The Gravitational Field: A New Approach

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In this paper, we consider the Einstein field equations with the cosmological term. If we assume that this term is slightly varying, it induces a vacuum background field filling the space. In this case, inspection shows that the gravitational field is no longer represented by a pseudo-tensor, but appears on the right hand side of the field equations as a true tensor together with the bare mass tensor thus restoring the same conservation condition as obeyed by the Einstein tensor.

Introduction

Soon after his theory of General Relativity was published in 1916, Einstein rapidly turned to the unifying of the gravitational field with electromagnetism (which at that time was considered as the second fundamental field).

The quest for such an universal scheme ended in 1955 with the Einstein-Schrödinger theory (see for example [1]) definitely abandoned since as the quantum field theories gained the increasing successes and have been long substantiated by numerous experimental confirmations.

Basically, the unified principle adopted by the successive authors (Kaluza-Klein, Weyl, Eddington, et al.) relied either on extra dimensions, or on an extension of the Riemannian theory with additional space-time curvatures introduced to yield the electromagnetic field characteristics, and where the stress-energy tensor regarded as provisional, will be eventually absent [2, 3, 4].

Total geometrization of matter and electromagnetism was anyhow the original focus.

To understand this long period of research, one should remember that Einstein always claimed that the energy-momentum tensor (s) which can appear in the right hand side of his field equations, was "clumsy"; in short, he considered this form as an unsatisfactory solution which had to fit differently in his equations.

Einstein's argument is actually strongly supported by the following fact: while his tensor exhibits a *conceptually* conserved property, any corresponding stress energy-tensor *does not*, which leaves the theory with a major inconsistency.

When pure matter is the source, the problem has been "cured" by introducing the so-called "pseudo-tensor" that "conveniently" describes the gravitational field of this mass so that the four-momentum of both matter and its gravity field is conserved.

Unfortunately by essence this pseudo-tensor cannot appear in the field equations, and so the obvious physical defect emphasized by Einstein, still remains to-day as a stumbling block.

In this paper, we tackle this problems by proceeding as follows: in contrast to the previous theories, the energy-momentum tensor of the source is here strengthened, although we restrict our study to neutral massive flow.

In this respect, it is shown that the gravitational field of a massive body is no longer described by a *pseudo-tensor*, but appears as a *true tensor* in the field equations as it should be, in order to balance the conceptually conserved property of the Einstein tensor.

To achieve this goal we do:

- We first formulate the field equations with a massive source in density notation;
- We write the conservation law for the Einstein tensor density derived from the Bianchi identities, which cannot apply to the energy-momentum tensor density as a source;
- We then include a variable term that supersedes the so-called cosmological term Λg_{ab} in the field equations, still complying with the conservation property of the Einstein tensor density in GR;
- Under this latter assumption, we will then formally show that the gravity field of a massive source is no longer described by a vanishing *pseudo tensor* but it reduces to a true tensor describing a *persistent* vacuum background field resulting from the existence of the variable term.

1 The field equations in General Relativity

1.1 The tensor representation

In the General Theory of Relativity (GR), it is well known that by varying the action

$$\mathcal{S} = \boldsymbol{L}_E\, d^4x\,,$$

where the *Lagrangian density* is given by

$$\boldsymbol{L}_E = \sqrt{-g}\, G^{ab}\left(\left\{{e \atop ab}\right\}\left\{{d \atop de}\right\} + \left\{{d \atop ae}\right\}\left\{{e \atop bd}\right\}\right), \tag{1.1}$$

$$g = \det\|g_{ab}\| \tag{1.2}$$

one infers the *symmetric Einstein tensor*

$$G_{ab} = R_{ab} - \frac{1}{2}\, g_{ab}R, \tag{1.3}$$

where

$$R_{bc} = \partial_a \left\{ {a \atop bc} \right\} - \partial_c \left\{ {a \atop ba} \right\} + \left\{ {d \atop bc} \right\} \left\{ {a \atop da} \right\} - \left\{ {d \atop ba} \right\} \left\{ {a \atop dc} \right\} \tag{1.4}$$

is the *Ricci tensor* with its contraction R, the *curvature scalar*, while $\left\{ {e \atop ab} \right\}$ denote the Christoffel Symbols of the second kind.

