Dark Matter and Dark Energy: Breaking the Continuum Hypothesis?

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In the present paper an attempt is made to develop a fractional integral and differential, deterministic and projective method based on the assumption of the essential discontinuity observed in real systems (note that more than 99% of the volume occupied by an atom in real space has no matter). The differential treatment assumes continuous behaviour (in the form of averaging over the recent past of the system) to predict the future time evolution, such that the real history of the system is "forgotten". So it is easy to understand how problems such as unpredictability (chaos) arise for many dynamical systems, as well as the great difficulty to connecting Quantum Mechanics (a probabilistic differential theory) with General Relativity (a deterministic differential theory). I focus here on showing how the present theory can throw light on crucial astrophysical problems like dark matter and dark energy.

1 Introduction

In 1999 I published [1] the preliminaries of a new theory: the General Interactivity. It was a sketched presentation of the mathematical basis of the theory, i. e. the fractional integral treatment of time evolution. In the present paper we extend the ideas of General Interactivity to the fractional derivatives, and so we can explain the outer flatness of rotation curves, last measures of SN Ia at high redshifts, the fluctuations in the CMB radiation and the classical cosmology theory.

In 1933 Zwicky [2] found that the Coma cluster of galaxies ought to contain more matter than is inferred from optical observations: many of the thousands of galaxies in the cluster move at speeds faster than the escape velocity expected from the amount of visible matter and from the Newton theory of gravitation. In the 1970's, many authors discovered that the speed of stars and clouds of hydrogen atoms rotating in a galactic disk is nearly constant all the way out to the edge of the galaxy [3, 4]. Using Newton's law of gravitation, this implied that the amount of matter at increasing radius is not falling away, against the observed star-light suggests. Over past two decades, the measured deflection of light from a distant star by a massive object like a galaxy (gravitational lens) points to a mass-to-light ratio for the lensing galaxies of about 150, and yet if galaxies contained only observed stars the expected value would be between 5 and 10 [5]. From the observed cosmic microwave background (CMB, the relic radiation of the Big Bang that fills the Universe) fluctuations, we need that 23% of the Universe is dark matter, and 73% is dark energy [6, 7, 8, 9, 10]. Recent observations of SN Ia brightness show that the expansion of the Universe has been speeding up. This unexpected acceleration is also ascribed to an amount of dark energy that is very similar than 73% of the Universe [11].

In Section 2 we show a review of the theory, in Section 3 we apply the theory to account for the observed dark matter and dark energy, and in Section 4 we develop the conclusions.

2 The model

I start from two hypothesis: (1) the irreversibility in time of natural systems and (2) the interactivity among all the systems in the Universe. These hypotheses imply an intricate, unsettled and discontinuous (and hence non-differentiable) space-time. The differential treatment projects a variable $X(t)$, whose value is known at a time t, to a successive time, $t+\Delta t$, through the assumption of a knowledge of their time derivative, $X'(t)$, as follows: $X(t+\Delta t) = X(t) + X'(t)\Delta t$. In many cases, to a good approximation, there is proportionality between $X(t)$ and $X'(t)$ so that $X'(t) \propto X(t+\Delta t)$. Here I extend this projection, but with two crucial modifications: (a) I project a complete distribution of real values (a set of measured values ordered in time) instead of individual values at one time, and (b) I generalize the derivative to the Liouville fractional derivative (to take into account the possibility of the discontinuous space-time of the system under study). This then gives the fundamental equation of the new dynamics:

$$\frac{d^{\beta}_{FRAC}}{dt} X(t_{past}) \propto X(t_{future}) \,. \tag{1}$$

$X(t_{past})$ being a table of values of the variable X until the present time, $X(t_{future})$ the same number of values of X but from the present time to the future (a projection), and β a value between 0 and 1 that includes the key information about the history of the system.

