Testing 5D Gravity with LIGO for Space Polarization by Scalar Field

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Whether LIGO detectors can directly detect the scalar field dark energy and thus test the five-dimensional (5D) gravity or not is examined analytically in terms of the author previously well-developed 5D fully covariant theory of gravitation with a scalar field. It is shown that an object with some thousand kilograms (e.g. 4700 kg), if electrically charged up to some ten kilovolts (e.g. 40 kV), can polarize the space or vacuum by the scalar field dark energy of the charged object and thus be able to extend the optical path length of a laser beam that travels through one LIGO arm with some hundred reflections (e.g. 280) by approximately 4×10^{-19} m (or the space-polarization strain of 10^{-22}), which is the amount of 4 times greater than that to be detected by the LIGO detectors. Switching on and off the power to the object, we can carry out tests of this 5D gravity by examining whether the converging laser beams become out of phase and thus the interference pattern varies or not. We can also apply a harmonically varying voltage with a frequency, e.g. 100 Hz, to charge the object and thus produce a varying optical length difference in the specific frequency range of LIGO detectors. Therefore, being added a highly charged sphere into the experimental setup, LIGO, which has recently detected first ever the gravitational waves from binary black hole mergers, can directly examine the existence of the scalar field dark energy of 5D gravity in a ground-base experiment. This study provides a design criterion for this new approach and experiment of discovering dark energy as well as testing 5D gravity.

1 Introduction

The observed acceleration of the present universe is generally attributed to the existence of dark energy throughout the universe [1-2]. A direct detection of the dark energy, whose true nature remains elusive, has become one of the most important issues in the modern astrophysics and cosmology since the discovery of acceleration of the universe. Two commonly accepted candidates of dark energy are the cosmological constant and the quintessence. Unlike the cosmological constant, which Albert Einstein first introduced into his general theory of relativity in order for the universe to be static, the quintessence is a scalar field Φ that varies throughout spacetime and has been modeled in various theories of gravitation such as the four-dimensional (4D) Brans-Dicke scalar-tensor gravity [3] and the five-dimensional (5D) Kaluza-Klein scalar-vector-tensor gravity (shortened by 5D gravity) [4-6].

The scalar field of 5D gravity, which has been recently related to the Higgs field of 4D particle physics in[7], were theoretically shown to be capable of polarizing the space or vacuum [8-9] and thus able to extend the optical path length of a laser beam that travels through the polarized vacuum. The vacuum polarization by a scalar field has been studied in the Schwarzschild spacetime [10] , in a waveguide [11], in the de Sitter spacetime with the presence of global monopole [12], and in a homogeneous space with an invariant metric [13]. Recently, the author, in terms of his 5D fully covariant theory of gravitation, has quantitatively determined the dielectric constant of the polarized vacuum in accordance with the charge-mass ratio of a charged object [14].

In this paper, we will further analytically demonstrate that the vacuum polarization by the scalar field dark energy of 5D gravity can increase the relative optical path length (i.e. the strain) above a factor of 10^{-22} and therefore can be directly detected via the extremely accurate LIGO detectors that have recently detected first ever the gravitational waves from the binary black hole merger as declared in [15]. We will use a harmonic voltage to charge the object, which leads to a varying optical length difference in the frequency range of the LIGO detection. A positive result of detecting the scalar field dark energy by LIGO will provide a fundamental test of 5D gravity.

