, $N = 10^6$, $\Omega_s = 400$ rad/sec, $a = 8 \times 10^6$ m, $e = 0.2$, $R_e = 6.378 \times 10^6$ meters we arrive at the following values for $\Delta\phi$:

$$\Delta\phi(\text{perigee}) = 3.57 \times 10^{-16} \text{ radians,}$$

$$\Delta\phi(\text{apogee}) = 2.44 \times 10^{-16} \text{ radians.}$$

These values are based on the dominant potential correction in (11) of section 3 which is the first term in (11) or the Newtonian correction:

$$\text{Newtonian correction} = 2.17 \times 10^{-16} \text{ radians.}$$

In comparison, the second and third terms in (11) are the oblateness and relativistic corrections respectively and they produce the following values based on the given parameters:

$$\text{Oblateness correction} = 8.52 \times 10^{-20},$$

$$\text{Relativistic correction} = 7.91 \times 10^{-26}.$$

So by comparison of the values above, the Newtonian correction is much easier to measure.

$$\Delta\phi_{rs} = \frac{8\pi^2 a_s^2 N \Omega_s \nu \left(a_s \Omega_s + \pi^{-2} \sqrt{\frac{GM_e}{(R_e + z_{orb})}} \right)}{(c^2 - a_s^2 \Omega_s^2) \left[1 + \frac{a_s \left[1 + (e^2 + e - 1) \cos \left[\frac{2\pi a_s \Omega_s}{c} \left(1 + \frac{a_s \Omega_s}{c} \right)^{-1} \right] \right]}{(1+e \cos \left[\frac{2\pi a_s \Omega_s}{c} \left(1 + \frac{a_s \Omega_s}{c} \right)^{-1} \right])} \left(\frac{GM_e}{r^2 c^2} - \frac{3GM_e R_e^2 J_2}{4r^4 c^2} - \frac{G^2 M_e^2}{r^3 c^4} \right) \right]} \quad (30)$$

$$\Delta\phi_{rs} = \frac{8\pi^2 a_s^2 N \Omega_s \nu \left(1 + \frac{a_s^2 \Omega_s^2}{c^2} \right) \left(a_s \Omega_s + \pi^{-2} \sqrt{\frac{GM_e}{R_e} \left(1 - \frac{z_{orb}}{R_e} \right)} \right)}{c^2 \left[1 + \frac{a_s \left[1 + (e^2 + e - 1) \cos \left[\frac{2\pi a_s \Omega_s}{c} \left(1 + \frac{a_s \Omega_s}{c} \right)^{-1} \right] \right]}{(1+e \cos \left[\frac{2\pi a_s \Omega_s}{c} \left(1 + \frac{a_s \Omega_s}{c} \right)^{-1} \right])} \left(\frac{GM_e}{R_e^2 c^2} \left(1 - \frac{2z_{orb}}{R_e} \right) - \frac{3GM_e R_e^2 J_2}{4R_e^4 c^2} \left(1 - \frac{4z_{orb}}{R_e} \right) - \frac{G^2 M_e^2}{R_e^3 c^4} \left(1 - \frac{3z_{orb}}{R_e} \right) \right) \right]} \quad (31)$$

$$\Delta\phi_{rs} = \frac{8\pi^2 a_s^2 N \Omega_s \nu \left(1 + \frac{a_s^2 \Omega_s^2}{c^2} \right) \left(a_s \Omega_s + \pi^{-2} \sqrt{\frac{GM_e}{a} \left(\frac{1+e^2+2e \cos f}{1-e^2} \right)} \right)}{c^2 \left[1 + \frac{a_s \left[1 + (e^2 + e - 1) \cos \left[\frac{2\pi a_s \Omega_s}{c} \left(1 + \frac{a_s \Omega_s}{c} \right)^{-1} \right] \right]}{(1+e \cos \left[\frac{2\pi a_s \Omega_s}{c} \left(1 + \frac{a_s \Omega_s}{c} \right)^{-1} \right])} \left(\frac{GM_e (1+e \cos f)^2}{c^2 a^2 (1-e^2)^2} - \frac{3GM_e R_e^2 J_2 (1+e \cos f)^4}{4c^2 a^4 (1-e^2)^4} - \frac{G^2 M_e^2 (1+e \cos f)^3}{c^4 a^3 (1-e^2)^3} \right) \right]} \quad (32)$$

$$\Delta\phi_{rs} (perigee) = \frac{8\pi^2 a_s^2 N \Omega_s \nu \left(1 + \frac{a_s^2 \Omega_s^2}{c^2} \right) \left(a_s \Omega_s + \pi^{-2} \sqrt{\frac{GM_e}{a} \left(\frac{1+e}{1-e} \right)} \right)}{c^2 \left[1 + \frac{a_s \left[1 + (e^2 + e - 1) \cos \left[\frac{2\pi a_s \Omega_s}{c} \left(1 + \frac{a_s \Omega_s}{c} \right)^{-1} \right] \right]}{(1+e \cos \left[\frac{2\pi a_s \Omega_s}{c} \left(1 + \frac{a_s \Omega_s}{c} \right)^{-1} \right])} \left(\frac{GM_e}{c^2 a^2 (1-e)^2} - \frac{3GM_e R_e^2 J_2}{4c^2 a^4 (1-e)^4} - \frac{G^2 M_e^2}{c^4 a^3 (1-e)^3} \right) \right]} \quad (33)$$

$$\Delta\phi_{rs} (apogee) = \frac{8\pi^2 a_s^2 N \Omega_s \nu \left(1 + \frac{a_s^2 \Omega_s^2}{c^2} \right) \left(a_s \Omega_s + \pi^{-2} \sqrt{\frac{GM_e}{a} \left(\frac{1-e}{1+e} \right)} \right)}{c^2 \left[1 + \frac{a_s \left[1 + (e^2 + e - 1) \cos \left[\frac{2\pi a_s \Omega_s}{c} \left(1 + \frac{a_s \Omega_s}{c} \right)^{-1} \right] \right]}{(1+e \cos \left[\frac{2\pi a_s \Omega_s}{c} \left(1 + \frac{a_s \Omega_s}{c} \right)^{-1} \right])} \left(\frac{GM_e}{c^2 a^2 (1+e)^2} - \frac{3GM_e R_e^2 J_2}{4c^2 a^4 (1+e)^4} - \frac{G^2 M_e^2}{c^4 a^3 (1+e)^3} \right) \right]} \quad (34)$$

The $\Delta\phi$ values given above may be more easily measured using a QPSK-modulator inserted in the CCW or CW beam path to improve phase resolution. Also, the use of higher wavelengths (factor of 10 higher in frequency) will increase resolution.

7 We suggest a Sagnac with an elliptic fiber loop

To attempt increasing the resolution of the phase difference of the Sagnac interferometer let us now propose a Sagnac loop, that has the shape of an ellipse that rotates with an angular velocity Ω . In this case it can be shown that the height difference between two points on the ellipse can be given by:

$$z = a \left[\frac{1 + (e^2 + e - 1) \cos \theta}{1 + e \cos \theta} \right]. \quad (28)$$

To check the validity of the formula we derived we can set $e=0$ which is the case of a circular Sagnac fiber optical path we can see that the (13) is now retrieved since

$R_{sag} = a_{loop(sag)} = a_s$ is the semi major axis of the elliptical fiber loop. When the ellipse spins with angular velocity Ω that would force it to trace out a circle whose radius r , will be that of the semi-major axis a of the ellipse, and therefore

we can finally write for (13):

$$z = \frac{a_s \left[1 + (e^2 + e - 1) \cos \left\{ \frac{2\pi a_s \Omega_s}{c} \left(1 + \frac{a_s \Omega_s}{c} \right)^{-1} \right\} \right]}{\left(1 + e \cos \left\{ \frac{2\pi a_s \Omega_s}{c} \left(1 + \frac{a_s \Omega_s}{c} \right)^{-1} \right\} \right)}. \quad (29)$$

8 Circular orbit formula for the phase difference of the Sagnac

Let now as before have a Sagnac interferometer be aboard a satellite in a circular polar orbit of inclination $i = 90$ degrees. If the inclination is 90 degrees the term $\sin^2 \frac{i}{2} - \frac{1}{3} = \frac{1}{6}$ and the orbital velocity at some height z above the surface of the Earth is $v_{orb(circ)} = \sqrt{\frac{GM_e}{(R_e + z_{orb})}}$ and (6) takes the form (30) that can be finally written as (31).

9 Sagnac in elliptical orbit of known inclination

If now a satellite is carrying a Sagnac device is in an elliptical orbit of eccentricity e and semi-major axis a we have that the radial orbital vector and the orbital velocity are given by (32).

At perigee the equation (32) becomes (33) and also (34).

For (33) and (34) above the following values are computed assuming $e = 0.2$, $\nu = 2 \times 10^{14}$ Hz, $a = 8 \times 10^6$ meters, $N = 1$ (because the orbit is the Sagnac loop), $R_{sag} = R_{perigee}$ or R_{apogee} as determined by (22), $\Omega_{perigee} = 0.001$ rad/sec, and $\Omega_{apogee} = 6 \times 10^{-4}$ rad/sec we find,

$$\Delta\phi(perigee) = 6.05 \times 10^{10} \text{ radians,}$$

$$\Delta\phi(apogee) = 2.36 \times 10^{10} \text{ radians.}$$

These values are for measuring the dominant Newtonian contribution as described in Section 6. To detect relativistic contribution which is 3.64×10^{-10} smaller than the Newtonian contribution the corresponding phase-shifts from (33) and (34) are:

$$\Delta\phi(perigee) = 22 \text{ radians,}$$

$$\Delta\phi(apogee) = 8.59 \text{ radians.}$$

Thus, the relativistic contribution in (11) of Section 3 is easily measurable using a Sagnac interferometer where the satellites in orbit are the Sagnac loop. In this scenario, the light path can be implemented by transmitting laser beams from one satellite to the next satellite in orbit ahead of it. Also, by using the maximum spacing possible between satellites in orbit this will allow line of site transmission while reducing the number of satellites required for the Sagnac loop. With the potential to measure such small relativistic corrections, the merit of using satellites to implement a large Sagnac loop of radius $R_s = R_{ap}$ or R_{per} is well worth considering.

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Resolving Spacecraft Earth-Flyby Anomalies with Measured Light Speed Anisotropy

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Doppler shift observations of spacecraft, such as Galileo, NEAR, Cassini, Rosetta and MESSENGER in earth flybys, have all revealed unexplained speed “anomalies” — that the Doppler-shift determined speeds are inconsistent with expected speeds. Here it is shown that these speed anomalies are not real and are actually the result of using an incorrect relationship between the observed Doppler shift and the speed of the spacecraft — a relationship based on the assumption that the speed of light is isotropic in all frames, *viz* invariant. Taking account of the repeatedly measured light-speed anisotropy the anomalies are resolved *ab initio*. The Pioneer 10/11 anomalies are discussed, but not resolved. The spacecraft observations demonstrate again that the speed of light is not invariant, and is isotropic only with respect to a dynamical 3-space. The existing Doppler shift data also offers a resource to characterise a new form of gravitational waves, the dynamical 3-space turbulence, that has also been detected by other techniques. The Einstein spacetime formalism uses a special definition of space and time coordinates that mandates light speed invariance for all observers, but which is easily misunderstood and misapplied.

1 Introduction

Planetary probe spacecraft (SC) have their speeds increased, in the heliocentric frame of reference, by a close flyby of the Earth, and other planets. However in the Earth frame of reference there should be no change in the asymptotic speeds after an earth flyby, assuming the validity of Newtonian gravity, at least in these circumstances. However Doppler shift observations of spacecraft, such as Galileo, NEAR, Cassini, Rosetta and MESSENGER in earth flybys, have all revealed unexplained speed “anomalies” — that the Doppler-shift determined speeds are inconsistent with expected speeds [1–6]. Here it is shown that these speed anomalies are not real and are actually the result of using an incorrect relationship between the observed Doppler shift and the speed of the spacecraft — a relationship based on the assumption that the speed of light is isotropic in all frames, *viz* invariant. Taking account of the repeatedly measured light-speed anisotropy the anomalies are resolved *ab initio*.

The speed of light anisotropy has been detected in at least 11 experiments [7–17], beginning with the Michelson-Morley 1887 experiment [7]. The interferometer observations and experimental techniques were first understood in 2002 when the Special Relativity effects and the presence of gas were used to calibrate the Michelson interferometer in gas-mode; in vacuum mode the Michelson interferometer cannot respond to light speed anisotropy [18, 19], as confirmed in vacuum resonant cavity experiments, a modern version of the vacuum-mode Michelson interferometer [20]. So far three different experimental techniques have given consistent results: gas-mode Michelson interferometers [7–11, 16],

coaxial cable RF speed measurements [12–14], and optical-fiber Michelson interferometers [15, 17]. This light speed anisotropy reveals the existence of a dynamical 3-space, with the speed of light being invariant only with respect to that 3-space, and anisotropic according to observers in motion relative to that ontologically real frame of reference — such a notion being conventionally known as “absolute motion”, a notion thought to have been rendered inappropriate by the early experiments, particularly the Michelson-Morley experiment. However that experiment was never null — they reported a speed of at least 8km/s [7] using Newtonian physics for the calibration. A proper calibration of the Michelson-Morley apparatus gives a light speed anisotropy of at least 300km/s. The spacecraft Doppler shift anomalies are shown herein to give another technique that may be used to measure the anisotropy of the speed of light, and give results consistent with previous detections.

The numerous light speed anisotropy experiments have also revealed turbulence in the velocity of the 3-space relative to the Earth. This turbulence amounts to the detection of sub-mHz gravitational waves — which are present in the Michelson and Morley 1887 data, as discussed in [21], and also present in the Miller data [8, 22] also using a gas-mode Michelson interferometer, and by Torr and Kolen [12], DeWitte [13] and Cahill [14] measuring RF speeds in coaxial cables, and by Cahill [15] and Cahill and Stokes [17] using an optical-fiber interferometer. The existing Doppler shift data also offers a resource to characterise this new form of gravitational waves.

There has been a long debate over whether the Lorentz 3-space *and* time interpretation or the Einstein spacetime inter-

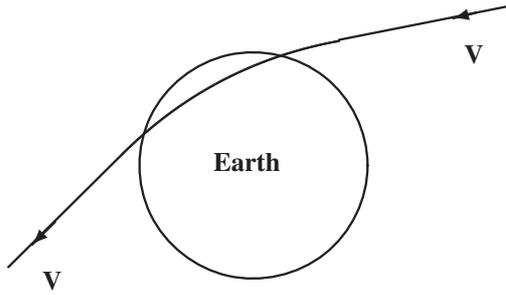


Fig. 1: Spacecraft (SC) earth flyby trajectory, with initial and final asymptotic velocity \mathbf{V} , differing only by direction. The Doppler shift is determined from Fig. 2 and (1). Assuming, as conventionally done, that the speed of light is invariant in converting measured Doppler shifts to deduced speeds, leads to the so-called flyby anomaly, namely that the incoming and outgoing asymptotic speeds appear to be differ, by ΔV_∞ . However this effect is yet another way to observe the 3-space velocity vector, as well as 3-space wave effects, with the speed of light being c and isotropic only with respect to this structured and dynamical 3-space. The flyby anomalies demonstrate, yet again, that the invariance of the speed of light is merely a definitional aspect of the Einstein spacetime formalism, and is not based upon observations. A *neo-Lorentzian* 3-space and time formalism is more physically appropriate.

pretation of observed SR effects is preferable or indeed even experimentally distinguishable. What has been discovered in recent years is that a dynamical structured 3-space exists, so confirming the Lorentz interpretation of SR [22, 24, 25], and with fundamental implications for physics. This dynamical 3-space provides an explanation for the success of the SR Einstein formalism. Indeed there is a mapping from the physical Lorentzian space and time coordinates to the non-physical spacetime coordinates of the Einstein formalism — but it is a singular map in that it removes the 3-space velocity with respect to an observer. The Einstein formalism transfers dynamical effects, such as length contractions and clock slowing effects, to the metric structure of the spacetime manifold, where these effects then appear to be merely perspective effects for different observers. For this reason the Einstein formalism has been very confusing. Developing the Lorentzian interpretation has lead to a new account of gravity, which turns out to be a quantum effect [23], and of cosmology [21, 22, 26, 27], doing away with the need for dark matter and dark energy. So the discovery of the flyby anomaly links this effect to various phenomena in the emerging new physics.

2 Absolute motion and flyby Doppler shifts

The motion of spacecraft relative to the Earth are measured by observing the direction and Doppler shift of the transponded RF transmissions. As shown herein this data gives another technique to determine the speed and direction of the dynamical 3-space, manifested as a light speed anisotropy. Up to now the repeated detection of the anisotropy of the speed of

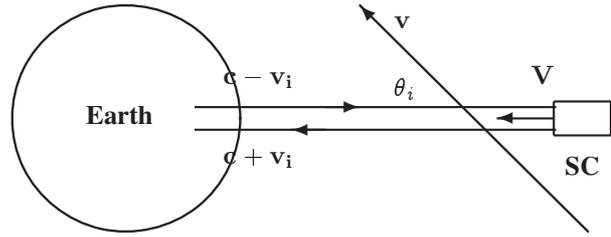


Fig. 2: Asymptotic flyby configuration in Earth frame-of-reference, with spacecraft (SC) approaching Earth with velocity \mathbf{V} . The departing asymptotic velocity will have a different direction but the same speed, as no force other than conventional Newtonian gravity is assumed to be acting upon the SC. The Dynamical 3-space velocity is $\mathbf{v}(\mathbf{r}, t)$, which causes the outward EM beam to have speed $c - v_i$, and inward speed $c + v_i$, where $v_i = v \cos(\theta_i)$, with θ_i the angle between \mathbf{v} and \mathbf{V} .

light has been ignored in analysing the Doppler shift data, causing the long-standing anomalies in the analysis [1–6].

In the Earth frame of reference, see Fig. 2, let the transmitted signal from earth have frequency f , then the corresponding outgoing wavelength is $\lambda_0 = (c - v_i)/f$, where $v_i = v \cos(\theta_i)$. This signal is received by the SC to have period $T_c = \lambda_0/(c - v_i + V)$ or frequency $f_c = (c - v_i + V)/\lambda_0$. The signal is re-transmitted with the same frequency, and so has wavelength $\lambda_i = (c + v_i - V)/f_c$, and is detected at earth with frequency $f_i = (c + v_i)/\lambda_i$. Then overall we obtain*

$$f_i = \frac{c + v_i}{c + v_i - V} \cdot \frac{c - v_i + V}{c - v_i} f. \quad (1)$$

Ignoring the projected 3-space velocity v_i , that is, assuming that the speed of light is invariant as per the usual literal interpretation of the Einstein 1905 light speed postulate, we obtain instead

$$f_i = \frac{c + V}{c - V} f. \quad (2)$$

The use of (2) instead of (1) is the origin of the putative anomalies. The Doppler shift data is usually presented in the form of speed anomalies. Expanding (2) we obtain

$$\frac{\Delta f_i}{f} = \frac{f_i - f}{f} = \frac{2V}{c} + \dots \quad (3)$$

From the observed Doppler shift data acquired during a flyby, and then best fitting the trajectory, the asymptotic hyperbolic speeds $V_{i\infty}$ and $V_{f\infty}$ are inferred, but incorrectly so, as in [1]. These inferred asymptotic speeds may be related to an inferred asymptotic Doppler shift:

$$\frac{\Delta f_i}{f} = \frac{f_i - f}{f} = \frac{2V_{i\infty}}{c} + \dots \quad (4)$$

*In practice the analysis is more complex as is the doppler shift technology. The analysis herein is sufficient to isolate and quantify the light-speed anisotropy effect.

Parameter	GLL-I	GLL-II	NEAR	Cassini	Rosetta	M'GER
Date	Dec 8, 1990	Dec 8, 1992	Jan 23, 1998	Aug 18, 1999	Mar 4, 2005	Aug 2, 2005
V_∞ km/s	8.949	8.877	6.851	16.010	3.863	4.056
α_i deg	266.76	219.35	261.17	334.31	346.12	292.61
δ_i deg	-12.52	-34.26	-20.76	-12.92	-2.81	31.44
α_f deg	219.97	174.35	183.49	352.54	246.51	227.17
δ_f deg	-34.15	-4.87	-71.96	-20.7	-34.29	-31.92
α_v deg(hrs)	108.8(7.25)	129.0(8.6)	108.8(7.25)	45.0(3.0)	130.5(8.7)	168.0(11.2)
δ_v deg	-76	-80	-76	-75	-80	-85
v km/s	420	420	450	420	420	420
θ_i deg	90.5	56.4	81.8	72.6	95.3	124.2
θ_f deg	61.8	78.2	19.6	76.0	60.5	55.6
(O) ΔV_∞ mm/s	3.92 ± 0.3	-4.6 ± 1.0	13.46 ± 0.01	-2 ± 1	1.80 ± 0.03	0.02 ± 0.01
(P) ΔV_∞ mm/s	3.92 ± 0.1	-4.60 ± 0.6	13.40 ± 0.1	-0.99 ± 1.0	1.77 ± 0.3	0.025 ± 0.03

Table 1: Earth flyby parameters from [1] for spacecraft Galileo (GLL: flybys I and II), NEAR, Cassini, Rosetta and MESSENGER (M'GER). V_∞ is the average osculating hyperbolic asymptotic speed, α and δ are the right ascension and declination of the incoming (i) and outgoing (f) osculating asymptotic velocity vectors, and (O) ΔV_∞ is the putative “excess speed” anomaly deduced by assuming that the speed of light is isotropic in modeling the doppler shifts, as in (4). The observed (O) ΔV_∞ values are from [1], and after correcting for atmospheric drag in the case of GLL-II, and thruster burn in the case of Cassini. (P) ΔV_∞ is the predicted “excess speed”, using (7), taking account of the known light speed anisotropy and its effect upon the doppler shifts, using α_v and δ_v as the right ascension and declination of the 3-space flow velocity, having speed v , which has been taken to be 420 km/s in all cases, except for NEAR, see Fig. 3. The \pm values on (P) ΔV_∞ indicate changes caused by changing the declination by 5% — a sensitivity indicator. The angles θ_i and θ_f between the 3-space velocity and the asymptotic initial/final SV velocity V are also given. The observed doppler effect is in exceptional agreement with the predictions using (7) and the previously measured 3-space velocity. The flyby doppler shift is thus a new technique to accurately measure the dynamical 3-space velocity vector, albeit retrospectively from existing data. Note: By fine tuning the α_v and δ_v values for each flyby a perfect fit to the observed (O) ΔV_∞ is possible. But here we have taken, for simplicity, the same values for GLL-I and NEAR.

However expanding (1) we obtain, for the same Doppler shift*

$$V_{i\infty} \equiv \frac{\Delta f_i}{f} \cdot \frac{c}{2} = \frac{f_i - f}{f} \cdot \frac{c}{2} = \left(1 + \frac{v_i^2}{c^2}\right) V + \dots \quad (5)$$

where V is the actual asymptotic speed. Similarly after the flyby we obtain

$$V_{f\infty} \equiv -\frac{\Delta f_f}{f} \cdot \frac{c}{2} = -\frac{f_f - f}{f} \cdot \frac{c}{2} = \left(1 + \frac{v_f^2}{c^2}\right) V + \dots \quad (6)$$

and we see that the “asymptotic” speeds $V_{i\infty}$ and $V_{f\infty}$ must differ, as indeed first noted in the data by [3]. We then obtain the expression for the so-called flyby anomaly

$$\begin{aligned} \Delta V_\infty &= V_{f\infty} - V_{i\infty} = \frac{v_f^2 - v_i^2}{c^2} V + \dots \\ &= \frac{v^2}{c^2} (\cos(\theta_f)^2 - \cos(\theta_i)^2) V_\infty + \dots \quad (7) \end{aligned}$$

where here $V \approx V_\infty$ to sufficient accuracy, where V_∞ is the average of $V_{i\infty}$ and $V_{f\infty}$. The existing data on \mathbf{v} permits

*We ignore terms of order vV/c^2 within the parentheses, as in practice they are much smaller than the v^2/c^2 terms.

ab initio predictions for ΔV_∞ , and as well a separate least-squares-fit to the individual flybys permits the determination of the average speed and direction of the 3-space velocity, relative to the Earth, during each flyby. These results are all remarkably consistent with the data from the 11 previous laboratory experiments that studied \mathbf{v} . Note that whether the 3-space velocity is $+\mathbf{v}$ or $-\mathbf{v}$ is not material to the analysis herein, as the flyby effect is 2nd order in v .

3 Earth flyby data analysis

Eqn. (7) permits the speed anomaly to be predicted as the direction and speed v of the dynamical 3-space is known, as shown in Fig. 3. The first determination of its direction was reported by Miller [8] in 1933, and based on extensive observations during 1925/1926 at Mt. Wilson, California, using a large gas-mode Michelson interferometer. These observations confirmed the previous non-null observations by Michelson and Morley [7] in 1887. The general characteristics of $\mathbf{v}(\mathbf{r}, t)$ are now known following the detailed analysis of the experiments noted above, namely its average speed, and removing the Earth orbit effect, is some 420 ± 30 km/s,

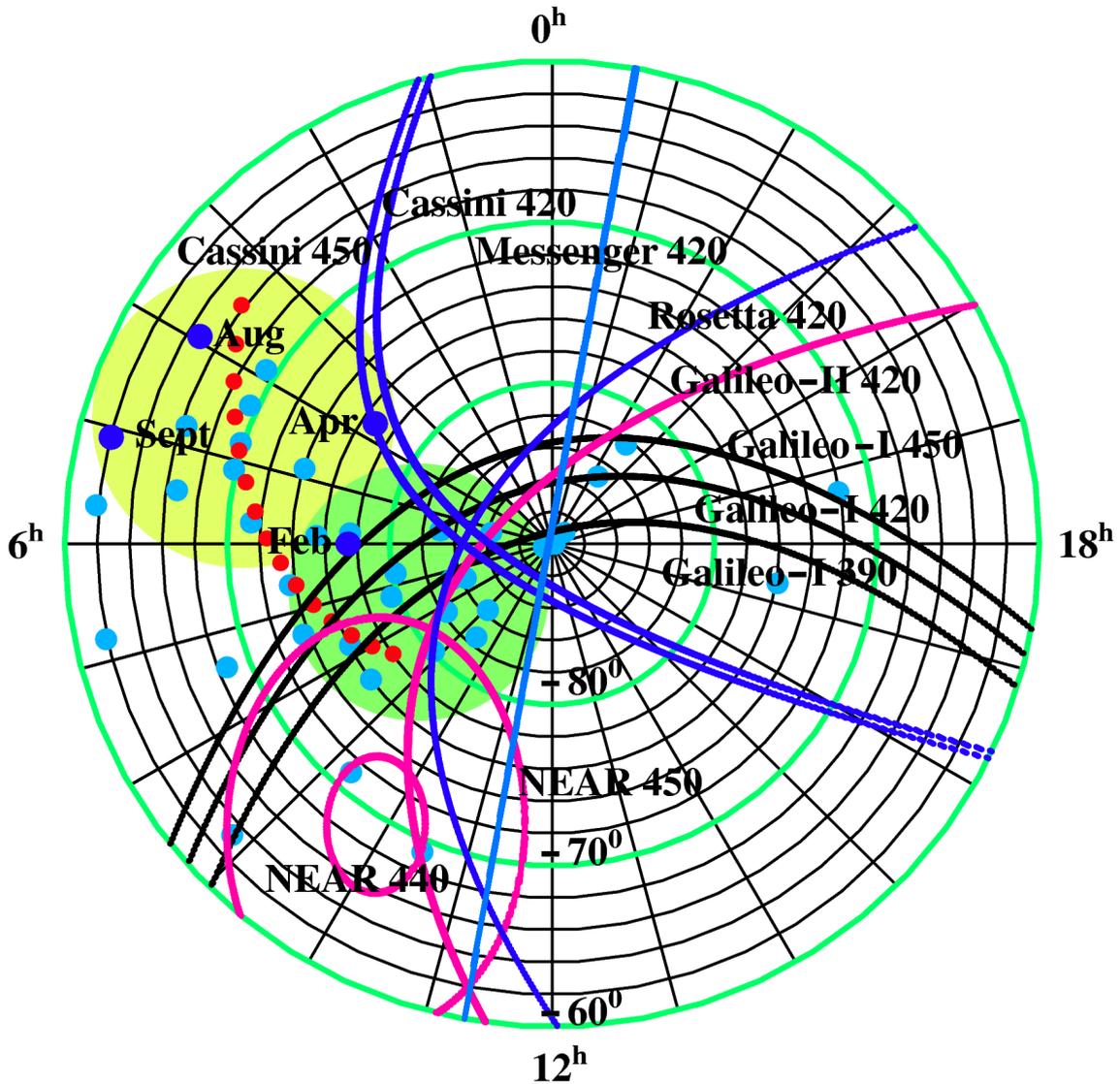


Fig. 3: Southern celestial sphere with RA and Dec shown. The 4 dark blue points show the consolidated results from the Miller gas-mode Michelson interferometer [8] for four months in 1925/1926, from [22]. The sequence of red points show the running daily average RA and Dec trend line, as determined from the optical fiber interferometer data in [17], for every 5 days, beginning September 22, 2007. The light-blue scattered points show the RA and Dec for individual days from the same experiment, and show significant turbulence/wave effects. The curved plots show iso-speed ΔV_∞ “anomalies”: for example for $v = 420$ km/s the RA and Dec of \mathbf{v} for the Galileo-I flyby must lie somewhere along the “Galileo-I 420” curve. The available spacecraft data in Table 1, from [1], does not permit a determination of a unique \mathbf{v} during that flyby. In the case of “Galileo-I” the curves are also shown for 420 ± 30 km/s, showing the sensitivity to the range of speeds discovered in laboratory experiments. We see that the “Galileo-I” December flyby has possible directions that overlap with the December data from the optical fiber interferometer, although that does not exclude other directions, as the wave effects are known to be large. In the case of NEAR we must have $v \geq 440$ km/s otherwise no fit to the NEAR ΔV_∞ is possible. This demonstrates a fluctuation in v of at least $+20$ km/s on that flyby day. This plot shows the remarkable concordance in speed and direction from the laboratory techniques with the flyby technique in measuring \mathbf{v} , and its fluctuation characteristics. The upper-left coloured disk (radius = 8°) shows concordance for September/August interferometer data and Cassini flyby data (MESSENGER data is outside this region — but has very small ΔV_∞ and large uncertainty), and the same, lower disk, for December/January/February/March data (radius = 6°). The moving concordance effect is understood to be caused by the earth’s orbit about the Sun, while the yearly average of 420 ± 30 km/s is a galaxy related velocity. Directions for each flyby \mathbf{v} were selected and used in Table 1.

from direction right ascension $\alpha_v = 5.5 \pm 2^{\text{hr}}$, declination $\delta_v = 70 \pm 10^\circ\text{S}$ — the center point of the Miller data in Fig. 3, together with large wave/turbulence effects, as illustrated in Fig. 4. Miller’s original calibration technique for the interferometer turned out to be invalid [22], and his speed of approximately 208 km/s was recomputed to be 420 ± 30 km/s in [19, 22], and the value of 420 km/s is used here as shown in Table 1. The direction of \mathbf{v} varies throughout the year due to the Earth-orbit effect and low frequency wave effects. A more recent determination of the direction was reported in [17] using an optical-fiber version of the Michelson interferometer, and shown also in Fig. 3 by the trend line and data from individual days. Directions appropriate to the date of each flyby were approximately determined from Fig. 3.

The SC data in Table 1 shows the values of V_∞ and ΔV_∞ after determining the osculating hyperbolic trajectory, as discussed in [1], as well as the right ascension and declination of the asymptotic SC velocity vectors $\mathbf{V}_{i\infty}$ and $\mathbf{V}_{f\infty}$. In computing the predicted speed “anomaly” ΔV_∞ using (7) it is only necessary to compute the angles θ_i and θ_f between the dynamical 3-space velocity vector and these SC incoming and outgoing asymptotic velocities, respectively, as we assume here that $|v| = 420$ kms, except for NEAR as discussed in Fig. 3 caption. So these predictions are essentially *ab initio* in that we are using 3-space velocities that are reasonably well known from laboratory experiments. The observed Doppler effects are in exceptional agreement with the predictions using (7) and the previously measured 3-space velocity. The flyby anomaly is thus a new technique to accurately measure the dynamical 3-space velocity vector, albeit retrospectively from existing data.

4 New gravitational waves

Light-speed anisotropy experiments have revealed that a dynamical 3-space exists, with the speed of light being c , in vacuum, only with respect to to this space: observers in motion “through” this 3-space detect that the speed of light is in general different from c , and is different in different directions. The dynamical equations for this 3-space are now known and involve a velocity field $\mathbf{v}(\mathbf{r}, t)$, but where only relative velocities are observable locally — the coordinates \mathbf{r} are relative to a non-physical mathematical embedding space. These dynamical equations involve Newton’s gravitational constant G and the fine structure constant α . The discovery of this dynamical 3-space then required a generalisation of the Maxwell, Schrödinger and Dirac equations. The wave effects already detected correspond to fluctuations in the 3-space velocity field $\mathbf{v}(\mathbf{r}, t)$, so they are really 3-space turbulence or wave effects. However they are better known, if somewhat inappropriately, as “gravitational waves” or “ripples” in “spacetime”. Because the 3-space dynamics gives a deeper understanding of the spacetime formalism we now know that the

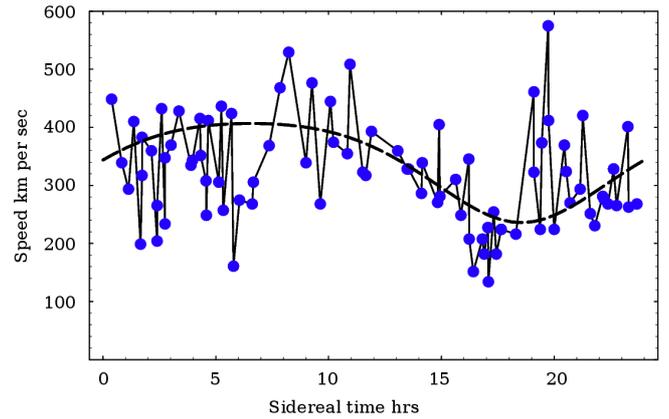


Fig. 4: Speeds v_P , of the 3-space velocity \mathbf{v} projected onto the horizontal plane of the Miller gas-mode Michelson interferometer, plotted against local sidereal time in hours, for a composite day, with data collected over a number of days in September 1925. The data shows considerable fluctuations, from hour to hour, and also day to day, as this is a composite day. The dashed curve shows the non-fluctuating best-fit variation over one day, as the Earth rotates, causing the projection onto the plane of the interferometer of the velocity of the average direction of the space flow to change. The maximum projected speed of the curve is 417 km/s, and the min/max occur at approximately 5 hrs and 17 hrs sidereal time (right ascension); see Fig. 3 for September. Analysing Millers’s extensive data set from 1925/26 gives average speed, after removing earth orbit effect, of 420 ± 30 km/s, and the directions for each month shown in Fig. 3.

metric of the induced spacetime, merely a mathematical construct having no ontological significance, is related to $\mathbf{v}(\mathbf{r}, t)$ according to [21, 22, 27]

$$ds^2 = dt^2 - \frac{(\mathbf{dr} - \mathbf{v}(\mathbf{r}, t)dt)^2}{c^2} = g_{\mu\nu} dx^\mu dx^\nu. \quad (8)$$

The gravitational acceleration of matter, a quantum effect, and of the structural patterns characterising the 3-space, are given by [21, 23]

$$\mathbf{g} = \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \quad (9)$$

and so fluctuations in $\mathbf{v}(\mathbf{r}, t)$ may or may not manifest as a gravitational acceleration. The flyby technique assumes that the SC trajectories are not affected — only the light speed anisotropy is significant. The magnitude of this turbulence depends on the timing resolution of each particular experiment, and was characterised to be sub-mHz in frequency by Cahill and Stokes [14]. Here we have only used asymptotic osculating hyperbolic trajectory data from [1]. Nevertheless even this data suggests the presence of wave effects. For example the NEAR data requires a speed in excess of 440 km/s, and probably closer to 450 km/s, whereas the other flybys are consistent with the average of 420 km/s from laboratory experiments. So here we see flyby evidence of fluctuations in the speed v .

Data exists for each full flyby, and analysis of that data using the new Doppler shift theory will permit the study and characterisation of the 3-space wave turbulence during each flyby: essentially the flybys act as gravitational wave detectors. These gravitational waves are much larger than predicted by general relativity, and have different properties.

5 Pioneer 10/11 anomalies

The Pioneer 10/11 spacecraft have been exploring the outer solar system since 1972/73. The spacecraft have followed escape hyperbolic orbits near the plane of the ecliptic, after earlier planet flybys. The Doppler shift data, using (2), have revealed an unexplained anomaly beyond 10 AU [28]. This manifests as an unmodelled increasing blue shift $\frac{d}{dt}(\frac{\Delta f}{f}) = (2.92 \pm 0.44) \times 10^{-18} \text{ s/s}^2$, corresponding to a constant inward sun-directed acceleration of $a = \frac{dV}{dt} = (8.74 \pm 1.33) \times 10^{-8} \text{ cm/s}^2$, averaged from Pioneer 10 and Pioneer 11 data. However the Doppler-shift data from these spacecraft has been interpreted using (2), instead of (1), in determining the speed, which in turn affects the distance data. Essentially this implies that the spacecraft are attributed with a speed that is too large by $\frac{v^2}{c^2} V_D$, where V_D is the speed determined using (2). This then implies that the spacecraft are actually closer to the Sun by the distance $\frac{v^2}{c^2} R_D$, where R_D is the distance determined using (2). This will then result in a computed spurious inward acceleration, because the gravitational pull of the Sun is actually larger than modelled, for distance R_D . However this correction to the Doppler-shift analysis appears not to be large enough to explain the above mention acceleration anomaly. Nevertheless re-analysis of the Pioneer 10/11 data should be undertaken using (1).

6 Conclusions

The spacecraft earth flyby anomalies have been resolved. Rather than actual relative changes in the asymptotic inward and outward speeds, which would have perhaps required the invention of a new force, they are instead direct manifestations of the anisotropy of the speed of light, with the Earth having a speed of some $420 \pm 30 \text{ km/s}$ relative to a dynamical 3-space, a result consistent with previous determinations using laboratory experiments, and dating back to the Michelson-Morley 1887 experiment, as recently reanalysed [18, 19, 21]. The flyby data also reveals, yet again, that the 3-space velocity fluctuates in direction and speed, and with results also consistent with laboratory experiments. Hence we see a remarkable concordance between three different laboratory techniques, and the newly recognised flyby technique. The existing flyby data can now be re-analysed to give a detailed characterisation of these gravitational waves. The detection of the 3-space velocity gives a new astronomical window on the galaxy, as the observed speeds are those relevant

to galactic dynamics. The dynamical 3-space velocity effect also produces very small vorticity effects when passing the Earth, and these are predicted to produce observable effects on the GP-B gyroscope precessions [29].

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The Neutrosophic Logic View to Schrödinger's Cat Paradox, Revisited

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The present article discusses Neutrosophic logic view to Schrödinger's cat paradox. We argue that this paradox involves some degree of indeterminacy (unknown) which Neutrosophic logic can take into consideration, whereas other methods including Fuzzy logic cannot. To make this proposition clear, we revisit our previous paper by offering an illustration using modified coin tossing problem, known as Parrondo's game.

1 Introduction

The present article discusses Neutrosophic logic view to Schrödinger's cat paradox. In this article we argue that this paradox involves some degree of indeterminacy (unknown) which Neutrosophic logic can take into consideration, whereas other methods including Fuzzy logic cannot.

In the preceding article we have discussed how Neutrosophic logic view can offer an alternative method to solve the well-known problem in Quantum Mechanics, i.e. the Schrödinger's cat paradox [1, 2], by introducing indeterminacy of the outcome of the observation.

In other article we also discuss possible re-interpretation of quantum measurement using Unification of Fusion Theories as generalization of Information Fusion [3, 4, 5], which results in proposition that one can expect to neglect the principle of "excluded middle"; therefore Bell's theorem can be considered as merely tautological. [6] This alternative view of Quantum mechanics as Information Fusion has also been proposed by G. Chapline [7]. Furthermore this Information Fusion interpretation is quite consistent with measurement theory of Quantum Mechanics, where the action of measurement implies information exchange [8].

In the first section we will discuss basic propositions of Neutrosophic probability and Neutrosophic logic. Then we discuss solution to Schrödinger's cat paradox. In subsequent section we discuss an illustration using modified coin tossing problem, and discuss its plausible link to quantum game.

While it is known that derivation of Schrödinger's equation is heuristic in the sense that we know the answer to which the algebra and logic leads, but it is interesting that Schrödinger's equation follows logically from de Broglie's *grande loi de la Nature* [9, p.14]. The simplest method to derive Schrödinger's equation is by using simple wave as [9]:

$$\frac{\partial^2}{\partial x^2} \exp(ikx) = -k^2 \cdot \exp(ikx). \quad (1)$$

By deriving twice the wave and defining:

$$k = \frac{2\pi mv}{h} = \frac{mv}{\hbar} = \frac{p_x}{\hbar}, \quad (2)$$

where p_x , \hbar represents momentum at x direction, and rationalised Planck constants respectively.

By introducing kinetic energy of the moving particle, T , and wavefunction, as follows [9]:

$$T = \frac{mv^2}{2} = \frac{p_x^2}{2m} = \frac{\hbar^2}{2m} k^2, \quad (3)$$

and

$$\psi(x) = \exp(ikx). \quad (4)$$

Then one has the time-independent Schrödinger equation from [1, 3, 4]:

$$-\frac{\hbar}{2m} \frac{\partial^2}{\partial x^2} \psi(x) = T \cdot \psi(x). \quad (5)$$

It is interesting to remark here that by convention physicists assert that "the wavefunction is simply the *mathematical function* that describes the wave" [9]. Therefore, unlike the wave equation in electromagnetic fields, one should not consider that equation [5] has any physical meaning. Born suggested that the square of wavefunction represents the probability to observe the electron at given location [9, p.56]. Although Heisenberg rejected this interpretation, apparently Born's interpretation prevails until today.

Nonetheless the founding fathers of Quantum Mechanics (Einstein, De Broglie, Schrödinger himself) were dissatisfied with the theory until the end of their lives. We can summarize the situation by quoting as follows [9, p.13]:

"The interpretation of Schrödinger's wave function (and of quantum theory generally) remains a matter of continuing concern and controversy among scientists who cling to philosophical belief that the natural world is basically logical and deterministic."

Furthermore, the "pragmatic" view of Bohr asserts that for a given quantum measurement [9, p.42]:

"A system does not possess objective values of its physical properties until a measurement of one of them is made; the act of measurement is asserted to force the system into an eigenstate of the quantity being measured."

In 1935, Einstein-Podolsky-Rosen argued that the axiomatic basis of Quantum Mechanics is incomplete, and subsequently Schrödinger was inspired to write his well-known cat paradox. We will discuss solution of his cat paradox in subsequent section.

2 Cat paradox and imposition of boundary conditions

As we know, Schrödinger's deep disagreement with the Born interpretation of Quantum Mechanics is represented by his cat paradox, which essentially questioning the "statistical" interpretation of the wavefunction (and by doing so, denying the physical meaning of the wavefunction). The cat paradox has been written elsewhere [1, 2], but the essence seems quite similar to coin tossing problem:

"Given $p=0.5$ for each side of coin to pop up, we will never know the state of coin before we open our palm from it; unless we know beforehand the "state" of the coin (under our palm) using ESP-like phenomena. Prop. (1)."

The only difference here is that Schrödinger asserts that the state of the cat is half alive and half dead, whereas in the coin problem above, we can only say that we don't know the state of coin until we open our palm; i.e. the state of coin is *indeterminate* until we open our palm. We will discuss the solution of this problem in subsequent section, but first of all we shall remark here a basic principle in Quantum Mechanics, i.e. [9, p.45]:

"Quantum Concept: The first derivative of the wavefunction Ψ of Schrödinger's wave equation must be single-valued everywhere. As a consequence, the wavefunction itself must be single-valued everywhere."

The above assertion corresponds to quantum logic, which can be defined as follows [10, p.30; 11]:

$$P \vee Q = P + Q - PQ. \quad (6)$$

As we will see, it is easier to resolve this cat paradox by releasing the aforementioned constraint of "single-valuedness" of the wavefunction and its first derivative. In fact, nonlinear fluid interpretation of Schrödinger's equation (using the level set function) also indicates that the physical meaning of wavefunction includes the notion of multivaluedness [12]. In other words, one can say that observation of spin-half electron at location x does not exclude its possibility to pop up somewhere else. This counter-intuitive proposition will be described in subsequent section.

3 Neutrosophic solution of the Schrödinger cat paradox

In the context of physical theory of information [8], Barrett has noted that "there ought to be a set theoretic language which applies directly to all quantum interactions". This is because the idea of a bit is itself straight out of *classical set*

theory, the definitive and unambiguous assignment of an element of the set $\{0,1\}$, and so the assignment of an information content of the photon itself is fraught with the same difficulties [8]. Similarly, the problem becomes more adverse because the fundamental basis of conventional statistical theories is the same classical set $\{0,1\}$.

For example the Schrödinger's cat paradox says that the quantum state of a photon can basically be in more than one place in the same time which, translated to the neutrosophic set, means that an element (quantum state) belongs and does not belong to a set (a place) in the same time; or an element (quantum state) belongs to two different sets (two different places) in the same time. It is a question of "alternative worlds" theory very well represented by the neutrosophic set theory. In Schrödinger's equation on the behavior of electromagnetic waves and "matter waves" in quantum theory, the wave function, which describes the superposition of possible states may be simulated by a neutrosophic function, i.e. a function whose values are not unique for each argument from the domain of definition (the vertical line test fails, intersecting the graph in more points).

Therefore the question can be summarized as follows [1]:

"How to describe a particle ζ in the infinite micro-universe that belongs to two distinct places P_1 and P_2 in the same time? $\zeta \in P_1$ and $\zeta \in \neg P_1$ is a true contradiction, with respect to Quantum Concept described above."

Now we will discuss some basic propositions in Neutrosophic logic [1].

3a Non-standard real number and subsets

Let T,I,F be standard or non-standard real subsets $\subseteq]-0, 1^+[$,

with $\sup T = t_{\sup}$, $\inf T = t_{\inf}$,

$\sup I = i_{\sup}$, $\inf I = i_{\inf}$,

$\sup F = f_{\sup}$, $\inf F = f_{\inf}$,

and $n_{\sup} = t_{\sup} + i_{\sup} + f_{\sup}$,

$n_{\inf} = t_{\inf} + i_{\inf} + f_{\inf}$.

Obviously, $t_{\sup}, i_{\sup}, f_{\sup} \leq 1^+$; and $t_{\inf}, i_{\inf}, f_{\inf} \geq -0$, whereas $n_{\sup} \leq 3^+$ and $n_{\inf} \geq -0$. The subsets T, I, F are not necessarily intervals, but may be any real subsets: discrete or continuous; single element; finite or infinite; union or intersection of various subsets etc. They may also overlap. These real subsets could represent the relative errors in determining t, i, f (in the case where T, I, F are reduced to points).

For interpretation of this proposition, we can use modal logic [10]. We can use the notion of "world" in modal logic, which is semantic device of what the world might have been like. Then, one says that the neutrosophic truth-value of a statement A, $NL_t(A) = 1^+$ if A is "true in all possible worlds." (syntagme first used by Leibniz) and all conjunctures, that one may call "absolute truth" (in the modal logic

it was named *necessary truth*, as opposed to possible truth), whereas $NL_t(A) = 1$ if A is true in at least one world at some conjuncture, we call this “relative truth” because it is related to a “specific” world and a specific conjuncture (in the modal logic it was named *possible truth*). Because each “world” is dynamic, depending on an ensemble of parameters, we introduce the sub-category “conjuncture” within it to reflect a particular state of the world.

In a formal way, let’s consider the world W as being generated by the formal system FS. One says that statement A belongs to the world W if A is a well-formed formula (*wff*) in W, i.e. a string of symbols from the alphabet of W that conforms to the grammar of the formal language endowing W. The grammar is conceived as a set of functions (formation rules) whose inputs are symbols strings and outputs “yes” or “no”. A formal system comprises a formal language (alphabet and grammar) and a deductive apparatus (axioms and/or rules of inference). In a formal system the rules of inference are syntactically and typographically formal in nature, without reference to the meaning of the strings they manipulate.

Similarly for the Neutrosophic falsehood-value, $NL_f(A) = 1^+$ if the statement A is false in all possible worlds, we call it “absolute falsehood”, whereas $NL_f(A) = 1$ if the statement A is false in at least one world, we call it “relative falsehood”. Also, the Neutrosophic indeterminacy value $NL_i(A) = 1$ if the statement A is indeterminate in all possible worlds, we call it “absolute indeterminacy”, whereas $NL_i(A) = 1$ if the statement A is indeterminate in at least one world, we call it “relative indeterminacy”.

3b Neutrosophic probability definition

Neutrosophic probability is defined as: “Is a generalization of the classical probability in which the chance that an event A occurs is $t\%$ true — where t varies in the subset T, $i\%$ indeterminate — where i varies in the subset I, and $f\%$ false — where f varies in the subset F. One notes that $NP(A) = (T, I, F)$ ”. It is also a generalization of the imprecise probability, which is an interval-valued distribution function.

The universal set, endowed with a Neutrosophic probability defined for each of its subset, forms a Neutrosophic probability space.

3c Solution of the Schrödinger’s cat paradox

Let’s consider a neutrosophic set a collection of possible locations (positions) of particle x . And let A and B be two neutrosophic sets. One can say, by language abuse, that any particle x neutrosophically belongs to any set, due to the percentages of truth/indeterminacy/falsity involved, which varies between -0 and 1^+ . For example: $x(0.5, 0.2, 0.3)$ belongs to A (which means, with a probability of 50% particle x is in a position of A, with a probability of 30% x is not in A, and the rest is *undecidable*); or $y(0, 0, 1)$ belongs to A (which

normally means y is not for sure in A); or $z(0, 1, 0)$ belongs to A (which means one does know absolutely nothing about z ’s affiliation with A).

More general, $x((0.2-0.3), (0.40-0.45) \cup [0.50-0.51], \{0.2, 0.24, 0.28\})$ belongs to the set A, which means:

- with a probability in between 20-30% particle x is in a position of A (one cannot find an exact approximate because of various sources used);
- with a probability of 20% or 24% or 28% x is not in A;
- the indeterminacy related to the appurtenance of x to A is in between 40–45% or between 50–51% (limits included).

The subsets representing the appurtenance, indeterminacy, and falsity may overlap, and $n_sup = 30\% + 51\% + 28\% > 100\%$ in this case.

To summarize our proposition [1, 2], given the Schrödinger’s cat paradox is defined as a state where the cat can be dead, or can be alive, or it is undecided (i.e. we don’t know if it is dead or alive), then herein the Neutrosophic logic, based on three components, truth component, falsehood component, indeterminacy component (T, I, F), works very well. In Schrödinger’s cat problem the Neutrosophic logic offers the possibility of considering the cat neither dead nor alive, but undecided, while the fuzzy logic does not do this. Normally indeterminacy (I) is split into uncertainty (U) and paradox (conflicting) (P).

We have described Neutrosophic solution of the Schrödinger’s cat paradox. Alternatively, one may hypothesize four-valued logic to describe Schrödinger’s cat paradox, see Rauscher *et al.* [13, 14].

In the subsequent section we will discuss how this Neutrosophic solution involving “possible truth” and “indeterminacy” can be interpreted in terms of coin tossing problem (albeit in modified form), known as Parrondo’s game. This approach seems quite consistent with new mathematical formulation of game theory [20].

4 An alternative interpretation using coin toss problem

Apart from the aforementioned pure mathematics-logical approach to Schrödinger’s cat paradox, one can use a well-known neat link between Schrödinger’s equation and Fokker-Planck equation [18]:

$$D \frac{\partial^2 p}{\partial z^2} - \frac{\partial \alpha}{\partial z} p - \alpha \frac{\partial p}{\partial z} - \frac{\partial p}{\partial t} = 0. \quad (7)$$

A quite similar link can be found between relativistic classical field equation and non-relativistic equation, for it is known that the time-independent Helmholtz equation and Schrödinger equation is formally identical [15]. From this reasoning one can argue that it is possible to explain Aharonov effect from pure electromagnetic field theory; and therefore it seems also possible to describe quantum mechan-

ical phenomena without postulating the decisive role of “observer” as Bohr asserted. [16, 17]. In idiomatic form, one can expect that quantum mechanics does not have to mean that “the Moon is not there when nobody looks at”.

With respect to the aforementioned neat link between Schrödinger’s equation and Fokker-Planck equation, it is interesting to note here that one can introduce “finite difference” approach to Fokker-Planck equation as follows. First, we can define local coordinates, expanded locally about a point (z_0, t_0) we can map points between a real space (z, t) and an integer or discrete space (i, j) . Therefore we can sample the space using linear relationship [19]:

$$(z, t) = (z_0 + i\lambda, t_0 + j\tau), \quad (8)$$

where λ is the sampling length and τ is the sampling time. Using a set of finite difference approximations for the Fokker-Planck PDE:

$$\frac{\partial p}{\partial z} = A_1 = \frac{p(z_0 + \lambda, t_0 - \tau) - p(z_0 - \lambda, t_0 - \tau)}{2\lambda}, \quad (9)$$

$$\begin{aligned} \frac{\partial^2 p}{\partial z^2} &= 2A_2 = \\ &= \frac{p(z_0 - \lambda, t_0 - \tau) - 2p(z_0, t_0 - \tau) + p(z_0 + \lambda, t_0 - \tau)}{\lambda^2}, \end{aligned} \quad (10)$$

and

$$\frac{\partial p}{\partial t} = B_1 = \frac{p(z_0, t_0) - p(z_0, t_0 - \tau)}{\tau}. \quad (11)$$

We can apply the same procedure to obtain:

$$\frac{\partial \alpha}{\partial z} = A_1 = \frac{\alpha(z_0 + \lambda, t_0 - \tau) - \alpha(z_0 - \lambda, t_0 - \tau)}{2\lambda}. \quad (12)$$

Equations (9–12) can be substituted into equation (7) to yield the required finite partial differential equation [19]:

$$\begin{aligned} p(z_0, t_0) &= a_{-1} \cdot p(z_0 - \lambda, t_0 - \tau) - a_0 \cdot p(z_0, t_0 - \tau) + \\ &+ a_{+1} \cdot p(z_0 + \lambda, t_0 - \tau). \end{aligned} \quad (13)$$

This equation can be written in terms of discrete space by using [8], so we have:

$$p_{i,j} = a_{-1} \cdot p_{i-1,j-1} + a_0 \cdot p_{i,j-1} + a_{+1} \cdot p_{i+1,j-1}. \quad (14)$$

Equation (14) is precisely the form required for Parrondo’s game. The meaning of Parrondo’s game can be described in simplest way as follows [19]. Consider a coin tossing problem with a biased coin:

$$p_{head} = \frac{1}{2} - \varepsilon, \quad (15)$$

where ε is an external bias that the game has to “overcome”. This bias is typically a small number, for instance 1/200. Now we can express equation (15) in finite difference equation (14) as follows:

$$p_{i,j} = \left(\frac{1}{2} - \varepsilon\right) \cdot p_{i-1,j-1} + 0 \cdot p_{i,j-1} + \left(\frac{1}{2} + \varepsilon\right) \cdot p_{i+1,j-1}. \quad (16)$$

Furthermore, the bias parameter can be related to an ap-

plied external field.

With respect to the aforementioned Neutrosophic solution to Schrödinger’s cat paradox, one can introduce a new “indeterminacy” parameter to represent conditions where the outcome may be affected by other issues (let say, apparatus setting of Geiger counter). Therefore equation (14) can be written as:

$$\begin{aligned} p_{i,j} &= \left(\frac{1}{2} - \varepsilon - \eta\right) \cdot p_{i-1,j-1} + \\ &+ a_0 \cdot p_{i,j-1} + \left(\frac{1}{2} + \varepsilon - \eta\right) \cdot p_{i+1,j-1}, \end{aligned} \quad (17)$$

where unlike the bias parameter ($\sim 1/200$), the indeterminacy parameter can be quite large depending on the system in question. For instance in the Neutrosophic example given above, we can write that:

$$\eta \sim 0.2 - 0.3 = k \left(\frac{d}{t}\right)^{-1} = k \left(\frac{t}{d}\right) \leq 0.50. \quad (18)$$

The only problem here is that in original coin tossing, one cannot assert an “intermediate” outcome (where the outcome is neither A nor B). Therefore one shall introduce modal logic definition of “possibility” into this model. Fortunately, we can introduce this possibility of intermediate outcome into Parrondo’s game, so equation (17) shall be rewritten as:

$$\begin{aligned} p_{i,j} &= \left(\frac{1}{2} - \varepsilon - \eta\right) \cdot p_{i-1,j-1} + \\ &+ (2\eta) \cdot p_{i,j-1} + \left(\frac{1}{2} + \varepsilon - \eta\right) \cdot p_{i+1,j-1}, \end{aligned} \quad (19)$$

For instance, by setting $\eta \sim 0.25$, then one gets the finite difference equation:

$$\begin{aligned} p_{i,j} &= (0.25 - \varepsilon) \cdot p_{i-1,j-1} + (0.5) \cdot p_{i,j-1} + \\ &+ (0.25 + \varepsilon) \cdot p_{i+1,j-1}, \end{aligned} \quad (20)$$

which will yield more or less the same result compared with Neutrosophic method described in the preceding section.

For this reason, we propose to call this equation (19): *Neutrosophic-modified Parrondo’s game*. A generalized expression of equation [19] is:

$$\begin{aligned} p_{i,j} &= (p_0 - \varepsilon - \eta) \cdot p_{i-1,j-1} + (z\eta) \cdot p_{i,j-1} + \\ &+ (p_0 + \varepsilon - \eta) \cdot p_{i+1,j-1}, \end{aligned} \quad (21)$$

where p_0, z represents the probable outcome in standard coin tossing, and a real number, respectively. For the practical meaning of η , one can think (by analogy) of this indeterminacy parameter as a variable that is inversely proportional to the “thickness ratio” (d/t) of the coin in question. Therefore using equation (18), by assuming $k = 0.2$, coin thickness = 1.0 mm, and coin diameter $d = 50$ mm, then we get $d/t = 50$, or $\eta = 0.2(50)^{-1} = 0.004$, which is negligible. But if we use a thick coin (for instance by gluing 100 coins altogether), then by assuming $k = 0.2$, coin thickness = 100 mm,

and coin diameter $d = 50$ mm, we get $d/t = 0.5$, or $\eta = 0.2(0.5)^{-1} = 0.4$, which indicates that chance to get out come neither A nor B is quite large. And so forth.

It is worth noting here that in the language of “modal logic” [10, p.54], the “intermediate” outcome described here is given name ‘possible true’, written $\diamond A$, meaning that “it is not necessarily true that not-A is true”. In other word, given that the cat cannot be found in location x , does not have to mean that it shall be in y .

Using this result (21), we can say that our proposition in the beginning of this paper (Prop. 1) has sufficient reasoning; i.e. it is possible to establish link from Schrödinger wave equation to simple coin toss problem, albeit in modified form. Furthermore, this alternative interpretation, differs appreciably from conventional Copenhagen interpretation.

It is perhaps more interesting to remark here that Heisenberg himself apparently has proposed similar thought on this problem, by introducing “potentia”, which means “*a world devoid of single-valued actuality but teeming with unrealized possibility*” [4, p.52]. In Heisenberg’s view an atom is certainly real, but its attributes dwell in an existential limbo “halfway between an idea and a fact”, a quivering state of attenuated existence. Interestingly, experiments carried out by J. Hutchison seem to support this view, that a piece of metal can come in and out from existence [23].

In this section we discuss a plausible way to represent the Neutrosophic solution of cat paradox in terms of Parrondo’s game. Further observation and theoretical study is recommended to explore more implications of this plausible link.

5 Concluding remarks

In the present paper we revisit the Neutrosophic logic view of Schrödinger’s cat paradox. We also discuss a plausible way to represent the Neutrosophic solution of cat paradox in terms of Parrondo’s game.

It is recommended to conduct further experiments in order to verify and explore various implications of this new proposition, including perhaps for the quantum computation theory.

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A Classical Model of Gravitation

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A classical model of gravitation is proposed with time as an independent coordinate. The dynamics of the model is determined by a proposed Lagrangian. Applying the canonical equations of motion to its associated Hamiltonian gives conservation equations of energy, total angular momentum and the z component of the angular momentum. These lead to a Keplerian orbit in three dimensions, which gives the observed values of perihelion precession and bending of light by a massive object. An expression for gravitational redshift is derived by accepting the local validity of special relativity at all points in space. Exact expressions for the GEM relations, as well as their associated Lorentz-type force, are derived. An expression for Mach's Principle is also derived.

1 Introduction

The proposed theory is based on two postulates that respectively establish the dynamics and kinematics of a system of particles subject to a gravitational force. The result is a closed particle model that satisfies the basic experimental observations of the force.

The details of applications and all derivations are included in the doctoral thesis of the author [1].

2 Postulates

The model is based on two postulates:

Postulate 1: The **dynamics** of a system of particles subject to gravitational forces is determined by the Lagrangian,

$$L = -m_0(c^2 + v^2) \exp \frac{R}{r}, \quad (1)$$

where m_0 is *gravitational rest mass* of a test body moving at velocity \mathbf{v} in the vicinity of a massive, central body of mass M , $\gamma = 1/\sqrt{1 - v^2/c^2}$, $R = 2GM/c^2$ is the Schwarzschild radius of the central body.

Postulate 2: Special Relativity (SR) is valid instantaneously and locally at all points in the reference system of the central massive body. This gives the **kinematics** of the system.

3 Conservation equations

Applying the canonical equations of motion to the Hamiltonian, derived from the Lagrangian, leads to three conservation equations:

$$E = m_0 c^2 \frac{e^{R/r}}{\gamma^2} = \text{total energy} = \text{constant}, \quad (2)$$

$$\mathbf{L} = e^{R/r} \mathbf{M}, \quad (3)$$

= total angular momentum = constant,

$$L_z = e^{R/r} m_0 r^2 \sin^2 \theta \dot{\phi}, \quad (4)$$

= z component of \mathbf{L} = constant,

where $\mathbf{M} = (\mathbf{r} \times m_0 \mathbf{v})$. Equations (2), (3) and (4) give the quadrature of motion:

$$\frac{d\Psi}{du} = \pm \left[\frac{e^{2Ru}}{L^2} - u^2 - \frac{Ee^{Ru}}{L^2} \right]^{-1/2}, \quad (5)$$

where $u = 1/r$, $L = |\mathbf{L}|$ and Ψ is defined by

$$|\mathbf{M}| = m_0 r^2 \frac{d\Psi}{dt}. \quad (6)$$

Expanding the exponential terms to second degree yields a differential equation of generalized Keplerian form,

$$\frac{d\Psi}{du} = (au^2 + bu + c)^{-1/2}, \quad (7)$$

where

$$\left. \begin{aligned} u &= \frac{1}{r} \\ a &= \frac{R^2(4 - E)}{2L^2} - 1 \\ b &= \frac{R(2 - E)}{L^2} \\ c &= \frac{1 - E}{L^2} \end{aligned} \right\}, \quad (8)$$

and the convention $m_0 = c = 1$ was used.

Integrating (7) gives the orbit of a test particle as a generalized conic,

$$u = K(1 + \epsilon \cos k\Psi), \quad (9)$$

where the angles are measured from $\Psi = 0$, and

$$k = (-a)^{\frac{1}{2}}, \quad (10)$$

$$K = -\frac{b}{2a}, \quad (11)$$

$$\epsilon = \left(1 - \frac{4ac}{b^2} \right)^{\frac{1}{2}}. \quad (12)$$

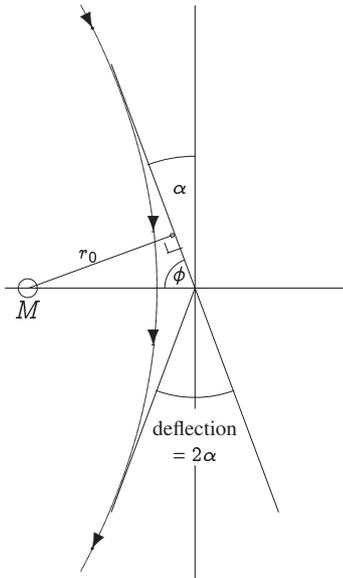


Fig. 1: Deflection of light.

4 Gravitational redshift

Assuming the validity of $\gamma d\tau = dt$ of SR at each point in space and taking frequencies as the inverses of time, (2) yields

$$\nu = \nu_0 e^{-R/2r} \quad (\nu_0 = \text{constant}), \quad (13)$$

which, to first approximation in $\exp(-R/2r)$, gives the observed gravitational redshift.

5 Perihelion precession

In the case of an ellipse ($\epsilon < 1$), the presence of the coefficient k causes the ellipse not to be completed after a cycle of $\theta = 2\pi$ radians, i.e. the perihelion is shifted through a certain angle. This shift, or precession, can be calculated as (see Appendix 9):

$$\Delta\phi = \frac{3\pi R}{\bar{a}(1-\epsilon^2)}, \quad (14)$$

where \bar{a} is the semi-major axis of the ellipse. This expression gives the observed perihelion precession of Mercury.

6 Deflection of light

We define a photon as a particle for which $v = c$. From (2) it follows that $E = 0$ and the eccentricity of the conic section is found to be (see Appendix 9)

$$\epsilon = \frac{r_0}{R}, \quad (15)$$

where r_0 is the impact parameter. Approximating r_0 by the radius of the sun, it follows that $\epsilon > 1$. From Fig. 1 we see that the trajectory is a hyperbola with total deflection equal to $2R/r_0$. This is in agreement with observation.

7 Lorentz-type force equation

The corresponding force equation is found from the associated Euler-Lagrange equations:

$$\dot{\mathbf{p}} = \mathbf{E}m + m_0 \mathbf{v} \times \mathbf{H}, \quad (16)$$

where

$$\mathbf{p} = m_0 \dot{\mathbf{r}} = m_0 \mathbf{v}, \quad (17)$$

$$m = \frac{m_0}{\gamma^2}, \quad (18)$$

$$\mathbf{E} = -\hat{\mathbf{r}} \frac{GM}{r^2}, \quad (19)$$

$$\mathbf{H} = \frac{GM(\mathbf{v} \times \mathbf{r})}{c^2 r^3}. \quad (20)$$

The force equation shows the deviation from Newton's law of gravitation. The above equations are analogous to the gravitoelectromagnetic (GEM) equations derived by Mashhoon [2] as a lowest order approximation to Einstein's field equations for $v \ll c$ and $r \gg R$.

8 Mach's Principle

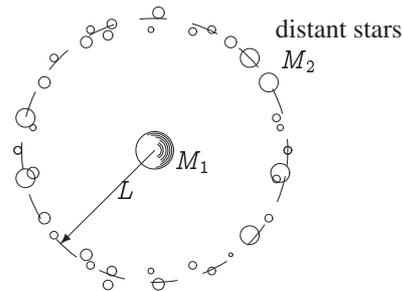
An *ad hoc* formulation for Mach's Principle has been presented as [3,4]

$$G \cong \frac{Lc^2}{M}, \quad (21)$$

where: L = radius of the universe,

M = mass of the universe \cong mass of the distant stars.

This relation can be found by applying the energy relation of (2) to the system of Fig. 2.

Fig. 2: Mutual gravitational interaction between a central mass M_1 and the distant stars of total mass M_2 .

The potential at M_2 due to M_1 is $\Phi_1 = GM_1/L = R_1 c^2/2L$ and the potential of the shell at M_1 is $\Phi_2 = GM_2/L = R_2 c^2/2L$. Furthermore, since M_1 and M_2 are in relative motion, the value of γ will be the same for both of them. Applying (2) to the mutual gravitational interaction between the shell of distant stars and the central body then gives

$$E = M_1 c^2 \exp \frac{R_2}{L} = M_2 c^2 \exp \frac{R_1}{L}.$$

Since $L > R_2 \gg R_1$ we can realistically approximate the exponential to first order in R_2/L . After some algebra we get $R_2 \approx L$, which gives the Mach relation,

$$\frac{2GM_2}{Lc^2} \approx 1.$$

9 Comparison with General Relativity

The equations of motion of General Relativity (GR) are approximations to those of the proposed Lagrangian. This can be seen as follows.

The conservation equations of (2), (3) and (4) can also be derived from a generalized metric,

$$ds^2 = e^{-R/r} dt^2 - e^{R/r} (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2). \quad (22)$$

Comparing this metric with that of GR,

$$ds^2 = \left(1 - \frac{R}{r}\right) dt^2 - \frac{1}{1 - \frac{R}{r}} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2, \quad (23)$$

we note that (23) is a first order approximation to the time and radial coefficients, and a zeroth order approximation to the angular coefficients of (22). It implies that all predictions of GR will be accommodated by the Lagrangian of (1) within the orders of approximation.

Comparing (5) with the corresponding quadrature of GR,

$$\frac{d\theta}{du} = \pm \left[\frac{1-E}{J^2} + \frac{uRE}{J^2} - u^2 + Ru^3 \right]^{-1/2}, \quad (24)$$

we note that it differs from the Newtonian limit, or the Keplerian form of (7), by the presence of the Ru^3 term. The form of this quadrature does not allow the conventional Keplerian orbit of (9).

Appendix

A.1 Precession of the perihelion

After one revolution of 2π radians, the perihelion of an ellipse given by the conic of (9) shifts through an angle $\Delta\phi = \frac{2\pi}{k} - 2\pi$ or, from (10), as

$$\Delta\phi = 2\pi [(-a)^{-1/2} - 1], \quad (25)$$

where a is given by (8). The constants of motion E and L are found from the boundary conditions of the system, i.e. $du/d\theta = 0$ at $u = 1/r_-$ and $1/r_+$, where r_+ and r_- are the maximum and minimum radii respectively of the ellipse. We find [1]

$$\left. \begin{aligned} E &\approx 1 + \frac{R}{2\bar{a}} \\ \frac{R^2}{L^2} &\approx \frac{2R}{\bar{a}(1-\epsilon^2)} \end{aligned} \right\}, \quad (26)$$

where $\bar{a} = (r_+ + r_-)/2$ is the semi-major axis of the approximate ellipse. Substituting these values in (8) gives

$$a = \frac{3R}{\bar{a}(1-\epsilon^2)} - 1. \quad (27)$$

Substituting this value in (25) gives (14).

A.2 Deflection of light

We first have to calculate the eccentricity ϵ of the conic for this case,

$$\epsilon = \left(1 - \frac{4ac}{b^2}\right)^{1/2}.$$

For a photon, setting $v = c$ in (8) gives

$$\epsilon^2 = \left[-1 + \frac{L^2}{R^2}\right]. \quad (28)$$

At the distance of closest approach, $r = r_0 = 1/u_0$, we have $d\theta/du = 0$, so that from (5):

$$L^2 = \frac{e^{2Ru_0}}{u_0^2} = r_0^2 e^{2R/r_0}. \quad (29)$$

From (28) and (29), and ignoring terms of first and higher order in R/r_0 , we find

$$\epsilon \approx \frac{r_0}{R}. \quad (30)$$

For a hyperbola $\cos \phi = 1/\epsilon$, so that (see Fig. 1):

$$\begin{aligned} \sin \alpha &= 1/\epsilon \\ \Rightarrow \alpha &\approx 1/\epsilon \\ \Rightarrow 2\alpha &\approx 2R/r_0 = \text{total deflection.} \end{aligned}$$

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On the Origin of the Dark Matter/Energy in the Universe and the Pioneer Anomaly

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Einstein's special relativity is a theory rich of paradoxes, one of which is the recently discovered *Relativistic Invariant Mass Paradox*. According to this Paradox, the relativistic invariant mass of a galaxy of moving stars exceeds the sum of the relativistic invariant masses of the constituent stars owing to their motion relative to each other. This excess of mass is the mass of *virtual matter* that has no physical properties other than positive relativistic invariant mass and, hence, that reveals its presence by no means other than gravity. As such, this virtual matter is the dark matter that cosmologists believe is necessary in order to supply the missing gravity that keeps galaxies stable. Based on the Relativistic Invariant Mass Paradox we offer in this article a model which quantifies the anomalous acceleration of Pioneer 10 and 11 spacecrafts and other deep space missions, and explains the presence of dark matter and dark energy in the universe. It turns out that the origin of dark matter and dark energy in the Universe lies in the Paradox, and that the origin of the Pioneer anomaly results from neglecting the Paradox. In order to appreciate the physical significance of the Paradox within the frame of Einstein's special theory of relativity, following the presentation of the Paradox we demonstrate that the Paradox is responsible for the extension of the kinetic energy theorem and of the additivity of energy and momentum from classical to relativistic mechanics. Clearly, the claim that the acceleration of Pioneer 10 and 11 spacecrafts is anomalous is incomplete, within the frame of Einstein's special relativity, since those who made the claim did not take into account the presence of the Relativistic Invariant Mass Paradox (which is understandable since the Paradox, published in the author's 2008 book, was discovered by the author only recently). It remains to test how well the Paradox accords with observations.

1 Introduction

Einstein's special relativity is a theory rich of paradoxes, one of which is the *Relativistic Invariant Mass Paradox*, which was recently discovered in [1], and which we describe in Section 5 of this article. The term mass in special relativity usually refers to the rest mass of an object, which is the Newtonian mass as measured by an observer moving along with the object. Being observer's invariant, we refer the Newtonian, rest mass to as the *relativistic invariant mass*, as opposed to the common *relativistic mass*, which is another name for energy, and which is observer's dependent. Lev B. Okun makes the case that the concept of relativistic mass is no longer even pedagogically useful [2]. However, T. R. Sandin has argued otherwise [3].

As we will see in Section 5, the Relativistic Invariant Mass Paradox asserts that the resultant relativistic invariant mass m_0 of a system S of uniformly moving N particles exceeds the sum of the relativistic invariant masses m_k , $k = 1, \dots, N$, of its constituent particles, $m_0 > \sum_{k=1}^N m_k$, since the contribution to m_0 comes not only from the masses m_k of the constituent particles of S but also from their speeds relative to each other. The resulting excess of mass in the

resultant relativistic invariant mass m_0 of S is the mass of virtual matter that has no physical properties other than positive relativistic invariant mass and, hence, that reveals itself by no means other than gravity. It is therefore naturally identified as the *mass* of *virtual* dark matter that the system S possesses. The presence of dark matter in the universe in a form of virtual matter that reveals itself only gravitationally is, thus, dictated by the Relativistic Invariant Mass Paradox of Einstein's special theory of relativity. Accordingly, (i) the fate of the dark matter particle(s) theories as well as (ii) the fate of their competing theories of modified Newtonian dynamics (MOND [4]) are likely to follow the fate of the eighteenth century phlogiston theory and the nineteenth century luminiferous ether theory, which were initiated as *ad hoc* postulates and which, subsequently, became obsolete.

Dark matter and dark energy are *ad hoc* postulates that account for the observed missing gravitation in the universe and the late time cosmic acceleration. The postulates are, thus, a synonym for these observations, as C. Lämmerzahl, O. Preuss and H. Dittus had to admit in [5] for their chagrin. An exhaustive review of the current array of dark energy theories is presented in [6].

The Pioneer anomaly is the anomalous, unmodelled acceleration of the spacecrafts Pioneer 10 and 11, and other spacecrafts, studied by J. D. Anderson *et al* in [7] and summarized by S. G. Turyshev *et al* in [8]. In [7], Anderson *et al* compared the measured trajectory of a spacecraft against its theoretical trajectory computed from known forces acting on the spacecraft. They found the small, but significant discrepancy known as the anomalous, or unmodelled, acceleration directed approximately towards the Sun. The inability to explain the Pioneer anomaly with conventional physics has contributed to the growing interest about its origin, as S. G. Turyshev, M. M. Nieto and J. D. Anderson pointed out in [9]. It is believed that no conventional force has been overlooked [5] so that, seemingly, new physics is needed. Indeed, since Anderson *et al* announced in [7] that the Pioneer 10 and 11 spacecrafts exhibit an unexplained anomalous acceleration, numerous articles appeared with many plausible explanations that involve new physics, as C. Castro pointed out in [10].

