4X1-Matrix Functions and Dirac's Equation

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All 4X1-matrix square integrable functions with restricted domain obey slightly generalized Dirac's equations. These equations give formulas similar to some gluon and gravity ones.

1 Significations

Denote:

$$1_2 := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, 0_2 := \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\beta^{[0]} := -\begin{bmatrix} 1_2 & 0_2 \\ 0_2 & 1_2 \end{bmatrix} = -1_4,$$

the Pauli matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

I call a set $\widetilde{C}$ of complex $n \times n$ matrices *a Clifford set of rank* n [1] if the following conditions are fulfilled:

— if $\alpha_k \in \widetilde{C}$ and $\alpha_r \in \widetilde{C}$ then $\alpha_k \alpha_r + \alpha_r \alpha_k = 2\delta_{k,r}$;

— if $\alpha_k \alpha_r + \alpha_r \alpha_k = 2\delta_{k,r}$ for all elements α_r of set $\widetilde{C}$ then $\alpha_k \in \widetilde{C}$.

If $n = 4$ then the Clifford set either contains 3 (*Clifford triplet*) or 5 matrices (*Clifford pentad*).

Here exist only six Clifford pentads [1]: one which I call

• *light pentad* β:

$$\beta^{[1]} := \begin{bmatrix} \sigma_1 & 0_2 \\ 0_2 & -\sigma_1 \end{bmatrix}, \quad \beta^{[2]} := \begin{bmatrix} \sigma_2 & 0_2 \\ 0_2 & -\sigma_2 \end{bmatrix}, \quad \beta^{[3]} := \begin{bmatrix} \sigma_3 & 0_2 \\ 0_2 & -\sigma_3 \end{bmatrix}, \tag{1}$$

$$\gamma^{[0]} := \begin{bmatrix} 0_2 & 1_2 \\ 1_2 & 0_2 \end{bmatrix}, \tag{2}$$

$$\beta^{[4]} := \mathrm{i} \cdot \begin{bmatrix} 0_2 & 1_2 \\ -1_2 & 0_2 \end{bmatrix}; \tag{3}$$

three *coloured* pentads:

• *the red pentad* ζ:

$$\zeta^{[1]} = \begin{bmatrix} -\sigma_1 & 0_2 \\ 0_2 & \sigma_1 \end{bmatrix}, \quad \zeta^{[2]} = \begin{bmatrix} \sigma_2 & 0_2 \\ 0_2 & \sigma_2 \end{bmatrix}, \quad \zeta^{[3]} = \begin{bmatrix} -\sigma_3 & 0_2 \\ 0_2 & -\sigma_3 \end{bmatrix}, \tag{4}$$

$$\gamma_\zeta^{[0]} = \begin{bmatrix} 0_2 & -\sigma_1 \\ -\sigma_1 & 0_2 \end{bmatrix}, \quad \zeta^{[4]} = \mathrm{i} \begin{bmatrix} 0_2 & \sigma_1 \\ -\sigma_1 & 0_2 \end{bmatrix}; \tag{5}$$

• *the green pentad* η:

$$\eta^{[1]} = \begin{bmatrix} -\sigma_1 & 0_2 \\ 0_2 & -\sigma_1 \end{bmatrix}, \quad \eta^{[2]} = \begin{bmatrix} -\sigma_2 & 0_2 \\ 0_2 & \sigma_2 \end{bmatrix}, \quad \eta^{[3]} = \begin{bmatrix} \sigma_3 & 0_2 \\ 0_2 & \sigma_3 \end{bmatrix}, \tag{6}$$

$$\gamma_\eta^{[0]} = \begin{bmatrix} 0_2 & -\sigma_2 \\ -\sigma_2 & 0_2 \end{bmatrix}, \quad \eta^{[4]} = \mathrm{i} \begin{bmatrix} 0_2 & \sigma_2 \\ -\sigma_2 & 0_2 \end{bmatrix}; \tag{7}$$

• *the blue pentad* θ:

$$\theta^{[1]} = \begin{bmatrix} \sigma_1 & 0_2 \\ 0_2 & \sigma_1 \end{bmatrix}, \quad \theta^{[2]} = \begin{bmatrix} -\sigma_2 & 0_2 \\ 0_2 & -\sigma_2 \end{bmatrix}, \quad \theta^{[3]} = \begin{bmatrix} -\sigma_3 & 0_2 \\ 0_2 & \sigma_3 \end{bmatrix}, \tag{8}$$

$$\gamma_\theta^{[0]} = \begin{bmatrix} 0_2 & -\sigma_3 \\ -\sigma_3 & 0_2 \end{bmatrix}, \quad \theta^{[4]} = \mathrm{i} \begin{bmatrix} 0_2 & \sigma_3 \\ -\sigma_3 & 0_2 \end{bmatrix}; \tag{9}$$

• two *gustatory* pentads: *the sweet pentad* $\underline{\Delta}$:

$$\underline{\Delta}^{[1]} = \begin{bmatrix} 0_2 & -\sigma_1 \\ -\sigma_1 & 0_2 \end{bmatrix}, \quad \underline{\Delta}^{[2]} = \begin{bmatrix} 0_2 & -\sigma_2 \\ -\sigma_2 & 0_2 \end{bmatrix},$$

$$\underline{\Delta}^{[3]} = \begin{bmatrix} 0_2 & -\sigma_3 \\ -\sigma_3 & 0_2 \end{bmatrix},$$

$$\underline{\Delta}^{[0]} = \begin{bmatrix} -1_2 & 0_2 \\ 0_2 & 1_2 \end{bmatrix}, \quad \underline{\Delta}^{[4]} = \mathrm{i} \begin{bmatrix} 0_2 & 1_2 \\ -1_2 & 0_2 \end{bmatrix}.$$

• *the bitter pentad* $\underline{\Gamma}$:

$$\underline{\Gamma}^{[1]} = \mathrm{i} \begin{bmatrix} 0_2 & -\sigma_1 \\ \sigma_1 & 0_2 \end{bmatrix}, \quad \underline{\Gamma}^{[2]} = \mathrm{i} \begin{bmatrix} 0_2 & -\sigma_2 \\ \sigma_2 & 0_2 \end{bmatrix},$$

$$\underline{\Gamma}^{[3]} = \mathrm{i} \begin{bmatrix} 0_2 & -\sigma_3 \\ \sigma_3 & 0_2 \end{bmatrix},$$

$$\underline{\Gamma}^{[0]} = \begin{bmatrix} -1_2 & 0_2 \\ 0_2 & 1_2 \end{bmatrix}, \quad \underline{\Gamma}^{[4]} = \begin{bmatrix} 0_2 & 1_2 \\ 1_2 & 0_2 \end{bmatrix}.$$

If A is a 2×2 matrix then

$$A1_4 := \begin{bmatrix} A & 0_2 \\ 0_2 & A \end{bmatrix} \text{ and } 1_4 A := \begin{bmatrix} A & 0_2 \\ 0_2 & A \end{bmatrix}.$$

And if B is a 4×4 matrix then

$$A + B := A1_4 + B,\; AB := A1_4 B$$

etc.

$$\underline{x} := \langle x_0, \mathbf{x}\rangle := \langle x_0, x_1, x_2, x_3\rangle ,$$
$$x_0 := \mathrm{c}t,$$

with $\mathrm{c} = 299792458$.

2 Planck's functions

Let $\mathrm{h} = 6.6260755 \times 10^{-34}$ and $\underline{\Omega}$ ($\underline{\Omega} \subset R^{1+3}$) be a domain such that if $\underline{x} \in \underline{\Omega}$ then $|x_r| < \frac{\mathrm{c}\pi}{\mathrm{h}}$ for $r \in \{0,1,2,3\}$.

Let $\Re_{\underline{\Omega}}$ be a set of functions such that for each element $\phi(\underline{x})$ of this set: if $\underline{x} \notin \underline{\Omega}$ then $\phi(\underline{x}) = 0$.

Hence:

$$\int_{(\underline{\Omega})} d\underline{x} \cdot \phi(\underline{x}) =$$
$$= \int_{-\frac{\mathrm{c}\pi}{\mathrm{h}}}^{\frac{\mathrm{c}\pi}{\mathrm{h}}} dx_0 \int_{-\frac{\mathrm{c}\pi}{\mathrm{h}}}^{\frac{\mathrm{c}\pi}{\mathrm{h}}} dx_1 \int_{-\frac{\mathrm{c}\pi}{\mathrm{h}}}^{\frac{\mathrm{c}\pi}{\mathrm{h}}} dx_2 \int_{-\frac{\mathrm{c}\pi}{\mathrm{h}}}^{\frac{\mathrm{c}\pi}{\mathrm{h}}} dx_3 \cdot \phi(\underline{x}),$$

and let for each element $\phi(\underline{x})$ of $\Re_{\underline{\Omega}}$ exist a number J_ϕ such that

$$J_\phi = \int_{(\underline{\Omega})} d\underline{x} \cdot \phi^*(\underline{x})\, \phi(\underline{x}).$$

Therefore, $\Re_{\underline{\Omega}}$ is unitary space with the following scalar product:

$$\widetilde{u} * \widetilde{v} := \int_{(\underline{\Omega})} d\underline{x} \cdot \widetilde{u}^*(\underline{x})\, \widetilde{v}(\underline{x}). \tag{10}$$

This space has an orthonormalised basis with the following elements:

$$\varsigma_{w,\mathbf{p}}(t,\mathbf{x}) :=$$
$$:= \begin{cases} \left(\frac{\mathrm{h}}{2\pi\mathrm{c}}\right)^2 \exp(\mathrm{i}\mathrm{h}wt) \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{p}\mathbf{x}\right) \text{ if} \\ -\frac{\pi\mathrm{c}}{\mathrm{h}} \leqslant x_k \leqslant \frac{\pi\mathrm{c}}{\mathrm{h}}; \\ 0, \text{ otherwise.} \end{cases} \tag{11}$$

with $k \in \{0,1,2,3\}$ and $x_0 := \mathrm{c}t$, and with natural w, p_1, p_2, p_3 (here: $\mathbf{p}\langle p_1, p_2, p_3\rangle$ and $\mathbf{p}\mathbf{x} = p_1x_1 + p_2x_2 + p_3x_3$).

I call elements of the space with this basis *Planck's functions*.

Let $j \in \{1,2,3,4\}$, $k \in \{1,2,3,4\}$ and denote:

$$\sum_{\mathbf{k}} := \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} \sum_{k_3=-\infty}^{\infty}.$$

Let a Fourier series for $\varphi_j(t,\mathbf{x})$ have the following form:

$$\varphi_j(t,\mathbf{x}) = \sum_{w=-\infty}^{\infty} \sum_{\mathbf{p}} c_{j,w,\mathbf{p}} \varsigma_{w,\mathbf{p}}(t,\mathbf{x}). \tag{12}$$

If denote: $\varphi_{j,w,\mathbf{p}}(t,\mathbf{x}) := c_{j,w,\mathbf{p}}\varsigma_{w,\mathbf{p}}(t,\mathbf{x})$ then a Fourier series for $\varphi_j(t,\mathbf{x})$ has the following form:

$$\varphi_j(t,\mathbf{x}) = \sum_{w=-\infty}^{\infty} \sum_{\mathbf{p}} \varphi_{j,w,\mathbf{p}}(t,\mathbf{x}). \tag{13}$$

Let $\langle t, \mathbf{x}\rangle$ be any space-time point.