The 10 *source free field equations* are

$$G_{ab} = 0. \tag{1.5}$$

The second rank Einstein tensor G_{ab} is symmetric and is only function of the metric tensor components g_{ab} and their first and second order derivatives.

The relation

$$\nabla_a G^a_b = 0 \tag{1.6}$$

is the conservation identities provided that the tensor G_{ab} has the form [5]

$$G_{ab} = k \left[R_{ab} - \frac{1}{2}\, g_{ab}(R - 2\Lambda) \right], \tag{1.7}$$

k is a constant, which is here taken 1, is usually named cosmological constant Λ.

When a source is present, the field equations become

$$G_{ab} = R_{ab} - \frac{1}{2}\, g_{ab}R - g_{ab}\Lambda = \varkappa\, T_{ab}, \tag{1.8}$$

where T_{ab} is the energy-momentum tensor of the source.

1.2 The tensor density representation

We first set

$$\boldsymbol{g}^{ab} = \sqrt{-g}\, g^{ab} \tag{1.9}$$

and the Einstein tensor density is

$$\boldsymbol{G}^{ab} = \sqrt{-g}\, G^{ab}, \quad \boldsymbol{G}^c_a = \sqrt{-g}\, G^c_a\,, \tag{1.10}$$

$$\boldsymbol{R}^{ab} = \sqrt{-g}\, R^{ab}. \tag{1.11}$$

In density notations, the field equations with the source (1.8) will read

$$\boldsymbol{G}^{ab} = \boldsymbol{R}^{ab} - \frac{1}{2}\, g^{ab}\boldsymbol{R} - g^{ab}\zeta = \varkappa\, \boldsymbol{T}^{ab}. \tag{1.12}$$

Here in place of the constant cosmological term Λ which should be here represented by $\Lambda\sqrt{-g}$, we have introduced a *scalar density* denoted as

$$\zeta = \Xi\sqrt{-g}. \tag{1.13}$$

Unlike Λ, the scalar Ξ is slightly variable and represents the *Lagrangian* characterizing a specific *vacuum background field* as will be shown below.

2 The conservation identities

2.1 Tensor version for the Einstein tensor

From the Bianchi identities applied to the Riemann tensor

$$R^{ci\cdot\cdot}_{\cdot\cdot bc\,;\,i} + R^{ci\cdot\cdot}_{\cdot\cdot ib\,;\,c} + R^{ci\cdot\cdot}_{\cdot\cdot ci\,;\,b} = 0 \tag{2.1}$$

we infer the conservation conditions which apply to the Einstein tensor without Ξ, and hereinafter denoted by

$${}^{\circ}G^a_b = R^a_b - \frac{1}{2}\, g^a_b R. \tag{2.2}$$

The Einstein tensor thus satisfies intrinsically the conservation law:

$$\nabla_a\, {}^{\circ}G^a_b = 0. \tag{2.3}$$

2.2 Tensor density version for the Einstein tensor

In the same way, we start with the Einstein tensor density without the cosmological term

$${}^{\circ}\boldsymbol{G}^{ab} = \boldsymbol{R}^{ab} - \frac{1}{2}\, g^{ab}\boldsymbol{R}. \tag{2.4}$$

With (2.3), let us write down

$$\nabla_a {}^{\circ}G^a_b = \partial_a {}^{\circ}G^a_b + \left\{ {a \atop ca} \right\} {}^{\circ}G^c_b - \left\{ {c \atop ba} \right\} {}^{\circ}G^a = \frac{\partial_a {}^{\circ}\boldsymbol{G}^a_b}{\sqrt{-g}} - \left\{ {c \atop ba} \right\} {}^{\circ}G^a_c = 0,$$

which is easily found to be

$$\frac{\partial_a {}^{\circ}\boldsymbol{G}^a_b}{\sqrt{-g}} - \frac{1}{2}\, {}^{\circ}G^{ea}\, \partial_b\, g_{ea} = 0 \tag{2.5}$$

using $dg_{ai} = -g_{ab}g_{ic}dg^{bc}$ and $dg^{ai} = -g^{ab}g^{ic}dg_{bc}$ the formula (2.5) can be also written as

$$\partial_a {}^{\circ}\boldsymbol{G}^a_b - \frac{1}{2}\, \boldsymbol{G}^{ea}\, \partial_b\, g_{ea} = 0. \tag{2.6}$$

The latter equation is the conservation condition for ${}^{\circ}\boldsymbol{G}^{ab}$ which is equivalent to (2.3).