But for more physical sense, one must take the inverse of equation (1), i. e.

$$X(t_{past}) \propto \frac{1}{\Gamma(\beta)} \int_{t_{past}}^{T} \frac{X(t_{future})}{(t_{past} - t_{future})^{1-\beta}} dt_{future} \tag{2}$$

which is the fractional integration (or the Riemann-Liouville integral) of $X(t_{future})$, T being a time-period characteristic of each system. The first hypothesis, irreversibility, suggests the necessity of projecting the values of $X(t)$, weighted by a function of time that must be similar to the function characteristic of critical points, such as observed in the well known irreversible phase transitions in Thermodynamics; for example, the form $(T_E - T_{EC})^{-0.64}$ for the time correlation length of an infinite set of spins with a temperature T_E near the critical temperature T_{EC} [12]. Compare this with the term $(t_{past} - t_{future})^{\beta-1}$ in equation (2). I call this weighting "generalized inertia"; it is characteristic of each system in the sense of incorporating into the β exponent the history of all the interactions suffered by the system, including those interactions avoided by the differential approximation (high order terms in Taylor expansions) due to its small values.

To use the fundamental equation (1) with maximum efficiency, I invert equation (2) because this is an Abel integral transform, and there is a technique developed by Simmoneau et al. [13] to optimize the inversion of Abel transforms. This technique consists in making a spectral expansion using a special kind of polynomials whose coefficients are obtained by means of numerical integration, thus avoiding the basic problem of amplification of the errors, a problem inherent in numerical differentiation; in the technique of Simmoneau et al., measurement errors are incorporated into the coefficients of the spectral expansion and then propagate with time without being amplified.

In the present context one can see the time as a critical variable, each "present" being an origin of time coordinates, with two time dimensions: the past and the future. We should note that in Quantum Mechanics two independent wave functions are needed (the real part and the imaginary part of the total wave function) to describe the state of a system at each moment in time.

One can view General Interactivity as a third approximation to reality: the first was the conception of continuous and flat space-time by Newton, the second was that of continuous and curved space-time by Einstein. Here I see a discontinuous space-time whose degree of intricacy measures the essential cause of changing. As in Newtonian Dynamics, where the forces are the causes of changing, and in General Relativity, where modifications of the metric of space-time are the cause of changes in the motion of all massive systems, in General Interactivity the exponent β gives us a measure of the intricacy of the space-time "seen" by each system through a given variable X. But how can we see Gravity from the new point of view of General Interactivity? From (differential) Potential Theory we know that the modulus of the gravity force per unit mass is the following function of mass distribution, $\rho(x)$, in space:

$$F_G(x) = G \int \frac{\rho(x')}{|x' - x|^2} d^3x' \tag{3}$$

and, comparing with the three-dimensional fractional integration of $\rho(x)$ we have:

$$R_\beta\left[\rho(x)\right] = \pi^{\beta - \frac{\pi}{2}} \frac{\Gamma\left(\frac{3-\beta}{2}\right)}{\Gamma\left(\frac{\beta}{2}\right)} \int \frac{\rho(x')}{|x - x'|^{3-\beta}} d^3x'; \tag{4}$$

$F_G(x)$ can be identified with the 1-integral of $\rho(x)$ in three-dimensional space ($\beta = 1$) except for a constant. So in the present context the gravity force can be interpreted as a one-dimensional projection of the three-dimensional continuous distribution of matter. It is not, then, a complete integral (this would be $\beta = 3$) and so the sum (integral) for obtaining the gravity is more intricate than the mass distribution (continuous by definition), i. e. the real discontinuity of mass distributions is transferred to the fractional integral instead of working with a discontinuous $\rho(x)$. Gravity, like the electrostatic force, whose expression is very similar to $F_G(x)$, is seen as an inertial reaction of space-time, which would tend to its initial (less intricate,i. e. simpler) state, towards a structure in which the masses were all held together without relative motions; both forces are seen as reactions against the action of progressive intricacy in the general expansion of the Universe following the Big Bang.

We take the total mass-energy of the Universe as the observable magnitude $X(t)$ to evolve in time using Eq. (2). The constancy of this variable gives $1 = \frac{1}{\beta^2}(T^2 - t_{past}^2)^\beta$ (where I take squared variables for simplicity in the use of Simmoneau et al.'s inversion technique). The greater past-time variable, t_{past}, less β indicating that the space-time is more intricate with time; this is the reason for integrating more fractionally (less β). So the parameter β can also be considered as a measure of the entropy of the Universe.