Let us denote:

$$A_k := \varphi_{k,w,\mathbf{p}}\big|_{\langle t,\mathbf{x}\rangle} \tag{14}$$

the value of function $\varphi_{k,w,\mathbf{p}}$ in this point, and by

$$C_j := \left(\frac{1}{\mathrm{c}}\partial_t \varphi_{j,w,\mathbf{p}} - \sum_{s=1}^{4}\sum_{\alpha=1}^{3} \beta_{j,s}^{[\alpha]} \partial_\alpha \varphi_{s,w,\mathbf{p}}\right)\Bigg|_{\langle t,\mathbf{x}\rangle} \tag{15}$$

the value of function

$$\left(\frac{1}{\mathrm{c}}\partial_t \varphi_{j,w,\mathbf{p}} - \sum_{s=1}^{4}\sum_{\alpha=1}^{3} \beta_{j,s}^{[\alpha]} \partial_\alpha \varphi_{s,w,\mathbf{p}}\right).$$

Here A_k and C_j are complex numbers. Hence, the following set of equations:

$$\begin{cases} \sum_{k=1}^{4} z_{j,k,w,\mathbf{p}} A_k = C_j\,, \\ z^*_{j,k,w,\mathbf{p}} = -z_{k,j,w,\mathbf{p}} \end{cases} \tag{16}$$

is a system of 14 algebraic equations with complex unknowns $z_{j,k,w,\mathbf{p}}$.

Because

$$\partial_t \varphi_{j,w,\mathbf{p}} = \partial_t c_{j,w,\mathbf{p}} \varsigma_{w,\mathbf{p}} = \mathrm{i}\mathrm{h}w c_{j,w,\mathbf{p}} \varsigma_{w,\mathbf{p}} = \mathrm{i}\mathrm{h}w \varphi_{j,w,\mathbf{p}}$$

and for $k \neq 0$:

$$\partial_k \varphi_{j,w,\mathbf{p}} = -\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}} p_k \varphi_{j,w,\mathbf{p}}.$$

then

$$C_j = \mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\left(w\varphi_{j,w,\mathbf{p}} + \sum_{s=1}^{4}\sum_{\alpha=1}^{3} \beta_{j,s}^{[\alpha]} p_\alpha \varphi_{s,w,\mathbf{p}}\right)\Bigg|_{\langle t,\mathbf{x}\rangle}.$$

Therefore, this system (16) has got the following form:

$$z_{1,1,w,\mathbf{p}}A_1 + z_{1,2,w,\mathbf{p}}A_2 + z_{1,3,w,\mathbf{p}}A_3 + z_{1,4,w,\mathbf{p}}A_4 =$$
$$= \mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}(w + p_3)A_1 + \mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}(p_1 - \mathrm{i}p_2)A_2\,,$$
$$z_{2,1,w,\mathbf{p}}A_1 + z_{2,2,w,\mathbf{p}}A_2 + z_{2,3,w,\mathbf{p}}A_3 + z_{2,4,w,\mathbf{p}}A_4 =$$
$$= \mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}(w - p_3)A_2 + \mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}(p_1 + \mathrm{i}p_2)A_1\,,$$
$$z_{3,1,w,\mathbf{p}}A_1 + z_{3,2,w,\mathbf{p}}A_2 + z_{3,3,w,\mathbf{p}}A_3 + z_{3,4,w,\mathbf{p}}A_4 =$$
$$= \mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}(w - p_3)A_3 - \mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}(p_1 - \mathrm{i}p_2)A_4\,,$$
$$z_{4,1,w,\mathbf{p}}A_1 + z_{4,2,w,\mathbf{p}}A_2 + z_{4,3,w,\mathbf{p}}A_3 + z_{4,4}A_{4,w,\mathbf{p}} =$$
$$= \mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}(w + p_3)A_4 - \mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}(p_1 + \mathrm{i}p_2)A_3\,,$$

$$z^*_{1,1,w,\mathbf{p}} = -z_{1,1,w,\mathbf{p}}\,,$$
$$z^*_{1,2,w,\mathbf{p}} = -z_{2,1,w,\mathbf{p}}\,,$$
$$z^*_{1,3,w,\mathbf{p}} = -z_{3,1,w,\mathbf{p}}\,,$$
$$z^*_{1,4,w,\mathbf{p}} = -z_{4,1,w,\mathbf{p}}\,,$$
$$z^*_{2,2,w,\mathbf{p}} = -z_{2,2,w,\mathbf{p}}\,,$$
$$z^*_{2,3,w,\mathbf{p}} = -z_{3,2,w,\mathbf{p}}\,,$$
$$z^*_{2,4,w,\mathbf{p}} = -z_{4,2,w,\mathbf{p}}\,,$$
$$z^*_{3,3,w,\mathbf{p}} = -z_{3,3,w,\mathbf{p}}\,,$$
$$z^*_{3,4,w,\mathbf{p}} = -z_{4,3,w,\mathbf{p}}\,,$$
$$z^*_{4,4,w,\mathbf{p}} = -z_{4,4,w,\mathbf{p}}\,.$$

This system can be transformed into a system of 8 linear real equations with 16 real unknowns $x_{s,k} := \mathrm{Re}\left(z_{s,k,w,\mathbf{p}}\right)$ for $s < k$ and $y_{s,k} := \mathrm{Im}\left(z_{s,k,w,\mathbf{p}}\right)$ for $s \leqslant k$:

$$\begin{aligned}
&-y_{1,1}b_1 + x_{1,2}a_2 - y_{1,2}b_2 + x_{1,3}a_3 - \\
&- y_{1,3}b_3 + x_{1,4}a_4 - y_{1,4}b_4 = \\
&= -\frac{\mathrm{h}}{\mathrm{c}}wb_1 - \frac{\mathrm{h}}{\mathrm{c}}p_3b_1 - \frac{\mathrm{h}}{\mathrm{c}}p_1b_2 + \frac{\mathrm{h}}{\mathrm{c}}p_2a_2\,, \\
&y_{1,1}a_1 + x_{1,2}b_2 + y_{1,2}a_2 + x_{1,3}b_3 + \\
&+ y_{1,3}a_3 + x_{1,4}b_4 + y_{1,4}a_4 = \\
&= \frac{\mathrm{h}}{\mathrm{c}}wa_1 + hp_3a_1 + \frac{\mathrm{h}}{\mathrm{c}}p_1a_2 + hp_2b_2\,, \\
&-x_{1,2}a_1 - y_{1,2}b_1 - y_{2,2}b_2 + x_{2,3}a_3 - \\
&- y_{2,3}b_3 + x_{2,4}a_4 - y_{2,4}b_4 = \\
&= -\frac{\mathrm{h}}{\mathrm{c}}wb_2 - \frac{\mathrm{h}}{\mathrm{c}}p_1b_1 - \frac{\mathrm{h}}{\mathrm{c}}p_2a_1 + \frac{\mathrm{h}}{\mathrm{c}}p_3b_2\,, \\
&-x_{1,2}b_1 + y_{1,2}a_1 + y_{2,2}a_2 + x_{2,3}b_3 + \\
&+ y_{2,3}a_3 + x_{2,4}b_4 + y_{2,4}a_4 = \\
&= \frac{\mathrm{h}}{\mathrm{c}}wa_2 + \frac{\mathrm{h}}{\mathrm{c}}p_1a_1 - \frac{\mathrm{h}}{\mathrm{c}}p_2b_1 - \frac{\mathrm{h}}{\mathrm{c}}p_3a_2\,, \\
&-x_{1,3}a_1 - y_{1,3}b_1 - x_{2,3}a_2 - y_{2,3}b_2 - \\
&- y_{3,3}b_3 + x_{3,4}a_4 - y_{3,4}b_4 = \\
&= -\frac{\mathrm{h}}{\mathrm{c}}wb_3 + \frac{\mathrm{h}}{\mathrm{c}}p_3b_3 + \frac{\mathrm{h}}{\mathrm{c}}p_1b_4 - \frac{\mathrm{h}}{\mathrm{c}}p_2a_4\,, \\
&-x_{1,3}b_1 + y_{1,3}a_1 - x_{2,3}b_2 + y_{2,3}a_2 + \\
&+ y_{3,3}a_3 + x_{3,4}b_4 + y_{3,4}a_4 = \\
&= \frac{\mathrm{h}}{\mathrm{c}}wa_3 - \frac{\mathrm{h}}{\mathrm{c}}p_3a_3 - \frac{\mathrm{h}}{\mathrm{c}}p_1a_4 - \frac{\mathrm{h}}{\mathrm{c}}p_2b_4\,, \\
&-x_{1,4}a_1 - y_{1,4}b_1 - x_{2,4}a_2 - y_{2,4}b_2 - \\
&-x_{3,4}a_3 - y_{3,4}b_3 - y_{4,4}b_4 = \\
&= -\frac{\mathrm{h}}{\mathrm{c}}wb_4 + \frac{\mathrm{h}}{\mathrm{c}}p_1b_3 + \frac{\mathrm{h}}{\mathrm{c}}p_2a_3 - \frac{\mathrm{h}}{\mathrm{c}}p_3b_4\,, \\
&-x_{1,4}b_1 + y_{1,4}a_1 - x_{2,4}b_2 + y_{2,4}a_2 - \\
&- x_{3,4}b_3 + y_{3,4}a_3 + y_{4,4}a_4 = \\
&= \frac{\mathrm{h}}{\mathrm{c}}wa_4 - \frac{\mathrm{h}}{\mathrm{c}}p_1a_3 + \frac{\mathrm{h}}{\mathrm{c}}p_2b_3 + \frac{\mathrm{h}}{\mathrm{c}}p_3a_4\,;
\end{aligned}$$

(here $a_k := \mathrm{Re}A_k$ and $b_k := \mathrm{Im}A_k$).

This system has solutions according to the Kronecker-Capelli theorem (rank of this system augmented matrix and rank of this system basic matrix equal to 7). Hence, such complex numbers $z_{j,k,w,\mathbf{p}}|_{\langle t,\mathbf{x}\rangle}$ exist in all points $\langle t,\mathbf{x}\rangle$.

From (16), (14), (15):

$$\sum_{k=1}^{4} z_{j,k,w,\mathbf{p}}\varphi_{k,w,\mathbf{p}}|_{\langle t,\mathbf{x}\rangle} = \\ = \left(\frac{1}{\mathrm{c}}\,\partial_t\varphi_{j,w,\mathbf{p}} - \sum_{s=1}^{4}\sum_{\alpha=1}^{3}\beta^{[\alpha]}_{j,s}\partial_\alpha\varphi_{s,w,\mathbf{p}}\right)|_{\langle t,\mathbf{x}\rangle}\,,$$

in every point $\langle t,\mathbf{x}\rangle$.

Therefore, from (16, 15, 14):

$$\frac{1}{\mathrm{c}}\,\partial_t\varphi_{j,w,\mathbf{p}} = \\ = \sum_{k=1}^{4}\left(\sum_{\alpha=1}^{3}\beta^{[\alpha]}_{j,k}\partial_\alpha\varphi_{k,w,\mathbf{p}} + z_{j,k,w,\mathbf{p}}\varphi_{k,w,\mathbf{p}}\right) \tag{17}$$

in every point $\langle t,\mathbf{x}\rangle$.

Let $\kappa_{w,\mathbf{p}}$ be linear operators on linear space, spanned of basic functions $\varsigma_{w,\mathbf{p}}\left(t,\mathbf{x}\right)$, such that

$$\kappa_{w,\mathbf{p}}\varsigma_{w',\mathbf{p}'} := \left\{\begin{array}{l} \varsigma_{w',\mathbf{p}'},\ \text{if } w = w',\ \mathbf{p} = \mathbf{p}'; \\ 0,\ \text{if } w \neq w' \text{ and/or } \mathbf{p} \neq \mathbf{p}'. \end{array}\right|$$

Let

$$Q_{j,k}|_{\langle t,\mathbf{x}\rangle} := \sum_{w,\mathbf{p}}\left(z_{j,k,w,\mathbf{p}}|_{\langle t,\mathbf{x}\rangle}\right)\kappa_{w,\mathbf{p}}$$

in every point $\langle t,\mathbf{x}\rangle$.