2.3 Conservation of the energy-momentum tensor

2.3.1 Problem statement

Let us consider the energy-momentum tensor for neutral matter density ρ:

$$T_{ab} = \rho\, u_a\, u_b \tag{2.7}$$

as the right hand side of the field equations

$${}^{\circ}G_{ab} = R_{ab} - \frac{1}{2}\, g_{ab}R = \varkappa T_{ab}. \tag{2.8}$$

The conservation condition for this tensor are written

$$\nabla_a T^a_b = \frac{1}{\sqrt{-g}}\, \partial_a \boldsymbol{T}^a_b - \frac{1}{2}\, T^{ac}\, \partial_b\, g_{ac} = 0 \tag{2.9}$$

with the tensor density

$$\mathfrak{T}^a_b = \sqrt{-g}\, T^a_b. \tag{2.10}$$

However, across a given hypersurface dS_b, the integral

$$P^a = \int T^{ab} \sqrt{-g}\, dS_b \tag{2.11}$$

is conserved only when

$$\partial_a \mathfrak{T}^a_b = 0. \tag{2.12}$$

From (2.6) inspection still shows that

$$\partial_a \mathfrak{T}^a_b = \frac{1}{2}\, \mathfrak{T}^{cd}\, \partial_b\, g_{cd} \tag{2.13}$$

but here, unlike the Einstein tensor $^\circ G_{ab}$ which is *conceptually conserved* ($\nabla_a {}^\circ G^a_b = 0$), the conditions

$$\nabla_a T^a_b = 0$$

or

$$\partial_a \mathfrak{T}^a_b = 0$$

are thus never satisfied in a general coordinates system.

Therefore, the Einstein tensor $^\circ G_{ab}$ which *intrinsically* obeys a conservation condition, is related with a massive tensor $T_{ab}(\rho)$ which obviously *fails to satisfy the same requirement*:

$$^\circ G_{ab} = \varkappa T_{ab}. \tag{2.14}$$

As a matter of fact, a correct formulation would consist of explicitly writing down the mass density with its gravity field, i.e. with a pseudo-tensor $(t_{ab})_{field}$.

As is known, the name *pseudo-tensor* is chosen since this quantity can be transformed away by a suitable choice of coordinates.

Hence, we should write

$$G_{ab} = \varkappa \left[(T_{ab})_{matter} + (t_{ab})_{field}\right]. \tag{2.15}$$

This is classically interpreted by requiring that the **total** 4-momentum vector P^a of *matter* with its *gravitational field*

$$P^a = \left[(T^{ab})_{matter} + t^{ab})_{field}\right] \sqrt{-g}\, dS_b \tag{2.16}$$

must be together conserved*

*Some authors [8] state that integrating $\nabla_k T^k_i = 0$ yields a conservation law for a vector $P^a = T^{ab} K_b$ when the metric admits a Killing vector $\boldsymbol{K}$: $P^a_{;a} = T^{ab}_{;a} K_b + T^{ab} K_{b;a}$ and since T^{ab} is symmetric, we have for the Lie derivative $K_{b;a} = \frac{1}{2} \boldsymbol{L}_K \boldsymbol{g}_{ab} = 0$, then $P^a_{;a} = 0$.

2.3.2 The gravity pseudo-tensor

In order to follow this way, Landau and Lifshitz [6] started from the unsuitable tensor equation (2.9)

$$\nabla_k T^k_i = \frac{1}{\sqrt{-g}}\, \partial_k \mathfrak{T}^k_i - \frac{1}{2}\, T^{kl}\, \partial_i\, g_{kl} = 0.$$

They thus consider a special choice of a set of the coordinates which cancels out all first derivatives of the g_{ik} at a given 4-space-time point.