Another key to understand General Interactivity comes from the classical Gaussian and Planckian distribution functions, to which real systems in equilibrium tend. The equilibrium distribution function for systems of particles, for collisional and for collisionless systems (in the non-degenerate limit [14]) is Gaussian; classical Brownian motion is an example [15]; the equilibrium distribution function for systems of waves is a Planckian, and a key example is blackbody radiation. If both distribution functions evolve in time, then, using the inversion of equation (2), we have the same final result: the Planckian distribution. This tells us that whatever the initial distribution at the beginning of the Big Bang (perhaps both the Planckian characteristic of interacting waves and the Gaussian characteristic of interacting particles coexisted), their time evolution leads to a Planckian distribution, thereby connecting with the actual observed spectrum of the Cosmic Microwave Background, which appears to be almost perfectly Planckian.

But, why is the Planckian more stable than the Gaussian with the passing of time? The answer I propose is that successive critical transitions (at each time), due to the complexity caused by interactions at large distances, tend to amplify the Gaussian distribution to all range of energies, making it flatter. This breaks the thermal homogeneity because of the very different time evolution of many regions, due to the delay in the transmission of information from any one zone to others that are far away (note that the speed of the light is a constant). This amplification goes preferentially to high energies because there is no limit, in contrast with lower energies, for which the limit is the vacuum energy.

In this context, then, the Universe is seen as an expansion of objects that emitting information (electromagnetic waves) in all directions, and one can differentiate between two basic kinds of interactions: (a) at small distances (the distance travelled by light during a time that is characteristic of each system) forming coupled systems showing macroscopic (ensemble) characteristics, such as temperature or density, well differentiated from those of their surroundings; and (b) at large distances, interfering one system from another in a complex manner due to the permanent change in the relative distances due to the constancy of the speed of light, the huge number of interactions and the internal variation of the sources themselves. Note that this distinction between small and large distances can be extended relative to each physical system. For instance, a cloud of water vapour (as in the Earth's atmosphere) constitutes a system of water molecules interacting over short distances, while the interaction between one cloud and another is considered to take place over a large distance. Inside a galaxy, the stars in a cluster are considered to interact over short distances, while the interactions between that cluster and the remaining stars and gas clouds in the galaxy are considered as interactions over large distances.

In General Relativity there are no point objects; instead, all the objects in Nature are considered as systems of other objects, even subatomic particles appearing to be composed of others yet smaller.

I now focus on one of the most puzzling interpretations of Quantum Mechanics: the wave-particle duality. In Quantum Mechanics the objects under study show a double behaviour depending on what type of experiment one makes. An electron behaves as a particle in collisions with other electrons, but the same electron passing through two gaps (enough small and enough near each other) behaves as a wave in that the outcoming electrons form an interference pattern. In General Interactivity each "particle" is considered as a system, and we know that the equilibrium distribution of random particles is Gaussian, and that after the time evolution given by Eq. (2) the distribution transforms into a Planckian (the interaction with the other systems "drives" the random set of particles) which is the distribution to which a set of interacting waves in a cavity naturally tend. Furthermore, the Planckian can be decomposed into a set of Gaussians, so that the double nature of matter/energy is ensured. The fact that a Planckian can be the result of the addition of Gaussians of different centres and amplitudes is interpreted as the Planckian representing an ensemble of random motions in turn represented by Gaussians, which find a series of walls to which resulting in certain reflexion and certain absorption. As already demonstrated [15], both processes, reflexion and absorption by a barrier, are equivalent to the addition and the subtraction, respectively, of two Gaussians: the main Gaussian and that which emerge as a consequence of the barrier (by displacing its centre to the other side of the barrier). A Gaussian, then, converts into a set of several other Gaussians at progressively smaller amplitudes as a consequence of the existence of barriers, and the envelope is a Planckian. There is a partial reflection at each barrier in the direction of higher energies, while the reflection is total to the lower energies and the absorption of unreflected part must be added to the left of the barrier. This argument can be applied to explain the Planckian distribution observed in the Cosmic Microwave Background Radiation: the energy barriers can be thought of as the consequence of the existence of wrinkles in space-time, caused by the finiteness of the Universe (closed box) and the uncoupled expansion of the content with respect to the box, or by breaking of the expansion because of the collision of the outer parts with another medium, or by the succession of several bangs at the beginning, instead of only one bang.

