Application of the Model of Oscillations in a Chain System to the Solar System

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A numerical analysis revealed that masses, radii, distances from the sun, orbital periods and rotation periods of celestial bodies can be expressed on the logarithmic scale though a systematic set of numbers: $4e$, $2e$, e, $\frac{e}{2}$, $\frac{e}{4}$, $\frac{e}{8}$ and $\frac{e}{16}$. We analyzed these data with a fractal scaling model originally published by Müller in this journal, interpreting physical quantities as proton resonances. The data were expressed in continued fraction form, where all numerators are Euler's number. From these continued fractions, we explain the volcanic activity on Venus, the absence of infrared emission of Uranus and why Jupiter and Saturn emit more infrared radiation than they receive as total radiation energy from the Sun. We also claim that the Kuiper cliff was not caused by a still unknown planet. It can be understood why some planets have an atmosphere and others not, as well as why the ice on dwarf planet Ceres does not evaporate into space through solar radiation. The results also suggest that Jupiter and Saturn have the principal function to capture asteroids and comets, thus protecting the Earth, a fact which is well-reflected in the high number of their irregular satellites.

1 Introduction

Recently in three papers of this journal, Müller [1–3] suggested a chain of similar harmonic oscillators as a general model to describe physical quantities as proton resonance oscillation modes. In this model, the spectrum of eigenfrequencies of a chain system of many proton harmonic oscillators is given by a continuous fraction equation [2]:

$$f = f_p \exp S \tag{1}$$

where f is any natural oscillation frequency of the chain system, f_p the oscillation frequency of one proton and S the continued fraction corresponding to f. S was suggested to be in the canonical form with all partial numerators equal 1 and the partial denominators are positive or negative integer values.

$$S = n_0 + \cfrac{1}{n_1 + \cfrac{1}{n_2 + \cfrac{1}{n_3 + \ldots}}} \tag{2}$$

Particularly interesting properties arise when the nominator equals 2 and all denominators are divisible by 3. Such fractions divide the logarithmic scale in allowed values and empty gaps, i.e. ranges of numbers which cannot be expressed with this type of continued fractions. He showed that these continued fractions generate a self-similar and discrete spectrum of eigenvalues [1], that is also logarithmically invariant. Maximum spectral density areas arise when the free link n_0 and the partial denominators n_i are divisible by 3.

In a previous article [5] we slightly modified this model, substituting all nominators by Euler's number. In that way we confirmed again that elementary particles are proton resonance states, since most masses were found to be located close to spectral nodes and definitively not random.

In this article we investigated various solar system data, such as masses, sizes and distances from the Sun, rotation and orbital periods of celestial bodies on the logarithmic scale. We showed that continued fractions with Euler's number as nominator are adequate to describe the solar system. From these continued fractions we derived claims regarding specific properties of planets. It became evident, that the solar system possesses a hidden fractal structure.

2 Data sources and computational details

All solar system data, such as distances, masses, radii, orbital and rotation periods of celestial bodies, were taken from the NASA web-site. The km was converted into the astronomical unit via 1 AU = $149,597,870.7$ km. The mean distance of an object from the central body is understood as $\frac{1}{2}(Aphelion - Periphelion)$. Numerical values of continued fractions were always calculated using the the Lenz algorithm as indicated in reference [4].

3 Results and discussion

3.1 Standard numerical analysis

Before doing any numerical analysis, one always has to be aware of the fact that the numerical value of a quantity depends on the physical unit. In this particular analysis we decided to choose practical units which were made exclusively by nature. Such units are the astronomical unit (AU) for lengths, the earth mass for planetary masses, as well as the year and the day for orbit and rotation periods. As can be seen, this particular choice leads to quite interesting regularities.

In a previous article [5], we had already done a similar analysis of elementary particle masses on the logarithmic scale and detected a set of systematic mass gaps: $2e$, e, $\frac{e}{2}$, $\frac{e}{4}$, $\frac{e}{8}$ and $\frac{e}{16}$. Therefore, our numerical analysis was focused on these numbers and in a similar way, we detected this set of expressions again.

When looking from the Earth in direction away from the Sun, it can be noted that there are two principal zones, where mass accumulation into heavy planets seems to be forbidden. The existing mass is scattered in the form of asteroids and large bodies cannot become more than dwarf planets. The first such zone is the so-called Asteroid belt, located between Mars and Jupiter. Its population has already been well investigated, especially to confirm the orbital resonance effects manifesting in the Kirkwood gaps. Most asteroids have semi-major axes between 2.1 and 3.5 AU.