Therefore, from (13) and (17), for every function φ_j here exists an operator $Q_{j,k}$ such that dependence of φ_j on t is described by the following differential equations:

$$\partial_t\varphi_j = \mathrm{c}\sum_{k=1}^{4}\left(\beta^{[1]}_{j,k}\partial_1 + \beta^{[2]}_{j,k}\partial_2 + \beta^{[3]}_{j,k}\partial_3 + Q_{j,k}\right)\varphi_k. \tag{18}$$

and

$$Q^*_{j,k} = \sum_{w,\mathbf{p}}\left(z^*_{j,k,w,\mathbf{p}}\right)\kappa_{w,\mathbf{p}} = \\ = \sum_{w,\mathbf{p}}\left(-z^*_{k,j,w,\mathbf{p}}\right)\kappa_{w,\mathbf{p}} = -Q_{k,j}\,.$$

Matrix form of formula (18) is the following:

$$\partial_t\varphi = \mathrm{c}\left(\beta^{[1]}\partial_1 + \beta^{[2]}\partial_2 + \beta^{[3]}\partial_3 + \widehat{Q}\right)\varphi \tag{19}$$

with

$$\varphi = \left[\begin{array}{c}\varphi_1\\ \varphi_2\\ \varphi_3\\ \varphi_4\end{array}\right]$$

and

$$\widehat{Q} := \begin{bmatrix} \mathrm{i}\vartheta_{1,1} & Q_{1,2} & Q_{1,3} & Q_{1,4} \\ -Q_{1,2}^{*} & \mathrm{i}\vartheta_{2,2} & Q_{2,3} & Q_{2,4} \\ -Q_{1,3}^{*} & -Q_{2,3}^{*} & \mathrm{i}\vartheta_{3,3} & Q_{3,4} \\ -Q_{1,4}^{*} & -Q_{2,4}^{*} & -Q_{3,4}^{*} & \mathrm{i}\vartheta_{4,4} \end{bmatrix} \tag{20}$$

with $Q_{k,s} := \mathrm{i}\vartheta_{k,s} - \varpi_{k,s}$ if $k \neq s$, and with $\varpi_{s,k} := \mathrm{Re}\,(Q_{s,k})$ and $\vartheta_{s,k} := \mathrm{Im}\,(Q_{s,k})$.

Let $\vartheta_{s,k}$ and $\varpi_{s,k}$ be terms of $\widehat{Q}$ (20) and let Θ_0, Θ_3, Υ_0 and Υ_3 be the solution of the following sets of equations:

$$\left\{ \begin{array}{l} -\Theta_0 + \Theta_3 - \Upsilon_0 + \Upsilon_3 = \vartheta_{1,1}; \\ -\Theta_0 - \Theta_3 - \Upsilon_0 - \Upsilon_3 = \vartheta_{2,2}; \\ -\Theta_0 - \Theta_3 + \Upsilon_0 + \Upsilon_3 = \vartheta_{3,3}; \\ -\Theta_0 + \Theta_3 + \Upsilon_0 - \Upsilon_3 = \vartheta_{4,4} \end{array} \right|,$$

and Θ_1, Υ_1, Θ_2, Υ_2, M_0, M_4, $M_{\zeta,0}$, $M_{\zeta,4}$, $M_{\eta,0}$, $M_{\eta,4}$, $M_{\theta,0}$, $M_{\theta,4}$ be the solutions of the following sets of equations:

$$\left\{ \begin{array}{l} \Theta_1 + \Upsilon_1 = \vartheta_{1,2}; \\ -\Theta_1 + \Upsilon_1 = \vartheta_{3,4}; \end{array} \right| \left\{ \begin{array}{l} -\Theta_2 - \Upsilon_2 = \varpi_{1,2}; \\ \Theta_2 - \Upsilon_2 = \varpi_{3,4}; \end{array} \right|$$

$$\left\{ \begin{array}{l} M_0 + M_{\theta,0} = \vartheta_{1,3}; \\ M_0 - M_{\theta,0} = \vartheta_{2,4}; \end{array} \right| \left\{ \begin{array}{l} M_4 + M_{\theta,4} = \varpi_{1,3}; \\ M_4 - M_{\theta,4} = \varpi_{2,4}; \end{array} \right|$$

$$\left\{ \begin{array}{l} M_{\zeta,0} - M_{\eta,4} = \vartheta_{1,4}; \\ M_{\zeta,0} + M_{\eta,4} = \vartheta_{2,3}; \end{array} \right| \left\{ \begin{array}{l} M_{\zeta,4} - M_{\eta,0} = \varpi_{1,4}; \\ M_{\zeta,4} + M_{\eta,0} = \varpi_{2,3} \end{array} \right|.$$

Thus the columns of $\widehat{Q}$ are the following:
— the first and the second columns:

$$\begin{array}{c} -\mathrm{i}\Theta_0 + \mathrm{i}\Theta_3 - \mathrm{i}\Upsilon_0 + \mathrm{i}\Upsilon_3 \\ \mathrm{i}\Theta_1 + \mathrm{i}\Upsilon_1 - \Theta_2 - \Upsilon_2 \\ \mathrm{i}M_0 + \mathrm{i}M_{\theta,0} + M_4 + M_{\theta,4} \\ \mathrm{i}M_{\zeta,0} - \mathrm{i}M_{\eta,4} + M_{\zeta,4} - M_{\eta,0} \end{array}$$

$$\begin{array}{c} \mathrm{i}\Theta_1 + \mathrm{i}\Upsilon_1 + \Theta_2 + \Upsilon_2 \\ -\mathrm{i}\Theta_0 - \mathrm{i}\Theta_3 - \mathrm{i}\Upsilon_0 - \mathrm{i}\Upsilon_3 \\ \mathrm{i}M_{\zeta,0} + \mathrm{i}M_{\eta,4} + M_{\zeta,4} + M_{\eta,0} \\ \mathrm{i}M_0 - \mathrm{i}M_{\theta,0} + M_4 - M_{\theta,4} \end{array}$$

— the third and the fourth columns:

$$\begin{array}{c} \mathrm{i}M_0 + \mathrm{i}M_{\theta,0} - M_4 - M_{\theta,4} \\ \mathrm{i}M_{\zeta,0} + \mathrm{i}M_{\eta,4} - M_{\zeta,4} - M_{\eta,0} \\ -\mathrm{i}\Theta_0 - \mathrm{i}\Theta_3 + \mathrm{i}\Upsilon_0 + \mathrm{i}\Upsilon_3 \\ -\mathrm{i}\Theta_1 + \mathrm{i}\Upsilon_1 + \Theta_2 - \Upsilon_2 \end{array}$$

$$\begin{array}{c} \mathrm{i}M_{\zeta,0} - \mathrm{i}M_{\eta,4} - M_{\zeta,4} + M_{\eta,0} \\ \mathrm{i}M_0 - \mathrm{i}M_{\theta,0} - M_4 + M_{\theta,4} \\ -\mathrm{i}\Theta_1 + \mathrm{i}\Upsilon_1 - \Theta_2 + \Upsilon_2 \\ -\mathrm{i}\Theta_0 + \mathrm{i}\Theta_3 + \mathrm{i}\Upsilon_0 - \mathrm{i}\Upsilon_3 \end{array}$$

Hence

$$\begin{aligned} \widehat{Q} = \mathrm{i}\Theta_0\beta^{[0]} + \mathrm{i}\Upsilon_0\beta^{[0]}\gamma^{[5]} &+ \\ + \mathrm{i}\Theta_1\beta^{[1]} + \mathrm{i}\Upsilon_1\beta^{[1]}\gamma^{[5]} &+ \\ + \mathrm{i}\Theta_2\beta^{[2]} + \mathrm{i}\Upsilon_2\beta^{[2]}\gamma^{[5]} &+ \\ + \mathrm{i}\Theta_3\beta^{[3]} + \mathrm{i}\Upsilon_3\beta^{[3]}\gamma^{[5]} &+ \\ + \mathrm{i}M_0\gamma^{[0]} + \mathrm{i}M_4\beta^{[4]} &- \\ - \mathrm{i}M_{\zeta,0}\gamma_\zeta^{[0]} + \mathrm{i}M_{\zeta,4}\zeta^{[4]} &- \\ - \mathrm{i}M_{\eta,0}\gamma_\eta^{[0]} - \mathrm{i}M_{\eta,4}\eta^{[4]} &+ \\ + \mathrm{i}M_{\theta,0}\gamma_\theta^{[0]} + \mathrm{i}M_{\theta,4}\theta^{[4]}&. \end{aligned}$$

From (19) the following equation is received:

$$\sum_{k=0}^{3} \beta^{[k]} \left(\partial_k + \mathrm{i}\Theta_k + \mathrm{i}\Upsilon_k\gamma^{[5]}\right)\varphi + \\ + \left(\begin{array}{c} +\mathrm{i}M_0\gamma^{[0]} + \mathrm{i}M_4\beta^{[4]} - \\ -\mathrm{i}M_{\zeta,0}\gamma_\zeta^{[0]} + \mathrm{i}M_{\zeta,4}\zeta^{[4]} - \\ -\mathrm{i}M_{\eta,0}\gamma_\eta^{[0]} - \mathrm{i}M_{\eta,4}\eta^{[4]} + \\ +\mathrm{i}M_{\theta,0}\gamma_\theta^{[0]} + \mathrm{i}M_{\theta,4}\theta^{[4]} \end{array} \right)\varphi = 0 \tag{21}$$

with real Θ_k, Υ_k, M_0, M_4, $M_{\zeta,0}$, $M_{\zeta,4}$, $M_{\eta,0}$, $M_{\eta,4}$, $M_{\theta,0}$, $M_{\theta,4}$ and with

$$\gamma^{[5]} := \begin{bmatrix} 1_2 & 0_2 \\ 0_2 & -1_2 \end{bmatrix}. \tag{22}$$

Because

$$\zeta^{[k]} + \eta^{[k]} + \theta^{[k]} = -\beta^{[k]}$$

with $k \in \{1, 2, 3\}$ then from (21):

$$\left(\begin{array}{c} -\left(\partial_0 + \mathrm{i}\Theta_0 + \mathrm{i}\Upsilon_0\gamma^{[5]}\right) + \\ \sum\limits_{k=1}^{3} \beta^{[k]}\left(\partial_k + \mathrm{i}\Theta_k + \mathrm{i}\Upsilon_k\gamma^{[5]}\right) \\ +2\left(\mathrm{i}M_0\gamma^{[0]} + \mathrm{i}M_4\beta^{[4]}\right) \end{array} \right)\varphi + \\ + \left(\begin{array}{c} -\left(\partial_0 + \mathrm{i}\Theta_0 + \mathrm{i}\Upsilon_0\gamma^{[5]}\right) \\ -\sum\limits_{k=1}^{3} \zeta^{[k]}\left(\partial_k + \mathrm{i}\Theta_k + \mathrm{i}\Upsilon_k\gamma^{[5]}\right) \\ +2\left(-\mathrm{i}M_{\zeta,0}\gamma_\zeta^{[0]} + \mathrm{i}M_{\zeta,4}\zeta^{[4]}\right) \end{array} \right)\varphi + \\ + \left(\begin{array}{c} \left(\partial_0 + \mathrm{i}\Theta_0 + \mathrm{i}\Upsilon_0\gamma^{[5]}\right) \\ -\sum\limits_{k=1}^{3} \eta^{[k]}\left(\partial_k + \mathrm{i}\Theta_k + \mathrm{i}\Upsilon_k\gamma^{[5]}\right) \\ +2\left(-\mathrm{i}M_{\eta,0}\gamma_\eta^{[0]} - \mathrm{i}M_{\eta,4}\eta^{[4]}\right) \end{array} \right)\varphi + \\ + \left(\begin{array}{c} -\left(\partial_0 + \mathrm{i}\Theta_0 + \mathrm{i}\Upsilon_0\gamma^{[5]}\right) \\ -\sum\limits_{k=1}^{3} \theta^{[k]}\left(\partial_k + \mathrm{i}\Theta_k + \mathrm{i}\Upsilon_k\gamma^{[5]}\right) \\ +2\left(\mathrm{i}M_{\theta,0}\gamma_\theta^{[0]} + \mathrm{i}M_{\theta,4}\theta^{[4]}\right) \end{array} \right)\varphi = 0.$$

It is a generalization of the Dirac equation with gauge field A:

$$\left(-\left(\partial_0+\mathrm{ie}A_0\right)+\sum_{k=1}^{3}\beta^{[k]}\left(\partial_k+\mathrm{ie}A_k\right)+\mathrm{i}m\gamma^{[0]}\right)\varphi=0.$$

Therefore, all Planck's functions obey to Dirac's type equations.