3 Dark Matter and Dark Energy

Another example of application of this theory is the generalization of one of the most important theorems in Field Theory, Gauss's theorem, leading to a possible solution (as a kind of Modified Newtonian Dynamics theory) of the well known problem of the "lost mass" of the Universe and its associated problem of "dark matter" [16]. Assuming the well known observation of the infinitesimal volume occupied by matter relative to holes in Nature (the nucleus of an atom occupy less than 1% of the atom's volume, and gas clouds in the interstellar medium have densities of 1 atom per cubic centimeter or less), one must consider the possibility of relaxing the continuum hypothesis. The Gauss's theorem can be expressed, simplified and for the gravitational field, as

$$\int_S g_N dS = -4\pi GM \,, \tag{5}$$

where g_N is the intensity of the gravity field over a closed surface, S, which contains the mass, M, which is the origin of the field, on the assumption of continuity, and G is the universal gravity constant. So, integrating Eq. (5) on the assumption of $g_N \simeq$ constant over S, we get $g_N = -4\pi GM/S$, with $S = \int_S dS$. If we take as the starting point the differential form of Gauss theorem, and then we take in Eq. (5) the fractional, instead of the full, integral and also assume $g \simeq$ const, we have

$$g(r) = \frac{-4\pi GM}{\frac{\pi^{\beta-1}\Gamma\left(\frac{2-\beta}{2}\right)}{\Gamma\left(\frac{\beta}{2}\right)} \int_S \frac{dX}{|S-X|^{2-\beta}}} \,. \tag{6}$$

Because β is less than 2, g is greater than g_N, and this result could explain the observational fact of g_N being very

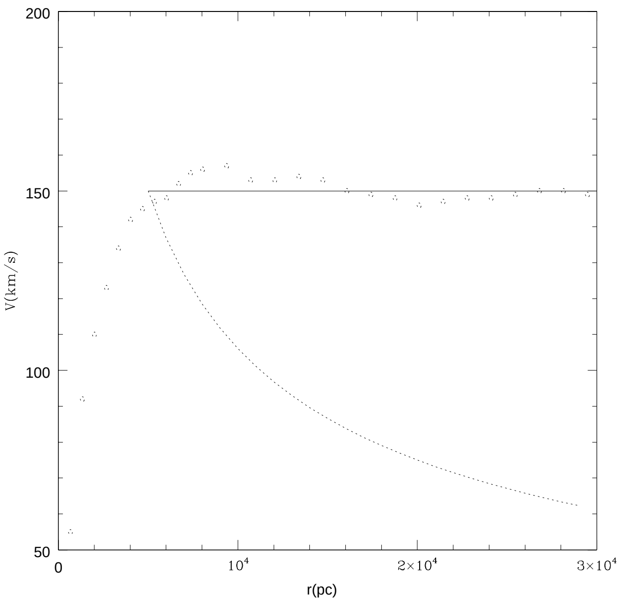

Fig. 1: Rotation curve (kms^{-1}) for NGC3198. Crosses are observational data points taken from van Albada et al. [17]. Full line is the prediction of the present theory, and dashed line is the prediction by the Newton law of gravitation.

small in explaining, for instance, many galactic rotation curves far from the central regions. For, assuming spherical simetry one has:

$$g(r) = g_N \frac{4\pi r^2}{\frac{\pi^{\beta-1}\Gamma\left(\frac{2-\beta}{2}\right)}{\Gamma\left(\frac{\beta}{2}\right)} \int_0^{2\pi} \frac{2\pi r^2 \sin\theta d\theta}{r^{2-\beta}}} . \tag{7}$$

And taking $\beta = 1$ one has $g(r) = g_N r$, which introduced in the classical centrifugal equilibrium identity $\frac{V^2}{r} = g$ leads to the amazingly $V = (GM)^{1/2} \simeq$ constant as is observed for flat rotation curves that needs dark matter (see Fig. 1).