The second scattered-mass zone is the Kuiper belt, located from the orbit of Neptune (30 AU) to 55 AU distance from the Sun.

The Oort cloud is also such a scattered-mass zone. Due to its giant distance from the center of the solar system, there is no well-confirmed lower and upper limit, so we did not include it into the numerical analysis.

Table 1: Mean distances of celestial bodies (d) from the Sun expressed through e on the logarithmic scale and absolute values of corresponding numerical errors.

Object d [AU] ln(d)	Expression	Numerical error
Mercury 0.3871044 -0.9491	$-\left(\frac{e}{4}+\frac{e}{8}\right)$	0.0703
Venus 0.723339 -0.3239	$-\frac{e}{8}$	0.0159
Earth 0.9999808 0.0000	$0e$	0.0000
Mars 1.523585 0.4211	$\frac{e}{8}$	0.0812
Ceres 2.7663 1.0175	$\frac{e}{4}+\frac{e}{8}$	0.0019
Jupiter 5.204419 1.6495	$\frac{e}{2}+\frac{e}{8}$	0.0494
Saturn 9.582516 2.2599	$\frac{e}{2}+\frac{e}{4}+\frac{e}{16}$	0.0513
Uranus 19.201209 2.9550	$e+\frac{e}{16}$	0.0668
Neptune 30.04762 3.4028	$e+\frac{e}{4}$	0.0049
Pluto 39.486178 3.6758	$e+\frac{e}{4}+\frac{e}{8}$	0.0618

Table 2: Equatorial radii (r) of celestial bodies expressed through e on the logarithmic scale and absolute values of corresponding numerical errors.

Object r [AU] ln(r)	Expression	Numerical error
Mercury 1.6308×10^{-5} -11.0238	$-\left(4e+\frac{e}{16}\right)$	0.0192
Venus 4.0454×10^{-5} -10.1154	$-\left(2e+e+\frac{e}{2}+\frac{e}{4}\right)$	0.0782
Earth 4.2635×10^{-5} -10.0628	$-\left(2e+e+\frac{e}{2}+\frac{e}{8}+\frac{e}{16}\right)$	0.0391
Mars 2.2708×10^{-5} -10.6928	$-\left(2e+e+\frac{e}{2}+\frac{e}{4}+\frac{e}{8}+\frac{e}{16}\right)$	0.0104
Ceres 3.2574×10^{-6} -12.6346	$-\left(4e+e+\frac{e}{2}+\frac{e}{8}\right)$	0.0625
Jupiter 4.7789×10^{-4} -7.6461	$-\left(2e+\frac{e}{2}+\frac{e}{4}+\frac{e}{16}\right)$	0.0010
Saturn 4.0287×10^{-4} -7.8169	$-\left(2e+\frac{e}{2}+\frac{e}{4}+\frac{e}{8}\right)$	0.0018
Uranus 1.709×10^{-4} -8.6747	$-\left(2e+e+\frac{e}{8}+\frac{e}{16}\right)$	0.0102
Neptune 1.6554×10^{-4} -8.7063	$-\left(2e+e+\frac{e}{8}+\frac{e}{16}\right)$	0.0418
Pluto 7.6940×10^{-6} -11.7751	$-\left(4e+\frac{e}{4}+\frac{e}{16}\right)$	0.0525
Sun 4.649×10^{-3} -5.3817	$-2e$	0.0549

Table 3: Sidereal orbital periods (T) of celestial bodies expressed through e on the logarithmic scale and absolute values of corresponding numerical errors.

Object T [y] ln(T)	Expression	Numerical error
Mercury 0.2408467 -1.4236	$-\frac{e}{2}$	0.0645
Venus 0.61519726 -0.4858	$-\left(\frac{e}{8}+\frac{e}{16}\right)$	0.0239
Earth 1.0000174 0.0000	$0e$	0.0000
Mars 1.8808476 0.6317	$\frac{e}{4}$	0.0479
Ceres 4.60 1.5261	$\frac{e}{2}+\frac{e}{16}$	0.0029
Jupiter 11.862615 2.4734	$\frac{e}{2}+\frac{e}{4}+\frac{e}{8}+\frac{e}{16}$	0.0750
Saturn 29.447498 3.3826	$e+\frac{e}{4}$	0.0153
Uranus 84.016846 4.4310	$e+\frac{e}{2}+\frac{e}{8}$	0.0138
Neptune 164.79132 5.1047	$e+\frac{e}{2}+\frac{e}{4}+\frac{e}{8}$	0.0079
Pluto 247.92065 5.5131	$2e$	0.0765