2 5D gravity and vacuum polarization by scalar field dark energy

2.1 5D gravity with scalar field and field solution

A 5D gravity is a Kaluza-Klein theory that unifies the 4D Einsteinian general relativity (GR) and Maxwellian electromagnetism (EM). Without a scalar field (i.e. $\Phi = 1$), the 5D unification is trivial because, in the (4+1) split form, it is identical to GR and EM. With a scalar field, however, a 5D gravity can lead to a sequence of new effects such as the space or vacuum polarization [8-9, 14], electric redshift [16], gravitational field shielding [17-18], gravitationless black hole [19], modified neutron star mass-radius relation [20], and so on. A 5D gravity with the Friedmann-Lemaitre-Robertson-Walker (FLRW) metric of the universe modifies the Friedmann equa-

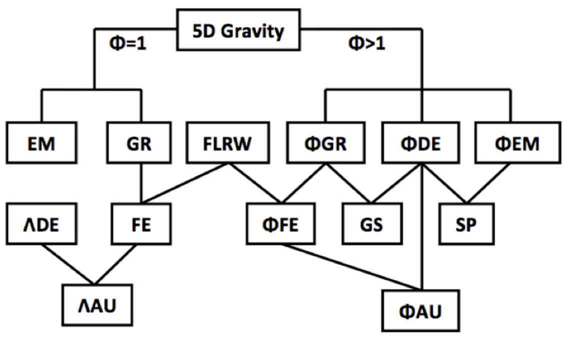

Fig. 1: Characteristics of 5D gravity with and without a scalar field dark energy (ΦDE). Without a scalar field (i.e. $\Phi = 1$), 5D gravity just trivially unifies the 4D Einsteinian general relativity (GR) and Maxwellian electromagnetism (EM). Combining with the Friedmann-Lemaitre-Robertson-Walker (FLRW) metric of 4D spacetime, the field equation given in GR derives the Friedmann equation (FE) that governs the dynamic and development of the universe. Including the cosmological constant dark energy (ΛDE), FE explains the acceleration of the universe (ΛAU). With a scalar field (i.e. $\Phi > 1$), 5D gravity modifies the general relativity (ΦGR) and electromagnetism (ΦEM) through the scalar field dark energy (ΦDE). These modifications lead to a sequence of new effects such as the space or vacuum polarization (SP) and the gravitational field shielding (GS). Combining with the FLRW metric of 4D spcatime, ΦGR derives a modified Friedmann equation (ΦFE), which can also explain the acceleration of the universe (ΦAU) but due to the scalar field dark energy (ΦDE). The space polarization (SP) or the effect on light by the ΦDE of 5D gravity can be significant enough for the accurate LIGO detectors to detect.

tion with a scalar field, which plays the role of dark energy and explains the acceleration of the universe [21-23]. These new effects are results of the scalar field that modulates both gravitational and electromagnetic fields as shown in the (4+1) split form of the 5D field equation or as seen in the field solutions [14, 24]. Figure 1 shows the characteristics of a 5D gravity with and without a scalar field dark energy and its role to the cosmology.

The metric of 5D spactime is usually given by [25]:

$$\bar{g}_{\alpha\beta} = \begin{pmatrix} g_{\mu\nu} + q^2\Phi^2 A_\mu A_\nu & q\Phi^2 A_\nu \\ q\Phi^2 A_\mu & \Phi^2 \end{pmatrix} \tag{1}$$

where α and β are the subscripts for the 5D coordinates, running through 0 - 4; μ and ν are the subscripts for the 4D coordinates, running through 0 - 3; $g_{\mu\nu}$ is the metric of 4D spacetime; A_μ is the standard 4D electromagnetic potential; Φ is the scalar field, which is an effectively massless 4D scalar; q is a scale constant defined by $q = 2\sqrt{G}$ with G the gravitational constant. The fifth dimension is compact [26]. In isotropic coordinates, the line element ds^2 of 4D spacetime can be represented according to the metric as [27]

$$ds^2 = -g_{\mu\nu}dx^\mu dx^\nu$$

$$ds^2 = -e^\nu dt^2 + e^\lambda \left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right), \tag{2}$$

where e^λ and e^ν are the metric $rr-$ and $tt-$components as functions of the radial distance r. Then, the exact static spherically symmetric solution of gravitational, electromagnetic, and scalar fields of a charged body is given by [24]

