ΔI=2 Nuclear Staggering in Superdeformed Rotational Bands

Madiha D. Okasha

Physics Department, Faculty of Science (Girls College), Al-Azhar University, Cairo, Egypt. E-mail: mady200315@yahoo.com

A four parameters model including collective rotational energies to fourth order is applied to reproduce the ΔI=2 staggering in transition energies in four selected super deformed rotational bands, namely, ^{148}Gd (SD6), ^{194}Hg (SD1, SD2, SD3). The model parameters and the spin of the bandhead have been extracted assuming various values to the lowest spin of the bandhead at nearest integer, in order to obtain a minimum root mean square deviation between calculated and the experimental transition energies. This allows us to suggest the spin values for the energy levels which are experimentally unknown. For each band a staggering parameter represent the deviation of the transition energies from a smooth reference has been determined by calculating the fourth order derivative of the transition energies at a given spin. The staggering parameter contains five consecutive transition energies which is denoted here as the five-point formula. In order to get information about the dynamical moment of inertia, the two point formula which contains only two consecutive transition energies has been also considered. The dynamical moment of inertia decreasing with increasing rotational frequency for $A \sim 150$, while increasing for $A \sim 190$ mass regions.

1 Introduction

The observation [1] of a very regular pattern of closely spaced γ-transitions in the spectrum of ^{152}Dy, which assigned to a rotational cascade between levels of spin ranging from $60\hbar$ to $24\hbar$ and excitation energy varying from ∼ 30 to 12 MeV may adopt a superdeformed (SD) at high angular momentum. The moment of inertia of the associated band was found to be close to that of a rigid rotor with a 2:1 axis rotation. Now more than 350 settled superdeformed rotational bands (SDRB's), in more than 100 nuclei have been studied in nuclei of mass A ∼ 30, 60, 80, 130, 150, 160, 190 [2, 3]. Such nuclei are associated with extremely large quadrupole $\beta_2 = 0.6$ in the mass A ∼ 150 region and $\beta_2 = 0.47$ in the mass A ∼ 190 region. Hence, they are expected to have a different structures to normal deformed nuclei.

Unfortunately, despite the rather large amount of experimental information on SDRB's, there are still a number of very interesting properties, which have not yet been measured. For example, the spin, parity and excitation energy relative to the ground state of the SD bands. The difficulty lies with observing the very weak discrete transitions which link SD levels with levels of normal deformation (ND). Several related approaches to assign the spins of SDRB's in terms of their observed γ-ray transition energies were proposed [4–10]. For all approaches an extrapolation fitting procedure was used.

It was found that some SDRB's show an unexpected ΔI=2 staggering in their γ-ray transition energies [11–20]. The SD energy levels are consequently separated into two sequences with spin values I, I+4, I+8, …and I+2, I+6, I+10, … respectively. The magnitude of splitting is found to be of some hundred eV to a few keV. Several theoretical explanation have been made. One of the earliest ones being based on the assumption of a C4 symmetry [21]. Also it was suggested that [22] the staggering is associated with the alignment of the total angular momentum along the axis perpendicular to the long deformation axis of a prolate nucleus. The staggering phenomenon was interpreted also as due to the mixing of a series of rotational bands differ by ΔI=4 [23] or arise from the mixing of two bands near yrast line [24] or by proposing phenomenological model [25, 26]. The main purpose of the present paper is to predict the spins of the bandhead of four SDRB's in A ∼ 150 and A ∼ 190 mass regions, and to examine the ΔI=2 staggering and the properties of the dynamical moments of inertia in framework of proposed four parameters collective rotational model.