It can be seen that the distance between Ceres (the largest Asteroid belt object) and Pluto (the largest Kuiper belt object) matches Euler's number quite accurately. Table 1 summarizes the mean distances of the most important celestial bodies from the Sun together with the corresponding natural logarithms. It was found that all logarithms can be expressed as a sum of $2e$, e, $\frac{e}{2}$, $\frac{e}{4}$, $\frac{e}{8}$ and $\frac{e}{16}$. Most distances could even expressed as multiples of $\frac{e}{8}$ since they do not contain the summand $\frac{e}{16}$. The numerical errors on the logarithmic scale are significantly lower than $\frac{e}{16}$.

Analogously, we expressed the equatorial radii, sidereal orbital periods, sidereal rotation periods and masses of celestial bodies on the logarithmic number line (see Tables 2–5).

Table 4: Sidereal rotation periods (T) of celestial bodies (retrograde rotation ignored) expressed through e on the logarithmic scale and absolute values of corresponding numerical errors.

Object T [d] ln(T)	Expression	Numerical error
Mercury 58.6462 4.0715	$e+\frac{e}{2}$	0.0059
Venus 243.018 5.4931	$2e$	0.0565
Earth 0.99726968 -0.0027	$0e$	0.0027
Mars 1.02595676 0.0256	$0e$	0.0256
Ceres 0.3781 -0.9726	$-\left(\frac{e}{4}+\frac{e}{8}\right)$	0.0468
Jupiter 0.41354 -0.8830	$-\left(\frac{e}{4}+\frac{e}{16}\right)$	0.0335
Saturn 0.44401 -0.8119	$-\left(\frac{e}{4}+\frac{e}{16}\right)$	0.0376
Uranus 0.71833 -0.3308	$-\frac{e}{8}$	0.0090
Neptune 0.67125 -0.3986	$-\frac{e}{4}$	0.0588
Pluto 6.3872 1.8543	$\frac{e}{2}+\frac{e}{8}+\frac{e}{16}$	0.0145
Sun 25.05 3.2209	$e+\frac{e}{8}+\frac{e}{16}$	0.0071

In very few cases it was necessary to introduce $4e$ into the set of summands.

From these results we conclude that all these numerical values of planetary data are definitively not a set of random numbers. The repeatedly occurring summands strongly support the idea of a self-similar, fractal structure as Müller already claimed in reference [2].

In the present form, these results are obtained only when considering nature-made units, which underlines their importance.

Table 5: Masses (m) of celestial bodies, rescaled by earth mass and expressed through e on the logarithmic scale and absolute values of corresponding numerical errors.

Object m [$\times 10^{24}$ kg] $\ln(\frac{m}{m_{Earth}})$	Expression	Numerical error
Mercury 0.330104 -2.8950	$-\left(e+\frac{e}{16}\right)$	0.0068
Venus 4.86732 -0.2046	$-\frac{e}{16}$	0.0347
Earth 5.97219 0.0000	$0e$	0.0000
Mars 0.641693 -2.2312	$-\left(\frac{e}{2}+\frac{e}{4}+\frac{e}{16}\right)$	0.0226
Ceres 0.000943 -8.7403	$-\left(2e+e+\frac{e}{8}+\frac{e}{16}\right)$	0.0758
Jupiter 1898.13 5.7615	$2e+\frac{e}{8}$	0.0148
Saturn 568.319 4.5556	$e+\frac{e}{2}+\frac{e}{8}+\frac{e}{16}$	0.0315
Uranus 86.8103 2.6766	e	0.0416
Neptune 102.410 2.8419	$e+\frac{e}{16}$	0.0463
Pluto 0.01309 -6.1193	$-\left(2e+\frac{e}{4}\right)$	0.0032
Sun 1989100 12.7161	$4e+\frac{e}{2}+\frac{e}{8}+\frac{e}{16}$	0.0258