However, we find in this article that no new physics is needed for the explanation of both the presence of dark matter/energy and the appearance of the Pioneer anomaly. Rather, what is needed is to cultivate the Relativistic Invariant Mass Paradox, which has recently been discovered in [1], and which is described in Section 5 below.

Accordingly, the task we face in this article is to show that the Relativistic Invariant Mass Paradox of Einstein's special relativity dictates the formation of dark matter and dark energy in the Universe and that, as a result, the origin of the Pioneer anomaly stems from the motions of the constituents of the Solar system relative to each other.

2 Einstein velocity addition vs. Newton velocity addition

The improved way to study Einstein's special theory of relativity, offered by the author in his recently published book [1], enables the origin of the dark matter/energy in the Universe and the Pioneer anomaly to be determined. The improved study rests on analogies that Einsteinian mechanics and its underlying hyperbolic geometry share with Newtonian mechanics and its underlying Euclidean geometry. In particular, it rests on the analogies that Einsteinian velocity addition shares with Newtonian velocity addition, the latter being just the common vector addition in the Euclidean 3-space \mathbb{R}^3 .

Einstein addition \oplus is a binary operation in the ball \mathbb{R}_c^3 of \mathbb{R}^3 ,

$$\mathbb{R}_c^3 = \{\mathbf{v} \in \mathbb{R}^3 : \|\mathbf{v}\| < c\} \quad (1)$$

of all relativistically admissible velocities, where c is the speed of light in empty space. It is given by the equation

$$\mathbf{u} \oplus \mathbf{v} = \frac{1}{1 + \frac{\mathbf{u} \cdot \mathbf{v}}{c^2}} \left\{ \mathbf{u} + \frac{1}{\gamma_{\mathbf{u}}} \mathbf{v} + \frac{1}{c^2} \frac{\gamma_{\mathbf{u}}}{1 + \gamma_{\mathbf{u}}} (\mathbf{u} \cdot \mathbf{v}) \mathbf{u} \right\} \quad (2)$$

where $\gamma_{\mathbf{u}}$ is the gamma factor

$$\gamma_{\mathbf{v}} = \frac{1}{\sqrt{1 - \frac{\|\mathbf{v}\|^2}{c^2}}} \quad (3)$$

in \mathbb{R}_c^3 , and where \cdot and $\|\cdot\|$ are the inner product and norm that the ball \mathbb{R}_c^3 inherits from its space \mathbb{R}^3 . Counterintuitively, Einstein addition is neither commutative nor associative.

Einstein gyrations $\text{gyr}[\mathbf{u}, \mathbf{v}] \in \text{Aut}(\mathbb{R}_c^3, \oplus)$ are defined by the equation

$$\text{gyr}[\mathbf{u}, \mathbf{v}]\mathbf{w} = \ominus(\mathbf{u} \oplus \mathbf{v}) \oplus (\mathbf{u} \oplus (\mathbf{v} \oplus \mathbf{w})) \quad (4)$$

for all $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}_c^3$, and they turn out to be automorphisms of the Einstein groupoid (\mathbb{R}_c^3, \oplus) . We recall that a groupoid is a non-empty space with a binary operation, and that an automorphism of a groupoid (\mathbb{R}_c^3, \oplus) is a one-to-one map f of \mathbb{R}_c^3 onto itself that respects the binary operation, that is, $f(\mathbf{u} \oplus \mathbf{v}) = f(\mathbf{u}) \oplus f(\mathbf{v})$ for all $\mathbf{u}, \mathbf{v} \in \mathbb{R}_c^3$. To emphasize that the gyrations of the Einstein groupoid (\mathbb{R}_c^3, \oplus) are automorphisms of the groupoid, gyrations are also called gyroautomorphisms.

Thus, $\text{gyr}[\mathbf{u}, \mathbf{v}]$ of the definition in (4) is the gyroautomorphism of the Einstein groupoid (\mathbb{R}_c^3, \oplus) , generated by $\mathbf{u}, \mathbf{v} \in \mathbb{R}_c^3$, that takes the relativistically admissible velocity \mathbf{w} in \mathbb{R}_c^3 into the relativistically admissible velocity $\ominus(\mathbf{u} \oplus \mathbf{v}) \oplus (\mathbf{u} \oplus (\mathbf{v} \oplus \mathbf{w}))$ in \mathbb{R}_c^3 .

The gyrations, which possess their own rich structure, measure the extent to which Einstein addition deviates from commutativity and associativity as we see from the following identities [1, 11, 12]:

$\mathbf{u} \oplus \mathbf{v} = \text{gyr}[\mathbf{u}, \mathbf{v}](\mathbf{v} \oplus \mathbf{u})$	Gyrocommutative Law
$\mathbf{u} \oplus (\mathbf{v} \oplus \mathbf{w}) = (\mathbf{u} \oplus \mathbf{v}) \oplus \text{gyr}[\mathbf{u}, \mathbf{v}]\mathbf{w}$	Left Gyroassociative
$(\mathbf{u} \oplus \mathbf{v}) \oplus \mathbf{w} = \mathbf{u} \oplus (\mathbf{v} \oplus \text{gyr}[\mathbf{u}, \mathbf{v}]\mathbf{w})$	Right Gyroassociative
$\text{gyr}[\mathbf{u}, \mathbf{v}] = \text{gyr}[\mathbf{u} \oplus \mathbf{v}, \mathbf{v}]$	Left Loop Property
$\text{gyr}[\mathbf{u}, \mathbf{v}] = \text{gyr}[\mathbf{u}, \mathbf{v} \oplus \mathbf{u}]$	Right Loop Property

Einstein addition is thus regulated by its gyrations so that Einstein addition and its gyrations are inextricably linked. Indeed, the Einstein groupoid (\mathbb{R}_c^3, \oplus) forms a group-like mathematical object called a *gyrocommutative gyrogroup* [13], which was discovered by the author in 1988 [14]. Interestingly, Einstein gyrations are just the mathematical abstraction of the relativistic *Thomas precession* [1, Sec. 10.3].

The rich structure of Einstein addition is not limited to its gyrocommutative gyrogroup structure. Einstein addition admits scalar multiplication, giving rise to the Einstein gyrovector space. The latter, in turn, forms the setting for the Beltrami-Klein ball model of hyperbolic geometry just as vector spaces form the setting for the standard model of Euclidean geometry, as shown in [1].

Guided by the resulting analogies that relativistic mechanics and its underlying hyperbolic geometry share with classical mechanics and its underlying Euclidean geometry, we

are able to present analogies that Newtonian systems of particles share with Einsteinian systems of particles in Sections 3 and 4. These analogies, in turn, uncover the Relativistic Invariant Mass Paradox in Section 5, the physical significance of which is illustrated in Section 6 in the frame of Einstein's special theory of relativity. Finally, in Sections 7 and 8 the Paradox reveals the origin of the dark matter/energy in the Universe as well as the origin of the Pioneer anomaly.

3 Newtonian systems of particles

In this section we set the stage for revealing analogies that a Newtonian system of N particles and an Einsteinian system of N particles share. In this section, accordingly, as opposed to Section 4, $\mathbf{v}_k, k = 0, 1, \dots, N$, are Newtonian velocities in \mathbb{R}^3 , and m_0 is the Newtonian resultant mass of the constituent masses $m_k, k = 1, \dots, N$ of a Newtonian particle system S .

Accordingly, let us consider the following well known classical results, (6)–(8) below, which are involved in the calculation of the Newtonian resultant mass m_0 and the classical center of momentum (CM) of a Newtonian system of particles, and to which we will seek Einsteinian analogs in Section 4. Thus, let

$$S = S(m_k, \mathbf{v}_k, \Sigma_0, N), \quad \mathbf{v}_k \in \mathbb{R}^3 \quad (5)$$

be an isolated Newtonian system of N noninteracting material particles the k -th particle of which has mass m_k and Newtonian uniform velocity \mathbf{v}_k relative to an inertial frame $\Sigma_0, k = 1, \dots, N$. Furthermore, let m_0 be the resultant mass of S , considered as the mass of a virtual particle located at the center of mass of S , and let \mathbf{v}_0 be the Newtonian velocity relative to Σ_0 of the Newtonian CM frame of S . Then,

$$1 = \frac{1}{m_0} \sum_{k=1}^N m_k \quad (6)$$

and

$$\left. \begin{aligned} \mathbf{v}_0 &= \frac{1}{m_0} \sum_{k=1}^N m_k \mathbf{v}_k \\ \mathbf{u} + \mathbf{v}_0 &= \frac{1}{m_0} \sum_{k=1}^N m_k (\mathbf{u} + \mathbf{v}_k) \end{aligned} \right\}, \quad (7)$$

$\mathbf{u}, \mathbf{v}_k \in \mathbb{R}^3, m_k > 0, k = 0, 1, \dots, N$. Here m_0 is the Newtonian mass of the Newtonian system S , supposed concentrated at the center of mass of S , and \mathbf{v}_0 is the Newtonian velocity relative to Σ_0 of the Newtonian CM frame of the Newtonian system S in (5).

It follows from (6) that m_0 in (6)–(7) is given by the Newtonian resultant mass equation

$$m_0 = \sum_{k=1}^N m_k. \quad (8)$$

The derivation of the second equation in (7) from the first equation in (7) is immediate, following (i) the distributive law of scalar-vector multiplication, and (ii) the simple relationship (8) between the Newtonian resultant mass m_0 and its constituent masses $m_k, k = 1, \dots, N$.

4 Einsteinian systems of particles

In this section we present the Einsteinian analogs of the Newtonian expressions (5)–(8) listed in Section 3. The presented analogs are obtained in [1] by means of analogies that result from those presented in Section 2.

In this section, accordingly, as opposed to Section 3, $\mathbf{v}_k, k = 0, 1, \dots, N$, are Einsteinian velocities in \mathbb{R}_c^3 , and m_0 is the Einsteinian resultant mass, yet to be determined, of the masses $m_k, k = 1, \dots, N$, of an Einsteinian particle system S .

In analogy with (5), let

$$S = S(m_k, \mathbf{v}_k, \Sigma_0, N), \quad \mathbf{v}_k \in \mathbb{R}_c^3 \quad (9)$$

be an isolated Einsteinian system of N noninteracting material particles the k -th particle of which has invariant mass m_k and Einsteinian uniform velocity \mathbf{v}_k relative to an inertial frame $\Sigma_0, k = 1, \dots, N$. Furthermore, let m_0 be the resultant mass of S , considered as the mass of a virtual particle located at the center of mass of S (calculated in [1, Chap. 11]), and let \mathbf{v}_0 be the Einsteinian velocity relative to Σ_0 of the Einsteinian center of momentum (CM) frame of the Einsteinian system S in (9). Then, as shown in [1, p. 484], the relativistic analogs of the Newtonian expressions in (6)–(8) are, respectively, the following Einsteinian expressions in (10)–(12),

$$\left. \begin{aligned} \gamma_{\mathbf{v}_0} &= \frac{1}{m_0} \sum_{k=1}^N m_k \gamma_{\mathbf{v}_k} \\ \gamma_{\mathbf{u} \oplus \mathbf{v}_0} &= \frac{1}{m_0} \sum_{k=1}^N m_k \gamma_{\mathbf{u} \oplus \mathbf{v}_k} \end{aligned} \right\} \quad (10)$$

and

$$\left. \begin{aligned} \gamma_{\mathbf{v}_0} \mathbf{v}_0 &= \frac{1}{m_0} \sum_{k=1}^N m_k \gamma_{\mathbf{v}_k} \mathbf{v}_k \\ \gamma_{\mathbf{u} \oplus \mathbf{v}_0} (\mathbf{u} \oplus \mathbf{v}_0) &= \frac{1}{m_0} \sum_{k=1}^N m_k \gamma_{\mathbf{u} \oplus \mathbf{v}_k} (\mathbf{u} \oplus \mathbf{v}_k) \end{aligned} \right\}, \quad (11)$$

$\mathbf{u}, \mathbf{v}_k \in \mathbb{R}_c^3, m_k > 0, k = 0, 1, \dots, N$. Here m_0 ,

$$m_0 = \sqrt{\left(\sum_{k=1}^N m_k \right)^2 + 2 \sum_{\substack{j,k=1 \\ j < k}}^N m_j m_k (\gamma_{\Theta \mathbf{v}_j \oplus \mathbf{v}_k} - 1)} \quad (12)$$

is the relativistic invariant mass of the Einsteinian system S , supposed concentrated at the relativistic center of mass of S

(calculated in [1, Chap. 11]), and \mathbf{v}_0 is the Einsteinian velocity relative to Σ_0 of the Einsteinian CM frame of the Einsteinian system S in (9).

5 The relativistic invariant mass paradox of Einstein's special theory of relativity

In analogy with the Newtonian resultant mass m_0 in (8), which follows from (6), it follows from (10) that the Einsteinian resultant mass m_0 in (10)–(11) is given by the elegant Einsteinian resultant mass equation (12), as shown in [1, Chap. 11].

The Einsteinian resultant mass equation (12) presents a Paradox, called the *Relativistic Invariant Mass Paradox*, since, in general, this equation implies the inequality

$$m_0 > \sum_{k=1}^N m_k \quad (13)$$

so that, paradoxically, the invariant resultant mass of a system may exceed the sum of the invariant masses of its constituent particles.

The paradoxical invariant resultant mass equation (12) for m_0 is the relativistic analog of the non-paradoxical Newtonian resultant mass equation (8) for m_0 , to which it reduces in each of the following two special cases:

- (i) The Einsteinian resultant mass m_0 in (12) reduces to the Newtonian resultant mass m_0 in (8) in the limit as $c \rightarrow \infty$; and
- (ii) The Einsteinian resultant mass m_0 in (12) reduces to the Newtonian resultant mass m_0 in (8) in the special case when the system S is rigid, that is, all the internal motions in S of the constituent particles of S relative to each other vanish. In that case $\ominus\mathbf{v}_j \oplus \mathbf{v}_k = \mathbf{0}$ so that $\gamma_{\ominus\mathbf{v}_j \oplus \mathbf{v}_k} = 1$ for all $j, k = 1, N$. This identity, in turn, generates the reduction of (12) to (8).

The second equation in (11) follows from the first equation in (11) in full analogy with the second equation in (7), which follows from the first equation in (7) by the distributivity of scalar multiplication and by the simplicity of (8). However, while the proof of the latter is simple and well known, the proof of the former, presented in [1, Chap. 11], is lengthy owing to the lack of a distributive law for the Einsteinian scalar multiplication (see [1, Chap. 6]) and the lack of a simple relation for m_0 like (8), which is replaced by (12). Indeed, the proof of the former, that the second equation in (11) follows from the first equation in (11), is lengthy, but accessible to undergraduates who are familiar with the vector space approach to Euclidean geometry. However, in order to follow the proof one must familiarize himself with a large part of the author's book [1] and with its "gyrolanguage", as indicated in Section 2.

It is therefore suggested that interested readers may corroborate numerically (using a computer software like

MATLAB) the identities in (10)–(12) in order to gain confidence in their validity, before embarking on reading several necessary chapters of [1].

6 The physical significance of the paradox in Einstein's special theory of relativity

In this section we present two classically physical significant results that remain valid relativistically owing to the Relativistic Invariant Mass Paradox, according to which the relativistic analog of the classical resultant mass m_0 in (8) is, paradoxically, the relativistic resultant mass m_0 in (12).

To gain confidence in the physical significance that results from the analogy between

- (i) the Newtonian resultant mass m_0 in (8) of the Newtonian system S in (5) and
- (ii) the Einsteinian invariant resultant mass m_0 in (12) of the Einsteinian system S in (9)

we present below two physically significant resulting analogies. These are:

- (1) *The Kinetic Energy Theorem* [1, p. 487]: According to this theorem,

$$K = K_0 + K_1, \quad (14)$$

where

- (i) K_0 is the relativistic kinetic energy, relative to a given observer, of a virtual particle located at the relativistic center of mass of the system S in (9), with the Einsteinian resultant mass m_0 in (12); and
- (ii) K_1 is the relativistic kinetic energy of the constituent particles of S relative to its CM; and
- (iii) K is the relativistic kinetic energy of S relative to the observer.

The Newtonian counterpart of (14) is well known; see, for instance, [15, Eq. (1.55)]. The Einsteinian analog in (14) was, however, unknown in the literature since the Einsteinian resultant mass m_0 in (12) was unknown in the literature as well till its first appearance in [1]. Accordingly, Oliver D. Johns had to admit for his chagrin that "The reader (of his book; see [15, p. 392]) will be disappointed to learn that relativistic mechanics does not have a theory of collective motion that is as elegant and complete as the one presented in Chapter 1 for Newtonian mechanics."

The proof that m_0 of (12) is compatible with the validity of (14) in Einstein's special theory of relativity is presented in [1, Theorem 11.8, p. 487].

- (2) *Additivity of Energy and Momentum*: Classically, energy and momentum are additive, that is, the total energy and the total momentum of a system S of particles is, respectively, the sum of the energy and the sum of momenta of its constituent particles. Consequently,

also the resultant mass m_0 of S is additive, as shown in (8). Relativistically, energy and momentum remain additive but, consequently, the resultant mass m_0 of S is no longer additive. Rather, it is given by (12), which is the relativistic analog of (8).

The proof that m_0 of (12) is compatible with the additivity of energy and momentum in Einstein's special theory of relativity is presented in [1, pp. 488–491].

Thus, the Einsteinian resultant mass m_0 in (12) of the Einsteinian system S in (9) is the relativistic analog of the Newtonian resultant mass m_0 in (8) of the Newtonian system S in (5). As such, it is the Einsteinian resultant mass m_0 in (12) that is responsible for the extension of the validity of (14) and of the additivity of energy and momentum from classical to relativistic mechanics.

However, classically, mass is additive. Indeed, the Newtonian resultant mass m_0 equals the sum of the masses of the constituent particles, $m_0 = \sum_{k=1}^N m_k$, as we see in (8). Relativistically, in contrast, mass is not additive. Indeed, the Einsteinian resultant mass m_0 may exceed the sum of the masses of the constituent particles, $m_0 \geq \sum_{k=1}^N m_k$, as we see from (12). Accordingly, from the relativistic viewpoint, the resultant mass m_0 in (12) of a galaxy that consists of stars that move relative to each other exceeds the sum of the masses of its constituent stars. This excess of mass reveals its presence only gravitationally and, hence, we identify it as the mass of dark matter. Dark matter is thus virtual matter with positive mass, which reveals its presence only gravitationally. In particular, the dark mass m_{dark} of the Einsteinian system S in (9), given by (16) below, is the mass of virtual matter called the dark matter of S . To contrast the real matter of S with its virtual, dark matter, we call the former *bright* (or, luminous, or, baryonic) matter. The total mass m_0 of S , which can be detected gravitationally, is the composition of the bright mass m_{bright} of the real, bright matter of S , and the dark mass m_{dark} of the virtual, dark matter of S . This mass composition, presented in (15)–(17) in Section 7 below, quantifies the effects of dark matter.

7 The origin of the dark matter

Let

$$m_{bright} = \sum_{k=1}^N m_k \quad (15)$$

and

$$m_{dark} = \sqrt{2 \sum_{\substack{j,k=1 \\ j < k}}^N m_j m_k (\gamma_{\ominus \mathbf{v}_j \oplus \mathbf{v}_k} - 1)} \quad (16)$$

so that the Einsteinian resultant mass m_0 in (12) turns out to be a composition of an ordinary, bright mass m_{bright} of real matter and a dark mass m_{dark} of virtual matter according to

the equation

$$m_0 = \sqrt{m_{bright}^2 + m_{dark}^2} \quad (17)$$

The mass m_{bright} in (15) is the Newtonian resultant mass of the particles of the Einsteinian system S in (9). These particles reveal their presence gravitationally, as well as by radiation that they may emit and by occasional collisions.

In contrast, the mass m_{dark} in (16) is the mass of virtual matter in the Einsteinian system S in (9), which reveals its presence only gravitationally. In particular, it does not emit radiation and it does not collide. As such, it is identified with the dark matter of the Universe.

In our expanding universe, with accelerated expansion [16], relative velocities between some astronomical objects are significantly close to the speed of light c . Accordingly, since gamma factors γ_v approach ∞ when their relative velocities $\mathbf{v} \in \mathbb{R}_c^3$ approach the speed of light, it follows from (16) that dark matter contributes an increasingly significant part of the mass of the universe.

8 The origin of the dark energy

Under different circumstances dark matter may appear or disappear resulting in gravitational attraction or repulsion. Dark matter increases the gravitational attraction of the region of each stellar explosion, a supernova, since any stellar explosion creates relative speeds between objects that were at rest relative to each other prior to the explosion. The resulting generated relative speeds increase the dark mass of the region, thus increasing its gravitational attraction. Similarly, relative speeds of objects that converge into a star vanish in the process of star formation, resulting in the decrease of the dark mass of a star formation region. This, in turn, decreases the gravitational attraction or, equivalently, increases the gravitational repulsion of any star formation inflated region. The increased gravitational repulsion associated with star formation results in the accelerated expansion of the universe, first observed in 1998; see [6, p. 1764], [17] and [18, 19]. Thus, according to the present special relativistic dark matter/energy model, the universe accelerated expansion is a late time cosmic acceleration that began at the time of star formation.

9 The origin of the Pioneer anomaly

The Einsteinian resultant mass m_0 of our Solar system is given by the composition (17) of the bright mass m_{bright} and the dark mass m_{dark} of the Solar system. The bright mass m_{bright} of the Solar system equals the sum of the Newtonian masses of the constituents of the Solar system. Clearly, it is time independent. In contrast, the dark mass m_{dark} of the Solar system stems from the speeds of the constituents of the Solar system relative to each other and, as such, it is time dependent.

The Pioneer 10 and 11 spacecrafts and other deep space missions have revealed an anomalous acceleration known as the Pioneer anomaly [7, 8]. The Pioneer anomaly, described in the introductory section, results from an unmodelled acceleration, which is a small constant acceleration on top of which there is a smaller time dependent acceleration. A brief summary of the Pioneer anomaly is presented by K. Tangen, who asks in the title of [20]: “Could the Pioneer anomaly have a gravitational origin?”

Our answer to Tangen’s question is affirmative. Our dark matter/energy model, governed by the Einsteinian resultant mass m_0 in (15)–(17), offers a simple, elegant model that explains the Pioneer anomaly. The motion of any spacecraft in deep space beyond the Solar system is determined by the Newtonian law of gravity where the mass of the Solar system is modelled by the Einsteinian resultant mass m_0 in (17) rather than by the Newtonian resultant mass m_0 in (8). It is the contribution of the dark mass m_{dark} to the Einsteinian resultant mass m_0 in (15)–(17) that generates the Pioneer anomaly.

Ultimately, our dark matter/energy model, as dictated by the paradoxical Einsteinian resultant mass m_0 in (12), will be judged by how well the model accords with astrophysical and astronomical observations. Since our model is special relativistic, only uniform velocities are allowed. Hence, the model can be applied to the solar system, for instance, under the assumption that, momentarily, the solar system can be viewed as a system the constituents of which move uniformly.

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A Critical Analysis of Universality and Kirchhoff's Law: A Return to Stewart's Law of Thermal Emission

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It has been advanced, on experimental (P.-M. Robitaille, *IEEE Trans. Plasma Sci.*, 2003, v. 31(6), 1263–1267) and theoretical (P.-M. Robitaille, *Progr. Phys.*, 2006, v. 2, 22–23) grounds, that blackbody radiation is not universal and remains closely linked to the emission of graphite and soot. In order to strengthen such claims, a conceptual analysis of the proofs for universality is presented. This treatment reveals that Gustav Robert Kirchhoff has not properly considered the combined effects of absorption, reflection, and the directional nature of emission in real materials. In one instance, this leads to an unintended movement away from thermal equilibrium within cavities. Using equilibrium arguments, it is demonstrated that the radiation within perfectly reflecting or arbitrary cavities does not necessarily correspond to that emitted by a blackbody.

1 Introduction

Formulated in 1858, Stewart's Law [1] states that when an object is studied in thermal equilibrium, its absorption is equal to its emission [1]. Stewart's formulation leads to the realization that the emissive power of any object depends on its temperature, its nature, and on the frequency of observation. Conversely, Gustav Kirchhoff [2–4] reaches the conclusion that the emissive power of a body is equal to a universal function, dependent only on its temperature and the frequency of interest, and independent of its nature and that of the enclosure. He writes: "*When a space is surrounded by bodies of the same temperature, and no rays can penetrate through these bodies, every pencil in the interior of the space is so constituted, with respect to its quality and intensity, as if it proceeded from a perfectly black body of the same temperature, and is therefore independent of the nature and form of the bodies, and only determined by the temperature*" (see [4], p. 96–97).

At the same time, Max Planck, in his *Theory of Heat Radiation*, reminds us that: "... *in a vacuum bounded by totally reflecting walls any state of radiation may persist*" (see [5], §51). Planck is aware that a perfect reflector does not necessarily produce blackbody radiation in the absence of a perfect absorber [6]. It is not simply a matter of waiting a sufficient amount of time, but rather the radiation will "*persist*" in a non-blackbody, or arbitrary, state. Planck re-emphasizes this aspect when he writes: "*Every state of radiation brought about by such a process is perfectly stationary and can continue infinitely long, subject, however, to the condition that no trace of an emitting or absorbing substance exists in the radiation space. For otherwise, according to Sec. 51, the distribution of energy would, in the course of time, change through the releasing action of the substance irreversibly, i.e., with an increase of the total entropy, into the stable distribu-*

tion corresponding to black radiation" (see [5], §91). Planck suggests that if an absorbing substance is present, blackbody radiation is produced. Such a statement is not supported scientifically. In fact, a perfect absorber, such as graphite or soot, is required [6–8].

Recently, I have stated [6–8] that cavity radiation was not universal and could only assume the normal distribution (i.e. that of the blackbody) when either the walls of the cavity, or the objects it contains, were perfectly absorbing. These ideas are contrary to the expressed beliefs of Kirchhoff and Planck. Therefore, they deserve further exposition by revisiting Kirchhoff's basis for universality. In combination with a historical review of blackbody radiation [8], such an analysis demonstrates that claims of universality were never justified [6–8].

2.1 Kirchhoff's first treatment of his law

Kirchhoff's first presentation of his law [2] involved two plates, C and c , placed before one another (see Fig. 1). Neither plate was perfectly absorbing, or black. Behind each plate, there were mirrors, R and r , which ensured that all the radiation remained between the plates. Kirchhoff assumed that one of the plates, c , was made of a special material which absorbed only one wavelength and transmitted all others. This assumption appears to have formed the grounds for the most strenuous objections relative to Kirchhoff's first derivation [9–11]. Kirchhoff moved to insist (see [9] for a treatment in English) that, under these conditions, at a certain temperature and wavelength, all bodies had the same ratio of emissive and absorptive powers.

The fallacy with Kirchhoff's argument lays not only in the need for a special material in the second plate, c , as so many have hinted [9–11]. The most serious error was that he did not consider the reflection from the plates themselves. He treated

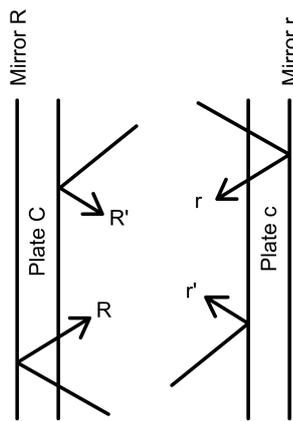


Fig. 1: Schematic representation of Kirchhoff's first proof [2]. C and c represented objects of a specified nature (see text). R and r corresponded to perfectly reflecting mirrors. Note that Kirchhoff had neglected the reflection from the surfaces of C and c denoted as R' and r' .

the reflection as coming only from the mirrors placed behind the plates. But this dealt with the problem of transmission, not reflection. As a result, Kirchhoff ignored the reflection produced by the surfaces of the plates.

The total radiation leaving from the surface of each plate, given thermal equilibrium, is obtained, not only by its emission, E (or e), but rather by the *sum* of its emission, E (or e), and reflection, R' (or r'). It is only when the plates are black that surface reflection can be neglected. Consequently, if Kirchhoff insists that surface reflection itself need not be addressed ($R' = r' = 0$), he simply proves that the ratio of emission to absorption is the same for all blackbodies, not for all bodies. The entire argument, therefore, is flawed because Kirchhoff ignored the surface reflection of each plate, and is considering all reflection as originating from the perfectly reflecting mirrors behind the plates. A proper treatment would not lead to universality, since the total radiation from plate C was $E + R'$ not simply E , where R' denotes the reflection from surface C (see Fig. 1). Similarly, the total radiation from plate c was $e + r'$, not simply e , where r' denotes the reflection from surface c . The mirrors, R and r , are actually dealing only with transmission through plates C and c . The conceptual difficulty when reviewing this work is that Kirchhoff apparently treats reflection, since mirrors are present. In fact, he dismisses the issue. The mirrors cannot treat the reflection off the surfaces of C and c . They deal with transmission. Kirchhoff's incorrect visualization of the effect of reflection is also a factor in his second proof.

2.2 Kirchhoff's second treatment of his law

Kirchhoff's second treatment of his law [3, 4] is much more interesting conceptually and any error will consequently be more difficult to locate. The proof is complex, a reality recy-

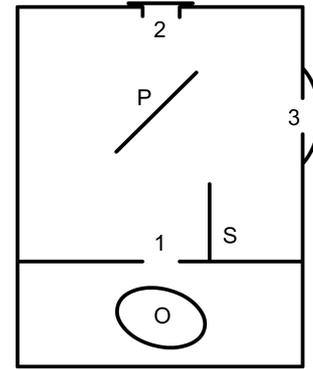


Fig. 2: Schematic representation of Kirchhoff's second proof [3, 4]. The cavity contained three openings, labeled 1, 2, and 3. There was also a plate, P , which was perfectly transmitting for the frequency and polarization of interest, and perfectly reflecting for all others. While the existence of such a plate can be the source of objections relative to Kirchhoff's proof [10], the discussion in this work does not center on the nature of the plate. Idealized objects can be assumed as valid as they represent (more or less) mathematical extensions of physical observations (see text). A black screen, S , was used to prevent radiation from traveling directly between openings 1 and 3. An object, which was either perfectly absorbing or arbitrary, was placed in the enclosure located behind opening 1. The key to Kirchhoff's proof relied on rapidly changing the covering of opening 3, from a perfect concave mirror to a perfectly absorbing surface. In Kirchhoff's initial presentation, the entire cavity was perfectly absorbing [3, 4]. However, Kirchhoff extended his result to be independent of the nature of the walls, making it acceptable to consider the entire cavity as perfectly reflecting (see text).

ognized by Stewart in his *Reply*: "*I may remark, however, that the proof of the Heidelberg Professor is so very elaborate that I fear it has found few readers either in his own country or in this*" [12].

Kirchhoff began by imagining a cavity whose walls were perfectly absorbing (see Fig. 2). In the rear of the cavity was an enclosure wherein the objects of interest were placed. There were three openings in the cavity, labeled 1, 2, and 3. He conceived that openings 2 and 3 could each be sealed with a perfectly absorbing surface. As a result, when Kirchhoff did this, he placed his object in a perfectly absorbing cavity [6]. He eventually stipulated that the experiment was independent of the nature of the walls, in which case the cavity could be viewed as perfectly reflecting [6]. Yet, as has been previously highlighted [6], the scenario with the perfectly reflecting cavity required, according to Planck, the introduction of a minute particle of carbon [5, 8]. Hence, I have argued that Kirchhoff's analysis was invalid on this basis alone [6]. By carefully considering Kirchhoff's theoretical constructs, the arguments against blackbody radiation, within a perfect reflecting enclosure, can now be made from a slightly different perspective.

Kirchhoff's analysis of his cavity (see Fig. 2) was ingenious. He set strict conditions for the positions of the walls

which linked the openings 1 and 2, and which contained opening 3. The key was in the manner wherein opening 3 was handled. Kirchhoff permitted opening 3 to be covered either with a perfect absorber or with a perfect concave mirror. He then assumed that equilibrium existed in the cavity and that he could instantaneously change the covering at opening 3. Since equilibrium was always preserved, Kirchhoff could then treat the rays within the cavity under these two different conditions and, hence, infer the nature of the radiation within the cavity at equilibrium.

Kirchhoff initially demonstrated that, if the enclosed object and the cavity were perfectly absorbing, the radiation was denoted by the universal function of blackbody radiation. He then replaced the object with an arbitrary one, and concluded, once again, that the radiation was black. Kirchhoff's presentation was elegant, at least when the cavity was perfectly absorbing. The Heidelberg Professor extended his findings to make them independent of the nature of the walls of the enclosure, stating that the derivation was valid, even if the walls were perfectly reflecting. He argued that the radiation within the cavity remained blackbody radiation. Let us revisit what Kirchhoff had done.

Since the walls can be perfectly reflecting, this state is adopted for our analysis. Opening 3 can once again be covered, either by a concave mirror or by a perfectly absorbing surface. An arbitrary object, which is not a blackbody, is placed in the cavity. The experiment is initiated with the perfect concave mirror covering opening 3. As shown in Section 3.1.2, under these conditions, the cavity contains radiation whose nature depends not on the cavity, but on the object. This radiation, in fact, is not black. This can be seen, if the object was taken as perfectly reflecting. The arbitrary radiation is weaker at all frequencies. Thus, when an arbitrary object is placed in the enclosure, the intensity of the radiation within the cavity, at any given frequency, does not correspond to that predicted by the Planckian function (see Section 3.1.2). However, when opening 3 is covered by a perfectly absorbing substance, the radiation in the cavity becomes black (see Sections 3.1.2 and 3.2). The emission from the object is that which the object emits and which it reflects. The latter originates from the surface of opening 3 (see Section 3.2). When the perfect absorber is placed over opening 3, the entire cavity appears to hold blackbody radiation. Therefore, by extending his treatment to the perfect reflector, Kirchhoff is inadvertently jumping from one form of cavity radiation (case 1: the concave mirror, object radiation) to another (case 2: the perfect absorber, blackbody radiation) when the covering on opening 3 is changed. At that moment, the cavity moves out of equilibrium.

Thus, Kirchhoff's proof is invalid. This is provided, of course, that the test began with the perfect concave mirror covering opening 3. Only under these circumstances would Kirchhoff's proof fail. Nonetheless, the experimental proof cannot be subject to the order in which manipulations are ex-

ecuted. This is because the validity of equilibrium arguments is being tested. Consequently, nothing is independent of the nature of the walls. This is the lesson provided to us by Balfour Stewart in his treatise when he analyzes radiation in a cavity temporarily brought into contact with another cavity [8]. Dynamic changes, not equilibrium, can be produced in cavities, if reflectors are used. This is the central error relative to Kirchhoff's second attempt at universality [3, 4].

There are additional minor problems in Kirchhoff's presentation [3, 4]. In §13 of his proof [3, 4], Kirchhoff is examining an arbitrary object within a perfectly absorbing cavity. It is true that the resultant cavity radiation will correspond to a blackbody, precisely because the walls are perfectly absorbing (see Section 3.1.1). However, Kirchhoff states: "*the law §3 is proved under the assumption that, of the pencil which falls from surface 2 through opening 1 upon the body C, no finite part is reflected by this back to the surface 2; further, that the law holds without limitation, if we consider that when the condition is not fulfilled, it is only necessary to turn the body C infinitely little in order to satisfy it, and that by such a rotation the quantities E and A undergo only and [sic] infinitely small change*" (see [4], p. 92). Of course, real bodies can have diffuse reflection. In addition, rotation does not ensure that reflection back to surface 2 will not take place. Real bodies also have directional spectral emission, such that the effect of rotation on E and A is not necessarily negligible. These complications are of little significance within a perfectly absorbing cavity. The radiation within such enclosures is always black (see Section 3.1.1). Conversely, the problems cannot be dismissed in the perfect reflector and the entire proof for universality, once again, is invalid.

For much of the 19th century, the understanding of blackbody radiation changed little, even to the time of Planck [11]. No laboratory proof of Kirchhoff's Law was ever produced, precisely because universality could not hold. Only theoretical arguments prevailed [10]. Yet, such findings cannot form the basis for a law of physics. Laws stem from experiments and are fortified by theory. They are not born *de novo*, using mathematics without further validation. It is not possible to ensure that black radiation exists, within a perfectly reflecting cavity, without recourse at least to a carbon particle [6, 8]. In fact, this is the route which Planck utilized in treating Kirchhoff's Law [5, 8].

3 Thermal equilibrium in cavities

A simple mathematical treatment of radiation, under conditions of thermal equilibrium, begins by examining the fate of the total incoming radiation, Γ , which strikes the surface of an object. The various portions of this radiation are either absorbed (A), reflected (R), or transmitted (T) by the object. If normalized, the sum of the absorbed, reflected, or transmitted radiation is equal to $\alpha + \rho + \tau = 1$. Here, absorptivity, α , corresponds to the absorbed part of the incoming radi-

tion/total incoming radiation. Similarly, the reflectivity, ρ , is the reflected part of the incoming radiation/total incoming radiation. Finally, the transmissivity, τ , involves the transmitted part of the incoming radiation/total incoming radiation. If all objects under consideration are fully opaque, then $1 = \alpha + \rho$.

Stewart's Law [1] states that, under conditions of thermal equilibrium, the ability of an object to absorb light, α , is exactly equal to its ability to emit light, ε . Nonetheless, for this presentation, Stewart's Law is not assumed to be valid [1]. The question arises only in the final Section 4.2, when two objects are placed within a perfectly reflecting cavity. Emissivity, ε , is standardized relative to lamp-black [8] and, for such a blackbody, it is equal to 1. For a perfect reflector, the emissivity, ε , is 0. All other objects hold values of emissivity between these two extremes. If thermal equilibrium is not established, then ε and α are not necessarily equal [8].

If a cubical cavity is considered with walls P^1 , P^2 , P^3 , P^4 , P^5 (top surface), and P^6 (bottom surface), the following can be concluded at thermal equilibrium: since P^1 and P^3 are equal in area and opposite one another, then the total radiation from these walls must be balanced, $\Gamma_{p1} - \Gamma_{p3} = 0$. Similarly, $\Gamma_{p2} - \Gamma_{p4} = 0$ and $\Gamma_{p5} - \Gamma_{p6} = 0$. As such, $\Gamma_{p1} = \Gamma_{p3}$ and $\Gamma_{p2} = \Gamma_{p4}$. If one considers pairs of adjacent walls, then $(\Gamma_{p1} + \Gamma_{p2}) - (\Gamma_{p3} + \Gamma_{p4}) = 0$. It is possible to conclude that $\Gamma_{p1} = \Gamma_{p2} = \Gamma_{p3} = \Gamma_{p4}$ and, using symmetry, it can finally be concluded that $\Gamma_{p1} = \Gamma_{p2} = \Gamma_{p3} = \Gamma_{p4} = \Gamma_{p5} = \Gamma_{p6}$. Consequently, with normalization, $\Gamma_c = \frac{1}{6}(\Gamma_{p1} + \Gamma_{p2} + \Gamma_{p3} + \Gamma_{p4} + \Gamma_{p5} + \Gamma_{p6})$. For an opaque cavity, the total radiation coming from the cavity, Γ_T , is given by $\Gamma_T = \varepsilon_c \Gamma_c + \rho_c \Gamma_c = \varepsilon_c \Gamma_c + (1 - \alpha_c) \Gamma_c$. This states that the total emission from the cavity must be represented by the sum of its internal emission and reflection. If the cavity is constructed from perfectly absorbing walls, $\alpha_c = 1$, $\rho_c = 0$, yielding $\Gamma_T = \varepsilon_c \Gamma_c$. The cavity is black and ε_c must now equal 1, by necessity. Stewart's Law [1] has now been proved for blackbodies. If the cavity is made from perfectly reflecting walls, at thermal equilibrium, $\varepsilon_c \Gamma_c + (1 - \alpha_c) \Gamma_c = 0$. There is also no source of radiation inside the cavity ($\varepsilon_c = 0$) and $(1 - \alpha_c) \Gamma_c = 0$, leading explicitly to $\Gamma_c = 0$. Because $\Gamma_c = 0$, the total radiation monitored $\Gamma_T = \varepsilon_c \Gamma_c + \rho_c \Gamma_c = 0$.

These conclusions can be extended to perfectly absorbing and reflecting cavities of rectangular (or arbitrary) shapes. The central point is that a perfectly reflecting cavity can sustain no radiation, a first hint that universality cannot be valid. Planck only obtains blackbody radiation, in such cavities, by invoking the action of a carbon particle [6, 8]. This special case will be treated in Sections 3.1.1 and 3.2.

3.1 An object in a perfect cavity

At thermal equilibrium, the total emission from the surface of the object, Γ_{so} , is equal to that from the surface of the cavity, Γ_{sc} . When normalizing, the total emission, Γ_T , will therefore be as follows: $\Gamma_T = \frac{1}{2} \Gamma_{so} + \frac{1}{2} \Gamma_{sc}$. The total ra-

diation from the surface of the object is equal to that which it emits plus that which it reflects, $\Gamma_{so} = [\varepsilon_o \Gamma_o + \rho_o \Gamma_c]$, and similarly for the surface of the cavity, $\Gamma_{sc} = [\varepsilon_c \Gamma_c + \rho_c \Gamma_o]$. Therefore, at equilibrium, $[\varepsilon_o \Gamma_o + \rho_o \Gamma_c] = [\varepsilon_c \Gamma_c + \rho_c \Gamma_o]$ or $\Gamma_o[\varepsilon_o - \rho_c] = \Gamma_c[\varepsilon_c - \rho_o]$. Solving for either Γ_o or Γ_c , we obtain that $\Gamma_o = \Gamma_c \frac{[\varepsilon_c - \rho_o]}{[\varepsilon_o - \rho_c]}$ and $\Gamma_c = \Gamma_o \frac{[\varepsilon_o - \rho_c]}{[\varepsilon_c - \rho_o]}$.

3.1.1 An arbitrary object in a perfectly absorbing cavity

In such a case $\varepsilon_c = 1$, $\rho_c = 0$. Since $\Gamma_T = \frac{1}{2} \Gamma_{so} + \frac{1}{2} \Gamma_{sc}$, then $\Gamma_T = \frac{1}{2} \left(\varepsilon_o \Gamma_o \frac{[\varepsilon_c - \rho_o]}{[\varepsilon_o - \rho_c]} + \rho_o \Gamma_c \right) + \frac{1}{2} \left(\varepsilon_c \Gamma_c + \rho_c \Gamma_o \frac{[\varepsilon_c - \rho_o]}{[\varepsilon_o - \rho_c]} \right)$. It is readily shown that $\Gamma_T = \Gamma_c$. Note that no use of Stewart's Law [1] was made in this derivation. In any case, when an object is placed within a cavity, which is perfectly absorbing, the emitted spectrum is independent of the object and depends only on the nature of the cavity. A blackbody spectrum is produced. This was the condition which prevailed over much of the 19th century when cavities were often lined with soot [8]. If the radiation was *independent of the nature of the walls, or of the object*, it was because the walls were coated with this material [8].

3.1.2 An arbitrary object in a perfectly reflecting cavity

In such a case $\varepsilon_c = 0$, $\rho_c = 1$. Since $\Gamma_T = \frac{1}{2} \Gamma_{so} + \frac{1}{2} \Gamma_{sc}$, then $\Gamma_T = \frac{1}{2} \left(\varepsilon_o \Gamma_o + \rho_o \Gamma_o \frac{[\varepsilon_o - \rho_c]}{[\varepsilon_c - \rho_o]} \right) + \frac{1}{2} \left(\varepsilon_c \Gamma_o \frac{[\varepsilon_o - \rho_c]}{[\varepsilon_c - \rho_o]} + \rho_c \Gamma_o \right)$. It is readily shown that $\Gamma_T = \Gamma_o$. Note, once again, that no use of Stewart's Law [1] was made in this derivation. When an object is placed within a cavity which is perfectly reflecting, the emitted spectrum is determined only by the object and is independent of the nature of the cavity. If the object is perfectly absorbing, like a carbon particle [6, 8], a blackbody spectrum will be obtained. Furthermore, if an arbitrary object is placed within a cavity, which is perfectly reflecting, the emitted spectrum *is dependent only on the nature of the object*. One observes *object radiation*, not blackbody radiation, because the object was never black *a priori*. This is the condition which Kirchhoff has failed to realize when he extended his treatment to be independent of the nature of the walls in his 1860 proof [3, 4], as seen in Section 2.

3.1.3 An arbitrary object in an arbitrary cavity

Consider such a general case. Since $\Gamma_T = \frac{1}{2} \Gamma_{so} + \frac{1}{2} \Gamma_{sc}$, then $\Gamma_T = \frac{1}{2} \left(\varepsilon_o \Gamma_o + \rho_o \Gamma_o \frac{[\varepsilon_o - \rho_c]}{[\varepsilon_c - \rho_o]} \right) + \frac{1}{2} \left(\varepsilon_c \Gamma_o \frac{[\varepsilon_o - \rho_c]}{[\varepsilon_c - \rho_o]} + \rho_c \Gamma_o \right)$ or alternatively, we have $\Gamma_T = \frac{1}{2} \left(\varepsilon_o \Gamma_c \frac{[\varepsilon_c - \rho_o]}{[\varepsilon_o - \rho_c]} + \rho_o \Gamma_c \right) + \frac{1}{2} \left(\varepsilon_c \Gamma_c + \rho_c \Gamma_c \frac{[\varepsilon_c - \rho_o]}{[\varepsilon_o - \rho_c]} \right)$. In this case, the expressions cannot be further simplified and the initial form, $\Gamma_T = \frac{1}{2} \Gamma_{so} + \frac{1}{2} \Gamma_{sc}$, can be maintained. Therefore, the total radiation emitted from such a cavity is a mixture *depending on both the characteristics of the object and the walls of the cavity*.

This highlights that cavities do not always contain black radiation and that universality is invalid [6–8].

3.2 An arbitrary object and a carbon particle in a perfectly reflecting cavity

If thermal equilibrium exists between an opaque object, o , a carbon particle, p , and a cavity, c , then $[\varepsilon_o\Gamma_o + \rho_o\Gamma_p + \rho_o\Gamma_c] - [\varepsilon_p\Gamma_p + \rho_p\Gamma_o + \rho_p\Gamma_c] + [\varepsilon_c\Gamma_c + \rho_c\Gamma_o - \rho_c\Gamma_p] = 0$. Since the cavity is perfectly reflecting, $\Gamma_c = 0$, $\varepsilon_c = 0$, and $\rho_c = 1$, yielding, $\varepsilon_o\Gamma_o + \rho_o\Gamma_p - \varepsilon_p\Gamma_p - \rho_p\Gamma_o + \Gamma_o - \Gamma_p = 0$, and with rearrangement, $(\varepsilon_o + \rho_o - 1)\Gamma_p - \varepsilon_p\Gamma_p + (1 - \rho_p)\Gamma_o = 0$. If we take Stewart's Law ($\varepsilon_p = \alpha_p$; $\varepsilon_o = \alpha_o$) as valid [1], we can see that $\varepsilon_o + \rho_o = 1$, and then $(1 - \rho_p)\Gamma_o = \varepsilon_p\Gamma_p$, leading directly to $\Gamma_o = \Gamma_p$. Alternatively, we may notice that, by definition, $\rho_o = 1 - \alpha_o$ and $\rho_p = 1 - \alpha_p$, then, $\Gamma_o = \frac{(\varepsilon_p - \varepsilon_o + \alpha_o)}{\alpha_p} \Gamma_p$. If we take the particle to be black, we can simplify to $\Gamma_o = (1 - \varepsilon_o + \alpha_o)\Gamma_p$. Therefore, if we then observe the radiation in the cavity and find it to be black, since the particle is also black, Stewart's law is verified. This is because Γ_o will be black and equal to Γ_p only when $\varepsilon_o = \alpha_o$.

The problem can be examined from a slightly different angle in order to yield a little more insight, but the same conclusions hold. Because the objects are in a perfect reflector, then the radiation coming off their surfaces can be expressed as $\Gamma_{so} = \varepsilon_o\Gamma_o + \rho_o\Gamma_p$ and $\Gamma_{sp} = \varepsilon_p\Gamma_p + \rho_p\Gamma_o$. Given thermal equilibrium, the production of radiation from each object must be equal, $\Gamma_{so} = \Gamma_{sp}$, and thus $\varepsilon_o\Gamma_o + \rho_o\Gamma_p = \varepsilon_p\Gamma_p + \rho_p\Gamma_o$. Consequently, $\Gamma_o = \frac{[\varepsilon_p - \rho_o]}{[\varepsilon_o - \rho_p]} \Gamma_p$ (see Section 3.1). If the particle is black, $\varepsilon_p = 1$ and $\rho_p = 0$, and $\Gamma_o = \frac{(1 - \rho_o)}{\varepsilon_o} \Gamma_p$. As a result of thermal equilibrium, the object must be producing a total emission which appears black in nature. Γ_o must equal Γ_p . All solutions involve $\rho_o + \varepsilon_o = 1$, which as stated above, is a proof of Stewart's Law ($\varepsilon_o = \alpha_o$). The object takes the appearance of a blackbody through the sum of its emission and reflection. The presence of completely black radiation within a cavity filled in this manner constitutes an explicit verification of Stewart's Law [1], as mentioned above. Since such cavities are known to be black, Stewart's Law has been proven. In fact, we have returned to the first portion of Section 3.1.2. The effect is the same as if the walls of the cavity were perfectly absorbing. This is the point Planck failed to realize when he placed the carbon particle within the perfectly reflecting cavity and gave it a catalytic function [5, 6, 8].

4 Conclusions

Nearly 150 years have now passed since Gustav Robert Kirchhoff first advanced his Law of Thermal Radiation. Kirchhoff's Law [2–4] was far reaching. Its universal nature had a profound effect on the scientists of the period. At the time, many of these men were trying to discover the most

general laws of nature. Hence, the concept of universality had great appeal and became ingrained in the physics literature. As a result, Kirchhoff's Law has endured, despite controversy [10], until this day. Recently, I have questioned universality [6, 7]. It is doubtful that Kirchhoff's Law can long survive the careful discernment of those physicists who wish to further pursue this issue.

At the same time, Kirchhoff's Law seems inseparably tied to Max Planck's equation [13]. As such, could a reevaluation of Kirchhoff's ideas compromise those of Max Planck [13]? In the end, it is clear that this cannot be the case [8]. Planck's solution to the blackbody problem remains valid for cavities which are perfectly absorbing. Thus, physics loses nothing of the Planck and Boltzmann constants, h and k , which were born from the study of heat radiation [1, 8]. That blackbody radiation loses universal significance also changes nothing, in fact, relative to the mathematical foundations of quantum theory. However, the same cannot be said relative to experimental findings [8]. In the end, the physics community may well be led to reconsider some of these positions [8].

Balfour Stewart [1] preceded Kirchhoff [2–4] by nearly two years in demonstrating, under equilibrium, the equality between absorptivity and emissivity. Stewart's treatment, unlike Kirchhoff's, does not lead to universality [1, 8, 9, 14] but, rather, shows that the emissive power of an object is dependent on its nature, its temperature, and the frequency of observation. This is true even within cavities, provided that they do not contain a perfect absorber. It is only in this special circumstance that the nature of the object is eliminated from the problem. Yet, this is only because the nature of the carbon itself controls the situation. Stewart also properly treats emission and reflection in his *Treatise* [14]. Despite popular belief to the contrary [9], Stewart's interpretation is the correct solution. Conversely, Kirchhoff's formulation, not only introduced error, but provided justification for setting temperatures inappropriately. I have repeatedly expressed concern in this area [6–8]. It can be argued that Stewart's analysis lacked mathematical sophistication [9]. Stewart himself [12] counters the point [8]. Nonetheless, it is doubtful that the important consequences of Stewart's work can continue to be ignored. Justice and the proper treatment of experimental data demand otherwise.

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Dedication

This work is dedicated to the memory of my beloved mother, Jacqueline Alice Roy (May 12, 1935 – December 02, 1996).

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Blackbody Radiation and the Carbon Particle

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Since the days of Kirchhoff, blackbody radiation has been considered to be a universal process, independent of the nature and shape of the emitter. Nonetheless, in promoting this concept, Kirchhoff did require, at the minimum, thermal equilibrium with an enclosure. Recently, the author stated (P.-M. Robitaille, *IEEE Trans. Plasma Sci.*, 2003, v. 31(6), 1263–1267; P.-M. Robitaille, *Progr. in Phys.*, 2006, v. 2, 22–23), that blackbody radiation is not universal and has called for a return to Stewart’s law (P.-M. Robitaille, *Progr. in Phys.*, 2008, v. 3, 30–35). In this work, a historical analysis of thermal radiation is presented. It is demonstrated that soot, or lampblack, was the standard for blackbody experiments throughout the 1800s. Furthermore, graphite and carbon black continue to play a central role in the construction of blackbody cavities. The advent of universality is reviewed through the writings of Pierre Prévost, Pierre Louis Dulong, Alexis Thérèse Petit, Jean Baptiste Joseph Fourier, Siméon Denis Poisson, Frédéric Hervé de la Provostaye, Paul Quentin Desain, Balfour Stewart, Gustav Robert Kirchhoff, and Max Karl Ernst Ludwig Planck. These writings illustrate that blackbody radiation, as experimentally produced in cavities and as discussed theoretically, has remained dependent on thermal equilibrium with at least the smallest carbon particle. Finally, Planck’s treatment of Kirchhoff’s law is examined in detail and the shortcomings of his derivation are outlined. It is shown once again, that universality does not exist. Only Stewart’s law of thermal emission, not Kirchhoff’s, is fully valid.

1 Introduction

If real knowledge is to be derived from an equation, it is often necessary to reassess the experiments that gave it life. A thorough evaluation of these developments, relative to Planck’s equation [1, 2], can be found in Hans Kangro’s *Early History of Planck’s Radiation Law* [3]. Kangro reminds us of the need to study important milestones relative to physical ideas: “*Only concern with details appearing in sources reveals — often unexpectedly — what has really happened historically, and allowed something to be divined from that history as to ‘how it really happened’*” [3; p. 3]. He then sets forth a fascinating account of the history of the law [1, 2] which gave birth to modern physics. Kangro’s work [3] is unique for its balance relative to experimental methods and theoretical foundations. It covers, in considerable detail, the period from Kirchhoff to Planck [3]. Hoffmann’s work [4] is also valuable since it is short, well written, and reviews the experiments from which Planck formulated his equation [1, 2]. Kuhn’s text [5] centers on the theoretical basis of Planck’s law. It has been the subject of substantial justified criticism, primarily for advancing that Planck was not the first to introduce quantized processes [6–8]. It is by using such works, and the collection of the scientific literature, that we may revisit the days of Planck [9–16] and judge, with perhaps greater insight than our forefathers, the soundness of the claims on which universality in blackbody radiation rests.

At the onset, it should be emphasized that the validity of Planck’s equation [1, 2], as a mathematical solution to the blackbody problem, is not being disputed in any way. The accuracy and merit of Planck’s equation [1, 2] has been established beyond question. Nonetheless, two aspects of Planck’s formulation are being brought to the forefront. First, that Planck [1, 2, 9–16], Einstein [17, 18], and all of physics have yet to ascribe a direct physical process for the production of blackbody radiation [19]. That is to say, blackbody radiation remains unlinked to a specific and identifiable physical entity (such as the nucleus, the electron, etc). Second, that blackbody radiation is not universal, contrary to what Kirchhoff has concluded [20–22] and Planck believed [1, 2, 9].

I have previously stated that Kirchhoff’s law [20–22], and, as a necessary result, Planck’s law [1, 2] and blackbody radiation, are not universal in nature [23–25]. Kirchhoff’s conclusions hold only for objects in thermal equilibrium with a perfectly absorbing enclosure [23]. Under these conditions, Kirchhoff’s cavities act, in essence, as transformers of light [23]. Any object placed within them will give a total emission which is the sum of its own emission and the reflection of the emission from the cavity wall. Consequently, the entire cavity appears black [23, 25]. Outside the restrictions imposed by such a cavity, universality does not exist [23–25]. As for Kirchhoff’s law, it holds only under very limited experimental conditions: the walls of these cavities, or the objects they contain, must be perfectly absorbing (see [25] for a proof).

Otherwise, Kirchhoff's law in its widest sense (i.e. universality) does not hold [23]. However, that section of Kirchhoff's law specifically addressing the equality between emissivity and absorptivity at equilibrium is valid. This is Stewart's law [26], not Kirchhoff's [20–22], as will be seen below.

In Planck's words (see [9; §44]), Kirchhoff's law of thermal emission holds that: "With these assumptions, according to equations (46), (45), and (43), Kirchhoff's law holds, $E/A = I = d\sigma \cos \theta d\Omega K_\nu d\nu$, i.e., the ratio of the emissive power to the absorbing power of any body is independent of the nature of the body". The implications of Kirchhoff's law are best summarized in the words of its originator: "When a space is surrounded by bodies of the same temperature, and no rays can penetrate through these bodies, every pencil in the interior of the space is so constituted, with respect to its quality and intensity, as if it proceeded from a perfectly black body of the same temperature, and is therefore independent of the nature and form of the bodies, and only determined by the temperature... In the interior of an opaque glowing hollow body of given temperature there is, consequently, always the same brightness whatever its nature may be in other respects" [22; §17]. Kirchhoff's law states that, for all bodies, the ratio of emissive to absorbing power is a function of only wavelength and temperature, given thermal equilibrium with an enclosure. All that Kirchhoff knew about his universal function, in 1859, was that its value was zero in the visible range at low temperatures, non-zero at high temperatures, and non-zero at the longer wavelengths at all temperatures [3; p. 7]. Planck [1, 2], in 1900, eventually defined the function on the right side of Kirchhoff's law [20–22].

Given thermal equilibrium within an enclosure, Kirchhoff's law [20–22] states that the ability of an object to emit a photon is equal to its ability to absorb one. This aspect of Kirchhoff's work [20–22], properly called Stewart's law [25, 26], is not being questioned. If equilibrium holds, the equality between emissivity and absorptivity has been experimentally demonstrated (see [25] for a complete discussion). It is only when objects are permitted to radiate freely, that equality may fail. Discussions on this issue have been published [27–29]. It has been argued that the equality between absorptivity and emissivity may, in fact, still be applicable for freely radiating bodies, provided that "the distribution over material states is the equilibrium condition" [27]. At the same time, it should be realized that, under all non-equilibrium conditions, these laws collapse [20–22, 25, 26].

The vast experimental knowledge relative to thermal emission reveals that virtually all materials fall far short of exhibiting blackbody behavior. Yet, Max Thiesen, a pupil of Kirchhoff, in 1900 stated that: "we have become accustomed to treat radiation independently of the emitting body" and therefore, this radiation should "be designated simply as black radiation" [3; p. 184]. Experimental reality illustrates that nothing in nature behaves like a blackbody. Kirchhoff's statement that: "In the interior of an opaque glowing hol-

low body of given temperature there is, consequently, always the same brightness whatever its nature may be in other respects" [22; Brace, p. 97] is incorrect without much further consideration. Even graphite and soot produce the desired result only over a limited range of conditions. It remains true that "different bodies ... radiate different kinds of heat" as published in the first issue of *Nature* in 1869 [30]. An examination of thermal emissivity plots is sufficient to confirm these statements [31]. Not a single object in nature is a blackbody. Hence, it is reasonable to wonder why this concept has so captivated physics. In studying blackbody radiation, it will be demonstrated that radiation within an enclosed body is not necessarily black [25], as Kirchhoff's law erroneously dictates [20–22].

If this subject matter remains important after all these years [1, 2, 20–22], it is because so much of physics, and more specifically astrophysics, is tied to the concept of universality in blackbody radiation. Agassi highlights the importance of Kirchhoff's law for astrophysics: "Browsing through the literature, one may find an occasional use of Kirchhoff's law in some experimental physics, but the only place where it is treated at all seriously today is in the astrophysical literature" [32]. As a result, in astrophysics, if a thermal spectrum is observed which displays, or even approximates, a Planckian (or normal) distribution, temperatures are immediately inferred. For this reason, the fall of universality heralds, in the most profound and far-reaching manner, a new dawn in this sub-discipline. Should universality be reconsidered, there are significant consequences for our models of the Sun and relative to the temperatures of the stars [33–35]. The validity of the ~ 3 K microwave background temperature would be questioned [36–41] and with it, perhaps, the entire framework of cosmology [33, 42]. Kirchhoff's law of thermal emission [20–22] may well be the simplest law in physics, but it is clear that, upon its validity, rests the very foundation of modern astrophysics.

Given these facts, it is unusual that Planck has advanced an equation [1, 2] which remains unlinked to any real physical process or object. Sadly, it is somewhat as a result of Kirchhoff's law that Planck remained unable to link his equation to a physical cause. The problem was an extremely serious one for Planck, and the fact that his hands were tied by universality is no more evident than in the helplessness he displays in the following quotation: "On the contrary, it may just as correctly be said that in all nature there is no process more complicated than the vibrations of black radiation. In particular, these vibrations do not depend in any characteristic manner on the special processes that take place in the centers of emission of the rays, say on the period or the damping of emitting particles; for the normal spectrum is distinguished from all other spectra by the very fact that all individual differences caused by the special nature of the emitting substances are perfectly equalized and effaced. Therefore to attempt to draw conclusions concerning the special properties of the parti-

cles emitting the rays from the elementary vibrations in the rays of the normal spectrum would be a hopeless undertaking" [9; §111].

Yet, it is primarily universality that makes this task a "hopeless undertaking". Planck, in fact, realized that vibrating atoms, electrons, or particles of some sort, must be responsible for the process of thermal emission. He specifically believed that the answer might be found by studying the electron and devoted much of his life to this topic [5; pp. 133–134, 198–199, 245]. But, unfortunately, Planck never makes the link to a real physical species, and the electron itself is not the proper lone candidate. Planck's belief that the answer lay in electron theory is explicitly contained in his letter to Paul Ehrenfest on July 6, 1905 in which he states: "*But perhaps it is not out of the question to make progress in the following way. If one assumes that resonator oscillations are produced by the motion of electrons...*" [5; p. 132]. Lorenz had already been successful, in deriving the radiation equation for long wavelengths (the Rayleigh-Jeans solution), using the analysis of electrons [5; p. 190].

Surprisingly, the real solution to the blackbody radiation problem has never been discovered [19]. Even Albert Einstein, in 1909, expressed frustration in this regard in a letter to H. A. Lorentz: "*I cherish the hope that you can find the right way, if indeed you find the reasons given in the paper for the untenability of the current foundations to be at all valid. But if you should deem those reasons to be invalid, then your counterarguments could perhaps furnish the key to the real solution of the radiation problem*" [18; p. 105]. The problem was never solved. As late as 1911, Einstein continues to express his frustration to Lorentz: "*I am working on the case of damped resonators; it involves quite a lot of calculation. The case of the electrons in the magnetic field, which I already mentioned in Brussels, is interesting, but not as much as I had thought in Brussels. Electrons in a spatially variable magnetic field are oscillators with variable frequency. If one neglects the radiation, then statistical mechanics yields the distribution law at every location if it is known at one location. If that location is field-free, then Maxwell's distribution holds there; from this one concludes it must hold everywhere. This leads of course to Jean's formula. Nevertheless, to me the thing seems to show that mechanics does not hold even in the case of the electron moving in the magnetic field. I am telling you this as an argument against the view that mechanics ceases to hold at the point where more than two things interact with each other. Anyway, the h-disease looks ever more hopeless*" [18; p. 228]. Blackbody radiation was never linked to a direct physical process. Yet, according to Kuhn, Einstein pointed out that "*not only the vibrations of electrons but also those of charged ions must, contribute to the blackbody problem*" [5; p. 210]. Nonetheless, Kuhn goes on to write that by the early 1910s "*while the nature of Planck's oscillators and of the corresponding emission process remained a mystery, the black-body problem could provide no further clues to*

physics" [5; p. 209]. In 1910, Peter Debye, derives Planck's law by quantizing the vibration modes of the electromagnetic field without recourse to oscillators [5; p. 210]. Albert Einstein would soon obtain it using his coefficients [17]. But the nature of the emitter was not identified [19]. In fact, in both cases, physics moved increasingly outside the realm of physical reality and causality.

Astrophysics believes that nothing of known physical origin is needed to obtain a blackbody spectrum. All that is required is a mathematical construct involving photons in thermal equilibrium and this, well outside the confines of a solid enclosure, as demanded by the experimental constraints surrounding blackbody radiation. Astrophysics has no need of the physical lattice, of some physical species vibrating within the confines of a structural physical assembly. But, if a thermal spectrum is to be produced, it is precisely this kind of physical restriction which must exist [19, 23]. However, as long as the idea that blackbody radiation is independent of the nature of the walls prevails, there can be no correction of this situation. It is the very formation of Kirchhoff's law [20–22] which must be brought into question, if any progress is to be made toward linking Planck's equation [1, 2] to the physical world and if astrophysics is to reform the manner in which it treats data. For these reasons, we now embark on the review of the findings which led to the concept of universality. Overwhelming evidence will emerge (see also [23–25]) that this concept is erroneous and should be reconsidered.

2 Experimental production of black radiation

2.1 The 19th century and the lampblack standard

Wedgwood published his delightful analysis on the production of light from heated substances in 1792 [43]. The works are noteworthy and pleasant to read because 1) they define the "state of the art" just prior to the 19th century, 2) they examine a plethora of substances, and 3) they possess wonderful historical descriptions of antecedent works. The experiments contained therein are nothing short of elegant for the period. Even at this time, the emission within a cylinder, either polished or blackened (presumably covered with lampblack), had already gained the attention of science [43]. Wedgwood realized that it did not matter, if heat entered the substance of interest through light, or through friction [43]. Much was already known about thermal radiation, but confusion remained.

The experimental aspects of the science of thermal radiation really began with the release of Leslie's *An Experimental Inquiry into the Nature and Propagation of Heat* [44]. In this classic work, Leslie describes how all objects emit light, but also that they have very different emissive powers, even at the same temperature [44; pp. 81, 90, 110]. This was well understood throughout the 19th century [45, 46]. Leslie opens his work as follows: "*The object I chiefly proposed, was to discover the nature, and ascertain the properties of*

what is termed *Radiant Heat*. No part of physical science appeared so dark, so dubious and neglected" [44; p. X]. Ironically, Leslie's last sentence rings somewhat true, even 200 years later.

Using reflectors made of tin, Leslie analyzed radiation emitted from the sides of a cube made of "block tin". At least one side was kept polished, one side was often coated with lampblack, and the other two were used to place miscellaneous substances, like tin foil, colored papers, or pigments [44; p. 8]. In order to maintain a constant temperature, the cubes were filled with water. The key to Leslie's experiments was a differential thermometer. By positioning various faces of the cube towards the reflector and placing his thermometer at the focal point, he soon discovered that polished metals give much less radiant heat than soot. He also realizes that the power to absorb or emit heat is somehow conjoined [44; p. 24]. It is interesting that, in his very first experiment, Leslie examines lampblack. It would become, for the rest of the 1800s, the means by which radiation would be calibrated.

Lampblack, the oxidation product of oil lamps, was not only a suitable material for coating surfaces and generating blackbodies over the course of the 1800s, it rapidly became the standard of radiation. By 1833, the Reverend Baden Powell, whose son was to form the Scouting movement, already writes that: "all experimenters have usually blackened their thermometer" [47; p. 276]. In 1848, G. Bird notes how lampblack has become a reference standard in the study of emission [48; p. 516]. Stewart refers repeatedly to lampblack invoking that soot had become the standard by which all radiation was to be measured: "The reason why lampblack was chosen as the standard is obvious; for, it is known from Leslie's observations, that the radiating power of a surface is proportional to its absorbing power. Lampblack, which absorbs all the rays that fall upon it, and therefore possesses the greatest possible absorbing power, will possess also the greatest possible radiating power" [26; §4]. He directly refers to lampblack heat [49; p. 191]. His experiments with lampblack are covered below in the context of the theoretical formulation of the law of radiation. Silliman's work is particularly valuable in that it was completed in 1861 [50]. It not only gives a well written and thorough account of the current state of knowledge in heat radiation, but it restates the central role of lampblack: "Lampblack is the only substance which absorbs all the thermal rays, whatever be the source of heat" [50; p. 442].

Langley re-emphasizes the extensive use of lampblack in his paper on solar and lunar spectra: "I may reply that we have lately found an admirable check upon the efficiency of our optical devices in the behavior of that familiar substance lampblack, which all physicists use either on thermometers, thermopiles, or bolometers" [51]. In 1893, Clerke writes of the "lampblack standard" in her tremendous work on the history of Astronomy [52; p. 271]. Tillman, in the 4th edition of his *Elementary Lessons in Heat*, summarizes well the be-

lief that prevailed throughout the 1800s: "Lampblack is the most perfect absorber and radiator, it being devoid of both reflecting and diffusive power. Its absorbing power is also most nearly independent of the source of heat. It absorbs all rays nearly alike, the luminous as well as the dark ones. Lampblack is accordingly taken as the standard surface of absorption, absorbing in the greatest degree every variety of ray which fall upon it. It is consequently, also, when hot, the typical radiator, giving out the maximum amount of heat which any substance at the same temperature could possibly give out; moreover, it gives out the maximum amount of each kind of heat that can be given out by any body at that temperature" [46; p. 92]. Tillman does recall Langley's discovery that, in the infrared, lampblack was nearly transparent [51]. In any event, the role of lampblack in thermal radiation was well established by the end of the 19th century.

In his textbook on physics, published for the 7th time in 1920, Watson provides an elaborate description of the use of lampblack in coating both thermometers and surfaces for the study of comparative emission between objects [53]. He describes the lampblack standard as follows: "Lampblack, although it does not absorb quite the whole of the incident radiation, yet possesses the property of absorbing very nearly, if not quite, the same proportion of the incident radiation whatever the wave-length, and so this substance is taken as a standard" [53; p. 301].

A review of the blackbody literature for the 19th century reveals that blackbodies were produced either from graphite itself or from objects covered with lampblack (soot) or paints, which contained soot or bone black [54]. That is not to say that other substances were not used. Kangro [3] outlines an array of studies where experimentalists, over a small region of the spectrum, used different materials (platinum black, copper oxide, iron oxide, thorium oxide, etc). Nonetheless, graphite and soot take precedence over all other materials, precisely because their absorbance extends over such a wide range of wavelengths. Conversely, all other materials exhibit disadvantages, either because of their suboptimal emissivity, or due to their limited frequency ranges [31]. There are problems in visualizing the infrared, even with platinum black. Kangro explains: "They (Lummer and Kurlbaum) changed to a platinum box as being more easily heated electrically and better suited to exact temperature measurement, then they used a platinum roll and finally a platinum cylinder the interior of which was blackened with iron oxide, and also divided by diaphragms the whole enclosed in a large asbestos cylinder" [3; p. 159]. They also report "the defective absorption of long wavelengths by Platinum black with which their bolometers were coated" as a possible source of error [3; p. 159]. Lummer and Kurlbaum made their 1898 cavity from platinum blackened with a mixture of chromium, nickel, and cobalt oxide [4]. Nonetheless, in order to properly visualize the longest wavelengths, the method of residual rays, developed by Rubens, was utilized [4]. These were critical experiments

for Planck. Yet, since platinum black could not reach elevated temperatures, in 1903, Lummer and Pringsheim would design a new blackbody with graphite walls [4]. This design has endured, essentially unchanged, until the present day [4].

2.2 The 19th century and the general state of knowledge

In 1833, Powell gave his excellent report on radiant heat [47]. By this time, the amount of radiation was known to be inversely related to conductive power [47; p. 266]. The more an object conducted thermal radiation, the better it acted as a reflector and the worse it was as an emitter/absorber. Based on the experiments of William Ritchie [55], it was also known that the absorptive power of a substance was directly related to its emissive power [47, p. 265]. Prévost's theory on thermal equilibrium, the famous *Theory of Exchanges* [56–58] was understood [47; p. 261]. Herschel's studies with infrared radiation were complete and the blocking action of glass was established [47; pp. 269–272]. While Herschel had discovered infrared radiation in 1800 [59], it was not until Langley, that infrared radiation could be accurately monitored [51]. At the time, Langley observed that lampblack was very nearly transparent to infrared radiation. Using prisms, it was also known that, on opposite sides of the spectrum, there existed “isothermal points” [47; p. 296]. Prisms played an important role in the early classification of the quality of light and heat by separation into colors [47; pp. 291–296]. Interestingly, Powell takes a sidestep relative to liquids and writes in his conclusion: “*In liquids, it has been disputed whether there can be radiation; and they are worse conductors than solids*” [47; p. 300]. Silliman notes that, even at the time of Kirchhoff, there remained some debate as to the relation between absorptive and emissive powers [50; p. 441], with de la Provostaye, Desains, and Melloni highlighting that these were not always equivalent. Given this general state of knowledge during the 19th century, we now move to the most important areas of experimentation, Prévost's *Theory of Exchanges* [56–58] and cavity radiation at thermal equilibrium.

2.3 The 19th century and cavity radiation

Pierre Prévost advanced his powerful *Theory of Exchanges* just as the 19th century came to life [56–58]. In formulating his law, Prévost invokes the enclosure: “*... I will suppose the two portions to be enclosed in an empty space, terminated on all sides by impenetrable walls*” [56; in Brace, p. 5]. He then moves to develop his *Theory of Exchanges* [56–58]. This theory was critical to Kirchhoff's thinking when the concept of universality was formulated [20–22]. As such, it is important to understand how Prévost's theory was viewed, not simply at the time of its formulation, but in the days of Kirchhoff. This knowledge can be gained by examining Balfour Stewart's summary of Prévost's theory. Stewart recounts the central ideas of equilibrium with an enclosure in his *Treatise* [49]. He summarizes Prévost's findings as follows: “*1. If an*

enclosure be kept at a uniform temperature, any substance surrounded by it on all sides will ultimately attain that temperature. 2. All bodies are constantly giving out radiant heat, at a rate depending upon their substance and temperature, but independent of the substance or temperature of the bodies that surround them. 3. Consequently when a body is kept at uniform temperature it receives back just as much heat as it gives out” [49; p. 215].

With Prévost, nearly 70 years before Kirchhoff, the real study of cavity radiation began. At the same time, the understanding of cavity radiation really grew near the 1820s. This was when the experimental work of Dulong and Petit [60] with cavities took place. Simultaneously, theoretical studies of heat were being forged by Fourier [61–67] and Poisson [68, 69]. Fourier's works are particularly important in that they represent the most far-reaching theoretical analysis of heat and cavities in this time frame.

The paper by Dulong and Petit [60] is a major milestone in experimental science and it is difficult to do it justice in a brief treatment. Thus, let us concentrate not on the first section dealing with the measurements of temperatures, the dilatation of solids, and the specific heats of materials, but rather on the second section. This section addresses the laws of cooling derived within an enclosure. Of course, Kirchhoff's law of thermal emission [20–22] deals with radiation under equilibrium conditions. Conversely, the results of Dulong and Petit examine a dynamic process [60]. While they do not directly apply, the studies by Dulong and Petit form the experimental basis for the works that follow and are crucial to understanding cavity radiation. Dulong and Petit recognized the importance of distinguishing the effects of gas particles and radiative emission in cooling [60]. By examining the cooling of water and liquids in enclosures of varying shapes, they conclude that the rate of cooling is independent of the shape of the walls of the enclosure, on its size, and on the nature of the liquid [60; p. 245]. Note how this conclusion is reminiscent of Kirchhoff's law [20–22]. Importantly, they observe that the rate of cooling is dependent on the state of the surface of the enclosure [60; p. 245].

Dulong and Petit continue their inquiry into the laws of cooling by building a copper enclosure, the inner surface of which they cover with lampblack [60; p. 247]. They place a thermometer at the center of the enclosure. The outer surface of the thermometer is either silvered or left in its glassy state [60; p. 250]. Using a pump, a balloon (containing various gases of interest), and a barometer attached to the enclosure, they deduce the law of cooling. Dulong and Petit accomplished their goal by varying the gas pressure within the enclosure while monitoring the drop in temperature of the previously heated thermometer. Initially, ignoring the effect of gases and working near vacuum, they quickly realize that the rate of cooling depends on the nature of the thermometer surface, and this even within the blackened cavity [60; p. 260]. The rates of cooling of the two thermometers were

proportional to one another, not equal [60; p. 260]. They arrive at a simple general law of cooling that applies to all bodies [60; p. 263]. Finally, by repeating the same experiments with gases at different pressures, they derive a law of cooling with two terms depending on radiation and the effect of the gas. They infer that the first term depends on the nature, the size, and the absolute temperature of the enclosure, while the second term depends only on the characteristics of the gas [60; p. 288]. Dulong and Petit's work is not revisited in a substantial manner until de la Provostaye and Desain publish their Mémoires [70–75].