2 Nuclear SDRB's in Framework of Four Parameters Collective Rotational Model

On the basis of collective rotational model [27] in adiabatic approximation, the rotational energy E for an axial symmetric nucleus can be expanded in powers of I(I+1), where I is the spin of state:

$$E(I) = A[I(I+1)]+B[I(I+1)]^2+C[I(I+1)]^3+D[I(I+1)]^4 \quad (1)$$

where A is the well-known rotational parameter for sufficiently small values of I and B, C, D are the corresponding higher order parameters. In the view of the above mentioned, it seems that the ground state energy bands of deformed even-even nucleus have quantum number K=0 (K is the projection of I along the symmetry axis), together with even parity and angular momentum. In SD nuclei, the experimentally determined quantities are the gamma ray transition energies between levels differing by two units of angular momentum, then we could obtain the reference transition energy

$$E_\gamma^{ref} = E(I) - E(I-2) \quad (2)$$

Table 1: The calculated adopted best parameters and the bandhead spins for the selected SD nuclei to investigate the $\Delta I = 2$ staggering.

SD-Band	A (keV)	B (keV) $\times 10^4$	C (keV) $\times 10^8$	D (keV) $\times 10^{12}$	I ($\hbar$)	E_γ (MeV)
^{148}Gd (SD-6)	4.33360	1.17108	0.001135	-0.04435	41	802.200
^{194}Hg (SD-1)	5.40524	-1.86747	0.000338	-0.00213	8	211.700
^{194}Hg (SD-2)	5.24253	-1.577380	0.003991	-0.00269	8	200.790
^{194}Hg (SD-3)	5.21638	-1.48121	0.0006129	-0.006501	9	222.000

$$E_\gamma^{ref} = 2(2I-1)[A+2(I^2-I+1)B + (3I^4-6I^3+13I^2-10I+4)C + 4(I^6-3I^5+10I^4-15I^3+15I^2-8I+2)D]. \tag{3}$$

The rotational frequency is not directly measurable but it is related to the observed excitation energy E.

Let us define the angular velocity of nuclear rotation as the derivative of the energy E with respect to the angular momentum I in analogy with classical mechanics. Instead of I it is convenient to use the quantum mechanical analogies $\sqrt{(I(I+1))}$

$$\hbar\omega = \frac{dE}{d\left(\sqrt{(I(I+1))}\right)} \tag{4}$$

$$= 2A[I(I+1)]^{1/2} + B[I(I+1)]^{3/2} + 6C[I(I+1)]^{5/2} + 8D[I(I+1)]^{7/2}. \tag{5}$$

The rotational energy spectra can be discussed in terms of the dynamical moment of inertia calculated from the reciprocal second order derivative:

$$\frac{J^{(2)}}{\hbar^2} = \left(\frac{d^2E}{d\left(\sqrt{I(I+1)}\right)^2}\right)^{-1} \tag{6}$$

$$= \left([2A + 12B[I(I+1)] + 30C[I(I+1)]^2 + 56D[I(I+1)]^3]\right)^{-1}. \tag{7}$$

The experimental $\hbar\omega$ and $J^{(2)}$ for the SDRB's are usually extracted from the observed energies of gamma transition between two consecutive transitions within the band from the following formulae:

$$\hbar\omega = [E_\gamma(I) + E_\gamma(I+2)]/4, \tag{8}$$

$$J^{(2)} = \frac{4}{E_\gamma(I+2) - E_\gamma(I)}. \tag{9}$$

We notice that $\hbar\omega$ and $J^{(2)}$ does not depend on the knowledge of the spin I, but only on the measured gamma ray energies.

In order to see the variation in the experimental transition energies $E_\gamma(I)$ in a band, we subtract from them a calculated reference. The corresponding five-point formula is the fourth order derivative of the transition energies at a given spin

$$\Delta^4 E_\gamma(I) = \frac{1}{16}[E_\gamma(I+4) - 4E_\gamma(I+2) + 6E_\gamma(I) - 4E_\gamma(I-2) + E_\gamma(I-4)]. \tag{10}$$

One can easily see that $\Delta^4 E_\gamma(I)$ vanishes if our model contains two parameters A and B, due to the fact that the five-point formula is a normalized discrete approximation of the fourth derivatives of the function $E_\gamma(I)$. We define the staggering parameter $S^{(4)}(I)$ as the difference between the experimental transition energies and the auxiliary reference.