3.2 Continued fraction analysis

Due to the fact that all the solar system data can be expressed by multiples of $\frac{e}{16}$, it is consistent to set all partial numerators in Müller's continued fractions (given in equation(2)) to Euler's number. We further follow the formalism of previous publications [5, 6] and introduce a phase shift p in equation (2). According to [6] the phase shift can only have the values 0 or ± 1.5. So we write for instance for the masses of the celestial bodies:

$$\ln \frac{mass}{proton\ mass} = p + S \tag{3}$$

where S is the continued fraction

$$S = n_0 + \cfrac{e}{n_1 + \cfrac{e}{n_2 + \cfrac{e}{n_3 + \ldots}}}. \tag{4}$$

We abbreviate $p + S$ as $[p; n_0 \mid n_1, n_2, n_3, \ldots]$. The free link n_0 and the partial denominators n_i are integers divisible by 3. For convergence reason, we have to include $|e{+}1|$ as allowed partial denominator. This means the free link n_0 is allowed to be $0, \pm 3, \pm 6, \pm 9 \ldots$ and all partial denominators n_i can take the values $e{+}1, -e{-}1, \pm 6, \pm 9, \pm 12 \ldots$.

Analogously we write for the planetary mean distances from the Sun:

$$\ln \frac{mean\ distance}{\lambda_C} = p + S \tag{5}$$

where $\lambda_C = \frac{h}{2\pi mc}$ is the reduced Compton wavelength of the proton with the numerical value $2.103089086 \times 10^{-16}$ m. Since the exact diameter or radius of the proton is unknown, some other proton related parameter is used, which can be determined accurately. The same applies for the equatorial radii. For orbital and rotational periods we write:

$$\ln \frac{time\ period}{\tau} = p + S \tag{6}$$

where $\tau = \frac{\lambda_C}{c}$ is the oscillation period of a hypothetical photon with the reduced Compton wavelength of the proton and traveling with light speed (numerical value $7.015150081 \times 10^{-25}$ s).

For the calculation of the continued fractions we did not consider any standard deviation of the published data. Practically, we developed the continued fraction and determined only 18 partial denominators. Next we calculated repeatedly the data value from the continued fraction, every time considering one more partial denominator. As soon as considering further denominators did not improve the experimental data value significantly (on the linear scale), we stopped considering further denominators and gave the resulting fraction in Tables 6-10. This means we demonstrate how accurately the published solar system data can be expressed through continued fractions. Additionally we gave also the numerical error, which is defined as absolute value of the difference between NASA's published data value and the value calculated from the continued fraction representation.

The continued fraction representations of the masses of celestial bodies are given in Table 6. As can be seen, the absolute value of the first partial denominator is frequently high, which locates the mass very close to the principal node.

Table 6: Continued fraction representation of masses (m) of celestial bodies according to equation (3) and absolute values of corresponding numerical errors.

Object m [kg]	Continued fraction representation Numerical error [kg]
Mercury 0.330104×10^{24}	[1.5; 114 | 9, -12, -e-1, e+1] $5.5e + 19$
Venus 4.86732×10^{24}	[1.5; 117 | -305223] 1.6×10^{14}
Earth 5.97219×10^{24}	[1.5; 117 | 12, e+1, -e-1, e+1] 3.0×10^{22}
Mars 0.641693×10^{24}	[0; 117 | -6, e+1, -6, 33, -60, -e-1, e+1, -e-1] 1.1×10^{15}
Ceres 9.43×10^{20}	[1.5; 108 | 6, 99, e+1, -e-1, e+1, -6, e+1, e-1] 3.4×10^{12}
Jupiter 1.89813×10^{27}	[1.5; 123 | -81, e+1, -e-1, -e-1, -e-1, e+1, -9, -e-1] 3.6×10^{18}
Saturn 5.68319×10^{26}	[0; 123 | 9, e+1, -e-1] 8.1×10^{24}
Uranus 8.68103×10^{25}	[1.5; 120 | -24, e+1, -e-1, e+1] 7.0×10^{22}
Neptune 1.0241×10^{26}	[1.5; 120 | 60, -e-1, e+1, -e-1] 3.9×10^{22}
Pluto 1.309×10^{22}	[1.5; 111 | 33, 9, -e-1, e+1, -18, e+1, e+1, -15] 3.2×10^{12}
Sun 1.9891×10^{30}	[0; 132 | -e-1, -e-1, e+1, -e-1, 12, -e-1] 5.0×10^{25} [1.5; 129 | e+1, -e-1, 15, e+1] 6.2×10^{26}

Table 7: Continued fraction representation of mean distances of celestial bodies from the Sun according to equation (5) and absolute values of corresponding numerical errors.