I call matrices $\gamma^{[0]}$, $\beta^{[4]}$, $\gamma_\zeta^{[0]}$, $\zeta^{[4]}$, $\gamma_\eta^{[0]}$, $\eta^{[4]}$, $\gamma_\theta^{[0]}$, $\theta^{[4]}$ *mass elements of pentads*.

3 Colored equation

I call the following part of (21):

$$\left(\begin{array}{c}\beta^{[0]}\left(-\mathrm{i}\partial_0+\Theta_0+\Upsilon_0\gamma^{[5]}\right)+\\ \beta^{[1]}\left(-\mathrm{i}\partial_1+\Theta_1+\Upsilon_1\gamma^{[5]}\right)+\\ \beta^{[2]}\left(-\mathrm{i}\partial_2+\Theta_2+\Upsilon_2\gamma^{[5]}\right)+\\ \beta^{[3]}\left(-\mathrm{i}\partial_3+\Theta_3+\Upsilon_3\gamma^{[5]}\right)-\\ -M_{\zeta,0}\gamma_\zeta^{[0]}+M_{\zeta,4}\zeta^{[4]}+\\ -M_{\eta,0}\gamma_\eta^{[0]}-M_{\eta,4}\eta^{[4]}+\\ +M_{\theta,0}\gamma_\theta^{[0]}+M_{\theta,4}\theta^{[4]}\end{array}\right)\varphi=0. \qquad (23)$$

a coloured moving equation.

Here (5), (7), (9):

$$\gamma_\zeta^{[0]}=-\left[\begin{array}{cccc}0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0\end{array}\right],\quad \zeta^{[4]}=\left[\begin{array}{cccc}0&0&0&\mathrm{i}\\0&0&\mathrm{i}&0\\0&-\mathrm{i}&0&0\\-\mathrm{i}&0&0&0\end{array}\right]$$

are mass elements of red pentad;

$$\gamma_\eta^{[0]}=\left[\begin{array}{cccc}0&0&0&\mathrm{i}\\0&0&-\mathrm{i}&0\\0&\mathrm{i}&0&0\\-\mathrm{i}&0&0&0\end{array}\right],\quad \eta^{[4]}=\left[\begin{array}{cccc}0&0&0&1\\0&0&-1&0\\0&-1&0&0\\1&0&0&0\end{array}\right]$$

are mass elements of green pentad;

$$\gamma_\theta^{[0]}=\left[\begin{array}{cccc}0&0&-1&0\\0&0&0&1\\-1&0&0&0\\0&1&0&0\end{array}\right],\quad \theta^{[4]}=\left[\begin{array}{cccc}0&0&-\mathrm{i}&0\\0&0&0&\mathrm{i}\\-\mathrm{i}&0&0&0\\0&\mathrm{i}&0&0\end{array}\right]$$

are mass elements of blue pentad.

I call:

- $M_{\zeta,0}$, $M_{\zeta,4}$ *red lower and upper mass members*;
- $M_{\eta,0}$, $M_{\eta,4}$ *green lower and upper mass members*;
- $M_{\theta,0}$, $M_{\theta,4}$ *blue lower and upper mass members*.

The mass members of this equation form the following matrix sum:

$$\widehat{M}:=\left(\begin{array}{c}-M_{\zeta,0}\gamma_\zeta^{[0]}+M_{\zeta,4}\zeta^{[4]}-\\ -M_{\eta,0}\gamma_\eta^{[0]}-M_{\eta,4}\eta^{[4]}+\\ +M_{\theta,0}\gamma_\theta^{[0]}+M_{\theta,4}\theta^{[4]}\end{array}\right)=$$

$$=\left[\begin{array}{cccc}0&0&-M_{\theta,0}&M_{\zeta,\eta,0}\\0&0&M_{\zeta,\eta,0}^{*}&M_{\theta,0}\\-M_{\theta,0}&M_{\zeta,\eta,0}&0&0\\M_{\zeta,\eta,0}^{*}&M_{\theta,0}&0&0\end{array}\right]+$$

$$+\mathrm{i}\left[\begin{array}{cccc}0&0&-M_{\theta,4}&M_{\zeta,\eta,4}^{*}\\0&0&M_{\zeta,\eta,4}&M_{\theta,4}\\-M_{\theta,4}&-M_{\zeta,\eta,4}^{*}&0&0\\-M_{\zeta,\eta,4}&M_{\theta,4}&0&0\end{array}\right]$$

with $M_{\zeta,\eta,0}:=M_{\zeta,0}-\mathrm{i}M_{\eta,0}$ and $M_{\zeta,\eta,4}:=M_{\zeta,4}-\mathrm{i}M_{\eta,4}$.

Elements of these matrices can be turned by formula of shape [2]:

$$\left(\begin{array}{cc}\cos\frac{\theta}{2}&\mathrm{i}\sin\frac{\theta}{2}\\ \mathrm{i}\sin\frac{\theta}{2}&\cos\frac{\theta}{2}\end{array}\right)\left(\begin{array}{cc}Z&X-\mathrm{i}Y\\X+\mathrm{i}Y&-Z\end{array}\right)\times$$

$$\times\left(\begin{array}{cc}\cos\frac{\theta}{2}&-\mathrm{i}\sin\frac{\theta}{2}\\ -\mathrm{i}\sin\frac{\theta}{2}&\cos\frac{\theta}{2}\end{array}\right)=$$

$$=\left(\begin{array}{cc}Z\cos\theta-Y\sin\theta & X-\mathrm{i}\left(\begin{array}{c}Y\cos\theta\\+Z\sin\theta\end{array}\right)\\ X+\mathrm{i}\left(\begin{array}{c}Y\cos\theta\\+Z\sin\theta\end{array}\right) & -Z\cos\theta+Y\sin\theta\end{array}\right).$$

Hence, if:

$$U_{2,3}(\alpha):=\left[\begin{array}{cccc}\cos\alpha&\mathrm{i}\sin\alpha&0&0\\ \mathrm{i}\sin\alpha&\cos\alpha&0&0\\0&0&\cos\alpha&\mathrm{i}\sin\alpha\\0&0&\mathrm{i}\sin\alpha&\cos\alpha\end{array}\right]$$

and

$$\widehat{M}':=\left(\begin{array}{c}-M'_{\zeta,0}\gamma_\zeta^{[0]}+M'_{\zeta,4}\zeta^{[4]}-\\ -M'_{\eta,0}\gamma_\eta^{[0]}-M'_{\eta,4}\eta^{[4]}+\\ +M'_{\theta,0}\gamma_\theta^{[0]}+M'_{\theta,4}\theta^{[4]}\end{array}\right):=U_{2,3}^{\dagger}(\alpha)\,\widehat{M}U_{2,3}(\alpha)$$

then

$$\begin{aligned}M'_{\zeta,0}&=M_{\zeta,0}\,,\\ M'_{\eta,0}&=M_{\eta,0}\cos2\alpha+M_{\theta,0}\sin2\alpha\,,\\ M'_{\theta,0}&=M_{\theta,0}\cos2\alpha-M_{\eta,0}\sin2\alpha\,,\\ M'_{\zeta,4}&=M_{\zeta,4}\,,\\ M'_{\eta,4}&=M_{\eta,4}\cos2\alpha+M_{\theta,4}\sin2\alpha\,,\\ M'_{\theta,4}&=M_{\theta,4}\cos2\alpha-M_{\eta,4}\sin2\alpha\,.\end{aligned}$$

Therefore, matrix $U_{2,3}(\alpha)$ makes an oscillation between green and blue colours.

Let us consider equation (21) under transformation $U_{2,3}(\alpha)$ where α is an arbitrary real function of time-space variables ($\alpha=\alpha(t,x_1,x_2,x_3)$):

$$U_{2,3}^{\dagger}(\alpha)\left(\frac{1}{\mathrm{c}}\,\partial_t+\mathrm{i}\Theta_0+\mathrm{i}\Upsilon_0\gamma^{[5]}\right)U_{2,3}(\alpha)\,\varphi=$$

$$= U^{\dagger}_{2,3}(\alpha)\begin{pmatrix} \beta^{[1]}\left(\partial_1 + \mathrm{i}\Theta_1 + \mathrm{i}\Upsilon_1\gamma^{[5]}\right) + \\ +\beta^{[2]}\left(\partial_2 + \mathrm{i}\Theta_2 + \mathrm{i}\Upsilon_2\gamma^{[5]}\right) + \\ +\beta^{[3]}\left(\partial_3 + \mathrm{i}\Theta_3 + \mathrm{i}\Upsilon_3\gamma^{[5]}\right) + \\ +\mathrm{i}M_0\gamma^{[0]} + \mathrm{i}M_4\beta^{[4]} + \widehat{M}\end{pmatrix} U_{2,3}(\alpha)\,\varphi\,.$$

Because

$$U^{\dagger}_{2,3}(\alpha)\,U_{2,3}(\alpha) = 1_4\,,$$
$$U^{\dagger}_{2,3}(\alpha)\,\gamma^{[5]}U_{2,3}(\alpha) = \gamma^{[5]}\,,$$
$$U^{\dagger}_{2,3}(\alpha)\,\gamma^{[0]}U_{2,3}(\alpha) = \gamma^{[0]}\,,$$
$$U^{\dagger}_{2,3}(\alpha)\,\beta^{[4]}U_{2,3}(\alpha) = \beta^{[4]}\,,$$
$$U^{\dagger}_{2,3}(\alpha)\,\beta^{[1]} = \beta^{[1]}U^{\dagger}_{2,3}(\alpha)\,,$$
$$U^{\dagger}_{2,3}(\alpha)\,\beta^{[2]} = \left(\beta^{[2]}\cos 2\alpha + \beta^{[3]}\sin 2\alpha\right)U^{\dagger}_{2,3}(\alpha)\,,$$
$$U^{\dagger}_{2,3}(\alpha)\,\beta^{[3]} = \left(\beta^{[3]}\cos 2\alpha - \beta^{[2]}\sin 2\alpha\right)U^{\dagger}_{2,3}(\alpha)\,,$$

then

$$\left(\frac{1}{\mathrm{c}}\,\partial_t + U^{\dagger}_{2,3}(\alpha)\,\frac{1}{\mathrm{c}}\,\partial_t U_{2,3}(\alpha) + \mathrm{i}\Theta_0 + \mathrm{i}\Upsilon_0\gamma^{[5]}\right)\varphi =$$