$$S^{(4)}(I) = 2^4[\Delta^4 E_\gamma^{exp}(I) - \Delta^4 E_\gamma^{ref}(I)] \tag{11}$$

3 Numerical Calculations and Discussions

The transition energies $E_\gamma(I)$ of equation (2) is used to fit the observed transition energies for our selected SDRB's with A, B, C, D and spin value of the bandhead I_0 as free parameters. I_0 is taken to the nearest integer of the fitting, the another fit is made to determine A, B, C and D by using a simulated search program [9] in order to obtain a minimum root mean square deviation

$$\chi = \left[\frac{1}{N}\sum_{i=1}^{N}\left(\frac{E_\gamma^{exp}(I) - E_\gamma^{Cal}(I)}{\triangle E_\gamma^{exp}(I)}\right)\right]^{1/2}$$

of the calculated transition energies E_γ^{Cal} from the measured energies E_γ^{exp}, where N is the number of data points considered, and ΔE_γ^{exp} is the uncertainty of the γ-transition energies. The experimental data for transition energies are taken from ref. [2]. Table (1) summarize the model parameters A, B, C, D and the correct bandhead lowest level spin I_0 and also the lowest γ- transition energies $E_\gamma(I_0 + 2 \rightarrow I_0)$ for our 4 SDRB's.

To investigate the appearance of staggering effects in the γ-transition energies of our selected SDRB's, for each band, the deviation of the γ-transition energies $E_\gamma(I)$ from a smooth reference (rigid rotor) was determined by calculating fourth-derivatives of $E_\gamma(I)$ (d^4E_γ/dI^4) at a given spin I by using the finite difference approximation. The resulting staggering parameters values against spin are presented in Figure (1). A significant ΔI=2 staggering was observed. At high spins the

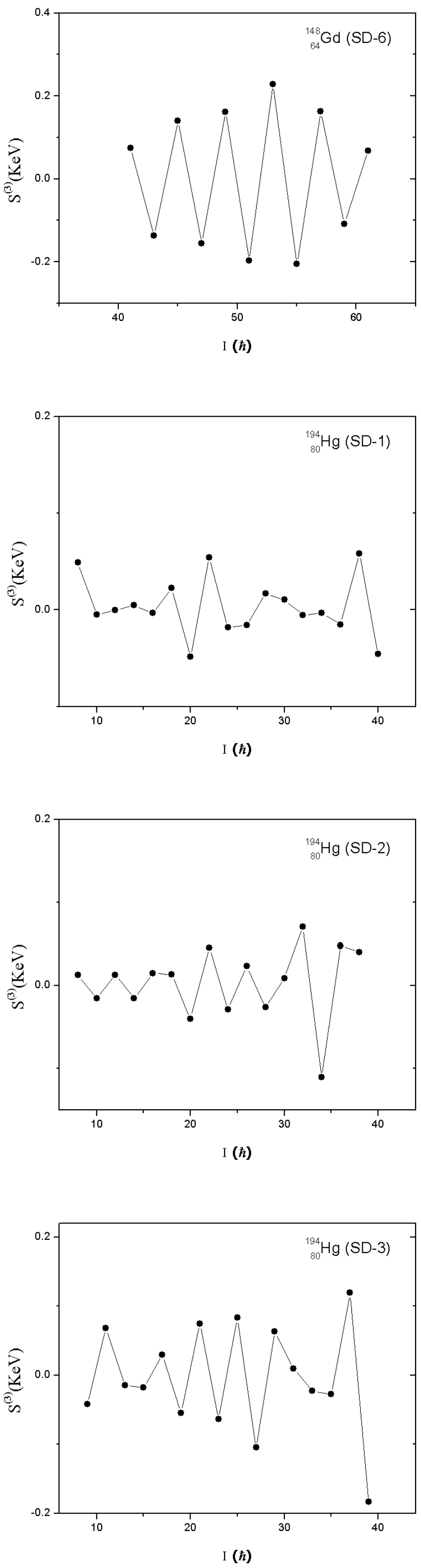

Fig. 1: The calculated $\Delta I = 2$ staggering parameters $S^{(4)}(I)$ obtained by five-point formula versus nuclear spin I for the SDRB's in ^{148}Gd and ^{194}Hg.