Object mean distance [km]	Continued fraction representation Numerical error
Mercury 57.91×10^6	[0; 60 | e+1, -e-1, -e-1, -e-1, 6, 6, -9, -e-1] 1 km
Venus 108.21×10^6	[1.5; 60 | 513, 6, -9, e+1] 260 m
Earth 149.595×10^6	[1.5; 60 | 9, -e-1, 51, e+1, 6, 6] 873 m
Mars 227.925×10^6	[0; 63 | -e-1, 30, -e-1, -15, 6, 9, -9] 0.4 m
Ceres 413.833×10^6	[0; 63 | -18, 9, e+1, -e-1, e+1, -e-1, e+1, -e-1] 5854 km
Jupiter 778.57×10^6	[0; 63 | 6, -9, 6, -e-1, e+1, -e-1, -6, 54] 372 m
Saturn 1433.525×10^6	[1.5; 63 | -6, -e-1, -e-1, -15, -48, e+1, -e-1] 8.7 km
Uranus 2872.46×10^6	no continued fraction found
Neptune 4495.06×10^6	[0; 66 | -e-1, 15, 15, 54, 9, -e-1, e+1, -e-1] 46 m [1.5; 63 | e+1, -597, -9, e+1] 181 km
Pluto 5906.375×10^6	[0; 66 | -6, 6, -e-1, -6, -15, -e-1, -12, -e-1] 7.2 km

In case of the Venus, the mass is almost exactly located in a node. Notably two low-weight bodies, Ceres and Mars, are most distant from the principal nodes. A preferred accumulation of planetary masses in nodes in agreement with results previously published by Müller [2]. This author published already a continued fraction analysis of planetary masses, however, the continued fractions were in the canonical form with all nominators equal 1. Interestingly, his result is principally not changed substituting the nominators for e. The only exception is the Sun, here even two continued fractions can be given and the mass is located in a non-turbulent zone between the principal nodes 129+1.5 and 132. This indicates that the probability of mass changes of the Sun is extremely low, so one can expect that all astrophysical parameters of the Sun will not show any evolution for a long time. We conclude that it seems to be a general property of mass to accumulate close to the nodes. Apparently no specific properties of the celestial bodies can be correlated to these data.

Table 7 displays the continued fraction representations of the mean distances from the Sun of the considered celestial bodies. When analyzing the denominators, it is directly clear that there is no general behavior of the planetary distances. For instance Venus is located almost in a node (n_1 very high), while Mercury, Mars and Neptune are far away from a node ($n_1 = e{+}1$ or $-e{-}1$). Uranus is even in a gap. Earth, Jupiter, Saturn and Pluto are moderately close to a node. This opens a door to associate a specific property of these bodies to the continued fraction representation. In this particular case we relate the mean distance to seismic activity of a solid object or heat release of a gas planet. The oscillation process inside Venus is turbulent, and it is known that Venus has an extreme

Table 8: Continued fraction representation of equatorial radii of celestial bodies according to equation (5) and absolute values of corresponding numerical errors.

Object Equatorial radius [km]	Continued fraction representation Numerical error
Mercury 2439.7	[0; 51 | -15, e+1, -e-1, e+1, -e-1] 1.6 km
Venus 6051.8	[0; 51 | e+1, 30, 9] 98 m
Earth 6378.14	[0; 51 | e+1, -15, -e-1, e+1, e+1] 57 m [1.5; 51 | -e-1, 207] 58 m
Mars 3397	[0; 51 | 21, -e-1, e+1, -e-1, e+1] 1.8 km
Ceres 487.3	[1.5; 48 | -9, 27, 9, 18] 0.01 m
Jupiter 71492	[0; 54 | 15, -18, -24, -6] 2 m
Saturn 60268	[0; 54 | 222, -6, -e-1] 46 m
Uranus 25559	[0; 54 | -e-1, 6, -e-1, -e-1, 9] 898 m [1.5; 51 | e+1, 6, 12, -e-1, e+1, -6] 44 m
Neptune 24764	[0; 54 | -e-1, e+1, e+1, 9, -e-1, 9] 22 m [1.5; 51 | e+1, e+1, 6, -6, -213] 0.05 m
Pluto 1151	[0; 51 | -e-1, e+1, -6, e+1, e+1] 475 m
Sun 6.955×10^5	[0; 57 | -6, e+1, -e-1, e+1, -e-1, -e-1, 12, -6] 49 m [1.5; 54 | e+1, -e-1, e+1, e+1, e+1 -e-1, e+1, -e-1, e+1] 21 km

volcanic activity [7,8]. Scientists also believe that the volcanism on Venus has been changing over time [7], so changes in trend may occur. The data also suggest that seismic activity on Earth is higher than on Mars, Mercury or Pluto.