$$= \begin{pmatrix} \beta^{[1]}\begin{pmatrix}\partial_1 + U^{\dagger}_{2,3}(\alpha)\,\partial_1 U_{2,3}(\alpha)\\ +\mathrm{i}\Theta_1 + \mathrm{i}\Upsilon_1\gamma^{[5]}\end{pmatrix} + \beta^{[2]}\times \\ \times\begin{pmatrix} (\cos 2\alpha\cdot\partial_2 - \sin 2\alpha\cdot\partial_3) \\ + U^{\dagger}_{2,3}(\alpha)\begin{pmatrix}\cos 2\alpha\cdot\partial_2\\ -\sin 2\alpha\cdot\partial_3\end{pmatrix}U_{2,3}(\alpha) \\ +\mathrm{i}\left(\Theta_2\cos 2\alpha - \Theta_3\sin 2\alpha\right) \\ +\mathrm{i}\left(\Upsilon_2\gamma^{[5]}\cos 2\alpha - \Upsilon_3\gamma^{[5]}\sin 2\alpha\right)\end{pmatrix} \\ +\beta^{[3]}\times \\ \times\begin{pmatrix} (\cos 2\alpha\cdot\partial_3 + \sin 2\alpha\cdot\partial_2) \\ + U^{\dagger}_{2,3}(\alpha)\begin{pmatrix}\cos 2\alpha\cdot\partial_3\\ +\sin 2\alpha\cdot\partial_2\end{pmatrix}U_{2,3}(\alpha) \\ +\mathrm{i}\left(\Theta_2\sin 2\alpha + \Theta_3\cos 2\alpha\right) \\ +\mathrm{i}\left(\Upsilon_3\gamma^{[5]}\cos 2\alpha + \Upsilon_2\gamma^{[5]}\sin 2\alpha\right)\end{pmatrix} \\ +\mathrm{i}M_0\gamma^{[0]} + \mathrm{i}M_4\beta^{[4]} + \widehat{M}'\end{pmatrix}\varphi\,. \quad (24)$$

Let x'_2 and x'_3 be elements of other coordinate system such that:

$$\frac{\partial x_2}{\partial x'_2} = \cos 2\alpha,$$
$$\frac{\partial x_3}{\partial x'_2} = -\sin 2\alpha,$$
$$\frac{\partial x_2}{\partial x'_3} = \sin 2\alpha.$$
$$\frac{\partial x_3}{\partial x'_3} = \cos 2\alpha,$$
$$\frac{\partial x_0}{\partial x'_2} = \frac{\partial x_1}{\partial x'_2} = \frac{\partial x_0}{\partial x'_3} = \frac{\partial x_1}{\partial x'_3} = 0.$$

Hence:

$$\begin{aligned}\partial'_2 &:= \frac{\partial}{\partial x'_2} = \\ &= \frac{\partial}{\partial x_0}\frac{\partial x_0}{\partial x'_2} + \frac{\partial}{\partial x_1}\frac{\partial x_1}{\partial x'_2} + \frac{\partial}{\partial x_2}\frac{\partial x_2}{\partial x'_2} + \frac{\partial}{\partial x_3}\frac{\partial x_3}{\partial x'_2} = \\ &= \cos 2\alpha\cdot\frac{\partial}{\partial x_2} - \sin 2\alpha\cdot\frac{\partial}{\partial x_3} = \\ &= \cos 2\alpha\cdot\partial_2 - \sin 2\alpha\cdot\partial_3\,,\end{aligned}$$

$$\begin{aligned}\partial'_3 &:= \frac{\partial}{\partial x'_3} = \\ &= \frac{\partial}{\partial x_0}\frac{\partial x_0}{\partial x'_3} + \frac{\partial}{\partial x_1}\frac{\partial x_1}{\partial x'_3} + \frac{\partial}{\partial x_2}\frac{\partial x_2}{\partial x'_3} + \frac{\partial}{\partial x_3}\frac{\partial x_3}{\partial x'_3} = \\ &= \cos 2\alpha\cdot\frac{\partial}{\partial x_3} + \sin 2\alpha\cdot\frac{\partial}{\partial x_2} = \\ &= \cos 2\alpha\cdot\partial_3 + \sin 2\alpha\cdot\partial_2\,.\end{aligned}$$

Therefore, from (24):

$$\left(\frac{1}{\mathrm{c}}\,\partial_t + U^{\dagger}_{2,3}(\alpha)\,\frac{1}{\mathrm{c}}\,\partial_t U_{2,3}(\alpha) + \mathrm{i}\Theta_0 + \mathrm{i}\Upsilon_0\gamma^{[5]}\right)\varphi =$$

$$= \begin{pmatrix} \beta^{[1]}\begin{pmatrix}\partial_1 + U^{\dagger}_{2,3}(\alpha)\,\partial_1 U_{2,3}(\alpha)\\ +\mathrm{i}\Theta_1 + \mathrm{i}\Upsilon_1\gamma^{[5]}\end{pmatrix} \\ +\beta^{[2]}\begin{pmatrix}\partial'_2 + U^{\dagger}_{2,3}(\alpha)\,\partial'_2 U_{2,3}(\alpha)\\ +\mathrm{i}\Theta'_2 + \mathrm{i}\Upsilon'_2\gamma^{[5]}\end{pmatrix} \\ +\beta^{[3]}\begin{pmatrix}\partial'_3 + U^{\dagger}_{2,3}(\alpha)\,\partial'_3 U_{2,3}(\alpha)\\ +\mathrm{i}\Theta'_3 + \mathrm{i}\Upsilon'_3\gamma^{[5]}\end{pmatrix} \\ +\mathrm{i}M_0\gamma^{[0]} + \mathrm{i}M_4\beta^{[4]} + \widehat{M}'\end{pmatrix}\varphi\,.$$

with

$$\Theta'_2 := \Theta_2\cos 2\alpha - \Theta_3\sin 2\alpha\,,$$
$$\Theta'_3 := \Theta_2\sin 2\alpha + \Theta_3\cos 2\alpha\,,$$
$$\Upsilon'_2 := \Upsilon_2\cos 2\alpha - \Upsilon_3\sin 2\alpha\,,$$
$$\Upsilon'_3 := \Upsilon_3\cos 2\alpha + \Upsilon_2\sin 2\alpha\,.$$

Therefore, the oscillation between blue and green colours curves the space in the x_2, x_3 directions.

Similarly, matrix

$$U_{1,3}(\vartheta) := \begin{bmatrix}\cos\vartheta & \sin\vartheta & 0 & 0\\ -\sin\vartheta & \cos\vartheta & 0 & 0\\ 0 & 0 & \cos\vartheta & \sin\vartheta\\ 0 & 0 & -\sin\vartheta & \cos\vartheta\end{bmatrix}$$

with an arbitrary real function $\vartheta(t, x_1, x_2, x_3)$ describes the oscillation between blue and red colours which curves the space in the x_1, x_3 directions. And matrix

$$U_{1,2}(\varsigma) := \begin{bmatrix}e^{-\mathrm{i}\varsigma} & 0 & 0 & 0\\ 0 & e^{\mathrm{i}\varsigma} & 0 & 0\\ 0 & 0 & e^{-\mathrm{i}\varsigma} & 0\\ 0 & 0 & 0 & e^{\mathrm{i}\varsigma}\end{bmatrix}$$

with an arbitrary real function $\varsigma(t, x_1, x_2, x_3)$ describes the oscillation between green and red colours which curves the space in the x_1, x_2 directions.

Now, let

$$U_{0,1}(\sigma) := \begin{bmatrix} \cosh\sigma & -\sinh\sigma & 0 & 0 \\ -\sinh\sigma & \cosh\sigma & 0 & 0 \\ 0 & 0 & \cosh\sigma & \sinh\sigma \\ 0 & 0 & \sinh\sigma & \cosh\sigma \end{bmatrix}.$$

and

$$\widehat{M}'' := \begin{pmatrix} -M''_{\zeta,0}\gamma_\zeta^{[0]} + M''_{\zeta,4}\zeta^{[4]} - \\ -M''_{\eta,0}\gamma_\eta^{[0]} - M''_{\eta,4}\eta^{[4]} + \\ +M''_{\theta,0}\gamma_\theta^{[0]} + M''_{\theta,4}\theta^{[4]} \end{pmatrix} := U^\dagger_{0,1}(\sigma)\,\widehat{M}U_{0,1}(\sigma)$$

then:

$$\begin{aligned} M''_{\zeta,0} &= M_{\zeta,0}\,, \\ M''_{\eta,0} &= \left(M_{\eta,0}\cosh 2\sigma - M_{\theta,4}\sinh 2\sigma\right), \\ M''_{\theta,0} &= M_{\theta,0}\cosh 2\sigma + M_{\eta,4}\sinh 2\sigma\,, \\ M''_{\zeta,4} &= M_{\zeta,4}\,, \\ M''_{\eta,4} &= M_{\eta,4}\cosh 2\sigma + M_{\theta,0}\sinh 2\sigma\,, \\ M''_{\theta,4} &= M_{\theta,4}\cosh 2\sigma - M_{\eta,0}\sinh 2\sigma\,. \end{aligned}$$

Therefore, matrix $U_{0,1}(\sigma)$ makes an oscillation between green and blue colours with an oscillation between upper and lower mass members.

Let us consider equation (21) under transformation $U_{0,1}(\sigma)$ where σ is an arbitrary real function of time-space variables ($\sigma = \sigma(t, x_1, x_2, x_3)$):

$$U^\dagger_{0,1}(\sigma)\left(\frac{1}{\mathrm{c}}\,\partial_t + \mathrm{i}\Theta_0 + \mathrm{i}\Upsilon_0\gamma^{[5]}\right)U_{0,1}(\sigma)\,\varphi =$$
$$= U^\dagger_{0,1}(\sigma)\begin{pmatrix} \beta^{[1]}\left(\partial_1 + \mathrm{i}\Theta_1 + \mathrm{i}\Upsilon_1\gamma^{[5]}\right) + \\ +\beta^{[2]}\left(\partial_2 + \mathrm{i}\Theta_2 + \mathrm{i}\Upsilon_2\gamma^{[5]}\right) + \\ +\beta^{[3]}\left(\partial_3 + \mathrm{i}\Theta_3 + \mathrm{i}\Upsilon_3\gamma^{[5]}\right) + \\ +\mathrm{i}M_0\gamma^{[0]} + \mathrm{i}M_4\beta^{[4]} + \widehat{M} \end{pmatrix}U_{0,1}(\sigma)\,\varphi\,.$$

