Addendum to "Phenomenological Derivation of the Schrödinger Equation"

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This addendum to the article [1] is crucial for understanding how the complex effective action, despite its derivation based on classical concepts, prevents quantal particles to move along extreme action trajectories. The reason relates to homogenous, isotropic and unpredictable impulses received from the environment. These random impulses allied to natural obedience to the dynamical principle imply that such particles are permanently and randomly passing from an extreme action trajectory to another; all of them belonging to the ensemble given by the stochastic Hamilton-Jacobi-Bohm equation. Also, to correct a wrong interpretation concerning energy conservation, it is shown that the remaining energy due to these permanent particle-medium interactions (absorption-emission phenomena) is the so-called quantum potential.

1 Introduction

The central subject of the article [1] is: Quantal particles (such as electrons), due to its interactions with the environment, move in accordance with the complex effective action

$$S_{eff} = S + i\,\frac{\hbar}{2}\,\ln P \tag{1}$$

which was obtained following the classical Hamilton's dynamical principle but considering the motion as a whole, that is, taking averages. The resulting canonical equations coincide with those extracted from the Schrödinger equation writing $\psi = \sqrt{P}\exp(iS/\hbar)$, namely:

$$\frac{\partial S}{\partial t} + \frac{\nabla S\cdot\nabla S}{2m} + V + Q = 0 \tag{2}$$

and

$$\frac{\partial P}{\partial t} + \nabla\cdot\left(P\frac{\nabla S}{m}\right) = 0\,, \tag{3}$$

where

$$Q = \frac{\hbar^2}{8m}\frac{\nabla P\cdot\nabla P}{P^2} - \frac{\hbar^2}{4m}\frac{\nabla\cdot\nabla P}{P} \tag{4}$$

is the quantum potential which, visibly, is the remaining energy of two distinct concurrent phenomena.

The main motivation for writing this addendum concerns the result

$$\int P\left(i\,\frac{\hbar}{2}\frac{1}{P}\frac{\partial P}{\partial t}\right)d^3r = 0 \tag{5}$$

which is not the expression of energy conservation, as argued in connection with Eg. 23 of the article. In true, the null value of this average means that the involved energy (the enclosed quantity) does not remain in the particle; it is radiation, as will be shown. In doing this, it is necessary to explain how Q — as an energy resulting from the particle-medium interactions — agrees with the energy conservation required by the so-called quantum equilibrium.

Also, in the mentioned article the meaning of the effective action (1) is not so clear. It was derived supposing a particle over a possible trajectory; what, in view of the results, must be true. On the other hand, a continuous trajectory of elementary particles is an experimentally discredited concept. So, there must be a link between these two opposing points of view. In true, there is, as will be seen. Indeed, it will be shown that quantal particles occupies, sequentially and instantly, just one point over different trajectories which are randomically chosen in the ensemble (2). This means that quantal particles don't move along extreme action trajectories but occupy trajectories (permited by the dynamical principle) just for a moment.

The interacting medium — primarily responsible for quantum effects — is the zero-point field (ZPF) which, according to the classical description of the Casimir's experiment, is viewed as a homogeneous and isotropic distribution of electromagnetic waves pervading all space. As the phases of these waves are randomically distributed in the range $[0, 2\pi]$, then electrical charged particles (balanced or not) are permanently receiving unpredictable impulses. This has two main consequences: First, the accelerated charged particles radiate all the absorbed energy. Second, the unpredictable impulses prevent quantal particles to follow predictable paths. Even so, the overall motion obeys the Hamilton's principle which is founded on trajectories. How can all this happen?

2 The quantum potential and the ensemble of virtual trajectories

The answer to the above question lay in the fact that the natural behavior of any moving particle, at any time, is obeying the classical dynamical principle. This must be interpreted as follows: In the absence of random forces, they move along extreme action trajectories. However, in the case of particles which are significantly affected by the ZPF the situation is drastically modified. Indeed, homogeneous, isotropic and random forces (including beck reaction forces) are not part of the traditional classical description of the motion.