For the gas planets Jupiter, Saturn and Neptune, it has been known that they produce more heat internally than they receive from the Sun [9, 10]. Contrary to this, Uranus is a relatively cold planet, radiating very little more energy than received. The principal source of this heating is believed to be a liberation of thermal energy from precipitation of Helium or other compounds in the interior of the planet while simultaneously gravitational potential energy is released.

Physically, such processes should exist in all gas planets, this means only the process kinetics can be associated to the continued fraction representation. We assume that the rate of this process is influenced by oscillations in the planet. For Uranus, which is located in a gap, the oscillation capability is low, which means the heat-releasing process occurred faster and is already almost completed. Jupiter and Saturn, located in proximity to the nodes 63 and 1.5+63, are in a fluctuation zone. So here the heat releasing process is disturbed and they are yet in a more early phase of process development, whereas Neptune (away from nodes) is in an already more advanced phase. From this we can predict that one day in future, first Neptune stops releasing excess heat, while Jupiter and Saturn will do this much later.

A very special situation is the continued fraction representation of dwarf planet Ceres. As can be seen, it has an exceptional high numerical error, actually this must be interpreted as "no continued fraction found". We report the fraction here only in order to demonstrate that the whole Asteroid belt is in a fluctuation zone around the node 63, which translates to $\lambda_C exp(63) = 3.22$ AU. This value is not acceptable as an average for the distances of the Asteroid belt objects from the Sun. Actually most Asteroids can be found between 2.1 and 3.5 AU. From this it can be concluded that most Asteroids accumulate in the compression zone before the principal node 63. Similarly is the situation for the Kuiper belt. All Kuiper belt objects are located before the node 66, $\lambda_C exp(66) = 64.77$ AU. The Astrophysics textbooks always teach the belt is located from the orbit of Neptune (30 AU) to 50 or 55 AU distance from the Sun. So again, the celestial bodies accumulate before a principal node.

Since Ceres is the largest Asteroid belt object, it is reasonable to claim Ceres is located in a gap, even inside a fluctuation zone. We interpret these fluctuations as the cause of the observed mass scattering in the whole Asteroid belt.

More research must still be done regarding the distribution of Kuiper belt objects. Brunini and Melita [11] suggested a Mars like object around 60 AU distance from the Sun in order to explain the Kuiper cliff, a sudden drop off of space rocks beyond 50 AU. Later, numerical simulations of Lykawka and Mukai showed that such a body would not reproduce the observed orbital distribution in the Kuiper belt [12], however these authors did not completely exclude the possibility of an unknown planet. Now, from our continued fraction analysis we suggest that there is indeed no unknown planet, it is just so that the compression zone before the principal node acts as accumulation site of these relatively light Kuiper belt objects. If there was such a solid planet in the fluctuation zone, it should possess volcanic activity similarly to Venus, and consequently should be very easy to detect, because of emission of infrared radiation. So this argument again confirms the absence of such a planet. Anyway, a detailed continued fraction analysis of Trans-Neptunian objects combined with Kuiper belt objects would be very useful.

Table 8 displays analogously the continued fraction representations of planetary equatorial radii. From these data, some statements regarding the atmosphere of solid planets can be derived. We interprete an atmosphere as an extension of a planet with the effect to increase its radius. On the other hand, an atmosphere is also governed by the chemical composition of a planet and its temperature and these parameters are more decisive. Such an analysis cannot be applied to gaseous planets, since they always have a very dense atmosphere, regardless of their radii.

The most dense atmospheres can be found on Earth and on Venus. The first partial denominator in the continued fraction representation of Venus is e+1. this means the radius of Venus is in an expansion zone and far away from the node. An increase in radius is favored and any probabilities of trend changes are low. This is in agreement with the observed high density of the atmosphere on Venus, with a pressure of 95 bar at the surface [8]. In the case of our planet Earth, two continued fractions can be given, so the radius is influenced by the two nodes 51 and 51+1.5. Both first partial denominators put the radius far away from the corresponding nodes into a non-fluctuation zone. Here does not exist any specific trend and the formation of the atmosphere is solely governed by chemical composition and temperature.

Pluto is with a negative first partial denominator in a compression zone, so the expansion of its radius by an atmosphere is not favored. Indeed Pluto has only a very thin atmosphere in the micro-bar range [13]. According to reference [14], Pluto's atmosphere at perihelion extends to depths greater than Earth's atmosphere and may even enclose the moon Charon. The atmosphere is thought to be actively escaping, so Pluto is the only planet in the solar system actively losing its atmosphere now.