Since:

$$\begin{aligned} &U^\dagger_{0,1}(\sigma)\,U_{0,1}(\sigma) = \left(\cosh 2\sigma - \beta^{[1]}\sinh 2\sigma\right), \\ &U^\dagger_{0,1}(\sigma) = \left(\cosh 2\sigma + \beta^{[1]}\sinh 2\sigma\right)U^{-1}_{0,1}(\sigma)\,, \\ &U^\dagger_{0,1}(\sigma)\,\beta^{[1]} = \left(\beta^{[1]}\cosh 2\sigma - \sinh 2\sigma\right)U^{-1}_{0,1}(\sigma)\,, \\ &U^\dagger_{0,1}(\sigma)\,\beta^{[2]} = \beta^{[2]}U^{-1}_{0,1}(\sigma)\,, \\ &U^\dagger_{0,1}(\sigma)\,\beta^{[3]} = \beta^{[3]}U^{-1}_{0,1}(\sigma)\,, \\ &U^\dagger_{0,1}(\sigma)\,\gamma^{[0]}U_{0,1}(\sigma) = \gamma^{[0]}, \\ &U^\dagger_{0,1}(\sigma)\,\beta^{[4]}U_{0,1}(\sigma) = \beta^{[4]}, \\ &U^{-1}_{0,1}(\sigma)\,U_{0,1}(\sigma) = 1_4\,, \\ &U^{-1}_{0,1}(\sigma)\,\gamma^{[5]}U_{0,1}(\sigma) = \gamma^{[5]}\,, \\ &U^\dagger_{0,1}(\sigma)\,\gamma^{[5]}U_{0,1}(\sigma) = \gamma^{[5]}\left(\cosh 2\sigma - \beta^{[1]}\sinh 2\sigma\right), \end{aligned}$$

then

$$\begin{pmatrix} U^{-1}_{0,1}(\sigma)\begin{pmatrix} \cosh 2\sigma\cdot\frac{1}{\mathrm{c}}\,\partial_t \\ +\sinh 2\sigma\cdot\partial_1 \end{pmatrix}U_{0,1}(\sigma) \\ +\left(\cosh 2\sigma\cdot\frac{1}{\mathrm{c}}\,\partial_t + \sinh 2\sigma\cdot\partial_1\right) \\ +\mathrm{i}\left(\Theta_0\cosh 2\sigma + \Theta_1\sinh 2\sigma\right) \\ +\mathrm{i}\left(\Upsilon_0\cosh 2\sigma + \sinh 2\sigma\cdot\Upsilon_1\right)\gamma^{[5]} - \\ -\beta^{[1]}\times \\ \times\begin{pmatrix} U^{-1}_{0,1}(\sigma)\begin{pmatrix} \cosh 2\sigma\cdot\partial_1 + \\ \sinh 2\sigma\cdot\frac{1}{\mathrm{c}}\,\partial_t \end{pmatrix}U_{0,1}(\sigma) \\ +\left(\cosh 2\sigma\cdot\partial_1 + \sinh 2\sigma\cdot\frac{1}{\mathrm{c}}\partial_t\right) \\ +\mathrm{i}\left(\Theta_1\cosh 2\sigma + \Theta_0\sinh 2\sigma\right) \\ +\mathrm{i}\left(\Upsilon_1\cosh 2\sigma + \Upsilon_0\sinh 2\sigma\right)\gamma^{[5]} \end{pmatrix} \\ -\beta^{[2]}\begin{pmatrix} \partial_2 + U^{-1}_{0,1}(\sigma)\left(\partial_2 U_{0,1}(\sigma)\right) \\ +\mathrm{i}\Theta_2 + \mathrm{i}\Upsilon_2\gamma^{[5]} \end{pmatrix} \\ -\beta^{[3]}\begin{pmatrix} \partial_3 + U^{-1}_{0,1}(\sigma)\left(\partial_3 U_{0,1}(\sigma)\right) \\ +\mathrm{i}\Theta_3 + \mathrm{i}\Upsilon_3\gamma^{[5]} \end{pmatrix} \\ -\mathrm{i}M_0\gamma^{[0]} - \mathrm{i}M_4\beta^{[4]} - \widehat{M}'' \end{pmatrix}\varphi = 0\,. \qquad (25)$$

Let t' and x'_1 be elements of other coordinate system such that:

$$\left.\begin{aligned} \frac{\partial x_1}{\partial x'_1} &= \cosh 2\sigma \\ \frac{\partial t}{\partial x'_1} &= \frac{1}{\mathrm{c}}\sinh 2\sigma \\ \frac{\partial x_1}{\partial t'} &= \mathrm{c}\sinh 2\sigma \\ \frac{\partial t}{\partial t'} &= \cosh 2\sigma \\ \frac{\partial x_2}{\partial t'} &= \frac{\partial x_3}{\partial t'} = \frac{\partial x_2}{\partial x'_1} = \frac{\partial x_3}{\partial x'_1} = 0 \end{aligned}\right\}. \qquad (26)$$

Hence:

$$\partial'_t := \frac{\partial}{\partial t'} = \frac{\partial}{\partial t}\frac{\partial t}{\partial t'} + \frac{\partial}{\partial x_1}\frac{\partial x_1}{\partial t'} + \frac{\partial}{\partial x_2}\frac{\partial x_2}{\partial t'} + \frac{\partial}{\partial x_3}\frac{\partial x_3}{\partial t'} =$$
$$= \cosh 2\sigma\cdot\frac{\partial}{\partial t} + \mathrm{c}\sinh 2\sigma\cdot\frac{\partial}{\partial x_1} =$$
$$= \cosh 2\sigma\cdot\partial_t + \mathrm{c}\sinh 2\sigma\cdot\partial_1\,,$$

that is

$$\frac{1}{\mathrm{c}}\,\partial'_t = \frac{1}{\mathrm{c}}\,\cosh 2\sigma\cdot\partial_t + \sinh 2\sigma\cdot\partial_1$$

and

$$\begin{aligned}\partial'_1 &:= \frac{\partial}{\partial x'_1} = \\ &= \frac{\partial}{\partial t}\frac{\partial t}{\partial x'_1} + \frac{\partial}{\partial x_1}\frac{\partial x_1}{\partial x'_1} + \frac{\partial}{\partial x_2}\frac{\partial x_2}{\partial x'_1} + \frac{\partial}{\partial x_3}\frac{\partial x_3}{\partial x'_1} = \\ &= \cosh 2\sigma \cdot \frac{\partial}{\partial x_1} + \sinh 2\sigma \cdot \frac{1}{\mathrm{c}}\frac{\partial}{\partial t} = \\ &= \cosh 2\sigma \cdot \partial_1 + \sinh 2\sigma \cdot \frac{1}{\mathrm{c}}\partial_t \,.\end{aligned}$$

Therefore, from (25):

$$\left(\begin{array}{c} \beta^{[0]}\left(\begin{array}{c} \frac{1}{\mathrm{c}}\partial'_t + U_{0,1}^{-1}(\sigma)\frac{1}{\mathrm{c}}\partial'_t U_{0,1}(\sigma) \\ +\mathrm{i}\Theta''_0 + \mathrm{i}\Upsilon''_0\gamma^{[5]} \end{array}\right) \\ +\beta^{[1]}\left(\begin{array}{c} \partial'_1 + U_{0,1}^{-1}(\sigma)\partial'_1 U_{0,1}(\sigma) \\ +\mathrm{i}\Theta''_1 + \mathrm{i}\Upsilon''_1\gamma^{[5]} \end{array}\right) \\ +\beta^{[2]}\left(\begin{array}{c} \partial_2 + U_{0,1}^{-1}(\sigma)\partial_2 U_{0,1}(\sigma) \\ +\mathrm{i}\Theta_2 + \mathrm{i}\Upsilon_2\gamma^{[5]} \end{array}\right) \\ +\beta^{[3]}\left(\begin{array}{c} \partial_3 + U_{0,1}^{-1}(\sigma)\partial_3 U_{0,1}(\sigma) \\ +\mathrm{i}\Theta_3 + \mathrm{i}\Upsilon_3\gamma^{[5]} \end{array}\right) \\ +\mathrm{i}M_0\gamma^{[0]} + \mathrm{i}M_4\beta^{[4]} + \widehat{M}'' \end{array}\right)\varphi = 0$$

with

$$\begin{aligned}\Theta''_0 &:= \Theta_0 \cosh 2\sigma + \Theta_1 \sinh 2\sigma \,, \\ \Theta''_1 &:= \Theta_1 \cosh 2\sigma + \Theta_0 \sinh 2\sigma \,, \\ \Upsilon''_0 &:= \Upsilon_0 \cosh 2\sigma + \sinh 2\sigma \cdot \Upsilon_1 \,, \\ \Upsilon''_1 &:= \Upsilon_1 \cosh 2\sigma + \Upsilon_0 \sinh 2\sigma \,.\end{aligned}$$

Therefore, the oscillation between blue and green colours with the oscillation between upper and lower mass members curves the space in the t, x_1 directions.

Similarly, matrix

$$U_{0,2}(\phi) := \begin{bmatrix} \cosh\phi & \mathrm{i}\sinh\phi & 0 & 0 \\ -\mathrm{i}\sinh\phi & \cosh\phi & 0 & 0 \\ 0 & 0 & \cosh\phi & -\mathrm{i}\sinh\phi \\ 0 & 0 & \mathrm{i}\sinh\phi & \cosh\phi \end{bmatrix}$$

with an arbitrary real function $\phi(t, x_1, x_2, x_3)$ describes the oscillation between blue and red colours with the oscillation between upper and lower mass members curves the space in the t, x_2 directions. And matrix

$$U_{0,3}(\iota) := \begin{bmatrix} e^{\iota} & 0 & 0 & 0 \\ 0 & e^{-\iota} & 0 & 0 \\ 0 & 0 & e^{-\iota} & 0 \\ 0 & 0 & 0 & e^{\iota} \end{bmatrix}$$

with an arbitrary real function $\iota(t, x_1, x_2, x_3)$ describes the oscillation between green and red colours with the oscillation between upper and lower mass members curves the space in the t, x_3 directions.

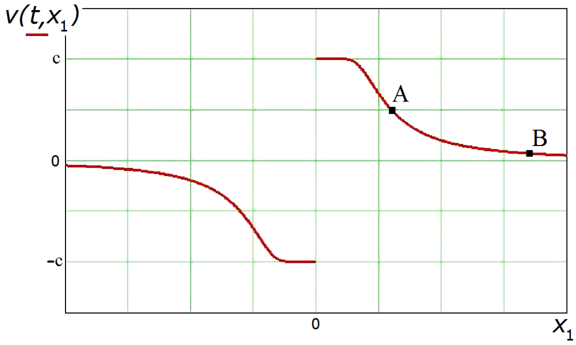

Fig. 1: It is dependency of $v(t, x_1)$ from x_1.

From (26):

$$\begin{aligned}\frac{\partial x_1}{\partial t'} &= \mathrm{c}\sinh 2\sigma \,, \\ \frac{\partial t}{\partial t'} &= \cosh 2\sigma \,.\end{aligned}$$

Because

$$\begin{aligned}\sinh 2\sigma &= \frac{v}{\sqrt{1 - \frac{v^2}{\mathrm{c}^2}}} \,, \\ \cosh 2\sigma &= \frac{1}{\sqrt{1 - \frac{v^2}{\mathrm{c}^2}}}\end{aligned}$$

with v is a velocity of system $\{t', x'_1\}$ as respects to system $\{t, x_1\}$ then

$$v = \tanh 2\sigma \,.$$

Let

$$2\sigma := \omega(x_1)\frac{t}{x_1}$$

with

$$\omega(x_1) = \frac{\lambda}{|x_1|} \,,$$

where λ is a real constant bearing positive numerical value.

In that case

$$v(t, x_1) = \tanh\left(\omega(x_1)\frac{t}{x_1}\right)$$

and if g is an acceleration of system $\{t', x'_1\}$ as respects to system $\{t, x_1\}$ then

$$\mathrm{g}(t, x_1) = \frac{\partial v}{\partial t} = \frac{\omega(x_1)}{\left(\cosh^2 \omega(x_1)\frac{t}{x_1}\right) x_1} \,.$$

Figure 1 shows the dependency of a system $\{t', x'_1\}$ velocity $v(t, x_1)$ on x_1 in system $\{t, x_1\}$.

This velocity in point A is not equal to one in point B. Hence, an oscillator, placed in B, has a nonzero velocity in respect to an observer, placed in point A. Therefore, from the Lorentz transformations, this oscillator frequency for observer, placed in point A, is less than own frequency of this oscillator (*red shift*).

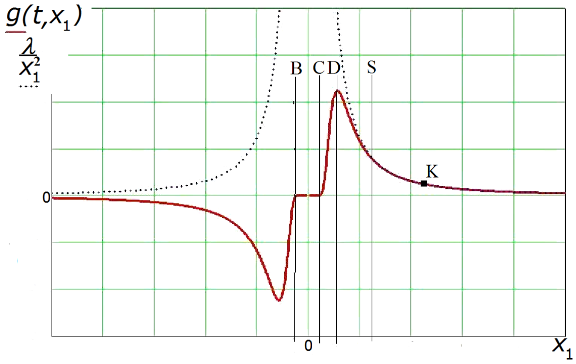

Fig. 2: It is dependency of $g(t, x_1)$ from x_1.