De la Provostaye and Desain published their second *Mémoire on the Radiation of Heat* in 1848, more than 10 years before the formulation of Kirchhoff's law of thermal emission [71]. The authors open their work by stating (all translations from French were made by the author): “We must know how the quantity of heat emitted by a surface of a determined size depends on its temperature, its proper nature, its state, on the direction of the emission” [71; p. 358]. They then highlight: “but that we (scientists) have not, up to this day, introduced into the solution questions of equilibrium and of movement of the heat” [71; p. 358].

The authors revisit Dulong and Petit's experiments with gases using a half liter cylinder, blackened interiorly with lampblack (*noir de fumée*), in which they can introduce gases. They were never able to confirm the exact relation of Dulong and Petit and, therefore, present a more elaborate equation to describe the law of cooling [71; p. 369]. The paper contains a relevant caveat in that the authors report that it is not always easy to obtain a black surface, even with lampblack paste. They resort to the flame of a lamp to resurface the object of interest in order that its emission becomes truly independent of angle of observation [71; p. 398]. However, the bulk of our concern is relative to their work on the approach towards thermal equilibrium within an enclosure [71; pp. 406–431].

They recall that Fourier has proved: “1) that within a blackened enclosure without reflective power, equilibrium is established from element to element, 2) that the equilibrium is maintained in the same manner if we restore to one of the elements a reflective power, as long as we admit, in the first instance, that the absorbing and reflecting powers are complementary; and in the second place, that the emissive power is equal to the absorptive power, 3) that the same will hold, if we restore a certain reflective power to all the elements” [71; p. 406].

De la Provostaye and Desain highlight that the enclosure must be blackened for Fourier's conclusions to hold, but the latter does not always specifically state if his cavity is blackened interiorly. Nonetheless, Fourier's derivations make the assumption that the wall of the enclosure follows Lambert's law [66]. As such, the objects can be viewed as placed within a perfectly absorbing cavity. De la Provostaye and Desain make the point as follows: “The demonstration supposes, what the author (Fourier) seems in fact to have

recognized for himself (*Annales de Chimie et de Physiques, tome XXVII, page 247* (see [66]) in his last *Mémoires, that the radiating body is stripped of all reflective power. It would therefore be not at all general...*” [71; p. 408].

De la Provostaye and Desain begin their studies by placing a hypothetical thermometer in a spherical cavity and make no assumptions other than stating that diffuse reflection does not occur. They permit, therefore, that both the cavity and the thermometer can sustain normal reflection and emission. Assuming that reflective power does not depend on the angle of incidence, they permit the rays to travel throughout the cavity and follow the progression of the rays over time, until equilibrium is reached. The authors conclude that the radiation inside such a cavity will not follow Lambert's law [71; p. 414]. The result is important because it directly contradicts Kirchhoff's assertion that the radiation inside all cavities must be black [20–22]. They then restrict their treatment to the consideration of angles below 60° or 70°, in order to reach a simplified form for the laws of cooling.

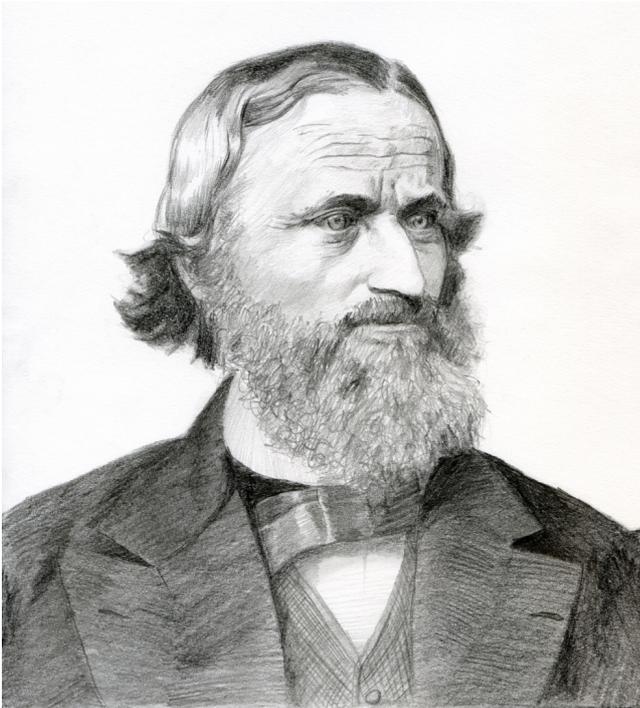
Like Dulong and Petit [60], de la Provostaye and Desain [70–75] are not concerned exclusively with thermal equilibrium, but rather, they are examining the velocity of cooling, the path to equilibrium. They provide important insight into the problem, as the following excerpt reveals: “When in an blackened enclosure with an invariable temperature t , we introduce a thermometer at the same temperature and a body either warmer or colder, but maintained always at the same degree T , the thermometer will warm or cool, and, following the reciprocal exchanges of heat, it will attain a final temperature θ , whose value, function of T and t , depends also on the emissive power E' of its surface, of that E of the source, and of their forms, sizes and reciprocal distances” [71; p. 424].

Siegel [76] highlights appropriately that de la Provostaye and Desains defined the emissive power E of a body as a fraction of the radiant emission of the blackbody where $f(t)$ is the emission of the blackbody, and the emission of the body is $E f(t)$ [74; p. 431]. In contrast, Kirchhoff defines emission simply as E , which, in fact, corresponds to de la Provostaye and Desain's $E f(t)$ [76]. Consequently, the universal function $f(t)$ is incorporated into Kirchhoff's law, even when it does not seem to be the case [76].

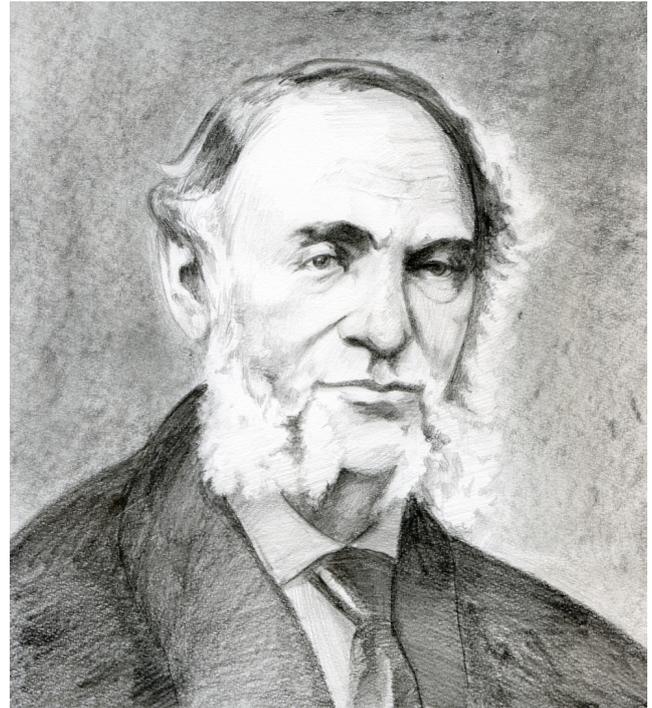
3 Cavity radiation

3.1 The Stewart-Kirchhoff dispute

Balfour Stewart [26] preceded Kirchhoff [20–22] by at least 2 years in the treatment of radiation at thermal equilibrium. Both Kirchhoff and Stewart built on the idea, initially advanced by Prévost [56–58], and expanded upon by Fourier [61–67], Poisson [68, 69], Dulong [60], Petit [60], de la Provostaye [70–75], Desains [70–75], and surely others, that thermal equilibrium existed between objects at the same temperature in the presence of confinement [49; p. 196]. The Stewart-Kirchhoff conflict is one of the darkest moments in



Gustav Robert Kirchhoff (12 March 1824 – 17 October 1887)



Balfour Stewart (1 November 1828 – 19 December 1887)

the history of science and it has been the subject of an excellent review [76]. This public quarrel is worth revisiting, not only because it is a powerful example of how science must not be performed, but also because it is very likely that the dispute between these men, and the international involvement of their collaborators [76] was directly responsible for the persistence of universality. If Stewart and Kirchhoff had better communicated, Kirchhoff might have yielded and the erroneous concept of universality, might have been retracted.

However, nationalistic passions were inflamed to such a measure that reason and scientific truth were moved to secondary positions. The animosity between Germany and British scientists would eventually reach the boiling point when, in 1914, Planck and 92 other learned men signed the *Appeal to the Cultured Peoples of the World* [16; pp. 70–ff]. Planck apparently signed the *Appeal* without examining its contents. Wien, for his part, insisted that British scientists “appropriated discoveries made in Germany, confused truth and falsehood, argued in bad faith, and . . . that England was the worst enemy of the Reich” [16; p. 72]. He urged that German scientists avoid, as much as possible, publication in British journals [16; p. 72]. Planck, for his part, refused to sign Wien’s manifesto [16; pp. 70–ff]. While the Stewart-Kirchhoff affair cannot bear all the responsibility for these tragic developments, and while other scientific battles also raged [76], it is relatively certain that the situation played an early role in the building of such misconceptions.

The papers from Stewart and Kirchhoff which caused this conflict were all published in *The London, Edinurgh, and*

Dublin Philosophical Magazine and Journal of Science. Kirchhoff was able to have access to the English literature, primarily through the assistance of F. Guthrie and Henry E. Roscoe. The latter translated many of Kirchhoff’s works into English for *Philosophical Magazine*. Roscoe had studied and published with Bunsen who, in turn, eventually became Kirchhoff’s key collaborator.

Stewart opens the discourse by publishing, in 1858, “*An account of some experiments on radiant heat, involving an extension of Prévost’s Theory of Exchanges*” [26]. It will be discovered below that, in fact, it is Stewart’s work which reached the proper conclusion, not Kirchhoff’s [25]. Yet, Stewart’s *Account* [26] has been forgotten, in large part, because, unlike Kirchhoff’s papers [20–22], it did not arrive at universality as Seigel emphasizes [76].

The battle really begins when F. Guthrie translates Kirchhoff’s paper and places it in *Philosophical Magazine* [21], the journal where Stewart’s work had appeared just two years earlier. Kirchhoff is rapidly criticized for failure to cite prior work, not only relative to Stewart, but relative to other seminal discoveries [76]. With the aid of Roscoe [77–80], he publishes in 1863, “*Contributions towards the history of spectrum analysis and of the analysis of the solar atmosphere*” [81] in which he seems to dismiss the importance of Stewart’s contributions. Kirchhoff writes: “*This proof cannot be a strict one, because experiments which have only taught us concerning more and less, cannot strictly teach us concerning equality*” [81]. Kirchhoff highlights that Stewart is not treating an enclosure in his experiments, but extends his con-

clusions to these objects [81]. In the end, Kirchhoff's *Contributions* [81] is not impolite... but it is tough.

For Stewart, Kirchhoff's *Contributions* [81] is viewed as an attack which must be immediately countered [82]. Stewart opens his rebuttal by stating: "*In the course of his remarks the learned author has reviewed in a somewhat disparaging manner some researches of mine on radiant heat, in consequence of which I am forced to reply, although very unwillingly, and desiring much to avoid a scientific controversy, especially with Professor Kirchhoff as an opponent*" [82]. In his own defense, Stewart then adds: "*nor did I omit to obtain the best possible experimental verification of my views, or to present this to men of science as the chief feature, grounding theory upon the experiments, rather than deducing the experiments from the theory*" [82]. This powerful charge by Stewart, in the end, forms the entire argument against Kirchhoff's proof [82]. Kirchhoff's results can never be validated by experiments, and Stewart, as an expert in heat radiation, must have recognized this to be the case [82].

Stewart closes his defense as follows: "*Although I preceded Kirchhoff nearly two years in my demonstration, I did not hesitate to acknowledge that his solution had been independently obtained; but, as a general principle, I cannot consent to admit that when a man of science has proved a new law and is followed by another who from the same premises deduces the same conclusions, the latter is justified in depreciating the labours of the former because he conceives that his solution is more complete. Will Kirchhoff himself willingly forego his own claims in favour of any one who shall in the future ages devise (if this be possible) a simpler and more convincing demonstration than that which has been given us by the Hiedelberg Professor? I feel, Sir, that, as an historian of science, you will acknowledge the justice of these remarks, and join me in regretting that one who has so eminently distinguished himself in original investigation should have chosen to superadd to his functions as a discoverer those of a severe and hostile critic upon the labours of those men who have worked at the same subject with himself, and by all of whom he has been treated with the utmost possible consideration*" [82].

The Stewart-Kirchhoff dispute reached such a magnitude that Kirchhoff, it seems, never again publishes in *Philosophical Magazine*, even though Bunsen, for his part, continues to utilize the journal. Stewart remained at a profound disadvantage, as he did not benefit from a relationship similar to that between Kirchhoff, Bunsen, and Roscoe. Roscoe would reprint Kirchhoff's infamous *Contributions* [81] in his *Spectrum Analysis* [80; pp. 115–122]. However, in this version [80; pp. 115–122], all text referring to Stewart has been removed without comment. It is impossible to understand Roscoe's motivation for the attenuated version. Roscoe may have suffered for having translated the letter. Alternatively, Kirchhoff's *Contributions* [81] might not fit in its entirety within the context of the other lectures. In any event,

Roscoe's *Spectrum Analysis* is a strange ode to Kirchhoff, which lacks broad scientific review. Regrettably, it seems that Roscoe made no attempt to reconcile the Kirchhoff-Stewart matter through proper and continuing scientific discourse.

In the end, Kirchhoff and Stewart each fell short of the mark. However, Kirchhoff's error was more serious [20–22], since it has theoretical consequences to this day. As for Balfour Stewart, had he presented a better theoretical case [26], the course of physics may have followed a different path. Kirchhoff, for example, correctly highlighted that Stewart's proof should not use the index of refraction, but rather, the square of the index [81]. Stewart conceded the point [76, 82]. For Kirchhoff, Stewart's proof was possibly true, not necessarily true [81]. Siegel elegantly clarified Kirchhoff's concerns [76]. These shortcomings in Stewart's derivation hinder the search for truth. Finally, had nationalistic sentiments not been aroused [76], it might have been easier to resolve the conflict.

3.2 Balfour Stewart

In examining Stewart's writings [26, 49, 82–85], we discover, as Brace highlights, "*the comprehensiveness of his mind and the originality of his genius*" [83; p. 72]. Many of Stewart's [26, 82–85] ideas are contained in his *Elementary Treatise on Heat* [49] and the later reflects his positions at the end of his life. As such, our discussion will begin first with the examination of this work and close with the review of his 1858 and 1859 papers [26, 83].

By the time Stewart writes his *Treatise*, he clearly recognizes that all substances display at least selective absorption of light [49; p. 191]. He comments on the probable identity of heat and light and writes: "*The facts detailed in this chapter all tend to shew that radiant light and heat are only varieties of the same physical agent, and also that when once the spectrum of a luminous object has been obtained, the separation of the different rays from one another is physically complete; so that if we take any region of the visible spectrum, its illuminating and heating effect are caused by precisely the same rays*" [49; p. 195]. He continues: "*Furthermore, we have reason to suppose that the physical distinction between different parts of the spectrum is one of wave length, and that rays of great wave length are in general less refracted than those of small wave length*" [49; p. 196].

Stewart's thoughts with respect to radiation within a cavity are important, not only because they provide us insight into the proper analysis of the enclosures, but also because they clearly outline what was known just prior to Planck. Stewart's comments relative to these experiments are summarized once again in his *Treatise*: "*... let us for our present purpose imagine to ourselves a chamber of the following kind. Let the walls which surround this chamber be kept at a constant temperature, say 100°C, and let them be covered with lampblack — a substance which reflects no heat, or at least*

very little; — also let there be a thermometer in the enclosure. It is well known that this thermometer will ultimately indicate the temperature of the surrounding walls. . . Suppose that the outside of the bulb of the thermometer of last article is covered with tinfoil, so that its reflecting power is considerable. Now according to the Theory of Exchanges this thermometer is constantly radiating heat towards the lampblack, but it is receiving just as much heat as it radiates. Let us call radiation of lampblack 100, and suppose that 80 of these 100 rays which strike the thermometer are reflected back from its tinfoil surface, while the remaining 20 are absorbed. Since therefore the thermometer is absorbing 20 rays, and since nevertheless its temperature is not rising, it is clear that it must be also radiating 20 rays, that is to say, under such circumstances its absorption and radiation must be equal to one another. If we now suppose the outside of the bulb to be blackened instead of being covered with tinfoil, the thermometer will absorb nearly all the 100 rays that fall upon it, and just as in the previous case, since its temperature is not rising, it must be radiating 100 rays. Thus we see that when covered with tinfoil it only radiated 20 rays, but when blackened it radiates 100. The radiation from a reflecting metallic surface ought therefore, if our theory be true, to be much less than from a blackened one. This has been proved experimentally by Leslie, who shewed that good reflectors of heat are bad radiators. Again, we have seen that in the case of the bulb covered with tinfoil 80 of the 100 rays which fell upon it were reflected back, and we have also seen that 20 were radiated by the bulb. Hence the heat reflected plus the heat radiated by this thermometer in the imaginary enclosure (author underscoring text) will be equal to 100, that is to say, it will be equal to the lampblack radiation from the walls of the enclosure. We may generalize this statement by saying that in an enclosure of constant temperature the heat reflected plus the heat radiated by any substance will be equal to the total lampblack radiation of that temperature, and this will be the case whether the reflecting substance be placed inside the enclosure or whether it form a part of the walls of the enclosure” [49; pp. 199–201].

Stewart reaches this conclusion for an enclosure whose walls have been covered with lampblack [49]. In that case, the heat inside the enclosure will correspond to that from lampblack, as I have shown [25]. In the pages which follow [49], Stewart goes on to explain that his law holds, in a manner which is independent of the nature of the walls, provided that both radiation and reflection are included. He also illustrates independence relative to wall shape. Importantly, he invokes the work of de la Provostaye and Desains with silver and lampblack to demonstrate that the total radiation inside an enclosure containing a silver surface will also be equal to 100, where 2.2 parts arise from the emission of silver itself and 97 parts from the reflection of lampblack. Stewart realizes that the value of 100 is only achieved in the presence of lampblack. The nature of the wall was immaterial simply because lampblack was always present. In fact, it appears that

Stewart was actually contemplating enclosures which contain both reflective surfaces and absorbing ones, as seen in his section 227: “It has already been stated (Art. 204) that the stream of radiant heat continually proceeding through an enclosure of which the walls are kept at a constant temperature depends only on the temperature of the walls, and not on the nature of the various substances of which they are composed; the only difference being that for metals this stream is composed partly of radiated and partly also of reflected heat, while for lampblack it is composed wholly of radiated heat. This may be expressed by saying that this stream depends upon or is a function of the temperature, and of it alone; but there is the following very important difference between a reflecting and lampblack surface, as representing this stream of radiant heat. It is only when a reflecting surface forms part of a complete enclosure of the same temperature as itself, that the radiated and reflected heat from this surface together represent the whole stream of heat; for if we bring it for a moment into another enclosure of lower temperature, the reflected heat is altered, and although the radiation will for a short time continue nearly constant, yet this radiation will not represent the whole stream of heat due to the temperature of the surface. On the other hand, if a lampblack surface be placed in the above position, since the stream of heat which flows from it is entirely independent of the reflexion due to neighboring bodies, the heat which it radiates when brought for a moment into an enclosure of lower temperature than itself will truly represent the stream of radiant heat due to the temperature of the lampblack” [49; pp. 221–222]. One can see that reflecting materials provide very different conditions than lampblack within enclosures. That is, within an enclosure under dynamic conditions, objects which are partially or fully reflecting cannot indefinitely support black radiation. They simply emit their own radiation and reflect the heat incident upon their surface. Through this discussion, Stewart demonstrates that thermal equilibrium would be disturbed when a perfect absorber is replaced with a reflector, bringing about dynamic rather than equilibrium conditions. This was an important insight relative to the analysis which I recently provided [25] of Kirchhoff’s second proof [21, 22].

In order to examine the velocity of temperature change, Stewart invokes a thin copper globe lined with lampblack: “Having now considered the law of cooling as representing with much accuracy the quantity of heat given out by a black substance at different temperatures, we come next to the relation between the temperature and the quality or nature of the heat given out. And here we may remark that the laws which connect the radiation of a black body with its temperature, both as regards to the quantity and the quality of the heat given out, hold approximately for bodies of indefinite thickness which are not black, — thus, for instance, they would hold for a metallic surface, which would represent very nearly a lampblack surface, with the radiation diminished a certain number of times. These laws would not, however, hold

exactly for a white surface, such as chalk; for this substance behaves like lampblack with respect to rays of low temperature, while it is white for rays of high temperature, and the consequence of this will be that its radiation will increase less rapidly than that of a lampblack surface. In like manner, these laws will not hold exactly for coloured surfaces” [49; p. 230]. Note how these statements are directly contradictory to what Kirchhoff requires. For Stewart, there is no universality and this is a major distinction between his work and that of his adversary [25].

With regards specifically to a black surface, Stewart writes (see page 231): “1. The spectrum of the radiant heat and light given out by a lampblack surface is continuous, embracing rays of all refrangibilities between certain limits on either side... 2. We have reason to think that as the temperature rises, the spectrum of a black substance is extended in the direction of greatest refrangibility, so as to embrace more and more of the violet and photographic rays” [49; p.231]. Stewart goes on to discuss thin plates of glass and explains how they cannot be compared to lampblack, as their radiation with increasing temperature will be substantially different [49; p. 232].

It is clear that if scientists of the period coated the walls of their enclosure with lampblack, that emission would be independent of the nature of the walls themselves, precisely because lampblack was coating these walls. After all, Stewart fully realizes that silver, for instance, has a total emission much below lampblack [49; pp. 201–206]. Stewart used an enclosure coated with lampblack to arrive at the following laws: “1. The stream of radiant heat is the same throughout, both in quantity and quality; and while it depends on the temperature it is entirely independent of the materials or shape of the enclosure. 2. This stream is unpolarized. 3. The absorption of a surface in such an enclosure is equal to its radiation and this holds for every kind of heat” [49; p. 206]. That is how the concept of independence of the nature of the walls entered the literature. Nothing, in fact, was independent. The walls were simply coated with lampblack [49; pp. 201–206]. This was such an obvious part of these experiments, during the 19th century, that it is likely that most scientists, unlike Balfour Stewart, simply neglected to report their common practice. As a result, future generations who followed the theoretical avenues of Kirchhoff, actually came to believe that the nature of the walls was unimportant and the vital role of the soot coating was forgotten.

Stewart’s law stated that absorption was equal to radiation for every kind of heat [26, 49, 76, 82]. This was true under equilibrium conditions. However, Kirchhoff objected [81] to this formulation by Stewart [26], since he believed that Stewart had inappropriately extended the results of his experimental finding to include equality whereas proportionality was all that had been proven [76, 81]. In any event, the fact remains that Stewart’s conclusion [26, 49, 82], not Kirchhoff’s [20–22], was correct. It alone was supported by the experimental

findings and, unlike Kirchhoff’s law [20–22], made no claims of universality [76].

The central portion of Stewart’s proof considers a continuous plate of rock salt positioned between two plates covered with lampblack [26; §12]. The idea is both simple and powerful. Stewart immediately reaches the result that “the absorption of a plate equals, its radiation, and that for every description of heat” [26; §19]. Then, Stewart considers radiation internal to a substance: “Let AB, and BC be two contiguous, equal, and similar plates in the interior of a substance of indefinite extent, kept at a uniform temperature” [26; §20]. Stewart is invoking the same restriction found for thermal equilibrium with an enclosure. However, he moves to the interior of a body, apparently in order to avoid dealing with surface reflection [82]. Siegel [76] highlights this point. Kirchhoff believes that Stewart has not properly treated the enclosure [81]. The point is weak as Stewart’s entire treatment is based on the ideas of Prévost [55–57].

Stewart is clearly working within the confines of Prévost’s *Theory of Exchanges* [26, 56–58]. Considering the equilibrium between lampblack and an arbitrary surface at thermal equilibrium, he writes “. . . hence the total quantity of heat radiated and reflected which leaves the surface. . . (is) the same as if the substance had been lampblack, the only difference being, that, in the case of lampblack, all this heat is radiated, whereas in other substances only part is radiated, the remainder being reflected heat” [26; §31]. He continues: “Although we have considered only one particular case, yet this is quite sufficient to make the general principle plain. Let us suppose we have an enclosure whose walls are of any shape, or any variety of substances (all at a uniform temperature), the normal or statical condition will be, that the heat radiated and reflected together, which leaves any portion of the surface, shall be equal to the radiated heat which would have left that same portion of the surface, if it had been composed of lampblack. . . Let us suppose, for instance, that the walls of this enclosure were of polished metal, then only a very small quantity of heat would be radiated; but this heat would be bandied backwards and forwards between the surfaces, until the total amount of radiated and reflected heat together became equal to the radiation of lampblack” [26; §32]. These passages are quite similar to Kirchhoff’s with the distinction that universality is never invoked. Stewart realizes that the lampblack surface within the enclosure is essential.

Stewart’s manner of addressing the problem is lacking, as Siegel highlights [76], especially for Kirchhoff [81]. A review of this work [76] provides a sufficient discussion. Stewart advances an initial attempt at the correct solution to the radiation puzzle, but the presentation was not sufficient, at least for his adversary. Surprisingly, in his *Reply* to Kirchhoff in 1863, Stewart seems embarrassed [76] relative to reflection writing: “I shall only add that it was attempted, as far as possible, to disengage the proof, theoretical and experimental, from the embarrassment of considering surface reflexion”

[82]. If reflection is neglected, however, almost by definition, the radiation must be black [25]. Consequently, all attempts to address the issue devoid of surface reflection can never yield the proper conclusion relative to the existence of universality. Stewart reaches the proper answer because he does include reflection in his papers [26, 83] and within his *Treatise* [49]. Within an enclosure containing a lampblack surface and another object, he reminds us that “*the reflection plus the radiation of the body at any temperature equals the lampblack radiation at that temperature*” [83; §44]. The proper consideration of reflection is key [25] and though Stewart may have had weaknesses in his presentation, he did ascertain the truth.

3.3 Gustav Kirchhoff and his law

It can be said that Kirchhoff’s law of thermal emission [20–22], through its claims of the universal nature of radiation within enclosures, represents one of the most profound dismissals of experimental science in the history of physics. The great mass of experimental evidence speaks against universality of radiation within cavities. Cavity radiation only assumes the normal distribution (i.e. that of the blackbody) when either the walls of the cavity, or at least one of the objects it contains, are perfectly absorbing [23, 25]. In fact, the proof that Kirchhoff’s law does not hold, in its universal form, does not require extensive mathematical or experimental arguments, only simple ones [23–25].

Schirmacher [86] emphasizes that, at the time Planck formulated his law, a solid proof of Kirchhoff’s remained absent. Furthermore, he highlights that, as late as 1912, Hilbert was arguing that Kirchhoff’s law still lacked proof [86]. Hilbert makes this statement in spite of Planck’s attempt to prove the law in his *Theory of Heat Radiation* [9]. Schirmacher also outlines that nearly all attempts to advance universality were met with a refutation [86; p. 16]. Sadly, these corrections never prevailed.

De la Provostaye was one of the first to offer an analysis of cavity radiation following Kirchhoff, in 1863 [87]. In his work, de la Provostaye deduces that the radiation within a perfectly absorbing cavity must be black [87]. He also infers that a cavity, a portion of whose walls are perfectly absorbing, and which contains an object of arbitrary emittance and reflectance, must also contain normal (or blackbody) radiation [87]. Like Kirchhoff, he attempts to extend his findings to a perfectly reflecting cavity. At first, he concedes that a fully reflecting cavity must be devoid of radiation. At this point, de la Provostaye should have ceased as the question was resolved; but strangely . . . he continues. Prompted perhaps by the quest for Kirchhoff’s universality [20–22], he permits radiation to enter the perfectly reflecting cavity and immediately moves to show that such radiation must be black [87]. As a result, de la Provostaye stumbles in a manner quite similar to Kirchhoff and his paper does not, in fact, form a refutation of Kirchhoff’s law [87]. De la Provostaye simply objected that Kirch-

hoff, by introducing perfect reflectors, essentially dictated the result which he sought [86].

De la Provostaye’s analysis of cavity radiation is particularly important, because he was an expert in the subject. He had dealt with enclosures on an experimental basis and must have known from the work of his own hands, that Kirchhoff’s law could not hold, in its universal form. This is why he presents the second case discussed above where at least a portion of the cavity walls remained perfectly absorbing. De la Provostaye did overreach in his conclusions [87] in a manner not dissimilar from Kirchhoff [20–22].

In any event, de la Provostaye’s theoretical objections relative to the absence of a perfectly reflecting mirror was not the central problem for Kirchhoff [25]. While many followed de la Provostaye’s initial objection, refutations always seemed to be based on arguments such as perfectly reflecting mirrors do not exist, neither do perfectly diathermanous (or transparent) bodies, or bodies which can only absorb one wavelength. Such idealized substances are utilized in various proofs of Kirchhoff’s law [86]. Unfortunately, since Kirchhoff’s law is based on a theoretical extension of experimental reality, the fact that idealized objects do not exist is not sufficient to overturn Kirchhoff’s position [25]. Hence, the law has prevailed, even though experimental reality is well established against its claims as de la Provostaye and Stewart must have realized.

The only way to refute Kirchhoff’s law is to show that some section of its treatment either fails to consider an essential aspect of physical reality or that, through its derivation, Kirchhoff himself violates the thermal equilibrium, which he required as a precondition [25]. Both of these complications have been brought to the forefront [25]. Kirchhoff’s law is not valid for two reasons: first, the importance of reflection is not properly included and second, Kirchhoff’s model gives rise, under certain conditions, to a violation of thermal equilibrium [25].

Physics is in a difficult position relative to Kirchhoff’s law, since the modern relationship between radiation and absorption, under equilibrium conditions, is based upon this work. At the same time, Kirchhoff’s claims of universality given enclosure are strictly invalid [25]. A perfect absorber must be present. The only means of rectifying this situation is to finally acknowledge the merit of Stewart’s contributions [26, 49, 83].

3.4 Max Planck and cavity radiation

3.4.1 Whence the carbon particle

In the first preface of his book *The Theory of Heat Radiation* Planck mentions that he has “*deviated frequently from the customary methods of treatment, wherever the matter presented or considerations regarding the form of presentation seems to call for it, especially in deriving Kirchhoff’s laws...*” [9; p.xi]. Yet, when one reads Planck’s text, the precise nature of the deviations cannot be ascertained and the

origin of the carbon particle remains a mystery. Since the exposition deals with Kirchhoff, one could be led to assume that the idea came from Kirchhoff [23]. Planck, after all, was a strict theoretician. He relied on experimentalists to give him insight in the particle used for the generation of blackbody radiation. Still, we are never told specifically that Kirchhoff invoked the carbon particle [23]. It is certain that, at the time of Kirchhoff, virtually all blackbodies were covered with lampblack. Hence, radiation in a cavity whose inner walls were coated with lampblack would have been observed to be independent of the nature of the walls. This simple observation may well have prompted Kirchhoff and Planck to reach for physically profound statements relative to universality while minimizing the role of soot.

The origin of the carbon particle is surely of historical interest. However, with regards to physics, its existence causes concern, not its historical origin. How a particle of carbon entered the perfectly reflecting cavity and involved the actions of Kirchhoff, Planck, or another scientist, alters nothing relative to the consequences for universality [23]. What remain critical are Kirchhoff's claims that blackbody radiation was independent of the nature of the walls of the cavity, whether these were absorbing, transparent or reflecting to radiation, provided that thermal equilibrium was maintained [21, 22]. Planck's invocation of the carbon particle [9] shatters all these arguments [23, 25] and, as such, it is important to repeat the many words of Planck relative to the need for a tiny piece of carbon.

We begin by recalling how Planck himself was well aware that real blackbodies are formed using lampblack. Nothing here is independent of the nature of the walls: "Now, since smooth non-reflecting surfaces do not exist . . . it follows that all approximately black surfaces which may be realized in practice (lampblack, platinum black) . . ." [9; §11]. Relative to the carbon particle itself, the first key passages come at the end of Part I: "Thus far all the laws derived in the preceding sections for diathermanous media hold for a definite frequency, and it is to be kept in mind that a substance may be diathermanous for one color and adiathermanous for another. Hence the radiation of a medium completely enclosed by absolutely reflecting walls is, when thermodynamic equilibrium has been established for all colors for which the medium has a finite coefficient of absorption, always the stable radiation corresponding to the temperature of the medium such as is represented by the emission of a black body. Hence this is briefly called "black" radiation. On the other hand, the intensity of colors for which the medium is diathermanous is not necessarily the stable black radiation, unless the medium is in a state of stationary exchange of radiation with an absorbing substance" [9; §50]. Planck recognizes that the presence of a perfectly absorbing substance is required within the perfect reflector. If this condition is not fulfilled, Planck reminds us immediately that: "... in a vacuum bounded by totally reflecting walls any state of radiation may persist"

[9; §51]. As such, Planck is fully aware that the perfect reflector can never produce blackbody radiation in the absence of a perfect absorber. It is not simply a matter of waiting a sufficient amount of time, but rather, the radiation will *persist* in a non-blackbody or arbitrary state. He re-emphasizes this aspect clearly "Every state of radiation brought about by such a process is perfectly stationary and can continue infinitely long, subject, however, to the condition that no trace of an emitting or absorbing substance exists in the radiation space. For otherwise, according to Sec. 51, the distribution of energy would, in the course of time, change through the releasing action of the substance irreversibly, i.e., with an increase of the total entropy, into the stable distribution corresponding to black radiation" [9; §91].

Planck soon brings the carbon particle front and center: "But as soon as an arbitrarily small quantity of matter is introduced into the vacuum, a stationary state of radiation is gradually established. In this the radiation of every color which is appreciably absorbed by the substance has intensity K_ν corresponding to the temperature of the substance and determined by the universal function (42) for $q = c$, the intensity of radiation of the other colors remaining intermediate. If the substance introduced is not diathermanous for any color, e.g., a piece of carbon however small, there exists at the stationary state of radiation in the whole vacuum for all colors the intensity K_ν of black radiation corresponding to the temperature of the substance. The magnitude of K_ν , regarded as a function of ν gives the spectral distribution of black radiation in a vacuum, or the so-called normal energy spectrum, which depends on nothing but the temperature. In the normal spectrum, since it is the spectrum of emission of a black body, the intensity of radiation of every color is the largest which a body can emit at that temperature at all" [9; §51].

"It is therefore possible to change a perfectly arbitrary radiation, which exists at the start in the evacuated cavity with perfectly reflecting walls under consideration, into black radiation by the introduction of a minute particle of carbon. The characteristic feature of this process is that the heat of the carbon particle may be just as small as we please, compared with the energy of radiation contained in the cavity of arbitrary magnitude. Hence, according to the principle of conservation of energy, the total energy of radiation remains essentially constant during the change that takes place, because the changes in the heat of the carbon particle may be entire neglected, even if its changes in temperature would be finite. Herein the carbon particle exerts only a releasing (auslösend) action. Thereafter the intensities of the pencils of different frequencies originally present and having different frequencies, directions, and different states of polarization change at the expense of one another, corresponding to the passage of the system from a less to a more stable state of radiation or from a state of smaller to a state of larger entropy. From a thermodynamic point of view this process is perfectly analogous, since the time necessary for the process is not essential,

to the change produced by a minute spark in a quantity of oxy-hydrogen gas or by a small drop of liquid in a quantity of supersaturated vapor. In all these cases the magnitude of the disturbance is exceedingly small and cannot be compared with the magnitude of the energies undergoing the resultant changes, so that in applying the two principles of thermodynamics the cause of the disturbance of equilibrium, viz., the carbon particle, the spark, or the drop, need not be considered. It is always a case of a system passing from a more or less unstable into a more stable state, wherein, according to the first principle of thermodynamics, the energy of the system remains constant, and, according to the second principle, the entropy of the system increases" [9; §52]. Planck views the carbon particle simply as a catalyst. He does not recognize that it has a vital function as a perfect absorber. This is a critical oversight, as demonstrated in my review of thermal equilibrium within a perfectly reflecting cavity containing a carbon particle [25].

Planck invokes the carbon particle repeatedly throughout his text. This issue is so central to the discussion at hand that all these sections must be brought forth. He writes: "For the following we imagine a perfectly evacuated hollow cylinder with an absolutely tight-fitting piston free to move in a vertical direction with no friction. A part of the walls of the cylinder, say the rigid bottom, should consist of a black body, which temperature T may be regulated arbitrarily from the outside. The rest of the walls including the inner surface of the piston may be assumed to be totally reflecting. Then, if the piston remains stationary and the temperature, T , constant, the radiation in the vacuum will, after a certain time, assume the character of black radiation (Sec. 50) uniform in all directions. The specific intensity, K , and the volume density, ν , depend only on the temperature, T , and are independent of the volume, V , of the vacuum and hence the position of the piston" [9; §61].

"Let us also consider a reversible adiabatic process. For this it is necessary not merely that the piston and the mantle but also that the bottom of the cylinder be assumed as completely reflecting, e.g., as white. Then the heat furnished on compression or expansion of the volume of radiation is $Q = 0$ and the energy of radiation changes only by the value $p dV$ of the external work. To insure, however, that in a finite adiabatic process the radiation shall be perfectly stable at every instant, i.e., shall have the character of black radiation, we may assume that inside the evacuated cavity there is a carbon particle of minute size. This particle, which may be assumed to possess an absorbing power differing from zero for all kinds of rays, serves merely to produce stable equilibrium of the radiation in the cavity (Sec. 51 et seq.) and thereby to ensure the reversibility of the process, while its heat contents may be taken as so small compared with the energy of radiation, U , that the addition of heat required for an appreciable temperature change of the particle is perfectly negligible. Then at every instant in the process there exists absolutely

stable equilibrium of radiation and the radiation has the temperature of the particle in the cavity. The volume, energy, and entropy of the particle may be entirely neglected" [9; §68].

"Let us finally, as a further example, consider a simple case of an irreversible process. Let the cavity of volume V , which is elsewhere enclosed by absolutely reflecting walls, be uniformly filled with black radiation. Now let us make a small hole through any part of the walls, e.g., by opening of a stopcock, so that the radiation may escape into another completely evacuated space, which may also be surrounded by rigid, absolutely reflecting walls. The radiation will at first be of a very irregular character; after some time, however, it will assume a stationary condition and will fill both communicating spaces uniformly, its total volume being, say, V' . The presence of a carbon particle will cause all conditions of black radiation to be satisfied in the new state" [9; §69].

"If the process of irreversible adiabatic expansion of the radiation from the volume V to the volume V' takes place as just described with the single difference that there is no carbon particle present in the vacuum, after the stationary state of radiation is established, as will be the case after a certain time on account of the diffuse reflection from the walls of the cavity, the radiation in the new volume V' will not any longer have the character of black radiation, and hence no definite temperature . . . If a carbon particle is afterwards introduced into the vacuum, absolutely stable equilibrium is established by a second irreversible process, and, the total energy as well as the total volume remaining constant, the radiation assumes the normal energy distribution of black radiation and the entropy increases to the maximum value S' . . ." [9; §70].

"Hence, on subsequent introduction of a carbon particle into the cavity, a finite change of the distribution of energy is obtained, and simultaneously the entropy increases further to the value S' calculated in (82)" [9; §103].

Throughout *The Theory of Heat Radiation*, Planck invokes the carbon particle as a vital determinant of blackbody radiation. Only in the section of the derivation of Wien's law does he try to minimize the importance of his catalyst. However, in this case, the derivation starts with the presence of a blackbody spectrum *a priori*. One could argue that Planck goes through great pains to explain that he does not need the particle when, in fact, he has already invoked it to produce the radiation he requires as a starting point. The discussion is well worth reading precisely for the number of times that the carbon particle is utilized: "The starting point of Wien's displacement law is the following theorem. If the black radiation contained in a perfectly evacuated cavity with absolutely reflecting walls is compressed or expanded adiabatically and infinitely slowly, as described above in Sec. 68, the radiation always retains the character of black radiation, even without the presence of a carbon particle. Hence the process takes place in an absolute vacuum just as was calculated in Sec. 68 and the introduction, as a precaution, of a carbon particle is shown to be superfluous. But this is true only in this special

case, not at all in the case described in Sec. 70. . .” [9; §71].

“Let the completely evacuated hollow cylinder, which is at the start filled with black radiation, be compressed adiabatically and infinitely slowly to a finite fraction of the original volume. If, now, the compression being completed, the radiation were no longer black, there would be no stable thermodynamic equilibrium of the radiation (Sec. 51). It would then be possible to produce a finite change at constant volume and constant total energy of radiation, namely, the change to the absolutely stable state of radiation, which would cause a finite increase of entropy. This change could be brought about by the introduction of a carbon particle, containing a negligible amount of heat as compared with the energy of radiation. This change, of course, refers only to the spectral density of the radiation u_ν , whereas the total density of the energy u remains constant. After this has been accomplished, we could, leaving the carbon particle in the space, allow the cylinder to return adiabatically and infinitely slowly to its original volume and then remove the carbon particle. The system will then have passed through a cycle without any external changes remaining. For heat has been neither added nor removed, and the mechanical work done on compression has been regained on expansion, because the latter, like the radiation pressure, depends only on the total density u of the energy of radiation, not on its spectral distribution. Therefore, according to the first principle of thermodynamics, the total energy of radiation is at the end just the same as at the beginning, and hence also the temperature of the black radiation is again the same. The carbon particle and its changes do not enter into the calculation, for its energy and entropy are vanishingly small compared with the corresponding quantities of the system. The process has therefore been reversed in all details; it may be repeated any number of times without any permanent change occurring in nature. This contradicts the assumption, made above, that a finite increase in entropy occurs; for such a finite increase, once having taken place, cannot in any way be completely reversed. Therefore no finite increase in entropy can have been produced by the introduction of the carbon particle in the space of radiation, but the radiation was, before the introduction and always, in the state of stable equilibrium” [9; §71].

In reading these sections, it is almost as if Planck has entered into a duel with the carbon particle. He tries to minimize its role, even though it is strictly necessary to his success. In any event, as I have shown [25], when Planck (or Kirchhoff) places the carbon particle inside the perfectly reflecting cavity, it is as if the entire cavity had been lined with soot [23]. Thermal equilibrium arguments are powerful, and one of their interesting aspects is that equilibrium does not depend on the extent of the interacting surfaces. This affects only the amount of time required to reach equilibrium, not the nature of the radiation present under equilibrium conditions. Planck’s catalyst is a perfect absorber, and therefore, given equilibrium, it controls the entire situation. The carbon parti-

cle does not simply lead to a distribution of radiation which would have occurred even in its absence.

3.4.2 Planck’s derivation of Kirchhoff’s law

Planck’s derivation of Kirchhoff’s law, as presented in *The Theory of Heat Radiation* [9; pp. 1–45], brings the reader to universality, precisely because reflection is not fully considered. Planck’s exposition is elegant and involves two distinct parts. The first deals with radiation within an object [9; §4–26] and is eerily similar to Stewart’s formulation [26, 82]. The second examines radiation between “two different homogeneous isotropic substances contiguous to each other . . . and enclosed in a rigid cover impermeable to heat” [9; §35–39]. By combining these two parts, Planck arrives at a relationship which is independent of the nature of the materials in a manner consistent with his belief in universality.

A cursory examination of this derivation [9; pp. 1–45], suggests that universality must be valid. Planck seems to properly include reflection, at least when discussing the interface between two separate materials [9; §35–39]. He arrives with ease at Kirchhoff’s law, $q^2(\varepsilon_\nu/\alpha_\nu) = q^2 K_\nu$, [9; Eq. 42], involving the square of the velocity of propagation, q , the coefficient of emission, ε_ν , the coefficient of absorption, α_ν , and the universal function, K_ν . This relationship simplifies to the familiar form $\varepsilon_\nu/\alpha_\nu = K_\nu$. *The Theory of Heat Radiation* focuses, later, on the definition of the universal function, which of course, is the right side of Planck’s famous equation [1, 2]:

$$\frac{\varepsilon_\nu}{\alpha_\nu} = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}.$$

Unfortunately, there is a difficulty at the very beginning of the Planck’s elucidation of Kirchhoff’s law.

In order to arrive at universality [20–22], Planck first examines the equilibrium of radiation within an object. He begins by considering only the emission from a single element $d\tau$ internal to the object and in so doing, is deliberately ignoring reflection. Planck writes, in deriving Eq. (1), that the “total energy in a range of frequency from ν to $\nu + d\nu$ emitted in the time dt in the direction of the conical element $d\Omega$ by a volume element $d\tau$ ” [9; §6] is equal to $dt d\tau d\Omega d\nu 2\varepsilon_\nu$. This will lead directly to Kirchhoff’s law. If Planck had properly weighed that the total radiation coming from the element $d\tau$ was equal to the *sum* of its emission and reflection, he would have started with $dt d\tau d\Omega d\nu 2(\varepsilon_\nu + \rho_\nu)$, which would not lead to universality.

Planck moves on to examine absorption, by imagining two elements $d\sigma$ and $d\sigma'$ which are exchanging radiation within the same substance [9; §20]. Finally, he views the total “space density of radiation” in a sphere at the center of which is a volume element, ν , receiving radiation from a small surface element, $d\sigma$ [9; §22]. In the end, by combining his results for emission and absorption, Planck demonstrates

that within an individual substance, $K_\nu = \varepsilon_\nu / \alpha_\nu$. He writes the powerful conclusion that “*in the interior of a medium in a state of thermodynamic equilibrium the specific intensity of radiation of a certain frequency is equal to the coefficient of emission divided by the coefficient of absorption of the medium for this frequency*” [9; §26]. This was the flaw in his presentation. Had Planck fully included reflection, he would have obtained $K_\nu = (\varepsilon_\nu + \rho_\nu) / (\alpha_\nu + \rho_\nu)$.

Yet, this is only the first portion of Planck’s walk to universality. In order to extend his deduction to all substances, he must first bring two differing materials in contact with one another. He accomplishes this correctly in §35–38. Properly treating reflection in this case, he is led, as was seen above, to $q^2(\varepsilon_\nu / \alpha_\nu) = q^2 K_\nu$ [9; §38], a statement of universality. The equation becomes completely independent of the nature of the substance. But if Planck had properly executed the first portion of his proof [9; §1–26], he would have been led, for every substance, once again to $K_\nu = (\varepsilon_\nu + \rho_\nu) / (\alpha_\nu + \rho_\nu)$.

In hindsight, there are many problems with Planck’s derivation. In the first section of his proof, he moves to the inside of an object. He advances that thermal equilibrium is achieved internally, not through conduction and the vibration of atoms, but rather through radiation. While it is true, as Planck believes, that in a state of thermal equilibrium there can be no net conduction, it cannot be said that there can be no conduction. In fact, modern condensed matter physics would surely argue that thermal equilibrium within objects is sustained through conduction, not radiation. Planck like Stewart before him [26, 76, 82] invokes internal radiation as a central component of his proof. He does so precisely to avoid dealing with reflection. He assumes that the volume elements $d\tau$, $d\sigma$ and $d\sigma'$ can sustain only emission, not reflection. In so doing, he predetermines the outcome he seeks, beginning as we have seen with his equation (1) [9; §6].

3.5 Graphite, carbon-black, and the modern age

Graphite and soot, whose commercial forms include carbon black [88] and black carbon [89], continue to be at the center of nearly all blackbody experiments conducted by the National Bureau of Standards and other laboratories. Nonetheless, certain metal blacks [88], namely platinum black and gold black [90–92], have a narrow range of uses as absorbers, especially at long wavelengths. Platinum black is usually prepared by electroplating the surface with platinum. Gold black is particularly interesting as a material. It is produced, by vaporizing the metal onto a substrate until thin gold films are generated. In this sense, the conductivity of gold is being structurally limited and the resulting material is black. In the end, the metal blacks are used primarily in the infrared, and their applications, while important, even in the days following Planck, are somewhat limited.

It remains the overwhelming case that the walls of many cavities are still made from graphite [93–97]. However, if

they are made of alternate materials (i.e. brass [98], copper [99], clay [93]), they are either blackened, or smoked with soot [98], or they are covered with black paint [93, 96, 98–104]. Some of these paints have proprietary contents. Nonetheless, it is relatively certain that they all contain the carbon black pigment [105, 106]. For instance, the author has been able to verify that Aeroglaze Z306 and Z302 both contain carbon black (private communication, Robert Hetzell, Lord Corporation, Erie, PA). The same can be ascertained relative to Nextel Velvet coating P/N101-C10 black. It is true that carbon black, with its extremely high carbon content remains the premium black pigment [105]. Graphite and soot (carbon black, black carbon) continue to absolutely dominate all work with experimental blackbodies.

Even fixed point blackbodies [95] which operate at the freezing points of elements such as gold [95], aluminum, zinc, and tin [100] rely either on graphite [107] walls or cavities coated with black paints. In these fixed point blackbodies, the metal freezing/melting point ensures that the entire surface of the emitter can be temporarily maintained at a unique temperature. Interestingly, the metals themselves appear to be relatively innocuous or transparent to emission by the graphitic, or carbon lined, surfaces of the cavity.

There are restrictions on the quality of freezing point blackbody cavities, and these have been outlined by Geist [108]: “*How well the actual radiance approaches the ideal radiance in a given blackbody is often referred to in a qualitative manner as the quality of the blackbody...The principle restriction on the concept of quality...is that it can only be defined for radiation from blackbodies with wall materials whose thermal radiative parameters are independent of wavelength. One important class of freezing point blackbody for which this is not a serious restriction is the class whose cavity walls are constructed from graphite.*” A mathematical treatment of laboratory blackbodies reveals that the production of a cavity whose performance will yield a high quality blackbody is not a trivial task [109].

In any event, it remains clear that whether a blackbody is designed to operate at the freezing point of an element or not, graphite [31, 107], or soot (carbon black [105, 106], or black carbon [89]) continue to dominate this field.

4 Conclusion

Through the exposition of Kirchhoff’s law, we have been able to highlight that universality does not hold in cavity radiation. The great bulk of experimental evidence leads to this conclusion. Indeed, if blackbody radiation was universal, there would be no need for the National Bureau of Standards to utilize graphite or soot in order to study such processes. The absence of cavities made of arbitrary walls (without any trace of a perfect absorber) is the best physical proof that universality does not hold. Our laboratories require carbon. Nothing further is needed to shatter Kirchhoff’s belief. Nonetheless, even

the simplest of mathematical considerations suffices to illustrate the point [25]. Perfectly reflecting cavities, containing no objects, emit no radiation [25]. Perfectly reflecting cavities which contain objects emit radiation which is characteristic of these objects [25]. Thus, if a carbon particle is placed within a perfectly reflecting cavity, the cavity will be black, irrespective of the size of the particle. This is a testament to the power of thermal equilibrium; but if the particle is small, it may take some time to reach this equilibrium. Perfectly absorbing cavities emit normal, or blackbody radiation [23, 25]. In such a cavity, the proper description of the radiation from an arbitrary object is $(\varepsilon_\nu + \rho_\nu)/(\alpha_\nu + \rho_\nu) = f(T, \nu)$ [25]. This equation echoes Stewart [26, 49, 82]. Conversely, Kirchhoff incorrectly advanced $\varepsilon_\nu/\alpha_\nu = f(T, \nu)$, leading to universality [20–22].

Consequently, when examining blackbody radiation, we are not dealing with a phenomenon of universal significance. Rather, we are dealing with a physical process which is extremely limited in its applications. Blackbodies are made of solids, and specifically relative to practical blackbodies, they are made of graphite. Nature knows no equivalent as is well demonstrated by the review of thermal emissivity tables [31]. Yet, even in the case of radiation from graphite, the physical cause of the process remains remarkably unknown to modern science. The physical species producing blackbody emission has not been concretely identified [19, 23].

If Planck's law [1, 2] has not been linked to a physical species, it is in part certain that the formulation of Kirchhoff's law [20–22], in its creation of universality, hindered the process. At the same time, there is a fundamental difficulty in providing a complete physical picture relative to thermal emission. This is because the nature of the oscillators, at the heart of thermal radiation, can change depending on the physical nature of the material being examined. The thermal emission profiles of metals are highly affected by their conduction electrons, at least in the sense that their presence acts to prevent emission and favor reflection. For each opaque material, a unique emission profile exists [31] and the answer to these problems will most likely involve the use of computational tools, not simple algebraic solutions. It may well be that entire lattices will have to be represented and processed in digital forms, in order to yield meaningful results. Yet, some thermal emission profiles, which provide Planck-like behavior, such as graphite, the microwave background (only apparent Planckian behavior), and the emission of the photosphere (only apparent Planckian behavior), may be capable of being solved analytically. A solution for one of these is likely to have broad implications for the others. At the same time, only graphite will remain truly Planckian in nature, as it is the only one restricted to a solid. The microwave background and the photosphere produce only apparent Planckian spectra. Since their physical sources are not solids, their relevant internal bonds (if any) are weak, and they support convection processes which alter the validity of

the temperatures they report [33].

For graphite or soot

$$\frac{\varepsilon_\nu}{\alpha_\nu} \sim \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

as Planck derived [1, 2]. Conversely, for the Sun and the microwave background, we can write that

$$\frac{\varepsilon_\nu}{\alpha_\nu} \sim \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT_{app}} - 1},$$

where T_{app} is constant. $T_{app} = T/\iota$, where T is the real temperature of the source and ι is a variable, with temperature dependence, whose value is $\sim 1,000$ for the photosphere and ~ 100 for the microwave background [33]. Thus, the real temperature of the photosphere is $\sim 1,000$ times higher than the currently accepted temperature [34, 35]. Similarly, the temperature for the source of the microwave background is ~ 100 times higher than the measured value [33, 39, 40]. These complications arise because we are dealing with non-solids outside the confines of enclosure [23, 33].

If a Planckian approach is used to analyze graphite, the carbon nucleus can be viewed as the mass and the carbon-carbon bond as the spring in an oscillator scenario [1, 2]. If the microwave background is confirmed to be from an oceanic source [33, 36–42], then the oscillators might be entire water molecules, linked through weak hydrogen bonding, vibrating within a fleeting lattice. In this regard, it remains interesting that water can become completely black. This occurs, for instance, when shock waves from nuclear explosions propagate in the sea. For the photosphere, if a hydrogen-based condensed Sun is contemplated [34, 35], the vibration of protons within a fleeting lattice field will have to be considered. In this case, the electrons might simply occupy conduction bands. Nonetheless, the nuclei should be viewed as being confined to a distinct condensed structure which, though fleeting, is being maintained, perhaps only by the need to sustain the quantum mechanical requirements to produce the conduction bands. Physicists versed in the properties of condensed liquid metallic hydrogen might consider these questions. Only the future can reveal how mankind moves forward on linking a given physical species to a center of emission.

With the loss of the universal function, the proper treatment of materials will involve the long recognized fact that the ratio of the emission, e , of an object to its absorption, a , is equal to a complex function dependent on its temperature, T , its nature, N , (its shape, the roughness of its surface, its specific heat, etc.), and the wavelengths of interest, namely $e/a = f(T, N, \lambda)$. Also, e and a , individually, are functions of these parameters, otherwise, as Agassi highlights [30], spectroscopy would be impossible. The aforementioned equation can be simplified to Kirchhoff's formulation $e/a = f(T, \lambda)$ only within a perfectly absorbing enclosure or within an enclosure where a perfect absorber is also present. In all these cases, the object never truly becomes a

blackbody. Along with its own emission, it simply reflects radiation in the cavity and appears to hold blackbody properties. It is difficult to envision how this scenario is of any use in modern physics.

The physics community has persisted in upholding Kirchhoff's law of thermal emission even though it has been refuted both recently [23–25] and in the past (see [86] for a discussion of the controversy surrounding Kirchhoff's law). This has occurred despite the fact that graphite and soot are uniquely positioned in all blackbody work with cavities. Nonetheless, some of this hesitance may be due to a certain respect, even reverence, for Kirchhoff and his work. In part, there is also the proximity to Planck himself. Such concerns are unjustified, in that even if Kirchhoff's law loses its universal status, nothing changes relative to Planck's derivation. Planck's law [1, 2] simply becomes devoid of universal significance. It maintains its value relative to the treatment of radiation within perfectly absorbing enclosures and within perfectly reflecting enclosures which contain a perfect absorber. Of course, Planck's equation will no longer extend to simple perfectly reflecting enclosures.

At the same time, the merit of k and h , at the heart of Planck's law, is not altered. The great changes simply involve the interdict of extending the laws of thermal emission [1, 2, 110, 111], without modification, to objects which are not solids [33–42] or enclosed within perfectly absorbing cavities [23–25].

Despite these facts, it may well be that physics remains unwilling to pronounce itself relative to the invalidity of Kirchhoff's treatment until the consequences of the error become so great that society demands retraction. The reassignment of the microwave background to the Earth [33, 36–42] should eventually provide sufficient motivation to act. On that day, a new age in astrophysics will spring forth [34, 35] and we may finally begin to write the long-awaited ode to Balfour Stewart.

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Dedication

This work is dedicated to my sister H el ene, her husband Gervais B edard, their children (Sonia, Karl, and Genevi eve) and their grandchildren (Megan Gagn e, Rapha el Turcotte[†], and Th eog enes Turcotte) on this, their 30th wedding anniversary (May 22, 1978).

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The Rôle of the Element Rhodium in the Hyperbolic Law of the Periodic Table of Elements

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The rôle of the element rhodium as an independent affirmation of calculations by the Hyperbolic Law and validity of all its relations is shown herein. The deviation in calculation by this method of the atomic mass of heaviest element is 0.0024%, and its coefficient of scaling 0.001–0.005%.

1 Introduction

The method of rectangular hyperbolas assumes that their peaks (i.e. vertices) should be determined with high accuracy. For this purpose the theorem of Lagrange and the coefficient of scaling calculated by the Author for transition from the system of coordinates of the image of a hyperbola, standard practice of the mathematician, and used in chemistry, are utilized. Such an approach provides a means for calculating the parameters of the heaviest element in the Periodic Table of D. I. Mendeleev [1].

In the first effect of the Hyperbolic Law it is shown that to each direct hyperbola corresponds an adjacent hyperbola: they intersect on the line $Y = 0.5$ at a point the abscissa of which is twice the atomic mass of an element [2]. This fact is clearly illustrated for Be, Ca, Cd in Fig. 1.

Upon close examination of the figure deeper relationships become apparent:

- From the centre of adjacent hyperbolas ($X = 0, Y = 1$) the secants have some points of crossing, the principal of which lie on the line $Y = 0.5$ and on the virtual axes (peaks);
- The secants intersect a direct hyperbola in two points, with gradual reduction of a segment with the increase in molecular mass;
- Behind the virtual axis of adjacent hyperbolas the secants cut a direct hyperbola in only one point;
- In conformity therewith, the magnitude of the abscissa, between a secant and a point of intersection of hyperbolas on the line $Y = 0.5$, also changes;
- For the element rhodium the secant becomes a tangent and also becomes the virtual axis of adjacent hyperbolas.

2 Mathematical motivation

On the basis of the presented facts, we have been led to calculations for 35 elements to establish the laws for the behavior of secants. The results are presented in the table for the following parameters:

- Atomic numbers of elements and their masses;
- Calculated coordinates of peaks of elements (the square root of the atomic mass and coefficient of scaling 20.2895 are used);
- Abscissas of secants on the line $Y = 0.5$ are deduced from the equation of a straight lines by two points

$$\frac{(X - X_1)}{(X_2 - X_1)} = \frac{(Y - Y_1)}{(Y_2 - Y_1)} \quad (\text{column 6});$$

- Points of intersection of direct and adjacent hyperbolas (column 7);
- Difference between the abscissas in columns 6 and 7 (column 8);
- Tangent of an inclination of a secant from calculations for column 6.

According to columns 6 and 7 in Fig. 2, dependences which essentially differ from each other are obtained. Abscissas of secants form a curve of complex form which can describe with high reliability (size of reliability of approximation $R^2 = 1$) only a polynomial of the fifth degree. The second dependency has a strictly linear nature ($Y = 2X$), and its straight line is a tangent to a curve at the point (102.9055, 205.811). For clarity the representation of a curve has been broken into two parts: increases in molecular mass (Fig. 3) and in return — up to hydrogen, inclusive (Fig. 4). The strongly pronounced maximum for elements B, C, N, O, F, Ne is observed.

At the end of this curve there is a very important point at which the ordinate is equal to zero, where (the line of rhodium in the table) the data of columns 6 and 7 coincide.

Thus it is unequivocally established that for rhodium the secant, tangent and the virtual axis for an adjacent hyperbola are represented by just one line, providing for the first time a means to the necessary geometrical constructions on the basis of only its atomic mass (**the only one in the Periodic Table**), for the proof of the Hyperbolic Law.

Graphical representation of all reasoning is reflected in Fig. 5 from which it is plain that the point with coordinates (205.811, 0.5) is the peak of both hyperbolas, and the peaks

of Ca and Ta are on both sides of it. Below are the calculations for the basic lines of rhodium on these data:

1. A secant: —

$$\frac{(X - 0)}{(205.811 - 0)} = \frac{(Y - 1)}{(0.5 - 1)},$$

whence

$$Y = -0.0024294134X + 1.$$

At $Y = 0$, $X = 411.622$; in this case coordinates of peak will be: $X = 205.811$, $Y = 0.5$.

2. A tangent:— the equation of a direct hyperbola,

$$Y = \frac{102.9055}{X},$$

its derivative at $X = 205.811$, so

$$Y' = -\frac{102.9055}{205.811^2} = -0.0024294134,$$

$$Y - 0.5 = -0.0024294134X + 0.5.$$

Finally,

$$Y = -0.0024294134X + 1;$$

at $Y = 0$, $X = 411.622$.

3. A normal: — (the virtual axis),

$$Y = 0.0024294134X;$$

at $Y = 1$, $X = 411.622$.

Here are the same calculations for the tabulated data presented:

1. A secant: —

$$\frac{X}{205.82145} = \frac{(Y - 1)}{(0.4999746 - 1)},$$

whence

$$Y = -0.0024294134X + 1;$$

$$Y = 1, \quad X = 411.622.$$

2. A tangent: —

$$Y = \frac{102.9055}{X},$$

the fluxion at $X = 205.821454$,

$$Y' = -\frac{102.9055}{205.82145^2} = -0.0024291667,$$

so

$$Y - 0.4999746 = -0.0024291667(X - 205.82145),$$

whence

$$Y = -0.0024291667X + 0.99994928,$$

$$Y = 0, \quad X = 411.6429.$$

3. A normal: —

$$Y = 0.0024291667X;$$

$$Y = 1, \quad X = 411.6638.$$

3 Comparative analysis calculations

For a secant the results are identical with the first set of calculations above, whereas for a tangent and normal there are some deviations, close to last element calculated.

By the first set of calculations above its atomic mass is 411.622; hence the deviation is $411.663243 - 411.622 = 0.041243$ (0.01%). By the second set the size of a tangent and a normal are close to one another (an average of 411.65335) and have a smaller deviation: $411.663243 - 411.65335 = 0.009893$ (0.0024%). This is due to the tangent of inclination of the virtual axis of a direct hyperbola in the first set is a little high.

Using rhodium (Fig. 5) we can check the propriety of a choice of coefficient of scaling. It is necessary to make the following calculations for this purpose:

- Take the square root of atomic mass of rhodium ($X = Y = 10.1442348$);
- Divide X_0 by X of the peak ($205.811/10.1442348 = 20.2885$);
- Divide $Y = 10.1442348$ by Y_0 of the peak (0.5): also gives 20.2885;
- The difference by X and Y with the coefficient obtained, 20.2895, yielding the same size at 0.001 or 0.005%.

Formulae for transition from one system of coordinates to another have been given in the first paper of this series.

Using data for peaks, from the table, we get the following results:

Coordinates of peak

$$X_0 = 205.8215, \quad Y_0 = 0.49997,$$

$$X = Y = 10.1442348,$$

then

$$\frac{X_0}{X} = 20.2895, \quad \frac{Y}{Y_0} = 20.2897,$$

i. e. absolute concurrence (maximum difference of 0.0009%).

4 The rôle of the element Rhodium

However, all these insignificant divergences do not belittle the most important conclusion: that the validity of the Hyperbolic Law is established because the data calculated above completely coincide with calculations for rhodium is proved, based only on its atomic mass.

All the calculations for the table were necessary in order to find a zero point for rhodium, for which it is possible to do so without calculating the secant, but using only its atomic mass, thereby verifying the Hyperbolic Law.

How to get the correct choice of abscissa of a secant is depicted in Fig. 6 (using beryllium as an example) where instead of its tabulated value, 35.7434, the value equal to twice the point of intersection (36.0488) has been used. Here we

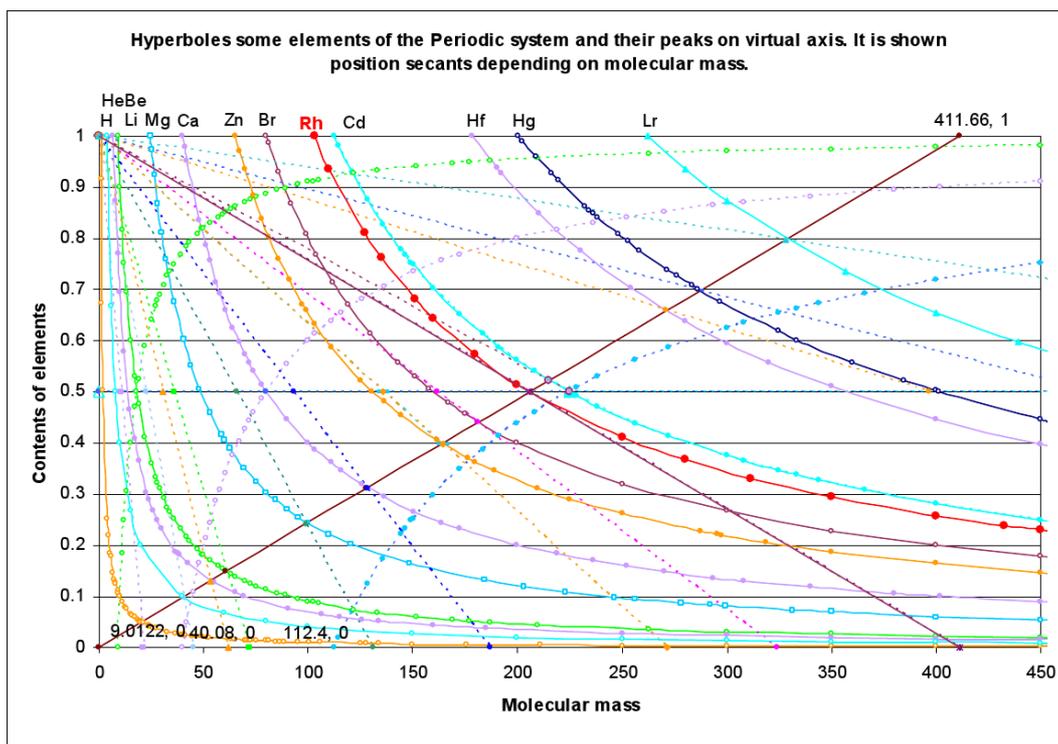


Fig. 1

tried to make a start from any fixed point not calculated (similar to the case for rhodium). It has proved to be impossible and has led to a mistake in the definition of the peak. In Fig. 7 the geometrical constructions for beryllium on the basis of correct settlement of data are given.

5 Conclusions

Previously we marked complexity of a choice of peak of a hyperbola of an element in the coordinates, satisfying the conditions $Y \leq 1$, $K \leq X$, as on an axis of ordinates the maximum value being a unit whilst the abscissa can take values in the hundreds of units. The problem has been solved by means of the theorem of Lagrange and the coefficient of scaling deduced. On the basis thereof our further conclusions depended, so it was very important to find a method not dependent on our calculations and at the same time allowing unequivocally to estimate the results. Owing to properties of the virtual axis of an rectangular hyperbola on which peaks of all elements lie, it is enough to have one authentic point.

Analyzing the arrangement of the virtual axes of direct and adjacent hyperbolas, we have paid attention to their point of intersection (205.83, 0.5), the abscissa of which is exactly half of atomic mass of the last element. As secants from the centre $X = 0$, $Y = 1$ cut direct hyperbolas any way (Fig. 1), we have been led to necessary calculations and have obtained a zero point at which the secant coincides with a tangent and

the valid axis. The divergence with tabular data is in the order of 0.004%–0.009%.

Thus rhodium provides an independent verification of the method of rectangular hyperbolas for the Periodic Table of elements of D. I. Mendeleev.

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1	2	3	4	5	6	7	8	9
El.	No.	At. mass	X ₀ peak	Y ₀ peak	Abs. secant	Cross. hyperb.	$\Delta = 6 - 7$	$\tan \alpha$, secant
H	1	1.0079	20.3695	0.04948	10.715	2.0158	8.6992	-0.046664
He	2	4.0026	40.5992	0.0986	22.5163	8.0052	14.5111	-0.0222
Li	3	6.941	53.4543	0.12985	30.7155	13.882	16.8335	-0.01628
Be	4	9.0122	60.9097	0.14976	35.7434	18.0244	17.719	-0.014
B	5	10.811	66.712	0.162055	39.80692	21.622	18.18492	-0.01256
C	6	12.0107	70.3162	0.1708	42.4	24.0214	18.3786	-0.0117923
N	7	14.0067	75.9345	0.184458	46.5546	28.0134	18.5412	-0.01074
O	8	15.9994	81.1565	0.197143	50.5423	31.9988	18.5435	-0.009893
F	9	18.9984	88.4362	0.21483	56.3163	37.9968	18.3195	-0.008878
Ne	10	20.1797	91.1441	0.2214	58.5311	40.3594	18.1717	-0.0085425
Mg	12	24.305	100.0274	0.242983	66.0669	48.61	17.4569	-0.007568
S	16	32.065	114.89125	0.27909	79.6849	64.13	15.5549	-0.006273
Ca	20	40.078	128.4471	0.31202	93.3508	80.156	13.1948	-0.005356
Cr	24	51.9961	146.3042	0.3554	113.484	103.9922	9.4918	-0.004406
Zn	30	65.409	164.093	0.3986	136.428	130.818	5.61	-0.003665
Br	35	79.904	181.366	0.44057	162.0982	159.808	2.29	-0.003085
Zr	40	91.224	193.7876	0.47074	183.075	182.448	0.627	-0.002731
Mo	42	95.94	198.7336	0.482757	192.1085	191.88	0.2285	-0.002603
Rh	45	102.906	205.82145	0.4999746	205.811	205.811	0	-0.00242941
Cd	48	112.411	215.1175	0.52256	225.26	224.822	0.458	-0.00221946
Ba	56	137.327	237.7658	0.577573	281.428	274.654	6.774	-0.001777
Nd	60	144.242	243.6785	0.591936	298.5785	288.484	10.09455	-0.0016746
Sm	62	150.36	248.7926	0.60436	314.417	300.72	13.7	-0.00159
Dy	66	162.5	258.6414	0.628283	347.9	325	22.9	-0.001437
Yb	70	173.04	266.8976	0.64834	379.48	346.08	33.4	-0.0013176
Hf	72	178.49	271.068	0.65847	396.843	356.98	39.863	-0.00126
Ta	73	180.948	272.928	0.663	404.923	361.896	43.027	-0.0012348
Re	75	186.207	276.8658	0.67255	422.7646	372.414	50.35	-0.0011827
Ir	77	192.217	281.2984	0.68332	444.1376	384.434	59.704	-0.0011258
Hg	80	200.59	287.3598	0.698	475.8318	401.18	74.6518	-0.00105
At	85	210	294.0228	0.71423	514.44	420	94.44	-0.000972
Fr	87	223	302.9868	0.736	573.85	446	127.85	-0.00087
Th	90	232.038	309.0658	0.75077	620.0472	464.07612	155.971	-0.000806
Am	95	243	316.282	0.7683	682.53	486	196.53	-0.0007326
Es	99	252	322.0858	0.7824	740.0874	504	236.0874	-0.0006756

- columns 4 and 5 contain coordinates of peaks of rectangular hyperbolas of elements;
- in a column 6 are presented abscissas the secants which are starting with the peak center (0,1) up to crossings with line $Y = 0.5$; at prolongation they cross the valid axis in points peaks;
- in a column 7 are resulted abscissa points of crossing of a direct and adjacent hyperbola each element presented here;
- the column 8 contains a difference between sizes of 6 and 7 columns;
- in a column 9 tangents of a corner of an inclination of secants are resulted; at an element "rhodium" this line crosses an axis X in a point with abscissa, equal 411.622, and its position coincides with tangent in peak; $411.66 - 411.62 = 0.04$ or nearly so 0.01% from atomic mass.