The same is true for Mercury. In agreement with the observations, Mercury does not have an atmosphere [8], which can also be alternatively explained by its high surface temperature.

Mars is with the positive number 21 of the first partial denominator in an expansion zone, so the formation of an atmosphere is favored. At the same time the radius is also close to the node 51 in a fluctuation zone. This means changes in process trends may occur. Considering the formation of an atmosphere as the relevant process, this process can be interrupted or inverted over long time periods. As a consequence, one would expect an atmosphere, but significantly thinner than that on Venus. Actually the surface pressure on Mars is close to 1% to that of the Earth and there are speculations that the atmosphere on Mars has experienced major changes in the past [8].

Ceres is a low density object consisting of rock and ice with mean density of only 2 g/cm^3, which supports the presence of a lot of ice. The "frost line" in our solar system — the distance where ice will not evaporate — is roughly at 5 AU from the Sun [15]. So one must ask why Ceres does not have already lost all his ice through sublimation. From the continued fraction representation, the radius of Ceres is in a compression zone and the formation of an atmosphere is not favored. Through evaporation of the ice, at least temporarily an atmosphere will form. For this reason we believe Ceres is able to continue for a long time as an icy dwarf planet.

When looking at the data it turns out that the gaseous planets seem to prefer radii that can be described by two con-

Table 9: Continued fraction representation of sidereal orbital periods of celestial bodies according to equation (6) and absolute values of corresponding numerical errors.

Planet T [s]	Continued fraction representation Numerical error
Mercury 7595370	[0; 72 | -6, e+1, -e-1, e+1, -30, -e-1, -33, -6] 0.002 s [1.5; 69 | e+1, -e-1, e+1, 6, -12, 6, -e-1, e+1, -15] 0.01 s
Venus 19400861	[0; 72 | 6, e+1, -6, 6, e+1, -e-1, e+1, -e-1, e+1] 128 s
Earth 31536549	[0; 72 | e+1, -e-1, -6, e+1, -6, -6, -e-1, 9, -6] 0.1 s [1.5; 72 | -e-1, -e-1, -12, 45, e+1, -6, -e-1, -24] 0.0003 s
Mars 59314410	[1.5; 72 | 183, -e-1, 12, -e-1, e+1, -e-1] 13 s
Ceres 145065600	[0; 75 | -e-1, -e-1, e+1, 6, -e-1, 6, -6, -18, e+1] 0.3 s [1.5; 72 | e+1, -e-1, -225, -e-1, e+1, -e-1, -9, -e-1] 0.06 s
Jupiter 374099427	no continued fraction found
Saturn 928656297	[1.5; 75 | -12, 6, e+1, -e-1, 33, e+1, -e-1, e+1, -e-1] 74 s
Uranus 2649555255	[0; 78 | -e-1, -12, e+1, -e-1, 12, -e-1, -69, -9] 0.9 s
Neptune 5196859068	[0; 78 | -225, e+1, -9, e+1, -6, e+1, 48] 0.04 s
Pluto 7818425618	no continued fraction found

tinued fractions. For the Sun, Uranus and Neptune it can be said that they are influenced by two neighbored nodes. This indicates their sizes will remain constant over a longe time. The only exceptions are Jupiter and Saturn, which are in an expansion zone. One would expect their sizes increasing. How could this be achieved in practice? There is only one possibility, Jupiter and Saturn must capture some asteroids or comets preferentially from the Kuiper belt. When looking at the number of their moons, it can be assumed that such a process has already been progressing for a long time. A moon can be interpreted as an incomplete capture, this means the object was captured without crashing into the planet and increasing its size. Indeed Jupiter and Saturn have 63 and 62 confirmed moons, while Uranus has 27, and Neptune only 13 moons. Normally one would expect that Uranus and Neptune should have the most moons, since they are much closer located to the Kuiper belt. Notably 55 of Jupiter's moons are irregular satellites with high eccentricities and inclinations, while Saturn has just 38 of such satellites. It is assumed that these irregular satellites were captured from other orbits.

In Table 9, the continued fraction representations of the orbital periods are given. When analyzing these fractions, their interpretation is problematic: One has to bear in mind that Kepler's 3rd law relates the orbital period to the semi-major axis (for most planets close to the mean distance), so these parameters are not independent from each other.