More complicated treatment can be made: in the integral treatment one can consider the basic constituents of matter (the atoms) and the infinitesimal size of the volume occupied by the atomic mass (the nucleus) with respect to the size of the atom, and then one finds the necessity of take into account that ubiquitous nature of the big holes existing inside the matter (the atoms and molecules inside the very low density galactic gas clouds amplify the hole effect respect to the whole cloud and then amplify the influence in the macroscopic gravitational (massive) behaviour). So one can consider the hypothesis of continuity as a first approximation, and one can re-examine the Gauss' theorem

$$\int_S g dS = -4\pi G \int_V \rho dV \,, \tag{8}$$

where g is the gravitational field over the surface S, S is any closed surface containing the massive object which is the source of the field, G is the gravitational constant, ρ is the density, and V is the volume contained within the surface S. And one can generalize Eq. (5) in the sense of take both integrals as fractional integrals (α and β respectively) which leads to normal integrals for some especial case. If one assumes, for simplicity, spherical symmetry for the gas mass distribution in the galaxy, one has:

$$\rho = \rho_0 \, e^{\frac{-(r-r_0)}{r'}} \tag{9}$$

and assuming $g \simeq$ constant over the now non-necessary continue surface (the fractional integration takes this into account) one has:

$$g = -\frac{16\pi^2 G \rho_0 C_2(\beta)}{f(\alpha) r^2 C_1(\alpha)} \, r^{2-\alpha} \int r^2 r^{\beta-3} e^{-\frac{r-r_0}{r'}} dr \,, \tag{10}$$

where $f(\alpha)$ is some function of α,

$$C_2(\beta) = \pi^{\beta-3/2} \frac{\Gamma\left(\frac{3-\beta}{2}\right)}{\Gamma\left(\frac{\beta}{2}\right)} \,, \tag{11}$$

$$C_1(\alpha) = \pi^{\alpha-1} \frac{\Gamma\left(\frac{2-\alpha}{2}\right)}{\Gamma\left(\frac{\alpha}{2}\right)} \,, \tag{12}$$

while

$$g_N = -\frac{4\pi G \rho_0}{r^2} \int r^2 e^{-\frac{r-r_0}{r'}} dr \,. \tag{13}$$

So, in the especial case when $\beta = 3$ and $\alpha = 2$ we have $g = g_N$. Then, expanding $r^{\beta-3} \simeq 1 + (\beta-3)\log r + \ldots$ as $\beta \to 3$ and $r^{2-\alpha} \simeq 1 - (\alpha-2)\log r + \ldots$ as $\alpha \to 2$, and including the expansions into Eq. (13) one has

$$g \simeq \frac{8\pi C_2(\beta)}{f(\alpha) C_1(\alpha)} \big(1 - (\alpha-2)\log r\big) \times \times \left(g_N - \frac{4\pi G \rho_0}{r^2} \int (\beta-3)(\log r) r^2 e^{-\frac{r-r_0}{r'}} dr \right). \tag{14}$$

And integrating by parts and taking very large values for r, we have

$$g \simeq g_N \frac{8\pi C_2(\beta)}{f(\alpha) C_1(\alpha)} \big(1 + (\alpha-2)(3-\beta)\log^2 r\big) \,. \tag{15}$$

And for typical values of observed flat rotation curves (5kpc $\leqslant r \leqslant$ 20kpc) we have that $g \propto g_N r$ represents a good approximation. So, for certain values of α and β (α less than 2 and β greater than 3) one has that outer rotation curves can be flat as observed.

But the most puzzling problem up-to-date in cosmology is the necessity of adding "ad hoc" a dark energy or negative pressure (the so called by Einstein cosmological constant) to the main equation of General Relativity to account for the last measures on supernovae Ia and the fluctuations in the cosmic microwave background radiation which implies a flat accelerating expanding universe. The field equation of General Relativity was formulated by Einstein as the generalization of the classic Poisson equation which relates the second derivative of the potential ϕ associated to the gravitational field with the assumed continuous mass distribution represented by the volume density ρ:

$$\Delta^2\phi + 4\pi G\rho = 0\,. \tag{16}$$

For comparison, the similar equation in General Relativity, which relates the mass and energy distribution with the differential changes in the geometry of the continuum space-time, is (see e. g. Einstein [18]):

$$\left(R_{\mu\nu} - \frac{1}{2}\,g_{\mu\nu}R\right) + \kappa\,T_{\mu\nu} = 0\,. \tag{17}$$