Table 1: Results of calculations for some elements of the Periodic Table

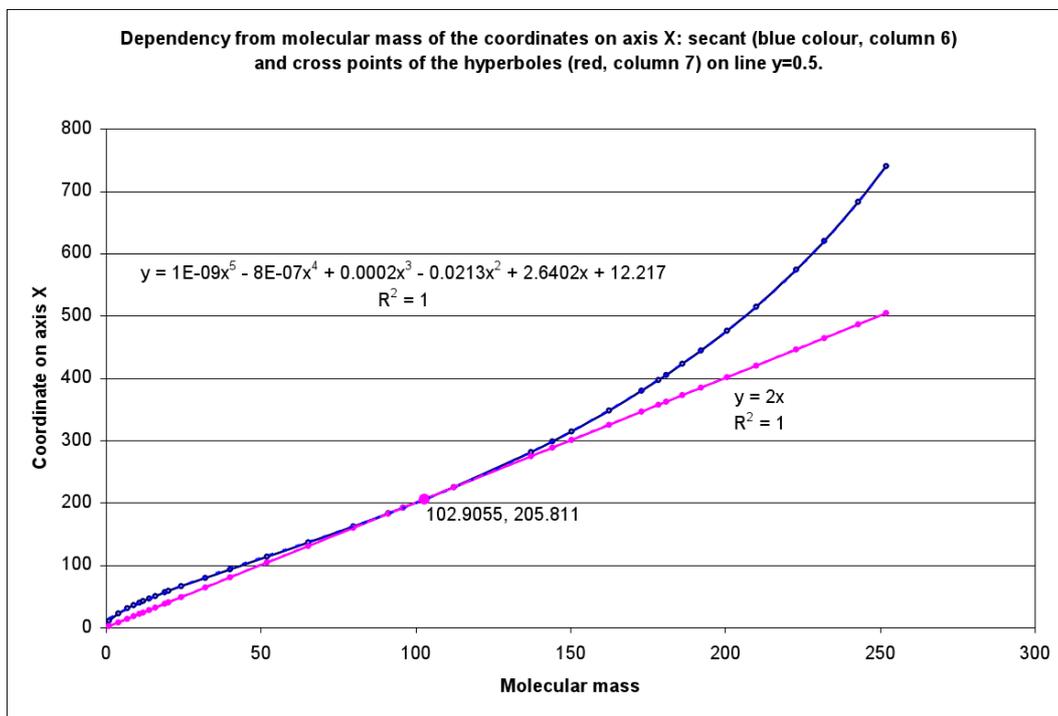


Fig. 2

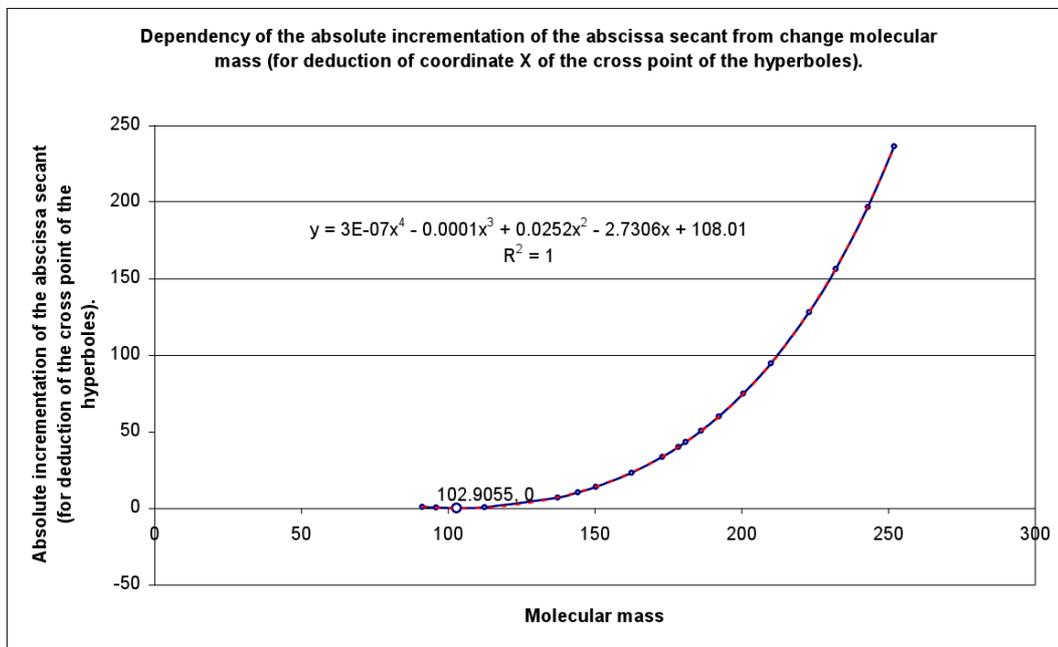


Fig. 3

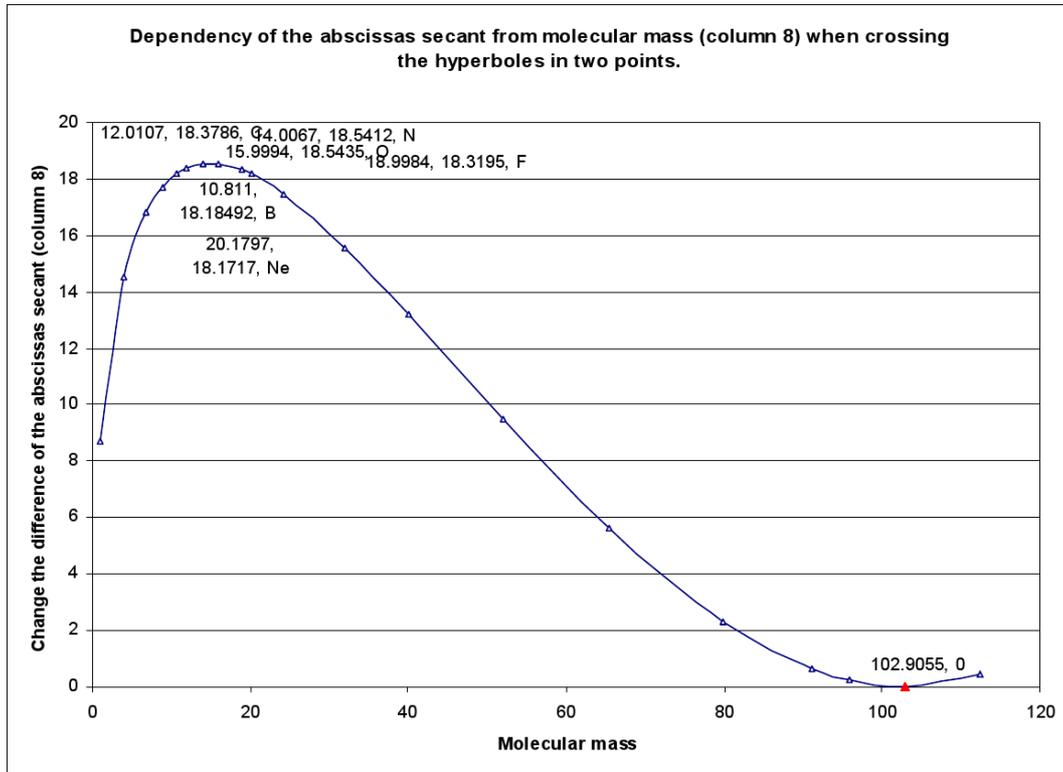


Fig. 4

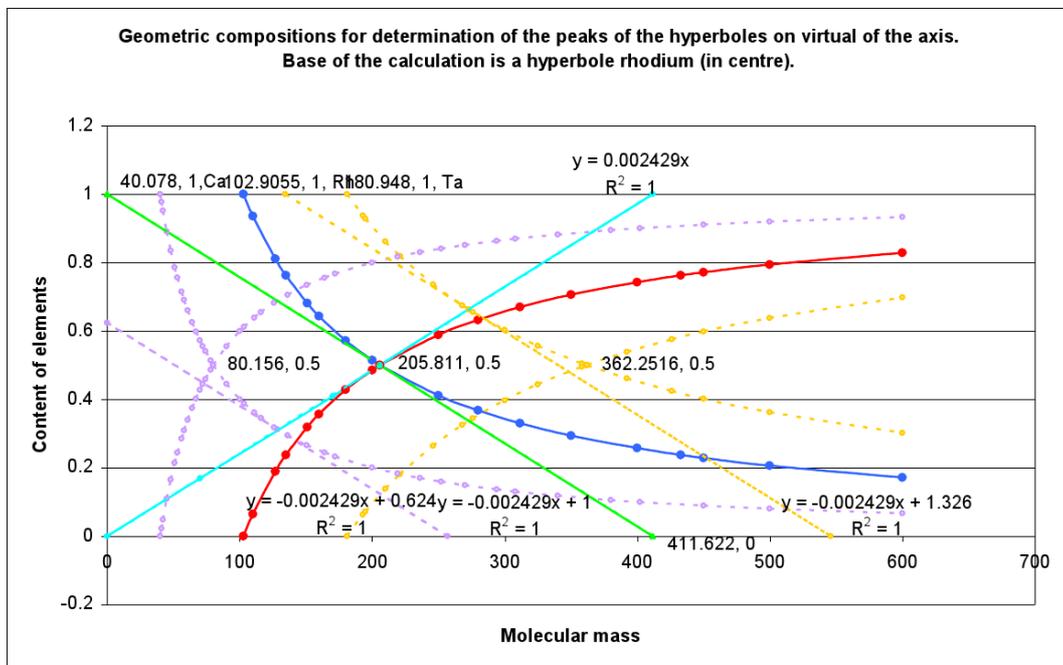


Fig. 5

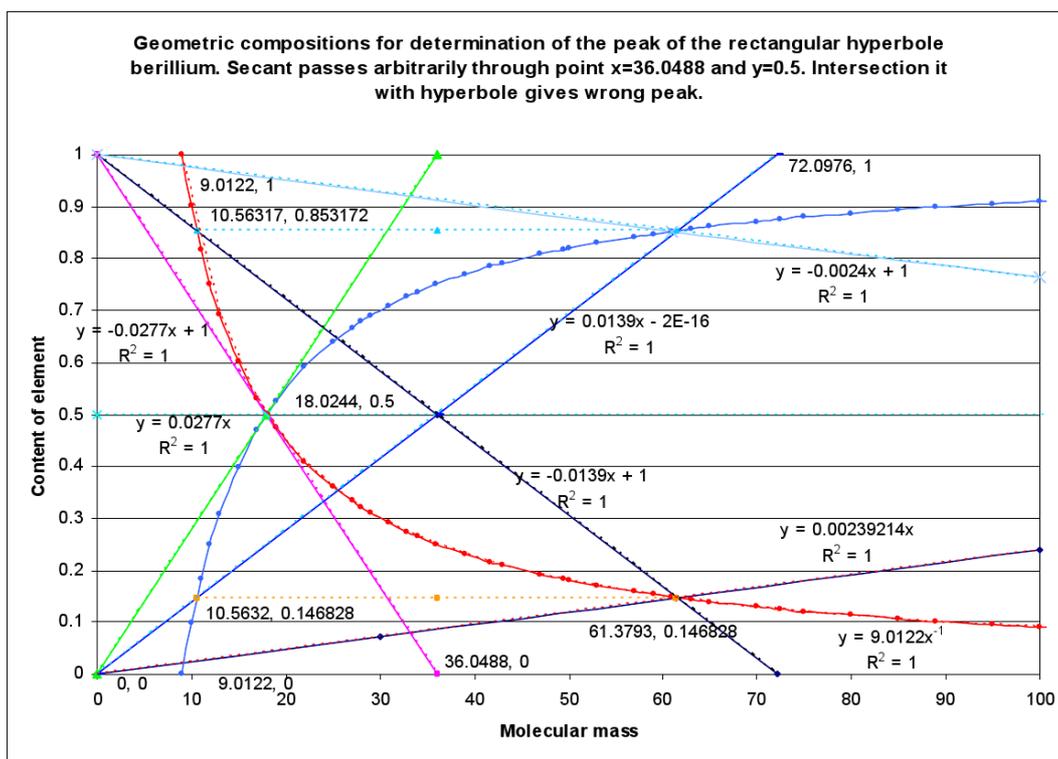


Fig. 6

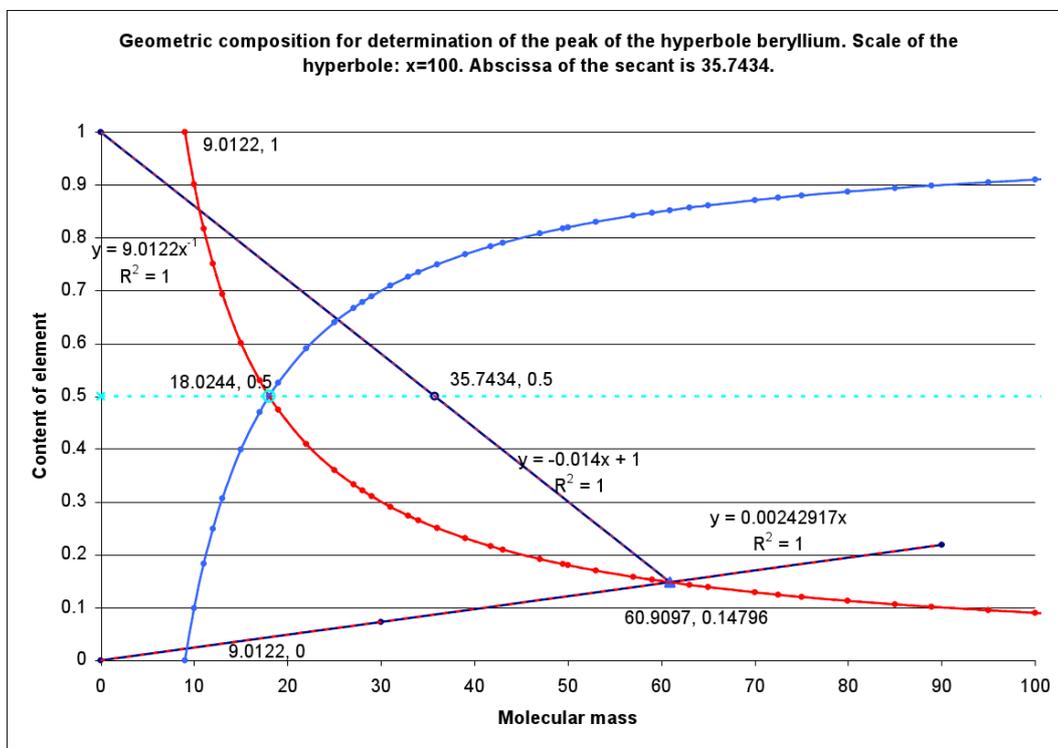


Fig. 7

What Gravity Is. Some Recent Considerations

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It is well-known, that when it comes to discussions among physicists concerning the meaning and nature of gravitation, the room temperature can be so hot. Therefore, for the sake of clarity, it seems worth that all choices were put on a table, and we consider each choice's features and problems. The present article describes a non-exhaustive list of such gravitation theories for the purpose of inviting further and more clear discussions.

1 Introduction

The present article summarizes a non-exhaustive list of gravitation theories for the purpose of inviting further and more clear discussions. It is well-known, that when it comes to discussions among physicists concerning the meaning and nature of gravitation, the room temperature can be so hot. Therefore, for the sake of clarity, it seems worth that all choices were put on a table, and we consider each choice's features and problems. Of course, our purpose here is not to say the last word on this interesting issue.

2 Newtonian and non-relativistic approaches

Since the days after Newton physicists argued what is the meaning of “action at a distance” (Newton term) or “spooky action” (Einstein term). Is it really possible to imagine how an apple can move down to Earth without a medium whatsoever?

Because of this difficulty, from the viewpoint of natural philosophy, some physicists maintained (for instance Euler with his impulsion gravity), that there should be “pervasive medium” which can make the attraction force possible. They call this medium “ether” though some would prefer this medium more like “fluid” instead of “solid”. Euler himself seems to suggest that gravitation is some kind of “external force” acting on a body, instead of *intrinsic* force:

“gravity of weight: It is a power by which all bodies are forced towards the centre of the Earth” [3].

But the Michelson-Morley experiment [37] opened the way for Einstein to postulate that ether hypothesis is not required at all in order to explain Lorentz's theorem, which was the beginning of Special Relativity. But of course, one can ask whether the Michelson-Morley experiment really excludes the so-called ether hypothesis. Some experiments after Michelson seem to indicate that “ether” is not excluded in the experiment setup, which means that there is Earth absolute motion [4, 5].

To accept that gravitation is external force instead of intrinsic force implies that there is distinction between gravitation and inertial forces, which also seem to indicate that inertial force can be modified externally via electromagnetic field [6].

The latter notion brings us to long-time discussions in various physics journals concerning the electromagnetic nature of gravitation, i.e. whether gravitation pulling force have the same properties just as electromagnetic field is described by Maxwell equations. Proponents of this view include Tajmar and de Matos [7, 8], Sweetser [9]. And recently Rabounski [10] also suggests similar approach.

Another version of Euler's hypothesis has emerged in modern way in the form of recognition that gravitation was carried by a boson field, and therefore gravitation is somehow related to low-temperature physics (superfluid as boson gas, superconductivity etc.). The obvious advantage of superfluidity is of course that it remains frictionless and invisible; these are main features required for true ether medium — i.e. no resistance will be felt by objects surrounded by the ether, just like the passenger will not feel anything inside the falling elevator. No wonder it is difficult to measure or detect the ether, as shown in Michelson-Morley experiment. The superfluid Bose gas view of gravitation has been discussed in a series of paper by Consoli et al. [11], and also Volovik [12].

Similarly, gravitation can also be associated to superconductivity, as shown by de Matos and Beck [29], and also in Podkletnov's rotating disc experiment. A few words on Podkletnov's experiment. Descartes conjectured that there is no gravitation without rotation motion [30]. And since rotation can be viewed as solution of Maxwell equations, one can say that there is no gravitation separated from electromagnetic field. But if we consider that equations describing superconductivity can be viewed as mere generalization of Maxwell equations (London field), then it seems we can find a modern version of Descartes' conjecture, i.e. *there is no gravitation without superconductivity rotation*. This seems to suggest the significance of Podkletnov's experiments [31, 32].

3 Relativistic gravitation theories

Now we will consider some alternative theories which agree with both Newton theory and Special Relativity, but differ either slightly or strongly to General Relativity. First of all, Einstein's own attempt to describe gravitation despite earlier gravitation theories (such as by Nordstrom [1]) has been inspired by his thought-experiment, called the "falling elevator" experiment. Subsequently he came up with conjecture that there is proper metric such that a passenger inside the elevator will not feel any pulling gravitation force. Therefore gravitation can be replaced by certain specific-chosen metric.

Now the questions are twofold: (a) whether the proper-metric to replace gravitation shall have non-zero curvature or it can be flat-Minkowskian; (b) whether the formulation of General relativity is consistent enough with Mach principle from where GTR was inspired. These questions inspired heated debates for several decades, and Einstein himself (with colleagues) worked on to generalize his own gravitation theories, which implies that he did find that his theory is not complete. His work with Strauss, Bergmann, Pauli, etc. (Princeton School) aimed toward such a unified theory of gravitation and electromagnetism.

There are of course other proposals for relativistic gravitation theories, such as by Weyl, Whitehead etc. [1]. Meanwhile, R. Feynman and some of his disciples seem to be more flexible on whether gravitation shall be presented in the General-Relativity "language" or not.

Recently, there is also discussion in online forum over the question: (a) above, i.e. whether curvature of the metric surface is identical to the gravitation. While most physicists seem to agree with this proposition, there is other argument suggesting that it is also possible to conceive General Relativity even with zero curvature [13, 14].

Of course, discussion concerning relativistic gravitation theories will not be complete without mentioning the PV-gravitation theory (Puthoff et al. [15]) and also Yilmaz theory [16], though Misner has discussed weaknesses of Yilmaz theory [17], and Yilmaz et al. have replied back [18]. Perhaps it would be worth to note here that General Relativity itself is also not without limitations, for instance it shall be modified to include galaxies' rotation curve, and also it is actually theory for one-body problem only [2], therefore it may be difficult to describe interaction between bodies in GTR.

Other possible approaches on relativistic gravitation theories are using the fact that the "falling-elevator" seems to suggest that it is possible to replace gravitation force with certain-chosen metric. And if we consider that one can find simplified representation of Maxwell equations with Special Relativity (Minkowski metric), then the next logical step of this "metrical" (some physicists prefer to call it "geometro-dynamics") approach is to represent gravitation with yet another special relativistic but with extra-dimension(s). This was first conjectured in Kaluza-Klein theory [19]. Einstein

himself considered this theory extensively with Strauss etc. [20]. There are also higher-dimensional gravitation theories with 6D, 8D and so forth.

In the same direction, recently these authors put forth a new proposition using Carmeli metric [21], which is essentially a "phase-space" relativity theory in 5-dimensions.

Another method to describe gravitation is using "torsion", which is essentially to introduce torsion into Einstein field equations. See also torsional theory developed by Hehl, Kiehn, Rapoport etc. cited in [21].

It seems worth to remark here, that relativistic gravitation does not necessarily exclude the possibility of "aether" hypothesis. B. Riemann extended this hypothesis by assuming (in 1853) that the gravitational aether is an incompressible fluid and normal matter represents "sinks" in this aether [34], while Einstein discussed this aether in his Leiden lecture *Ether and Relativity*.

A summary of contemporary developments in gravitation theories will not be complete without mentioning Quantum Gravity and Superstring theories. Both are still major topics of research in theoretical physics and consist of a wealth of exotic ideas, some or most of which are considered controversial or objectionable. The lack of experimental evidence in support of these proposals continues to stir a great deal of debate among physicists and makes it difficult to draw definite conclusions regarding their validity [38]. It is generally alleged that signals of quantum gravity and superstring theories may occur at energies ranging from the mid or far TeV scale all the way up to the Planck scale.

Loop Quantum Gravity (LQG) is the leading candidate for a quantum theory of gravitation. Its goal is to combine the principles of General Relativity and Quantum Field Theory in a consistent non-perturbative framework [39]. The features that distinguish LQG from other quantum gravity theories are: (a) background independence and (b) minimality of structures. Background independence means that the theory is free from having to choose an a priori background metric. In LQG one does not perturb around any given classical background geometry, rather arbitrary fluctuations are allowed, thus enabling the quantum "replica" of Einstein's viewpoint that gravity is geometry. Minimality means that the general covariance of General Relativity and the principles of canonical quantization are brought together without new concepts such as extra dimensions or extra symmetries. It is believed that LQG can unify all presently known interactions by implementing their common symmetry group, the four-dimensional diffeomorphism group, which is almost completely broken in perturbative approaches.

The fundamental building blocks of String Theory (ST) are one-dimensional extended objects called strings [40, 41]. Unlike the "point particles" of Quantum Field Theories, strings interact in a way that is almost uniquely specified by mathematical self-consistency, forming an allegedly valid quantum theory of gravity. Since its launch as a dual res-

onance model (describing strongly interacting hadrons), ST has changed over the years to include a group of related superstring theories (SST) and a unifying picture known as the M-theory. SST is an attempt to bring all the particles and their fundamental interactions under one umbrella by modeling them as vibrations of super-symmetric strings.

In the early 1990s, it was shown that the various superstring theories were related by dualities, allowing physicists to map the description of an object in one superstring theory to the description of a different object in another superstring theory. These relationships imply that each of SST represents a different aspect of a single underlying theory, proposed by E. Witten and named M-theory. In a nut-shell, M-theory combines the five consistent ten-dimensional superstring theories with eleven-dimensional supergravity. A shared property of all these theories is the holographic principle, that is, the idea that a quantum theory of gravity has to be able to describe physics occurring within a volume by degrees of freedom that exist on the surface of that volume. Like any other quantum theory of gravity, the prevalent belief is that true testing of SST may be prohibitively expensive, requiring unprecedented engineering efforts on a large-system scale. Although SST is falsifiable in principle, many critics argue that it is un-testable for the foreseeable future, and so it should not be called science [38].

One needs to draw a distinction in terminology between string theories (ST) and alternative models that use the word “string”. For example, Volovik talks about “cosmic strings” from the standpoint of condensed matter physics (topological defects, superfluidity, superconductivity, quantum fluids). Beck refers to “random strings” from the standpoint of statistical field theory and associated analytic methods (space-time fluctuations, stochastic quantization, coupled map lattices). These are not quite the same as ST, which are based on “brane” structures that live on higher dimensional space-time.

There are other contemporary methods to treat gravity, i.e. by using some advanced concepts such as group(s), topology and symmetries. The basic idea is that Nature seems to prefer symmetry, which lead to higher-dimensional gravitation theories, Yang-Mills gravity etc.

Furthermore, for the sake of clarity we have omitted here more advanced issues (sometimes they are called “fringe research”), such as faster-than-light (FTL) travel possibility, warpdrive, wormhole, cloaking theory (Greenleaf et al. [35]), antigravity (see for instance Naudin’s experiment) etc. [36].

4 Wave mechanical method and diffraction hypothesis

The idea of linking gravitation with wave mechanics of Quantum Mechanics reminds us to the formal connection between Helmholtz equation and Schrödinger equation [22].

The use of (modified) Schrödinger equation has become so extensive since 1970s, started by Wheeler-DeWitt (despite

the fact that the WDW equation lacks observation support). And recently Nottale uses his scale relativistic approach based on stochastic mechanics theory in order to generalize Schrödinger equation to describe wave mechanics of celestial bodies [23]. His scale-relativity method finds support from observations both in Solar system and also in exo-planets.

Interestingly, one can also find vortex solution of Schrödinger equation, and therefore it is worth to argue that the use of wave mechanics to describe celestial systems implies that there are vortex structure in the Solar system and beyond. This conjecture has also been explored by these authors in the preceding paper. [24] Furthermore, considering formal connection between Helmholtz equation and Schrödinger equation, then it seems also possible to find out vortex solutions of Maxwell equations [25, 26, 27]. Interestingly, experiments on plasmoid by Bostick et al. seem to vindicate the existence of these vortex structures [28].

What’s more interesting in this method, perhaps, is that one can expect to consider gravitation and wave mechanics (i.e. Quantum Mechanics) in equal footing. In other words, the quantum concepts such as ground state, excitation, and zero-point energy now can also find their relevance in gravitation too. This “classical” implications of Wave Mechanics has been considered by Ehrenfest and also Schrödinger himself.

In this regards, there is a recent theory proposed by Gulko [33], suggesting that matter absorbs from the background small amounts of energy and thus creates a zone of reduced energy, and in such way it attracts objects from zones of higher energy.

Another one, by Glenn E. Perry, says that gravity is diffraction (due to the changing energy density gradient) of matter or light as it travels through the aether [33].

We can remark here that Perry’s Diffraction hypothesis reminds us to possible production of energy from physical vacuum via a small fluctuation in it due to a quantum indeterminacy (such a small oscillation of the background can be suggested in any case because the indeterminacy principle). On the average the background vacuum does not radiate — its energy is constant. On the other hand, it experiences small oscillation. If an engine built on particles or field interacts with the small oscillation of the vacuum, or at least “senses the oscillation, there is a chance to get energy from them. Because the physical vacuum is eternal capacity of energy, it is easy to imagine some possible techniques to be discovered in the future to extract this energy.

Nonetheless, diffraction of gravity is not a “new hot topic” at all. Such ideas were already proposed in the 1920’s by the founders of relativity. They however left those ideas, even unpublished but only mentioned in memoirs and letters. The main reason was that (perhaps) almost infinitely small energy which can be extracted from such background per second. (In the mean time, there are other various proposals suggesting that it is possible to ‘extract’ energy from gravitation field).

About Glenn Perry and his theory. There is a drawback that that matter he called “aether” was not properly determined by him. In such a way like that, everything can be “proven”. To produce any calculation for practical purpose, we should have exact data on the subject of this calculation, and compare it with actual experiments.

On the other hand, such an idea could be put into another field — the field of Quantum Mechanics. That is, to study diffraction not gravitational radiation (gravitational waves which is so weak that not discovered yet), but waves of the field of the gravitational force — in particular those can be seismic-like waves travelling in the cork of the Earth (we mean not the earthquakes) but in the gravitational field of the planet. These seismic-like oscillations (waves) of the gravitational force are known to science, and they aren't weak: everyone who experienced an earthquake knows this fact.

Other hint from wave aspect of this planet is known in the form of Schumann resonance, that the Earth produces vibration at very-low frequency, which seems to support the idea that planetary mass vibrates too, just as hypothesized in Wave Mechanics (de Broglie's hypothesis). Nonetheless, there are plenty of things to study on the large-scale implications of the Wave Mechanics.

5 Concluding remarks

The present article summarizes a non-exhaustive list of gravitation theories for the purpose of inviting further and more clear discussions. Of course, our purpose here is not to say the last word on this interesting issue. For the sake of clarity, some advanced subjects have been omitted, such as faster-than-light (FTL) travel possibility, warpdrive, wormhole, cloaking theory (Greenleaf et al.), antigravity etc. As to the gravitation research in the near future, it seems that there are multiple directions which one can pursue, with which we're not so sure. The only thing that we can be sure is that everything changes (Heraclitus of Ephesus), including how we define “what the question is” (Wheeler's phrase), and also what we mean with “metric”, “time”, and “space”. Einstein himself once remarked that ‘distance’ itself is merely an illusion.

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Models for Quarks and Elementary Particles — Part III: What is the Nature of the Gravitational Field?

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The first two parts of this article series dealt with the questions: What is a quark? and What is mass? While the present models lead to a *physical idea* of the mass, the geometrical theory of the general relativity only shows the *effect* of mass. From the physical idea of mass, from the idea of the resultant vector (EV) as electric flux $\not\epsilon$ and from the ideas relating to the magnetic monopole (MMP) it follows that the gravitational field is an electrical field. The share of the electrical gravitational flux $\not\epsilon_\Gamma$ on the entire electrical flux $\not\epsilon$ of a quark is determined from Newton's empirical gravitational constant G . The superposition of the $\not\epsilon_\Gamma$ -fluxes of two quark collectives produces the gravitational force effect between two quark collectives. Gravitational fields reach infinitely far according to our current ideas. Connected with the quark oscillations hinted in the Parts I and II this results in the idea of the $\not\epsilon$ - $\not\epsilon_\Gamma$ -flux spreading with infinite speed, having enormous consequences.

1 Introduction

In Parts I and II separate reference is made to the most productive assumptions or ideas relating to the development of the models. In Part I the formal assumptions/ideas are shown, which include the vectors in the constellation of the outer product of a vector with certain angular movements. At the end of Part II it transpires that the locus loop created by the EV is a physical central-symmetrical sinus oscillation in the mass-affected three-quark particle. Other productive ideas are the orthogonal, hyperbolic space with two real axes and an imaginary axis as well as the identification of the formal EV with a physical meaning. The EV identified as electrical flux $\not\epsilon$ with the dimension [Vm] results in the idea of the MMP. The absolute number of $\not\epsilon$ amounts to $\not\epsilon = 1.8095 \times 10^{-8}$ [Vm] according to the network of constants, see [1, page 143]. The massless MMP is an important idea to recognise on the one hand what mass is and on the other hand to develop the quark structure of the massless photon(-likes) from the quark composition of the electron.

2 The meaning of the “fountain”

In Part II the model idea for the composition of the MMP with the surrounding electrical field is shown with Fig. 1. Thus, the decisive physical components of a quark are introduced with Part II, not considering the dynamics of these components in mass-affected and massless particles. Relating to Fig. 1 it was not explained what the $\not\epsilon_\Gamma$ -field is. This is done now.

During the course of the development of the models attempts were made to look behind the facade of Newton's gravitational equation wherein obviously there was no shortage of incorrect estimates, one way streets and wrong tracks.

Newton's gravitational constant G included in the equation is one of the many independent quantities of the standard model of physics to be determined empirically.

In Part II it is shown what mass is. The route there commences with the equations of $E = m \times c^2$ and $E = h \times \nu$, resulting in equation 1 of Part II, which can also be described as equation (8–II) of [1]: $m = \frac{elt}{2^e \alpha \times \lambda_C \times c^2}$. If this form is introduced in Newton's gravitational equation $K_\Gamma = G \times \frac{m_a \times m_b}{l^2}$, G can be determined with the correct dimension [m^5/VAs^5]:

$$G = \frac{4^e \alpha^2 \times c^4 \lambda_C a \lambda_C b}{elt \times n_1 \times n_2} . \quad (1)$$

In it $e\alpha$ are the fine structure constant, λ_C the Compton wavelengths of the elementary particles involved and elt see below. On the route to clarifying the gravitational equation the aim is to find what the quantities n_1 and n_2 are and how large they are. If equation (1) is solved for n_1 and n_2 and the Compton wavelengths of the nucleons (as mass-richest elementary particles) are substituted for λ_C , the empirical numerical value $\sqrt{n_1 \times n_2} = n_i = 3.939 \times 10^{18}$ is obtained.

The n thus are gigantic numbers. What do these gigantic numbers stand for?

At this point it is highly productive to use Fig. 1 of Part II. Visible is the MMP that occurs with highest frequencies, which is enclosed by the electrical source flux $\not\epsilon$. Here, by far the predominant part of this source flux $\not\epsilon$ is closely connected with the magnetic flux Φ (Maxwell). Only the minute share $\not\epsilon_\Gamma$ of the total flux $\not\epsilon$ leads to the outside. This share is expressed in the simple relationship:

$$\not\epsilon_\Gamma = \frac{1}{n} \times \not\epsilon . \quad (2)$$

If the gigantic numbers n are substituted in the equation (2) (see [1, page 172, equation (8–XIII)]), it follows:

$$\begin{aligned}\zeta_{\Gamma} &= \frac{1}{3.939 \times 10^{18}} \times \zeta = \\ &= 2.539 \times 10^{-19} \times 1.8095 \times 10^{-8} = 4.594 \times 10^{-27} \text{ [Vm]}.\end{aligned}$$

This is the minute share of the ζ -field ζ_{Γ} (ζ_{Γ} -field or gravitational field), leaving the quarks of a three-quark particle (3QT). ζ_{Γ} is shown as a symbolic line in Fig. 1, Part II.

In addition to Newton's gravitational equation there are further important equations of physics with a similar structure, such as the equations of Coulomb (elec. charges), Rydberg (spectral series) and Schrödinger (waves). These equations are different forms of the universal equation from [1, page 157]:

$$elt \times elt \times n_1 \times n_2 = a \times b \times elt_a \times elt_b. \quad (3)$$

In it the universal constant elt has the dimension [VAsm], [1, page 141]. It can be composed of many kinds of constants, e.g.: $elt = {}^N h \times c$ [VAsm] with ${}^N h = h \times 2^e \alpha$.

Equation (3) can be paraphrased with some considerations in a further equation (4), which can be written next to the equations of Coulomb (charges), Newton (gravitation), Rydberg (spectral series) and Schrödinger (waves): With $elt = K \times l^2$ and according to [1, Fig. 8-1c], $elt = \zeta^2 \times \epsilon_0$ it follows from equation (3):

$$K = \frac{\epsilon_0}{n_1 \times n_2} \times \frac{a \zeta \times b \zeta}{l^2}. \quad (4)$$

If the relationship $\zeta_{\Gamma} = \frac{1}{n} \times \zeta$ of equation (2) is substituted in equation (4) and if some more considerations are examined, the following is obtained:

$$K = \epsilon_0 \times \frac{a \zeta_{\Gamma} \times b \zeta_{\Gamma}}{l^2}, \quad (4a)$$

$$K = \frac{\epsilon_0}{0.8 \pi} \times \frac{a \zeta_{\Gamma} \times b \zeta_{\Gamma}}{l^2}. \quad (4b)$$

Thus the following is realised:

1. The meaning of the gigantic numbers n_1 and n_2 in Newton's empirical, gravitational constant G analysed with equation (1) is seen as follows. With the product of the inverse of the number n_i and of the electrical source flux ζ the minute fraction of the electrical source flux, that is to say ζ_{Γ} , of each "3QT" is described, where ζ_{Γ} is leaving the quarks of a "3QT". The minute fraction of ζ accounts for the ζ_{Γ} -field of a quark or a "3QT";
2. The quantity of said fraction of the ζ -field of a "3QT" is $\frac{1}{3.939 \times 10^{18}} = 2.539 \times 10^{-19}$ or inverted $3.939 \times 10^{18} \times \zeta_{\Gamma} = \zeta$. ζ_{Γ} has the empirical value $\zeta_{\Gamma} = 1.8095 \times 10^{-8}$ [Vm] $\times 2.539 \times 10^{-19} = 4.594 \times 10^{-27}$ [Vm] as absolute number. These numbers apply to our galactic environment;

3. The equations (4a) and (4b) signify that the superposition of the ζ_{Γ} -fields of two quarks or two quark collectives (a and b) produces the gravitational force effect between two quark collectives;
4. These considerations have made the "gravitation" a superposition of physical namely electrical ζ_{Γ} -fields of highest frequency!

3 Some aspects relating to the ζ_{Γ} -fields

In Part II it is explained by means of Shapiro's experiments how electrical fields and thus the gravitational fields influence the photon(-likes). This physical substantiation for example for the reduction of the speed of light ("refractive index of the vacuum") is to be preferred compared to an substantiation through the geometrical theory of the general relativity.

Gravitational fields reach infinitely far according to our current ideas. The loci of the quarks (sinus oscillations) of which we and our environment consist, are traversed within 10^{-20} (electrons) to 10^{-25} (nucleons) seconds. This means the ζ_{Γ} -field of a quark expands into infinity and contracts again within this absurdly short time. The propagation speed of the ζ_{Γ} -field is thus infinitely large. (Of course this has an effect on large research projects as e.g. LISA with which the allegedly wave-shaped and light-speed propagation of the gravitational field according to the standard physics is to be investigated.)

The infinitely fast propagation of the ζ_{Γ} -field has "natural" consequences everywhere. If the composition of the quarks according to Fig. 1 of Part II applies — which is assumed in these models — the electrical field ζ enclosing the MMPs also expands at infinite speed. This means the ζ -fields of the mass-affected particles occur instantaneously. The range of the ζ -fields is approximately congruent with the range within the Maginpar or the range of the ζ -fields is congruent with the confinement. The confinement located inside a particle is marked off from the outer range by a spherical shell around the coordinate centre with approximately the radius of the Maginpar. ***No causality applies any longer in the small range of the ζ -field within the confinement!***

The infinitely fast propagation of the ζ -field undoubtedly also influences the uncertainty principle. The latter is valid for the range outside the confinement and therefore for electromagnetic processes. In the outer range with causality — with Δt between two events — applies e.g. $\Delta t \times \Delta E = h$ or $\Delta x \times \Delta p = h$.

Inside the confinement the ranges for the toroidal magnetic field Φ and the electric source field ζ are distinguished, where $\Delta t = 0$ applies because of the instantaneous propagation of the ζ -field. Some relation for the interior of the confinement corresponding to the uncertainty principle looks different; the input quantities are certain: ${}^N \Theta \times \nu \times \lambda = {}^N h$. The product from inertia quantum ${}^N \Theta$ times frequency ${}^N \Theta \times \nu$

corresponds to the impulse p or Δp and λ corresponds to the x or Δx . (Otherwise ${}^N\Theta = {}^N h/c$ is the definition equation for the natural constant ${}^N\Theta$.)

Entirely different aspects are touched by the infinitely fast propagation of the \notin_{Γ} -field, which are merely mentioned here but not discussed: A) The infinitely fast propagation of the \notin_{Γ} -field revitalises the Mach principle according to which the local behaviour of matter is based on the influences of the remainder of the universe. B) The universal structure of galactic chains and dark bubbles and the synchronised creation of galaxies are based on the infinitely fast propagation. C) According to the models the centres of the galaxies are quantum objects. The considerations relating to causality and uncertainty also apply to these. D) The Planck length, [1, page 178], is determined through the interaction of MMP and \notin_{Γ} -field. E) The experiments of A. Zeilinger for teleportation are based on the infinitely fast propagation of the \notin -field in the rapidly enlarging confinement of polarisation-entangled photons (12QT).

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Models for Quarks and Elementary Particles — Part IV: How Much Do We Know of This Universe?

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Essential laws and principles of the natural sciences were discovered at the high aggregation levels of matter such as molecules, metal crystals, atoms and elementary particles. These principles reappear in these models in modified form at the fundamental level of the quarks. However, the following is probably true: since the principles apply at the fundamental level of the quarks they also have a continuing effect at the higher aggregation levels. In the manner of the law of mass action, eight processes for weak interaction are formulated, which are also called Weak Processes here. Rules for quark exchange of the reacting elementary particles are named and the quasi-Euclidian or complex spaces introduced in Part I associated with the respective particles. The weak processes are the gateway to the “second” strand of this universe which we practically do not know. The particles with complex space, e.g. the neutrino, form this second strand. According to the physical model of gravitation from Part III the particles of both strands have \notin_T -fields and are thus subject to the superposition, which results in the attraction by gravity of the particles of both strands. The weak processes (7) and (8) offer a fair chance for the elimination of highly radioactive waste.

1 Introduction

The first parts of this series of papers have headline questions which are answered within the scope of the models [1]: I) What is a quark? II) What is mass? III) What is the nature of the gravitational field?

Which of the three questions will a physicist representing the current standard model be able to answer positively without hesitation? The standard model of physics combines huge quantities of analyses, conformities with natural laws and theories. However, too many independent quantities that can only be captured empirically still enter the standard model of physics and inconsistencies between individual theories are known. For this reason, theoreticians are looking for new physics especially in the field of the strings, loops and branes; however, they have been unable to establish any reference to reality. The standard model of cosmology has the general theory of relativity (GTR) as thread, wherein the GTR is a geometrical and not a physical theory. Despite this deficit the mainstream of cosmologists is absolutely convinced of the big bang model which is based on the GTR, wherein the big bang is a central part of the standard model of cosmology. The physical model of gravitation presented in Part III opens up a new interpretation of our universe. The perspectives of Part III render a Part V for cosmology — the utmost level of organisation — unnecessary. But there is a Part V in preparation concerning the magnetic load, which leads to the undermost level of organisation of our universe. Although many relationships are better recognizable with this model than in the past, there is certainly a lot we do not know of our universe.

2 The weak interaction

The equations of the weak interaction which in the following are also called “Weak Processes” are the central content of the present Part IV. Physics books present equations relating to the weak interaction. These equations are considered correct although the authors have no exact idea of what a quark is, although they are uncertain as to the mass possessed for instance by a neutrino, although they should have doubts in the uniformity of so-called “elementary particles”, although they are looking for additional particles that could be included in the equations.

An often-quoted equation in the literature is formulated thus:

$$p^+ + \bar{\nu}_e \rightarrow n^0 + e^+. \quad (1)$$

According to Table 1 of Part I, each of the four elementary particles involved is a three-quark particle (3QT). If this is used to make a quark equation — which cannot happen in the standard model of physics — according to the models to date equation (1) must read as follows:

$$\underbrace{uu}_{||} d + \underbrace{\bar{d}\bar{d}}_{\perp} \bar{u} \rightarrow \underbrace{dd}_{||} u + \underbrace{\bar{d}\bar{d}}_{\perp} \bar{d}.$$

As can be seen, the quarks on both sides do not agree in **number and type**. If the left side is correct, an \bar{u} and an u are missing on the right, instead there are a d and an \bar{d} too many on the right; the charge balance would be correct as in equation (1).

The literature equation (1) cannot be corrected because it is wrong. To get onto the right track here are some fundamental remarks concerning equations with particles.

From [1], Chapter 8.5, page 202: The (A) law of mass action, the (B) Pauli principle, the (C) superconductivity and the (D) uncertainty principle were found at higher aggregation levels of the particle world and applied to (A) molecules, (B) atoms, (C) metal crystals and (D) elementary particles. All four can be found again in these models in modified form at the fundamental level of the quarks, e.g. in the following (A) weak processes or with the (B) configurations of the nuclei in [1], Chapter 7.5 or in the (C) “fountain”, Fig. 1 in Part I, or in the definition of the natural constant of the (D) inertia quantum $N\Theta$, see penultimate paragraph of Part III. Probably the effect of such laws and principles has to be seen differently: *Since they apply at the fundamental level they continue to have an effect also at the higher aggregation levels.*

The following is an example using the (B) Pauli’s principle. The Pauli principle states for a complete atom — i.e. for a higher aggregation level — that a shell (K, L, M etc. with the sub-shells s, p, d etc.) of the atomic shell cannot be occupied by two electrons.

In Part I, Table 1 in line A shows the particles $\underbrace{dd}_{\parallel} d \equiv e_e$ and $\underbrace{uu}_{\parallel} u \equiv \Delta_{\Delta}$ for the fundamental level of the quarks. In addition, Fig. 12 in Part I shows the loci for a $\underbrace{dd}_{\perp} - Zk$. (A definition of the “dual-coordination” or briefly “Zk” is given in Part I, page 74, paragraph 5.) If the locus of a third d-quark were to be placed in the level of this Zk, either space I or space III would be occupied with two loci. Such double occupancy is demanded for the particles e_e and Δ_{Δ} by the \parallel symbol. According to the Pauli principle this means at the fundamental level of the quarks that the particles e_e and Δ_{Δ} are prohibited, see Table 2! Allowed are only the electron $\underbrace{dd}_{\perp} d \equiv e^-$ and the deldopon $\underbrace{uu}_{\perp} u \equiv (\Delta^{++})$, where each quark assumes a different position.

Another example relates to the (A) law of mass action. This law primarily applies to the fundamental quark equations, but was initially discovered by us by means of the chemical reactions at the high aggregation level. The equation of a chemical reaction is formulated in the same manner as a fundamental quark equation. All constituents entering a fundamental reaction again come out of the reaction in a changed composition. Nothing disappears or is added. In this regard, some of the equations for the weak interaction offered in physics books are totally unsatisfactory, since the particles on both sides of the equations lack a common basis. This is also evident from the above equation (1): for the nucleons there is the quark representation in the standard model, not for the leptons.

3 The eight weak processes

Reading the following is not easy, the subject however highly interesting for the understanding of our universe. The comments regarding the equations are intended to facilitate this understanding.

Eight processes with the construction

Starting particle \rightarrow (Quarkpool) \rightarrow Reaction products

$$p^+ + e^- \rightarrow \rightarrow \rightarrow \rightarrow n^0 + \nu_e \quad (2)$$

$$\underbrace{uu}_{\parallel} d + \underbrace{dd}_{\perp} d \rightarrow \left(\begin{array}{c} uu_{\parallel} d: dd_{\perp} d \\ \uparrow \text{-----} \uparrow \end{array} \right) \rightarrow \underbrace{dd}_{\parallel} u + \underbrace{dd}_{\perp} u \quad (2a)$$

Space type **qeR** **qeR** \rightarrow \rightarrow \rightarrow \rightarrow **qeR** **koR**

$$n^0 + \nu_e \rightarrow \rightarrow \rightarrow \rightarrow p^+ + e^- \quad (3)$$

$$\underbrace{dd}_{\parallel} u + \underbrace{dd}_{\perp} u \rightarrow \left(\begin{array}{c} dd_{\parallel} u: dd_{\perp} u \\ \uparrow \text{-----} \uparrow \end{array} \right) \rightarrow \underbrace{uu}_{\parallel} d + \underbrace{dd}_{\perp} d \quad (3a)$$

Space type **qeR** **koR** \rightarrow \rightarrow \rightarrow \rightarrow **qeR** **qeR**

The equations (2) and (3) count among the best known of the weak interaction. For the formulations according to the standard model the common basis of the particles mentioned above is absent. As quark equation (2a) and (3a), they correspond to the characteristics of the law of mass action. Details for a “quark pool” are included in the quark equations. This quark pool stands for the physical process of the reaction of the particles involved which requires a finite time and during which exchange processes take place. The signs within the brackets explain this exchange. During both the above processes a quark from the Zk of the baryon/nucleon involved is exchanged for the singular quark of the lepton, while the quark from the Zk of the baryon does not belong to the $u_{\parallel}d$ -group.

It can also be seen that the structure symbols in the equations are retained. A \parallel and a \perp symbol each are present on the left and on the right side of the equation. This is to be correlated with the retention of the baryon and lepton number of the standard model. This means there are fixed rules for the reactions during the weak processes.

In each third line for each reaction the space type **qeR** or **koR**, see [5], Part I, page 72/73, of the elementary particle is noted. If two particles from “our” quasi-Euclidian space (qeR) react with each other the probability of the reaction substantially depends on a resonance possibility, i.e. the size of the particles MAGINPARs. In addition to this probability for a reaction there is obviously also a second one. This depends on the space type. This means, two particles with the same space type react with each other with far greater probability than particles with different space type.

We are aware of this in the case of the hugely plentiful neutrinos with the complex space type qeR which hit the particles of earth with the space type qeR with only an extremely

3QT	dd d	dd u	uu d	uu u
Locus level of singular quark parallel () to the locus level of the Zk †	e _e †	n ⁰	p ⁺	Δ _Δ †
Space type of particle	koR1 †	qeR1	qeR1	koR1 †
Locus level of singular quark vertically (⊥) on locus level of Zk	e ⁻	ν _e	? ⁺	(Δ ⁺⁺)
Space type of particle	qeR2	koR2	koR2	qeR2

koR ≡ “complex” space, qeR ≡ quasi-Euclidian space.

The number 1 or 2 designates the number of the ρ-rotation levels per particle.

†Elementary particles prohibited by the Pauli principle.

Table 2: Space structures of the elementary particles

low probability. The probabilities for a reaction are called MAGINPAR and space type probability. All eight weak processes are characterized in that at least one particle of a process has the space type koR. Thus the space type probability applies to the eight here treated processes which is why we talk about the “weak” interaction.

$$p^+ + ?^+ \rightarrow \rightarrow \rightarrow \rightarrow n^0 + (\Delta^{++}) \quad (4)$$

$$\underbrace{uu}_{||} d + \underbrace{uu}_{\perp} d \rightarrow \left(\begin{array}{c} uu_{||} d : uu_{\perp} d \\ \uparrow \text{-----} \uparrow \\ \underbrace{uu}_{\perp} u \end{array} \right) \rightarrow \underbrace{dd}_{||} u + \quad (4a)$$

Space type **qeR koR** → → → → **qeR qeR**

$$n^0 + (\Delta^{++}) \rightarrow \rightarrow \rightarrow \rightarrow p^+ + ?^+ \quad (5)$$

$$\underbrace{dd}_{||} u + \underbrace{uu}_{\perp} u \rightarrow \left(\begin{array}{c} dd_{||} u : uu_{\perp} u \\ \uparrow \text{-----} \uparrow \\ \underbrace{uu}_{\perp} d \end{array} \right) \rightarrow \underbrace{uu}_{||} d + \quad (5a)$$

Space type **qeR qeR** → → → → **qeR koR**

A hypothesis (here the models under consideration) establishes new predictions/expansions unknown to date for the (physical) teaching applicable to that point, which have to be verified. Such predictions are made by Table 1 and the still to follow Table 2 with some of the particles noted there, which also occur in the equations (4) and (5). For the sake of brevity the particles of the Tables that have not been found yet will not be further commented upon at this point. Reference is only made to the respective exchange of the quarks in the quark pool, which corresponds to the fixed rules for the reactions mentioned above.

To facilitate the association of the space types with the individual elementary particles Table 2 is inserted.

The best known equation to describe the “β-decay” is the following:

$$n^0 \rightarrow p^+ + e^- + \bar{\nu}_e \quad (6)$$

Under the aspect of the standard theories such an equation is possible because four totally independent particles are present, the electric charges involved are correct and the n⁰ has the greatest mass/energy so that it can decay into the three other particles of lower energy. Under the aspect of the models developed here the two sides of this process cannot be brought into line even from the number of the quarks involved. The right side of the equation comprises nine quarks, the left side three quarks. In other words six quarks have to be added to the left side, while a 6QT or boson is obvious.

The following arguments speak for the photon-like ν-gamma (ν - γ) as trigger of the process — incompletely — described with the above non-equation:

1. The particle is not yet known which is why it is not named so far on the left side of the process;
2. Because of its space type koR the particle — based on ν_e (Table 2) — is difficult to discover;
3. ν-gamma brings with it the necessary number and type of quarks and of structure signs || respectively ⊥.

The almost known “β-decay” according to the standard model as fully formulated weak process according to these models then becomes the following as particle and quark equation:

$$n^0 + \nu - \gamma \rightarrow \rightarrow \rightarrow \rightarrow p^+ + e^- + \bar{\nu}_e \quad (8)$$

$$\underbrace{dd}_{||} u + \underbrace{dd}_{\perp} \underline{u\bar{u}}_{\perp} \underline{d\bar{d}} \rightarrow \left(\begin{array}{c} d \text{ from } n^0 \Leftrightarrow \\ u \text{ from } Bk \end{array} \right) \rightarrow \rightarrow \underbrace{uu}_{||} d + d_{\perp} \underbrace{dd}_{\perp} + \bar{u}_{\perp} \underline{d\bar{d}} \quad (8a)$$

Space type **qeR koR** → → → → **qeR qeR koR**

The central part $\underline{u\bar{u}}$ of the $\nu - \gamma$ within the quark-equation (8a) is called a “binding coordination”, briefly “Bk”.

The fixed rules for the quark reactions need only be modified slightly for the reaction type (8) relative to the reaction type (2) and (3) or (4) and (5):

- A baryon each reacts with a photon-like 6QT (instead of 3QT lepton).
- From the original particles, a formally singular quark each (not anti-quark) of a lepton and now part of a Bk in the photon-like is exchanged for a Zk-quark (not from the u||d-group) of the baryon.
- In addition to the type and number of the quarks involved the type and number of the structure signs || and \perp now agree on both sides of equation (8) as well.

Since equation (8) relative to the non-equation (6) has been explained, equation (7) is now added where $e-\gamma$ (our “normal” photon) has to be additionally considered compared with the standard version.

$$p^+ + e - \gamma \rightarrow \rightarrow \rightarrow \rightarrow n^0 + e^+ + \nu_e \quad (7)$$

$$\underbrace{uu}_{||} d + \underbrace{dd}_{\perp} \underbrace{d\bar{d}}_{\perp} \underbrace{d\bar{d}}_{\perp} \rightarrow \left(\begin{array}{l} u \text{ from } p^+ \Leftrightarrow \\ d \text{ from } Bk \end{array} \right) \rightarrow$$

$$\rightarrow \underbrace{dd}_{||} u + \underbrace{d\bar{d}}_{\perp} \bar{d} + \underbrace{dd}_{\perp} u \quad (7a)$$

$$\text{Space type } \text{qeR} \quad \text{qeR} \rightarrow \rightarrow \rightarrow$$

$$\rightarrow \text{qeR} \quad \text{qeR} \quad \text{koR}$$

Since the following equations (9) and (10) contain particles not yet found from the systematic of Table 1 and 2 here they will not be further commented upon. In structure they correspond to the type of the equations (7) and (8) and complete the set of the weak processes according to these models:

$$p^+ + ? - \gamma \rightarrow \rightarrow \rightarrow \rightarrow n^0 + (\Delta^{2+}) + ?^+ \quad (9)$$

$$\underbrace{uu}_{||} d + \underbrace{uu}_{\perp} \underbrace{d\bar{d}}_{\perp} \underbrace{u\bar{u}}_{\perp} \rightarrow \left(\begin{array}{l} u \text{ from } p^+ \Leftrightarrow \\ d \text{ from } Bk \end{array} \right) \rightarrow$$

$$\rightarrow \underbrace{dd}_{||} u + \underbrace{uu}_{\perp} u + \underbrace{u\bar{u}}_{\perp} \bar{d} \quad (9a)$$

$$n^0 + (\Delta) - \gamma \rightarrow \rightarrow \rightarrow \rightarrow p^+ + ?^+ + (\Delta^{2-}) \quad (10)$$

$$\underbrace{dd}_{||} u + \underbrace{uu}_{\perp} \underbrace{u\bar{u}}_{\perp} \underbrace{u\bar{u}}_{\perp} \rightarrow \left(\begin{array}{l} d \text{ from } n^0 \Leftrightarrow \\ u \text{ from } Bk \end{array} \right) \rightarrow$$

$$\rightarrow \underbrace{uu}_{||} d + d_{\perp} \underbrace{uu}_{\perp} + \bar{u}_{\perp} \underbrace{u\bar{u}}_{\perp} \quad (10a)$$

The weak processes are the gateway to the “second” strand of this universe. The particles having a complex space (koR) form this second strand. “Our” particles with quasi-Euclidian space (qeR) from the “first” strand overlap those from the second strand without problems, which is why the spaces also overlap without problems. (What is a “space” being created in our imagination?) The “spaces” do not interact with each other.

In contrast with this, the physical = electric \notin -fields from qeR and koR interact very well with each other so that their superposition results in the mutual attraction, see [1], page 186, line 18. Measured by the undiscovered particles of Tables 1 and 2 there is much to be discovered behind the gate to the “second” strand of this universe. Judging by the ratio of the gravitational effects of the visible matter and the dark matter what can be discovered behind the gate is a multiple of what we already know.

4 The Meaning of the Weak Processes (7) and (8)

Equations (7) and (8) contain some fascinating technical potential. H. Stumpf deals with nuclear reaction rates of the electroweak interaction [2] and at the end of his paper he refers to L. I. Urutskov and other Russian authors, who perform experiments regarding this item. The potential of those works includes finding new routes for the elimination of highly radioactive waste. In a few years this waste from hundreds of disused nuclear reactors will pile up in many states of our earth. The final storage of this waste is not clarified and costs for a long time storage with e. g. sarcophagus as in Tschernobyl would be enormous. The duration of storage follows from natural β -decay half-life periods of different elements or their isotopes which can last for up to 1.5×10^{24} years for $^{128}_{52}\text{Te}$, [4], page 34, which mankind cannot live to see.

Equation (7) respectively (7a) demonstrates, that the protons of radioactive elements can have resonance and can react with very short waved photons ($e-\gamma$) into neutrons, positrons and neutrinos. Thereby the structure and the therewith combined beat of the photon shown in Part II, page 77, left column, point (2) and the storage of the photon in an electron (resonance), Part II, page 77, right column, penultimate paragraph are called to mind.

Equation (7) is confirmed by two aspects of the above mentioned experiments of L. I. Urutskov et al. [3]. First aspect: The central incidents of the experiments are electric discharges between metallic foils in vessels filled with various fluids, [2], page 455. My interpretation is, that by the discharges those short waved $e-\gamma$ of process (7) are generated, which can have resonance and reaction with the protons of the (radioactive) elements. Second aspect: The possibility of “low-energy nuclear transformation” is reported in [3]. If an electron and a visible photon have a comparable COMPTON-wavelength and therefore have resonance, then the photon has an energy of multiplier 10^5 less than the electron, [1], Chapter 8.2.3, page 163. With weak interaction nuclei emit short waved $e-\gamma$ in the range of a few keV up to a few MeV. That means nuclei are in the position to have resonance with those short waved $e-\gamma$. If such short waved $e-\gamma$ arrives at a nucleus and hit (a neutron or) a proton then there is the possibility for the “low-energy” exchange of quarks in a quark-pool according to the rules of page 72, middle of right column, and page 74, upper part left column. By the exchange of quarks

in accordance with equation (7) the proton transforms into a neutron and by this a new element respectively a new isotope takes shape. New elements respectively isotopes were detected by the authors of [3].

Following are comments on the peculiarities of the weak process (7):

1. The Standard Model of Physics treats the β -“decay” as statistical phenomenon or as happening by chance. The model under consideration especially the weak process (7) presents a dosed bombardment of protons by $e-\gamma$. The transformation of the protons into products of reaction happens not by chance instead the reaction is determined by the efficiency of the law of mass action.
2. Without the knowledge of the weak process (7) Urutskoev et al. with exotic experiments strive for the realisation of reactions according to this process. With knowledge of equation (7) different experiments are possible:

Possibly one could observe the weak process cease when the bombardment of protons by photons, which can have resonance, is prevented completely. Nevertheless the “radioactive decay” of a specimen with an outer screening could continue because a radiation could be released “from the interior” of this specimen. The latter could stem from the less probable but possible opposite reaction of equation (7): $n^0 + e^+ + \nu_e \rightarrow p^+ + e-\gamma$. The $e-\gamma$ originating in the interior of the atomic nucleus would be absorbed after flying a very short distance in the specimen because of a high probability of resonance. By this the weak process (7) would be caused “from the interior”.

3. The weak process (7) cannot be observed in nature, [4], S. 38.

Following are comments on the peculiarities of the weak process (8):

1. The very common but not applicable non-equation (6) claims that the neutrons of radioactive elements would “decay” into protons, electrons and anti-neutrinos. As with equation (7) the law of mass action is valid with equation (8);
2. By the exchange of quarks according equation (8) a new element respectively a new isotope takes shape. The problem is, until now we still do not know the $\nu - \gamma$ because of its complex space koR and beyond this we cannot shield it from the outside or handle it at all. From that point of view we would be dependent on the sun, on space or on nuclear reactors as generators for $\nu - \gamma$ of whatever intensity and wavelength to shorten half-life periods by chance.

Eventually a possibility on the basis of the opposite reaction of the weak process (8) will be revealed. Those $\nu - \gamma$ originating from $p^+ + e^- + \bar{\nu}_e \rightarrow n^0 + \nu - \gamma$ would be absorbed

after flying the shortest distance because of a high probability of resonance. By this the weak process (8) would be released “from the interior”. The opposite reaction of the weak process (8) should be reinforced by proper conditions in such a way that the reaction rates are of sufficient size.

In summary: Though till now we do not know the $\nu - \gamma$ radiation so far and, much less, we can control it, there is hope to transform the neutrons of radioactive elements by $\nu - \gamma$ via the opposite reaction of equation (8). The construction of some technical apparatus for short waved $e-\gamma$ -radiation as e.g. X-rays of 10^3 to 10^6 eV is feasible. By the reaction of proton and $e-\gamma$ (photon) according to the weak process (7) natural, partly very long time half-life periods can be shortened down to seconds using a technical apparatus! The use of both types of radiation, $\nu - \gamma$ and $e-\gamma$, would be decisive steps for the elimination of highly radioactive waste.

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The Generalized Conversion Factor in Einstein's Mass-Energy Equation

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Einstein's September 1905 paper is origin of light energy-mass inter conversion equation ($L = \Delta mc^2$) and Einstein speculated $E = \Delta mc^2$ from it by simply replacing L by E . From its critical analysis it follows that $L = \Delta mc^2$ is only true under special or ideal conditions. Under general cases the result is $L \propto \Delta mc^2$ ($E \propto \Delta mc^2$). Consequently an alternate equation $\Delta E = A c^2 \Delta M$ has been suggested, which implies that energy emitted on annihilation of mass can be equal, less and more than predicted by $\Delta E = \Delta mc^2$. The total kinetic energy of fission fragments of U^{235} or Pu^{239} is found experimentally 20–60 MeV less than Q -value predicted by Δmc^2 . The mass of particle Ds (2317) discovered at SLAC, is more than current estimates. In many reactions including chemical reactions $E = \Delta mc^2$ is not confirmed yet, but regarded as true. It implies the conversion factor than c^2 is possible. These phenomena can be explained with help of generalized mass-energy equation $\Delta E = A c^2 \Delta M$.

1 Introduction

Mass energy inter-conversion processes are the oldest in nature and constitute the basis of various phenomena. Before Einstein's work, Newton [1] stated that "Gross bodies and light are convertible into one another...". Einstein derived light energy-mass inter-conversion equation for Newton's perception as $L = \Delta mc^2$. Before Einstein scientists such as S. Tolver Preston [2] Olinto De Pretto [3], Fritz Hasenohrl [4, 5] Frederick Soddi [6] contributed to the topic.

Einstein's derivation of $L = \Delta mc^2$ (from which Einstein speculated $E = \Delta mc^2$), is true under special conditions (where selective values of variables are taken). Under general conditions (when all possible values of parameters are taken) equations like $L = 0.0011\Delta mc^2$, $L = 0.999988\Delta mc^2$ etc. are obtained i.e. $L \propto \Delta mc^2$. Thus conversion factor other than c^2 is possible in Einstein's derivation. Further the generalized mass-energy equation $\Delta E = A c^2 \Delta M$, is derived, and $E = \Delta mc^2$ is special case of the former depending upon value of A (depends upon the characteristics conditions of the process). Thus apart from theoretical limitations, $E = \Delta mc^2$ has experimental limitations e.g. sometimes experimental results differ from it and in many cases it is not confirmed. Under such cases $\Delta E = A c^2 \Delta M$ is widely useful and applicable. The fission fragments result from U^{235} and Pu^{239} have total kinetic energy 20–60 MeV less than Q -value (200 MeV) of reaction predicted by $\Delta E = \Delta mc^2$ [7–9]. Palano [10] has confirmed that mass of particle Ds (2317) has been found more than current estimates based upon $\Delta E = \Delta mc^2$. Also $\Delta E = \Delta mc^2$ does not give consistent results in explaining the binding energy, as it violates the universal equality of masses of nucleons.

All these facts can be explained by $\Delta E = A c^2 \Delta M$ with value of A less or more than one. $\Delta E = \Delta mc^2$ is not confirmed in many processes such chemical reactions, atom

bomb explosions, volcanic reactions etc. Whatever may be the case $\Delta E = A c^2 \Delta M$ is capable of explaining the phenomena. Thus conversion factor other than c^2 is possible, in Einstein's September 1905 derivation and confirmed experimentally also.

2 Einstein's light energy — mass equation $L = \Delta mc^2$ and its hidden aspects

Einstein [11] perceived that let there be a luminous body at rest in co-ordinate system (x, y, z) . The system (ξ, η, ζ) is in uniform parallel translation w.r.t. system (x, y, z) ; and origin of system (ξ, η, ζ) moves along x -axis with relative velocity v . Let a system of plane light waves have energy ℓ relative to system (x, y, z) , the ray direction makes angle ϕ with x -axis of the system (ξ, η, ζ) . The quantity of light measured in system (ξ, η, ζ) has the energy [11, 12].

$$\ell_* = \ell \frac{\left(1 - \frac{v}{c} \cos \phi\right)}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (1)$$

Einstein has given Eq. (1) in his paper known as Special Theory of Relativity [12] and called Eq. (1) as Doppler principle for any velocities whatever.

Let E_0 and H_0 are energies in coordinate system (x, y, z) and system (ξ, η, ζ) before emission of light energy, further E_1 and H_1 are the energies of body in the both systems after it emits light energy. Thus Einstein wrote various equations as Energy of body in system (x, y, z)

$$E_0 = E_1 + 0.5L + 0.5L = E_1 + L; \quad (2)$$

Energy of body in system (ξ, η, ζ)

$$H_0 = H_1 + 0.5\beta L \left[\left(1 - \frac{v}{c} \cos \phi\right) + \left(1 + \frac{v}{c} \cos \phi\right) \right] \quad (3)$$

where $\beta = 1/[1 - v^2/c^2]^{1/2}$;

$$H_0 = H_1 + \beta L; \quad (4)$$

or

$$(H_0 - E_0) - (H_1 - E_1) = L(\beta - 1). \quad (5)$$

Einstein calculated, kinetic energy of body before emission of light energy, $K_0(m_b v^2/2)$ and kinetic energy of body after emission of light energy, $K(m_a v^2/2)$ as

$$K_0 - K = L \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right) \quad (6)$$

Einstein considered the velocity in classical region thus applying binomial theorem,

$$K_0 - K = L \left(1 + \frac{v^2}{2c^2} + \frac{3v^4}{8c^4} + \frac{15v^6}{48c^6} + \frac{105v^8}{384c^8} + \dots - 1 \right). \quad (7)$$

Further Einstein quoted [16] "Neglecting magnitudes of fourth and higher orders, we may place"

$$K_0 - K = L \frac{v^2}{2c^2} \quad (8)$$

$$M_b \frac{v^2}{2} - M_a \frac{v^2}{2} = L \frac{v^2}{2c^2} \quad (9)$$

or

$$L = (M_b - M_a) c^2 = \Delta m c^2, \quad (10)$$

or Mass of body after emission (M_a) = Mass of body before emission ($M_b - L/c^2$).

Now replacing L (light energy) by E (total energy or every energy) Einstein wrote

$$E = (M_b - M_a) c^2 = \Delta m c^2 \quad (11)$$

or Mass of body after emission (M_a) = Mass of body before emission ($M_b - E/c^2$).

Thus Einstein derived that conversion factor between mass and light energy is precisely equal to c^2 , this aspect is elaborated by Fadner [13]. But Einstein's this derivation has been critically discussed by many such a Planck [14], Stark [15], Ives [16], Stachel [17], Okun [18] and N. Hamdan [19] etc. At the same time in some references [20, 21] it is expressed that Einstein has taken hints to derive equation $E = \Delta m c^2$ and from existing literature without acknowledging the work of preceding scientists. Max Born [22] has expressed that Einstein should have given references of existing literature.

Thus Einstein's work on the topic has been critically analyzed by scientists since beginning, in views of its scientific and procedural aspects.

3 The conversion factor between mass-energy other than c^2 is also supported by Einstein's derivation under general conditions

As already mentioned Einstein's September 1905 derivation of $\Delta L = \Delta m c^2$ is true under special or ideal conditions (selected values of parameters is taken) only, this aspect is studied critically with details by the author [23–36] discussing those aspects which have not been raised earlier. Thus the value of conversion factor other than c^2 is also supported from Einstein's derivation under general conditions (all possible values of variables). The law or phenomena of interconversion of mass and energy holds good in all cases for all bodies and energies under all conditions.

In the derivation of $\Delta L = \Delta m c^2$ there are FOUR variables e.g.

- Number of waves emitted,
- l magnitude of light energy,
- Angle ϕ at which light energy is emitted and
- Uniform velocity, v .

Einstein has taken special values of parameters and in general for complete analysis the derivation can be repeated with all possible values of parameters i.e. under general conditions taking in account the momentum conservation (which is discussed in next sub-section).

- The body can emit large number of light waves but Einstein has taken only TWO light waves emitted by luminous body.
- The light waves emitted may have different magnitudes but Einstein has taken EQUAL magnitudes
- Body may emit large number of light waves of different magnitudes of energy making DIFFERENT ANGLES (other than 0° and 180°) assumed by Einstein.
- Einstein has taken velocity in classical region ($v \ll c$) has not at all used velocity in relativistic region. If velocity is regarded as in relativistic region (v is comparable with c), then equation for relativistic variation of mass with velocity i.e.

$$M_{rel} = \frac{M_{rest}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (12)$$

is taken in account. It must be noted that before Einstein's work this equation was given by Lorentz [37,38] and firstly confirmed by Kaufman [39] and afterwards more convincingly by Bucherer [40]. Einstein on June 19, 1948 wrote a letter to Lincoln Barnett [41] and advocated abandoning relativistic mass and suggested that is better to use the expression for the momentum and energy of a body in motion, instead of relativistic mass.

It is strange suggestion as Einstein has used relativistic mass in his work including in the expression of relativistic kinetic energy [12] from which rest mass energy is derived.

- (E) In addition Einstein has assumed that body remains at rest before and after emission of light energy. But the body may be at rest i.e. $v = 0$, velocity may be in classical region and velocity may be in relativistic region ($v \sim c$), the law of inter-conversion of mass and energy holds good under all conditions.

In electron-positron annihilation, the material particles are in motion before and after annihilation. In materialization of energy, a gamma ray photon is converted to electron positron pair, which move in opposite directions to conserve momentum. In nuclear fission and fusion particles remain in motion in the process of mass energy inter conversion. The thermal neutron which causes fission has velocity 2185 m/s.

4 $L \propto \Delta mc^2$ is mathematically consistent in Einstein's derivation, under general conditions

Under general conditions (all possible values of variables) the value of conversion factor other than c^2 can be easily justified mathematically in Einstein's derivation [23–36]. This aspect is not touched by the preceding authors [13–21].

- (a) In Einstein's derivation if one wave is regarded as to form angle 0.5° rather than 0° then

$$H_0 = H_1 + 0.5\beta L \times \left[\left(1 - \frac{v}{c} \cos 0.5^\circ\right) + \left(1 - \frac{v}{c} \cos 180^\circ\right) \right], \quad (13)$$

or

$$H_0 = H_1 + \beta L \left(1 + 0.000018038 \frac{v}{c}\right),$$

or

$$K_0 - K = 0.000019038 l L \frac{v}{c} + L \frac{v^2}{2c^2},$$

or

$$\Delta m (M_b - M_a) = 0.000038077 \frac{L}{cv} + \frac{L}{c^2}, \quad (14)$$

or

$$L = \frac{\Delta mc^2}{1141} = 0.000876 \Delta mc^2, \quad (15)$$

$$\Delta L \propto \Delta mc^2.$$

Further, M_a (mass after emission of light energy) = M_b (mass before emission of light energy): $0.000038077 L/cv = L/c^2$ in (14).

According to Einstein if body emits two light waves of energy $0.5L$ each in opposite directions then decrease in mass is given by Eq. (10) i.e. $\Delta m = L/c^2$ and in this case decrease

in mass is $(0.000038077 L/cv + L/c^2)$ thus there is no definite value of decrease in mass in Einstein's derivation. In this case decrease in mass is more than as predicted by Einstein, hence again the conversion factor other than c^2 is confirmed i.e. $\Delta L \propto \Delta mc^2$. Like this there are many examples of this type.

- (b) The central equation in Einstein's derivation is Eq. (1) and binomial theorem is equally applicable to it at any stage i.e. in the beginning or end. Einstein applied binomial theorem in the end and obtained $L = \Delta mc^2$, but the same equation is not obtained if binomial theorem is applied in the beginning. The binomial theorem is simply a mathematical tool and its application at any stage should not affect results i.e. make or mar equation $L = \Delta mc^2$.

The reason is that typical nature of derivation and Eq. (1) is different from other relativistic equations. The energy is scalar quantity and independent of direction but Eq. (1) is directional in nature due to angle ϕ . In contrast if binomial theorem is applied to Relativistic Kinetic Energy in the beginning or at the end then result is same i.e. classical form of kinetic energy ($m_{rest} v^2/2$). So there is inconsistency in applications in this case.

Applying binomial theorem to Eq. (1) and repeating the calculations as Einstein did, altogether different results are obtained,

$$\ell^* = \ell \left(1 - \frac{v}{c} \cos \phi\right) \left(1 + \frac{v^2}{2c^2} + \frac{3v^4}{8c^4} + \dots\right). \quad (16)$$

Here $v/c \ll 1$, hence v^2/c^2 and higher terms can be neglected. Thus

$$\ell^* = \ell \left(1 - \frac{v}{c} \cos \phi\right)$$

or

$$(H_0 - E_0) - (H_1 - E_1) = 0,$$

or

$$K_b - K_a = 0,$$

or

$$\frac{1}{2} M_b v^2 - \frac{1}{2} M_a v^2 = 0,$$

or

$$\begin{aligned} \text{Mass of body before emission } (M_b) &= \\ &= \text{Mass of body after emission } (M_a). \end{aligned} \quad (17)$$

Thus light energy is being emitted, but under this condition Einstein's this derivation does not provide any relationship (equality or proportionality) between mass annihilated and energy created. Similar is the situation if velocity $v = 0$. Hence Einstein's derivation gives decrease in mass of body equal to L/c^2 only under certain conditions. Thus in this case derivation is not valid.

Sr. No	Values of various parameters	Whether $L = \Delta mc^2$ or $L \propto \Delta mc^2$
1	$0.5L, 0.5L, \phi = 0^\circ, \phi = 180^\circ$	$L = \Delta mc^2$
2	$0.5L, 0.5L, \phi = 0.5^\circ, \phi = 180^\circ$	$L = \Delta mc^2/901$ or $L \propto \Delta mc^2$
3	$0.5001L, 0.49999L, \phi = 0^\circ, \phi = 180^\circ$	$L = 0.9999988\Delta mc^2$ or $L \propto \Delta mc^2$
4	$0.5L, 0.5L, \phi = 0^\circ, \phi = 180^\circ$ but $v = 0$	No relation between L and Δm
5	$0.5L, 0.5L, \phi = 0^\circ, \phi = 180^\circ$ but Binomial Theorem is applied in beginning.	No relation between L and Δm
6	For other energies than light	Equations not considered

Table 1: Einstein's Sep 1905 derivation gives $L = \Delta mc^2$ under certain conditions and $L \propto \Delta mc^2$ under general conditions

(c) Let the body emits two light waves of slightly different energies i.e. $0.5001L$ and $0.4999L$ in opposite directions and other parameters remain the same as assumed by Einstein. In this case

$$H_0 = H_1 + 0.4999\beta L \left(1 - \frac{v}{c} \cos 0^\circ\right) + 0.5001\beta L \left(1 - \frac{v}{c} \cos 180^\circ\right). \tag{18}$$

Now proceeding in the same way as Einstein did

$$K_0 - K = 0.0002L \frac{v}{c} + L \frac{v^2}{2c^2} \tag{19}$$

or

$$\begin{aligned} \Delta m &= \text{Mass of body before emission}(M_b) - \text{Mass of body after emission}(M_a), \\ &= 0.0004 \frac{L}{cv} + \frac{L}{c^2} \end{aligned} \tag{20}$$

or

$$M_a = 0.004 \frac{L}{cv} - \frac{L}{c^2} + M_b$$

or

$$L = \frac{\Delta mc^2}{\left(0.0004 \frac{v}{c} + 1\right)}.$$

The velocity v is in classical region, say 10 m/s,

$$L = \Delta mc^2 [0.000083], \tag{21}$$

$$\Delta L \propto \Delta mc^2.$$

Thus, $\Delta E \propto \Delta mc^2$. Hence conversion factor other than c^2 follows from Einstein's derivation under general conditions.

(d) Energy emitted in various reactions. In his September 1905 paper Einstein derived Eq. (10) i.e. $\Delta L = \Delta mc^2$ and then replaced L (light energy) by E (total energy) and speculated

$$\Delta E = \Delta mc^2. \tag{11}$$

In Eq. (11) E stands for all possible energies of the universe e.g.: (i) sound energy, (ii) heat energy, (iii) chemical energy, (iv) nuclear energy, (v) magnetic energy, (vi) electrical energy, (vii) energy emitted in form of invisible radiations,

(viii) energy emitted in cosmological and astrophysical phenomena, (ix) energy emitted volcanic reactions, (x) energies co-existing in various forms etc., etc.

Now Eq. (1) i.e.

$$\ell^* = \ell \frac{\left(1 - \frac{v}{c} \cos \phi\right)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

is put forth for light energy by Einstein in June 1905 paper (ℓ^* is light energy in moving frame), it is not meant for other possible energies as quoted above.

Einstein never justified Eq. (1) for all the energies cited above. The parameters used in Einstein's equation are defined for light energy only, not for all the energies. Thus by this derivation $L = \Delta mc^2$ is derived under special conditions for light energy only and replacing L by E in Eq. (10) is not justified.

There are evidences that Einstein worked hurriedly in other case also e.g. in theory of static universe the introduction of cosmological constant proved to be incorrect and Einstein accepted the mistake later as quoted by Gamow [42]. The various cases when $\Delta E \propto \Delta mc^2$ is justified are shown in Table 1.