Here, it will be proved that the quantal motion occurs as follows: Immediately before any particle-field random interaction the particle is over a given trajectory (obedience to the dynamical principle), but upon receiving an unpredictable impulse it is withdrawing from this trajectory to an unpredictable place. Again, in the new position it continues obeying the dynamical principle; that is, the particle is over another trajectory. As this is a permanent process, then the particle occupies these possible trajectories only instantly (virtual trajectories). In a sense, we can say that the unpredictable impulse has created initial conditions (arbitrary) for a new trajectory.

In the light of the foregoing, at each position actually occupied by the particle pass an infinite number of such virtual trajectories. This assumption is in agreement with the following facts: First, Eq. (2) represents an ensemble of unpredictable trajectories; $P(\mathbf{x}, t)$ — preserving its uniqueness — can take any value at $\mathbf{x}$. Moreover, ∇P is not deterministic. Second, energy and momentum in quantum mechanics are independent of coordinates. This means that everywhere there are equivalent ensembles of partial derivatives $\partial S/\partial t$ and ∇S — requiring continuous virtual functions S — which on average give the corresponding observed quantities. This statement implies the same uncertainty everywhere (non locality). Thirty, Probability density, classically, is defined over trajectories; it is canonically conjugate to the action function S (this remains valid in the equations above). Over extreme action trajectories $\partial P/\partial t = 0$ (we know where the particle is at the time t). Therefore, if $\partial P/\partial t \neq 0$, then it means that the particle was "banned" from its trajectory.

To formally prove that the trajectories represented by the virtual ensemble (2) are instantly visited by the particle, it is necessary finding a valid expression which leads to the idea that such trajectories (or momenta ∇S) are randomly chosen (or induced) where the particle is. This is better made after knowing the meaning of the quantum potential.

If a moving particle is not actuated by random forces, then, given the potential V and the initial conditions, we can predict its extreme action trajectory. However, the presence of random forces — exactly like that found in the ZPF — modify this classical way to see the motion. This rupture relates to the fact that now there is only a probability of finding the particle at a given position at the time t.

Whenever a particle is removed from a given position by random forces, the probability of find it there is diminished. Consequently, as probability is a conserved quantity, this decrease of probability leads to the emergence of an outgoing compensatory probability current. Formally, following standard techniques and considering the ZPF properties, at each position there is a diffusion of probability density currents $(P\mathbf{v})$, in such a way that

$$\frac{\partial P}{\partial t} + \nabla \cdot (P\mathbf{v}) = 0\,. \tag{6}$$

In true, $P\mathbf{v}$ represents all possible local outflows of matter whose velocities $\mathbf{v}$ have the directions of the vectors ∇P. Therefore, all currents obey

$$P\mathbf{v} = \alpha \nabla P, \tag{7}$$

where α is a proportionality factor, to be determined.

Being the matter-field interaction conservative, then there is no net momentum transfer to the particle. This implies that the average probability density current is zero, i.e.

$$\int P\,(P\mathbf{v})\; d^3r = \int P\,(\alpha \nabla P)\; d^3r = 0\,. \tag{8}$$

Integrating the second form by parts and considering that $P \to 0$ at infinity, we find that its null value is plenty satisfied if α is a constant. In true, it is an imaginary diffusion constant because there is no effective dislocation of matter in all directions (this is a single-particle description). In fact, in accordance with the imaginary part of the effective action (1), the unpredictable impulses received by the particle are given by

$$m\mathbf{v} = \nabla\left(i\,\frac{\hbar}{2}\,\ln P\right) = i\,\frac{\hbar}{2}\,\frac{\nabla P}{P}\,, \tag{9}$$

which, compared with (7), implies that $\alpha = i\hbar/2m$.

The consequent average kinetic energy induced by the ZPF on the particle is

$$\langle T_{ZPF}\rangle = \int P\left(\frac{1}{2}\,m|\mathbf{v}|^2\right) d^3r\,, \tag{10}$$

which considering (9), reads

$$\langle T_{ZPF}\rangle = \frac{\hbar^2}{8m}\int \frac{(\nabla P)^2}{P}\; d^3r. \tag{11}$$

However, the implicated acceleration makes the electrical charge radiates. So, we must appeal to the general rule concerning accelerated charged particles, namely: The change in the kinetic energy in the absorption-emission process is equal to the work done by the field minus the radiated energy. This is the energy conservation implicit in the determination of the Abraham-Lorentz back reaction force.