In this system, the energy-momentum tensor expression is given by

$$T^{ik} = \frac{1}{2\varkappa}\, \partial_e(-g)^{-1} \left[\partial_d\,(-g)\left(g^{ik} g^{ed} - g^{ie} g^{kd}\right)\right]. \tag{2.17}$$

As $\left\{ {i \atop ke} \right\}$ are postulated to be zero at the considered point, we may extract the factor $(-g)^{-1}$ from the derivative in the latter equation, so

$$(-g)\, T^{ik} = \partial_e \boldsymbol{H}^{ike} = \frac{1}{2\varkappa}\, \partial_e \left(\partial_d \boldsymbol{H}^{iked}\right).$$

The quantity

$$\boldsymbol{H}^{iked} = (-g)\left(g^{ik} g^{ed} - g^{ie} g^{kd}\right) \tag{2.18}$$

can be regarded as a "double tensor density" and is often referred to, as the "superpotential of Landau-Lifshitz" [7]. Now, in any other arbitrary system, generally

$$\partial_e \boldsymbol{H}^{ike} - (-g)\, T^{ik} \neq 0,$$

and so, we will have to bring a small tensor correction t^{ik}_{LL} (Landau-Lifshitz pseudo-tensor) which is accepted as representing the gravitational field of matter:

$$\partial_e \boldsymbol{H}^{ike} = (-g)\left(T^{ik} + t^{ik}_{LL}\right).$$

This equation implies the condition

$$\partial_k \left[(-g)\left(T^{ik} + t^{ik}_{LL}\right)\right] = 0, \tag{2.19}$$

which is the conservation law for the classical total four-momentum vector density of both matter and gravitational field written as

$$\boldsymbol{P}^i = \int \left[(-g)\left(T^{ik} + t^{ik}_{LL}\right)\right] dS_k, \tag{2.20}$$

(compare with (2.11)).

After a tedious calculation, the final form of the symmetric tensor t^{ik}_{LL} as a function of the g_{ik}, is found to be

$$\begin{aligned}(-g)\, t^{ik}_{LL} = \frac{1}{2\varkappa}\Big[&\boldsymbol{g}^{ik}_{,l}\, \boldsymbol{g}^{lm}_{,m} - \boldsymbol{g}^{il}_{,l}\, \boldsymbol{g}^{km}_{,m} + \frac{1}{2}\, g^{ik} g_{lm}\, \boldsymbol{g}^{ln}_{,p}\, \boldsymbol{g}^{pm}_{,n} - \\ &- \left(g^{il} g_{mn}\, \boldsymbol{g}^{kn}_{,p}\, \boldsymbol{g}^{mp}_{,l} + g^{kl} g_{mn}\, \boldsymbol{g}^{in}_{,p}\, \boldsymbol{g}^{mp}_{,l}\right) + g_{lm}\, g^{np} \boldsymbol{g}^{il}_{,n}\, \boldsymbol{g}^{km}_{,p} + \\ &+ \frac{1}{8}\left(2 g^{il} g^{km} - g^{ik} g^{lm}\right)\left(2 g_{np}\, g_{qr} - g_{pq}\, g_{nr}\right) \boldsymbol{g}^{nr}_{,l}\, \boldsymbol{g}^{pq}_{,m}\Big]. \end{aligned} \tag{2.21}$$

Therefore, the Einstein field equations can be eventually written in the form:

$$\boldsymbol{H}^{iked}_{\dots,kd} = 2\varkappa\,(-g)\left(T^{ie} + t^{ie}_{LL}\right). \qquad (2.22)$$

Unfortunately, the quantity t^{ie}_{LL} which now appears on the right hand side of the field equations as it should be, is not a *true tensor*.

Hence, we are once more faced with a contradiction: the left hand side of the field equations for a massive source is a true tensor, while the right hand side is not, which reveals a major inconsistency within the theory.

2.4 Introduction of a background field tensor

Let us now try to remove this ambiguity.