5 Conservation of momentum in general cases

The momentum is conserved irrespective of the fact that body remains at rest or recoils or tends to recoil after emission of light energy [43]. The law of conservation of momentum can be used to calculate the velocity of recoil in this case also. Let the body of mass 10 kg emits two waves of energy in visible region of wavelength 5000Å it corresponds to energy 7.9512×10^{-19} J. This energy is emitted in two waves i.e., as obvious, $0.5001L$ ($3.97639512 \times 10^{-19}$ J) and $0.4999L$ ($3.97480488 \times 10^{-19}$ J). Applying the conservation of momentum [43] the recoil velocity, recoil momentum and recoil kinetic energy comes out to be -5.3×10^{-32} m/s, 5.3×10^{-31} kg m/s and 1.404×10^{-62} J respectively. This recoil velocity (V_r) will change the uniform velocity v as $V_r + v$, but it will not make any difference to final result of change in mass as in Eq. (21), due to negligible value of V_r [27]. Hence in the law of conservation of momentum is obeyed in this case also.

6 Experimental feasibility with conversion factors other than c^2

(a) Dirac [44] was one of the first physicists to suggest that, in connection with his theory of large numbers, fundamental dimensional constants may vary in time during the expansion of the universe. The idea of variation of the speed of light is suggested in various cosmological models [45, 46] and has been the subject of attention by physicists in investigations of extra dimensions, strings and branes [47]. Webb [48] has reported variations in fine structure constant over cosmological time scales and hence variations in c . This suggestion implies $\Delta E \propto \Delta mc^2$.

(b) Einstein has derived $L = \Delta mc^2$ (conversion factor between mass and energy is precisely equal to c^2) under the extremely special or ideal conditions, which are even difficult to attain practically. The work of scientists before Einstein also justifies $\Delta E \propto \Delta mc^2$.

This discussion does not confront with existing experimental situation but addresses those theoretical and experimental issues for which $\Delta E = \Delta mc^2$ is not analyzed yet. The mass energy inter-conversion equation, with conversion factor equal to c^2 i.e. $\Delta E = \Delta mc^2$ has been confirmed in nuclear physics and is also basis of nuclear physics. Even elementary units of atomic mass (1 amu) or and energy (eV) are based upon it. Thus it will remain standard in measurements as seven days in a week; its validity in this regard is not doubted at all.

The aim is to discuss experimentally those phenomena in which $\Delta E = \Delta mc^2$ is not applied yet. The mass energy conversion processes are weird in nature and all have not been at all studied in view of $\Delta E = \Delta mc^2$. The conversion factor other than c^2 is discussed for such elusive cases, not for those it is already confirmed. Hence there is no confrontation with the established experimental situation at all, but aim is to open a mathematical front ($\Delta E \propto \Delta mc^2$) for numerous experimentally unstudied phenomena in nature. This development can be discussed as below.

7 Most abundant chemical reactions

(i) **Unconfirmed chemical reactions.** When Einstein derived $E = \Delta mc^2$, chemical reactions were the most abundant sources of energy in nature. Till date $E = \Delta mc^2$ is not confirmed in the chemical reaction and the reason cited for this is that equipments are not enough sensitive [49, 50]. Consider burning of 1kg straw or paper or petrol in controlled way i.e. in such a way that masses, ashes, gases and energy produced can be estimated. Even if 0.001 kg or 1gm of matter is annihilated then energy equal to 9×10^{13} J (can drive a truck of mass 1000 kg to distance of 9×10^7 km) will be produced. Until the equation is not confirmed in such reactions, then scientifically $E = \Delta mc^2$ may not be regarded as precisely true in such cases. It is equally possible that energy emitted may be

less than predicted by $E = \Delta mc^2$ i.e. $E \propto \Delta mc^2$ is feasible, it is an open possibility unless ruled out.

Reactions in nuclear physics

(ii) **Less efficiency:** The efficiency of the nuclear weapons as well as nuclear reactors is far less than the theoretical value predicted by $E = \Delta mc^2$. Robert Serber (member of first American team entered Hiroshima and Nagasaki in September 1945 to assess losses), has indicated [51] that the efficiency of "Little Boy" weapon (U^{235} , 49 kg) that was used against Hiroshima was about 2% only. It is assumed that all the atoms don't undergo fission, thus material is wasted. But no such waste material is specifically measured quantitatively. Thus the waste material (nuclear reactor or weapon) must be measured and corresponding energy be calculated, and it must quantitatively explain that why efficiency is less. It may require the measurements of all types of energies (may co-exist in various forms) in the processes and experimental errors. Until such experiments are specifically conducted and $E = \Delta mc^2$ is confirmed, $\Delta E \propto \Delta mc^2$ is equally feasible.

(iii) **Less energy:** In laboratory it is confirmed [7, 52, 53] that using thermal neutrons the total kinetic energy (TKE) of fission fragments that result from U^{235} and Pu^{239} is 20–60 MeV less than Q -value (200 MeV) of reaction predicted by $\Delta E = \Delta mc^2$. This observation is nearly four decades old. Bakhom [7] has explained it on the basis of equation $H = mv^2$ (energy emitted is less than $E = \Delta mc^2$). Hence here $E \propto \Delta mc^2$ is justified.

(iv) **More mass:** Palano [10] has confirmed that mass of particle Ds (2317) has been found more than current estimates based upon $\Delta E = \Delta mc^2$. Thus in this case $E \propto \Delta mc^2$ is justified.

(v) **Binding energy and mass defect in deuteron:** There are two inherent observations [23, 28, 29] about nucleus: firstly, masses of nucleons are fundamental constants, i.e. they are the same universally (inside and outside the nucleus in all cases); and secondly nuclei possess Binding Energy ($BE = \Delta mc^2$) owing to a mass defect. To explain these observations, in the case of the deuteron ($BE = 2.2244$ MeV), the mass defect of nucleons must be 0.002388 amu or about 0.11854% of the mass of nucleons, i.e., nucleons must be lighter in the nucleus. This is not experimentally justified, as masses of nucleons are universal constants. Thus observations and predictions based upon $\Delta E = \Delta mc^2$ are not justified, hence $\Delta E \propto \Delta mc^2$ is equally feasible.

8 Mathematical form of extended equation

Until $E = \Delta mc^2$ is not precisely confirmed experimentally in ALL CASES, it is equally feasible to assume that the energy emitted may be less than $E = \Delta mc^2$ (or $E \propto \Delta mc^2$). It does not have any effect on those cases where $E = \Delta mc^2$ is confirmed, it simply scientifically stresses confirmation of $E = \Delta mc^2$ in all cases. Also when reactants are in bulk

amount and various types of energies are simultaneously emitted and energies may co-exist. Thus both the possibilities are equally probable until one is not specifically ruled out. In view of weirdness in reactions emitting energy in universe, some theoretical inconsistencies in the derivation and non-availability of data, one can explore the second possibility even as a postulate. All the equations in science are regarded as confirmed when specifically justified in all experiments time and again. The reactions involving inter-conversion of mass and energy are utmost diverse, weird and new phenomena are being added regularly, thus $E = mc^2$ needs to be confirmed in all cases. Thus in general, in view of above proportionality it may be taken in account as

$$dE \propto c^2 dm.$$

The above proportionality $dE \propto c^2 dm$ can be changed into equation by introducing a constant of proportionality. The inception of proportionality constant is consistent with centuries old perception of constant of proportionality in physics since days of Aristotle and Newton. In second law of motion ($F = kma$) the value of constant of proportionality, k is always unity (like universal constant) i.e. $F = ma$. When more and more complex phenomena were studied or values of constants of proportionality were determined then it showed dependence on the inherent characteristics of the phenomena. In case constant of proportionality varies from one situation to other then it is known as co-efficient of proportionality e.g. co-efficient of thermal conductivity or viscosity etc. Thus removing the proportionality between dE and $c^2 dm$, we get

$$dE = A c^2 dm, \quad (22)$$

where A is (a co-efficient) used to remove that sign of proportionality; it depends upon inherent characteristics of the processes in which conversion of mass to energy takes place and it is dimensionless. It has nature precisely like Hubble's constant (50 and 80 kilometers per second-Megaparsec, Mpc) or coefficient of viscosity (1.05×10^{-3} poise to 19.2×10^{-6} poise) or co-efficient of thermal conductivity ($0.02 \text{ Wm}^{-1}\text{K}^{-1}$ to $400 \text{ Wm}^{-1}\text{K}^{-1}$) etc. Thus, in fact Hubble's constant may be regarded Hubble's variable constant or Hubble's coefficient, as it varies from one heavenly body to other. If " A " is equal to one, then we will get $dE = dm c^2$ i.e. same as Einstein's equation.

In Eq. (22) " A " is regarded as conversion factor as it describes feasibility and extent of conversion of mass into energy. For example out of bulk mass, the mass annihilated to energy is maximum in matter-antimatter annihilation, apparently least in chemical reactions, undetermined in volcanic reactions and cosmological reactions. It (the co-efficient A) depends upon the characteristic conditions of a particular process. It may be constant for a particular process and varies for the other depending upon involved parameters or experimental situation. Thus " A " cannot be regarded as universal

constant, just like universal gravitational constant G and k in Newton's Second Law of Motion. The reason is that mass energy inter-conversion are the bizarre processes in nature and not completely studied.

Now consider the case that when mass is converted into energy. Let in some conversion process mass decreases from M_i (initial mass) to M_f (final mass), correspondingly energy increases from E_i (initial energy) to E_f (final energy). The Eq. (22) gives infinitesimally small amount of energy dE created on annihilation of mass dm . To get the net effect the Eq. (22) can be integrated similarly Einstein has obtained the relativistic form of kinetic energy in June 1905 paper [18]

$$\int dE = A c^2 \int dm,$$

Initial limit of mass = M_i , Initial limit of Energy = E_i ,

Final limit of mass = M_f , Final limit of Energy = E_f .

Initially when mass of body is M_i , then E_i is the initial energy of the system. When mass (initial mass, M_i) is converted into energy by any process under suitable circumstances the final mass of system reduces to M_f . Consequently, the energy of system increases to E_f the final energy. Thus M_f and E_f are the quantities after the conversion. Hence, Eq. (22) becomes

$$E_f - E_i = A c^2 (M_f - M_i) \quad (23)$$

or

$$\Delta E = A c^2 \Delta m \quad (24)$$

$$\text{Energy evolved} = A c^2 (\text{decrease in mass}). \quad (25)$$

If the characteristic conditions of the process permit then whole mass is converted into energy i.e. after the reaction no mass remains ($M_f = 0$)

$$\Delta E = - A c^2 M_i \quad (26)$$

In this case energy evolved is negative implies that energy is created at the cost of annihilation of mass and the process is exo-energetic nature (energy is emitted which may be in any form). Energy is scalar quantity having magnitude only, thus no direction is associated with it.

Thus the generalized mass-energy equivalence may be stated as

"The mass can be converted into energy or vice-versa under some characteristic conditions of the process, but conversion factor may or may not always be c^2 ($9 \times 10^{16} \text{ m}^2/\text{s}^2$) or c^{-2} ."

9 Applications of generalized mass energy inter conversion equation $\Delta E = A c^2 \Delta m$

(i) It is already mentioned in section (3) that if 0.001 kg or 1 gm of matter is annihilated then energy equal to 9×10^{13} J

(can drive a truck of mass 1000 kg to distance of 9×10^7 km) will be produced. Such or similar predictions are not experimentally confirmed and energy emitted can be found less than predictions.

Let the energy observed is 4.5×10^{13} J corresponding to mass annihilated 0.001 kg, then value of A from $\Delta E = A c^2 \Delta m$ will be 0.5 i.e.

$$A = \frac{\Delta E}{c^2 \Delta m} = \frac{4.5 \times 10^{13} \text{ J}}{9 \times 10^{16}} = 0.5. \quad (27)$$

Thus in this case mass energy inter-conversion equation becomes

$$\Delta E = 0.5 c^2 \Delta m. \quad (28)$$

(ii) Let the TKE of fission fragments of U^{235} and Pu^{239} is 175 MeV (as experimentally it is observed less), instead of expected 200 MeV. It can be explained with help of $\Delta E = A c^2 \Delta m$ with value of A is equal to 0.875 i.e.

$$A = \frac{\Delta E}{c^2 \Delta m} = \frac{175}{200} = 0.875. \quad (29)$$

Thus energy of fission fragments of U^{235} and Pu^{239} is given by

$$\Delta E = 0.875 c^2 \Delta m. \quad (30)$$

Thus value of A less than one is justified experimentally in this case.

(iii) The anomalous observation of excess mass of Ds(2317) can be understood with help of $\Delta E = A c^2 \Delta m$, as mass of the observed particle is found more [10] than predictions of $E = \Delta m c^2$. In this case value of A will be less than one. For understanding consider energy equal to 10^6 J is converted into mass, then corresponding mass must be 1.11×10^{-11} kg. We are considering the case that mass is found more than this. Let the mass be 1.12×10^{-11} kg. The value of A this case is 0.992, as calculated from $\Delta E = A c^2 \Delta m$ i.e.

$$A = \frac{10^6}{1.08 \times 10^5} = 0.992. \quad (31)$$

Thus in this mass energy inter conversion equation becomes

$$\Delta E = 0.992 c^2 \Delta m \quad \text{or} \quad \Delta m = 1.008 \Delta E. \quad (32)$$

Thus corresponding to small energy more mass is emitted. (vi) $\Delta E = A c^2 \Delta m$ is useful in explaining the binding energy (2.2244 MeV or 3.55904×10^{-13} J), mass defect (0.002388 amu or 2.388×10^{-3} amu) and universal equality of mass of nucleons ($m_n = 1.008664$ amu, $m_p = 1.006082$ amu). Obviously neutron and protons contribute equally towards the mass defect (0.001194 amu), then mass of neutron inside nucleus must be 1.00747 amu (mass outside nucleus i.e. in Free State is 1.008664 amu). Similarly corresponding mass of proton in the nucleus must be 1.006082 amu (mass of proton outside nucleus 1.007274 amu). But decrease in mass of nucleons inside nucleus is not justified, as masses of nucleons are universally same [23, 28, 29].

Thus mass defect of deuteron must be **infinitesimally small**, only then masses of nucleons are same inside **nucleus and outside nucleus**. Also binding energy must be 2.2244 MeV as experimentally observed. Both these experimentally confirmed facts can be explained with help of $\Delta E = A c^2 \Delta m$.

Let in this case the mass defect is negligibly small i.e. 2.388×10^{-13} amu or 3.9653×10^{-40} kg. Then value of A (coefficient of proportionality or mass energy inter conversion coefficient) is 10^{10} i.e. for annihilation of infinitesimally small mass exceptionally large amount of energy is liberated. Thus

$$A = \frac{\Delta E}{c^2 \Delta m} = \frac{3.5634 \times 10^{-13}}{9 \times 10^{16} \times 3.9653 \times 10^{-40}} = 10^{10}, \quad (33)$$

$$\Delta E = 10^{10} c^2 \Delta m. \quad (34)$$

(v) Webb [48] has reported results for time variability of the fine structure constant or Sommerfeld fine structure constant (α) using absorption systems in the spectra of distant quasars. The variation in magnitude of alpha has been observed as

$$\frac{\Delta \alpha}{\alpha} = \frac{(\alpha_{then} - \alpha_{now})}{\alpha_{now}} = -0.9 \times 10^{-5}. \quad (35)$$

According to CODATA currently accepted value of alpha (α_{now}) is 7.297352×10^{-3} . Hence from Eq. (35),

$$\alpha_{then} = 0.007296. \quad (36)$$

Now corresponding to the reduced value of α ($\alpha_{then} = 0.007296$) the the speed of light can be determined from equation

$$c_{then} = \frac{e^2}{2 \alpha_{then} \epsilon h} \quad (37)$$

as 2.994×10^8 m/s (where all terms have usual meanings). Currently accepted value of the speed of light is 2.99729×10^8 m/s.

To explain the energy emitted with this value of the speed of light is the value A ($\Delta E = A c^2 \Delta M$)

$$A = \frac{c^2}{c_{then}^2} = 1.001. \quad (38)$$

Thus in this case mass energy inter conversion equation becomes

$$\Delta E = 1.001 c^2 \Delta m. \quad (39)$$

Hence $\Delta E \propto \Delta m c^2$ has both experimental and theoretical support, with emergence of new experimental data its significance will increase.

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On the Necessity of Aprioristic Thinking in Physics

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The thinking which encompasses both reasoning-in-itself and reasoning-for-itself, called “aprioristic thinking” by Hegel, is the freest form of thinking. This form of thinking is imparted to the physical sciences by philosophy. Only under this condition can physics obtain deeper scientific knowledge.

In the beginning of the last century, the renowned scientist Anri Bergson [1] gave an advanced notice: “We experience now one of the greatest crises; all our thinking, all ethics, all life, all our spiritual and moral existence are in a condition of intellectual fermentation...”. This fermentation, according to the opinion of the known philosopher Edmund Husserl [2], occurs due to installation dominant in positivistic and naturalistic philosophy. This installation of ordinary consciousness contrasts the human consciousness and being to each other, and, therefore, not taking into account consciousness, can lead to more crisis the European sciences. As pointed out by Husserl, the sciences about the nature can be founded only by means of phenomenology, as a strict philosophy, which is oriented towards a first-hand experience of consciousness. Though many years have already passed since then, as these scientists have written, resolute turn in this question is not yet present. Even, in spite of the fact that in one of the achievements of modern physics — in quantum physics — the consciousness of the observer has found a place for itself. In the interpretation of quantum mechanics, the most important upshot of this for physicists is that this problem is related to the problem of consciousness — an interdisciplinary problem concerning not only physicists, but also philosophers, psychologists, physiologists and biologists. Its solution will result in deeper scientific knowledge. But all the same, for some reason, scientists very often in case of scientific cognition neglect questions of the interaction between our consciousness and the surrounding world. If we wish to reach fuller scientific knowledge, we should not deal with physical phenomena and thinking (consciousness) itself separately. The well-known physicist Wigner [3] maintains that the separation between our perception and the laws of nature is no more than simplification. And though we are convinced that it has a harmless character, to nevertheless merely forget about it should not be the case. It is clear that deeper scientific knowledge should include in itself a problem of the theory of cognition — a problem of the origin of knowledge and a logical substantiation of the relevant system of knowledge.

In deciding upon this problem, the cognition theory considers the connection between “I”, my consciousness and an external world, and says that the decision is concealed in the interaction between sensuality and reason. Reason transforms our feelings into thoughts and it means that the representa-

tions are replaced with concepts. If science does not wish to be, as it was described by Hegel [4], a simple unit of data then, of course, it should have concepts and should operate with them. But, if science also does not wish to be positivistic (all sciences, except philosophy, are positivistic) then it should have a rational basis and beginning. Only in this case, does the sole purpose (affair) of science become the concept of the concept. (Hegel has distinguished between the sciences as follows: 1) sciences, as a simple unit of data, 2) the extremely positive sciences, 3) positive sciences, 4) philosophy. Positivism of a physical science is that it does not know that its definitions are final).

Physics, certainly, has a rational basis which is intimately connected with philosophy too. But what prevents a physical science from becoming a “mere” philosophy? Hegel has elaborated on the notion of a positivistic side of the sciences. In physics, this positivism is characterized by the lack of knowledge that its definitions are final and therefore there is no transition into the higher sphere. This finiteness is connected with the finiteness of the cognition (feeling, belief, authority of others, and authority of external and internal contemplation).

However, it is perhaps meant so to happen, as described by Hegel, that thoughtful contemplation, lowering casual conditions and organizing everything, will present the general outline before a detailed intellectual exposition. It is clear then that an intellectual physical science will picture a rational science of Nature in the form of an image which is the external image of Nature. This image is called a physical picture of the world, or, as called by Max Planck [5], the world of a physical science. Planck has explained further about it: “... We are compelled to recognize behind the sensual world the second, real world which leads independent existence independent of the person, — the world which we not can comprehend directly, but we comprehend via the sensual world, via known symbols which he informs us, as if we would consider a interesting subject only through the glasses, optical properties of which are absolutely unknown for us”.

Thus, according to Planck, there are three worlds: the real world, the sensual world and the world of a physical science or a physical picture of the world. The real world is the world outside us, it exists irrespective of our understanding of its laws, i.e. irrespective of our consciousness and therefore it is the objective world. The sensual world is our world because

we perceive it through our bodies of perception: eyes, hearing, charm etc., and it is subjective (it is possible to tell that it is illusion). A physical picture of the world is the world in which can be reflected both real and the sensual world. This world is a bridge for us with which help we study the world around. Reflection of the real world in the world of a physical science is a physical picture of the real world; it is also possible to describe the quantum world and the science studying this world is the quantum physics. The reason why the real world is the quantum world is because the so-called world of atoms and electrons, as Planck has given above, exists independently of the person. Reflection of the sensual world in a physical picture of the world is a physical picture of the sensual world (the classical world) and the corresponding science is the classical physics. Thus, only in case of the thoughtful contemplation can the physics can be concerned with the philosophy of nature.

But when will it be possible to tell, whether the physical science is not simply concerned with philosophy, and even enters into it, to a certain extent it? Based on a well-known classification of all sciences by Hegel, the nature philosophy is a science about an idea in another-being. Hegel has thus said: “what is real, is reasonable”, referring to understanding in the context of the reality of a reasonable idea. Such a reality is the maintenance of Hegel’s philosophy. Hegel writes that phenomena, being unstable (random) and existing in continuous fluidity, are in contrast to the idea and do not enter into it. Therefore Hegel takes the idea as the maintenance of his philosophy. In the ancient time, Plato too spoke about ideas [2]. He wrote: “In a horse, in the house or in the fine woman there is nothing real. The reality is concluded as a universal type (idea) of a horse, the house, the fine woman” [6]. Plato confirms the continuous fluidity of all existing forms and asks the question: can the philosophy be within continuous and chaotic fluidity? As a result, the human knowledge is possible only under the condition of the existence of steady ideas, and with the help of it, is possible to distinguish between things based on fluid validity and to plan in it any logical order. Hegel understands that an idea will be steady, if it will be the reality of a “reasonable”. After all, only reason is steady, absolute. But this is not only because it is so ingenious to define ideas in the way Hegel did it. In “Metaphysics”, Aristotle, criticizing Plato, asserts that the idea of a thing explains nothing in the thing itself, even provided that the idea relates to the thing, as found for example, in the fact that whiteness concerns a white subject. Aristotle did not actually deny the independent existence of ideas, but attributed to them the existence within things themselves. Namely, Hegel’s idea — the reality of the “reasonable” — satisfies Aristotle’s requirement. Because, in such determination, the idea is taken from the reality itself. But against Hegel’s reality the mind at once acts. The mind says to us that ideas are no existing chimeras. If science does not want to conceptualize its concept then it, of course, will agree with the mind. Then, very figuratively, it

is described by Hegel as follows: just as meal process is ungrateful to the meal (simply eats it, not giving instead of anything), similarly, thinking process will be ungrateful to a posteriori experience, and will simply give nothing in exchange. In order to receive something from thinking process, it is necessary to make the thinking itself by the subject of thinking. Reflection transforms our representations into concepts. And further reflections of concepts transform concepts into concepts, i.e. it becomes clear as a concept. Only under such conditions can the science understand its concept. However, only in philosophy do we find that the subject of thinking is the thinking itself (for example, for the mathematician, it is numbers, spaces etc.). The thinking, opposing with itself to itself, is the reasoning-for-itself. Process thinking nevertheless is inside and consequently it is the reasoning-in-itself. As a result, the “in itself” and “for itself” reasoning is the most substantial form of free thinking and it is defined by Hegel, as aprioristic thinking. Only by aprioristic thinking can the generality and authenticity be found. Namely, in this thinking, philosophy informs the maintenance of empirical sciences. The obligation of the sciences is not to refuse this process, because it is a very noble act for a science to reach the concept of the concept. But the mind, objecting again, speaks to us: “But what it can give to the physics? ”. At all times, there have been physicists who, knowing about the finiteness of the knowledge of their science, have spoken about deeper scientific knowledge [8–15]. They envision when it will be possible to speak about the physicist and about the consciousness of the observer simultaneously.

Hegel has very interestingly written: “In the physicist we too get acquainted with the general, with essence, the only distinction between physics and the philosophy of nature is that the philosophy of nature leads up us to the comprehension of the true forms of the concept of natural things”. But doesn’t it mean that in deeper scientific cognition the physical science has transited into a higher circle which is not present in physics because of its positivism? And the answer to this question is, of course, yes, it does. Thus, only under the condition of deeper scientific knowledge can we claim that the physical science is the philosophy of nature (in the sense that, for example, the apple is a fruit).

Hegel defines the philosophy of nature, as a science about an idea in its another being. As he writes, in philosophy we do not learn anything else, except ideas, but the ideas exist here as exterior forms. An exterior form of an idea is its another being. Because the being of an idea (reasoning-in-itself and reasoning-for-itself) takes place in the reason itself. Nature receives its exterior, that exterior which we see, in the exterior process of an idea. In fact, Hegel’s slogan “what is reasonable, is real” is confirmed.

Unwittingly, we could as well resolve one more problem. The maintenance of philosophy, as Hegel writes, is an idea which excludes from itself, the phenomenon, chance. But the maintenance of physics is Nature, its phenomena. At the

same time we may ask, “when can the physical science become the philosophy of nature?” All becomes clear when we agree with Hegel, that Nature is connected with an idea, in the sense that it is an idea in its another being. The laws of Nature, discovered by our thinking about physics, are also ideas – reasonables of reality.

Thus, as in the past, philosophy will continue to play an important role related to the necessity for the sciences to enter a higher level. Only in this case can the sciences avoid the crisis about which Husserl has always warned us. As Bergson continues that which has been said in the beginning of this article: “... The new system, more general, wider should become the doctrine for many decades and even centuries. These new principles should direct all our life on a new way on which the mankind will approach to cognition of true and to happiness increase at the Earth”.

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Potential Energy Surfaces of the Even-Even $^{230-238}\text{U}$ Isotopes

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Nuclear structure of $^{230-238}\text{U}$ isotopes have been studied in the frame work of the interacting boson approximation model ($IBM - 1$). The contour plot of the potential energy surfaces, $V(\beta, \gamma)$, shows that all nuclei are deformed and have rotational characters, $SU(3)$. Levels energy spectra belonging to the gsb, β, γ bands, electromagnetic transition rates $B(E1)$ and $B(E2)$, quadrupole moment Q_0 , deformation parameter β_2 and the strength of the electric monopole transitions $X(E0/E2)$ are calculated. The calculated values are compared with the available theoretical and experimental data and show reasonable agreement.

1 Introduction

The observation of a large quadrupole moments to $^{230-238}\text{U}$ isotopes had led to the suggestion that these nuclei might be deformed and have to be confirmed by the measurement of their nuclear properties as well as the observation of their rotational band structures. It is noticed that the level schemes of uranium isotopes are characterized by the existence of two bands of opposite parity and lay in the region of octupole deformations. The primary evidence for this octupole deformation comes from the parity-doublet bands, fast electric transition ($E1$) between the negative and positive parity bands and the low-lying $1^-, 0_2^+$ and 2_2^+ excitation energy states. Many authors have studied $^{230-238}\text{U}$ isotopes theoretically using different models. The relativistic Mean Field Model has employed [1–4] to obtain the densities of the cluster and daughter nuclei. Also, a systematic α -decay properties of the even-even heavy and superheavy nuclei have been investigated. The energy of the deformed nuclei in the actinide region has been determined in the frame work of the macroscopic — microscopic approach. The Yukawa folding procedure has used [5] together with the Liquid Drop Model [6].

The properties of the states of the alternating parity bands in actinides are analyzed within the Cluster Model. The model has been used successfully in calculating levels energy, quadrupole moments and half-lives of cluster radioactivity. A comparison was mad between the predicted data [7–13] and the calculated values by other models and show good agreement.

The band heads, energy spacings within bands and a number of interband as well as intraband $B(E2)$ transition rates are well reproduced [14] for all actinide nuclei using the Exactly Separable Davidson (ESD) solution of the Bohr Hamiltonian.

The potential energy surfaces are calculated [15] to ^{230}U using the most advanced asymmetric two-center shell model

that are added to the Yukawa-plus-exponential model.

Until now scarce informations are available about the actinide region in general and this is due to the experimental difficulties associated with this mass region. In the present article we used the Interacting Boson Model ($IBM - 1$) which is a theoretical model and differ than all the previous models used with the actinid nuclei. The aim of the present work is to process calculation for the follows:

1. For the potential energy surfaces, $V(\beta, \gamma)$, for all $^{230-238}\text{U}$ nuclei;
2. For levels energy;
3. For the electromagnetic transition rates $B(E1)$ and also calculation for $B(E2)$;
4. For the electric quadrupole moment Q_0 ;
5. For the deformation parameter β_2 ;
6. For the strength of the electric monopole transitions $X(E0/E2)$.

2 (IBA-1) model

2.1 Level energies

The IBA-1 model was applied to the positive and negative parity low-lying states in even-even $^{230-238}\text{U}$ isotopes. The proton, π , and neutron, ν , bosons are treated as one boson and the system is considered as an interaction between s -bosons and d -bosons. Creation ($s^\dagger d^\dagger$) and annihilation ($s\tilde{d}$) operators are for s and d bosons. The Hamiltonian [16] employed for the present calculation is given as:

$$\begin{aligned}
 H = & EPS \cdot n_d + PAIR \cdot (P \cdot P) + \\
 & + \frac{1}{2} ELL \cdot (L \cdot L) + \frac{1}{2} QQ \cdot (Q \cdot Q) + \\
 & + 5OCT \cdot (T_3 \cdot T_3) + 5HEX \cdot (T_4 \cdot T_4),
 \end{aligned} \tag{1}$$

nucleus	<i>EPS</i>	<i>PAIR</i>	<i>ELL</i>	<i>QQ</i>	<i>OCT</i>	<i>HEX</i>	<i>E2SD(eb)</i>	<i>E2DD(eb)</i>
²³⁰ U	0.2000	0.000	0.005	-0.0150	0.0000	0.0000	0.2060	-0.6094
²³² U	0.2000	0.000	0.0050	-0.0150	0.0000	0.0000	0.1890	-0.5591
²³⁴ U	0.2000	0.0000	0.0044	-0.0150	0.0000	0.0000	0.1782	-0.5271
²³⁶ U	0.2000	0.0000	0.0055	-0.0150	0.0000	0.0000	0.1720	-0.5088
²³⁸ U	0.2000	0.0000	0.0057	-0.0150	0.0000	0.0000	0.1630	-0.4822

Table 1: Table 1: Parameters used in IBA-1 Hamiltonian (all in MeV).

where

$$P \cdot P = \frac{1}{2} \left[\begin{array}{c} \left\{ (s^\dagger s^\dagger)_0^{(0)} - \sqrt{5} (d^\dagger d^\dagger)_0^{(0)} \right\} x \\ \left\{ (s s)_0^{(0)} - \sqrt{5} (\tilde{d} \tilde{d})_0^{(0)} \right\} \end{array} \right]_0^{(0)}, \quad (2)$$

$$L \cdot L = -10 \sqrt{3} \left[(d^\dagger \tilde{d})^{(1)} x (d^\dagger \tilde{d})^{(1)} \right]_0^{(0)}, \quad (3)$$

$$Q \cdot Q = \sqrt{5} \left[\begin{array}{c} \left\{ (S^\dagger \tilde{d} + d^\dagger s)^{(2)} - \frac{\sqrt{7}}{2} (d^\dagger \tilde{d})^{(2)} \right\} x \\ \left\{ (s^\dagger \tilde{d} + \tilde{d} s)^{(2)} - \frac{\sqrt{7}}{2} (d^\dagger \tilde{d})^{(2)} \right\} \end{array} \right]_0^{(0)}, \quad (4)$$

$$T_3 \cdot T_3 = -\sqrt{7} \left[(d^\dagger \tilde{d})^{(2)} x (d^\dagger \tilde{d})^{(2)} \right]_0^{(0)}, \quad (5)$$

$$T_4 \cdot T_4 = 3 \left[(d^\dagger \tilde{d})^{(4)} x (d^\dagger \tilde{d})^{(4)} \right]_0^{(0)}. \quad (6)$$

In the previous formulas, n_d is the number of boson; $P \cdot P$, $L \cdot L$, $Q \cdot Q$, $T_3 \cdot T_3$ and $T_4 \cdot T_4$ represent pairing, angular momentum, quadrupole, octupole and hexadecupole interactions between the bosons; *EPS* is the boson energy; and *PAIR*, *ELL*, *QQ*, *OCT*, *HEX* is the strengths of the pairing, angular momentum, quadrupole, octupole and hexadecupole interactions.

2.2 Transition rates

The electric quadrupole transition operator [16] employed in this study is given by:

$$T^{(E2)} = E2SD \cdot (s^\dagger \tilde{d} + d^\dagger s)^{(2)} + \frac{1}{\sqrt{5}} E2DD \cdot (d^\dagger \tilde{d})^{(2)}. \quad (7)$$

The reduced electric quadrupole transition rates between $I_i \rightarrow I_f$ states are given by

$$B(E_2, I_i \rightarrow I_f) = \frac{[\langle I_f || T^{(E2)} || I_i \rangle]^2}{2I_i + 1}. \quad (8)$$

3 Results and discussion

3.1 The potential energy surface

The potential energy surfaces [17], $V(\beta, \gamma)$, for uranium isotopes as a function of the deformation parameters β and γ

have been calculated using :

$$\begin{aligned} E_{N_\pi N_\nu}(\beta, \gamma) &= \langle N_\pi N_\nu; \beta \gamma | H_{\pi\nu} | N_\pi N_\nu; \beta \gamma \rangle = \\ &= \zeta_d (N_\nu N_\pi) \beta^2 (1 + \beta^2) + \beta^2 (1 + \beta^2)^{-2} \times \\ &\times \{ k N_\nu N_\pi [4 - (\bar{X}_\pi \bar{X}_\nu) \beta \cos 3\gamma] \} + \\ &+ \left\{ [\bar{X}_\pi \bar{X}_\nu \beta^2] + N_\nu (N_\nu - 1) \left(\frac{1}{10} c_0 + \frac{1}{7} c_2 \right) \beta^2 \right\}, \end{aligned} \quad (9)$$

where

$$\bar{X}_\rho = \left(\frac{2}{7} \right)^{0.5} X_\rho \quad \rho = \pi \text{ or } \nu. \quad (10)$$

The calculated potential energy surfaces, $V(\beta, \gamma)$, for uranium series of isotopes are presented in Fig. 1 and Fig. 2. It shows that all nuclei are deformed and have rotational-like characters. The two wells on both oblate and prolate sides are not equal but the prolate is deeper in all nuclei.. The energy and electromagnetic transition rates are calculated considering uranium series of isotopes a rotational-like nuclei.

3.2 Energy spectra

IBA-1 model has been used in calculating the energy of the positive and negative parity low -lying levels of uranium series of isotopes. In many deformed actinide nuclei the negative parity bands have been established and these nuclei are considered as an octupole deformed. A simple means to examine the nature of the band is to consider the ratio R which for octupole band, $R > 1$, and defined as [18]:

$$R = \frac{E(I+3) - E(I-1)_{NPB}}{E(I) - E(I-2)_{GSB}}. \quad (11)$$

In the present calculations all values of R for uranium series of isotopes are > 1 , and we treated them as octupole deformed nuclei.

A comparison between the experimental spectra [19–23] and our calculations, using values of the model parameters given in Table 1 for the ground and octupole bands, are illustrated in Fig. 3. The agreement between the calculated levels energy and their correspondence experimental values for all uranium nuclei are reasonable, but slightly higher especially for the higher excited states. We believe this is due to the change of the projection of the angular momentum which

$I_i^+ I_f^+$	^{230}U	^{232}U	^{234}U	^{236}U	^{238}U
$0_1 \text{ Exp. } 2_1$	9.70(12)	10.0(10)	10.66(20)	11.61(15)	12.09(20)
$0_1 \text{ Theor. } 2_1$	9.7128	10.0163	10.6479	11.6506	12.1143
$2_1 0_1$	1.9426	2.0033	2.1296	2.3301	2.4229
$2_2 0_1$	0.0107	0.0113	0.0104	0.0095	0.0081
$2_2 0_2$	1.2419	1.3677	1.5411	1.7598	1.8855
$2_3 0_1$	0.0190	0.0131	0.0099	0.0082	0.0066
$2_3 0_2$	0.0027	0.0095	0.0131	0.0144	0.0139
$2_3 0_3$	0.0245	0.0085	0.0031	0.0013	0.0007
$2_4 0_3$	0.7577	0.8679	1.0291	1.2308	1.3730
$2_4 0_4$	0.0508	0.0415	0.1309	0.0710	0.0022
$4_1 2_1$	2.7740	2.8443	3.0182	3.3014	3.4336
$4_1 2_2$	0.0699	0.0480	0.0352	0.0276	0.0213
$4_1 2_3$	0.0046	0.0019	0.0010	0.0007	0.0005
$6_1 4_1$	3.0183	3.0849	3.2707	3.5790	3.7256
$6_1 4_2$	0.0706	0.0532	0.0412	0.0333	0.0260
$6_1 4_3$	0.0128	0.0066	0.0039	0.0026	0.0018
$8_1 6_1$	3.0670	3.1387	3.3335	3.6548	3.8121
$8_1 6_2$	0.0618	0.0503	0.0415	0.0351	0.0381
$8_1 6_3$	0.0201	0.0117	0.0073	0.0049	0.0034
$10_1 8_1$	2.9919	3.0827	3.2910	3.6237	3.7930
$10_1 8_2$	0.0510	0.0439	0.0383	0.0340	0.0280

Table 2: Table 2: Values of the theoretical reduced transition probability, $B(E2)$ (in $e^2 b^2$).

$I_i^- I_f^+$	^{230}U	^{232}U	^{234}U	^{236}U	^{238}U
$1_1 0_1$	0.1353	0.1602	0.1824	0.2071	0.2294
$1_1 0_2$	0.0531	0.0512	0.0492	0.0475	0.0449
$3_1 2_1$	0.2509	0.2811	0.3075	—	—
$3_1 2_2$	0.0811	0.0763	0.0711	—	—
$3_1 2_3$	0.0013	0.0002	0.0000	—	—
$5_1 4_1$	0.3628	0.3913	—	—	—
$5_1 4_2$	0.0862	0.0831	—	—	—
$5_1 4_3$	0.0020	0.0006	—	—	—
$7_1 6_1$	0.4809	0.5064	—	—	—
$7_1 6_2$	0.0816	0.0811	—	—	—
$9_1 8_1$	0.6043	0.6267	—	—	—
$9_1 8_2$	0.0736	0.0749	—	—	—

Table 3: Table 3: Values of the theoretical reduced transition probability, $B(E1)$ (in $\mu e^2 b$).

<i>nucleus</i>	^{230}U	^{232}U	^{234}U	^{236}U	^{238}U
Q_0	9.920	10.020	10.340	10.800	11.020
β_2	0.263	0.264	0.272	0.282	0.286

Table 4: Table 4: The calculated electric quadrupole moment Q_0 and deformation parameter β_2 .

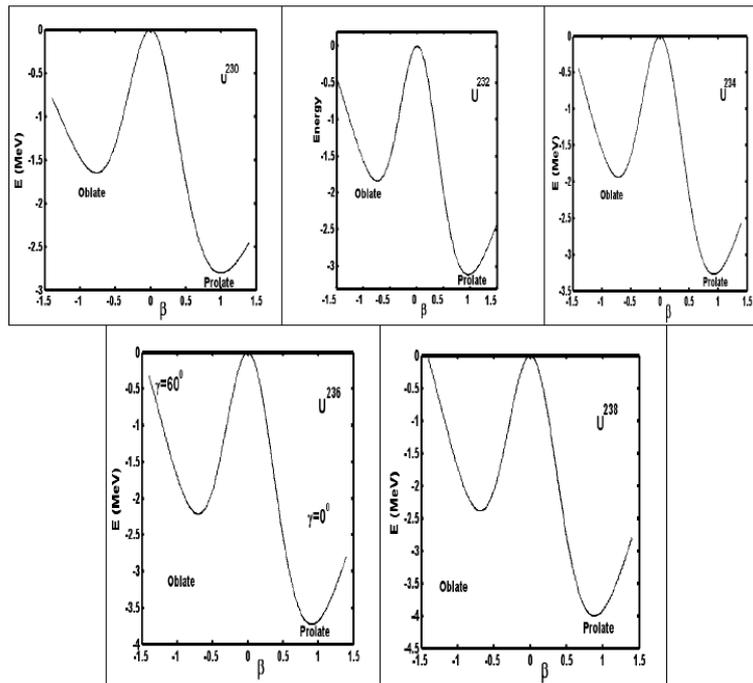


Fig. 1: Potential energy surfaces for $^{230-238}\text{U}$ nuclei at $\gamma = 0^\circ$ (prolate) and 60° (oblate).

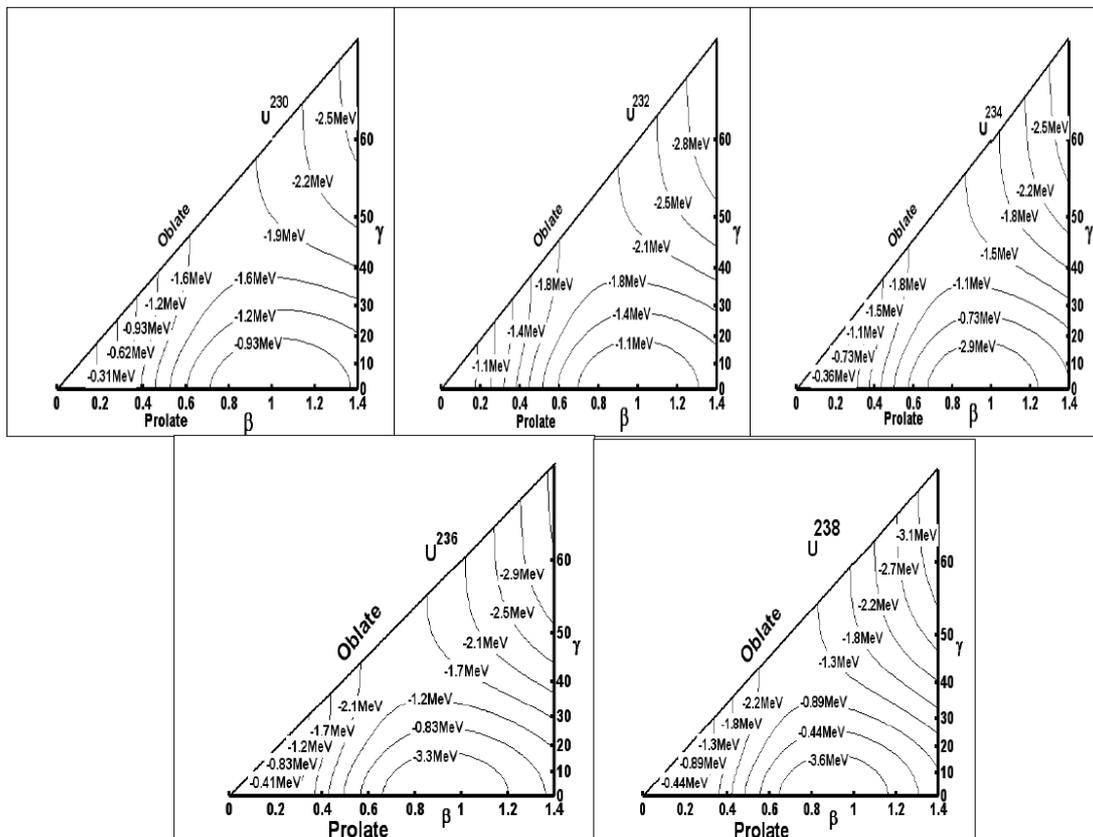


Fig. 2: Contour plot of the potential energy surfaces for $^{230-238}\text{U}$ nuclei.

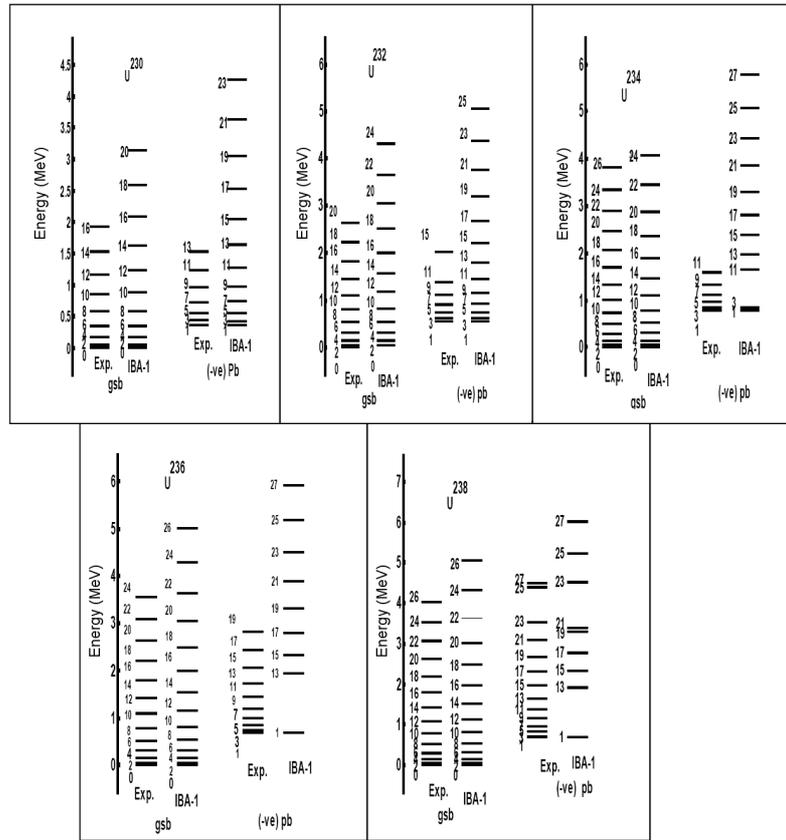


Fig. 3: Comparison between experimental (Exp.) [19–23] and theoretical (IBA-1) energy levels in $^{230-238}\text{U}$.

is due to band crossing and octupole deformation. From γ -bands [24] octupole deformation has been observed at $I = 14$ (for ^{232}U), $I = 10$ (for ^{234}U), $I = 15$ (for ^{236}U) and $I = 10$ (for ^{238}U) respectively.

Unfortunately there is not enough measurements of electromagnetic transition rates $B(E2)$ or $B(E1)$ for these series of nuclei. The only measured $B(E2, 0_1^+ \rightarrow 2_1^+)$'s are presented, in Table 2 for comparison with the calculated values. The parameters $E2SD$ and $E2DD$ used in the present calculations are displayed in Table 1.

The calculated [equations 12, 13] electric quadrupole moment Q_0 and deformation parameter β_2 are given in Table 4. It is clear that both values are increasing with the increase of the neutron number of uranium isotopes.

$$Q_0 = \left[\frac{16\pi B(E2)_{exp.}}{5} \right]^{1/2}, \quad (12)$$

$$\beta_2 = \frac{[B(E2)_{exp.}]^{1/2}}{\frac{3ZR_0^2}{4\pi}} \quad (13)$$

3.3 Electric monopole transitions

The electric monopole transitions, $E0$, are normally occurring between two states of the same spin and parity by trans-

ferring energy and zero unit of angular momentum. The strength of the electric monopole transitions, $X_{if'f}(E0/E2)$, [25] are calculated using equations (14, 15) and presented in Table 5.

$$X_{if'f}(E0/E2) = \frac{B(E0, I_i - I_f)}{B(E2, I_i - I_f)}, \quad (14)$$

$$X_{if'f}(E0/E2) = (2.54 \times 10^9) A^{3/4} \times \frac{E_\gamma^5(\text{MeV})}{\Omega_{KL}} \alpha(E2) \frac{T_e(E0, I_i - I_f)}{T_e(E2, I_i - I_f)}. \quad (15)$$

3.4 Conclusions

The IBA-1 model has been applied successfully to $^{230-238}\text{U}$ isotopes and we have got:

1. The ground state and octupole bands are successfully reproduced;
2. The potential energy surfaces are calculated and show rotational behavior to $^{230-238}\text{U}$ isotopes where they are mainly prolate deformed nuclei;
3. Electromagnetic transition rates $B(E1)$ and $B(E2)$ are calculated;

I_i^+	I_f^+	$I_{f'}^+$	^{230}U	^{232}U	^{234}U	^{226}U	^{238}U
0 ₂	0 ₁	2 ₁	0.660	0.560	0.001	0.920	1.470
0 ₃	0 ₁	2 ₁	13.370	1.300	15.910	0.282	—
0 ₃	0 ₁	2 ₂	2.400	0.410	3.000	1.960	212.500
0 ₃	0 ₁	2 ₃	3.510	0.280	2.420	1.240	0.520
0 ₃	0 ₂	2 ₁	0.620	0.590	0.660	0.500	—
0 ₃	0 ₂	2 ₂	0.110	0.180	0.120	0.001	1.500
0 ₃	0 ₂	2 ₃	0.180	0.130	0.100	0.001	3.720
0 ₄	0 ₁	2 ₂	1.960	7.750	0.001	0.230	7.250
0 ₄	0 ₁	2 ₃	1.320	0.250	—	0.250	0.190
0 ₄	0 ₁	2 ₄	32.660	0.330	1.000	0.170	0.460
0 ₄	0 ₂	2 ₂	—	0.020	0.0000	0.060	3.250
0 ₄	0 ₂	2 ₃	—	0.750	—	0.070	0.080
0 ₄	0 ₂	2 ₄	—	—	0.000	0.100	0.2000
0 ₄	0 ₃	2 ₁	0.330	—	—	24.000	19.000
0 ₄	0 ₃	2 ₂	0.020	0.080	0.110	0.330	4.750
0 ₄	0 ₃	2 ₃	0.010	2.750	—	0.360	0.130
0 ₄	0 ₃	2 ₄	0.330	—	17.000	0.520	0.300

Table 5: Table 5. Theoretical $X_{if'f}$ ($E0/E2$) ratios for $E0$ transitions in Ra isotopes.

4. Electric quadrupole moment Q_0 are calculated;
5. Deformation parameter β_2 are calculated.

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LETTERS TO PROGRESS IN PHYSICS**A Brief Note on “Un-Particle” Physics**

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The possibility of a hidden sector of particle physics that lies beyond the energy range of the Standard Model has been recently advocated by many authors. A bizarre implication of this conjecture is the emergence of a continuous spectrum of massless fields with non-integral scaling dimensions called “un-particles”. The purpose of this Letter is to show that the idea of “un-particles” was considered in at least two previous independent publications, prior to its first claimed disclosure.

The Standard Model (SM) is a highly successful theoretical framework that describes the relationships among all known elementary particles and the attributes of three of the four forces that act on these particles — electromagnetism, the strong force and the weak force. SM covers an energy range upper limited by the weak interaction scale of approx. 300 GeV. Despite the remarkable success of SM, it seems likely that a much deeper understanding of nature will be achieved as physicists continue to probe the fundamental constituents of matter at increasingly higher energies. Both theory and experiments strongly indicate that new phenomena await discovery beyond the SM range and reaching into the Terascale region. The Large Hadron Collider (LHC) at CERN is based on high energy proton beams and is scheduled to begin operation later this year. Moreover, further exploiting the Terascale physics will be possible in the near future with a new accelerator known as the International Linear Collider (ILC). It is believed that running both LHC and ILC will provide clues on how to go about solving many of the open questions challenging the current SM.

The possibility of a yet-unseen sector that lies in the Terascale range and is weakly coupled to SM has been recently advocated by many authors [1–6]. A bizarre implication of this conjecture is the emergence of a continuous spectrum of massless states with non-integral scaling dimensions called “un-particles”. In classical physics, the energy, linear momentum and mass of a free point particle are linked through the relativistic connection ($c = 1$):

$$E^2 = p^2 + m^2. \quad (1)$$

Quantum mechanics converts (1) into a dispersion relation for the corresponding quantum waves, with the mass m fixing the low frequency cut-off ($\hbar = 1$):

$$\omega^2 = k^2 + m^2. \quad (2)$$

Unlike (1) or (2), un-particles are conjectured to emerge as streams of fractional objects, something that has never been either imagined or seen before. A possible signal of

un-particles at either LHC or ILC may show up as “missing” energy in certain decay channels [1–6].

The purpose of this Letter is to set the record straight and point out that the idea of “un-particles”, first claimed in [1, 2], was previously considered elsewhere. To the best of our knowledge, there are at least two publications where a similar or identical concept was introduced and discussed:

1. In 2005, Prof. F. Smarandache has launched the term *un-matter* as part of his novel mathematical framework of Neutrosophy and Fuzzy Logic [7, 8];
2. In 2006, the author has formulated the concept of *fractional number of field quanta* in connection with the development of quantum field theory using complex dynamics [9].

It is unfortunate that neither one of [1–6] have referenced these contributions.

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LETTERS TO PROGRESS IN PHYSICS**International Injustice in Science**

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In the scientific research, it is important to keep our freedom of thinking and not being yoked by others' theories without checking them, no matter where they come from. *Cogito, ergo sum* (I think, therefore I am), said Descartes (1596–1650), and this Latin aphorism became his first principle in philosophy.

Inspired by D. Rabounski [1] and M. Apostol [2] I read more articles about injustices in science (for example [3]) and in arts and letters occurring in contemporary societies. The poet Plautus (254-184 B.C.) had once exclaimed that *homo homini lupus* (man is a wolf for man), so people make problems to people. In this short letter to the editor, I would like to list some inconvenient cases that manifest today:

There exist reviewing and indexing publications and institutes made just for a propagandistic way, and not reviewing all relevant literature on the topics, but reviewing their people and their ideas while ignoring, boycotting, denigrating, or discrediting other people and ideas. They exercise an international traffic of influence by manipulations and falsifications of information (such as biographies, history of events, etc.), discourage people for working on topics different from theirs, and use subversive techniques in their interest of hegemony in science, arts, and letters.

The science, art, and literature of the powerful are like that: If you don't cite them, it is your fault as if you have not read them. However, if they don't cite you, it's your fault too as if you did not deserve to be cited because you have published in so-called by them "obscure publications", even if these people have "borrowed" your idea without acknowledgement. They categorize as "obscure, unimportant, not by establishment" those journals, publishing houses, cultural centers and researchers or creators that do not obey to them or that dare to be independent thinkers, in order that these people with power positions stigmatize them in the public's eye (because they can not control these publications). While the publications and centers of research they control they proclaim as "the best". The science/art & letters establishments continue to ignore or minimize the research and creation done outside the establishment. It became a common procedure that people who control the so-called "high" publications abuse their power and they "take" ideas from less circulated publications and publish them in these "high" publications without citation, as their own ideas!

There are journals using hidden peer-reviewers that delay the publication until someone else from their house get credit for your paper's ideas.

Secret groups and services ignore and even boycott per-

sonalities who are independent in thinking and don't follow the establishment or don't obey to them; they manipulate national and international awards in science, arts, literature, also they manipulate university positions, high research jobs, funding; they try to confiscate the whole planet's thought by making biased so-called "reference sites" (as the self-called "encyclopedias", "dictionaries", "handbooks", etc.) where they slander independent thinkers, while blocking other sites they don't like; that's why the whole human history of science, arts, letters has to be re-written; the search engines bring these "reference sites" amongst the first pages in a search, even they are not the most relevant to the search topic, and since most of the hurry readers browse only the beginning pages [they don't spend time to look at all of them], it is a high probability that the populace is manipulated according to the biased information of these so-called "free" (just because they are not free!) reference sites; these groups try to confiscate the Internet at the global scale; always, during history, there were and unfortunately there still are intentions from some secret groups or services to dominate others... They try to transform other countries in spiritual colonies by brain washing. Secret groups and services do not only politic, economic, or military espionage, but also scientific, artistic, literary manipulations in the profit of their people.

Unfortunately, big cultures continue to destroy small cultures and to delete the collective memory of small nations. History is written by winners, says the aphorism, but this is not correct, history should be written by all parts. International organisms are created who unfortunately only serve the interests of a few powers, not of the whole world.

There are people believing they detain the **absolute truth**, and if somebody dares to have a different opinion from them, he or she is blacklisted, slandered, banned from various publications, etc.

The public opinion is provoked, manipulated through propaganda, publicity, dissemination by those who detain the power or control the mass media and the national and international awards, and these awards have been created in purpose to impose some people and ideologies.

There exist scientific, artistic, literary, or cultural associations/organizations whose hidden goal is to manipulate peo-

ple in their propagandistic interest and indoctrinate them. The literature they start to send (after collecting your membership money!) reflects only their ideas and praise only their people, while ignoring or boycotting others'. *Nolens volens* (unwilling or willing) the "member" of such association becomes their spiritual slave. Consequently, you are yoked to this association's propaganda. Better to be independent and not belonging to any association/organization.

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A Unified Theory of Interaction: Gravitation and Electrodynamics

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A theory is proposed from which the basic equations of gravitation and electromagnetism are derived from a single Lagrangian. The total energy of an atom can be expressed in a power series of the fine structure constant, α . Specific selections of these terms yield the relativistic correction to the Bohr values of the hydrogen spectrum and the Sommerfeld-Dirac equation for the fine structure spectrum of the hydrogen atom. Expressions for the classical electron radius and some of the Large Number Coincidences are derived. A Lorentz-type force equation is derived for both gravitation and electrodynamics. Electron spin is shown to be an effect of fourth order in α .

1 Introduction

In a previous article [2] in this journal we presented a classical Lagrangian characterizing the *dynamics* of gravitational interaction,

$$L = -m_0(c^2 + v^2) \exp R/r, \quad (1)$$

where we denote:

m_0 = *gravitational rest mass* of a test body moving at velocity \mathbf{v} in the vicinity of a massive, central body of mass M ,

γ = $1/\sqrt{1 - v^2/c^2}$,

R = $2GM/c^2$ is the Schwarzschild radius of the central body.

The following conservation equations follow:

$$E = mc^2 e^{R/r} = \text{total energy} = \text{constant}, \quad (2)$$

$$\mathbf{L} = e^{R/r} \mathbf{M} = \text{constant}, \quad (3)$$

$$\begin{aligned} L_z &= M_z e^{R/r} = e^{R/r} m_0 r^2 \sin^2 \theta \dot{\phi} \\ &= z \text{ component of } \mathbf{L} = \text{constant}, \end{aligned} \quad (4)$$

where $m = m_0/\gamma^2$ and

$$\mathbf{M} = (\mathbf{r} \times m_0 \mathbf{v}) \quad (5)$$

is the total angular momentum of the test body.

It was shown that the tests for perihelion precession and the bending of light by a massive body are satisfied by the equations of motion derived from the conservation equations.

The *kinematics* of the system is determined by assuming the local and instantaneous validity of special relativity (SR). This leads to an expression for gravitational redshift:

$$\nu = \nu_0 e^{-R/2r}, \quad (\nu_0 = \text{constant}), \quad (6)$$

which agrees with observation.

Electrodynamics is described by the theory of special relativity. If the motion of a particle is dynamically determined

by the above Lagrangian, then a description of the kinematics of its motion in terms of special relativity should yield equations of motion analogous to those of electrodynamics. This, in principle, should allow the simultaneous manifestation of gravitation and electrodynamics in one model of interaction.

We follow this approach and show, amongst others, that electrical charge arises from a mathematical necessity for bound motion. Other expressions, such as the classical electron radius and expressions of the Large Number Hypothesis follow.

The total energy for the hydrogen atom can be expressed in terms of a power series of the fine structure constant, α . Summing the first four terms yields the Sommerfeld-Dirac expression for the total energy. For higher order terms the finite radius of the nucleus must be taken into account. This introduces a factor analogous to “electron spin”.

Details of all calculations are given in the PhD thesis of the author [1].

2 Gravitation and Special Relativity

Einstein’s title of his 1905 paper, *Zur Elektrodynamik bewegter Körper* indicates that electrodynamics and SR are interrelated, with SR giving an explanation for certain properties of electrodynamics. Red-shift is such a property, combining both gravitation and electromagnetism in a single formulation, and should provide us with a dynamical link between these two phenomena. To do this, we substitute the photoelectric effect,

$$h\nu = \tilde{m}c^2, \quad (7)$$

where $\tilde{m} = \gamma\tilde{m}_0$ and \tilde{m}_0 is the *electromagnetic rest mass* of a particle, into (6). This gives

$$\left. \begin{aligned} E &= \tilde{m}c^2 e^{R/2r} = \tilde{m}_0 c^2 \frac{e^{R/2r}}{\sqrt{1 - v^2/c^2}} = \tilde{E} e^{R/2r} \\ &= \tilde{m}_0 c^2 + \tilde{m}_0 v^2/2 + \tilde{m}_0 R c^2/2r + \\ &\quad + \tilde{m}_0 R v^2/4r + \dots \end{aligned} \right\}, \quad (8)$$

where E is another constant of energy and $\tilde{E} = \tilde{m}c^2$ is the total energy of the theory of special relativity.

Let us compare this expansion with the expansion of (2) for the gravitational energy,

$$\frac{m_0c^2 - E}{2} = \frac{m_0v^2}{2} - \frac{GMm_0}{r} + \frac{m_0v^2R}{2r} - \frac{m_0c^2R^2}{4r^2} + \frac{m_0v^2R^2}{4r^2} + \dots \quad (9)$$

The negative sign of the second right hand term in (9) ensures attractive, or bound, motion under gravitation. In order for the motion determined by (8) to be bounded, the third right hand term must similarly be negative and inversely proportional to r . To ensure this we let

$$\tilde{m}_0c^2 = -e^2/r_e, \quad (10)$$

where e^2 is an arbitrary constant and

$$r_e = R/2. \quad (11)$$

Eq.(8) can then be rewritten as

$$E = \tilde{m}c^2 e^{r_e/r}. \quad (12)$$

As we shall see for the hydrogen atom, e represents the electron charge, r_e represents the classical electron radius and (11) yields some of the numbers of Dirac's Large Number Hypothesis.

The choice of a positive sign in (10) gives repulsive motion. Such a freedom of choice is not possible for the gravitational energy of (9).

2.1 Hamiltonian formulation

Confirmation of the above conclusions can be found by examining the predictions for the hydrogen spectrum. We follow a classical approach based on the principles of action variables [3].

Using the identity $\gamma^2 = 1 + \gamma^2 v^2/c^2$ to separate the kinetic and potential energies in (8), a corresponding Lagrangian can be found:

$$L = -\tilde{m}_0c^2 \sqrt{1 - v^2/c^2} \exp(r_e/r). \quad (13)$$

We obtain the conjugate momenta:

$$p_r = \tilde{m}v \exp(r_e/r), \quad (14)$$

$$p_\theta = \tilde{m}r^2\dot{\theta} \exp(r_e/r), \quad (15)$$

$$p_\phi = \tilde{m}r^2 \sin^2\theta \dot{\phi} \exp(r_e/r). \quad (16)$$

The associated Hamiltonian can be derived from the formula $H = \sum \dot{q}_i p_i - L$ as follows

$$H = \left[\tilde{m}_0^2 c^4 \exp(r_e/r) + c^2 (p_r^2 + p_\theta^2/r^2 + p_\phi^2/r^2 \sin^2\theta) \right]^{1/2}. \quad (17)$$

From the canonical equations

$$\dot{p}_i = \frac{\partial H}{\partial q_i}, \quad (18)$$

we find the following conservation equations:

$$L^2 \equiv M^2 \exp(2r_e/r) = p_\theta^2 + p_\phi^2/\sin^2\theta, \quad (19)$$

$$L_z \equiv M_z \exp(r_e/r) = p_\phi, \quad (20)$$

where L^2 and L_z are constants and

$$\mathbf{M} = (\mathbf{r} \times \tilde{m}\mathbf{v}), \quad (21)$$

is the total angular momentum of the orbiting particle.

It should be noted that (12), (19), (20) and (21) have respectively the same forms as for the gravitational equations (2), (3), (4), (5), but with $m = m_0/\gamma^2$ replaced by $\tilde{m} = \gamma\tilde{m}_0$ and R by $r_e = R/2$.

3 The hydrogen spectrum

In order to determine an expression for the energy levels of the H-atom, two different approaches can be followed: (i) Analogously to the Wilson-Sommerfeld model, one can apply the procedures of action angle variables, or (ii) perturbation theory, where the contribution of each energy term is evaluated separately.

To generalize our discussion we shall, where appropriate, use a general potential $\Phi = Rc^2/2r = r_e c^2/r$.

3.1 Method of action angle variables

The theory of action angle variables originated in the description of periodic motion in planetary mechanics [4, Ch.9]. From that theory Wilson and Sommerfeld postulated the quantum condition:

For any physical system in which the coordinates are periodic functions of time, there exists a quantum condition for each coordinate. These quantum conditions are

$$J_i = \oint p_i dq_i = n_i h, \quad (22)$$

where q_i is one of the coordinates, p_i is the momentum associated with that coordinate, n_i is a quantum number which takes on integral values, and the integral is taken over one period of the coordinate q_i .

Applying these quantization rules to the conjugate momenta of (14), (15) and (16) gives [3]

$$L_z = M_z \exp(r_e/r) = n_\phi \hbar, \quad (23)$$

$$L = M \exp(r_e/r) = (n_\theta + n_\phi) \hbar = k \hbar, \quad (24)$$

$$\oint [E^2/c^2 - \tilde{m}_0^2 c^2 \exp(r_e/r) - k^2 \hbar^2/r^2]^{1/2} dr = n_r \hbar, \quad (25)$$

where n_θ, n_ϕ, k and n_r have the values 0, 1, 2, ...

To determine the atomic spectrum we need to evaluate the integral of (25). Because of the finite radius of the nucleus we choose an arbitrary effective nuclear radius of gr_e . The potential term in the exponentials is then written as

$$\exp\left(\frac{2\Phi}{c^2}\right) = \exp\left(\frac{2r_e}{r - gr_e}\right), \quad (26)$$

so that

$$\begin{aligned} \exp(2\Phi/c^2) &= \\ &= 1 + 2\frac{r_e}{r} + 2\frac{r_e^2}{r^2}(g+1) + 3\frac{r_e^3}{r^3}g(g+1) + \dots \end{aligned} \quad (27)$$

For convenience we also define a parameter f such that

$$f = 2(g+1). \quad (28)$$

We shall subsequently see that the value of g , or f , is related to the concept of electron spin.

Approximating (27) to second order in r_e/r , substituting this approximation in (25) and integrating gives

$$E_m^2 = 1 - \frac{\alpha^2}{\left[n - k + \sqrt{k^2 + f\alpha^2}\right]^2}, \quad (29)$$

where $E_m = E/\tilde{m}_0c^2$, $n = n_r + k$ and $\alpha = e^2/\hbar c$ is the fine structure constant. This expression is simplified by expanding to fourth order in α :

$$E_m \cong 1 - \frac{\alpha^2}{2n^2} \left[1 + \frac{\alpha^2}{n} \left(\frac{1}{4n} - \frac{f}{k} \right) \right]. \quad (30)$$

The corresponding Sommerfeld/Dirac expressions are respectively

$$E_m^2 = \left(1 + \frac{\alpha^2}{\left[n - k + \sqrt{k^2 - \alpha^2}\right]^2} \right)^{-1} \quad (31)$$

and

$$E_m \cong 1 - \frac{\alpha^2}{2n^2} \left[1 + \frac{\alpha^2}{n} \left(\frac{1}{k} - \frac{3}{4n} \right) \right], \quad (32)$$

where $k = j + \frac{1}{2}$ for the Dirac expression, and $j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots, \frac{(n-1)}{2}$.

The difference between the energy given by our model E_W , as given by (30), and that of the Sommerfeld-Dirac model, E_D , as given by (32), is

$$(E_D - E_W)/\tilde{m}_0c^2 = \frac{\alpha^4}{2n^3} \left[\frac{1}{k}(f+1) - \frac{1}{n} \right]. \quad (33)$$

We shall show below that this difference corresponds to the energy associated with the "spin-orbit" interaction of our model.

4 Perturbation method

We use this method as applied by Born and others [3, Ch. 4].

To apply the perturbation method we need to express the energy \tilde{E} in terms of the momentum:

$$E = (p^2c^2 + \tilde{m}_0^2c^4)^{1/2} \exp(\Phi/c^2), \quad (34)$$

where $\mathbf{p} = \tilde{m}\mathbf{v}$. Again, taking the finite radius of the nucleus into account, we choose for the potential,

$$\exp(\Phi/c^2) = \exp(r_e/(r - gr_e)), \quad (35)$$

so that the potential term can be written as

$$\exp(\Phi/c^2) = 1 + \frac{r_e}{r} + w\frac{r_e^2}{r^2} + \left(w^2 - \frac{1}{4}\right)\frac{r_e^3}{r^3} + \dots, \quad (36)$$

where

$$w = (g + 1/2) = (f - 1)/2. \quad (37)$$

With this form for the potential, and using $\tilde{m}_0c^2r_e = -e^2$, (34) can be expanded as

$$\begin{aligned} E &= \underbrace{\tilde{m}_0c^2}_{E_0} + \underbrace{\frac{p^2}{2\tilde{m}_0}}_{E_1} - \underbrace{\frac{e^2}{r}}_{E_2} - \underbrace{\frac{p^4}{8\tilde{m}_0^3c^2}}_{E_3} + \underbrace{\frac{p^2r_e}{2\tilde{m}_0r}}_{E_3} \\ &+ \underbrace{w\frac{r_e^2\tilde{m}_0c^2}{r^2}}_{E_4} + \underbrace{w\frac{p^2r_e^2}{2\tilde{m}_0r^2}}_{E_5} - \underbrace{\frac{p^4r_e}{8\tilde{m}_0^3c^2r}}_{E_6} + \\ &+ \underbrace{\tilde{m}_0c^2\left(w^2 - \frac{1}{4}\right)\frac{r_e^3}{r^3}}_{E_7} + \dots \end{aligned} \quad (38)$$

Applying the unperturbed Bohr theory to each braced term, we find the following quantized expressions:

4.1 E_0 : rest mass energy

The first term on the right is the rest mass energy, which we denote by E_0 :

$$E_0 = \tilde{m}_0c^2. \quad (39)$$

4.2 E_1 : Bohr energy

The next two terms represent the unperturbed Coulomb energy of the hydrogen atom, which we indicate by E_1 :

$$E_1 = p^2/2\tilde{m}_0 - e^2/r. \quad (40)$$

According to the method of the Bohr theory,

$$E_1 = -R_e/n^2, \quad n = 1, 2, \dots \quad (41)$$

where

$$\left. \begin{aligned} R_e &= R_y\hbar c = e^2/2a_0 = \alpha^2\tilde{m}_0c^2/2 \\ a_0 &= \text{Bohr radius} = \hbar^2/\tilde{m}_0e^2 \\ R_y &= \text{Rydberg constant} = 2\pi^2e^4\tilde{m}_0/ch^3 = \alpha/4\pi a_0 \end{aligned} \right\} \quad (42)$$

4.3 E_2 : relativistic correction

The third term is denoted by E_2 . It can be shown that [1]

$$E_2 = -p^4/8\tilde{m}_0^3c^2, \quad (43)$$

$$= -\frac{\alpha^2 R_e}{n^3} \left[\frac{1}{k} - \frac{3}{4n} \right]. \quad (44)$$

This is the ‘‘relativistic correction’’ of the Bohr-Sommerfeld model [3, §33]. This energy term is similar to that contained in the Dirac expression of (32). The sum of E_0 , E_1 and E_2 gives an expression identical to that of Sommerfeld and similar to that of Dirac.

It is well-known that Sommerfeld’s result was fortuitous as the effect of spin-orbit coupling was ignored in his model. This effect is incorporated in the Dirac model. In our model we shall see below that E_3 is an orbit-interaction term and that E_4 is related to ‘electron spin’. These two terms, missing in the Sommerfeld model, can now be added to $E_0 + E_1 + E_2$ of the Sommerfeld energy expression.

4.4 E_3 : orbital magnetic energy

We denote the fourth term by E_3 :

$$E_3 = p^2 r_e / 2\tilde{m}_0 r. \quad (45)$$

Applying the unperturbed Bohr theory, we find from (40):

$$\begin{aligned} E_3 &= (E_1 + e^2/r)r_e/r \\ &= r_e(E_1/r + e^2/r^2). \end{aligned} \quad (46)$$

Using (41) and the average values [3, p144],

$$\overline{1/r} = 1/n^2 a_0, \quad (47)$$

$$\overline{1/r^2} = 1/a_0^2 n^3 k, \quad k = 1, 2, \dots, n \quad (48)$$

as well as

$$r_e/a_0 = a^2, \quad (49)$$

we get

$$E_3 = \frac{\alpha^2 R_e}{n^3} \left(\frac{2}{k} - \frac{1}{n} \right) = \frac{\alpha^4 \tilde{m}_0 c^2}{2n^3} \left(\frac{2}{k} - \frac{1}{n} \right). \quad (50)$$

The physical interpretation of E_3 is that it is the energy due to the magnetic interaction of an electron moving in orbit about a proton. This can be seen as follows.

Substituting $\mathbf{p} = \tilde{m}\mathbf{v}$ and $r_e = -e^2/\tilde{m}_0 c^2$ into (45) gives

$$\begin{aligned} E_3 &= -\frac{e^2 v^2}{2rc^2} \left(\frac{\tilde{m}}{\tilde{m}_0} \right)^2 \\ &\approx -\frac{e^2 v^2}{2rc^2} \quad \text{in the non-relativistic limit.} \end{aligned} \quad (51)$$

It corresponds to the classical form of the magnetic energy due to orbital motion, as given by (70) below:

4.5 E_4 : ‘‘electron spin’’

$$\begin{aligned} E_4 &= w r_e^2 \tilde{m}_0 c^2 / r^2, \\ &= w e^4 / \tilde{m}_0 c^2 r^2. \end{aligned} \quad (52)$$

Applying (48) gives

$$E_4 = \frac{w 2\alpha^2 R_e}{n^3 k} = w \alpha^4 \tilde{m}_0 c^2 \frac{1}{n^3 k}. \quad (53)$$

We consider the significance of the factor w . We note that the potential energy expression (36) can be truncated after the quadratic term in r_e/r by letting $w^2 - \frac{1}{4} = 0$. As such, truncation can be considered as the limit to the resolution of the apparatus used for spectral observation. With this condition, we find that

$$w = \pm \frac{1}{2} \quad (54)$$

gives the spectrum due to all interactions up to second degree in r/r_e . Therefore, from (42) and (53):

$$E_4 = \pm \frac{1}{2} \frac{e^8 \tilde{m}_0}{\hbar^4 c^2} \frac{1}{n^3 k}. \quad (55)$$

The above expression for E_4 corresponds to the quantum mechanical result for the energy due to electron spin. Except for the quantum numbers, Eisberg and Resnick [6, Example 8–3] find a similar result for the energy due to spin-orbit interaction.

The equivalence of (55) to the result of Eisberg and Resnick also confirms the implicit value $g_s = 2$ in E_4 .

In this study E_4 corresponds to the energy due to quantum mechanical spin only. Combining E_3 and E_4 gives the corresponding total spin-orbit energy.

For $k = 1$ the expression for E_4 is equal to the Darwin term of the Dirac theory. In the Dirac theory the Darwin term has to be introduced separately for $\ell = 0$ states, whereas in our model E_4 already provides for $\ell = 0$ through the degeneracy ($\ell = 0, 1$) associated with the $k = 1$ level.

In summary, ‘electron spin’ represents a second order contribution r_e^2/r^2 to the total energy of the atom.

The above reasoning also applies to higher orders of approximation. Expanding (35) to fourth degree in r_e/r gives:

$$\begin{aligned} \exp(\Phi/c^2) &= 1 + \frac{r_e}{r} + \frac{r_e^2}{r^2} w + \frac{r_e^3}{r^3} (w^2 - \frac{1}{2}) + \\ &+ \frac{r_e^4}{r^4} \left(w^2 - \frac{1}{4} \right) w + \dots \end{aligned} \quad (56)$$

The coefficient of r_e^4/r^4 is zero if $(w^2 - \frac{1}{4})w = 0$, or

$$w = \frac{1}{2}, -\frac{1}{2}, 0. \quad (57)$$

A next higher resolution to r_e^3/r^3 therefore introduces an additional value of $w = 0$, giving a triplet symmetrical about this value.

For a comprehensive survey of the conceptual developments surrounding electron spin we refer to the text by Tomonaga [7].