Regarding oscillation properties, it is clearly visible that the continued fraction representations of the orbital periods do not provide a similar image of planetary features than the representations of the corresponding mean distances. For instance, the orbital periods of Mars and Neptune are located in a highly turbulent zone. This is contrary to to the continued fraction representation of its mean distances given in Table 7, where both planets are far away from a node. Since for the mean distances a meaningful continued fraction representation exists, the orbital periods do not fit anymore in this model and their mathematical representation in continued fractions, as presented here, is physically meaningless.

Luckily, the situation is easier for the rotation periods of the celestial bodies (see Table 10). As can be seen, the rotation periods prefer values far away from the nodes in non-fluctuating zones. There are only three exceptions: Jupiter Saturn and Ceres have periods located in a principal node. This means the rotation periods are in an early stage of development, which can be justified with a specific process inside the celestial bodies.

For the gas planets Jupiter and Saturn it has been known that heat is generated from precipitation of Helium or other compounds in the interior of the planet while simultaneously gravitational potential energy is released. Through such a process, the moment of inertia of the planet changes gradually and the rotation period evolves. From the analysis of the mean distances of Jupiter and Saturn, we have already stated that their heat release processes are still in an early phase of development. Exactly the same can be derived from the analysis of rotation periods. The rotation of the Sun is also not yet completely evolved, however here this effect is minor. Any internal structuring of plasma fluxes could be responsible for this.

Ceres has an unusual location inside the Asteroid belt, which is a turbulent zone as can be derived from the continued fraction analysis of its mean distance from the Sun. Knowing this, we speculate that the evolution of its rotation period could have been influenced by the fluctuating population of the belt through collisions of an early Ceres with many smaller asteroids over a long time. According to reference [15], there are possibly volatile compounds in the interior of Ceres. Ceres could have accreted from rocky and icy planetesimals. This has taken some time, we speculate that

Table 10: Continued fraction representation of sidereal rotation periods (T) of celestial bodies according to equation (6) and absolute values of corresponding numerical errors.

Planet T [s]	Continued fraction representation Numerical error
Mercury 5067032	[0; 72 | -e-1, e+1, -6, 6, -15, -e-1, e+1, -e-1, e+1] 3 s
Venus 20996755	[0; 72 | 6, -9, -12, 18, -9, e+1] 0.1 s
Earth 86164	[0;66 | e+1, -e-1, e+1, -6, e+1, e+1, -e-1, 21] 0.02 s [1.5; 66 | -6, e+1, -15, -e-1, -6] 0.07 s
Mars 88643	[1.5; 66 | -6, 6, -18, -12] 0.04 s
Ceres 32668	[0; 66 | 255, -e-1, e+1, -e-1, e+1] 0.17 s
Jupiter 35730	[0; 66 | 27, 27, -21] 0.005 s
Saturn 38362	[0; 66 | 15, e+1, -e-1, -e-1, e+1, -e-1] 2 s
Uranus 62064	[0; 66 | e+1, 6, 39, -12] 0.02 s [1.5; 66 | -e-1, 6, -e-1, -9, -e-1, 48] 0.001 s
Neptune 57996	[0; 66 | e+1, e+1, -e-1, 9, -9, -18] 0.003 s [1.5; 66 | -e-1, e+1, -30, -e-1, e+1, -e-1] 4 s
Pluto 551854	no continued fraction found
Sun 2164320	[1.5; 69 | -9, -15, e+1, e+1, -6, 9] 0.003 s

possibly Ceres had less time for the evolution of its rotation than other planets.

An other reference [16] speculates regarding a subsurface ocean and mentions a modeling predicting that ice in the outer 10 km of Ceres would always remain frozen, although the frozen crust would be gravitationally unstable and likely overturn, melt, and re-freeze. Such repeatedly occurring movements of heavy masses on Ceres could have interfered with the evolution of its rotation period.

4 Conclusions

Numerical investigation of solar system data revealed that masses, radii, distances of celestial bodies from the Sun, orbital periods and rotation periods can be expressed as multiples of $\frac{e}{16}$ on the logarithmic number line, which proves that they are not a set of random numbers. Through application of a fractal scaling model, we set these numerical values in relation to proton resonances and correlated numerous features of celestial bodies with their oscillation properties. From this it can be concluded that the continued fraction representations with all nominators equal e are adequate and Müller's fractal model turned out to be a powerful tool to explain the fractal nature of the solar system. If some day in future, a further planet will be discovered in our solar system, it should be possible to derive analogously some of its features from its orbital parameters.

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