$$e^\lambda = \left(1 - \frac{B^2}{r^2}\right)^2 \Psi^{-2}, \tag{3}$$

$$e^\nu = \Psi^2 \Phi^{-2}, \tag{4}$$

$$H_{01} = -H_{10} = -\frac{Q}{r^2} e^{(\nu-\lambda)/2}, \tag{5}$$

$$\Phi^2 = a_1 \Psi^{p_1} + a_2 \Psi^{p_2}, \tag{6}$$

where the function Ψ is defined by

$$\Psi = \left(\frac{r-B}{r+B}\right)^{C/2B}, \tag{7}$$

and the seven constants (K, p_1, p_2, B, C, a_1, and a_2) are constrained by the following five relations:

$$K = 4(4B^2 - C^2)C^{-2}, \tag{8}$$

$$a_1 + a_2 = 1, \tag{9}$$

$$p_1 = 1 + \sqrt{1+K}, \tag{10}$$

$$p_2 = 1 - \sqrt{1+K}, \tag{11}$$

$$Q^2 = -a_1 a_2 C^2 (1+K) G^{-1}. \tag{12}$$

Here H_{01} and H_{10} are non-zero components of the effective 4D electromagnetic field $H_{\mu\nu} \equiv \phi^3 F_{\mu\nu}$ with $F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$. At $r \to \infty$, the limits of e^λ, e^ν, and Φ are the unity. The parameter Q denotes the electric charge. It is obvious that the above 5D solution of the fields includes two independent constants.

In a traditional 5D gravity, one usually assumes or hypothetically forms the fifteenth component ($\bar{T}^{44}$) of the 5D energy-momentum tensor by including an undetermined parameter called scalar charge S, e.g. $\bar{T}^{44} = S\rho$ as done by [24] with ρ the density of matter. Since it lacks of any measurement and short of any observational support, the undetermined parameter makes all results obtained from the traditional 5D gravity to be non-decisive and hence non-conclusive in comparison with other theories of gravitation, observations, and experiments. Describing the matter to be also covariant in the 5D spacetime as the fields are, however, this author analytically derived the fifteenth component of the 5D energy-momentum tensor without assuming any unknown parameter ([14] and references therein such as the early studies by the author [28-29]),

$$\bar{T}^{44} = \frac{\rho\alpha^2}{\Phi^2\sqrt{\Phi^2+\alpha^2}}, \tag{13}$$

where α is a non-dimensional constant (or charge-mass ratio) defined by

$$\alpha = \frac{Q}{2\sqrt{G}M}, \tag{14}$$

with M the mass of matter, and therefore analytically determined all the constants in the solution as follows

$$K = 8, \quad p_1 = 4, \quad p_2 = -2, \tag{15}$$

$$a_1 = -\alpha^2, \quad a_2 = 1 + \alpha^2, \tag{16}$$

$$C = \frac{2GM}{3c^2\sqrt{1+\alpha^2}}, \quad B = \frac{GM}{c^2\sqrt{3(1+\alpha^2)}}. \tag{17}$$

Here the cgs or Gaussian unit system is adapted. This set of constants is the simplest and most elegant, because of $K = 8$ that leads to p_1 and p_2 to be whole numbers, for the solution to be non-trivial. Therefore, according to this solution with the constants obtained, the gravitational, electromagnetic, and scalar fields of a charged spherically symmetric object are completely determined from the charge and mass of the object.