4.6 E_5 : radiative reaction

$$E_5 = w \frac{p^2 r_e^2}{2\tilde{m}_0 r^2} \quad (58)$$

$$= \pm \frac{1}{2} \alpha^4 R_e \left[\frac{1}{n^5 k} - \frac{2}{n^3 k^3} \right]. \quad (59)$$

Substituting $\mathbf{p} = \tilde{m}\mathbf{v}$ in (58) gives

$$E_5 = \pm \frac{1}{2} \frac{v^2 e^4}{2\tilde{m}_0 c^4 r^2} \left(\frac{\tilde{m}}{\tilde{m}_0} \right)^2. \quad (60)$$

In the non-relativistic limit, $\tilde{m} \approx \tilde{m}_0$, the above term corresponds to the last RHS term of (69), i.e. the classical energy resulting from radiative reaction. Its value is too small ($\sim 10^{-8}$ eV) to affect the values of the fine-spectrum.

4.7 Summary

$$\begin{aligned} E_0 &= m_0 c^2 & : & \text{rest mass energy,} \\ E_1 &= -\frac{R_e}{n^2} & : & \text{Bohr energy,} \\ E_2 &= -\frac{\alpha^2 R_e}{n^3} \left[\frac{1}{k} - \frac{3}{4n} \right] & : & \text{relativistic correction,} \\ E_3 &= \frac{\alpha^2 R_e}{n^3} \left[\frac{2}{k} - \frac{1}{n} \right] & : & \text{orbital magnetic energy,} \\ E_4 &= w \frac{2\alpha^2 R_e}{n^3 k} & : & \text{electron spin energy,} \\ E_5 &= w \alpha^4 R_e \left[\frac{1}{n^5 k} - \frac{2}{n^3 k^3} \right] & : & \text{Radiative reaction,} \end{aligned}$$

where $w = \pm \frac{1}{2}$.

The sum of the energy terms $\sum E_i = E_0 + E_1 + E_2 + E_3 + E_4 + E_5$ is:

$$\sum E_i / \tilde{m}_0 c^2 = 1 - \frac{\alpha^2}{2n^2} \left[1 - \frac{\alpha^2}{n} \left(\frac{f}{k} - \frac{1}{4n} \right) \right], \quad (61)$$

which, as expected, is the same as (30).

Each term in (38) can be related to a standard electrodynamic effect. It is significant that although (38) does not explicitly contain any vector quantities, such as the vector potential \mathbf{A} , this potential is implicit, as shown in the discussion of E_3 and the comparison with (69).

An explanation for the difference (33) between the spectrum of the proposed model and that of Dirac-Sommerfeld can be seen as follows:

Consider the sum

$$E_3 + E_4 = \frac{\alpha^2 R_e}{n^3} \left[\frac{2}{k} (w + 1) - \frac{1}{n} \right] \quad (62)$$

or, since $w = (f - 1)/2$,

$$E_3 + E_4 = \frac{\alpha^2 R_e}{n^3} \left[\frac{1}{k} (f + 1) - \frac{1}{n} \right]. \quad (63)$$

The above equation corresponds to (33), the difference between the Sommerfeld-Dirac expression and that of our model. The expression (30) therefore already incorporates the spin-orbit interaction.

The energy $E_3 + E_4$ therefore represents a perturbation to the Sommerfeld-Dirac values. The only candidate for this perturbation is the Lamb-shift. For the $(n, k) = (2, 1)$ level and for $w = -0.5$ the value of $E_3 + E_4$ is $4.5283178 e-5$ eV. The Lamb-shift for this level is $4.3738019 e-6$ eV, which is an order 10 smaller. It would be overly ambitious to find the observed Lamb-shift from the present simple model. At this degree of spectral resolution one would have to look at a modification of the effective nuclear radius to $r - a_1 r_e - a_2 r_e^2 - \dots$

4.8 Comparison with classical electromagnetic energy

In order to compare the results of this study with those of conventional electromagnetic theory, we give a brief summary of the energy relations of classical electrodynamic theory.

The Hamiltonian describing the interaction of an electron with fields \mathbf{H} and \mathbf{E} is given by [8, p. 124]

$$H_{\text{classical}} = e\Phi + \left(\mathbf{p} - \frac{e}{c}\mathbf{A} \right)^2 / 2\tilde{m}, \quad (64)$$

where Φ and \mathbf{A} are respectively the electrostatic and vector potentials of the system.

It is important to note that \mathbf{A} and Φ do not merely represent the external fields in which the particle moves, but also the particle's own fields. This implies that the force of radiative reaction is automatically included.

The corresponding classical Lagrangian is

$$L_{\text{classical}} = \frac{p^2}{2\tilde{m}} - e\Phi + \frac{e}{c}\mathbf{A} \cdot \mathbf{v}. \quad (65)$$

For an electron moving under the influence of a magnetic field,

$$\mathbf{H} = e(\mathbf{v} \times \mathbf{r})/cr^3, \quad (66)$$

a vector potential \mathbf{A} can be found as

$$\mathbf{A} = \frac{1}{2}(\mathbf{H} \times \mathbf{r}) = e\mathbf{v}/2cr. \quad (67)$$

Substituting this expression for \mathbf{A} and using $\mathbf{p} = \tilde{m}\mathbf{v}$, yields

$$\left(\mathbf{p} - \frac{e}{c}\mathbf{A} \right)^2 = p^2 - \frac{e^2 v^2 \tilde{m}}{c^2 r} + \frac{e^4 v^2}{4c^4 r^2}. \quad (68)$$

Since the Hamiltonian of (64) does not contain t explicitly, we may equate it to the total energy. Consequently, substituting (68), and $e\Phi = -e^2/r$, in (64) gives the classical

energy

$$E_{\text{classical}} = -\frac{e^2}{r} + \frac{p^2}{2\tilde{m}} - \frac{e^2 v^2}{2c^2 r} + \frac{e^4 v^2}{8\tilde{m}c^4 r^2}. \quad (69)$$

The third RHS term is the magnetic energy due to the orbital motion of the electron:

$$E_{\text{orbital}} = \boldsymbol{\mu}_\ell \cdot \mathbf{H} = -\frac{g_\ell e^2 v^2}{2rc^2}, \quad (70)$$

where $\boldsymbol{\mu}_\ell$ = magnetic moment, g_ℓ = Landé g factor = 1, and \mathbf{M} and \mathbf{H} are parallel to one another. This energy corresponds to that of E_3 above.

The fourth RHS term of (69) represents radiative reaction, which corresponds to our E_5 as given by (60).

The standard relativistic Hamiltonian is given by:

$$H_{\text{relativistic}} = [(\mathbf{p} - q\mathbf{A}/c)^2 c^2 + \tilde{m}_0^2 c^4]^{\frac{1}{2}} + q\Phi. \quad (71)$$

The Hamiltonians of (64) and (71) must be compared to ours of (17).

It is well-known that the Bohr model for the atom fails because of radiative reaction; in our model this loss is compensated for by the additional and associated potential term, E_4 . This term can also be interpreted as a modification of Coulomb's law. It is significant that this energy term can also be interpreted as arising from electron spin.

It is also significant that the Sommerfeld relativistic correction term, E_2 , does not appear in either (69) or (71).

We can consider the electromagnetic energy arising from the Hamiltonians of (64) and (71) as approximations to that of our Hamiltonian of (17).

We also note that the energy derived from the Hamiltonian of (64), which is normally derived from a Lagrangian containing the vector potential \mathbf{A} , appears as an approximation to our model, which does not explicitly contain a vector potential. A vector potential arises in our theory because of the variation of mass according to (12).

5 The large number coincidences

Dirac postulated that the large dimensionless ratios ($\sim 10^{40}$) of certain universal constants underlie a fundamental relationship between them. A theoretical explanation for these ratios has not yet been found, but it became known as Dirac's Large Number Hypothesis (LNH). [9] Some of these relations are derivable from (11).

Taking R as the Schwarzschild radius of the proton, $R_p = 2GM_p/c^2$, we rewrite (11) as

$$\begin{aligned} -\frac{e^2}{\tilde{m}_0 c^2} &= \frac{GM_p}{c^2} \\ \text{or } -\frac{e^2}{GM_p \tilde{m}_0} &= 1. \end{aligned} \quad (72)$$

Defining the relationship between the *gravitational mass* M_p and the *electromagnetic rest mass* \tilde{m}_{0p} of the proton as

$$M_p = N_D \tilde{m}_{0p}, \quad (73)$$

where N_D is a dimensionless number, we can write (72) as

$$-\frac{e^2}{G \tilde{M}_{0p} \tilde{m}_0} = N_D, \quad (74)$$

which, if the absolute value is taken, is the basic relationship of the LNH.

6 Lorentz force

The force equation for a particle, mass \tilde{m} and velocity \mathbf{v} is found by applying the Euler-Lagrange equations to (13). This gives

$$\dot{\mathbf{p}} = \hat{\mathbf{r}} \frac{\tilde{m} r_e c^2}{r^2} + \frac{\tilde{m} r_e}{r^3} \mathbf{v} \times (\mathbf{v} \times \mathbf{r}). \quad (75)$$

Defining

$$\mathbf{E} = \hat{\mathbf{r}} \frac{r_e c^2}{r^2}, \quad (76)$$

$$\mathbf{H} = \frac{r_e \mathbf{v} \times \mathbf{r}}{r^3}, \quad (77)$$

we can write (75) as

$$\text{Electromagnetic } \dot{\mathbf{p}} = \tilde{m} [\mathbf{E} + \mathbf{v} \times \mathbf{H}]. \quad (78)$$

For $v \ll c$, $\tilde{m} r_e c^2 \rightarrow \tilde{m}_0 r_e c^2 = e^2$ and then (75) approaches the classical Lorentz form.

7 Unifying gravitation and electromagnetism

Equation (16) of reference [1] can be combined with (78) in one formulation:

$$\dot{\mathbf{p}} = \tilde{m} [k\mathbf{E} + \mathbf{v} \times \mathbf{H}], \quad (79)$$

where for

$$\begin{aligned} \text{Gravitation} &: k = -1, \\ \text{Electromagnetism} &: k = 1. \end{aligned}$$

The same equation gives either planetary or atomic motion, where the vectors \mathbf{E} and \mathbf{H} are respectively given by

$$\mathbf{E} = \hat{\mathbf{r}} \frac{GM}{r^2} = \hat{\mathbf{r}} \frac{r_e c^2}{r^2}, \quad (80)$$

$$\mathbf{H} = \frac{GM(\mathbf{v} \times \mathbf{r})}{c^2 r^3} = \frac{r_e \mathbf{v} \times \mathbf{r}}{r^3}. \quad (81)$$

8 Summary

<u>Gravitation</u>	<u>Electromagnetism</u>
$R = 2GM/c^2$	$r_e = R/2$
m_0	$\tilde{m}_0 = m_0/N$
$L = -m_0(c^2 + v^2)e^{R/r}$	$L = -(\tilde{m}_0 c^2 / \gamma) e^{r_e/r}$
$E = mc^2 e^{R/r}$	$E = \tilde{m} c^2 e^{r_e/r}$
$m = m_0 / \gamma^2$	$\tilde{m} = \gamma \tilde{m}_0$
$L^2 = M^2 e^{2R/r} = \text{constant}$	$L^2 = M^2 e^{2r_e/r} = \text{constant}$
$L_z = M_z e^{R/r} = \text{constant}$	$L_z = M_z e^{r_e/r} = \text{constant}$
$\mathbf{M} = (\mathbf{r} \times m_0 \mathbf{v})$	$\mathbf{M} = (\mathbf{r} \times \tilde{m}_0 \mathbf{v})$
$\dot{\mathbf{p}} = m\mathbf{E} + m_0 \mathbf{v} \times \mathbf{H}$	$\dot{\mathbf{p}} = \tilde{m}[\mathbf{E} + \mathbf{v} \times \mathbf{H}]$
$\mathbf{p} = m_0 \mathbf{v}$	$\mathbf{p} = \tilde{m} \mathbf{v}$
$\mathbf{E} = -\hat{\mathbf{r}} GM/r^2$	$\mathbf{E} = \hat{\mathbf{r}} r_e c^2 / r^2$
$\mathbf{H} = GM(\mathbf{v} \times \mathbf{r})/r^3 c^2$	$\mathbf{H} = r_e(\mathbf{v} \times \mathbf{r})/r^3$

9 Nuclear force

In a subsequent article we shall show that equations for the nuclear force, such as the Yukawa potential, can be derived by considering the forms of both the energy equations (2) and (8) at $r \approx R/2 = r_e$.

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On Emergent Physics, “Unparticles” and Exotic “Unmatter” States

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Emergent physics refers to the formation and evolution of collective patterns in systems that are nonlinear and out-of-equilibrium. This type of large-scale behavior often develops as a result of simple interactions at the component level and involves a dynamic interplay between order and randomness. On account of its universality, there are credible hints that emergence may play a leading role in the Tera-ElectronVolt (TeV) sector of particle physics. Following this path, we examine the possibility of hypothetical high-energy states that have fractional number of quanta per state and consist of arbitrary mixtures of particles and antiparticles. These states are similar to “un-particles”, massless fields of non-integral scaling dimensions that were recently conjectured to emerge in the TeV sector of particle physics. They are also linked to “unmatter”, exotic clusters of matter and antimatter introduced few years ago in the context of Neutrosophy.

1 Introduction

Quantum Field Theory (QFT) is a framework whose methods and ideas have found numerous applications in various domains, from particle physics and condensed matter to cosmology, statistical physics and critical phenomena [1, 2]. As successful synthesis of Quantum Mechanics and Special Relativity, QFT represents a collection of *equilibrium* field theories and forms the foundation for the Standard Model (SM), a body of knowledge that describes the behavior of all known particles and their interactions, except gravity. Many broken symmetries in QFT, such as violation of parity and CP invariance, are linked to either the electroweak interaction or the physics beyond SM [3–5]. This observation suggests that unitary evolution postulated by QFT no longer holds near or above the energy scale of electroweak interaction ($\approx 300\text{GeV}$) [6,7]. It also suggests that progress on the theoretical front requires a framework that can properly handle *non-unitary evolution* of phenomena beyond SM. We believe that fractional dynamics naturally fits this description. It operates with derivatives of non-integer order called *fractal operators* and is suitable for analyzing many complex processes with long-range interactions [6–9]. Building on the current understanding of fractal operators, we take the dimensional parameter of the regularization program $\varepsilon = 4 - d$ to represent the order of fractional differentiation in physical space-time (alternatively, $\varepsilon = 1 - d$ in one-dimensional space) [10, 11]. It can be shown that ε is related to the reciprocal of the cutoff scale $\varepsilon \approx (\mu_0/\Lambda)$, where μ_0 stands for a finite and arbitrary reference mass and Λ is the cutoff energy scale. Under these circumstances, ε may be thought as an infinitesimal parameter that can be continuously tuned and drives the departure from equilibrium. The approach to scale invariance demands that the choice of this parameter is completely arbitrary, as

long as $\varepsilon \ll 1$. Full scale invariance and equilibrium field theory are asymptotically recovered in the limit of physical space-time ($d = 4$) as $\varepsilon \rightarrow 0$ or $\Lambda \rightarrow \infty$ [11, 12].

2 Definitions

We use below the Riemann-Liouville definition for the one-dimensional left and right fractal operators [13]. Consider for simplicity a space-independent scalar field $\varphi(t)$. Taking the time coordinate to be the representative variable, one writes

$${}_0D_L^\alpha \varphi(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t (t-\tau)^{-\alpha} \varphi(\tau) d\tau, \quad (1)$$

$${}_0D_R^\alpha \varphi(t) = \frac{1}{\Gamma(1-\alpha)} \left(-\frac{d}{dt}\right) \int_t^0 (\tau-t)^{-\alpha} \varphi(\tau) d\tau. \quad (2)$$

Here, fractional dimension $0 < \alpha < 1$ denotes the order of fractional differentiation. In general, it can be shown that α is linearly dependent on the dimensionality of the space-time support [8]. By definition, α assumes a continuous spectrum of values on fractal supports [11].

3 Fractional dynamics and ‘unparticle’ physics

The classical Lagrangian for the free scalar field theory in 3+1 dimensions reads [1–2, 14]

$$L = \partial^\mu \varphi \partial_\mu \varphi - m^2 \varphi^2, \quad (3)$$

and yields the following expression for the field momentum

$$\pi = \frac{\partial L}{\partial \left(\frac{\partial \varphi}{\partial t}\right)} = \frac{\partial \varphi}{\partial t}. \quad (4)$$

It is known that the standard technique of canonical quantization promotes a classical field theory to a quantum field theory by converting the field and momentum variables into operators. To gain full physical insight with minimal complications in formalism, we work below in 0+1 dimensions. Ignoring the left/right labels for the time being, we define the field and momentum operators as

$$\varphi \rightarrow \hat{\varphi} = \varphi, \quad (5)$$

$$\pi \rightarrow \hat{\pi}^\alpha = -i \frac{\partial^\alpha}{\partial |\varphi|^\alpha} \equiv -i D^\alpha. \quad (6)$$

Without the loss of generality, we set $m = 1$ in (3). The Hamiltonian becomes

$$H \rightarrow \hat{H}^\alpha = -\frac{1}{2} D^{2\alpha} + \frac{1}{2} \varphi^2 = \frac{1}{2} (\hat{\pi}^{2\alpha} + \varphi^2). \quad (7)$$

By analogy with the standard treatment of harmonic oscillator in quantum mechanics, it is convenient to work with the destruction and creation operators defined through [1–2, 14]

$$\hat{a}^\alpha \doteq \frac{1}{\sqrt{2}} [\hat{\varphi} + i \hat{\pi}^\alpha], \quad (8)$$

$$\hat{a}^{+\alpha} \doteq \frac{1}{\sqrt{2}} [\hat{\varphi} - i \hat{\pi}^\alpha]. \quad (9)$$

Straightforward algebra shows that these operators satisfy the following commutation rules

$$[\hat{a}, \hat{a}] = [\hat{a}^{+\alpha}, \hat{a}^{+\alpha}] = 0, \quad (10)$$

$$[\hat{a}^{+\alpha}, \hat{a}^\alpha] = i [\hat{\varphi}, \hat{\pi}^\alpha] = -\alpha \hat{\pi}^{(\alpha-1)}. \quad (11)$$

The second relation of these leads to

$$\hat{H}^\alpha = \hat{a}^{+\alpha} \hat{a}^\alpha + \frac{1}{2} \alpha \hat{\pi}^{(\alpha-1)}. \quad (12)$$

In the limit $\alpha = 1$ we recover the quantum mechanics of the harmonic oscillator, namely

$$\hat{H} = \hat{a}^+ \hat{a} + \frac{1}{2}. \quad (13)$$

It was shown in [6] that the fractional Hamiltonian (12) leads to a continuous spectrum of states having non-integer numbers of quanta per state. These unusual flavors of particles and antiparticles emerging as fractional objects were named “*complexons*”. Similar conclusions have recently surfaced in a number of papers where the possibility of a scale-invariant “hidden” sector of particle physics extending beyond SM has been investigated. A direct consequence of this setting is a continuous spectrum of massless fields having non-integral scaling dimensions called “un-particles”. The reader is directed to [15–21] for an in-depth discussion of “un-particle” physics.

4 Mixing properties of fractal operators

Left and *right* fractal operators (L/R) are natural analogues of *chiral* components associated with the structure of quantum fields [8, 9]. The goal of this section is to show that there is an inherent mixing of (L/R) operators induced by the fractional dynamics, as described below. An equivalent representation of (1) is given by

$${}_0 D_L^\alpha \varphi(t) = \frac{1}{\Gamma(1-\alpha)} \left(-\frac{d}{dt}\right) \int_t^0 [-(\tau-t)]^{-\alpha} \varphi(\tau) d\tau, \quad (14)$$

or

$$\begin{aligned} {}_0 D_L^\alpha \varphi(t) &= \frac{(-1)^{-\alpha}}{\Gamma(1-\alpha)} \left(-\frac{d}{dt}\right) \int_t^0 (\tau-t)^{-\alpha} \varphi(\tau) d\tau = \\ &= (-1)^{-\alpha} {}_0 D_R^\alpha \varphi(t), \end{aligned} \quad (15)$$

$${}_0 D_R^\alpha = (-1)^\alpha {}_0 D_L^\alpha = \exp(i\pi\alpha) {}_0 D_L^\alpha. \quad (16)$$

Starting from (2) instead, we find

$${}_0 D_L^\alpha = (-1)^\alpha {}_0 D_R^\alpha = \exp(i\pi\alpha) {}_0 D_R^\alpha. \quad (17)$$

Consider now the one-dimensional case $d = 1$, take $\alpha = \varepsilon = 1 - d$ and recall that continuous tuning of ε does not impact the physics as a consequence of scale invariance. Let us iterate (16) and (17) a finite number of times ($n \geq 1$) under the assumption that $n\varepsilon \ll 1$. It follows that the fractal operator of any infinitesimal order may be only defined up to an arbitrary dimensional factor $\exp(i\pi n\varepsilon) \approx 1 + (i\pi n\varepsilon) = 1 - i\tilde{\varepsilon}$, that is,

$${}_0 D_{L,R}^\varepsilon \varphi(t) \approx [{}_0 D_{L,R}^0 - i\tilde{\varepsilon}] \varphi(t) \quad (18)$$

or

$$i {}_0 D_{L,R}^\varepsilon \varphi(t) = [i {}_0 D_{L,R}^0 + \tilde{\varepsilon}] \varphi(t), \quad (19)$$

where

$$\lim_{\varepsilon \rightarrow 0} D_{L,R}^\varepsilon \varphi(t) = \varphi(t). \quad (20)$$

Relations (18–20) indicate that fractional dimension $\tilde{\varepsilon}$ induces: (a) a new type of mixing between chiral components of the field and (b) an ambiguity in the very definition of the field, fundamentally different from measurement uncertainties associated with Heisenberg principle. Both effects are *irreversible* (since fractional dynamics describes irreversible processes) and of *topological nature* (being based on the concept of continuous dimension). They do not have a counterpart in conventional QFT.

5 Emergence of “unmatter” states

Using the operator language of QFT and taking into account (6), (18) can be presented as

$$\hat{\pi}^\varepsilon \varphi(t) = \hat{\pi}^\varepsilon \varphi(t) - \tilde{\varepsilon} \hat{\varphi}(t). \quad (21)$$

Relation (21) shows that the fractional momentum operator $\hat{\pi}^\varepsilon$ and the field operator $\hat{\varphi}(t) = \varphi(t)$ are no longer independent entities but linearly coupled through fractional dimension $\tilde{\varepsilon}$. From (11) it follows that the destruction and creation operators are also coupled to each other. As a result, particles and antiparticles can no longer exist as linearly independent objects. Because $\tilde{\varepsilon}$ is continuous, they emerge as an *infinite spectrum of mixed states*. This surprising finding is counterintuitive as it does not have an equivalent in conventional QFT. Moreover, arbitrary mixtures of particles and antiparticles may be regarded as a manifestation of “unmatter”, a concept launched in the context of Neutrosophic Logic [22–24].

6 Definition of unmatter

In short, unmatter is formed by matter and antimatter that bind together [23, 24].

The building blocks (most elementary particles known today) are 6 quarks and 6 leptons; their 12 antiparticles also exist.

Then *unmatter* will be formed by at least a building block and at least an antibuilding block which can bind together.

Let’s start from neutrosophy [22], which is a generalization of dialectics, i.e. not only the opposites are combined but also the neutralities. Why? Because when an idea is launched, a category of people will accept it, others will reject it, and a third one will ignore it (don’t care). But the dynamics between these three categories changes, so somebody accepting it might later reject or ignore it, or an ignorant will accept it or reject it, and so on. Similarly the dynamicity of $\langle A \rangle$, $\langle \text{anti}A \rangle$, $\langle \text{neut}A \rangle$, where $\langle \text{neut}A \rangle$ means neither $\langle A \rangle$ nor $\langle \text{anti}A \rangle$, but in between (neutral). Neutrosophy considers a kind not of di-alectics but tri-alectics (based on three components: $\langle A \rangle$, $\langle \text{anti}A \rangle$, $\langle \text{neut}A \rangle$).

Hence unmatter is a kind of intermediary (not referring to the charge) between matter and antimatter, i.e. neither one, nor the other.

Neutrosophic Logic (NL) is a generalization of fuzzy logic (especially of intuitionistic fuzzy logic) in which a proposition has a degree of truth, a degree of falsity, and a degree of neutrality (neither true nor false); in the normalized NL the sum of these degrees is 1.

7 Exotic atom

If in an atom we substitute one or more particles by other particles of the same charge (constituents) we obtain an exotic atom whose particles are held together due to the electric charge. For example, we can substitute in an ordinary atom one or more electrons by other negative particles (say π^- , anti-Rho meson, D^- , D_s^- , muon, tau, Ω^- , Δ^- , etc., generally clusters of quarks and antiquarks whose total charge is negative), or the positively charged nucleus replaced by other

positive particle (say clusters of quarks and antiquarks whose total charge is positive, etc.).

8 Unmatter atom

It is possible to define the unmatter in a more general way, using the exotic atom.

The classical unmatter atoms were formed by particles like (a) electrons, protons, and antineutrons, or (b) antielectrons, antiprotons, and neutrons.

In a more general definition, an unmatter atom is a system of particles as above, or such that one or more particles are replaced by other particles of the same charge.

Other categories would be (c) a matter atom with where one or more (but not all) of the electrons and/or protons are replaced by antimatter particles of the same corresponding charges, and (d) an antimatter atom such that one or more (but not all) of the antielectrons and/or antiprotons are replaced by matter particles of the same corresponding charges.

In a more composed system we can substitute a particle by an unmatter particle and form an unmatter atom.

Of course, not all of these combinations are stable, semi-stable, or quasi-stable, especially when their time to bind together might be longer than their lifespan.

9 Examples of unmatter

During 1970–1975 numerous pure experimental verifications were obtained proving that “atom-like” systems built on nucleons (protons and neutrons) and anti-nucleons (anti-protons and anti-neutrons) are real. Such “atoms”, where nucleon and anti-nucleon are moving at the opposite sides of the same orbit around the common centre of mass, are very unstable, their life span is no more than 10^{-20} sec. Then nucleon and anti-nucleon annihilate into gamma-quanta and more light particles (pions) which can not be connected with one another, see [6, 7, 8]. The experiments were done in mainly Brookhaven National Laboratory (USA) and, partially, CERN (Switzerland), where “proton–anti-proton” and “anti-proton–neutron” atoms were observed, called them $\bar{p}p$ and $\bar{p}n$ respectively.

After the experiments were done, the life span of such “atoms” was calculated in theoretical way in Chapiro’s works [9, 10, 11]. His main idea was that nuclear forces, acting between nucleon and anti-nucleon, can keep them far way from each other, hindering their annihilation. For instance, a proton and anti-proton are located at the opposite sides in the same orbit and they are moved around the orbit centre. If the diameter of their orbit is much more than the diameter of “annihilation area”, they are kept out of annihilation. But because the orbit, according to Quantum Mechanics, is an actual cloud spreading far around the average radius, at any radius between the proton and the anti-proton there is a probability

that they can meet one another at the annihilation distance. Therefore nucleon—anti-nucleon system annihilates in any case, this system is unstable by definition having life span no more than 10^{-20} sec.

Unfortunately, the researchers limited the research to the consideration of $\bar{p}p$ and $\bar{p}n$ nuclei only. The reason was that they, in the absence of a theory, considered $\bar{p}p$ and $\bar{p}n$ “atoms” as only a rare exception, which gives no classes of matter.

The unmatter does exist, for example some mesons and antimessons, through for a trifling of a second lifetime, so the pions are unmatter (which have the composition $u\bar{d}$ and $d\bar{u}$, where by u we mean anti-up quark, d = down quark, and analogously u = up quark and d = anti-down quark, while by \bar{u} means anti), the kaon K^+ ($u\bar{s}$), K^- ($\bar{u}s$), Φ ($s\bar{s}$), D^+ ($c\bar{d}$), D^0 ($c\bar{u}$), D_s^+ ($c\bar{s}$), J/Ψ ($c\bar{c}$), B^- ($\bar{b}u$), B^0 ($\bar{d}b$), B_s^0 ($\bar{s}b$), $Upsilon$ ($b\bar{b}$), where c = charm quark, s = strange quark, b = bottom quark, etc. are unmatter too.

Also, the pentaquark Theta-plus (Θ^+), of charge $+1$, $uudd\bar{s}$ (i.e. two quarks up, two quarks down, and one anti-strange quark), at a mass of 1.54 GeV and a narrow width of 22 MeV, is unmatter, observed in 2003 at the Jefferson Lab in Newport News, Virginia, in the experiments that involved multi-GeV photons impacting a deuterium target. Similar pentaquark evidence was obtained by Takashi Nakano of Osaka University in 2002, by researchers at the ELSA accelerator in Bonn in 1997–1998, and by researchers at ITEP in Moscow in 1986.

Besides Theta-plus, evidence has been found in one experiment [25] for other pentaquarks, Ξ_5^- ($ddssu\bar{u}$) and Ξ_5^+ ($uusd\bar{s}$).

D. S. Carman [26] has reviewed the positive and null evidence for these pentaquarks and their existence is still under investigation.

In order for the paper to be self-contained let's recall that the *pionium* is formed by a π^+ and π^- mesons, the *positronium* is formed by an antielectron (positron) and an electron in a semi-stable arrangement, the *protonium* is formed by a proton and an antiproton also semi-stable, the *antiprotonic helium* is formed by an antiproton and electron together with the helium nucleus (semi-stable), and *muonium* is formed by a positive muon and an electron.

Also, the *mesonic atom* is an ordinary atom with one or more of its electrons replaced by negative mesons.

The *strange matter* is a ultra-dense matter formed by a big number of strange quarks bounded together with an electron atmosphere (this strange matter is hypothetical).

From the exotic atom, the pionium, positronium, protonium, antiprotonic helium, and muonium are unmatter.

The mesonic atom is unmatter if the electron(s) are replaced by negatively-charged antimessons.

Also we can define a mesonic antiatom as an ordinary antiatomic nucleus with one or more of its antielectrons replaced by positively-charged mesons. Hence, this mesonic

antiatom is unmatter if the antielectron(s) are replaced by positively-charged mesons.

The strange matter can be unmatter if these exist at least an antiquark together with so many quarks in the nucleus. Also, we can define the strange antimatter as formed by a large number of antiquarks bound together with an antielectron around them. Similarly, the strange antimatter can be unmatter if there exists at least one quark together with so many antiquarks in its nucleus.

The bosons and antibosons help in the decay of unmatter. There are 13+1 (Higgs boson) known bosons and 14 antibosons in present.

10 Chromodynamics formula

In order to save the colorless combinations prevailed in the Theory of Quantum Chromodynamics (QCD) of quarks and antiquarks in their combinations when binding, we devise the following formula:

$$Q - A \in \pm M3, \quad (22)$$

where $M3$ means multiple of three, i.e. $\pm M3 = \{3 \cdot k | k \in \mathbb{Z}\} = \{\dots, -12, -9, -6, -3, 0, 3, 6, 9, 12, \dots\}$, and Q = number of quarks, A = number of antiquarks.

But (22) is equivalent to:

$$Q \equiv A \pmod{3} \quad (23)$$

(Q is congruent to A modulo 3).

To justify this formula we mention that 3 quarks form a colorless combination, and any multiple of three ($M3$) combination of quarks too, i.e. 6, 9, 12, etc. quarks. In a similar way, 3 antiquarks form a colorless combination, and any multiple of three ($M3$) combination of antiquarks too, i.e. 6, 9, 12, etc. antiquarks. Hence, when we have hybrid combinations of quarks and antiquarks, a quark and an antiquark will annihilate their colors and, therefore, what's left should be a multiple of three number of quarks (in the case when the number of quarks is bigger, and the difference in the formula is positive), or a multiple of three number of antiquarks (in the case when the number of antiquarks is bigger, and the difference in the formula is negative).

11 Quantum chromodynamics unmatter formula

In order to save the colorless combinations prevailed in the Theory of Quantum Chromodynamics (QCD) of quarks and antiquarks in their combinations when binding, we devise the following formula:

$$Q - A \in \pm M3, \quad (24)$$

where $M3$ means multiple of three, i.e. $\pm M3 = \{3 \cdot k | k \in \mathbb{Z}\} = \{\dots, -12, -9, -6, -3, 0, 3, 6, 9, 12, \dots\}$, and Q = number of quarks, A = number of antiquarks, with $Q \geq 1$ and $A \geq 1$.

But (24) is equivalent to:

$$\mathbf{Q} \equiv \mathbf{A} \pmod{3} \quad (25)$$

(\mathbf{Q} is congruent to \mathbf{A} modulo 3), and also $\mathbf{Q} \geq 1$ and $\mathbf{A} \geq 1$.

12 Quark-antiquark combinations

Let's note by $q = \text{quark} \in \{\text{Up, Down, Top, Bottom, Strange, Charm}\}$, and by $a = \text{antiquark} \in \{\text{Up, Down, Top, Bottom, Strange, Charm}\}$.

Hence, for combinations of n quarks and antiquarks, $n \geq 2$, prevailing the colorless, we have the following possibilities:

- if $n = 2$, we have: qa (biquark — for example the mesons and antimesons);
- if $n = 3$, we have qqq, aaa (triquark — for example the baryons and antibaryons);
- if $n = 4$, we have $qqaa$ (tetraquark);
- if $n = 5$, we have $qqqqa, aaaaq$ (pentaquark);
- if $n = 6$, we have $qqqaaa, qqqqq, aaaaa$ (hexaquark);
- if $n = 7$, we have $qqqqaa, qqaaaa$ (septiquark);
- if $n = 8$, we have $qqqqaaa, qqqqqaa, qqaaaaa$ (octoquark);
- if $n = 9$, we have $qqqqqqq, qqqqqaa, qqaaaaa, aaaaaaa$ (nonaquark);
- if $n = 10$, obtain $qqqqaaaa, qqqqqaa, qqaaaaa$ (decaquark);
- etc.

13 Unmatter combinations

From the above general case we extract the unmatter combinations:

- For combinations of 2 we have: qa (unmatter biquark), (mesons and antimsons); the number of all possible unmatter combinations will be $6 \cdot 6 = 36$, but not all of them will bind together.

It is possible to combine an entity with its mirror opposite and still bound them, such as: $u\bar{u}, d\bar{d}, s\bar{s}, c\bar{c}, b\bar{b}$ which form mesons.

It is possible to combine, unmatter + unmatter = unmatter, as in $u\bar{d} + u\bar{s} = u\bar{d}\bar{s}$ (of course if they bind together);

- For combinations of 3 (unmatter triquark) we can not form unmatter since the colorless can not hold.
- For combinations of 4 we have: $qqaa$ (unmatter tetraquark); the number of all possible unmatter combinations will be $6^2 \cdot 6^2 = 1,296$, but not all of them will bind together;

- For combinations of 5 we have: $qqqqa, or aaaaq$ (unmatter pentaquarks); the number of all possible unmatter combinations will be $6^4 \cdot 6 + 6^4 \cdot 6 = 15,552$, but not all of them will bind together;

- For combinations of 6 we have: $qqqaaa$ (unmatter hexaquarks); the number of all possible unmatter combinations will be $6^3 \cdot 6^3 = 46,656$, but not all of them will bind together;

- For combinations of 7 we have: $qqqqaa, qqaaaa$ (unmatter septiquarks); the number of all possible unmatter combinations will be $6^5 \cdot 6^2 + 6^2 \cdot 6^5 = 559,872$, but not all of them will bind together;

- For combinations of 8 we have: $qqqqaaa, qqqqqqa, qaaaaaa$ (unmatter octoquarks); the number of all possible unmatter combinations will be $6^4 \cdot 6^4 + 6^7 \cdot 6^1 + 6^1 \cdot 6^7 = 5,038,848$, but not all of them will bind together;

- For combinations of 9 we have: $qqqqqaaa, qqaaaaaa$ (unmatter nonaquarks); the number of all possible unmatter combinations will be $6^6 \cdot 6^3 + 6^3 \cdot 6^6 = 2 \cdot 6^9 = 20,155,392$, but not all of them will bind together;

- For combinations of 10: $qqqqqqqaa, qqqqqaaaa, qaaaaaaaa$ (unmatter decaquarks); the number of all possible unmatter combinations will be $3 \cdot 6^{10} = 181,398,528$, but not all of them will bind together;

— etc.

I wonder if it is possible to make infinitely many combinations of quarks/antiquarks and leptons/antileptons. . . Unmatter can combine with matter and/or antimatter and the result may be any of these three.

Some unmatter could be in the strong force, hence part of hadrons.

14 Unmatter charge

The charge of unmatter may be positive as in the pentaquark Theta-plus, 0 (as in positronium), or negative as in anti-Rho meson, i.e. $u\bar{d}$, (M. Jordan).

15 Containment

I think for the containment of antimatter and unmatter it would be possible to use electromagnetic fields (a container whose walls are electromagnetic fields). But its duration is unknown.

16 Summary and conclusions

It is apparent from these considerations that, in general, both "unmatter" and "unparticles" are non-trivial states that may become possible under conditions that substantially deviate from our current laboratory settings. Unmatter can be thought

as *arbitrary* clusters of ordinary matter and antimatter, unparticles contain *fractional numbers of quanta* per state and carry *arbitrary spin* [6]. They both display a much richer dynamics than conventional SM doublets, for example mesons (quark-antiquark states) or lepton pairs (electron-electron antineutrino). Due to their unusual properties, “unmatter” and “unparticles” are presumed to be highly unstable and may lead to a wide range of symmetry breaking scenarios. In particular, they may violate well established conservation principles such as electric charge, weak isospin and color. Future observational evidence and analytic studies are needed to confirm, expand or falsify these tentative findings.

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The Dark Energy Problem

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The proposal for dark energy based on Type Ia Supernovae redshift is examined. It is found that the linear and non-Linear portions in the Hubble Redshift are easily explained by the use of the Hubble Sphere model, where two interacting Hubble spheres sharing a common mass-energy density result in a decrease in energy as a function of distance from the object being viewed. Interpreting the non-linear portion of the redshift curve as a decrease in interacting volume between neighboring Hubble Spheres removes the need for a dark energy.

1 Introduction

The discovery in 1998 of fainter than expected Type Ia supernova resulted in the hypothesis of an apparent acceleration in our expanding universe [1]. Type Ia supernovas have a previously determined standard-candle distance which has shown to be the same as their redshift distance for low z values. However, their fainter brightness at far distances indicate that they are further away than expected when compared with their redshift distance. This lead to the conclusion that the standard candle distance is correct but that there is an apparent acceleration in the expansion of the universe occurring in the range where the Type Ia supernovas were measured. This explanation was designed the preserve the linearity of Hubble's Law while explaining the further distance of the Type Ia supernova. The existence of dark energy, a repulsive gravitational field that is a manifestation of the cosmological constant, was theorized as the likely cause of the acceleration [2]. Experimentalists are now embarking on the task of proving the existence of dark energy with little examination or critical analysis of the cause and effect of the initial observations. We can show that the observed effects of the Type Ia supernova redshift are explainable by another phenomena which satisfies known laws of physics.

2 Assumptions

We begin by making the following assumptions:

Assumption 1: *The gravitational and electromagnetic force ranges are not infinite.*

Although there is as of yet no widely accepted model of unifying the gravitational and electromagnetic (QED) forces, they both follow an inverse-square law and have similar divergence properties so we assume they are fairly equivalent in nature but by no means infinite in range. We assume the gravitational and electromagnetic force ranges have a steep

decline in effect similar to the profile for the strong nuclear force but at a range $= 10^{26}$ meters $= R_u/2$ which BB theorists currently estimate as the radius of the Universe. We will call the sphere that is centered around our point of observation on Earth as our Hubble sphere, and it encompasses what we see out to the radius $R_u/2$ which we assume as the limit of the gravitational and electromagnetic forces. Likewise, objects at a distant d from us on Earth also have a Hubble sphere that is centered on their point of observation.

Assumption 2: *The Universe is bigger than the Hubble sphere and is perhaps infinite.*

When we refer to the Universe we are referring to all space including what lies beyond our Hubble sphere, which we cannot view because light is infinitely redshifted at the boundary of our sphere due to the steep decay of the gravitational and EM forces at a distance $R_u/2$. We currently accept that a decrease in energy between two points can cause a redshift in photons. This explanation should be adequate for the purposes of our discussion on how the apparent redshift-acceleration may be the cause of two overlapping Hubble spheres, each with their own center of observation. This explanation also answers Olber's Paradox in which an infinite Universe would contain so many stars that the darkness of night would be overwhelmed with starlight. The answer to the paradox is that there is no starlight that can reach us beyond our Hubble sphere radius because of the limit of the electromagnetic force range.

Assumption 3: *If one views an object at a distance d from Earth, the light from that object is affected by the mass-energy density of our local Hubble sphere interacting with the mass-energy density of the distant object's Hubble sphere.*

The intersecting volumes of two neighboring Hubble spheres correspond to a common mass-energy density between the spheres that decreases as the distance between the centers of the spheres increases, resulting in less common volume.

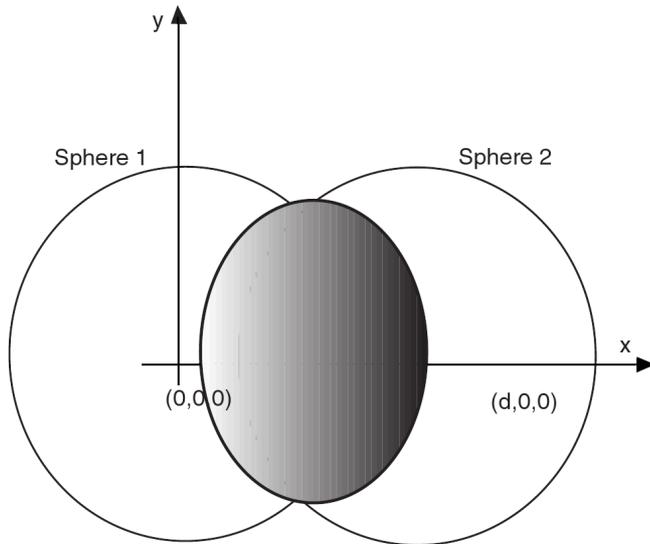


Fig. 1: Hubble sphere's 1 and 2 intersect sharing a volume (shaded gray).

The decrease in common mass-energy density between the spheres results in a redshift of photons emitted from the center of either Hubble sphere to the center of the other Hubble sphere. Regardless of which direction we look, we always see a redshift because there is matter all around the outside of our Hubble sphere that gravitationally attracts the matter inside our Hubble sphere. The Hubble sphere by this account is a three-dimensional Euclidean sphere, which is assumed to have a constant mass-energy density.

3 The common energy of Hubble spheres

If we examine Figure 1, we see the intersection of two Hubble spheres with their centers separated by a distance d . The shaded gray area is the intersecting volume, which also represents common mass-energy between the spheres. The center of sphere 1 can be imagined as our viewpoint from Earth and the center of sphere 2 can be the distant object we are viewing.

From Figure 1 we can find the ratio of intersecting volume between the spheres to the volume in our sphere as:

$$\frac{Volume_{common}}{Volume_{local}} = \frac{\pi(16R_u^3 - 12dR_u^2 + d^3)}{\frac{4}{3}\pi R_u^3} = \frac{3}{48} \left(\frac{d^3}{R_u^3} - 12 \frac{d}{R_u} + 16 \right), \quad (1)$$

where $Volume_{common}$ is the intersecting volume between the spheres and $Volume_{local}$ is the volume of our own sphere.

If we assume homogenous mass-energy throughout both spheres, then the ratio of common mass-energy between the spheres to the energy in our own sphere is proportional to the

ratio of the intersecting volume between the spheres to our sphere's volume. We also know that the mass-energy in a given sphere is proportional to the $h\nu$, so we arrive at:

$$\frac{\nu_2}{\nu_1} = \frac{Volume_{common}}{Volume_{local}} = \frac{3}{48} \left(\frac{d^3}{R_u^3} - 12 \frac{d}{R_u} + 16 \right) = 1 - \frac{3d}{4R_u} + \frac{d^3}{16R_u^3}, \quad (2)$$

The change in frequency $\Delta\nu/\nu_1 = (\nu_2 - \nu_1)/\nu_1$ is the similar to the measured value of z with respect to wavelength λ large, but we now look at it with respect to ν and $\Delta\nu/\nu$ is found to be:

$$\frac{\Delta\nu}{\nu} = -\frac{3d}{4R_u} + \frac{d^3}{16R_u^3}. \quad (3)$$

From (3) we see that the energy viewed from our observation point decreases with the distance d to the object (which is also the distance between the centers of the spheres), and is essentially linear for $d \ll R_u$ where R_u is the radius of each Hubble sphere. This linear decrease in energy is interpreted as an increase in redshift or a linear increase in velocity with distance by Big Bang (BB) theorists and amounts to the linear portion of Hubble's Law. For situations where d gets close to R_u there is a slight increase in energy resulting from the d^3 term in (3), suggesting to the BB theorist that the object being viewed is decelerating and is closer to us than would be expected from the previously linear Hubble slope when $d \ll R_u$.

Instead of accepting a non-linearity in the Hubble curve, BB theorists believe that the curve is still linear and that the shorter distance computed at larger d based on measured wavelength is still correct. The fainter-than-expected brightness of the Type Ia supernova is then a result of an apparent acceleration in the object due to some unknown "dark energy" with a negative gravitational force. In reality, the Hubble Law coincides fairly well with standard candle observations until d approaches R_u , where it then becomes non-linear and produces a result that mimics acceleration of the viewed object, if one still believes that Hubble's Law is linear. The d^3 term in (3) results in an apparent acceleration of the object viewed at larger distances and in fact this acceleration is not a real but instead is a non-linearity in Hubble's Law.

4 Conclusions

The results of the analysis of intersecting Hubble spheres shows that a linear redshift results by assuming that the gravitational and electromagnetic forces have a finite range, R_u . The linear relationship for smaller d explains Hubble's Law without requiring an expansion of the Universe or our own Hubble sphere. The derivation also explains the apparent acceleration of objects as our distance d to them approaches R_u . Therefore, a simpler explanation of a non-expanding Universe exists which to current knowledge is at least the size

of $2R_u$ and possibly much bigger. The Cosmic Microwave Background Radiation (CMBR) has been shown by others to be a result of absorption and scattering of the intergalactic medium [3]. The additional production of Helium and other element ratios is easily found by allowing the Universe as much time as it needs to produce these results in stellar cores. The proposed explanation is a far simpler one than the requirement to balance photon to proton ratios in the theorized early Universe of the Big Bang, with the added concern of an inflationary period to allow smoothness in the CMBR.

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Unravelling Lorentz Covariance and the Spacetime Formalism

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We report the discovery of an exact mapping from Galilean time and space coordinates to Minkowski spacetime coordinates, showing that Lorentz covariance and the spacetime construct are consistent with the existence of a dynamical 3-space, and “absolute motion”. We illustrate this mapping first with the standard theory of sound, as vibrations of a medium, which itself may be undergoing fluid motion, and which is covariant under Galilean coordinate transformations. By introducing a different non-physical class of space and time coordinates it may be cast into a form that is covariant under “Lorentz transformations” wherein the speed of sound is now the “invariant speed”. If this latter formalism were taken as fundamental and complete we would be led to the introduction of a pseudo-Riemannian “spacetime” description of sound, with a metric characterised by an “invariant speed of sound”. This analysis is an allegory for the development of 20th century physics, but where the Lorentz covariant Maxwell equations were constructed first, and the Galilean form was later constructed by Hertz, but ignored. It is shown that the Lorentz covariance of the Maxwell equations only occurs because of the use of non-physical space and time coordinates. The use of this class of coordinates has confounded 20th century physics, and resulted in the existence of a “flowing” dynamical 3-space being overlooked. The discovery of the dynamics of this 3-space has led to the derivation of an extended gravity theory as a quantum effect, and confirmed by numerous experiments and observations.

1 Introduction

It is commonly argued that the manifest success of Lorentz covariance and the spacetime formalism in Special Relativity (SR) is inconsistent with the anisotropy of the speed of light, and indeed the existence of absolute motion, that is, a detectable motion relative to an actual dynamical 3-space, despite the repeated experimental detection of such effects over, as we now understand, more than 120 years. This apparent incompatibility between a preferred frame, *viz* a dynamical 3-space, and the spacetime formalism is explicitly resolved by the discovery of an exact mapping from Galilean time and space coordinates to Minkowski spacetime coordinates*, showing that Lorentz covariance and the spacetime construct are indeed consistent with Galilean covariance, but that they suppress any account of an underlying dynamical 3-space.

In the neo-Galilean formalism, known also as the Lorentzian interpretation of SR, length contraction and clock effects are real effects experienced by objects and clocks in motion relative to an actual 3-space, whereas in the Minkowski-Einstein spacetime formalism these effects are transferred to the metric of the mathematical spacetime, and then appear to be merely perspective effects for different observers. Experiments, however, have shown that the Galilean space and time coordinates competently describe reality, whereas the Minkowski-Einstein spacetime construct is merely a mathe-

matical artifact, and that various observable phenomena cannot be described by that formalism. We thus arrive at the dramatic conclusion that the neo-Galilean formalism is the valid description of reality, and that it is a superior more encompassing formalism than the Minkowski-Einstein formalism in terms of both mathematical clarity and ontology.

Physics arrived at the Minkowski-Einstein formalism because of two very significant accidents of history, first that Maxwell’s unification of electric and magnetic phenomena failed to build in the possibility of an actual 3-space, for which the speed of light is only c relative to that space, and not relative to observers in general, and 2nd that the first critical test of the Maxwell EM unification by Michelson using interferometry actually suffered a fundamental design flaw, causing the instrument to be almost 2000 times less sensitive than Michelson had assumed. A related issue is that the Newtonian theory of gravity used an acceleration field for the description of gravitational phenomena, when a velocity field description would have immediately lead to a richer description, and for which notions such as “dark matter” and “dark energy” are not needed.

We illustrate the properties of this new mapping first with the standard theory of sound, as vibrations of a medium which itself may be undergoing fluid motion, and which is covariant under Galilean coordinate transformations, which relate the observations by different observers who may be in motion wrt the fluid and wrt one another. Here we show that by introducing a different non-physical class of space and time co-

*See [1] and Damour [2] for discussion of Minkowski’s work.

ordinates, essentially the Minkowski coordinates, the sound vibration dynamics may be cast into a form that is covariant under “Lorentz transformations”, wherein the speed of sound is now the invariant speed. If this latter formalism were taken as fundamental and complete we would be lead to the introduction of a pseudo-Riemannian “spacetime” formalism for sound with a metric characterised by the invariant speed of sound, and where “sound cones” would play a critical role.

This analysis is an allegory for the development of 20th century physics, but where the Lorentz covariant Maxwell equations were constructed first, and the Galilean form was later suggested by Hertz, but ignored. It is shown that the Lorentz covariance of the Maxwell equations only occurs because of the use of degenerate non-physical space and time coordinates. The conclusion is that Lorentz covariance and the spacetime formalism are artifacts of the use of peculiar non-physical space and time coordinates. The use of this class of coordinates has confounded 20th century physics, and lead to the existence of a “flowing” dynamical 3-space being overlooked. The dynamics of this 3-space, when coupled to the new Schrödinger and Dirac equations, has lead to the derivation of an extended gravity theory confirmed by numerous experiments and observations. This analysis also shows that Lorentz symmetry is consistent with the existence of a preferred frame, namely that defined by the dynamical 3-space. This dynamical 3-space has been repeatedly detected over more than 120 years of experiments, but has always been denied because of the obvious success of the Lorentz covariant formalism, where there the Lorentz transformations are characterised by the so-called invariant speed of light. Einstein’s fundamental principle that ‘the speed of light is invariant’ is not literally true, it is only valid if one uses the non-physical space and time coordinates.

As with sound waves, the non-invariance or speed anisotropy of the actual speed of light in vacuum is relatively easy to measure, and is also relatively large, being approximately 1 part in 1000 when measured on earth, with the direction of the “flowing space” known since the 1925/26 experiment by Miller [3]. Successful direct and sufficiently accurate measurements of the one-way speed of light have never been made simply because the speed of light is so fast that accurate timing for laboratory-sized speed measurements are not possible. For that reason indirect measurements have always been used. One of the first was the Michelson interferometer. However a subtlety always arises for indirect measurements — namely that the anisotropy of the speed of light also affects the operation of the experimental apparatus in ways that have not always been apparent. The Michelson interferometer, for example, has a major design flaw that renders it nearly 2000 times less sensitive than believed by Michelson, who used Newtonian physics in calibrating his instrument. It was only in 2002 [5, 6] that the correct calibration of the Michelson interferometer was derived, and analysis of the non-null fringe shift data from that Michelson-Morley 1887 experiment was

analysed and shown to reveal a “flowing space” with a speed in excess of 300km/s. The 2002 analysis [5, 6] showed that the presence of a gas in the Michelson interferometer was a key component of its operation — for in vacuum mode the instrument is totally defective as a detector of light speed anisotropy. This is merely because different unrelated effects just happen to cancel when the Michelson interferometer is used in vacuum mode — a simple design flaw that at least Michelson could not have known about. It so happens that having a gas in the light paths causes this cancellation to be incomplete. The sensitivity of the instrument varies as $n - 1$, where n is the refractive index. For gases this calibration factor is very small — for air at STP $n - 1 = 0.00029$, whereas Michelson, using Newtonian physics, used a calibration coefficient of value 1. However if we use optical fibers in place of air $n - 1 \approx 0.5$, and the detector is some 2000 times more sensitive, and the use of such detectors has lead to the detailed characterisation of turbulence in the 3-space flow — essentially gravitational waves*.

There are now four different experimental techniques for detecting light speed anisotropy: (1) gas-mode Michelson interferometer [3, 4, 7–10], (2) one-way RF speed in coaxial cables [11–13], (3) optical fiber interferometer [14, 15], and (4) doppler-shift effects in earth-flyby of spacecraft [16]. These consistent light-speed anisotropy experiments reveal earth rotation and orbit effects, and sub-mHz gravitational waves. The detection of gravitational wave effects, it now turns out, dates back to the pioneering work of Michelson and Morley in 1887 [4], as discussed in [20], and detected again by Miller [3] also using a gas-mode Michelson interferometer, and by Torr and Kolen [11], DeWitte [12] and Cahill [13] using RF waves in coaxial cables, and by Cahill [14] and Cahill and Stokes [15] using an optical-fiber interferometer design, and also present in the spacecraft flyby doppler shifts [16].

2 Sound wave Galilean covariant formalism

Let us first use the example of sound waves to discuss the mapping from Galilean space and time coordinates to Minkowski-Einstein spacetime coordinates — as in this case the underlying physics is well understood. The standard formulation for sound waves in a moving fluid is

$$\left(\frac{\partial}{\partial t} + \mathbf{v}(\mathbf{r}, t) \cdot \nabla \right)^2 \phi(\mathbf{r}, t) = c^2 \nabla^2 \phi(\mathbf{r}, t), \quad (1)$$

where $\nabla = \left\{ \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\}$. The physical time coordinate t and Euclidean space coordinates $\mathbf{r} = \{x, y, z\}$ are used by

*The design flaw of the vacuum-mode Michelson interferometer has been repeated in the large and expensive terrestrial gravitational wave detectors such as LIGO, and also in the vacuum-mode resonant cavity interferometers [17]. These cavity experiments are based on two mistaken notions: (i) that a breakdown of Lorentz symmetry is related to the existence of a preferred frame, and (ii) that vacuum-mode Michelson interferometers can detect a light speed anisotropy associated with such a preferred frame.

an observer O to label the readings of a clock and the location in space where the “wind” or “fluid flow” has velocity $\mathbf{v}(\mathbf{r}, t)$, and small pressure variations $\phi(\mathbf{r}, t)$, relative to the background pressure. Clearly the “fluid flow” and “pressure fluctuations” are different aspects of the same underlying phenomena — namely the dynamics of some macroscopic system of atoms and/or molecules, but separated into very low frequency effects, — the flow, and high frequency effects, — the sound waves. The dynamics for the flow velocity $\mathbf{v}(\mathbf{r}, t)$ is not discussed here. As well the symbol c is the speed of sound waves relative to the fluid. In (1) the coordinates $\{t, x, y, z\}$ ensure that the dynamical flow \mathbf{v} is correctly related to the pressure fluctuation ϕ , at the same time and space. Whenever we separate some unified phenomenon into two or more related phenomena we must introduce a “coordinate system” that keeps track of the connection. To demonstrate this we find plane-wave solutions of (1) for the case where the fluid flow velocity is time and space independent, *viz* uniform,

$$\phi(\mathbf{r}, t) = A \sin(\mathbf{k} \cdot \mathbf{r} - \omega t), \quad (2)$$

$$\omega(\mathbf{k}, \mathbf{v}) = c|\vec{\mathbf{k}}| + \mathbf{v} \cdot \mathbf{k}. \quad (3)$$

The sound wave group velocity is then

$$\mathbf{v}_g = \vec{\nabla}_k \omega(\mathbf{k}, \mathbf{v}) = c\hat{\mathbf{k}} + \mathbf{v}, \quad (4)$$

and we see that the wave has velocity \mathbf{v}_g relative to the observer, with the fluid flowing at velocity \mathbf{v} also relative to the observer, and so the speed of sound is c in direction $\hat{\mathbf{k}}$ relative to the fluid itself. This corresponds to a well known effect, namely that sound travels slower up-wind than down-wind. This “sound speed anisotropy” effect can be measured by means of one-way sound travel times, or indirectly by means of doppler shifts for sound waves reflected from a distant object separated by a known distance from the observer.

Next consider two observers, O and O' , in relative motion. Then the physical time and space coordinates of each are related by the Galilean transformation

$$\begin{aligned} t' &= t, \\ x' &= x - Vt, \quad y' = y, \quad z' = z. \end{aligned} \quad (5)$$

We have taken the simplest case where V is the relative speed of the two observers in their common x directions. Then the derivatives are related by

$$\begin{aligned} \frac{\partial}{\partial t} &= \frac{\partial}{\partial t'} - V \frac{\partial}{\partial x}, \\ \frac{\partial}{\partial x} &= \frac{\partial}{\partial x'}, \quad \frac{\partial}{\partial y} = \frac{\partial}{\partial y'}, \quad \frac{\partial}{\partial z} = \frac{\partial}{\partial z'}. \end{aligned} \quad (6)$$

Then (1) becomes for the 2nd observer, with $v' = v - V$,

$$\left(\frac{\partial}{\partial t'} + \mathbf{v}'(\mathbf{r}', t') \cdot \nabla' \right)^2 \phi'(\mathbf{r}', t') = c^2 \nabla'^2 \phi'(\mathbf{r}', t'). \quad (7)$$

For sound waves $\phi'(\mathbf{r}', t') = \phi(\mathbf{r}, t)$. If the flow velocity $\mathbf{v}(\mathbf{r}, t)$ is not uniform then we obtain refraction effects for the

sound waves. Only for an observer at rest in a time independent and uniform fluid does v' disappear from (7).

3 Sound wave Lorentz covariant formalism

The above Galilean formalism for sound waves is well known and uses physically sensible choices for the time and space coordinates. Of course we could choose to use spherical or cylindrical space coordinates if we so desired. This would cause no confusion. However we could also choose to use a new class of time and space coordinates, indicated by upper-case symbols T, X, Y, Z , that mixes the above time and space coordinates. One such new class of coordinates is

$$\begin{aligned} T &= \gamma(v) \left(\left(1 - \frac{v^2}{c^2} \right) t + \frac{vx}{c^2} \right), \\ X &= \gamma(v) x; \quad Y = y; \quad Z = z, \end{aligned} \quad (8)$$

where $\gamma(v) = 1/\sqrt{1 - v^2/c^2}$. Note that this is not a Lorentz transformation. The transformations for the derivatives are then found to be

$$\begin{aligned} \frac{\partial}{\partial t} &= \gamma(v) \left(1 - \frac{v^2}{c^2} \right) \frac{\partial}{\partial T}, \\ \frac{\partial}{\partial x} &= \gamma(v) \left(\frac{v}{c^2} \frac{\partial}{\partial T} + \frac{\partial}{\partial X} \right), \quad \frac{\partial}{\partial y} = \frac{\partial}{\partial Y}, \quad \frac{\partial}{\partial z} = \frac{\partial}{\partial Z}. \end{aligned} \quad (9)$$

We define $\bar{\nabla} = \left\{ \frac{\partial}{\partial X}, \frac{\partial}{\partial Y}, \frac{\partial}{\partial Z} \right\}$. Then (1) becomes, for uniform v ,

$$\left(\frac{\partial}{\partial T} \right)^2 \bar{\phi}(\mathbf{R}, T) = c^2 \bar{\nabla}^2 \bar{\phi}(\mathbf{R}, T), \quad (10)$$

with $\mathbf{R} = \{X, Y, Z\}$ and $\bar{\phi}(\mathbf{R}, T) = \phi(\mathbf{r}, t)$. This is a remarkable result. In the new class of coordinates the dynamical equation no longer contains the flow velocity \mathbf{v} — it has been mapped out of the dynamics. Eqn.(10) is now covariant under Lorentz transformations*,

$$\begin{aligned} T' &= \gamma(V) \left(T + \frac{VX}{c^2} \right), \\ X' &= \gamma(V)(X - VT), \quad Y' = Y, \quad Z' = Z, \end{aligned} \quad (11)$$

where we have taken the simplest case, and where V is a measure of the relative speed of the two observers in their common X directions.

There is now no reference to the underlying flowing fluid system — for an observer using this class of space and time coordinates the speed of sound relative to the observer is always c and so invariant — there will be no sound speed anisotropy. We could also introduce a “spacetime” construct with pseudo-Riemannian metric $ds^2 = c^2 dT^2 - d\mathbf{R}^2$,

*Lorentz did not construct the “Lorentz transformation” — and this nomenclature is very misleading as Lorentz held to a different interpretation of the so-called relativistic effects.

and sound cones along which $ds^2 = 0$. As well pairs of spacetime events could be classified into either time-like or space-like, with the time ordering of spacelike events not being uniquely defined.

However this sound-speed invariance is purely an artifact of the non-physical space and time coordinates introduced in (8). The non-physical nature of this inferred “invariance” would have been easily exposed by doing measurements of the speed of sound in different directions. However in a bizarre imaginary world the Lorentz-covariant sound formalism could have been discovered first, and the spacetime formalism might have been developed and become an entrenched belief system. If later experiments had revealed that the speed of sound was actually anisotropic then the experimentalist involved might have been applauded, or, even more bizarrely, their discoveries denied and suppressed, and further experiments stopped by various means. The overwhelming evidence is that this bizarre possibility is precisely what happened for electromagnetics, for Maxwell essentially introduced the Lorentz covariant electromagnetism formalism, and experiments that detected the light speed anisotropy.

4 Dynamical 3-space theory

Here we briefly review the dynamics of the 3-space that is the analogue of the “flowing fluid” in the sound allegory. For zero vorticity we have [19–21]

$$\begin{aligned} \nabla \cdot \left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right) + \frac{\alpha}{8} ((\text{tr} D)^2 - \text{tr}(D^2)) &= -4\pi G\rho, \\ \nabla \times \mathbf{v} &= \mathbf{0}, \quad D_{ij} = \frac{1}{2} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right), \end{aligned} \quad (12)$$

where $\rho(\mathbf{r}, t)$ is the matter and EM energy densities expressed as an effective matter density. Experiment and astrophysical data has shown that $\alpha \approx 1/137$ is the fine structure constant to within observational errors [19–22]. For a quantum system with mass m the Schrödinger equation must be generalised [22] with the new terms required to maintain that the motion is intrinsically wrt to the 3-space and that the time evolution is unitary

$$\begin{aligned} i\hbar \frac{\partial \psi(\mathbf{r}, t)}{\partial t} &= \\ &= -\frac{\hbar^2}{2m} \nabla^2 \psi(\mathbf{r}, t) - i\hbar \left(\mathbf{v} \cdot \nabla + \frac{1}{2} \nabla \cdot \mathbf{v} \right) \psi(\mathbf{r}, t). \end{aligned} \quad (13)$$

The space and time coordinates $\{t, x, y, z\}$ in (12) and (13) ensure that the separation of a deeper and unified process into different classes of phenomena — here a dynamical 3-space and a quantum system, is properly tracked and connected. As well the same coordinates may be used by an observer to also track the different phenomena. However it is important to realise that these coordinates have no ontological significance — they are not real. Nevertheless it is imperative not to use a degenerate system of coordinates that suppresses

the description of actual phenomena. The velocities \mathbf{v} have no ontological or absolute meaning relative to this coordinate system — that is in fact how one arrives at the form in (12), and so the “flow” is always relative to the internal dynamics of the 3-space. So now this is different to the example of sound waves.

A wave packet propagation analysis gives the acceleration induced by wave refraction to be [22]

$$\mathbf{g} = \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} + (\nabla \times \mathbf{v}) \times \mathbf{v}_R, \quad (14)$$

$$\mathbf{v}_R(\mathbf{r}_0(t), t) = \mathbf{v}_0(t) - \mathbf{v}(\mathbf{r}_0(t), t), \quad (15)$$

is the velocity of the wave packet relative to the 3-space, where \mathbf{v}_0 and \mathbf{r}_0 are the velocity and position relative to the observer, and the last term in (14) generates the Lense-Thirring effect as a vorticity driven effect. Together (12) and (14) amount to the derivation of gravity as a quantum effect, explaining both the equivalence principle (\mathbf{g} in (14) is independent of m) and the Lense-Thirring effect. Overall we see, on ignoring vorticity effects, that

$$\nabla \cdot \mathbf{g} = -4\pi G\rho - \frac{\alpha}{8} ((\text{tr} D)^2 - \text{tr}(D^2)), \quad (16)$$

which is Newtonian gravity but with the extra dynamical term whose strength is given by α . This new dynamical effect explains the spiral galaxy flat rotation curves (and so doing away with the need for “dark matter”), the bore hole g anomalies, the black hole “mass spectrum”. Eqn.(12), even when $\rho = 0$, has an expanding universe Hubble solution that fits the recent supernovae data in a parameter-free manner without requiring “dark matter” nor “dark energy”, and without the accelerating expansion artifact [21]. However (16) cannot be entirely expressed in terms of \mathbf{g} because the fundamental dynamical variable is \mathbf{v} . The role of (16) is to reveal that if we analyse gravitational phenomena we will usually find that the matter density ρ is insufficient to account for the observed \mathbf{g} . Until recently this failure of Newtonian gravity has been explained away as being caused by some unknown and undetected “dark matter” density. Eqn.(16) shows that to the contrary it is a dynamical property of 3-space itself.

Another common misunderstanding is that the success of the Dirac equation implies that a preferred frame cannot exist. This belief is again easily demolished. The generalised Dirac equation which uses the Galilean class of space-time coordinates is

$$i\hbar \frac{\partial \psi}{\partial t} = -i\hbar \left(c\vec{\alpha} \cdot \nabla + \mathbf{v} \cdot \nabla + \frac{1}{2} \nabla \cdot \mathbf{v} \right) \psi + \beta mc^2 \psi, \quad (17)$$

where $\vec{\alpha}$ and β are the usual Dirac matrices. This equation shows that the Dirac spinor propagates wrt to the 3-space, and that there are dynamical effects associated with that that are not in the generalised Schrödinger equation (13). As shown elsewhere (17) gives rise to relativistic gravitational effects*, that go beyond those in (14).

*Meaning when an object has speed comparable to c wrt the 3-space.

5 Galilean covariant electromagnetic theory

Hertz in 1890 [18] noted that Maxwell had overlooked the velocity field that accompanies time derivatives, as in (1), and presented an improved formalism, and the minimal source-free form is

$$\begin{aligned} \mu \left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \mathbf{H} &= -\nabla \times \mathbf{E}, \\ \epsilon \left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \mathbf{E} &= +\nabla \times \mathbf{H}, \\ \nabla \cdot \mathbf{H} &= \mathbf{0}, \quad \nabla \cdot \mathbf{E} = \mathbf{0}, \end{aligned} \quad (18)$$

with $\mathbf{v}(\mathbf{r}, t)$ being the dynamical 3-space velocity field as measured* by some observer using time and space coordinates $\{t, x, y, z\}$, although Hertz did not consider a time and space dependent \mathbf{v} . Again for uniform and time-independent \mathbf{v} (18) has plane wave solutions

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}, \quad \mathbf{H}(\mathbf{r}, t) = \mathbf{H}_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}, \quad (19)$$

$$\omega(\mathbf{k}, \mathbf{v}) = c|\vec{\mathbf{k}}| + \mathbf{v} \cdot \mathbf{k}, \quad \text{where } c = 1/\sqrt{\mu\epsilon}. \quad (20)$$

Then the EM group velocity is

$$\mathbf{v}_{EM} = \vec{\nabla}_{\mathbf{k}} \omega(\mathbf{k}, \mathbf{v}) = c\hat{\mathbf{k}} + \mathbf{v}. \quad (21)$$

So, like the analogy of sound, the velocity of EM radiation \mathbf{v}_{EM} has magnitude c only with respect to the 3-space, and in general not with respect to the observer if the observer is moving through that 3-space, as experiment has indicated again and again, as discussed above. Eqns.(18) give, for uniform \mathbf{v} ,

$$\begin{aligned} \left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right)^2 \mathbf{E} &= c^2 \nabla^2 \mathbf{E}, \\ \left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right)^2 \mathbf{H} &= c^2 \nabla^2 \mathbf{H}. \end{aligned} \quad (22)$$

on using the identity $\nabla \times (\nabla \times \mathbf{E}) = -\nabla^2 \mathbf{E} + \nabla(\nabla \cdot \mathbf{E})$ and $\nabla \cdot \mathbf{E} = 0$, and similarly for the \mathbf{H} field. Transforming to the Minkowski-Einstein T, X, Y, Z coordinates using (8) and (9) we obtain the form of the source-free “standard” Maxwell equations

$$\frac{\partial^2 \mathbf{E}}{\partial T^2} = c^2 \nabla^2 \mathbf{E}, \quad \frac{\partial^2 \mathbf{H}}{\partial T^2} = c^2 \nabla^2 \mathbf{H}, \quad (23)$$

which is again covariant under Lorentz transformation (11). It is important to emphasize that the transformation from the Galilean covariant Hertz-Maxwell equations (18) to the Lorentz covariant Maxwell equations (23) is *exact*. It is usually argued that the Galilean transformations (5) are the non-relativistic limit of the Lorentz transformations (11). While this is technically so, as seen by taking the limit $v/c \rightarrow 0$, this

*Earth based light speed anisotropy experiments show that v has value $\approx 420 \pm 30$ km/s in a known direction [20], and is not to be confused with the CMB velocity.

misses the key point that they are related by the new mapping in (8). Also we note that for the Galilean space-time class the speed of light is anisotropic, while it is isotropic for the Minkowski-Einstein space-time class. It is only experiment that can decide which of the two classes of coordinates is the more valid space-time coordinate system. As noted above, and since 1887, experiments have detected that the speed of light is indeed anisotropic.

Again when using the Minkowski-Einstein coordinates there is now no reference to the underlying dynamical 3-space system — for an observer using this class of space and time coordinates the speed of light relative to the observer is always c and so invariant. We could then be tricked into introducing a “spacetime” construct with pseudo-Riemannian metric $ds^2 = c^2 dT^2 - d\mathbf{R}^2$, and light cones along which $ds^2 = 0$. As well pairs of spacetime events could be classified into either time-like or space-like, with the time ordering of spacelike events not being uniquely defined. This loss of the notion of simultaneity is merely a consequence of the degenerate nature of the Minkowski-Einstein spacetime coordinates. This has confounded progress in physics for more than a century.

Hence the Minkowski-Einstein space-time coordinates are degenerate in that they map out the existence of the dynamical 3-space. So the development of 20th century physics has been misled by two immensely significant “accidents”, 1st that Maxwell failed to include the velocity \mathbf{v} , and the 2nd that the Michelson interferometer in gas-mode is some 2000 times less sensitive than Michelson had assumed, and that the observed fringe shifts actually indicate a large value for v in excess of 300km/s. These two accidents stopped physics from discovering the existence of a dynamical 3-space, until recently, and that the dynamical 3-space displays wave effects. Also again this transformation between the two classes of space-time coordinates explicitly demonstrates that “Lorentz covariance” coexists with a preferred frame, contrary to the aims of the experiments in [17]. Furthermore vacuum-mode Michelson interferometers, such as the vacuum cavity resonators, cannot even detect the long-standing light speed anisotropy. We can apply the inverse mapping, from the Minkowski-Einstein class to the Galilean class of coordinates, but in doing so we have lost the value of the velocity field. In this sense the Minkowski-Einstein class is degenerate — it cannot be used to analyse light speed anisotropy experiments for example.

6 Conclusions

We have reported herein the discovery of an exact and invertible mapping from Galilean time and space coordinates to Minkowski-Einstein spacetime coordinates. This mapping removes the effects of the velocity of the dynamical 3-space relative to an observer, and so in this sense the Minkowski-Einstein coordinates are degenerate — they stop the usual

Special Relativity formalism from being able to say anything about the existence of a preferred frame, a real 3-space, and from describing experiments that have detected light speed anisotropy. The Minkowski-Einstein formalism has nevertheless been very successful in describing other effects. The spacetime formalism, with its spacetime metric and Lorentz covariance, is really an artifact of the degenerate Minkowski-Einstein coordinates, and we have shown how one may unravel these mathematical artifacts, and display the underlying dynamics. The new mapping shows that relativistic effects are caused by motion relative to an actual 3-space — and which has been observed for more than 120 years. This was Lorentz's proposition. The belief that spacetime actually described reality has led to numerous misconceptions about the nature of space and time. These are distinct phenomena, and are not fused into some 4-dimensional entity. Indeed time is now seen to have a cosmic significance, and that all observers can measure that time — for by measuring their local absolute speed relative to their local 3-space they can correct the ticking rate of their clocks to remove the local time dilation effect, and so arrive at a measure of the ticking rate of cosmic time*. This changes completely how we might consider modelling deeper reality — one such proposition is *Process Physics* [19–21].

The Special Relativity formalism asserts that only relative descriptions of phenomena between two or more observers have any meaning. In fact we now understand that all effects are dynamically and observationally relative to an ontologically real, that is, detectable dynamical 3-space. Ironically this situation has always been known as an “absolute effect”. The most extraordinary outcome of recent discoveries is that a dynamical 3-space exists, and that from the beginning of Physics this has been missed — that a most fundamental aspect of reality has been completely overlooked.

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*Uncorrected Earth-based clocks lose approximately 0.085s per day compared to cosmic time, because $v \approx 420$ km/s.

Derivation of the Newton's Law of Gravitation Based on a Fluid Mechanical Singularity Model of Particles

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The main purpose of this paper is to seek a mechanical interpretation of gravitational phenomena. We suppose that the universe may be filled with a kind of fluid which may be called the $\Omega(0)$ substratum. Thus, the inverse-square law of gravitation is derived by methods of hydrodynamics based on a sink flow model of particles. The first feature of this theory of gravitation is that the gravitational interactions are transmitted by a kind of fluidic medium. The second feature is the time dependence of gravitational constant G and gravitational mass. The Newton's law of gravitation is arrived if we introduce an assumption that G and the masses of particles are changing so slowly that they can be treated as constants.

1 Introduction

The Newton's law of gravitation can be written as

$$\mathbf{F}_{12} = -G \frac{m_1 m_2}{r^2} \hat{\mathbf{r}}_{21}, \quad (1)$$

where m_1 and m_2 are the masses of two particles, r is the distance between the two particles, G is the gravitational constant, \mathbf{F}_{12} is the force exerted on the particle with mass m_2 by the particle with mass m_1 , $\hat{\mathbf{r}}_{21}$ denotes the unit vector directed outward along the line from the particle with mass m_1 to the particle with mass m_2 .

The main purpose of this paper is to derive the Newton's law of gravitation by means of fluid mechanics based on sink flow model of particles.

The motive of this paper is to seek a mechanism of gravitational phenomena. The reasons why new models of gravity are interesting may be summarized as follows.

Firstly, there exists some astronomical phenomena that could not be interpreted by the present theories of gravitation, for instance, the Titius-Bode law [1]. New theories of gravity may view these problems from new angles.

Secondly, whether the gravitational constant G depends on time and space is still unknown [2–8]. It is known that the gravitational constant G is a constant in the Newton's theory of gravitation and in theory of general relativity.

Thirdly, the mechanism of the action-at-a-distance gravitation remains an unsolved problem in physics for more than 300 years [9–11]. Although theory of general relativity is a field theory of gravity [12], the concept of field is different from that of continuum mechanics [13–16] because of the absence of a continuum in theory of general relativity. Thus, theory of general relativity can only be regarded as a phenomenological theory of gravity.