We start by writing the global energy-momentum tensor density of the massive source splitting up bare matter and pure field:

$$\boldsymbol{T}^a_b = (\boldsymbol{T}^a_b)_{matter} + (\boldsymbol{t}^a_b)_{field}. \qquad (2.23)$$

The field tensor density $(\boldsymbol{t}^a_b)_{field}$ is in turn composed of two parts: *gravity field + vacuum background field*

$$(\boldsymbol{t}^a_b)_{field} = (\boldsymbol{t}^a_b)_{gravity} + (\boldsymbol{t}^a_b)_{background\,field} \qquad (2.24)$$

with

$$(\boldsymbol{t}_{ab})_{background\,field} = \frac{\varsigma}{2\varkappa}\,g_{ab} = \frac{\Xi\sqrt{-g}}{2\varkappa}\,g_{ab}\,. \qquad (2.25)$$

According to the standard theory, we next re-formulate the field equations with a *bare* massive source

$$\boldsymbol{G}^{ab} = \boldsymbol{R}^{ab} - \frac{1}{2}\,g^{ab}\boldsymbol{R} - g^{ab}\varsigma = \varkappa(\boldsymbol{T}^{ab})_{matter} \qquad (2.26)$$

under the form

$$\boldsymbol{G}^{ab} = \boldsymbol{R}^{ab} - \frac{1}{2}\,g^{ab}\boldsymbol{R} = \varkappa(\boldsymbol{T}^{ab})_{matter} + g^{ab}\varsigma. \qquad (2.27)$$

3 Expliciting the field equations in density notation

3.1 Taking account of the Lagrangian Ξ

Reverting to (2.13), we now write for the *bare* matter tensor density

$$\partial_a(\boldsymbol{T}^a_b)_{matter} = \frac{1}{2}\,(\boldsymbol{T}^{cd})_{matter}\,\partial_b\,g_{cd}. \qquad (3.1)$$

Inspection then shows that

$$R_{il}\,d\boldsymbol{g}^{il} = \sqrt{-g}\left[-R^{ie} + \frac{1}{2}\,g^{ie}R\right]dg_{ie} = = -\varkappa(\boldsymbol{T}^{ie})_{matter}\,dg_{ie}. \qquad (3.2)$$

Taking now into account the Lagrangian formulation for R_{il}, which is

$$R_{il} = \frac{\delta \boldsymbol{L}_E}{\partial \boldsymbol{g}^{il}} = \partial_k\frac{\partial \boldsymbol{L}_E}{\partial(\partial_k \boldsymbol{g}^{il})} - \frac{\partial \boldsymbol{L}_E}{\partial \boldsymbol{g}^{il}}, \qquad (3.3)$$

we obtain

$$-\varkappa(\boldsymbol{T}^{il})_{matter}\,dg_{il} = \partial_k\frac{\partial \boldsymbol{L}_E}{\partial(\partial_k \boldsymbol{g}^{il})} - \frac{\partial \boldsymbol{L}_E}{\partial \boldsymbol{g}^{il}}\,d\boldsymbol{g}^{il} = = \partial_k\frac{\partial \boldsymbol{L}_E\,d\boldsymbol{g}^{il}}{\partial(\partial_k \boldsymbol{g}^{il})} - \partial \boldsymbol{L}_E,$$

that is

$$-\varkappa(\boldsymbol{T}^{il})_{matter}\,\partial_m g_{il} = \partial_k\left[\frac{\partial \boldsymbol{L}_E\partial_m(\partial \boldsymbol{g}^{il})}{\partial(\partial_k \boldsymbol{g}^{il})} - \delta^k_m\,\boldsymbol{L}_E\right] = = 2\varkappa\,\partial_k(\boldsymbol{t}^k_m)_{field}, \qquad (3.4)$$

where $(\boldsymbol{t}^k_m)_{field}$ denotes the field tensor density extracted from

$$2\varkappa(\boldsymbol{t}^k_m)_{field} = \frac{\partial \boldsymbol{L}_E\partial_m(\partial \boldsymbol{g}^{il})}{\partial(\partial_k \boldsymbol{g}^{il})} - \delta^k_m\boldsymbol{L}_E \qquad (3.5)$$