In the Einstein frame, this field solution simply reduces to the Schwarzschild solution of the Einsteinian general relativity when matter is neutral and fields are weak [14,17]. This guarantees that the fundamental tests of the Einsteinian general relativity in the case of weak fields are also the tests of this 5D gravity. In the case of strong fields, especially when matter is electrically charged, however, the results obtained from this 5D gravity are significantly different from the Einsteinian general relativity. These new strong field effects include the space polarization [8, 14], electric redshift [16], gravitational field shielding [17-18], and so on. At $\Phi = 1$, the 5D gravity is trivially equivalent to GR and EM, where the Reissner-Nordstrom solution determines the standard GR metric of a charged, massive particle [30-31]. The solution of this 5D gravity Eq. (3) is obtained at $\Phi \neq 1$ and thus cannot be limited to the Reissner-Nordstrom solution for a charged, massive particle. But when fields are weak and matter is weakly charged, the effect of the scalar field on both gravitational and electromagnetic fields are negligible.

2.2 Vacuum polarization by scalar field

In terms of this 5D gravity and the field solution obtained, the electric field of a charged body can be defined as

$$E \equiv H_{10} = -H_{01} = \frac{Q}{r^2}e^{(\nu-\lambda)/2}, \tag{18}$$

and then the dielectric constant (or relative permittivity) ϵ_r of the vacuum that is polarized by the scalar field can be determined by

$$\epsilon_r \equiv \frac{E_C}{E} = e^{(\lambda-\nu)/2} = \left(1 - \frac{B^2}{r^2}\right)\Phi\Psi^{-2}, \tag{19}$$

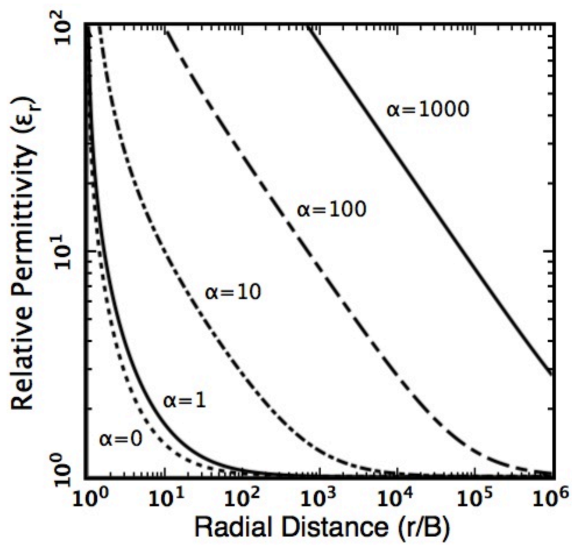

Fig. 2: The relative permittivity ϵ_r or the electric field ratio E_C/E versus the normalized radial distance r/B for a charged object with $\alpha = 0, 1, 10, 100, 1000$, respectively.

where $E_C = Q/r^2$ is the Coulomb electric field of the charged object. To see how significant the space or vacuum polarization is, we plot, in Figure 2, the relative permittivity ϵ_r as a function of the normalized radial distance r/B for a charged object with five different charge-mass ratios $\alpha = 0, 1, 10, 100, 1000$.

The result indicates that the electric field of the charged object asymptotically approaches the Coulomb electric field (i.e. $\epsilon_r \to 1$), when r is getting larger ($r \gg B$) or approaches infinity. When r becomes small, however, the electric field significantly deviates from the Coulomb electric field (i.e. $\epsilon_r \gg 1$) due to the vacuum space to be extensively polarized by the strong scalar field. When r tends to B, the relative permittivity approaches infinity and the electric field becomes weaker and weaker as compared with the strength of the Coulomb electric field, especially when the object is highly charged. In the limit case of $\epsilon_r = \infty$, the vacuum space is completely polarized by the extremely strong scalar field. It should be noted that a big deviation at $r \sim B$ still exists even if the object is weakly charged ($\alpha \ll 1$) or neutral. The deviation increases as the charge increases. For instance, at $\alpha = 100$ and $r/B = 10^3$, the electric field is only 10% of the Coulomb electric field. The electric field is significantly weakened as compared with the strength of the Coulomb electric field and the vacuum space is greatly polarized, especially when the object is highly charged.