But for the last equation be coherent with the last independent measures of SN Ia and fluctuations of CMB radiation, we need to add a term $g_{\mu\nu}\Lambda$ to the left side of equation which represents near 73% of all the other terms. This problem is avoided naturally if we consider a discontinuous space-time, and then we re-formulate the equations by using the fractional derivative instead the full derivative. In that case, the second derivative is less than the full derivative, and then the cosmological constant is not needed at all to equilibrate the equations. In fact the μ-fractional derivative of the function r^λ is given by [19]:

$$D^\mu r^\lambda = \frac{\Gamma(\lambda+1)}{\Gamma(\lambda-\mu+1)}\,r^{\lambda-\mu} \tag{18}$$

for λ greater than -1, μ greater than 0. But as $\lambda \to -1$, $r^\lambda \to \propto \phi$ being ϕ the gravitational potential. And as one can see, taken a fixed value of λ, as μ increase, the μ-derivative decrease. Or to be more precise, if we assume that the constant to be added to the left side of Eq. (17) represents the 73% of all the matter and energy in the Universe, one has:

$$\lim_{\lambda\to-1} \frac{\Gamma(\lambda-\mu+1)R^{-\lambda-2}}{\Gamma(\lambda-2+1)R^{-\lambda-\mu}} \simeq 1.73\,, \tag{19}$$

where R is a characteristic scale-length of the Universe. And the relation (19) works for values of μ greater but very near 2, being 2 the value corresponding to the usual second derivative. So we conclude that taking a value, for the derivatives in the field equations, slightly greater than the usual 2, we are able of include the cosmological constant inside the new fractional derivative of the classical field equations.

4 Conclusions

The new theory of the General Interactivity can be applied to many fields of natural science and constitutes a new step forward in the approximation to the real behaviour of Nature. It assumes the necessity of explicitly taking into account the real history of a system and projecting to the future. However, it also takes into account the non-uniformity of the distribution of holes in the Nature and is therefore a theory of discontinuity. The new theory can account for naturally the needed amounts of dark matter and dark energy as a light modification of classical field equations.

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References

1. Casuso E. *I.J.M.P.A.*, 1999, v. 14, No. 20, 3239.
2. Zwicky F. *Hel. Phys. Acta*, 1933, v. 6, 110.
3. Sancisi R., van Albada T. S. In: *Dark Matter in the Universe*, (eds. Knapp G., Kormendy J.), I.A.U. Symp. No. 117, 67 Reidel, Dordrecht, 1987.
4. Battaner E., Florido E. *Fund. of Cosmic Phys.*, 2000, v. 21, 1.
5. Mc Kay T. A. et al. *ApJ*, 2002, v. 571, L85.
6. Riess A. G. et al. *AJ*, 1998, v. 116, 1009.
7. Perlmutter S. et al. *ApJ*, 1999, v. 517, 565.
8. de Bernardis P. et al. *Nature*, 2000, v. 404, 955.
9. Hanany S. et al. *ApJ*, 2000, v. 545, L5.
10. Halverson N. W. et al. *ApJ*, 2002, v. 568, 38.
11. Kirshner R. P. *Science*, 2003, v. 300, 1914.
12. Wilson K. G., Kogut J. *Physics Reports*, 1974, v. 12, No. 2, 75.
13. Simmoneau E., Varela A. M., Munoz-Tunon C. *J.Q.S.R.T.*, 1993, v. 49, 149.
14. Nakamura J. *ApJ*, 2000, v. 531, 739.
15. Chandrasekhar S. *Rev. Mod. Phys.*, 1943, v. 15(1), 20.
16. Binney J., Tremaine S. In: *Galactic Dynamics*, Princeton Univ. Press, 1987.
17. van Albada T. S., Bahcall J. N., Begeman K., Sancisi R. *ApJ*, 1985, v. 295, 305.
18. Einstein A. In: *El Significado de la Relatividad*, ed. Planeta-Agostini, Spain, 1985.
19. Oldham K. B., Spanier J. In: *The Fractional Calculus: Integrations and Differentiations of Arbitrary Order*, New York, Academic Press, 1974.