so, that we have the explicit canonical form

$$(\boldsymbol{t}^k_m)_{field} = \frac{1}{2\varkappa}\left[\frac{\partial \boldsymbol{L}_E\partial_m(\partial \boldsymbol{g}^{il})}{\partial(\partial_k \boldsymbol{g}^{il})} - \delta^k_m\boldsymbol{L}_E\right] \qquad (3.6)$$

and where

$$\partial_k(\boldsymbol{T}^k_i)_{matter} = \frac{1}{2}\,(\boldsymbol{T}^{ek})_{matter}\,\partial_k g_{ei} = -\partial_k(\boldsymbol{t}^k_i)_{field}.$$

that is, the required conservation relation

$$\partial_k\left[(\boldsymbol{T}^k_i)_{matter} + (\boldsymbol{t}^k_i)_{field}\right] = 0. \qquad (3.7)$$

Then, re-instating the term ζ according to (2.24) and (2.25), the gravitational field tensor density now reads:

$$(\boldsymbol{t}^k_m)_{gravity} = \frac{1}{2\varkappa}\left[\frac{\partial \boldsymbol{L}_E\partial_m(\partial \boldsymbol{g}^{il})}{\partial(\partial_k \boldsymbol{g}^{il})}\right] - \delta^k_m(\boldsymbol{L}_E - \zeta). \qquad (3.8)$$

The presence of the scalar density ζ characterizing the background field is here of central importance, as it means that $(\boldsymbol{t}^k_m)_{gravity}$ can never be zero in contrast to the classical theory, and as a result, it constitutes a *true tensor*. Such a gravity field never completely cancels out, but far from its matter source, it sharply decreases down to the level of the background field described by the tensor density $(\boldsymbol{t}^{ab})_{background\,field}$.

In addition, we clearly see that ζ represents the *lagrangian density* characterizing the background field, thus lending support to our initial hypothesis regarding the lagrangian Ξ.

In this picture, the vacuum is permanently filled with this homogeneous background energy field ensuring a smooth continuity with the gravitational field of a neighbouring mass.

3.2 Classical formulation

When the term Ξ is kept constant like the cosmological term Λ, the tensor density (3.8) reduces to

$$(\boldsymbol{t}^k_m)_{pseudogravity} = \frac{1}{2\varkappa}\left[\frac{\partial \boldsymbol{L}_E\partial_m(\partial \boldsymbol{g}^{il})}{\partial(\partial_k \boldsymbol{g}^{il})} - \delta^k_m\boldsymbol{L}_E\right], \qquad (3.9)$$

which is just the classical *gravity pseudo-tensor density* that may now vanish in a given space-time point.

In this case, expressed with the explicit form of the Lagrangian density $\boldsymbol{L}_E$ written in (1.1), the expression (3.9) becomes:

$$(\boldsymbol{t}^k_m)_{pseudogravity} = \frac{1}{2\varkappa}\left[\left\{{k \atop i\,l}\right\}\partial_m \boldsymbol{g}^{il} - \left\{{i \atop i\,l}\right\}\partial_m \boldsymbol{g}^{lk} - \delta^k_m \boldsymbol{L}_E\right]. \quad (3.10)$$

This is the *mixed Einstein-Dirac pseudo-tensor density* [9] which is not symmetric on k and m, and is therefore not suitable for basing a definition of angular momentum on.

3.3 Field equations

The field equations with a massive source, which are

$$\boldsymbol{G}^{ab} = \boldsymbol{R}^{ab} - \frac{1}{2}\, g^{ab}\boldsymbol{R} - g^{ab}\zeta = \varkappa(\boldsymbol{T}^{ab})_{matter}, \quad (3.11)$$

may be now eventually re-written

$$^{\circ}\boldsymbol{G}^{ab} = \boldsymbol{R}^{ab} - \frac{1}{2}\, g^{ab}\boldsymbol{R} = \varkappa\left[(\boldsymbol{T}^{ab})_{matter} + (\boldsymbol{t}^{ab})_{gravity}\right] \quad (3.12)$$

with the explicit appearance of the gravity field as defined in (3.8) and which is now represented by a *true* tensor density.