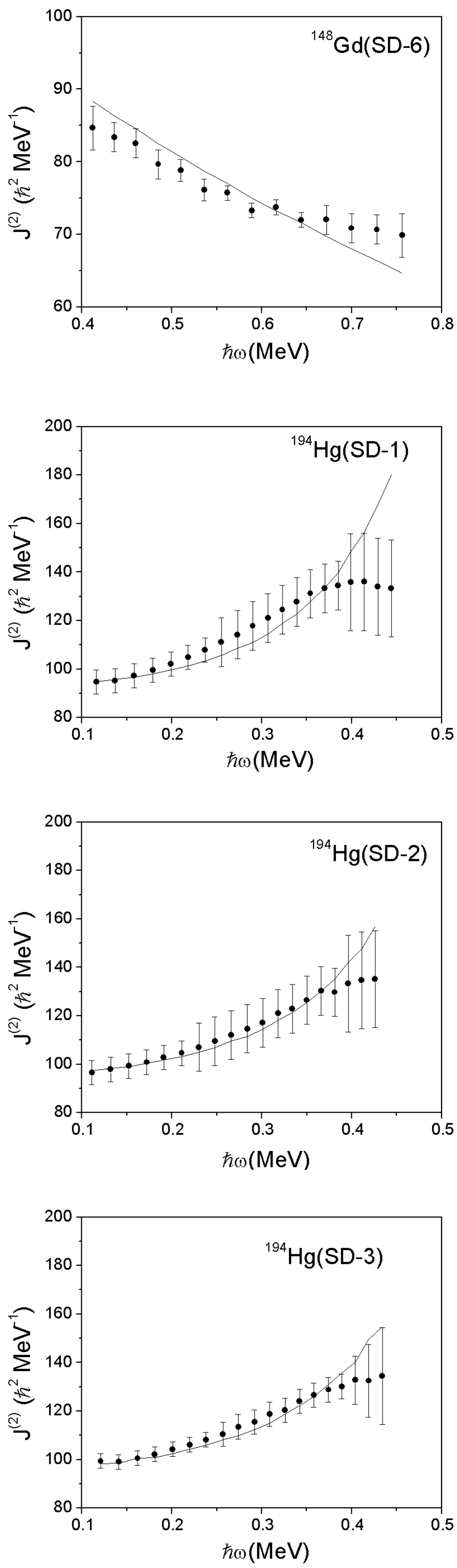

Fig. 2: The dynamical moment of inertia $J^{(2)}$ plotted as a function of the rotational frequency $\hbar\omega$ for the SDRB's in ^{148}Gd and ^{194}Hg nuclei. The solid curve represents the calculated results extracted from the proposed four parameters model. The experimental solid circles with error bars are presented for comparison.

ΔI=2 rotational band is perturbed and two ΔI=4 rotational sequences emerge with an energy splitting of some hundred eV. That is the E2 cascades obtained from our model exhibit for spins I, I+4, I+8, ... and I+2, I+6, I+10, ... staggering behavior.

The systematic behavior of the dynamic moment of inertia $J^{(2)}$ is very useful to understand the properties and structure of SDRB's. Our best fitted parameters were used to calculate the theoretical $J^{(2)}$. The evolution of the dynamical moment of inertia $J^{(2)}$ against rotational frequency $\hbar\omega$ are illustrated in Figure (2). It is seen that the agreement between the calculated (solid lines) and the values extracted from the observed data (closed circles) are excellent. For A∼190, the SDRB's have nearly the same $J^{(2)}$ which typically increase smoothly as rotational frequency increases due to gradual angular momentum alignment of a pair of nucleons occupying specific high-N intruder orbitals and the disappearance of pairing correlations. For A∼150 a smooth decrease of $J^{(2)}$ with increasing $\hbar\omega$ is reproduced well.

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