Figure 2 shows a dependency of a system $\{t', x'_1\}$ acceleration $g(t, x_1)$ on x_1 in system $\{t, x_1\}$.

If an object immovable in system $\{t, x_1\}$ is placed in point K then in system $\{t', x'_1\}$ this object must move to the left with acceleration g and g $\simeq \frac{\lambda}{x_1^2}$.

I call:

- interval from S to ∞ the *Newton Gravity Zone*,
- interval from B to C the *the Asymptotic Freedom Zone*,
- and interval from C to D *the Confinement Force Zone*.

Now let

$$\widetilde{U}(\chi) := \begin{bmatrix} e^{i\chi} & 0 & 0 & 0 \\ 0 & e^{i\chi} & 0 & 0 \\ 0 & 0 & e^{2i\chi} & 0 \\ 0 & 0 & 0 & e^{2i\chi} \end{bmatrix}$$

and

$$\widehat{M}' := \begin{pmatrix} -M'_{\zeta,0}\gamma_\zeta^{[0]} + M'_{\zeta,4}\zeta^{[4]} - \\ -M'_{\eta,0}\gamma_\eta^{[0]} - M'_{\eta,4}\eta^{[4]} + \\ +M'_{\theta,0}\gamma_\theta^{[0]} + M'_{\theta,4}\theta^{[4]} \end{pmatrix} := \widetilde{U}^\dagger(\chi)\,\widehat{M}\widetilde{U}(\chi)$$

then:

$$\begin{aligned} M'_{\zeta,0} &= (M_{\zeta,0}\cos\chi - M_{\zeta,4}\sin\chi)\,, \\ M'_{\zeta,4} &= (M_{\zeta,4}\cos\chi + M_{\zeta,0}\sin\chi)\,, \\ M'_{\eta,4} &= (M_{\eta,4}\cos\chi - M_{\eta,0}\sin\chi)\,, \\ M'_{\eta,0} &= (M_{\eta,0}\cos\chi + M_{\eta,4}\sin\chi)\,, \\ M'_{\theta,0} &= (M_{\theta,0}\cos\chi + M_{\theta,4}\sin\chi)\,, \\ M'_{\theta,4} &= (M_{\theta,4}\cos\chi - M_{\theta,0}\sin\chi)\,. \end{aligned}$$

Therefore, matrix $\widetilde{U}(\chi)$ makes an oscillation between upper and lower mass members.

Let us consider equation (23) under transformation $\widetilde{U}(\chi)$ where χ is an arbitrary real function of time-space variables ($\chi = \chi(t, x_1, x_2, x_3)$):

$$\widetilde{U}^\dagger(\chi)\left(\frac{1}{c}\partial_t + \mathrm{i}\Theta_0 + \mathrm{i}\Upsilon_0\gamma^{[5]}\right)\widetilde{U}(\chi)\,\varphi =$$

$$= \widetilde{U}^\dagger(\chi)\begin{pmatrix} \beta^{[1]}\left(\partial_1 + \mathrm{i}\Theta_1 + \mathrm{i}\Upsilon_1\gamma^{[5]}\right) + \\ +\beta^{[2]}\left(\partial_2 + \mathrm{i}\Theta_2 + \mathrm{i}\Upsilon_2\gamma^{[5]}\right) + \\ +\beta^{[3]}\left(\partial_3 + \mathrm{i}\Theta_3 + \mathrm{i}\Upsilon_3\gamma^{[5]}\right) + \\ +\widehat{M} \end{pmatrix}\widetilde{U}(\chi)\,\varphi\,.$$

Because

$$\gamma^{[5]}\widetilde{U}(\chi) = \widetilde{U}(\chi)\,\gamma^{[5]}\,,$$

$$\begin{aligned} \beta^{[1]}\widetilde{U}(\chi) &= \widetilde{U}(\chi)\,\beta^{[1]}\,, \\ \beta^{[2]}\widetilde{U}(\chi) &= \widetilde{U}(\chi)\,\beta^{[2]}\,, \\ \beta^{[3]}\widetilde{U}(\chi) &= \widetilde{U}(\chi)\,\beta^{[3]}\,, \end{aligned}$$

$$\widetilde{U}^\dagger(\chi)\,\widetilde{U}(\chi) = 1_4\,,$$

then

$$\left(\frac{1}{c}\partial_t + \frac{1}{c}\widetilde{U}^\dagger(\chi)\left(\partial_t\widetilde{U}(\chi)\right) + \mathrm{i}\Theta_0 + \mathrm{i}\Upsilon_0\gamma^{[5]}\right)\varphi =$$

$$= \begin{pmatrix} \beta^{[1]}\begin{pmatrix} \partial_1 + \widetilde{U}^\dagger(\chi)\left(\partial_1\widetilde{U}(\chi)\right) \\ +\mathrm{i}\Theta_1 + \mathrm{i}\Upsilon_1\gamma^{[5]} \end{pmatrix} + \\ +\beta^{[2]}\begin{pmatrix} \partial_2 + \widetilde{U}^\dagger(\chi)\left(\partial_2\widetilde{U}(\chi)\right) \\ +\mathrm{i}\Theta_2 + \mathrm{i}\Upsilon_2\gamma^{[5]} \end{pmatrix} + \\ +\beta^{[3]}\begin{pmatrix} \partial_3 + \widetilde{U}^\dagger(\chi)\left(\partial_3\widetilde{U}(\chi)\right) \\ +\mathrm{i}\Theta_3 + \mathrm{i}\Upsilon_3\gamma^{[5]} \end{pmatrix} + \\ +\widetilde{U}^\dagger(\chi)\,\widehat{M}\widetilde{U}(\chi) \end{pmatrix}\varphi\,.$$

Now let:

$$\widehat{U}(\kappa) := \begin{bmatrix} e^{\kappa} & 0 & 0 & 0 \\ 0 & e^{\kappa} & 0 & 0 \\ 0 & 0 & e^{2\kappa} & 0 \\ 0 & 0 & 0 & e^{2\kappa} \end{bmatrix}$$

and

$$\widehat{M}' := \begin{pmatrix} -M'_{\zeta,0}\gamma_\zeta^{[0]} + M'_{\zeta,4}\zeta^{[4]} - \\ -M'_{\eta,0}\gamma_\eta^{[0]} - M'_{\eta,4}\eta^{[4]} + \\ +M'_{\theta,0}\gamma_\theta^{[0]} + M'_{\theta,4}\theta^{[4]} \end{pmatrix} := \widehat{U}^{-1}(\kappa)\,\widehat{M}\widehat{U}(\kappa)$$

then:

$$\begin{aligned} M'_{\theta,0} &= (M_{\theta,0}\cosh\kappa - \mathrm{i}M_{\theta,4}\sinh\kappa)\,, \\ M'_{\theta,4} &= (M_{\theta,4}\cosh\kappa + \mathrm{i}M_{\theta,0}\sinh\kappa)\,, \\ M'_{\eta,0} &= (M_{\eta,0}\cosh\kappa - \mathrm{i}M_{\eta,4}\sinh\kappa)\,, \\ M'_{\eta,4} &= (M_{\eta,4}\cosh\kappa + \mathrm{i}M_{\eta,0}\sinh\kappa)\,, \\ M'_{\zeta,0} &= (M_{\zeta,0}\cosh\kappa + \mathrm{i}M_{\zeta,4}\sinh\kappa)\,, \\ M'_{\zeta,4} &= (M_{\zeta,4}\cosh\kappa - \mathrm{i}M_{\zeta,0}\sinh\kappa)\,. \end{aligned}$$

Therefore, matrix $\widehat{U}(\kappa)$ makes an oscillation between upper and lower mass members, too.

Let us consider equation (23) under transformation $\widehat{U}(\kappa)$ where κ is an arbitrary real function of time-space variables ($\kappa = \kappa(t, x_1, x_2, x_3)$):

$$\widehat{U}^{-1}(\kappa)\left(\frac{1}{c}\partial_t + \mathrm{i}\Theta_0 + \mathrm{i}\Upsilon_0\gamma^{[5]}\right)\widehat{U}(\kappa)\varphi =$$

$$= \widehat{U}^{-1}(\kappa)\left(\begin{array}{c} \beta^{[1]}\left(\partial_1 + \mathrm{i}\Theta_1 + \mathrm{i}\Upsilon_1\gamma^{[5]}\right) + \\ +\beta^{[2]}\left(\partial_2 + \mathrm{i}\Theta_2 + \mathrm{i}\Upsilon_2\gamma^{[5]}\right) + \\ +\beta^{[3]}\left(\partial_3 + \mathrm{i}\Theta_3 + \mathrm{i}\Upsilon_3\gamma^{[5]}\right) + \\ +\widehat{M} \end{array}\right)\widehat{U}(\kappa)\varphi$$

Because

$$\gamma^{[5]}\widehat{U}(\kappa) = \widehat{U}(\kappa)\gamma^{[5]},$$

$$\widehat{U}^{-1}(\kappa)\beta^{[1]} = \beta^{[1]}\widehat{U}^{-1}(\kappa),$$
$$\widehat{U}^{-1}(\kappa)\beta^{[2]} = \beta^{[2]}\widehat{U}^{-1}(\kappa),$$
$$\widehat{U}^{-1}(\kappa)\beta^{[3]} = \beta^{[3]}\widehat{U}^{-1}(\kappa),$$

$$\widehat{U}^{-1}(\kappa)\widehat{U}(\kappa) = 1_4,$$

then

$$\left(\frac{1}{c}\partial_t + \widehat{U}^{-1}(\kappa)\left(\frac{1}{c}\partial_t\widehat{U}(\kappa)\right) + \mathrm{i}\Theta_0 + \mathrm{i}\Upsilon_0\gamma^{[5]}\right)\varphi =$$

$$= \left(\begin{array}{c} \beta^{[1]}\left(\begin{array}{c}\partial_1 + \widehat{U}^{-1}(\kappa)\left(\partial_1\widehat{U}(\kappa)\right) \\ +\mathrm{i}\Theta_1 + \mathrm{i}\Upsilon_1\gamma^{[5]}\end{array}\right) + \\ +\beta^{[2]}\left(\begin{array}{c}\partial_2 + \widehat{U}^{-1}(\kappa)\left(\partial_2\widehat{U}(\kappa)\right) \\ +\mathrm{i}\Theta_2 + \mathrm{i}\Upsilon_2\gamma^{[5]}\end{array}\right) + \\ +\beta^{[3]}\left(\begin{array}{c}\partial_3 + \widehat{U}^{-1}(\kappa)\left(\partial_3\widehat{U}(\kappa)\right) \\ +\mathrm{i}\Theta_3 + \mathrm{i}\Upsilon_3\gamma^{[5]}\end{array}\right) + \\ +\widehat{U}^{-1}(\kappa)\widehat{M}\widehat{U}(\kappa) \end{array}\right)\varphi.$$