Therefore, varying the average kinetic energy, that is, taking the functional derivative of (11) with respect to P, we find that the remaining energy due to particle-field interactions is

$$\delta\langle T_{ZPF}\rangle = \frac{\hbar^2}{8m}\left(\frac{\nabla P\cdot\nabla P}{P^2} - 2\,\frac{\nabla\cdot\nabla P}{P}\right), \tag{12}$$

where, therefore, the first term relates to absorption of radiation, and the second to emission.

Coincidentally, this remaining energy is the quantum potential (4) which, therefore, is the expression of the required energy conservation implied in the so-called quantum equilibrium.

At this point we have sufficient valid information to prove that extreme action trajectories are randomly chosen at each position actually occupied by the particle.

Indeed, the probability density conservation (6), considering (9), reads

$$\frac{\partial P}{\partial t} + i\,\frac{\hbar}{2m}\nabla^2 P = 0\,, \tag{13}$$

which has the shape of a diffusion equation; local diffusion of probability density currents or virtual outflows of matter at the actual particle position.

The validity of this equation is unquestionable. In fact, it is absolutely equivalent to Eq. (3), or

$$\frac{\partial(\psi^*\psi)}{\partial t} + i\,\frac{\hbar}{2m}\,\nabla\cdot(\psi^*\,\nabla\psi - \psi\,\nabla\,\psi^*) = 0\,, \tag{14}$$

as can be proven from $|\nabla\psi|^2 = -\psi^*\nabla\psi$ and the parameterized forms of S and P in terms of ψ.

Very important, the equations (13) and (3) represent the same diffusion at each position **x** actually occupied. Equivalently, these two ways of expressing probability conservation contain implicitly all possibilities for the particle flow at **x**. As Eq. (3) expresses this in terms of ∇S, then ∇S must represent all possible momenta at **x**. However, as these partial derivatives require continuous action functions, then there pass multiple virtual trajectories. One of them infallibly will be occupied, but only for a moment because in the next position the same phenomenon is repeated.

In this sence, the obedience to the dynamical principle, implicit in the effective action (1), is traduced as follows: At a given time the particle is over a trajectory represented by the action S (real part), but at this very moment there is a choice for the next motion, which is dictated by the probability dependent local action (imaginary part). In other words, the imaginary part chose the next action function (S) representing another trajectory to be occupied during an infinitesimal time; and so on.

Now, it is possible to correct the interpretation given to (5) in the article [1]. Just rewrite Eq. (13) in the energy form

$$i\,\frac{\hbar}{2}\frac{1}{P}\frac{\partial P}{\partial t} = \frac{\hbar^2}{4m}\frac{\nabla\cdot\nabla P}{P}\,, \tag{15}$$

which implies that

$$\int_{all} P\left(i\,\frac{\hbar}{2}\frac{1}{P}\frac{\partial P}{\partial t}\right)d^3r = \int_{all} P\left(\frac{\hbar^2}{4m}\frac{\nabla\cdot\nabla P}{P}\right)d^3r = 0\,. \tag{16}$$

Being the second member of (15) the emitted energy of the balance (12), then the result (5) means that the involved energy doesn't remain in the particle.

3 Conslusion

The subsequent particle's positions, randomly chosen in the interactions, are on different trajectories. Therefore, there are continuous trajectories, but never followed by quantal particles. They simply represent the obedience to the mechanical principle, regardless of where the particle is. Nevertheless, as these virtual trajectories are inherent to the Schrödinger picture, then it is expected that they — properly determined and used as statistical tools - can give the same predictions. However, the convenience of such procedure needs to be better discussed.

On the other hand, were highlighted permanent emissions and absorptions of radiation, meaning that particles are actuated by forces and back reaction forces, which, on average, are zero. This explains why the interactions become transparent in the quantum description. Nevertheless, speculating, these permanent absorptions and emissions of electromagnetic waves (a delicate asymmetry accompanying particles everywhere) may be important to interpret certain properties of matter.

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References

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