Fourthly, we do not have a satisfactory quantum theory of gravity presently [17–21]. One of the challenges in theoretic-

all physics is to reconcile quantum theory and theory of general relativity [17, 22]. New theories of gravity may open new ways to solve this problem.

Fifthly, one of the puzzles in physics is the problem of dark matter and dark energy [23–31]. New theories of gravity may provide new methods to attack this problem [24, 25].

Finally, we do not have a successful unified field theory presently. Great progress has been made towards an unification of the four fundamental interactions in the universe in the 20th century. However, gravitation is still not unified successfully. New theories of gravity may shed some light on this puzzle.

To conclude, it seems that new considerations on gravitation is needed. It is worthy keeping an open mind with respect to all the theories of gravity before the above problems been solved.

Now let us briefly review the long history of mechanical interpretations of gravitational phenomena. Many philosophers and scientists, such as Laozi [32], Thales, Anaximenes, believed that everything in the universe is made of a kind of fundamental substance [9]. Descartes was the first to bring the concept of aether into science by suggesting that it has mechanical properties [9]. Since the Newton's law of gravitation was published in 1687 [33], this action-at-a-distance theory was criticized by the French Cartesian [9]. Newton admitted that his law did not touch on the mechanism of gravitation [34]. He tried to obtain a derivation of his law based on Descartes' scientific research program [33]. Newton himself even suggested an explanation of gravity based on the action of an aetherial medium pervading the space [34, 35]. Euler attempted to explain gravity based on some hypotheses of a fluidic aether [9].

In a remarkable paper published in 1905, Einstein abandoned the concept of aether [36]. However, Einstein's assertion did not cease the explorations of aether [9, 37–46]. Einstein changed his view later and introduced his new con-

cept of ether [47, 48]. I regret to admit that it is impossible for me to mention all the works related to this field in history. Adolphe Martin and Roy Keys [49–51] proposed a fluidic cosmic gas model of vacuum to explain the physical phenomena such as electromagnetism, gravitation, quantum mechanics and the structure of elementary particles.

Inspired by the aforementioned thoughts and others [52–56], we show that the Newton's law of gravitation is derived based on the assumption that all the particles are made of singularities of a kind of ideal fluid.

During the preparation of the manuscript, I noticed that John C. Taylor had proposed an idea that the inverse-square law of gravitation may be explained based on the concept of source or sink [65].

2 Forces acting on sources and sinks in ideal fluids

The purpose of this section is to calculate the forces between sources and sinks in inviscid incompressible fluids which is called ideal fluids usually.

Suppose the velocity field \mathbf{u} of an ideal fluid is irrotational, then we have [16, 54–59],

$$\mathbf{u} = \nabla \phi, \quad (2)$$

where ϕ is the velocity potential, $\nabla = \mathbf{i} \frac{\partial}{\partial x} + \mathbf{j} \frac{\partial}{\partial y} + \mathbf{k} \frac{\partial}{\partial z}$ is the Hamilton operator.

It is known that the equation of mass conservation of an ideal fluid becomes Laplace's equation [54–59],

$$\nabla^2 \phi = 0, \quad (3)$$

where ϕ is velocity potential, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the Laplace operator.

Using spherical coordinates (r, θ, φ) , a general form of solution of Laplace's equation (3) can be obtained by separation of variables as [56]

$$\phi(r, \theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos \theta), \quad (4)$$

where A_l and B_l are arbitrary constants, $P_l(x)$ are Legendre's function of the first kind which is defined as

$$P_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l} (x^2 - 1)^l. \quad (5)$$

If there exists a velocity field which is continuous and finite at all points of the space, with the exception of individual isolated points, then these isolated points are called singularities usually.

Definition 1 Suppose there exists a singularity at point $P_0 = (x_0, y_0, z_0)$. If the velocity field of the singularity at point $P = (x, y, z)$ is

$$\mathbf{u}(x, y, z, t) = \frac{Q}{4\pi r^2} \hat{\mathbf{r}}, \quad (6)$$

where $r = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}$, $\hat{\mathbf{r}}$ denotes the unit vector directed outward along the line from the singularity to the point $P = (x, y, z)$, then we call this singularity a source if $Q > 0$ or a sink if $Q < 0$. Q is called the strength of the source or sink.

Suppose a static point source with strength Q locates at the origin $(0, 0, 0)$. In order to calculate the volume leaving the source per unit time, we may enclose the source with an arbitrary spherical surface S with radius a . A calculation shows that

$$\oint_S \mathbf{u} \cdot \mathbf{n} dS = \oint_S \frac{Q}{4\pi a^2} \hat{\mathbf{r}} \cdot \mathbf{n} dS = Q, \quad (7)$$

where \mathbf{n} denotes the unit vector directed outward along the line from the origin of the coordinates to the field point (x, y, z) . Equation (7) shows that the strength Q of a source or sink evaluates the volume of the fluid leaving or entering a control surface per unit time.

From (4), we see that the velocity potential $\phi(r, \theta)$ of a source or sink is a solution of Laplace's equation $\nabla^2 \phi = 0$.

Theorem 2 Suppose (1) there exists an ideal fluid (2) the ideal fluid is irrotational and barotropic, (3) the density ρ is homogeneous, that is $\partial \rho / \partial x = \partial \rho / \partial y = \partial \rho / \partial z = \partial \rho / \partial t = 0$, (4) there are no external body forces exerted on the fluid, (5) the fluid is unbounded and the velocity of the fluid at the infinity is approaching to zero. Suppose a source or sink is stationary and is immersed in the ideal fluid. Then, there is a force

$$\mathbf{F}_Q = -\rho Q \mathbf{u}_0 \quad (8)$$

exerted on the source by the fluid, where ρ is the density of the fluid, Q is the strength of the source or the sink, \mathbf{u}_0 is the velocity of the fluid at the location of the source induced by all means other than the source itself.

Proof Only the proof of the case of a source is needed. Let us select the coordinates that is attached to the static fluid at the infinity.

We set the origin of the coordinates at the location of the source. We surround the source by an arbitrary small spherical surface S . The surface S is centered at the origin of the coordinates with radius r . The outward unit normal to the surface S is denoted by \mathbf{n} . Let $\tau(t)$ denotes the mass system of fluid enclosed in the volume between the surface S and the source at time t . Let \mathbf{F}_Q denotes the hydrodynamic force exerted on the source by the mass system τ , then a reaction of this force must act on the the fluid enclosed in the mass system τ . Let \mathbf{F}_S denotes the hydrodynamic force exerted on the mass system τ due to the pressure distribution on the surface S , \mathbf{K} denotes momentum of the mass system τ .

As an application of the Newton's second law of motion to the mass system τ , we have

$$\frac{D\mathbf{K}}{Dt} = -\mathbf{F}_Q + \mathbf{F}_S, \quad (9)$$

where D/Dt represents the material derivative in the lagrangian system [16, 54–59]. The expressions of the momentum \mathbf{K} and the force \mathbf{F}_S are

$$\mathbf{K} = \iiint_{\tau} \rho \mathbf{u} dV, \quad \mathbf{F}_S = \oint_S (-p) \mathbf{n} dS, \quad (10)$$

where the first integral is volume integral, the second integral is surface integral, \mathbf{n} denotes the unit vector directed outward along the line from the origin of the coordinates to the field point (x, y, z) .

Since the velocity field is irrotational, we have the following relation

$$\mathbf{u} = \nabla \phi, \quad (11)$$

where ϕ is the velocity potential.

According to Ostrogradsky–Gauss theorem (see, for instance, [54–56, 58, 59]), we have

$$\iiint_{\tau} \rho \mathbf{u} dV = \iiint_{\tau} \rho \nabla \phi dV = \oint_S \rho \phi \mathbf{n} dS. \quad (12)$$

Note that the mass system τ does not include the singularity at the origin. According to Reynolds’ transport theorem [54–56, 58, 59], we have

$$\frac{D}{Dt} \iiint_{\tau} \rho \mathbf{u} dV = \frac{\partial}{\partial t} \iiint_V \rho \mathbf{u} dV + \oint_S \rho \mathbf{u} (\mathbf{u} \cdot \mathbf{n}) dS, \quad (13)$$

where V is the volume fixed in space which coincide with the mass system $\tau(t)$ at time t , that is $V = \tau(t)$.

Then, using (13), (10) and (12), we have

$$\frac{D\mathbf{K}}{Dt} = \oint_S \rho \frac{\partial \phi}{\partial t} \mathbf{n} dS + \oint_S \rho \mathbf{u} (\mathbf{u} \cdot \mathbf{n}) dS. \quad (14)$$

According to Lagrange–Cauchy integral [54–56, 58, 59], we have

$$\frac{\partial \phi}{\partial t} + \frac{(\nabla \phi)^2}{2} + \frac{p}{\rho} = f(t), \quad (15)$$

where $f(t)$ is an arbitrary function of time t . Since the velocity \mathbf{u} of the fluid at the infinity is approaching to zero, and noticing (4), $\phi(t)$ must be of the following form

$$\phi(r, \theta, t) = \sum_{l=0}^{\infty} \frac{B_l(t)}{r^{l+1}} P_l(\cos \theta), \quad (16)$$

where $B_l(t), l \geq 0$ are functions of time t . Thus, we have the following estimations at the infinity of the velocity field

$$\phi = O\left(\frac{1}{r}\right), \quad \frac{\partial \phi}{\partial t} = O\left(\frac{1}{r}\right), \quad r \rightarrow \infty, \quad (17)$$

where $\varphi(x) = O(\psi(x)), x \rightarrow a$ stands for $\overline{\lim}_{x \rightarrow a} |\varphi(x)| / \psi(x) = k, (0 \leq k < +\infty)$.

Applying (15) at the infinity and using (17), we have $|\mathbf{u}| \rightarrow 0, \partial \phi / \partial t \rightarrow 0$ and $p = p_{\infty}$, where p_{∞} is a constant. Thus, $f(t) = p_{\infty} / \rho$. Therefore, according to (15), we have

$$p = p_{\infty} - \rho \frac{\partial \phi}{\partial t} - \frac{\rho (\mathbf{u} \cdot \mathbf{u})}{2}. \quad (18)$$

Using (10) and (18), we have

$$\mathbf{F}_S = \oint_S \rho \frac{\partial \phi}{\partial t} \mathbf{n} dS + \oint_S \frac{\rho (\mathbf{u} \cdot \mathbf{u}) \mathbf{n}}{2} dS. \quad (19)$$

Using (9), (14), (19), we have

$$\mathbf{F}_Q = \oint_S \left[\frac{1}{2} \rho (\mathbf{u} \cdot \mathbf{u}) \mathbf{n} - \rho \mathbf{u} (\mathbf{u} \cdot \mathbf{n}) \right] dS. \quad (20)$$

Now let us calculate this velocity \mathbf{u} in order to obtain \mathbf{F}_Q . Since the velocity field induced by the source Q is (6), then according to the superposition principle of velocity field of ideal fluids, the velocity on the surface S is

$$\mathbf{u} = \frac{Q}{4\pi r^2} \mathbf{n} + \mathbf{u}_0, \quad (21)$$

where \mathbf{n} denotes the unit vector directed outward along the line from the origin of the coordinates to the field point (x, y, z) . Using (20) and (21), we have

$$\mathbf{F}_Q = \rho \oint_S \left[-\frac{Q^2}{32\pi^2 r^4} \mathbf{n} + \frac{1}{2} (\mathbf{u}_0 \cdot \mathbf{u}_0) \mathbf{n} - \frac{Q}{4\pi r^2} \mathbf{u}_0 - (\mathbf{u}_0 \cdot \mathbf{n}) \mathbf{u}_0 \right] dS. \quad (22)$$

Since the radius r can be arbitrarily small, the velocity \mathbf{u}_0 can be treated as a constant in the integral of (22). Thus, (22) turns out to be

$$\mathbf{F}_Q = -\rho \oint_S \frac{Q}{4\pi r^2} \mathbf{u}_0 dS. \quad (23)$$

Since again \mathbf{u}_0 can be treated as a constant, (23) turns out to be (8). This completes the proof. \square

Remark Lagally [52], Landweber and Yih [53, 54], Faber [55] and Currie [56] obtained the same result of Theorem 2 for the special case where the velocity field is steady.

Theorem 2 only considers the situation that the sources or sinks are at rest. Now let us consider the case that the sources or sinks are moving in the fluid.

Theorem 3 Suppose the presuppositions (1), (2), (3), (4) and (5) in Theorem 2 are valid and a source or a sink is moving in the fluid with a velocity \mathbf{v}_s , then there is a force

$$\mathbf{F}_Q = -\rho Q (\mathbf{u}_f - \mathbf{v}_s) \quad (24)$$

is exerted on the source by the fluid, where ρ is the density of the fluid, Q is the strength of the source or the sink, \mathbf{u}_f is the

velocity of the fluid at the location of the source induced by all means other than the source itself.

Proof The velocity of the fluid relative to the source at the location of the source is $\mathbf{u}_f - \mathbf{v}_s$. Let us select the coordinates that is attached to the source and set the origin of the coordinates at the location of the source. Then (24) can be arrived following the same procedures in the proof of Theorem 2. \square

Applying Theorem 3 to the situation that a source or sink is exposed to the velocity field of another source or sink, we have:

Corollary 4 Suppose the presuppositions (1), (2), (3), (4) and (5) in Theorem 2 are valid and a source or a sink with strength Q_2 is exposed to the velocity field of another source or sink with strength Q_1 , then the force \mathbf{F}_{21} exerted on the singularity with strength Q_2 by the velocity field of the singularity with strength Q_1 is

$$\mathbf{F}_{21} = -\rho Q_2 \frac{Q_1}{4\pi r^2} \hat{\mathbf{r}}_{21} + \rho Q_2 \mathbf{v}_2, \quad (25)$$

where $\hat{\mathbf{r}}_{21}$ denotes the unit vector directed outward along the line from the singularity with strength Q_1 to the singularity with strength Q_2 , r is the distance between the two singularities, \mathbf{v}_2 is the velocity of the source with strength Q_2 .

3 Derivation of inverse-square-law of gravitation

Since quantum theory shows that vacuum is not empty and has physical effects, e.g., the Casimir effect [45, 60–62], it is valuable to probe vacuum by introducing the following hypotheses:

Assumption 5 Suppose the universe is filled by an ideal fluid named $\Omega(0)$ substratum; the ideal fluid fulfil the conditions (2), (3), (4), (5) in Theorem 2.

This fluid may be named $\Omega(0)$ substratum in order to distinguish with Cartesian aether. Following Einstein, Infeld and Hoffmann, who introduced the idea that particles may be looked as singularities in fields [63, 64], and noticing (25), it is nature to introduce the following:

Assumption 6 All the microscopic particles were made up of a kind of elementary sinks of $\Omega(0)$ substratum. These elementary sinks were created simultaneously. The initial masses and the strengths of the elementary sinks are the same.

We may call these elementary sinks as monads.

Suppose a particle with mass m is composed of N monads. Then, according to Assumption 6, we have:

$$m_0(t) = m_0(0) + \rho q_0 t, \quad (26)$$

$$Q = -Nq_0, \quad m(t) = Nm_0(t) = -\frac{Q}{q_0} m_0(t), \quad (27)$$

$$\frac{dm_0}{dt} = \rho q_0, \quad \frac{dm}{dt} = -\rho Q, \quad (28)$$

where $m_0(t)$ is the mass of monad at time t , $-q_0 (q_0 > 0)$ is the strength of a monad, $m(t)$ is the mass of a particle at time

t , Q is the strength of the particle, N is the number of monads that make up the particle, ρ is the density of the $\Omega(0)$ substratum, $t \geq 0$.

From (28), we see that the mass m_0 of a monad is increasing since q_0 evaluates the volume of the $\Omega(0)$ substratum fluid entering the monad per unit time. From (28), we also see that the mass of a monad or a particle is increasing linearly.

Based on Assumption 5 and Assumption 6, the motion of a particle is determined by:

Theorem 7 The equation of motion of a particle is

$$m(t) \frac{d\mathbf{v}}{dt} = \frac{\rho q_0}{m_0(t)} m(t) \mathbf{u} - \frac{\rho q_0}{m_0(t)} m(t) \mathbf{v} + \mathbf{F}, \quad (29)$$

where $m_0(t)$ is the mass of monad at time t , $-q_0$ is the strength of a monad, $m(t)$ is the mass of a particle at time t , \mathbf{v} is the velocity of the particle, \mathbf{u} is the velocity of the $\Omega(0)$ substratum at the location of the particle induced by all means other than the particle itself, \mathbf{F} denotes other forces.

Proof Applying the Newton's second law and Theorem 3 to this particle, we have $m d\mathbf{v}/dt = -\rho Q (\mathbf{u} - \mathbf{v}) + \mathbf{F}$. Noticing (27), we get (29). \square

Formula (29) shows that there exists a universal damping force

$$\mathbf{F}_d = -\frac{\rho q_0}{m_0} m \mathbf{v} \quad (30)$$

exerted on each particle.

Now let us consider a system consists of two particles. Based on Assumption 6, applying Theorem 7 to this system, we have:

Corollary 8 Suppose there is a system consists of two particles and there are no other forces exerted on the particles, then the equations of motion of this system are

$$m_1 \frac{d\mathbf{v}_1}{dt} = -\frac{\rho q_0}{m_0} m_1 \mathbf{v}_1 - \frac{\rho q_0^2}{4\pi m_0^2} \frac{m_1 m_2}{r^2} \hat{\mathbf{r}}_{12} \quad (31)$$

$$m_2 \frac{d\mathbf{v}_2}{dt} = -\frac{\rho q_0}{m_0} m_2 \mathbf{v}_2 - \frac{\rho q_0^2}{4\pi m_0^2} \frac{m_1 m_2}{r^2} \hat{\mathbf{r}}_{21}, \quad (32)$$

where $m_{i=1,2}$ is the mass of the particles, $\mathbf{v}_{i=1,2}$ is the velocity of the particles, m_0 is the mass of a monad, $-q_0$ is the strength of a monad, ρ is the density of the $\Omega(0)$ substratum, $\hat{\mathbf{r}}_{12}$ denotes the unit vector directed outward along the line from the particle with mass $m_2(t)$ to the particle with mass $m_1(t)$, $\hat{\mathbf{r}}_{21}$ denotes the unit vector directed outward along the line from the particle with mass $m_1(t)$ to the particle with mass $m_2(t)$.

Ignoring the damping forces in (32), we have:

Corollary 9 Suppose (1) $\mathbf{v}_{i=1,2} \ll \mathbf{u}_{i=1,2}$, where \mathbf{v}_i is the velocity of the particle with mass m_i , \mathbf{u}_i is the velocity of the $\Omega(0)$ substratum at the location of the particle with mass m_i induced by the other particle, (2) there are no other forces exerted on the particles, then the force $\mathbf{F}_{21}(t)$ exerted on the

particle with mass $m_2(t)$ by the velocity field of $\Omega(0)$ substratum induced by the particle with mass $m_1(t)$ is

$$\mathbf{F}_{21}(t) = -G(t) \frac{m_1(t)m_2(t)}{r^2} \hat{\mathbf{r}}_{21}, \quad (33)$$

where $G = \rho q_0^2 / (4\pi m_0^2(t))$, $\hat{\mathbf{r}}_{21}$ denotes the unit vector directed outward along the line from the particle with mass $m_1(t)$ to the particle with mass $m_2(t)$, r is the distance between the two particles.

Corollary 9 is coincide with the Newton's inverse-square-law of gravitation (1) except for two differences. The first difference is that $m_{i=1,2}$ are constants in the Newton's law (1) while in (1) while in Corollary are functions of time t . The second difference is that G is a t . The second difference is that G is a constant in the Newton's

Let us now introduce an assumption that G and the masses of particles are changing so slowly relative to the time scale of human beings that they can be treated as constants approximately. Thus, the Newton's law (1) of gravitation may be considered as a result of Corollary 9 based on this assumption.

4 Superposition principle of gravitational field

The definition of gravitational field \mathbf{g} of a particle with mass m is $\mathbf{g} = \mathbf{F}/m_{test}$, where m_{test} is the mass of a test point mass, \mathbf{F} is the gravitational force exerted on the test point mass by the gravitational field of the particle with mass m . Based on Theorem 7 and Corollary 9, we have

$$\mathbf{g} = \frac{\rho q_0}{m_0} \mathbf{u}, \quad (34)$$

where ρ is the density of the $\Omega(0)$ substratum, m_0 is the mass of a monad, q_0 is the strength of a monad, \mathbf{u} is the velocity of the $\Omega(0)$ substratum at the location of the test point mass induced by the particle mass m . From (34), we see that the superposition principle of gravitational field is deduced from the superposition theorem of the velocity field of ideal fluids.

5 Time dependence of gravitational constant G and mass

According to Assumption 6 and Corollary 9, we have we have

$$G = \frac{\rho q_0^2}{4\pi m_0^2(t)}, \quad (35)$$

where $m_0(t)$ is the mass of monad at time t , $-q_0$ is the strength of a monad, ρ is the density of the $\Omega(0)$ substratum. The time dependence of gravitational mass can be seen from (35) and (28).

6 Conclusion

We suppose that the universe may be filled with a kind of fluid which may be called the $\Omega(0)$ substratum. Thus, the inverse-

square law of gravitation is derived by methods of hydrodynamics based on a sink flow model of particles. There are two features of this theory of gravitation. The first feature is that the gravitational interactions are transmitted by a kind of fluidic medium. The second feature is the time dependence of gravitational constant and gravitational mass. The Newton's law of gravitation is arrived if we introduce an assumption that G and the masses of particles are changing so slowly that they can be treated as constants. As a byproduct, it is shown that there exists a universal damping force exerted on each particle.

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A Method of Successive Approximations in the Framework of the Geometrized Lagrange Formalism

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It is shown that in the weak field approximation the new geometrical approach can lead to the linear field equations for the several independent fields. For the stronger fields and in the second order approximation the field equations become non-linear, and the fields become dependent. This breaks the superposition principle for every separate field and produces the interaction between different fields. The unification of the gravitational and electromagnetic field theories is performed in frames of the geometrical approach in the pseudo-Riemannian space and in the curved Berwald-Moor space.

1 Introduction

In paper [1] the new (geometrical) approach was suggested for the field theory. It is applicable for any Finsler space [2] for which in any point of the main space x^1, x^2, \dots, x^n the indicatrix volume $(V_{ind}(x^1, x^2, \dots, x^n))_{ev}$ can be defined, provided the tangent space is Euclidean. Then the action I for the fields present in the metric function of the Finsler space is defined within the accuracy of a constant factor as a volume of a certain n -dimensional region V :

$$I = \text{const} \cdot \int_V^{(n)} \frac{dx^1 dx^2 \dots dx^n}{(V_{ind}(x^1, x^2, \dots, x^n))_{ev}}. \quad (1)$$

Thus, the field Lagrangian is defined in the following way

$$L = \text{const} \cdot \frac{1}{(V_{ind}(x^1, x^2, \dots, x^n))_{ev}}. \quad (2)$$

In papers [3,4] the spaces conformally connected with the Minkowski space and with the Berwald-Moor space were regarded. These spaces have a single scalar field for which the field equation was written and the particular solutions were found for the spherical symmetry and for the rhombododecaedron symmetry of the space.

The present paper is a continuation of those papers dealing with the study and development of the geometric field theory.

2 Pseudo-Riemannian space with the signature (+ ---)

Let us consider the pseudo-Riemannian space with the signature (+ ---) and select the Minkowski metric tensor $\overset{\circ}{g}_{ij}$ in the metric tensor $g_{ij}(x)$, of this space explicitly

$$g_{ij}(x) = \overset{\circ}{g}_{ij} + h_{ij}(x). \quad (3)$$

Let us suppose that the field $h_{ij}(x)$ is weak, that is

$$|h_{ij}(x)| \ll 1. \quad (4)$$

According to [1], the Lagrangian for a pseudo-Riemannian space with the signature (+ ---) is equal to

$$L = \sqrt{-\det(g_{ij})}. \quad (5)$$

Let us calculate the value of $[-\det(g_{ij})]$ within the accuracy of $|h_{ij}(x)|^2$:

$$-\det(g_{ij}) \simeq 1 + L_1 + L_2, \quad (6)$$

$$L_1 = \overset{\circ}{g}^{ij} h_{ij} \equiv h_{00} - h_{11} - h_{22} - h_{33}, \quad (7)$$

$$L_2 = -h_{00}(h_{11} + h_{22} + h_{33}) + h_{11}h_{22} + h_{11}h_{33} + h_{22}h_{33} - h_{12}^2 - h_{13}^2 - h_{23}^2 + h_{03}^2 + h_{02}^2 + h_{01}^2. \quad (8)$$

The last formula can be rewritten in a more convenient way

$$L_2 = - \left| \begin{array}{cc} h_{00} & h_{01} \\ h_{01} & h_{11} \end{array} \right| - \left| \begin{array}{cc} h_{00} & h_{02} \\ h_{02} & h_{22} \end{array} \right| - \left| \begin{array}{cc} h_{00} & h_{03} \\ h_{03} & h_{33} \end{array} \right| + \left| \begin{array}{cc} h_{11} & h_{12} \\ h_{12} & h_{22} \end{array} \right| + \left| \begin{array}{cc} h_{11} & h_{13} \\ h_{13} & h_{33} \end{array} \right| + \left| \begin{array}{cc} h_{22} & h_{23} \\ h_{23} & h_{33} \end{array} \right|, \quad (9)$$

then

$$L \simeq 1 + \frac{1}{2}L_1 + \frac{1}{2} \left[L_2 - \frac{1}{4}L_1^2 \right]. \quad (10)$$

To obtain the field equations in the first order approximation, one should use the Lagrangian L_1 , and to do the same in the second order approximation — the Lagrangian $(L_1 + L_2 - \frac{1}{4}L_1^2)$.

3 Scalar field

For the single scalar field $\varphi(x)$ the simplest representation of tensor $h_{ij}(x)$ has the form:

$$h_{ij}(x) \equiv h_{ij}^{(\varphi)}(x) = \pm \frac{\partial \varphi}{\partial x^i} \frac{\partial \varphi}{\partial x^j}. \quad (11)$$

That is why

$$L_\varphi = \sqrt{-\det(g_{ij})} = \sqrt{1 \pm L_1} \simeq 1 \pm \frac{1}{2} L_1 - \frac{1}{8} L_1^2, \quad (12)$$

where

$$L_1 = \left(\frac{\partial \varphi}{\partial x^0} \right)^2 - \left(\frac{\partial \varphi}{\partial x^1} \right)^2 - \left(\frac{\partial \varphi}{\partial x^2} \right)^2 - \left(\frac{\partial \varphi}{\partial x^3} \right)^2. \quad (13)$$

In the first order approximation, we can use the Lagrangian L_1 to obtain the following field equation

$$\frac{\partial^2 \varphi}{\partial x^0 \partial x^0} - \frac{\partial^2 \varphi}{\partial x^1 \partial x^1} - \frac{\partial^2 \varphi}{\partial x^2 \partial x^2} - \frac{\partial^2 \varphi}{\partial x^3 \partial x^3} = 0, \quad (14)$$

which presents the wave equation. The stationary field that depends only on the radius

$$r = \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2}, \quad (15)$$

will satisfy the equation

$$\frac{d}{dr} \left(r^2 \frac{d\varphi}{dr} \right) = 0, \quad (16)$$

the integration of which gives

$$\frac{d\varphi}{dr} = -C_1 \frac{1}{r^2} \Rightarrow \varphi(r) = C_0 + C_1 \frac{1}{r}. \quad (17)$$

In the second order approximation one should use the Lagrangian $(L_1 - \frac{1}{4} L_1^2)$ to obtain the field equation in the second order approximation

$$\overset{\circ}{g}{}^{ij} \frac{\partial}{\partial x^i} \left[\left(\pm 1 - \frac{1}{2} L_1 \right) \frac{\partial \varphi}{\partial x^j} \right] = 0; \quad (18)$$

this equation is already non-linear.

The strict field equation for the tensor $h_{ij}(x)$ (11) is

$$\overset{\circ}{g}{}^{ij} \frac{\partial}{\partial x^i} \left(\frac{\frac{\partial \varphi}{\partial x^j}}{\sqrt{1 \pm L_1}} \right) = 0, \quad (19)$$

then the stationary field depending only on the radius must satisfy the equation

$$\frac{d}{dr} \left(r^2 \frac{\frac{d\varphi}{dr}}{\sqrt{1 \mp \left(\frac{d\varphi}{dr} \right)^2}} \right) = 0. \quad (20)$$

Integrating it, we get

$$\begin{aligned} \frac{d\varphi}{dr} &= -\frac{C_1}{\sqrt{r^4 \pm C_1^2}} \Rightarrow \\ \Rightarrow \varphi(r) &= C_0 + \int_r^\infty \frac{C_1}{\sqrt{r^4 \pm C_1^2}} dr. \end{aligned} \quad (21)$$

The field with the upper sign and the field with the lower sign differ qualitatively: the upper sign “+” in Eq. (11) gives a finite field with no singularity in the whole space, the lower sign “-” in Eq. (11) gives a field defined everywhere but for the spherical region

$$0 \leq r \leq \sqrt{|C_1|}, \quad (22)$$

in which there is no field, while

$$r > \sqrt{|C_1|}, r \rightarrow \sqrt{|C_1|} \Rightarrow \frac{d\varphi}{dr} \rightarrow -C_1 \cdot \infty. \quad (23)$$

At the same time in the infinity ($r \rightarrow \infty$) both solutions $\varphi_\pm(r)$ behave as the solution of the wave equation Eq. (17).

If we know the Lagrangian, we can write the energy-momentum tensor T_j^i for the these solutions and calculate the energy of the system derived by the light speed c :

$$P_0 = \text{const} \int^{(3)} T_0^0 dV. \quad (24)$$

To obtain the stationary spherically symmetric solutions, we get

$$T_0^0 = -\frac{r^2}{\sqrt{r^4 \pm C_1^2}}, \quad (25)$$

that is why for both upper and lower signs $P_0 \rightarrow \infty$.

The metric tensor of Eq. (3,11) is the simplest way to “insert” the gravity field into the Minkowski space — the initial flat space containing no fields. Adding several such terms as in Eq. (11) to the metric tensor, we can describe more and more complicated fields by tensor $h_{ij} = h_{ij}^{(grav)}$.

4 Covariant vector field

To construct the twice covariant symmetric tensor $h_{ij}(x)$ with the help of a covariant field $A_i(x)$ not using the connection objects, pay attention to the fact that the alternated partial derivative of a tensor is a tensor too

$$F_{ij} = \frac{\partial A_j}{\partial x^i} - \frac{\partial A_i}{\partial x^j}, \quad (26)$$

but a skew-symmetric one. Let us construct the symmetric tensor on the base of tensor F_{ij} . To do this, first, form a scalar

$$\begin{aligned} L_A &= \overset{\circ}{g}{}^{ij} \overset{\circ}{g}{}^{km} F_{ik} F_{jm} = \\ &= 2 \overset{\circ}{g}{}^{ij} \overset{\circ}{g}{}^{km} \left(\frac{\partial A_k}{\partial x^i} \frac{\partial A_m}{\partial x^j} - \frac{\partial A_k}{\partial x^i} \frac{\partial A_j}{\partial x^m} \right), \end{aligned} \quad (27)$$

which gives the following expressions for two symmetric tensors

$$h_{ij}^{(1)} = \overset{\circ}{g}{}^{km} \left(2 \frac{\partial A_k}{\partial x^i} \frac{\partial A_m}{\partial x^j} - \frac{\partial A_k}{\partial x^i} \frac{\partial A_j}{\partial x^m} - \frac{\partial A_k}{\partial x^j} \frac{\partial A_i}{\partial x^m} \right), \quad (28)$$

$$h_{ij}^{(2)} = \overset{\circ}{g}{}^{km} \left(2 \frac{\partial A_i}{\partial x^k} \frac{\partial A_j}{\partial x^m} - \frac{\partial A_i}{\partial x^k} \frac{\partial A_m}{\partial x^j} - \frac{\partial A_j}{\partial x^k} \frac{\partial A_m}{\partial x^i} \right). \quad (29)$$

Notice, that not only F_{ij} and L_A but also the tensors $h_{ij}^{(1)}$, $h_{ij}^{(2)}$ are gradient invariant, that is they don't change with transformations

$$A_i \rightarrow A_i + \frac{\partial f(x)}{\partial x^i}, \quad (30)$$

where $f(x)$ is an arbitrary scalar function.

Let

$$h_{ij} \equiv h_{ij}^{(A_k)} = \chi(x) h_{ij}^{(1)} + [1 - \chi(x)] h_{ij}^{(2)}, \quad (31)$$

where $\chi(x)$ is some scalar function. Then in the first order approximation we get

$$L_1 = 2 \overset{\circ}{g}{}^{ij} \overset{\circ}{g}{}^{km} \left(\frac{A_k}{\partial x^i} \frac{\partial A_j}{\partial x^m} \right) \equiv L_A, \quad (32)$$

and the first order approximation for the field $A_i(x)$ gives Maxwell equations

$$\overset{\circ}{g}{}^{ij} \frac{\partial^2}{\partial x^i \partial x^j} A_k - \frac{\partial}{\partial x^k} \left(\overset{\circ}{g}{}^{ij} \frac{\partial A_j}{\partial x^i} \right) = 0. \quad (33)$$

For Lorentz gauge

$$\overset{\circ}{g}{}^{ij} \frac{\partial A_j}{\partial x^i} = 0, \quad (34)$$

the equations Eqs. (33) take the form

$$\square A_k = 0. \quad (35)$$

It is possible that Eq. (31) is not the most general form for tensor h_{ij} which in the first order approximation gives the field equations coinciding with Maxwell equations.

To obtain Maxwell equations not for the free field but for the field with sources $j_i(x)$, one should add to $h_{ij}^{(A_k)}$ Eq. (31) the following tensor

$$h_{ij}^{(j_k)} = \left(\frac{16\pi}{c} \right) \cdot \frac{1}{2} (A_i j_j + A_j j_i). \quad (36)$$

This means that the metric tensor Eq. (3) with tensor

$$h_{ij} = h_{ij}^{(Max)} \equiv h_{ij}^{(A_k)} + h_{ij}^{(j_k)} \quad (37)$$

describes the weak electromagnetic field with source $j_k(x)$. We must bear in mind that we use the geometrical approach to the field theory, and we have to consider $j_k(x)$ to be given and not obtained from the field equations.

So, the metric tensor Eq. (3) with tensor

$$h_{ij} = \mu h_{ij}^{(A_k)} + \gamma h_{ij}^{(grav)}, \quad (38)$$

where μ, γ are the fundamental constants in frames of the unique pseudo-Riemannian geometry describes simultaneously the free electromagnetic field and the free gravitational field. To include the sources, $j_k(x)$, of the electromagnetic field, the metric tensor must either include not only $j_k(x)$ but the partial derivatives of this field too or the field $j_k(x)$ must be expressed by the other fields as shown below.

If the gravity field is "inserted" in the simplest way as shown in the previous section, then the sources of the electromagnetic field can be expressed by the scalar field as follows

$$j_i(x) = q \frac{\partial \varphi}{\partial x^i}. \quad (39)$$

In this case the first order approximation for Lorentz gauge gives

$$\square A_k = \frac{4\pi}{c} j_k, \quad (40)$$

$$\square \varphi = 0. \quad (41)$$

Since the density of the current has the form of Eq. (39), the Eq. (41) gives the continuity equation

$$\overset{\circ}{g}{}^{ij} \frac{\partial j_i}{\partial x^j} = 0. \quad (42)$$

5 Several weak fields

The transition from the weak fields to the strong fields may lead to the transition from the linear equations for the independent fields to the non-linear field equations for the mutually dependent interacting fields $\varphi(x)$ and $\psi(x)$ "including" gravity field in the Minkowski space.

Let

$$h_{ij} = \varepsilon_\varphi \frac{\partial \varphi}{\partial x^i} \frac{\partial \varphi}{\partial x^j} + \varepsilon_\psi \frac{\partial \psi}{\partial x^i} \frac{\partial \psi}{\partial x^j}, \quad (43)$$

where $\varepsilon_\varphi, \varepsilon_\psi$ are independent sign coefficients. Then the strict Lagrangian can be written as follows

$$L_{\varphi, \psi} = \sqrt{1 + L_1 + L_2}, \quad (44)$$

and

$$L_1 = \overset{\circ}{g}{}^{ij} \left(\varepsilon_\varphi \frac{\partial \varphi}{\partial x^i} \frac{\partial \varphi}{\partial x^j} + \varepsilon_\psi \frac{\partial \psi}{\partial x^i} \frac{\partial \psi}{\partial x^j} \right), \quad (45)$$

where

$$L_2 = \varepsilon_\varphi \varepsilon_\psi \left[\begin{aligned} & - \left(\frac{\partial \varphi}{\partial x^0} \frac{\partial \psi}{\partial x^1} - \frac{\partial \varphi}{\partial x^1} \frac{\partial \psi}{\partial x^0} \right)^2 - \\ & - \left(\frac{\partial \varphi}{\partial x^0} \frac{\partial \psi}{\partial x^2} - \frac{\partial \varphi}{\partial x^2} \frac{\partial \psi}{\partial x^0} \right)^2 - \\ & - \left(\frac{\partial \varphi}{\partial x^0} \frac{\partial \psi}{\partial x^3} - \frac{\partial \varphi}{\partial x^3} \frac{\partial \psi}{\partial x^0} \right)^2 + \\ & + \left(\frac{\partial \varphi}{\partial x^1} \frac{\partial \psi}{\partial x^2} - \frac{\partial \varphi}{\partial x^2} \frac{\partial \psi}{\partial x^1} \right)^2 + \\ & + \left(\frac{\partial \varphi}{\partial x^1} \frac{\partial \psi}{\partial x^3} - \frac{\partial \varphi}{\partial x^3} \frac{\partial \psi}{\partial x^1} \right)^2 + \\ & + \left(\frac{\partial \varphi}{\partial x^2} \frac{\partial \psi}{\partial x^3} - \frac{\partial \varphi}{\partial x^3} \frac{\partial \psi}{\partial x^2} \right)^2 \end{aligned} \right] \quad (46)$$

This formula, Eq. (46), can be obtained from Eq. (9) most easily, if one uses the following simplifying formula

$$\begin{aligned} \left| \begin{array}{cc} h_{ii-} & h_{i-j-} \\ h_{i-j-} & h_{jj-} \end{array} \right| &= \pm \left| \begin{array}{cc} \frac{\partial \varphi}{\partial x^i} & \frac{\partial \psi}{\partial x^i} \\ \frac{\partial \varphi}{\partial x^j} & \frac{\partial \psi}{\partial x^j} \end{array} \right|^2 = \\ &= \pm \left(\frac{\partial \varphi}{\partial x^i} \frac{\partial \psi}{\partial x^j} - \frac{\partial \varphi}{\partial x^j} \frac{\partial \psi}{\partial x^i} \right)^2. \end{aligned}$$

In the first order approximation for the Lagrangian, the expression L_1 should be used. Then the field equations give the system of two independent wave equations

$$\left. \begin{aligned} \frac{\partial^2 \varphi}{\partial x^0 \partial x^0} - \frac{\partial^2 \varphi}{\partial x^1 \partial x^1} - \frac{\partial^2 \varphi}{\partial x^2 \partial x^2} - \frac{\partial^2 \varphi}{\partial x^3 \partial x^3} &= 0 \\ \frac{\partial^2 \psi}{\partial x^0 \partial x^0} - \frac{\partial^2 \psi}{\partial x^1 \partial x^1} - \frac{\partial^2 \psi}{\partial x^2 \partial x^2} - \frac{\partial^2 \psi}{\partial x^3 \partial x^3} &= 0 \end{aligned} \right\}$$

Here the fields $\varphi(x)$ and $\psi(x)$ are independent and the superposition principle is fulfilled.

Using the strict Lagrangian for the two scalar fields Eq. (44) we get a system of two non-linear differential equations of the second order

$$\begin{aligned} \overset{\circ}{g}{}^{ij} \frac{\partial}{\partial x^i} \left[\frac{\varphi_{,j} \left(1 \pm \overset{\circ}{g}{}^{rs} \psi_{,r} \psi_{,s} \right) \mp \psi_{,j} \overset{\circ}{g}{}^{rs} \varphi_{,r} \varphi_{,s}}{\sqrt{1 + L_1 + L_2}} \right] &= 0, \\ \overset{\circ}{g}{}^{ij} \frac{\partial}{\partial x^i} \left[\frac{\psi_{,j} \left(1 + \overset{\circ}{g}{}^{rs} \varphi_{,r} \varphi_{,s} \right) - \varphi_{,j} \overset{\circ}{g}{}^{rs} \varphi_{,r} \psi_{,s}}{\sqrt{1 + L_1 + L_2}} \right] &= 0, \end{aligned}$$

where comma means the partial derivative. Here the fields $\varphi(x)$, $\psi(x)$ depend on each other, and the superposition principle is not fulfilled. The transition from the last but one equations to the last equations may be regarded as the transition from the weak fields to the strong fields.

6 Non-degenerate polynumbers

Consider a certain system of the non-degenerate polynumbers P_n [5], that is n -dimensional associative commutative non-degenerated hyper complex numbers. The corresponding coordinate space x^1, x^2, \dots, x^n is a Finsler metric flat space with the length element equal to

$$ds = \sqrt[n]{\overset{\circ}{g}{}_{i_1 i_2 \dots i_n} dx^{i_1} dx^{i_2} \dots dx^{i_n}}, \quad (47)$$

where $\overset{\circ}{g}{}_{i_1 i_2 \dots i_n}$ is the metric tensor which does not depend on the point of the space. The Finsler spaces of this kind can be found in literature (e.g. [6–9]) but the fact that all the non-degenerated polynumber spaces belong to this type of Finsler spaces was established beginning from the papers [10, 11] and the subsequent papers of the same authors, especially in [5].

The components of the generalized momentum in geometry corresponding to Eq. (47) can be found by the formulas

$$p_i = \frac{\overset{\circ}{g}{}_{i j_2 \dots j_n} dx^{j_2} \dots dx^{j_n}}{\left(\overset{\circ}{g}{}_{i_1 i_2 \dots i_n} dx^{i_1} dx^{i_2} \dots dx^{i_n} \right)^{\frac{n-1}{n}}}. \quad (48)$$

The tangent equation of the indicatrix in the space of the non-degenerated polynumbers P_n can be always written [5] as follows

$$\overset{\circ}{g}{}^{i_1 i_2 \dots i_n} p_{i_1} p_{i_2} \dots p_{i_n} - \mu^n = 0, \quad (49)$$

where μ is a constant. There always can be found such a basis (and even several such bases) and such a $\mu > 0$ that

$$\left(\overset{\circ}{g}{}^{i_1 i_2 \dots i_n} \right) = \left(\overset{\circ}{g}{}_{i_1 i_2 \dots i_n} \right). \quad (50)$$

Let us pass to a new Finsler geometry on the base of the space of non-degenerated polynumbers P_n . This new geometry is not flat, but its difference from the initial geometry is infinitely small, and the length element in this new geometry is

$$ds = \sqrt[n]{\left[\overset{\circ}{g}{}_{i_1 i_2 \dots i_n} + \varepsilon h_{i_1 i_2 \dots i_n}(x) \right] dx^{i_1} dx^{i_2} \dots dx^{i_n}}, \quad (51)$$

where ε is an infinitely small value. If in the initial space the volume element was defined by the formula

$$dV = dx^{i_1} dx^{i_2} \dots dx^{i_n}, \quad (52)$$

in the new space within the accuracy of ε in the first power we have

$$dV_h \simeq \left[1 + \varepsilon \cdot C_0 \overset{\circ}{g}{}^{i_1 i_2 \dots i_n} h_{i_1 i_2 \dots i_n}(x) \right] dx^{i_1} dx^{i_2} \dots dx^{i_n}.$$

That is according to [1], the Lagrangian of the weak field in the space with the length element Eq. (51) in the first order

approximation is

$$L_1 = \overset{\circ}{g}{}^{i_1 i_2 \dots i_n} h_{i_1 i_2 \dots i_n}(x). \quad (53)$$

This expression generalizes formula Eq. (7).

7 Hyper complex space H_4

In the physical (“orthonormal” [5]) basis in which every point of the space is characterized by the four real coordinates x^0, x^1, x^2, x^3 the fourth power of the length element ds_{H_4} is defined by the formula

$$\begin{aligned} (ds_{H_4})^4 &\equiv \overset{\circ}{g}{}_{ijkl} dx^0 dx^1 dx^2 dx^3 = \\ &= (dx^0 + dx^1 + dx^2 + dx^3)(dx^0 + dx^1 - dx^2 - dx^3) \times \\ &\times (dx^0 - dx^1 + dx^2 - dx^3)(dx^0 - dx^1 - dx^2 + dx^3) = \\ &= (dx^0)^4 + (dx^1)^4 + (dx^2)^4 + (dx^3)^4 + 8dx^0 dx^1 dx^2 dx^3 - \\ &- 2(dx^0)^2(dx^1)^2 - 2(dx^0)^2(dx^2)^2 - 2(dx^0)^2(dx^3)^2 - \\ &- 2(dx^1)^2(dx^2)^2 - 2(dx^1)^2(dx^3)^2 - 2(dx^2)^2(dx^3)^2. \end{aligned} \quad (54)$$

Let us compare the fourth power of the length element ds_{H_4} in the space of polynumbers H_4 with the fourth power of the length element $ds_{M_{in}}$ in the Minkowski space

$$\begin{aligned} (ds_{M_{in}})^4 &= (dx^0)^4 + (dx^1)^4 + (dx^2)^4 + (dx^3)^4 - \\ &- 2(dx^0)^2(dx^1)^2 - 2(dx^0)^2(dx^2)^2 - 2(dx^0)^2(dx^3)^2 - \\ &+ 2(dx^1)^2(dx^2)^2 + 2(dx^1)^2(dx^3)^2 + 2(dx^2)^2(dx^3)^2. \end{aligned} \quad (55)$$

This means

$$\begin{aligned} (ds_{H_4})^4 &= (ds_{M_{in}})^4 + 8dx^0 dx^1 dx^2 dx^3 - \\ &- 4(dx^1)^2(dx^2)^2 - 4(dx^1)^2(dx^3)^2 - 4(dx^2)^2(dx^3)^2, \end{aligned} \quad (56)$$

and in the covariant notation we have

$$\begin{aligned} (ds_{H_4})^4 &= \left(\overset{\circ}{g}{}_{ij} \overset{\circ}{g}{}_{kl} + \frac{1}{3} \overset{\circ}{g}'{}_{ijkl} - \overset{\circ}{G}{}_{ijkl} \right) \times \\ &\times dx^i dx^j dx^k dx^l, \end{aligned} \quad (57)$$

where

$$\overset{\circ}{g}'{}_{ijkl} = \begin{cases} 1, & \text{if } i, j, k, l \text{ are all different} \\ 0, & \text{else} \end{cases} \quad (58)$$

$$\overset{\circ}{G}{}_{ijkl} = \begin{cases} 1, & \text{if } i, j, k, l \neq 0 \text{ and } i = j \neq k = l, \\ & \text{or } i = k \neq j = l, \\ & \text{or } i = l \neq j = k \\ 0, & \text{else} \end{cases} \quad (59)$$

The tangent equation of the indicatrix in the H_4 space can be written in the physical basis as in [5]:

$$\begin{aligned} (p_0 + p_1 + p_2 + p_3)(p_0 + p_1 - p_2 - p_3) \times \\ \times (p_0 - p_1 + p_2 - p_3)(p_0 - p_1 - p_2 + p_3) - 1 = 0, \end{aligned} \quad (60)$$

where p_i are the generalized momenta

$$p_i = \frac{\partial ds_{H_4}}{\partial(dx^i)}. \quad (61)$$

Comparing formula Eq. (60) with formula Eq. (61), we have

$$\overset{\circ}{g}{}^{ijkl} p_i p_j p_k p_l - 1 = 0. \quad (62)$$

Here

$$\overset{\circ}{g}{}^{ijkl} = \overset{\circ}{g}{}^{ij} \overset{\circ}{g}{}^{kl} + \frac{1}{3} \overset{\circ}{g}'{}^{ijkl} - \overset{\circ}{G}{}^{ijkl}, \quad (63)$$

and

$$\left. \begin{aligned} \left(\overset{\circ}{g}{}^{ijkl} \right) &= \left(\overset{\circ}{g}{}_{ijkl} \right) \\ \left(\overset{\circ}{g}'{}^{ijkl} \right) &= \left(\overset{\circ}{g}'{}_{ijkl} \right) \\ \left(\overset{\circ}{G}{}^{ijkl} \right) &= \left(\overset{\circ}{G}{}_{ijkl} \right) \end{aligned} \right\}. \quad (64)$$

To get the Lagrangian for the weak field in the first order approximation, we have to get tensor h_{ijkl} in Eq. (53). In the simplified version it could be splitted into two additive parts: gravitational part and electromagnetic part. The gravitational part can be constructed analogously to Sections 3 and 5 with regard to the possibility to use the two-index number tensors, since now tensors $\overset{\circ}{g}{}^{ijkl}, h_{ijkl}$ have four indices. The construction of the electromagnetic part should be regarded in more detail.

Since we would like to preserve the gradient invariance of the Lagrangian and to get Maxwell equations for the free field in the H_4 space, let us write the electromagnetic part of tensor h_{ijkl} in the following way

$$h_{ijkl}^{A_k} = \chi(x) h_{ijkl}^{(1)} + [1 - \chi(x)] h_{ijkl}^{(2)}, \quad (65)$$

where the tensors $h_{ijkl}^{(1)}, h_{ijkl}^{(2)}$ are the tensors present in the round brackets in the r.h.s. of formulas Eqs. (28,29). Then

$$\begin{aligned} L_A &= \overset{\circ}{g}{}^{ijkl} h_{ijkl}^{A_k} \equiv \\ &\equiv \overset{\circ}{g}{}^{ij} \overset{\circ}{g}{}^{km} \left(\frac{\partial A_k}{\partial x^i} \frac{\partial A_m}{\partial x^j} - \frac{\partial A_k}{\partial x^i} \frac{\partial A_j}{\partial x^m} \right). \end{aligned} \quad (66)$$

To obtain Maxwell equations not for the free field but for the field with a source $j_i(x)$, one should add to the tensor $h_{ijkl}^{(A_k)}$ Eq. (65) the following tensor

$$h_{ijkl}^{(j_k)} = \left(\frac{8}{3\pi} \right) \left(2A_i j_j \overset{\circ}{g}{}_{kl} - A_i \overset{\circ}{g}{}_{jk} j_l - j_i \overset{\circ}{g}{}_{jk} A_l \right),$$

symmetrized in all indices, that is tensor

$$h_{ijkl} = h_{ijkl}^{M_{ax}} \equiv h_{ijkl}^{(A_k)} + h_{ijkl}^{(j_k)}$$

describes the weak electromagnetic field with the sources $j_i(x)$, where

$$j_i = \sum_b q_{(a)} \frac{\partial \psi_{(b)}}{\partial x^i}, \quad (67)$$

and $\psi_{(b)}$ are the scalar components of the gravitational field.

To obtain the unified theory for the gravitational and electromagnetic fields one should take the linear combination of tensor $h_{ijkl}^{(Max)}$ corresponding to the electromagnetic field in the first order approximation, and tensor $h_{ijkl}^{(grav)}$ corresponding to the gravitational field in the first order approximation

$$h_{ijkl} = \mu h_{ijkl}^{(Max)} + \gamma h_{ijkl}^{(grav)}, \quad (68)$$

where μ, γ are constants. Tensor $h_{ijkl}^{(grav)}$ may be, for example, constructed in the following way

$$h_{ijkl}^{grav} = \sum_{a=1}^N \varepsilon_{(a)} \frac{\partial \varphi_{(a)}}{\partial x^i} \frac{\partial \varphi_{(a)}}{\partial x^j} \frac{\partial \varphi_{(a)}}{\partial x^k} \frac{\partial \varphi_{(a)}}{\partial x^l} + \sum_{b=1}^M \varepsilon_{(b)} \frac{\partial \psi_{(b)}}{\partial x^i} \frac{\partial \psi_{(b)}}{\partial x^j} \overset{\circ}{g}_{kl}, \quad (69)$$

where $\varepsilon_{(a)}, \varepsilon_{(b)}$ are the sign coefficients, and $\varphi_{(a)}, \psi_{(b)}$ are the scalar fields. The whole number of scalar fields is equal to $(N + M)$.

8 Conclusion

In this paper it was shown that the geometrical approach [1] to the field theory in which there usually appear the non-linear and non-splitting field equations could give a system of independent linear equations for the weak fields in the first order approximation. When the fields become stronger the superposition principle (linearity) breaks, the equations become non-linear and the fields start to interact with each other. We may think that these changes of the equations that take place when we pass from the weak fields to the strong fields are due to the two mechanisms: first is the qualitative change of the field equations for the free fields in the first order approximation; second is the appearance of the additional field sources, that is the generation of the field by the other fields.

In frames of the geometrical approach to the field theory [1] the unification of the electromagnetic and gravitational fields is performed both for the four-dimensional pseudo-Riemannian space with metric tensor $g_{ij}(x)$ and for the four-dimensional curved Berwald-Moor space with metric tensor $g_{ijkl}(x)$.

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Instant Interpretation of Quantum Mechanics

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We suggest a new interpretation of Quantum Mechanics, in which the system state vectors are identified with q-instants — new elements of reality that are similar to time instants but can be overlapped with each other. We show that this new interpretation provides a simple and objective solution to the measurement problem, while preserving the general validity of the Schrodinger equation as well as the superposition principle in Quantum Mechanics.

1 Introduction

In spite of the extraordinary practical successes of Quantum Mechanics, the foundations of the theory contain unresolved problems, of which the most commonly cited is the measurement problem. In standard Quantum Mechanics, the quantum state evolves according to the Schrodinger equation into a linear superposition of different states, but the actual measurements always find the physical system in a single state, with some probability given by Quantum Mechanics. To bridge this gap between theory and observed reality, different interpretations of Quantum Mechanics have been suggested, ranging from the conventional Copenhagen interpretation to Hidden-variables and Many-worlds interpretations.

The Copenhagen interpretation of Quantum Mechanics proposed a process of *collapse* which is responsible for the reduction of the superposition into a single state. This postulate of wavefunction collapse was widely regarded as artificial, ad-hoc and does not represent a satisfactory solution to the measurement problem. Hidden-variable theories are proposed as alternative interpretations in which the behavior of measurement could be understood by the assumptions on the existence of inaccessible local variables with definite values which determine the measurement outcome. However, Bell's celebrated inequality [1], and the more recent GHZ argument [2], show that a Hidden-variable theory which is consistent with Quantum Mechanics would have to be non-local and therefore contradictory to Relativity. The best known of such theory is Bohmian mechanics [3, 4], to which many physicists feel that it looks contrived. It was deliberately designed to give predictions which are in all details identical to conventional Quantum Mechanics.

In Everett's *Relative State* formulation [5], also known as the Many-worlds interpretation [6], one insists on the general validity of the superposition principle. The final state after the measurement is considered to be the full superposition state, and the measurement process is interpreted as the splitting of the system + apparatus into various branches (these are often called Everett branches) only one of which we observe. All measurement outcomes in the superposition thus coexist as separate real world outcomes. This means that, in some sense,

there is a very large, perhaps infinite, number of universes. Most physicists find this extremely unattractive. Moreover, in this interpretation it is not clear how to recover the empirical quantum mechanical probabilities.

In this paper we suggest a new interpretation of Quantum Mechanics, called Instant interpretation, which can give a simple, objective solution to the measurement problem and does not have the difficulties mentioned above. It assumes, as in the Everett interpretations, the general validity of the Schrodinger equation as well as the superposition principle of Quantum Mechanics. Basically, it consists in the introduction of the concept of *q-instant* (or *quantum instant*), and the interpretation of the system state vectors as the q-instants at which the quantum system is present or occurred. The q-instant, being a new concept of instant, is an element of reality that has the same role as time instants in classical physics: quantum events take place at different q-instants similarly to that classical events take place at different time instants. However, q-instants have new properties, especially the superposition, that are fundamentally different to time instants. Mathematically, q-instants are vector-like instants, while time instants are point-like instants. The difference in behavior of quantum and classical objects is essentially due to such differences between q-instants and time instants.

A particularly intriguing consequence of the linear time evolution of the quantum system in the context of Instant interpretation is that it leads, in quantum observation, to the *apparent collapse* phenomenon, or the *apparent unique measurement outcome*, an illusion that happens to any conscious-being observer. This is the key point to resolve the measurement problem by the Instant interpretation.

The outline of the article is as follows. We start with a preliminary introduction of the concept of quantum instant in Quantum Mechanics. In Section 3, we present the Instant interpretation and the formalism of Quantum Mechanics in this interpretation, named as *Instant Quantum Mechanics*. In Section 4, we show how the new interpretation can provide a simple and objective solution to the problem of definite outcome in quantum measurement theory, i.e. the problem related to the fact that a particular experiment on a quantum system always gives a unique result. Finally, in Section 5,

we give some conclusion remarks on the instant formalism of Quantum Mechanics and the role of quantum decoherence in this new interpretation.

2 Preliminary Concept of Quantum Instant

Before introducing the concept of q-instants in Quantum Mechanics, we shall describe briefly the basic meaning and property of its closed concept — the time instant notion.

From the physical viewpoint, time is part of the fundamental structure of the universe, a dimension in which events occur. A time instant or time point in this dimension is thus considered as a *holder* for the presence of events and objects. Each of the object's presences is called an occurrence of the object. A physical object at two different instants is considered as the same object, and not as two objects. Similarly, the worlds at different instants in the past, present and future are different occurrences of a single world, not of multiple worlds. We consider this as the basic meaning of the instant notion.

One particular property of time instant is its *distinctness*: Different time instants are strictly distinguished in the sense that when a physical object is being present in a given time instant, it is not present in other time instants. In other words, due to this separateness, the object completely leaves one time instant, before it can occur in another time instant.

The notion of q-instants that we use to interpret the wave function state in Quantum Mechanics has the same basic meaning as time instants, that is, q-instants are new *holders* for the presences of a physical system.

We shall illustrate the introduction of this new concept of instant in Quantum Mechanics by means of a simple example. Let ψ be a state vector such that

$$\psi = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2), \quad (1)$$

where ψ_1 and ψ_2 are two orthogonal state vectors (correspond to two eigenstates of some observable F).

What it really means a physical system in such a superposed state ψ ? It seems likely that the system is half in the state ψ_1 and half in ψ_2 , a property of quantum objects that is usually considered as weird and inexplicable (as it is typically expressed for the behavior of the particle in the two-slit experiment).

Using the concept of instants, however, we can explain the superposition in (1) as describing the occurrences of the system at two different *instants*: one associated with the state vector ψ_1 and other with ψ_2 .

Note that we do not intend to add some hidden-time τ associated with the system states by some mapping $f(\tau_i) = \psi_i$. Instead of introducing such classical extra hidden-variables that control the occurrences of the state ψ_i , we identify the state ψ_i with the *instant* itself. We then try to know what

are the properties of this new kind of instant, which we call *quantum instant* or *q-instant*.

In fact, by considering the state vectors ψ , ψ_1 and ψ_2 in the superposition (1) as q-instants, we see that the q-instant concept exhibits intriguing new properties, compared with conventional time instants: different q-instants can be superposed or overlapped, in contrast with the distinctness property mentioned above of time instants.

In our example, the q-instant ψ is a superposition of two q-instants ψ_1 and ψ_2 , it overlaps with each of these two q-instants. On the contrary, the two q-instants ψ_1 and ψ_2 are orthogonal, they are distinct and do not overlap with each other as in the case of two different time instants. The overlap of two q-instants has the consequence that when an object is being present in one instant, one of its occurrences can be found in another instant.

Mathematically, q-instants are vector-like instants, while time instants are point-like instants. In fact, due to its superposition property, quantum instant has the structure of a vector and is not represented by a point on the real line R like a time instant. The inner product of two vectors can then be used to measure the overlap of the two corresponding q-instants.

3 Formalism of Quantum Mechanics in Instant Interpretation

In the above section, we have illustrated the introduction of the notion of q-instant in Quantum Mechanics. For the sake of simplicity, we have identified the state vector of a physical object with the q-instant at which the object located. Taking into account the time dimension, we see that the state vector of a physical object evolves in time, while the q-instants are rather something independent with time. Indeed, in the Instant interpretation, we will consider that, for each physical system, besides the time dimension, there exists independently a continuum of q-instants in which the system takes its presences. Quantum events take place in time dimension as well as in the q-instant continuum. The state vector, in the Instant interpretation, is then considered as the *representation* of a q-instant at a time t . So the q-instant itself is independent with time, but its representation, i.e. the state vector, evolves in time according to the Schrodinger equation. Note that, in this sense, the q-instant corresponds to the state vector in the Heisenberg representation of Quantum Mechanics.

The axioms of Quantum Mechanics in the Instant interpretation are as follows:

- A1 Every physical system S is associated to a Hilbert space H_S and a q-instant continuum \mathbb{Q}_S in which the system takes its presences.
- A2 Each q-instant Q of the continuum \mathbb{Q}_S is described, at each time t , by a normalized vector $|\psi\rangle$ of H_S . The time evolution of the q-instant representation, i.e. the

vector $|\psi\rangle$ representing the instant Q , is governed by the Schrodinger equation:

$$i\hbar \frac{\partial |\psi(t)\rangle}{\partial t} = H |\psi(t)\rangle. \quad (2)$$

The operator H is the Hamiltonian of the system S .

A3 Let Q_1, Q_2 be two q-instants of the continuum \mathbb{Q}_S represented, at some given time, respectively by two vectors ψ_1, ψ_2 of H_S , then $|\langle\psi_1|\psi_2\rangle|^2$ is the measure of presence of q-instant Q_1 in q-instant Q_2 .

A4 Each physical observable O of the system S is represented by a self-adjointed operator in H_S . If a q-instant Q of the system S is described, at some time t , by an eigenvector $|O_n\rangle$ of an observable O then the value of the observable O of the system S at q-instant Q and time t is o_n , where o_n is the corresponding eigen-value of $|O_n\rangle$.

Quantum Mechanics based on these axioms is called *Instant Quantum Mechanics*. In the following, we will give some remarks about its axioms and the underlying concept q-instants. In particular, we will show how the notion of probability can be defined in the context of the Instant interpretation.

(R1) For each q-instant Q , we denote by $Q(t)$ the vector $|\psi\rangle$ of H_S that describes it at time t . We say that the system S at time t and q-instant Q is in the *state* ψ . Let U be the time unitary evolution of the system, then:

- at time t_0 and q-instant Q , the system is in the state $Q(t_0) \equiv |\psi_0\rangle$, and
- at time t and q-instant Q , the system is in the state $Q(t) \equiv |\psi\rangle = U(t) |\psi_0\rangle$.

Thus, according to Instant Quantum Mechanics, the state of a physical system is determined by a time instant and a q-instant. This is in contrast with standard Quantum Mechanics in which only the time t determines the *state* ψ of a physical system. In standard Quantum Mechanics, one basic axiom is that the physical system at each time t is described by a state vector ψ . This axiom seems evident, and the practical successes of Quantum Mechanics confirm it. However, as we shall show in the next sections, this is just apparently true, and the description of *state* in Instant Quantum Mechanics is not in contradiction with practical observations. While in standard Quantum Mechanics, to fix an initial system setting, we use the expression “*Suppose at time t_0 , the system S is in the state ψ* ”, in the Instant interpretation, we can equivalently express this by “*Consider the system S at time t_0 and q-instant Q such that $Q(t_0) = \psi$* ”.

(R2) Similar to the state space, the q-instant continuum \mathbb{Q}_S has also the structure of a Hilbert vector space. This structure is defined as follows.

Let, at some given time t , $|\psi\rangle, |\psi_1\rangle$ and $|\psi_2\rangle$ be the state vectors that describe respectively the q-instants Q, Q_1 and Q_2 . Then, we define:

- $Q = c_1 Q_1 + c_2 Q_2$ if $|\psi\rangle = c_1 |\psi_1\rangle + c_2 |\psi_2\rangle$,
- the inner product $\langle Q_1 | Q_2 \rangle = \langle \psi_1 | \psi_2 \rangle$.

Due to the linearity and unitarity of the time evolution of the q-instants representation, it is easy to see that the above definitions are consistent, that is, they are time-independent.

Let $|\psi\rangle = \sum_{i=1}^n c_i |\psi_i\rangle$ and $Q, Q_i, 1 \leq i \leq n$, be q-instants such that $Q(t) = |\psi\rangle, Q_i(t) = |\psi_i\rangle$, then we have the following facts:

- q-instant Q is a superposition of the q-instants Q_i :
 $Q = \sum_{i=1}^n c_i Q_i$,
- at time t and q-instant Q_i , the state of the system is $|\psi_i\rangle$, for $1 \leq i \leq n$,
- at time t and q-instant Q , the state of the system is $|\psi\rangle$.

(R3) Since q-instants are vectors, there is no order relation between them as in the case of time instants. There is thus no concept of *next* q-instant of a q-instant. If the system is being present at instant Q , it makes no sense to ask *what q-instant it will be present next?* Instead, there is a superposition between the different instants of the q-instant continuum. Between any two q-instants Q_α and Q_β there is a *weight* $w_{\alpha\beta} = |\langle Q_\alpha | Q_\beta \rangle|^2$, which is the *measure of presence or overlap* of the instant Q_α in the instant Q_β , defined in the axiom A3. If Q_α and Q_β are overlapped, i.e. $w_{\alpha\beta} \neq 0$, then when the system is present in instant Q_α , it is present also in Q_β . If $w_{\alpha\beta} = 0$, we say that the two instants Q_α and Q_β are *orthogonal*, that is, when the system is present in one instant, it is not present in the other instant.

(R4) The notion of *current instant*, having a straightforward meaning in the case of time instants, is not directly defined for the case of q-instants. It is not globally defined for the whole q-instant continuum and it makes no sense to ask *which is the current q-instant of the q-instant continuum?* In fact, in its usual sense, the *current* instant means the instant that the system is being present at and not elsewhere. This has sense only if the so-called current instant is orthogonal with all the others, a requirement which is impossible if we consider the whole q-instant continuum. The notion of *current q-instant* is thus defined *only* with respect to a *context* in which this orthogonality requirement is satisfied. We define it as follows: A context is a pair (Q, E) , where Q is a q-instant and $E = \{Q_i\}$ is an orthogonal basis. Suppose that $Q = \sum_i c_i Q_i$ is the expansion of Q in this basis. So when the system is present at instant Q , it will present also at all instants Q_i with $c_i \neq 0$. But as the instants Q_i 's of the basis E are pairwise orthogonal, there is always only one instant Q_i of E that the system is *currently* present, this Q_i is called the *current q-instant* of the context (Q, E) . As the system will present in all the above instants Q_i 's, all these instants will become the current q-instant while the system under consideration is in the context (Q, E) . The role of the current instant is thus alternatively played by each of the q-instants of E . This notion of current q-instant is therefore similar to that

of current time instant for the time dimension. However, different to the case of time instants, in which all time instants equally take the role of current instant during the flow of time, the assignment of this role in the case of q-instants is proportional to the measure of presence of each q-instant Q_i in the context q-instant Q . The measure of presence $|\langle Q|Q_i\rangle|^2$ determines therefore the *probability* that the q-instant Q_i becomes the current instant of (Q, E) . One can understand the intuition behind this probability notion by means of the following thought experiment: Imagine a person who lives in the q-instant dimension E , in which he takes a long sleep and then wakes up at some q-instant of E . Suppose that before sleeping the person does not know at which instant he will wake up. He knows it only when he wakes up and opens his eyes, at that moment he realizes that he is currently at some instant Q_i . So, before opening his eyes, the person can only predict with a certain probability which instant Q_i he is currently at. This probability for an instant Q_i is the *probability* that Q_i becomes the current instant, and it is proportional to the measure of presence of Q_i .

4 The measurement process and the apparent collapse phenomenon

In this section, we recall briefly first the standard description of the measurement process within traditional Quantum Mechanics and the problem arising from it, usually referred as *the measurement problem* in the literature. We then show how our Instant interpretation of Quantum Mechanics can give a simple and objective solution to this problem.

4.1 Measurements in traditional Quantum Mechanics — the problem of definite outcome

A standard scheme using pure Quantum Mechanics to describe the measurement process is the one devised by von Neumann (1932). In this schema, both the measured system and the apparatus are considered as quantum objects.

Let H_S be the Hilbert space of the measured system S and $\{|e_i\rangle\}$ be the eigenvectors of the operator F representing the observable to be measured. Let H_A be the Hilbert space of the apparatus A and $\{|a_i\rangle\}$ be the basis vectors of H_A , where the $|a_i\rangle$'s are assumed to correspond to macroscopically distinguishable *pointer* positions that correspond to the outcome of a measurement if S is in the state $|e_i\rangle$. The apparatus A is in the initial *ready* state $|a_0\rangle$.

The total system $S \otimes A$, assumed to be represented by the Hilbert product space $H_{SA} = H_S \otimes H_A$, evolves according to the Schrodinger equation. Let U be the time evolution of the total system from the initial state to the final state of the measuring process.

Suppose that the measured system S is initially in one of the eigenvector state $|e_i\rangle$ then $U(|e_i\rangle|a_0\rangle) = |e_i\rangle|a_i\rangle$ where $|\phi_f\rangle = |e_i\rangle|a_i\rangle$ is the final state of the total system + appara-

tus $S \otimes A$. The outcome $|a_i\rangle$ of the apparatus A can be predicted with certainty merely from the unitary dynamics.

Now, consider the case of measurement in which the system S is initially prepared not in the eigenstate $|e_i\rangle$ but in a superposition of the form $\sum_i c_i |e_i\rangle$. Due to the linearity of the Schrodinger equation, the final state $|\phi_f\rangle$ of total system is:

$$|\phi_f\rangle = U\left(\sum_i c_i |e_i\rangle |a_0\rangle\right) = \sum_i c_i |e_i\rangle |a_i\rangle. \quad (3)$$

So the final state $|\phi_f\rangle$ describes a state that does not correspond to a definite state of the apparatus. This is in contrast to what is actually perceived at the end of the measurement: in actual measurements, the observer always finds the apparatus in a definite pointer state $|a_i\rangle$, for some i , but not in a superposition of these states. The difficulty to understand this fact is typically referred to as the measurement problem in the literature.

Von Neumann's approach (like all other standard presentations of Quantum Mechanics) assumes that after the first stage of the measurement process, described as above, a second non-linear, indeterministic process takes place, the *reduction* (or *collapse*) of the wave packet, that involves $S \otimes A$ *jumping* from the entangled state $\sum_i c_i |e_i\rangle |a_i\rangle$ into the state $|e_i\rangle |a_i\rangle$ for some i . It's obvious that the wave-packet reduction postulate, abandoning the general validity of the Schrodinger equation without specifying any physical conditions under which the linear evolution fails, is *ad hoc* and does not consequently represent a satisfactory solution to the measurement problem.

In the last few decades, some important advances related to a theoretical understanding of the collapse process have been made. This new theoretical framework, called quantum decoherence, supersedes previous notions of instantaneous collapse and provides an explanation for the absence of quantum coherence after measurement [7–11]. While this theory correctly predicts the form and probability distribution of the final eigenstates, it does not explain the observation of a unique stable pointer state at the end of a measurement [12, 13].

4.2 Solution from Instant Quantum Mechanics

We will show how the Instant interpretation based on the concept of q-instants allows a simple and objective solution to the measurement problem. The above description of the measurement process can be reformulated in Instant Quantum Mechanics as follows:

According to the Instant interpretation, the combined system $S \otimes A$ takes its presences in a continuum \mathbb{Q}_{SA} of q-instants, each of which is represented at each time t by a normalized vector of the Hilbert product space $H_{SA} = H_S \otimes H_A$.

Following the remark (RI) of Section 3, the initial setting (according to standard Quantum Mechanics) in which

the combined system $S \otimes A$ is in the state

$$|\phi_0\rangle = \sum_i c_i |e_i\rangle |a_0\rangle . \quad (4)$$

is equivalent to the initial setting (according to the Instant interpretation) in which the combined system $S \otimes A$ under consideration is being present at the q-instant Q of the continuum \mathbb{Q}_{SA} such that

$$Q(t_0) = |\phi_0\rangle , \quad (5)$$

where $|\phi_0\rangle$ is defined in (4).

For each i , let Q_i be the q-instant of $S \otimes A$ such that

$$Q_i(t_0) = |\phi_i\rangle = |e_i\rangle |a_0\rangle . \quad (6)$$

The instants Q_i 's are, hence, orthogonal one with another. Following the remark (R2) of Section 3, the instant Q is spanned over this set of instants as follows:

$$Q = \sum_i c_i Q_i . \quad (7)$$

Following the axiom A3, the $|c_i|^2$ is the measure of presence of the system $S \otimes A$ in instant Q_i as long as the system is being present in instant Q .

The state vectors, representing the instants Q and Q_i 's, evolve independently in time following the Schrodinger equation. At the end of the measurement process, we have:

$$Q_i(t_f) = |e_i\rangle |a_i\rangle , \quad (8)$$

$$Q(t_f) = \sum_i c_i |e_i\rangle |a_i\rangle . \quad (9)$$

From (8), (9) we see that after measurement:

- at time t_f and q-instant Q , the state of the combined system is $\sum_i c_i |e_i\rangle |a_i\rangle$;
- at time t_f and q-instant Q_i , the state of the combined system is $|e_i\rangle |a_i\rangle$, hence the state of the apparatus is $|a_i\rangle$.

Thus, after measurement, the observer sees different outcomes $|a_i\rangle$'s, at different instants Q_i 's. So far, the description still seems to be in contrast to what is actually perceived by the observer at the end of the measurement, i.e. to the following fact:

Fact 1. The observer always sees the apparatus in one definite state $|a_i\rangle$, for some i , after the measurement.

The difficulty to explain Fact 1 is usually referred as the *problem of definite outcome* in quantum measurement theory. However, we will show that Fact 1 is intriguingly not true, it is an illusion of the observer. More precisely, we will show, according to the Instant interpretation, the following *apparent "collapse"* phenomenon (or the phenomenon of *apparent unique measurement outcome*):

Fact 2. The observer does see different measurement outcomes, but it seems to him that there is only one unique measurement outcome and the apparatus is in one definite state $|a_i\rangle$, for some i , after the measurement.

Proof. To prove this fact we will take into account the presence of the observer in the measurement process by considering him as a component of the total system. We will see that the illusion in Fact 2 comes from the property of time evolution independence of different q-instants in the measurement process and its impacts on the observer's recognition of the world.

To be consistent and objective, we will treat the observer quantum mechanically, that is, as a quantum object. We can simply write $|O_i\rangle$ to denote the state of the observer seeing the apparatus in position $|a_i\rangle$. However, to well understand the illusion, we need to consider the cognitive aspect of the observer in a little more detail. A conscious being can observe the world and use his brain cells to stock information he perceived. What make he feels that he is seeing an event, is the result of a process of recognition during which the brain cells "memorize" the event.

Let C be the set of memory cells that the observer uses in the recognition of the apparatus state, and C_i be the content of C when the observer perceives that the apparatus state is $|a_i\rangle$. This content C_i is considered as the *proof* that the observer perceives the apparatus in position $|a_i\rangle$.

Suppose that at some instant the content of the cells is C_i and the observer actually perceives that the apparatus state is $|a_i\rangle$. If the cells contents are later destroyed, not only the observer will not see the apparatus being in the state $|a_i\rangle$, but as his concerned memory data is lost, he will feel that he has never seen the apparatus being in the state $|a_i\rangle$. If, alternatively, the cell contents are changed and replaced by C_j , not only the observer will see that the apparatus is now in the state $|a_j\rangle$, but as his old data is lost, for him the apparatus have never been in the state $|a_i\rangle$.

This is what happens to the observer in the measurement process according to Instant Quantum Mechanics.

In fact, including the observer in the measurement process, the Hilbert space of the total system will be the product $H_S \otimes H_A \otimes H_O$, where H_O is the Hilbert space of the observer. We assume that H_O is spanned over the basis of state vectors $\{|O_i, C_i\rangle\}$ where the $|O_i, C_i\rangle$ describes the state of the observer seeing the apparatus in position $|a_i\rangle$ and having his memory cells contents C_i .