If denote:

$$\Lambda_1 := \begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$\Lambda_2 := \begin{bmatrix} 0 & \mathrm{i} & 0 & 0 \\ \mathrm{i} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathrm{i} \\ 0 & 0 & \mathrm{i} & 0 \end{bmatrix},$$

$$\Lambda_3 := \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix},$$

$$\Lambda_4 := \begin{bmatrix} 0 & \mathrm{i} & 0 & 0 \\ -\mathrm{i} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\mathrm{i} \\ 0 & 0 & \mathrm{i} & 0 \end{bmatrix},$$

$$\Lambda_5 := \begin{bmatrix} -\mathrm{i} & 0 & 0 & 0 \\ 0 & \mathrm{i} & 0 & 0 \\ 0 & 0 & -\mathrm{i} & 0 \\ 0 & 0 & 0 & \mathrm{i} \end{bmatrix},$$

$$\Lambda_6 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\Lambda_7 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix},$$

$$\Lambda_8 := \begin{bmatrix} \mathrm{i} & 0 & 0 & 0 \\ 0 & \mathrm{i} & 0 & 0 \\ 0 & 0 & 2\mathrm{i} & 0 \\ 0 & 0 & 0 & 2\mathrm{i} \end{bmatrix},$$

then

$$U_{0,1}^{-1}(\sigma)\left(\partial_s U_{0,1}(\sigma)\right) = \Lambda_1\partial_s\sigma,$$
$$U_{2,3}^{-1}(\alpha)\left(\partial_s U_{2,3}(\alpha)\right) = \Lambda_2\partial_s\alpha,$$
$$U_{1,3}^{-1}(\vartheta)\left(\partial_s U_{1,3}(\vartheta)\right) = \Lambda_3\partial_s\vartheta,$$
$$U_{0,2}^{-1}(\phi)\left(\partial_s U_{0,2}(\phi)\right) = \Lambda_4\partial_s\phi,$$
$$U_{1,2}^{-1}(\varsigma)\left(\partial_s U_{1,2}(\varsigma)\right) = \Lambda_5\partial_s\varsigma,$$
$$U_{0,3}^{-1}(\iota)\left(\partial_s U_{0,3}(\iota)\right) = \Lambda_6\partial_s\iota,$$
$$\widehat{U}^{-1}(\kappa)\left(\partial_s\widehat{U}(\kappa)\right) = \Lambda_7\partial_s\kappa,$$
$$\widetilde{U}^{-1}(\chi)\left(\partial_s\widetilde{U}(\chi)\right) = \Lambda_8\partial_s\chi.$$

Let $\grave{U}$ be the following set:

$$\grave{U} := \left\{U_{0,1}, U_{2,3}, U_{1,3}, U_{0,2}, U_{1,2}, U_{0,3}, \widehat{U}, \widetilde{U}\right\}.$$

Because

$U_{2,3}^{-1}(\alpha)\Lambda_1 U_{2,3}(\alpha) = \Lambda_1$

$U_{1,3}^{-1}(\vartheta)\Lambda_1 U_{1,3}(\vartheta) = (\Lambda_1\cos 2\vartheta + \Lambda_6\sin 2\vartheta)$

$U_{0,2}^{-1}(\phi)\Lambda_1 U_{0,2}(\phi) = (\Lambda_1\cosh 2\phi - \Lambda_5\sinh 2\phi)$

$U_{1,2}^{-1}(\varsigma)\Lambda_1 U_{1,2}(\varsigma) = \Lambda_1\cos 2\varsigma - \Lambda_4\sin 2\varsigma$

$U_{0,3}^{-1}(\iota)\Lambda_1 U_{0,3}(\iota) = \Lambda_1\cosh 2\iota + \Lambda_3\sinh 2\iota$

$\widehat{U}^{-1}(\kappa)\Lambda_1\widehat{U}(\kappa) = \Lambda_1$

$\widetilde{U}^{-1}(\chi)\Lambda_1\widetilde{U}(\chi) = \Lambda_1$

========

$\widetilde{U}^{-1}(\chi)\,\Lambda_2\widetilde{U}(\chi)=\Lambda_2$
$\widehat{U}^{-1}(\kappa)\,\Lambda_2\widehat{U}(\kappa)=\Lambda_2$
$U_{0,3}^{-1}(\iota)\,\Lambda_2U_{0,3}(\iota)=\Lambda_2\cosh 2\iota-\Lambda_4\sinh 2\iota$
$U_{1,2}^{-1}(\varsigma)\,\Lambda_2U_{1,2}(\varsigma)=\Lambda_2\cos 2\varsigma-\Lambda_3\sin 2\varsigma$
$U_{0,2}^{-1}(\phi)\,\Lambda_2U_{0,2}(\phi)=\Lambda_2\cosh 2\phi+\Lambda_6\sinh 2\phi$
$U_{1,3}^{-1}(\vartheta)\,\Lambda_2U_{1,3}(\vartheta)=\Lambda_2\cos 2\vartheta+\Lambda_5\sin 2\vartheta$
$U_{0,1}^{-1}(\sigma)\,\Lambda_2U_{0,1}(\sigma)=\Lambda_2$
=========
$U_{0,1}^{-1}(\sigma)\;\Lambda_3U_{0,1}(\sigma)=\Lambda_3\cosh 2\sigma-\Lambda_6\sinh 2\sigma$
$U_{2,3}^{-1}(\alpha)\;\Lambda_3U_{2,3}(\alpha)=\Lambda_3\cos 2\alpha-\Lambda_5\sin 2\alpha$
$U_{0,2}^{-1}(\phi)\;\Lambda_3U_{0,2}(\phi)=\Lambda_3$
$U_{1,2}^{-1}(\varsigma)\;\Lambda_3U_{1,2}(\varsigma)=\Lambda_3\cos 2\varsigma+\Lambda_2\sin 2\varsigma$
$U_{0,3}^{-1}(\iota)\;\Lambda_3U_{0,3}(\iota)=\Lambda_3\cosh 2\iota+\Lambda_1\sinh 2\iota$
$\widehat{U}^{-1}(\kappa)\;\Lambda_3\widehat{U}(\kappa)=\Lambda_3$
$\widetilde{U}^{-1}(\chi)\;\Lambda_3\widetilde{U}(\chi)=\Lambda_3$
==========
$\widetilde{U}^{-1}(\chi)\;\Lambda_4\widetilde{U}(\chi)=\Lambda_4$
$\widehat{U}^{-1}(\kappa)\;\Lambda_4\widehat{U}(\kappa)=\Lambda_4$
$U_{0,3}^{-1}(\iota)\;\Lambda_4U_{0,3}(\iota)=\Lambda_4\cosh 2\iota-\Lambda_2\sinh 2\iota$
$U_{1,2}^{-1}(\varsigma)\;\Lambda_4U_{1,2}(\varsigma)=\Lambda_4\cos 2\varsigma+\Lambda_1\sin 2\varsigma$
$U_{1,3}^{-1}(\vartheta)\;\Lambda_4U_{1,3}(\vartheta)=\Lambda_4$
$U_{2,3}^{-1}(\alpha)\;\Lambda_4U_{2,3}(\alpha)=\Lambda_4\cos 2\alpha-\Lambda_6\sin 2\alpha$
$U_{0,1}^{-1}(\sigma)\;\Lambda_4U_{0,1}(\sigma)=\Lambda_4\cosh 2\sigma+\Lambda_5\sinh 2\sigma$
==========
$U_{0,1}^{-1}(\sigma)\;\Lambda_5U_{0,1}(\sigma)\;=\Lambda_5\cosh 2\sigma+\Lambda_4\sinh 2\sigma$
$U_{2,3}^{-1}(\alpha)\;\Lambda_5U_{2,3}(\alpha)\;=\Lambda_5\cos 2\alpha+\Lambda_3\sin 2\alpha$
$U_{1,3}^{-1}(\vartheta)\;\Lambda_5U_{1,3}(\vartheta)\;=(\Lambda_5\cos 2\vartheta-\Lambda_2\sin 2\vartheta)$
$U_{0,2}^{-1}(\phi)\;\Lambda_5U_{0,2}(\phi)\;=\Lambda_5\cosh 2\phi-\Lambda_1\sinh 2\phi$
$U_{0,3}^{-1}(\iota)\;\Lambda_5U_{0,3}(\iota)\;=\Lambda_5$
$\widehat{U}^{-1}(\kappa)\;\Lambda_5\widehat{U}(\kappa)\;=\Lambda_5$
$\widetilde{U}^{-1}(\chi)\,\Lambda_5\widetilde{U}(\chi)\;=\Lambda_5$
===========
$\widetilde{U}^{-1}(\chi)\,\Lambda_6\widetilde{U}(\chi)\;=\Lambda_6$
$\widehat{U}^{-1}(\kappa)\,\Lambda_6\widehat{U}(\kappa)\;=\Lambda_6$
$U_{1,2}^{-1}(\varsigma)\,\Lambda_6U_{1,2}(\varsigma)\;=\Lambda_6$
$U_{0,2}^{-1}(\phi)\,\Lambda_6U_{0,2}(\phi)\;=\Lambda_6\cosh 2\phi+\Lambda_2\sinh 2\phi$
$U_{1,3}^{-1}(\vartheta)\,\Lambda_6U_{1,3}(\vartheta)\;=\Lambda_6\cos 2\vartheta-\Lambda_1\sin 2\vartheta$
$U_{2,3}^{-1}(\alpha)\,\Lambda_6U_{2,3}(\alpha)\;=\Lambda_6\cos 2\alpha+\Lambda_4\sin 2\alpha$
$U_{0,1}^{-1}(\sigma)\,\Lambda_6U_{0,1}(\sigma)=\Lambda_6\cosh 2\sigma-\Lambda_3\sinh 2\sigma$
========
$\widetilde{U}^{-1}(\chi)\;\Lambda_7\widetilde{U}(\chi)=\Lambda_7$
$U_{0,3}^{-1}(\iota)\;\Lambda_7U_{0,3}(\iota)=\Lambda_7$
$U_{1,2}^{-1}(\varsigma)\;\Lambda_7U_{1,2}(\varsigma)=\Lambda_7$
$U_{0,2}^{-1}(\phi)\;\Lambda_7U_{0,2}(\phi)=\Lambda_7$
$U_{1,3}^{-1}(\vartheta)\;\Lambda_7U_{1,3}(\vartheta)=\Lambda_7$
$U_{2,3}^{-1}(\alpha)\;\Lambda_7U_{2,3}(\sigma)=\Lambda_7$
$U_{0,1}^{-1}(\sigma)\;\Lambda_7U_{0,1}(\sigma)=\Lambda_7$
=========
$U_{0,1}^{-1}(\sigma)\;\Lambda_8U_{0,1}(\sigma)=\Lambda_8$
$U_{2,3}^{-1}(\alpha)\;\Lambda_8U_{2,3}(\alpha)=\Lambda_8$
$U_{1,3}^{-1}(\vartheta)\;\Lambda_8U_{1,3}(\vartheta)=\Lambda_8$
$U_{0,2}^{-1}(\phi)\;\Lambda_8U_{0,2}(\phi)=\Lambda_8$
$U_{1,2}^{-1}(\varsigma)\;\Lambda_8U_{1,2}(\varsigma)=\Lambda_8$
$U_{0,3}^{-1}(\iota)\;\Lambda_8U_{0,3}(\iota)=\Lambda_8$
$\widehat{U}^{-1}(\kappa)\;\Lambda_8\widehat{U}(\kappa)=\Lambda_8$

then for every product U of $\grave{U}$'s elements real functions $G_s^r(t,x_1,x_2,x_3)$ exist such that

$$U^{-1}(\partial_s U)=\frac{g_3}{2}\sum_{r=1}^{8}\Lambda_r G_s^r$$

with some real constant g_3 (similar to 8 gluons).

4 Conclusion

Therefore, unessential restrictions on 4X1 matrix functions give Dirac's equations, and it seems that some gluon and gravity phenomena can be explained with the help of these equations.

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References

1. Madelung E. Die Mathematischen Hilfsmittel des Physikers. Springer Verlag, 1957, 29.
2. Ziman J.M. Elements of advanced quantum theory. Cambridge University Press, 1969, formula (6.59).