Like we emphasized above, far from the mass, the "source free" field equations should always retain a non zero right hand side

$$^{\circ}\boldsymbol{G}^{ab} = \boldsymbol{R}^{ab} - \frac{1}{2}\, g^{ab}\boldsymbol{R} = \varkappa(\boldsymbol{t}^{ab})_{background\,field}, \quad (3.13)$$

which are the analogue of (1.7):

$$\boldsymbol{G}^{ab} = \boldsymbol{R}^{ab} - \frac{1}{2}\, g^{ab}\boldsymbol{R} - g^{ab}\zeta = 0. \quad (3.14)$$

In this case, the conservation law applied to the right hand side of the tensor field equations is straightforward:

$$\nabla_a (t^a_b)_{background\,field} = \nabla_a\left(\frac{\Xi}{2\varkappa}\,\delta^a_b\right) = 0, \quad (3.15)$$

from which readily follows

$$\partial_a (\boldsymbol{t}^a_b)_{background\,field} = \partial_a\left(\frac{\zeta}{2\varkappa}\,\delta^a_b\right) = 0. \quad (3.16)$$

3.4 Physical description

We would like now to give a simple but instructive picture of the situation where a static mass is placed in the vacuum background energy field. Let us write the energy-momentum tensor for matter and its gravitational field as in (3.12):

$$T_{ab} = (\rho\, u_a u_b)_{matter} + (t_{ab})_{gravity}. \quad (3.17)$$

In virtue of the principle of equivalence, any *bare mass* of volume *V together with its gravitational field*, can be expressed through the time component of a 4-momentum P^a according to

$$P^0 = \int \left(T^1_1 + T^2_2 + T^3_3 - T^0_0\right)\sqrt{-g}\; dV, \quad (3.18)$$

where T^a_a are the skew components of the energy-momentum tensor (3.17), which implicitly contains the gravity field [10].

Now, we formulate (3.18) under the equivalent form:

$$P^0 = P_0 = \int \left(\boldsymbol{T}^1_1 + \boldsymbol{T}^2_2 + \boldsymbol{T}^3_3 - \boldsymbol{T}^0_0\right) dV. \quad (3.19)$$

In the immediate vicinity of the mass, it is easy, to show that generalizing (3.19) leads to the 4-momentum vector that includes the right hand side of (3.12):

$$P_a = \int \left[(\boldsymbol{T}^b_a)_{matter} + (\boldsymbol{t}^b_a)_{gravity}\right] dS_b. \quad (3.20)$$

Far from the source, we have obviously

$$(P_a)_{background\,field} = \int \left[(\boldsymbol{t}^b_a)_{background\,field}\right] dS_b, \quad (3.21)$$

where $(\boldsymbol{t}^b_a)_{background\,field}$ is a true tensor density, and the conservation law applied to P^a holds for all configurations, in accordance with (3.7) and (3.16).

4 Conclusions and outlook

In this short paper, we have sketched here a possible way out of the gravitational field pseudo-tensor.

From the beginning of General Relativity, the cosmological constant Λ has played an unsavory role. Einstein included this constant in his theory, because he wanted to have a cosmological model of the Universe which he wrongly thought static.

But to-day, a cosmological term seems to be badly needed to explain some astronomical observed clues, within the basic dynamical expanding model of Robertson-Walker [11], even though its occurrence was never clearly explained.

However, there is no reason *à priori* to consider this cosmological term as constant everywhere.

In this respect, the background field hypothesis is rewarding in terms of several physical advantages:

- The ill-defined gravitational pseudo-tensor is now a true tensor, and it appears explicitly in the field equations with a massive source;
- The background persistent homogeneous energy field is then formally shown to be a consequence of the above derivation and it is actually regarded as the (sharply decreasing) continuation of any mass gravity field tensor;
- The inferred global energy-momentum tensor intrinsically satisfies the conservation law as well as the background field alone in the source free field equations, without introducing any other arbitrary ingredients or modification of the General Theory of Relativity.

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