Initially, the total system $S \otimes A \otimes O$ under consideration is being present at the q-instant Q of the continuum \mathbb{Q}_{SAO} such that

$$Q(t_0) = |\phi_0\rangle = \sum_i c_i |e_i\rangle |a_0\rangle |O_0, C_0\rangle . \quad (10)$$

The total system containing the measured system, the apparatus and the observer with his memory cells evolves in

time following the Schrodinger equation during the measurement process.

At the end of measurement, at time t_f , similar to (8), (9), we have:

$$Q_i(t_f) = |e_i\rangle |a_i\rangle |O_i, C_i\rangle, \quad (11)$$

$$Q(t_f) = \sum c_i |e_i\rangle |a_i\rangle |O_i, C_i\rangle. \quad (12)$$

So, after the measurement process, similar to the apparatus and the observer, the cells C takes its presences in different q-instants, and at q-instant Q_i , the observer memory cells content is C_i . Note that, due to the time evolution independence of the cells contents in different q-instants, the content of the cells C in one q-instant is not influenced by its contents in other q-instants.

We consider the impact of this on the observer behavior. After measurement, at instant Q_i , the cells content is C_i , but at another instant Q_j , the cells content is replaced by C_j . So at instant Q_j , the observer loses all information of his memory cells at instant Q_i . Due to the time evolution independence of the cells contents at Q_i and Q_j , basing on his memory cells information at Q_j , the observer has no trace or proof that he has ever lived in instant Q_i . By consequence, at instant Q_j , the observer sees the apparatus in position $|a_j\rangle$, but he absolutely *forgets* that he has ever lived in q-instant Q_i and seen the apparatus in position $|a_i\rangle$. In other words, after each measurement, the observer does see different outcomes at different q-instants, but he believes that there is only one outcome, the one that he is currently seeing. \square

So we have proved Fact 2 and solve therefore the problem of definite outcome. How about the probability of an outcome? Objectively, all outcomes are present after the measurement, so the probability of an outcome $|a_i\rangle$ here must be understood as the probability that an outcome $|a_i\rangle$ becomes the one that is *currently* perceived (and illusorily considered as unique) by the observer. In other words, the probability of an outcome $|a_i\rangle$ is the probability that the corresponding instant Q_i is the *current q-instant* in which the observer presents. As we have remarked in *R4* of Section 3, this notion of *current q-instant* is defined with respect to a context. In our case, corresponding to the setting of the measurement process, this context is (Q, E) , where Q is the q-instant under consideration of the total system at time t_0 , and $E = \{Q_i\}$ is the set of orthogonal instants Q_i in which the measured observable F has a definite value. So from *R4* of Section 3, we see that the probability of the outcome $|a_i\rangle$ is the measure of presence of the instant Q_i in instant Q which, from (7), is equal to $|c_i|^2$.

5 Concluding remarks

1. We note that the phenomenon of apparent unique outcome in the measurement process (Fact 2 of Section 4.2) illustrates

also a remark about the definition of *state* in the Instant interpretation in *R1* of Section 3: the state of a physical system is dependent not only on time but also on q-instant. In fact, as we have seen in Section 4.2, the *state* of the total system at the end of measurement is dependent on the q-instants at which the system presents. But, as demonstrated there, the observer is unconscious about this, for him the *state* of a quantum object is always unique at any time instant. The description of state in the Instant interpretation is thus not in contradiction with practical observations.

2. In the Instant interpretation, we consider that, like microscopic objects, a macroscopic object, e.g. an apparatus, also takes its presences in a q-instant continuum which supports the superposition principle. If Q_1 and Q_2 are two q-instants in which the object can present, then it can also present in a q-instant which is a superposition of Q_1 and Q_2 . The question is why can we observe a macroscopic object such as an apparatus in q-instants in which its pointer position is either *up* or *down*, but never in a q-instant in which its pointer is in a superposition of these positions.

This is the problem of *classicality* of macroscopic objects, to which decoherence theory, in particular the environment-induced decoherence, can provide an explanation. In fact, recent development in this domain [7–9, 11, 14–16] has shown that there exists, for macroscopic objects, certain *preferred sets of states*, often referred to as *pointer states* that are robust. These states are determined by the form of the interaction between the system and its environment and are suggested to correspond to the *classical* states of our experience. Thus, for a macroscopic object, one can not observe all of its Hilbert state vector space but only a small subset of it. In the context of Instant interpretation, this means that, while a macroscopic object can present in all q-instants of the continuum, we can observe it only in q-instants that are described by these robust classical states.

In summary, with respect to the measurement problem in Quantum Mechanics, decoherence theory can provide an explanation to the *classicality appearance* of the measurement outcomes, while the Instant interpretation allows to explain the observation of *an unique outcome* at the end of a measurement.

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Thermoelastic Property of a Semi-Infinite Medium Induced by a Homogeneously Illuminating Laser Radiation

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The problem of thermoelasticity, based on the theory of Lord and Shulman with one relaxation time, is used to solve a boundary value problem of one dimensional semi-infinite medium heated by a laser beam having a temporal Dirac distribution. The surface of the medium is taken as traction free. The general solution is obtained using the Laplace transformation. Small time approximation analysis for the stresses, displacement and temperature are performed. The convolution theorem is applied to get the response of the system on temporally Gaussian distributed laser radiation. Results are presented graphically. Concluding that the small time approximation has not affected the finite velocity of the heat conductivity.

1 Introduction

The classical theory (uncoupled) of thermoelasticity based on the conventional heat conduction equation. The conventional heat conduction theory assumes that the thermal disturbances propagate at infinite speeds. This prediction may be suitable for most engineering applications but it is a physically unacceptable situation, especially at a very low temperature near absolute zero or for extremely short-time responses.

Biot [1] formulated the theory of coupled thermoelasticity to eliminate the shortcoming of the classical uncoupled theory. In this theory, the equation of motion is a hyperbolic partial differential equation while the equation of energy is parabolic. Thermal disturbances of a hyperbolic nature have been derived using various approaches. Most of these approaches are based on the general notion of relaxing the heat flux in the classical Fourier heat conduction equation, thereby, introducing a non Fourier effect.

The first theory, known as theory of generalized thermoelasticity with one relaxation time, was introduced by Lord and Shulman [2] for the special case of an isotropic body. The extension of this theory to include the case of anisotropic body was developed by Dhaliwal and Sherief [4].

In view of the experimental evidence available in favor of finiteness of heat propagation speed, generalized thermoelasticity theories are supposed to be more realistic than the conventional theory in dealing with practical problems involving very large heat fluxes and/or short time intervals, like those occurring in laser units and energy channels.

The purpose of the present work is to study the thermoelastic interaction caused by heating a homogeneous and isotropic thermoelastic semi-infinite body induced by a Dirac pulse having a homogeneous infinite cross-section by employing the theory of thermo-elasticity with one relaxation time. The problem is solved by using the Laplace transform technique. Approximate small time analytical solutions to

stress, displacement and temperature are obtained. The convolution theorem is applied to get the spatial and temporal temperature distribution induced by laser radiation having a temporal Gaussian distribution. At the end of this work we present the computed results obtained from the theoretical relations applied on a Cu target.

2 Formulation of the problem

We consider a thermoelastic, homogeneous, isotropic semi-infinite target occupying the region $z \geq 0$, and initially at uniform temperature T_0 . The surface of the target $z = 0$ is heated homogeneously by a laser beam and assumed to be traction free. The Cartesian coordinates (x, y, z) are considered in the solution and z -axis pointing vertically into the medium. The equation of motion in the absence of the body forces has the form

$$\sigma_{ji,j} = \rho \ddot{u}_i, \quad (1)$$

where σ_{ij} is the components of stress tensor, u_i is the components of displacement vector and ρ is the mass density. Due to the Lord and Shalman theory of coupled thermoelasticity [2] (L-S) who considered a wave-type heat equation by postulating a new law of heat conduction equation to replace the Fourier's law

$$\rho c_E \left(\frac{\partial T}{\partial t} + t_0 \frac{\partial^2 T}{\partial t^2} \right) + \gamma T_0 \operatorname{div} \left(\frac{\partial u}{\partial t} + t_0 \frac{\partial^2 u}{\partial t^2} \right) = k \nabla^2 T, \quad (2)$$

where T_0 is a uniform reference temperature, $\gamma = (3\lambda + 2\mu)\alpha_t$, λ , and μ are Lamé's constants. α_t is the linear thermal expansion coefficient, c_E is the specific heat at constant strain and k is the thermal conductivity. The boundary conditions:

$$\sigma_{zz} = 0, \quad z = 0, \quad (3)$$

$$-k \frac{dT}{dz} = A_0 q_0 \delta(t), \quad z = 0, \quad (4)$$

where A_0 is an absorption coefficient of the material, q_0 is the intensity of the laser beam and $\delta(t)$ is the Dirac delta function [5]. The initial conditions:

$$\left. \begin{aligned} T(z, 0) &= T_0 \\ \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial t^2} = \frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial t^2} &= 0, \quad \text{at } t = 0, \quad \forall z \end{aligned} \right\}. \quad (5)$$

Due to the symmetry of the problem and the external applied thermal field, the displacement vector u has the components:

$$u_x = 0, \quad u_y = 0, \quad u_z = w(z, t). \quad (6)$$

From equation (6) the strain components e_{ij} , and the relation of the strain components to the displacement read;

$$\left. \begin{aligned} e_{xx} = e_{yy} = e_{xy} = e_{xz} = e_{yz} &= 0 \\ e_{zz} &= \frac{\partial w}{\partial z} \\ e_{ij} &= \frac{1}{2} (u_{i,j} + u_{j,i}) \end{aligned} \right\}. \quad (7)$$

The volume dilation e takes the form

$$e = e_{xx} + e_{yy} + e_{zz} = \frac{\partial w}{\partial z}. \quad (8)$$

The stress components are given by:

$$\left. \begin{aligned} \sigma_{xx} &= \lambda e - \gamma(T - T_0) \\ \sigma_{yy} &= \lambda e - \gamma(T - T_0) \\ \sigma_{zz} &= 2\mu \frac{\partial w}{\partial z} + \lambda e - \gamma(T - T_0) \end{aligned} \right\}, \quad (9)$$

where

$$\left. \begin{aligned} \sigma_{xy} &= 0 \\ \sigma_{xz} &= 0 \\ \sigma_{yz} &= 0 \end{aligned} \right\}. \quad (10)$$

The equation of motion (1) will be reduces to

$$\sigma_{zz,z} + \sigma_{xz,x} + \sigma_{yz,y} = \rho \ddot{u}_z. \quad (11)$$

Substituting from (9) and (10) into the last equation and using $\theta = T - T_0$ we get,

$$(2\mu + \lambda) \frac{\partial^2 w}{\partial z^2} - \gamma \frac{\partial \theta}{\partial z} = \rho \frac{\partial^2 w}{\partial t^2}, \quad (12)$$

where θ is the temperature change above a reference temperature T_0 . Differentiating (12) with respect to z and using (8), we obtain

$$(2\mu + \lambda) \frac{\partial^2 e}{\partial z^2} - \gamma \frac{\partial^2 \theta}{\partial z^2} = \rho \frac{\partial^2 e}{\partial t^2}. \quad (13)$$

The energy equation can be written in the form:

$$\left. \begin{aligned} \left(\frac{\partial}{\partial t} + t_0 \frac{\partial^2}{\partial t^2} \right) (\rho c_E \theta + \gamma T_0 e) &= k \nabla^2 T \\ \nabla^2 &\equiv \frac{\partial^2}{\partial z^2} \end{aligned} \right\}. \quad (14)$$

For convenience, the following non-dimensional quantities are introduced

$$\left. \begin{aligned} z^* &= c_1 \eta z, \quad w^* = c_1 \eta w, \quad t^* = c_1^2 \eta t \\ t_0^* &= c_1^2 \eta t_0, \quad \sigma_{ij}^* = \frac{\sigma_{ij}}{\mu}, \quad \theta^* = \frac{T - T_0}{T_0} \\ \eta &= \frac{\rho c_E}{k}, \quad c_1^2 = \sigma_{ij}^* = \frac{\lambda + 2\mu}{\rho} \end{aligned} \right\}. \quad (15)$$

Substituting from (15) into (12) we get after dropping the asterisks and adopting straight forward manipulation

$$\left. \begin{aligned} \nabla^2 e - g_1 \nabla^2 \theta &= \frac{\partial^2 e}{\partial t^2} \\ \nabla^2 \theta &= \left(\frac{\partial}{\partial t} + t_0 \frac{\partial^2}{\partial t^2} \right) (\theta + g_2 e) \end{aligned} \right\}, \quad (16)$$

where $g_1 = \frac{\gamma T_0}{(2\mu + \lambda)}$ and $g_2 = \frac{\gamma}{\rho c_E}$.

Substituting from (15) into (9) we get,

$$\left. \begin{aligned} \sigma_{xx} = \sigma_{yy} &= \beta e - \lambda_1 \theta \\ \sigma_{zz} &= \alpha e - \lambda_1 \theta \end{aligned} \right\}, \quad (17)$$

where $\alpha = \frac{\gamma(2\mu + \lambda)}{\mu}$, $\beta = \frac{\lambda}{\mu}$ and $\lambda_1 = \frac{\gamma T_0}{\mu}$. We now introduce the Laplace transform defined by the formula:

$$\bar{f}(z, s) = \int_0^\infty e^{-st} f(z, t) dt. \quad (18)$$

Applying (18) to both sides of equation (16) we get,

$$(\nabla^2 - s^2) \bar{e} - g_1 \nabla^2 \bar{\theta} = 0, \quad (19)$$

$$(\nabla^2 - s(1 + t_0 s)) \bar{\theta} - s(1 + t_0 s) g_2 \bar{e} = 0. \quad (20)$$

Eliminating $\bar{\theta}$ and \bar{e} between equation (19) and (20) we get the following fourth-order differential equations satisfied by \bar{e} and $\bar{\theta}$; respectively

$$(\nabla^4 - A \nabla^2 + C) \bar{e} = 0, \quad (21)$$

$$(\nabla^4 - A \nabla^2 + C) \bar{\theta} = 0, \quad (22)$$

with $A = s^2 + s(1 + t_0 s)(1 + g_1 g_2)$ and $C = s^3(1 + t_0 s)$. One can solve these fourth order ordinary differential equations by using e^{-kz} and finding the roots of the indicial equation

$$k^4 - A k^2 + C = 0, \quad (23)$$

suppose that k_i ($i = 1, 2$) are the positive roots, then the solution of (23) for $z \geq 0$ and $k_i > 0$ are; respectively

$$\bar{e}(z, s) = \sum_{i=1}^2 A_i e^{-k_i z} \tag{24}$$

and

$$\bar{\theta}(z, s) = \sum_{i=1}^2 A'_i e^{-k_i z}, \tag{25}$$

where $A_i = A_i(s)$ and $A'_i = A'_i(s)$ are some parameters depending only on s and k_i are functions of s . Substituting by (24) and (25) into (20) we get the relation,

$$A'_i = \frac{s(1+t_0s)g_2}{k_i^2 - s(1+t_0s)} A_i, \tag{26}$$

while Laplace transform of Equation (8) and integration w.r.t. z we obtain

$$\bar{w}(z, s) = - \sum_{i=1}^2 \frac{A_i}{k_i} e^{-k_i z}. \tag{27}$$

Substituting from Equation (24) and Equation (26) into (17) we get the stresses,

$$\sigma_{xx} = \sigma_{yy} = \sum_{i=1}^2 A_i e^{-k_i z} \times \frac{\beta(k_i^2 - s(1+t_0s)) - s(1+t_0s)\lambda_1 g_2}{k_i^2 - s(1+t_0s)}. \tag{28}$$

$$\sigma_{zz} = \sum_{i=1}^2 A_i e^{-k_i z} \times \frac{\alpha(k_i^2 - s(1+t_0s)) - s(1+t_0s)\lambda_1 g_2}{k_i^2 - s(1+t_0s)}. \tag{29}$$

Therefore it is easy to determine A_i and A'_i for $i = 1, 2$

$$A_1 = \frac{-A_0 q_0 (k_1^2 - s(1+t_0s)) B_1(s)}{g_2 s (1+t_0s) [-k_1 B_2(s) + k_2 B_3(s)]}, \tag{30}$$

$$A_2 = \frac{A_0 q_0 (k_2^2 - s(1+t_0s)) B_1(s)}{g_2 s (1+t_0s) [-k_1 B_2(s) + k_2 B_3(s)]}, \tag{31}$$

$$A'_1 = \frac{-A_0 q_0 B_1(s)}{[-k_1 B_2(s) + k_2 B_3(s)]}, \tag{32}$$

$$A'_2 = \frac{A_0 q_0 B_1(s)}{[-k_1 B_2(s) + k_2 B_3(s)]}, \tag{33}$$

where $B_1(s) = \alpha(k_2^2 - s(1+t_0s))(\alpha + \lambda_1 g_2)$, $B_2(s) = \alpha k_2^2 - s(1+t_0s)(\alpha + \lambda_1 g_2)$, and also $B_3(s) = \alpha k_1^2 - s(1+t_0s)(\alpha + \lambda_1 g_2)$.

3 Small time approximation

We now determine inverse transforms for the case of small values of time (large values of s). This method was used by

Hetnarski [6] to obtain the fundamental solution for the coupled thermelasticity problem and by Sherief [7] to obtain the fundamental solution for generalized thermoelasticity with two relaxation times for point source of heat. k_1 and k_2 are the positive roots of the characteristic equation (23), given by

$$k_1 = \left(\frac{s}{2} \left[s + (1+t_0s)(1+\epsilon) + \sqrt{s^2 + 2s(\epsilon-1)(1+t_0s) + (1+t_0s)^2(1+\epsilon)^2} \right] \right)^{\frac{1}{2}}, \tag{34}$$

$$k_2 = \left(\frac{s}{2} \left[s + (1+t_0s)(1+\epsilon) - \sqrt{s^2 + 2s(\epsilon-1)(1+t_0s) + (1+t_0s)^2(1+\epsilon)^2} \right] \right)^{\frac{1}{2}}, \tag{35}$$

where $\epsilon = g_1 g_2 = \frac{\alpha^2 (3\lambda + 2\mu)^2 T_0}{\rho c_E (2\mu + \lambda)}$. Setting $v = \frac{1}{s}$, equations (34) and (35) can be expressed in the following form

$$k_i = v^{-1} [f_i(v)]^{\frac{1}{2}}, \quad i = 1, 2, \tag{36}$$

where

$$f_1(v) = \frac{1}{2} \left[1 + (v+t_0)(1+\epsilon) + \sqrt{1 + 2(\epsilon-1)(v+t_0) + (v+t_0)^2(1+\epsilon)^2} \right], \tag{37}$$

$$f_2(v) = \frac{1}{2} \left[1 + (v+t_0)(1+\epsilon) - \sqrt{1 + 2(\epsilon-1)(v+t_0) + (v+t_0)^2(1+\epsilon)^2} \right]. \tag{38}$$

Expanding $f_1(v)$ and $f_2(v)$ in the Maclaurin series around $v = 0$ and consider only the first four terms, can be written $f_i(v)$ ($i = 1, 2$) as

$$f_i(v) = a_{i0} + a_{i1}v + a_{i2}v^2 + a_{i3}v^3, \quad i = 1, 2, \tag{39}$$

where the coefficients of the first four terms are given by

$$\left. \begin{aligned} a_{10} &= \frac{1+(1+\epsilon)t_0 + \sqrt{1+2(\epsilon-1)t_0 + (1+\epsilon)^2 t_0^2}}{2} \\ a_{20} &= \frac{1+(1+\epsilon)t_0 - \sqrt{1+2(\epsilon-1)t_0 + (1+\epsilon)^2 t_0^2}}{2} \\ a_{11} &= \frac{1}{2} \left[(1+\epsilon) - \frac{(\epsilon-1)t_0 + (1+\epsilon)^2 t_0}{\sqrt{1+2(\epsilon-1)t_0 + (1+\epsilon)^2 t_0^2}} \right] \\ a_{21} &= \frac{1}{2} \left[(1+\epsilon) + \frac{(\epsilon-1)t_0 + (1+\epsilon)^2 t_0}{\sqrt{1+2(\epsilon-1)t_0 + (1+\epsilon)^2 t_0^2}} \right] \\ a_{12} &= \frac{\epsilon}{[1+2(\epsilon-1)t_0 + (1+\epsilon)^2 t_0^2]^{\frac{3}{2}}} \\ a_{22} &= -\frac{\epsilon}{[1+2(\epsilon-1)t_0 + (1+\epsilon)^2 t_0^2]^{\frac{3}{2}}} \\ a_{13} &= \frac{-\epsilon(-1+\epsilon + (1+\epsilon)^2 t_0)}{[1+2(\epsilon-1)t_0 + (1+\epsilon)^2 t_0^2]^{\frac{3}{2}}} \\ a_{23} &= \frac{\epsilon(-1+\epsilon + (1+\epsilon)^2 t_0)}{[1+2(\epsilon-1)t_0 + (1+\epsilon)^2 t_0^2]^{\frac{3}{2}}} \end{aligned} \right\}. \tag{40}$$

Next, we expand $[f_i(v)]^{\frac{1}{2}}$ in the Maclaurin series around $v = 0$ and retaining the first three terms, we obtain finally the expressions for k_1 and k_2 which can be written in the form

$$k_i = v^{-1} (b_{i0} + b_{i1}v + b_{i2}v^2), \quad i = 1, 2, \quad (41)$$

where

$$b_{i0} = \sqrt{a_{i0}},$$

$$b_{i1} = \frac{a_{i1}}{2\sqrt{a_{i0}}},$$

and

$$b_{i2} = \frac{1}{8a_{i0}^{\frac{3}{2}}(9a_{i2}a_{i0} - a_{i0}^2)}.$$

Consider k_i to be written as

$$k_i = b_{i0}s + b_{i1}, \quad i = 1, 2. \quad (42)$$

Applying Maclaurin series expansion around $v = 0$ of the following expressions;

$$\frac{1}{k_i} A_i, \quad \frac{s(1+t_0s)g_2}{k_i^2 - s(1+t_0s)} A_i, \\ \left[\frac{\beta(k_i^2 - s(1+t_0s)) - s(1+t_0s)\lambda_1g_2}{k_i^2 - s(1+t_0s)} \right] A_i, \\ \left[\frac{\alpha(k_i^2 - s(1+t_0s)) - s(1+t_0s)\lambda_1g_2}{k_i^2 - s(1+t_0s)} \right] A_i,$$

$$i = 1, 2.$$

We find that $\bar{\theta}$, \bar{w} , $\bar{\sigma}_{xx}$, $\bar{\sigma}_{yy}$, and $\bar{\sigma}_{zz}$ can be written in the following form

$$\bar{\theta} = \left(\frac{c_{\theta 0}}{s} + \frac{c_{\theta 1}}{s^2} + \frac{c_{\theta 2}}{s^3} \right) e^{-k_1z} + \left(\frac{c_{\theta 3}}{s} + \frac{c_{\theta 4}}{s^2} + \frac{c_{\theta 5}}{s^3} \right) e^{-k_2z}, \quad (43)$$

$$\bar{w} = \left(\frac{c_{w0}}{s^2} + \frac{c_{w1}}{s^3} + \frac{c_{w2}}{s^4} \right) e^{-k_1z} + \left(\frac{c_{w3}}{s^2} + \frac{c_{w4}}{s^3} + \frac{c_{w5}}{s^4} \right) e^{-k_2z}, \quad (44)$$

$$\bar{\sigma}_{xx} = \bar{\sigma}_{yy} = \left(\frac{c_{\sigma 0}}{s} + \frac{c_{\sigma 1}}{s^2} + \frac{c_{\sigma 2}}{s^3} \right) e^{-k_1z} + \left(\frac{c_{\sigma 3}}{s} + \frac{c_{\sigma 4}}{s^2} + \frac{c_{\sigma 5}}{s^3} \right) e^{-k_2z}, \quad (45)$$

$$\bar{\sigma}_{zz} = \left(\frac{c_{z0}}{s} + \frac{c_{z1}}{s^2} + \frac{c_{z2}}{s^3} \right) e^{-k_1z} + \left(\frac{c_{z3}}{s} + \frac{c_{z4}}{s^2} + \frac{c_{z5}}{s^3} \right) e^{-k_2z}, \quad (46)$$

where

$$\left. \begin{aligned} c_{\theta 0} &= \frac{y_1}{f_0} = 0.00002466 \\ c_{\theta 1} &= \frac{y_2}{f_0} - \frac{f_1 y_1}{f_0^2} = 0.000666 \\ c_{\theta 2} &= \frac{y_3}{f_0} + \frac{f_1^2 y_1}{f_0^3} - \frac{f_2 y_1 + f_1 y_2}{f_0^2} = -0.911471 \\ c_{\theta 3} &= \frac{y_1^*}{f_0} = 0.705 \\ c_{\theta 4} &= \frac{y_2^*}{f_0} - \frac{f_1 y_1^*}{f_0^2} = -1.7696 \\ c_{\theta 5} &= \frac{y_3^*}{f_0} + \frac{f_1^2 y_1^*}{f_0^3} - \frac{f_2 y_1^* + f_1 y_2^*}{f_0^2} = 50.6493 \\ c_{w0} &= \frac{A_1}{R_0} = -0.0007519 \\ c_{w1} &= -\frac{R_1 A_1}{R_0^2} + \frac{A_2}{R_0} = 0.18 \\ c_{w2} &= \frac{R_1^2 A_1}{R_0^3} - \frac{R_2 A_1 + R_1 A_1}{R_0^2} + \frac{A_3}{R_0} = 26.90 \\ c_{w3} &= \frac{A_1^*}{R_0} = 0.000106 \\ c_{w4} &= \frac{-R_1^* A_1^*}{R_0^{*2}} + \frac{A_2^*}{R_0^*} = -0.000493 \\ c_{w5} &= \frac{R_1^{*2} A_1^*}{R_0^{*3}} - \frac{R_2^* A_1^* + R_1^* A_1^*}{R_0^{*2}} + \frac{A_3^*}{R_0^*} = 194.0138 \\ c_{\sigma 0} &= \frac{x_1}{\eta_1} = 0.001511 \\ c_{\sigma 1} &= \frac{x_2}{\eta_1} - \frac{\eta_2 x_1}{\eta_1^2} = -0.03623 \\ c_{\sigma 2} &= \frac{x_1}{\eta_1} - \frac{\eta_2 x_2}{\eta_1^2} - \frac{x_1 \eta_3}{\eta_1^2} = -54.064 \\ c_{\sigma 3} &= \frac{x_1^*}{\eta_1} = -0.002985 \\ c_{\sigma 4} &= \frac{x_2^*}{\eta_1} - \frac{\eta_2 x_1^*}{\eta_1^2} = 0.07314 \\ c_{\sigma 5} &= \frac{x_1^*}{\eta_1} - \frac{\eta_2 x_2^*}{\eta_1^2} - \frac{x_1^* \eta_3}{\eta_1^2} = 53.02 \\ c_{z0} &= \frac{L_1}{\eta_1} = 0.003015 \\ c_{z1} &= \frac{L_2}{\eta_1} - \frac{\eta_2 L_1}{\eta_1^2} = -0.0722 \\ c_{z2} &= \frac{L_1}{\eta_1} - \frac{\eta_2 L_2}{\eta_1^2} - \frac{L_1 \eta_3}{\eta_1^2} = -107.88 \\ c_{z3} &= \frac{L_1^*}{\eta_1} = -0.003 \\ c_{z4} &= \frac{L_2^*}{\eta_1} - \frac{\eta_2 L_1^*}{\eta_1^2} = 0.0722 \\ c_{z5} &= \frac{L_1^*}{\eta_1} - \frac{\eta_2 L_2^*}{\eta_1^2} - \frac{L_1^* \eta_3}{\eta_1^2} = 107.88 \end{aligned} \right\} \quad (47)$$

From equation (39), we obtain

$$e^{-k_1 z} = e^{-(b_{10}s + b_{11})z} = e^{-b_{11}z} e^{-b_{10}sz},$$

and

$$e^{-k_2 z} = e^{-(b_{20}s + b_{21})z} = e^{-b_{21}z} e^{-b_{20}sz}.$$

Applying the inverse Laplace transform for equations (43, 44, 45, 46) we get θ , w , σ_{xx} , σ_{yy} and σ_{zz} in the following form

$$\theta = e^{-b_{11}z} \Theta_1 H(t - b_{10}z) + e^{-b_{21}z} \Theta_2 H(t - b_{20}z), \quad (48)$$

where

$$\Theta_1 = \left[c_{\theta 0} + c_{\theta 1}(t - b_{10}z) + \frac{c_{\theta 2}}{2}(t - b_{10}z)^2 \right],$$

$$\Theta_2 = \left[c_{\theta 3} + c_{\theta 4}(t - b_{20}z) + \frac{c_{\theta 5}}{2}(t - b_{20}z)^2 \right],$$

and also

$$w = e^{-b_{11}z} W_1 H(t - b_{10}z) + e^{-b_{21}z} W_2 H(t - b_{20}z), \quad (49)$$

where

$$W_1 = \left[c_{w0}(t - b_{10}z) + \frac{c_{w1}(t - b_{10}z)^2}{2} + \frac{c_{w2}(t - b_{10}z)^3}{6} \right],$$

$$W_2 = \left[c_{w3}(t - b_{20}z) + \frac{c_{w4}(t - b_{20}z)^2}{2} + \frac{c_{w5}(t - b_{20}z)^3}{6} \right],$$

and also

$$\begin{aligned} \sigma_{xx} = \sigma_{yy} = \\ = e^{-b_{11}z} \Sigma_1 H(t - b_{10}z) + e^{-b_{21}z} \Sigma_2 H(t - b_{20}z), \end{aligned} \quad (50)$$

where

$$\Sigma_1 = \left[c_{\sigma 0} + c_{\sigma 1}(t - b_{10}z) + c_{\sigma 2} \frac{(t - b_{10}z)^2}{2} \right],$$

$$\Sigma_2 = \left[c_{\sigma 3} + c_{\sigma 4}(t - b_{20}z) + c_{\sigma 5} \frac{(t - b_{20}z)^2}{2} \right],$$

and also

$$\sigma_{zz} = e^{-b_{11}z} Z_1 H(t - b_{10}z) + e^{-b_{21}z} Z_2 H(t - b_{20}z), \quad (51)$$

where

$$Z_1 = \left[c_{z0} + c_{z1}(t - b_{10}z) + c_{z2} \frac{(t - b_{10}z)^2}{2} \right],$$

$$Z_2 = \left[c_{z3} + c_{z4}(t - b_{20}z) + c_{z5} \frac{(t - b_{20}z)^2}{2} \right],$$

and $H(t - b_{i0}z)$ is Heaviside's unit step functions. By using the convolution theorem $h(t) = \int_0^t f(\tau)g(t - \tau)d\tau$ for (48), (49), (50) and (51) we obtain under the assumption that $f(\tau) = e^{-\frac{(t_b - \tau)^2}{\varphi^2}}$; which represents the time behavior of the

intensity of the laser radiation, where t_b is the time at which $f(\tau)$ has maximum. Here φ is the time at which the intensity of the laser radiation reduces to $\frac{1}{e}$

$$\begin{aligned} \theta = e^{-b_{11}z} \left[\left(c_{\theta 0} + c_{\theta 1}(t - b_{10}z) + c_{\theta 2} \frac{(t - b_{10}z)^2}{2} + \right. \right. \\ \left. \left. + c_{\theta 2} \frac{(t - b_{10}z)^2}{2} + \frac{\varphi^2 c_{\theta 2}}{4} \right) \frac{\sqrt{\pi}}{2} \operatorname{erf} \left(\frac{t}{\varphi} \right) - c_{\theta 2} \frac{t\varphi}{4} e^{-\frac{t^2}{\varphi^2}} + \right. \\ \left. + (c_{\theta 1} + c_{\theta 2}(t - b_{10}z)) \frac{\varphi^2}{2} \left(1 - e^{-\frac{t^2}{\varphi^2}} \right) \right] + \\ + e^{-b_{21}z} \left[\left(c_{\theta 3} + c_{\theta 4}(t - b_{10}z) + c_{\theta 5} \frac{(t - b_{10}z)^2}{2} + \right. \right. \\ \left. \left. + \frac{\varphi^2 c_{\theta 5}}{4} \right) \frac{\sqrt{\pi}}{2} \operatorname{erf} \left(\frac{t}{\varphi} \right) - c_{\theta 2} \frac{t\varphi}{4} e^{-\frac{t^2}{\varphi^2}} + \right. \\ \left. + (c_{\theta 4} + c_{\theta 5}(t - b_{10}z)) \frac{\varphi^2}{2} \left(1 - e^{-\frac{t^2}{\varphi^2}} \right) \right], \end{aligned} \quad (52)$$

$$\begin{aligned} w = e^{-b_{11}z} \left[\left(c_{w0}(t - b_{10}z) + \frac{c_{w1}}{2} \left((t - b_{10}z)^2 + \frac{\varphi^2}{2} \right) + \right. \right. \\ \left. \left. + c_{w2} \left(\frac{(t - b_{10}z)^3}{6} + \frac{\varphi^2}{4} \right) \right) \frac{\sqrt{\pi}}{2} \operatorname{erf} \left(\frac{t}{\varphi} \right) - \right. \\ \left. - \left(c_{w0} + c_{w1}(t - b_{10}z) - \frac{\varphi^2 c_{w2}}{12} (\varphi^2 - (t^2 - \varphi^2)) e^{-\frac{t^2}{\varphi^2}} \right) + \right. \\ \left. + \frac{c_{w2}}{2} (t - b_{10}z)^2 \right) \frac{\varphi^2}{2} \left(1 - e^{-\frac{t^2}{\varphi^2}} \right) - \right. \\ \left. - \frac{1}{4} (c_{w1} + c_{w2}(t - b_{10}z)) t \varphi^2 e^{-\frac{t^2}{\varphi^2}} \right] + \\ + e^{-b_{21}z} \left[\left(c_{w3}(t - b_{20}z) + \frac{c_{w4}}{2} \left((t - b_{10}z)^2 + \frac{\varphi^2}{2} \right) + \right. \right. \\ \left. \left. + c_{w5} \left(\frac{(t - b_{10}z)^3}{6} + \frac{\varphi^2}{4} \right) \right) \frac{\sqrt{\pi}}{2} \operatorname{erf} \left(\frac{t}{\varphi} \right) - \right. \\ \left. - \left(c_{w3} + c_{w4}(t - b_{10}z) - \frac{\varphi^2 c_{w5}}{12} (\varphi^2 - (t^2 - \varphi^2)) e^{-\frac{t^2}{\varphi^2}} \right) + \right. \\ \left. + \frac{c_{w5}}{2} (t - b_{10}z)^2 \right) \frac{\varphi^2}{2} \left(1 - e^{-\frac{t^2}{\varphi^2}} \right) - \right. \\ \left. - \frac{1}{4} (c_{w4} + c_{w5}(t - b_{10}z)) t \varphi^2 e^{-\frac{t^2}{\varphi^2}} \right], \end{aligned} \quad (53)$$

$$\begin{aligned} \sigma_{zz} = e^{-b_{11}z} \left[\left(c_{z0} + c_{z1}(t - b_{10}z) + \frac{c_{z2}}{2} (t - b_{10}z)^2 + \right. \right. \\ \left. \left. + \frac{\varphi^2 c_{z2}}{4} \right) \frac{\sqrt{\pi}}{2} \operatorname{erf} \left(\frac{t}{\varphi} \right) - \frac{c_{z2}}{4} t \varphi e^{-\frac{t^2}{\varphi^2}} + \right. \\ \left. + (c_{z1} + c_{z2}(t - b_{10}z)) \frac{\varphi^2}{2} \left(1 - e^{-\frac{t^2}{\varphi^2}} \right) \right] + \\ + e^{-b_{11}z} \left[\left(c_{z3} + c_{z4}(t - b_{10}z) + \frac{c_{z5}}{2} (t - b_{10}z)^2 + \right. \right. \\ \left. \left. + \frac{\varphi^2 c_{z5}}{4} \right) \frac{\sqrt{\pi}}{2} \operatorname{erf} \left(\frac{t}{\varphi} \right) - \frac{c_{z5}}{4} t \varphi e^{-\frac{t^2}{\varphi^2}} + \right. \\ \left. + (c_{z4} + c_{z5}(t - b_{10}z)) \frac{\varphi^2}{2} \left(1 - e^{-\frac{t^2}{\varphi^2}} \right) \right], \end{aligned} \quad (54)$$

$$\begin{aligned} \sigma_{xx} = \sigma_{yy} = e^{-b_{11}z} & \left[\left(c_{\sigma 0} + c_{\sigma 1}(t - b_{10}z) + \right. \right. \\ & + \frac{c_{\sigma 2}}{2}(t - b_{10}z)^2 + \frac{\varphi^2 c_{\sigma 2}}{4} \left. \right) \frac{\sqrt{\pi}}{2} \operatorname{erf} \left(\frac{t}{\varphi} \right) - \frac{c_{\sigma 2} \varphi}{4} t e^{-\frac{t^2}{\varphi^2}} + \\ & + (c_{\sigma 1} + c_{\sigma 2}(t - b_{10}z)) \frac{\varphi^2}{2} \left(1 - e^{-\frac{t^2}{\varphi^2}} \right) \left. \right] + \\ & + e^{-b_{21}z} \left[\left(c_{\sigma 3} + c_{\sigma 4}(t - b_{20}z) + \frac{c_{\sigma 5}}{2}((t - b_{10}z)^2 + \right. \right. \\ & + \frac{c_{\sigma 5} \varphi^2}{4}) \varphi \frac{\sqrt{\pi}}{2} \operatorname{erf} \left(\frac{t}{\varphi} \right) - \frac{\varphi c_{\sigma 5}}{4} t e^{-\frac{t^2}{\varphi^2}} + \\ & + (c_{\sigma 4} + c_{\sigma 5}(t - b_{10}z)) \frac{\varphi^2}{2} \left(1 - e^{-\frac{t^2}{\varphi^2}} \right) \left. \right]. \end{aligned} \quad (55)$$

4 Computation and discussions

We have calculated the spatial temperature, displacement and stress θ , w , σ_{xx} , σ_{yy} and σ_{zz} with the time as a parameter for a heated target with a spatial homogeneous laser radiation having a temporally Gaussian distributed intensity with a width of (10E-3 s). We have performed the computation for the physical parameters $T_0 = 293$ K, $\rho = 8954$ Kg/m³,

$$\begin{aligned} A &= 0.01, \quad c_E = 383.1 \text{ J/kgK}, \\ \varphi &= 10^{-3} \text{ s}, \quad \epsilon = g_1 g_2 = 0.01680089, \\ \alpha_t &= 1.78(10^{-5}) \text{ K}^{-1}, \quad k = 386 \text{ W/mK}, \\ \lambda &= 7.76(10^{10}) \text{ kg/m sec}^2, \quad \mu = 3.86(10)^{10} \text{ kg/m sec}^2 \end{aligned}$$

and

$$t_0 = 0.02 \text{ sec}$$

for Cu as a target. We obtain the results displayed in the following figures.

Considering surface absorption the obtained results in Figure 1 show the temperature θ , Figure 2 display the temporal temperature distribution and the temporal behavior of the laser radiation, Figure 3 for the displacement w , Figure 4 for the stress σ_{zz} and Figure 5 for the stresses σ_{xx} and σ_{yy} .

The coupled system of differential equations describing the thermoelasticity treated through the Laplace transform of a temporally Dirac distributed laser radiation illuminating homogeneous a semi-infinite target and absorbed at its irradiated surface. Since the system is linear the response of the system on the Dirac function was convoluted with a temporally Gaussian distributed laser radiation. The theoretical obtained results were applied on the Cu target. Figure 1 illustrates the calculated spatial distribution of the temperature per unit intensity at different values of the time parameter ($t = 0.005, 0.007, 0.01, 0.015, \text{ and } 0.02$). From the curves it is evident that the temperature has a finite velocity expressed through the strong gradient of the temperature which moves deeper in the target as the time increases.

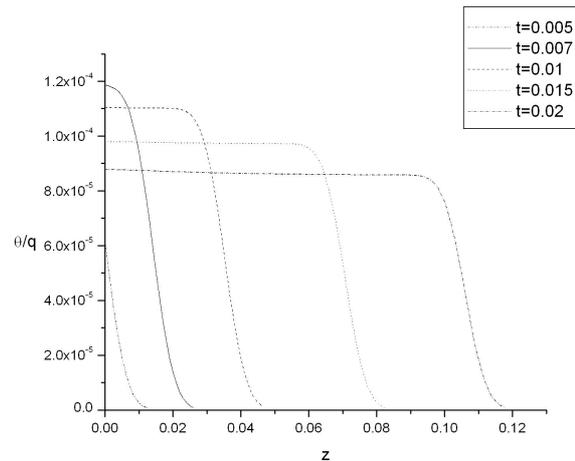


Fig. 1: The temperature distribution θ per unit intensity versus z with the time as a parameter.

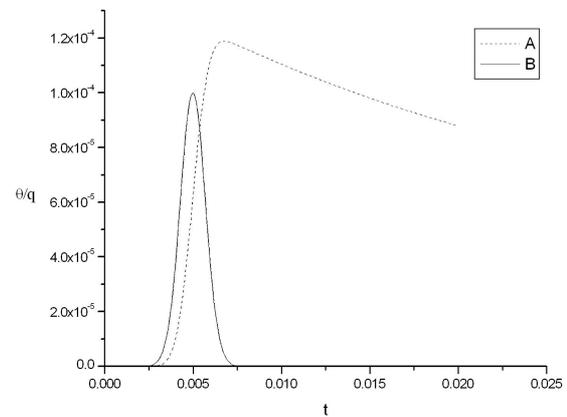


Fig. 2: (A) The temporal temperature distribution θ per unit intensity form the. (B) The temporal behavior of the laser radiation which is assumed to have a Gaussian distribution with width $\varphi = 10^{-3}$ s.

Figure 2 represent the calculated front temporal temperature distribution per unit intensity (curve A); as a result of the temporal behavior of the laser radiation which is assumed to have a Gaussian distribution with a width equals to (10E-3 s) (curve B). From the figure it is evident that the temperature firstly increases with increasing the time this can be attributed to the increased absorbed energy which over compensates the heat losses given by the heat conductivity inside the material. As the absorbed power equals the conducted one inside the material the temperature attains its maximum value. the maximum of the temperature occurs at later time than the maximum of the radiation this is the result of the heat conductivity of Cu and the relatively small gradient of the temperature in the vicinity of $z = 0$ as seen from Figure 1. After the radiation becomes weak enough such that it can not compensate the diffused power inside the material the temperature decreases monotonically with increasing time.

Figure 3 shows the calculated spatial displacement per unit intensity at different times(0.01,0.015 and 0.02). The

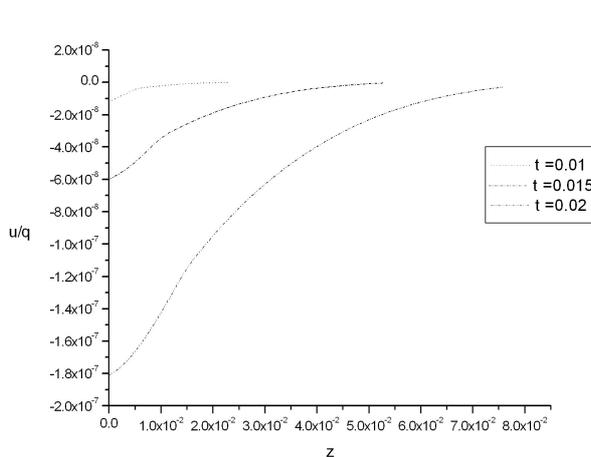


Fig. 3: The displacement distribution u per unit intensity versus z with the time as a parameter.

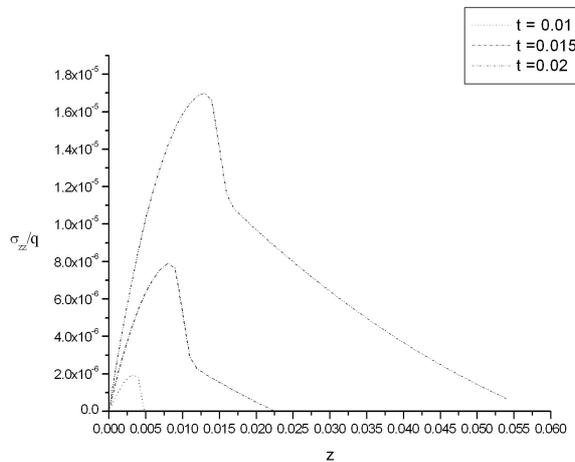


Fig. 4: The stress σ_{zz} distribution per unit intensity versus z with the time as a parameter.

displacement increases monotonically with time. It attains smaller gradient with increasing z . Both effects can be attributed to the temperature behavior. The negative displacement results from the co-ordinate system which is located at the front surface with positive direction of the z -axis pointing down words.

Figure 4 illustrates the spatial distribution of stress σ_{zz} per unit intensity at the times (0.01, 0.015 and 0.02). Since, $\sigma_{zz} = \alpha e - \lambda_1 \theta$, thus from Figure 3 σ_{zz} attains maxima at the locations for which the gradient of the displacement exhibits maxima and this is practically at the same points for which σ_{zz} is maximum. The calculations showed that σ_{xx} and σ_{yy} have the same behavior as σ_{zz} .

5 Results and conclusions

The thermoelasticity problem formulated by a coupled linear system of partial differential equations was discussed. The system was decoupled to provide a fourth order linear differential equations which were solved analytically using Laplace

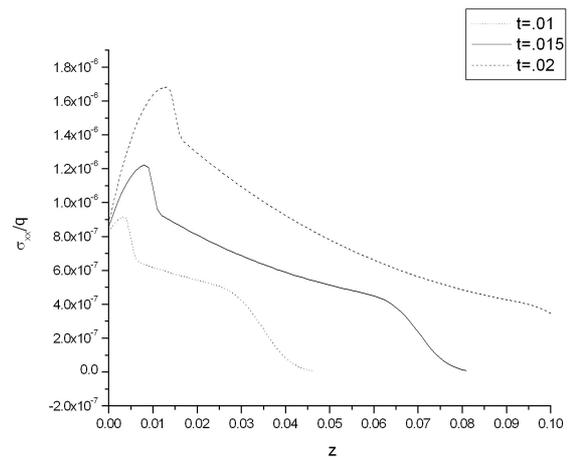


Fig. 5: The stress distribution σ_{xx} and σ_{yy} per unit intensity versus z with the time as a parameter.

transform. The small time approximation analysis was performed for the solution of temperature, displacement and for the stresses; showing that the finite velocity of the temperature described by the D.Es system was not affected by the small time approximation.

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Nature of the Excited States of the Even-Even $^{98-108}\text{Ru}$ Isotopes

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The positive and negative parity states of the even-even $^{98-108}\text{Ru}$ isotopes are studied within the frame work of the interacting boson approximation model ($IBA-1$). The calculated levels energy, potential energy surfaces, $V(\beta, \gamma)$, and the electromagnetic transition probabilities, $B(E1)$ and $B(E2)$, show that ruthenium isotopes are transitional nuclei. Staggering effect, $\Delta I = 1$, has been observed between the positive and negative parity states in some of ruthenium isotopes. The electric monopole strength, $X(E0/E2)$, has been calculated. All calculated values are compared with the available experimental and theoretical data where reasonable agreement has been obtained.

1 Introduction

The mass region $A = 100$ has been of considerable interest for nuclear structure studies as it shows many interesting features. These nuclei show back bending at high spin and shape transitions from vibrational to γ -soft and rotational characters. Many attempts have been made to explore these structural changes which are due mainly to the n-p interactions.

Experimentally, the nuclear reaction $^{100}\text{Mo}(\alpha, xn)$ [1] has been used in studying the level energy of ^{100}Ru . Angular distribution, γ - γ coincidences were measured, half-life time has been calculated and changes to the level scheme were proposed. Also, the double beta decay rate of ^{100}Mo to the first excited 0^+ state of ^{100}Ru has been measured experimentally [2] using the γ - γ coincidence technique.

Doppler-shift attenuation measurements following the $^{100}\text{Ru}(n, n'\gamma)$ reaction [3] have been used to measure the life times of the excited states in ^{100}Ru . Absolute transition rates were extracted and compared with the interacting boson model description. The $2^+(2240.8 \text{ keV})$ state which decays dominantly to the 2^+ via 1701 keV transition which is almost pure $M1$ in nature is considered as a mixed-symmetry state. Again ^{100}Ru has been studied [4] experimentally and several levels were seen where some new ones are detected below 3.2 MeV.

The excited states of ^{102}Ru have been investigated using $^{96}\text{Zr}(^{10}\text{B}, p3n)$ reaction [5] at a beam energy of 42 MeV and the emitted γ rays were detected. The analysis indicated that the nucleus is a γ -soft and the band crossing as well as the staggering effect have been observed.

Theoretically many models have been applied to ruthenium isotopes. Yukawa folded mean field [6] has been applied to ^{100}Ru nucleus while the microscopic vibrational model has been applied to ^{104}Ru and some other nuclei with their daughters [7]. The latter model was successful in describing the yrast 0^+ and 2^+ states of most of these nuclei and also some of their half-lives.

The very high-spin states of nuclei near $A \approx 100$ are investigated

by the Cranked Strutinsky method [8] and many very extended shape minima are found in this region. The interacting boson model has been used in studying Ru isotopes using a $U(5)-O(6)$ transitional Hamiltonian with fixed parameters [9, 10] except for the boson number N . The potential arising from a coherent-state analysis indicates that ^{104}Ru is close to the critical point between spherical and γ -unstable structures.

Hartree-Fock Bogoliubov [11] wave functions have been tested by comparing the theoretically calculated results for ^{100}Mo and ^{100}Ru nuclei with the available experimental data. The yrast spectra, reduced $B(E2, 0^+ \rightarrow 2^+)$ transition probabilities, quadrupole moments $Q(2^+)$ and g factors, $g(2^+)$ are computed. A reasonable agreement between the calculated and observed values has been obtained.

The microscopic anharmonic vibrator approach (MAVA) [12] has been used in investigating the low-lying collective states in $^{98-108}\text{Ru}$. Analysis of the level energies and electric quadrupole decays of the two-phonon type of states indicated that ^{100}Ru can be interpreted as being a transitional nucleus between the spherical anharmonic vibrator ^{98}Ru and the quasirotational $^{102-106}\text{Ru}$ isotopes.

A new empirical approach has been proposed [13] which is based on the connection between transition energies and spin. It allows one to distinguish vibrational from rotational characters in atomic nuclei. The cranked interacting boson model [14] has been used in estimating critical frequencies for the rotation-induced spherical-to-deformed shape transition in $A = 100$ nuclei. The predictions show an agreement with the back bending frequencies deduced from experimental yrast sequences in these nuclei.

The aim of the present work is to use the $IBA-1$ [15-17] for the following tasks:

- (1) calculating the potential energy surfaces, $V(\beta, \gamma)$, to know the type of deformation that exists;
- (2) calculating level energy, electromagnetic transition rates $B(E1)$ and $B(E2)$;

nucleus	<i>EPS</i>	<i>PAIR</i>	<i>ELL</i>	<i>QQ</i>	<i>OCT</i>	<i>HEX</i>	<i>E2SD(eb)</i>	<i>E2DD(eb)</i>
⁹⁸ Ru	0.6280	0.000	0.0090	-0.0010	0.0000	0.0000	0.1250	-0.3698
¹⁰⁰ Ru	0.5950	0.000	0.0085	-0.0200	0.0000	0.0000	0.1160	-0.3431
¹⁰² Ru	0.5650	0.0000	0.0085	-0.0200	0.0000	0.0000	0.1185	-0.3505
¹⁰⁴ Ru	0.4830	0.0000	0.0085	-0.0200	0.0000	0.0000	0.1195	-0.3535
¹⁰⁶ Ru	0.4560	0.0000	0.0085	-0.0200	0.0000	0.0000	0.1020	-0.3017
¹⁰⁸ Ru	0.4540	0.0000	0.0085	-0.0200	0.0000	0.0000	0.1035	-0.3062

Table 1: Parameters used in IBA-1 Hamiltonian (all in MeV).

- (3) studying the relation between the angular momentum I , the rotational angular frequency $\hbar\omega$ for bending in ruthenium isotopes;
- (4) calculating staggering effect and beat patterns to detect any interactions between the (+ve) and (-ve) parity states; and
- (5) calculating the electric monopole strength $X(E0/E2)$.

2 (IBA-1) model

2.1 Level energies

IBA-1 model was applied to the positive and negative parity states in even-even ⁹⁸⁻¹⁰⁸Ru isotopes. The Hamiltonian employed in the present calculation is:

$$\begin{aligned}
H = & EPS \cdot n_d + PAIR \cdot (P \cdot P) \\
& + \frac{1}{2} ELL \cdot (L \cdot L) + \frac{1}{2} QQ \cdot (Q \cdot Q) \\
& + 5OCT \cdot (T_3 \cdot T_3) + 5HEX \cdot (T_4 \cdot T_4),
\end{aligned} \quad (1)$$

where

$$P \cdot P = \frac{1}{2} \left[\begin{array}{c} \left\{ (s^\dagger s^\dagger)_0^{(0)} - \sqrt{5} (d^\dagger d^\dagger)_0^{(0)} \right\} x \\ \left\{ (ss)_0^{(0)} - \sqrt{5} (\tilde{d}\tilde{d})_0^{(0)} \right\} \end{array} \right]_0, \quad (2)$$

$$L \cdot L = -10 \sqrt{3} \left[(d^\dagger \tilde{d})^{(1)} x (d^\dagger \tilde{d})^{(1)} \right]_0, \quad (3)$$

$$Q \cdot Q = \sqrt{5} \left[\begin{array}{c} \left\{ (S^\dagger \tilde{d} + d^\dagger s)^{(2)} - \frac{\sqrt{7}}{2} (d^\dagger \tilde{d})^{(2)} \right\} x \\ \left\{ (s^\dagger \tilde{d} + \tilde{d}s)^{(2)} - \frac{\sqrt{7}}{2} (d^\dagger \tilde{d})^{(2)} \right\} \end{array} \right]_0, \quad (4)$$

$$T_3 \cdot T_3 = -\sqrt{7} \left[(d^\dagger \tilde{d})^{(2)} x (d^\dagger \tilde{d})^{(2)} \right]_0, \quad (5)$$

$$T_4 \cdot T_4 = 3 \left[(d^\dagger \tilde{d})^{(4)} x (d^\dagger \tilde{d})^{(4)} \right]_0. \quad (6)$$

In the previous formulas, n_d is the number of boson; $P \cdot P$, $L \cdot L$, $Q \cdot Q$, $T_3 \cdot T_3$ and $T_4 \cdot T_4$ represent pairing, angular momentum, quadrupole, octupole and hexadecupole interactions

between the bosons; *EPS* is the boson energy; and *PAIR*, *ELL*, *QQ*, *OCT*, *HEX* is the strengths of the pairing, angular momentum, quadrupole, octupole and hexadecupole interactions.

2.2 Transition rates

The electric quadrupole transition operator employed in this study is:

$$\begin{aligned}
T^{(E2)} = & E2SD \cdot (s^\dagger \tilde{d} + d^\dagger s)^{(2)} + \\
& + \frac{1}{\sqrt{5}} E2DD \cdot (d^\dagger \tilde{d})^{(2)}.
\end{aligned} \quad (7)$$

The reduced electric quadrupole transition rates between $I_i \rightarrow I_f$ states are given by

$$B(E2, I_i - I_f) = \frac{[\langle I_f || T^{(E2)} || I_i \rangle]^2}{2I_i + 1}. \quad (8)$$

3 Results and discussion

3.1 The potential energy surfaces

The potential energy surfaces [18], $V(\beta, \gamma)$, as a function of the deformation parameters β and γ are calculated using:

$$\begin{aligned}
E_{N_\pi N_\nu}(\beta, \gamma) = & \langle N_\pi N_\nu; \beta \gamma | H_{\pi\nu} | N_\pi N_\nu; \beta \gamma \rangle = \\
= & \zeta_d (N_\nu N_\pi) \beta^2 (1 + \beta^2) + \beta^2 (1 + \beta^2)^{-2} \times \\
& \times \{ k N_\nu N_\pi [4 - (\bar{X}_\pi \bar{X}_\nu) \beta \cos 3\gamma] \} + \\
& + \left\{ [\bar{X}_\pi \bar{X}_\nu \beta^2] + N_\nu (N_\nu - 1) \left(\frac{1}{10} c_0 + \frac{1}{7} c_2 \right) \beta^2 \right\},
\end{aligned} \quad (9)$$

where

$$\bar{X}_\rho = \left(\frac{2}{7} \right)^{0.5} X_\rho \quad \rho = \pi \text{ or } \nu. \quad (10)$$

The calculated potential energy surfaces, $V(\beta, \gamma)$, are presented in Fig. 1. It shows that ⁹⁸Ru is a vibrational — like nucleus while ¹⁰⁰⁻¹⁰⁴Ru are γ -soft where the two wells on the oblate and prolate sides are equal. ^{106,108}Ru are rotational - like where they are prolate deformed.

$I_i^+ I_f^+$	^{98}Ru	^{100}Ru	^{102}Ru	^{104}Ru	^{106}Ru	^{108}Ru
$0_1 \text{ Exp}^*. 2_1$	0.392(12)	0.490(5)	0.630(10)	0.820(12)	0.770(20)	1.010(15)
$0_1 \text{ Theor. } 2_1$	0.3930	0.4853	0.6279	0.8274	0.7737	1.0110
$2_1 0_1$	0.0786	0.0970	0.1256	0.1655	0.1547	0.2022
$2_2 0_1$	0.0000	0.0006	0.0012	0.0027	0.0032	0.0040
$2_2 0_2$	0.0226	0.0405	0.0548	0.0826	0.0870	0.1257
$2_3 0_1$	0.0000	0.0000	0.0000	0.0002	0.0006	0.0017
$2_3 0_2$	0.0658	0.0759	0.0993	0.1135	0.0853	0.0830
$2_3 0_3$	0.0093	0.0087	0.0121	0.0207	0.0264	0.0402
$2_4 0_3$	0.0041	0.0066	0.0121	0.0286	0.0448	0.0795
$2_4 0_4$	0.0565	0.0588	0.0712	0.0786	0.0530	0.0448
$4_1 2_1$	0.1260	0.1683	0.2257	0.3071	0.2912	0.3791
$4_1 2_2$	0.0092	0.0142	0.0190	0.0271	0.0267	0.0360
$4_1 2_3$	0.0269	0.0319	0.0424	0.0498	0.0384	0.0386
$6_1 4_1$	0.1420	0.2039	0.2838	0.3897	0.3681	0.4747
$6_1 4_2$	0.0172	0.0179	0.0228	0.0285	0.0256	0.0323
$6_1 4_3$	0.0208	0.0242	0.0333	0.0382	0.0292	0.0300
$8_1 6_1$	0.1264	0.2032	0.2998	0.4208	0.4012	0.5194
$8_1 6_2$	0.0247	0.0183	0.0223	0.0256	0.0217	0.0265
$8_1 6_3$	0.0113	0.0157	0.0239	0.0286	0.0228	0.0247
$10_1 8_1$	0.0791	0.1678	0.2768	0.4081	0.3997	0.5264
$10_1 8_2$	0.0319	0.0175	0.0207	0.0224	0.0183	0.0217

*Ref. 19.

Table 2: Values of the theoretical reduced transition probability, $B(E2)$ (in $e^2 b^2$).

$I_i^+ I_f^+$	^{98}Ru	^{100}Ru	^{102}Ru	^{104}Ru	^{106}Ru	^{108}Ru
$1_1 0_1$	0.0000	0.0030	0.0050	0.0104	0.0176	0.0261
$1_1 0_2$	0.1084	0.1280	0.1285	0.1280	0.1258	0.1227
$3_1 2_1$	0.1055	0.1211	0.1219	0.1306	0.1432	0.1564
$3_1 2_2$	0.0470	0.0415	0.0471	0.0544	0.0618	0.0712
$3_1 2_3$	0.0013	0.0002	0.0000	—	0.7737	—
$3_2 2_1$	0.0158	0.0024	0.0018	0.0029	0.0067	0.0130
$3_2 2_2$	0.0347	0.0197	0.0136	0.0102	0.0104	0.0121
$3_2 2_3$	0.1600	0.2126	0.2119	0.1943	0.1660	0.1352
$5_1 4_1$	0.2261	0.2533	0.2533	0.2605	0.2737	0.2881
$5_1 4_2$	0.0608	0.0480	0.0563	0.0648	0.0714	0.0784
$5_1 4_3$	0.0020	0.0006	—	—	0.7737	—
$7_1 6_1$	0.3657	0.3950	0.3912	0.3970	0.4083	0.4213
$7_1 6_2$	0.0609	0.0446	0.0551	0.0641	0.0701	0.0757
$9_1 8_1$	0.5276	0.5439	0.5367	0.5386	0.5465	0.5568
$9_1 8_2$	0.0425	0.0342	0.0472	0.0574	0.0640	0.0695
$11_1 10_1$	0.7143	0.6983	0.6872	0.6845	0.6882	0.6951

Table 3: Values of the theoretical reduced transition probability, $B(E1)$ (in $\mu e^2 b$).

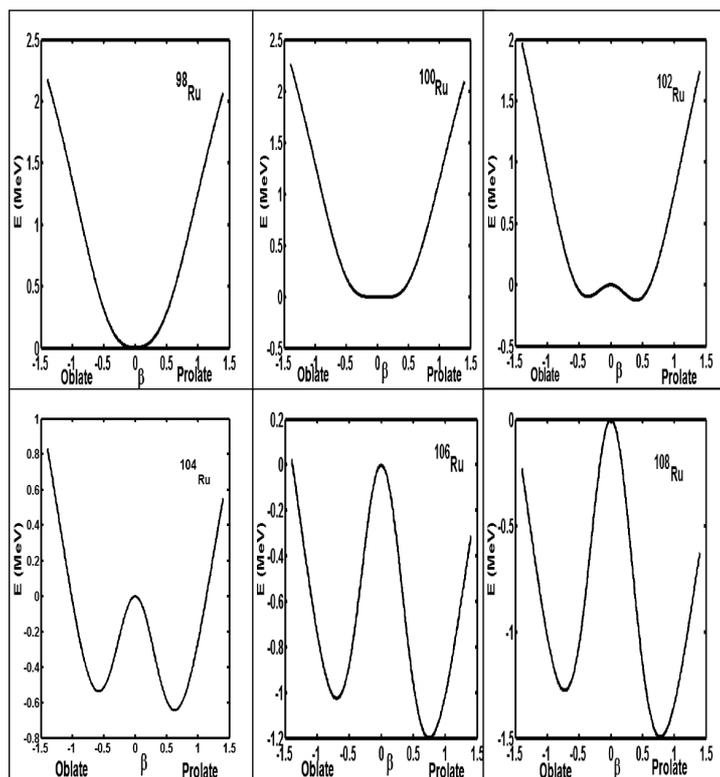


Fig. 1: Potential energy surfaces for $^{98-108}\text{Ru}$ nuclei.

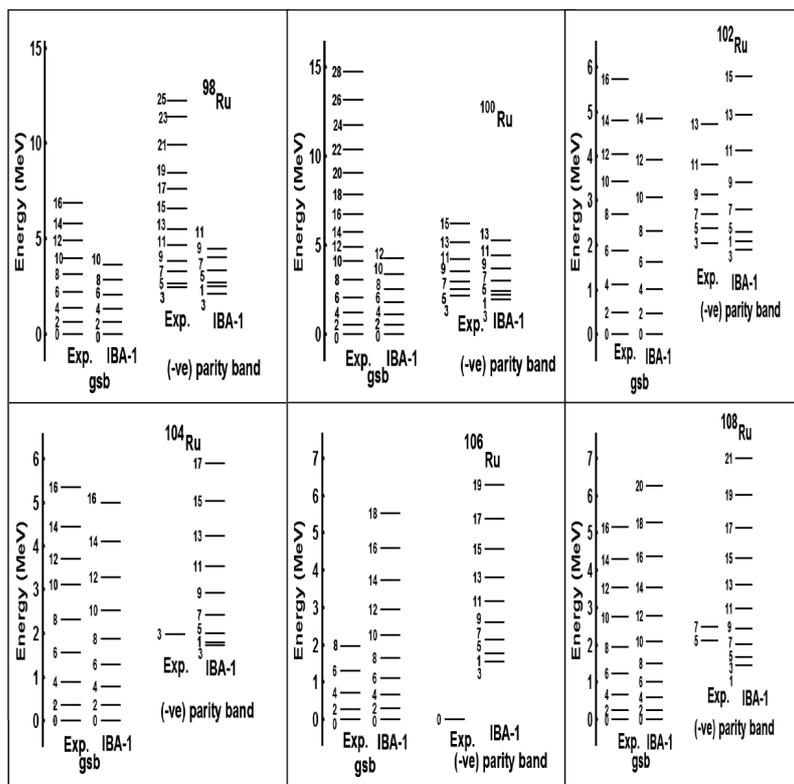


Fig. 2: Comparison between exp. [21–26] and theoretical (IBA-1) energy levels.

I_i^+	I_f^+	I_{if}^+	^{98}Ru	^{100}Ru	^{102}Ru	^{104}Ru	^{106}Ru	^{108}Ru
0 ₂	0 ₁	2 ₁	0.011	0.027	0.057	0.166	0.213	0.227
0 ₃	0 ₁	2 ₁	0.250	0.347	1.333	0.894	1.076	1.328
0 ₃	0 ₁	2 ₂	0.001	0.009	0.005	0.010	0.086	0.112
0 ₃	0 ₁	2 ₃	1.000	0.042	0.026	0.024	0.043	0.130
0 ₃	0 ₂	2 ₁	—	0.086	0.500	0.421	0.184	0.171
0 ₃	0 ₂	2 ₂	—	0.002	0.002	0.004	0.014	0.014
0 ₃	0 ₂	2 ₃	—	0.010	0.010	0.011	0.007	0.016
0 ₄	0 ₁	2 ₂	1.600	0.010	0.046	—	—	—
0 ₄	0 ₁	2 ₃	0.024	0.010	0.003	—	—	—
0 ₄	0 ₁	2 ₄	0.363	0.113	0.003	—	—	—
0 ₄	0 ₂	2 ₂	1.200	0.030	0.097	—	—	—
0 ₄	0 ₂	2 ₃	0.018	0.034	0.070	0.114	0.476	0.808
0 ₄	0 ₂	2 ₄	0.272	0.340	0.142	1.035	3.696	2.082
0 ₄	0 ₃	2 ₁	0.111	0.454	—	—	0.558	0.458
0 ₄	0 ₃	2 ₂	0.600	0.010	0.010	—	0.002	0.611
0 ₄	0 ₃	2 ₃	0.009	0.011	0.007	—	0.074	0.058
0 ₄	0 ₃	2 ₄	0.136	0.113	0.015	—	0.575	0.150

Table 4. Theoretical $X_{if} (E0/E2)$ in Ru isotopes.

3.2 Energy spectra

The energy of the positive and negative parity states of ruthenium series of isotopes are calculated using computer code PHINT [20]. A comparison between the experimental spectra [21–26] and our calculations, using values of the model parameters given in Table 1 for the ground state band are illustrated in Fig. 2. The agreement between the calculated levels energy and their correspondence experimental values for all nuclei are slightly higher especially for the higher excited states. We believe this is due to the change of the projection of the angular momentum which is due mainly to band crossing.

Unfortunately there is no enough measurements of electromagnetic transition rates $B(E1)$ or $B(E2)$ for these series of nuclei. The only measured $B(E2, 0_1^+ \rightarrow 2_1^+)$'s are presented, in Table 2 for comparison with the calculated values. The parameters $E2SD$ and $E2DD$ are used in the computer code NPBEM [20] for calculating the electromagnetic transition rates after normalization to the available experimental values and displayed in Table 1.

No new parameters are introduced for calculating electromagnetic transition rates $B(E1)$ and $B(E2)$ of intraband and interband. Some of the calculated values are presented in Fig. 3 and show bending at $N = 60, 62$ which means there is an interaction between the $(+ve)$ and $(-ve)$ parity states of the ground state band.

The moment of inertia I and angular frequency $\hbar\omega$ are calculated using equations (11, 12):

$$\frac{2I}{\hbar^2} = \frac{4I - 2}{\Delta E(I \rightarrow I - 2)}, \quad (11)$$

$$(\hbar\omega)^2 = (I^2 - I + 1) \left[\frac{\Delta E(I \rightarrow I - 2)}{(2I - 1)} \right]^2. \quad (12)$$

The plots in Fig. 4 show back bending at angular momentum $I^+ = 10$ for $^{98-108}\text{Ru}$ except ^{106}Ru where there is no experimental data available. It means, there is a crossing between the $(+ve)$ and $(-ve)$ parity states in the ground state band which confirmed by calculating staggering effect to these series of nuclei and the bending observed in Fig. 3.

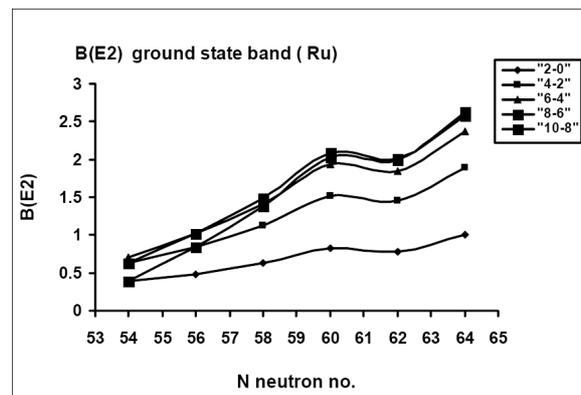


Fig. 3: The calculated $B(E2)$'s for the ground state band.

3.3 Electric monopole transitions

The electric monopole transitions, $E0$, are normally occurring between two states of the same spin and parity by transferring energy and zero unit of angular momentum. The strength of the electric monopole transition, $X_{if} (E0/E2)$, [27] can be calculated using equations (13, 14) and presented

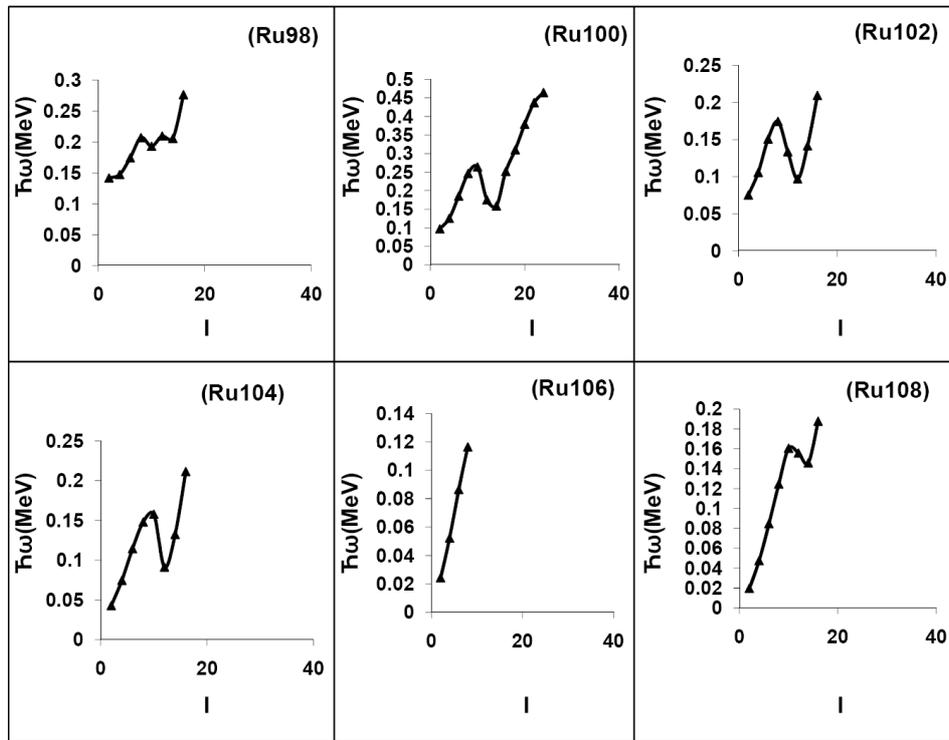


Fig. 4: Angular momentum I as a function of $(\hbar\omega)$.

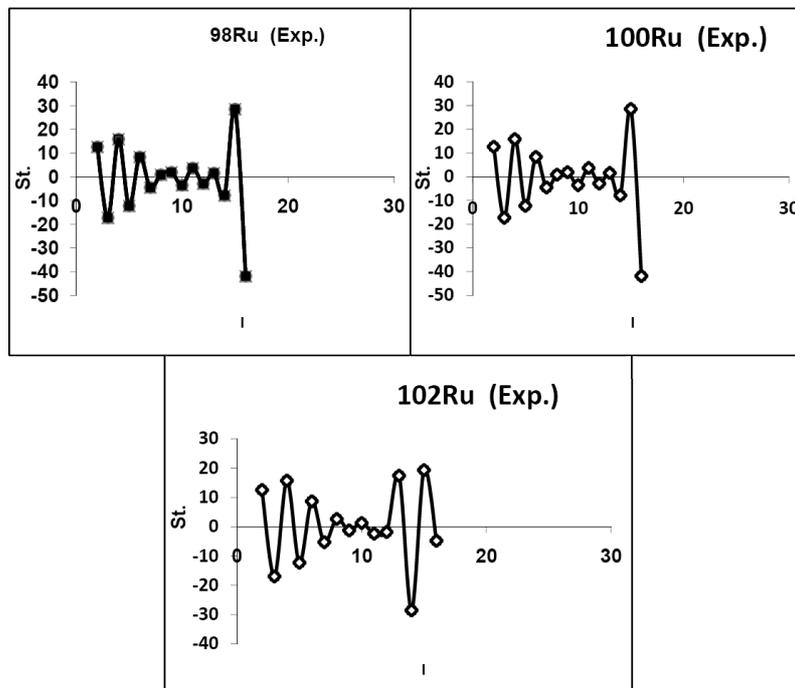


Fig. 5: $\Delta I = 1$, staggering patterns for $^{98-102}\text{Ru}$ isotopes.

in Table 4

$$X_{if'f}(E0/E2) = \frac{B(E0, I_i - I_f)}{B(E2, I_i - I_f)}, \quad (13)$$

$$X_{if'f}(E0/E2) = (2.54 \times 10^9) A^{3/4} \times \frac{E_\gamma^5(\text{MeV})}{\Omega_{KL}} \alpha(E2) \frac{T_e(E0, I_i - I_f)}{T_e(E2, I_i - I_f)}. \quad (14)$$

3.4 The staggering

The presence of $(+ve)$ and $(-ve)$ parity states has encouraged us to study staggering effect [28–30] for $^{98-108}\text{Ru}$ series of isotopes using staggering function equations (15, 16) with the help of the available experimental data [21–26].

$$\text{Stag}(I) = 6\Delta E(I) - 4\Delta E(I-1) - 4\Delta E(I+1) + \Delta E(I+2) + \Delta E(I-2), \quad (15)$$

with

$$\Delta E(I) = E(I+1) - E(I). \quad (16)$$

The calculated staggering patterns are illustrated in Fig. 5 and show an interaction between the $(+ve)$ and $(-ve)$ parity states for the ground state of $^{98-102}\text{Ru}$ nuclei.

3.5 Conclusions

IBA-1 model has been applied successfully to $^{98-108}\text{Ru}$ isotopes and we have got:

1. The levels energy are successfully reproduced;
2. The potential energy surfaces are calculated and show vibrational-like to ^{98}Ru , γ -soft to $^{100-104}\text{Ru}$ and rotational characters to $^{106-108}\text{Ru}$ isotopes where they are mainly prolate deformed nuclei;
3. Electromagnetic transition rates $B(E1)$ and $B(E2)$ are calculated;
4. Bending for $^{98-108}\text{Ru}$ has been observed at angular momentum $I^+ = 10$ except for ^{106}Ru , where there is no experimental data are available;
5. Electromagnetic transition rates $B(E1)$ and $B(E2)$ are calculated;
6. Strength of the electric monopole transitions $X_{if'f}(E0/E2)$ are calculated; and
7. Staggering effect has been calculated and beat patterns are obtained which show an interaction between the $(-ve)$ and $(+ve)$ parity states for $^{98-102}\text{Ru}$.

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