Only for a massive, compact and charged object, we can have a B not to be too small in comparison with its radius and can see a significant polarization of the vacuum. For a lab-sized object, the polarization of the vacuum can only be extremely weak. Figure 3 plots the deviation of the rel-

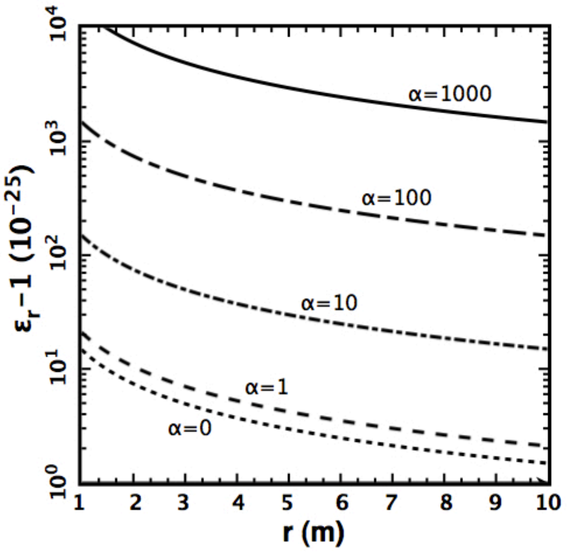

Fig. 3: The change of the relative permittivity $\epsilon_r - 1$ or the change of relative electric field $(E_c - E)/E$ versus the radial distance r for a charged object with mass of 1000-kg and charge-mass ratio $\alpha = 0$, 1, 10, 100, 1000, respectively.

ative permittivity of the vacuum from the unity, $\epsilon_r - 1$, due to the polarization as a function of the radial distance r for a charged object with mass of 1000 kilograms and charge in a range of $\alpha = 0 - 1000$. It is seen that, because the fields of a non-massive object are too weak, the polarization of the vacuum by the scalar field dark energy of 5D gravity is very very small and thus extremely difficult to be detected in laboratory, except for us to have an extremely accurate detector with an appropriate approach. In the following section, we will examine whether the LIGO detectors can detect such small vacuum polarization or not. The answer as shown in the next section is positive when the charge-mass ratio of the charged body is much greater than 1.

3 Can LIGO detect the scalar field dark energy?

In accordance with the relative permittivity determined above, we can find the refractive index of the vacuum that is polarized by the scalar field of 5D gravity as,

$$n \equiv \sqrt{\epsilon_r} = e^{(\lambda-\nu)/4}. \tag{20}$$

For the non-polarized vaccum, we have $n = 1$ and $\epsilon_r = 1$. Substituting Eqs. (3) and (4) into Eq. (20), we have

$$n = \Phi^{1/2}\Psi^{-1}\left(1 - \frac{B^2}{r^2}\right)^{1/2}. \tag{21}$$

In the case of weak fields, we can obtain the change of the refractive index for the polarized vacuum as,

$$\delta n = n - 1 \simeq \frac{\sqrt{1+\alpha^2}\, GM}{c^2 r}, \tag{22}$$

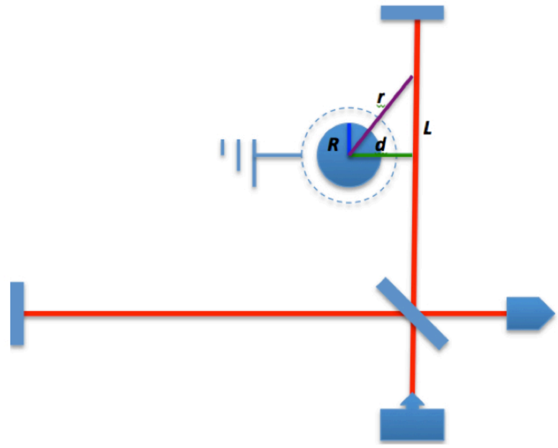

Fig. 4: A schematic diagram for LIGO with a charged object to detect the scalar field dark energy of 5D gravity. When we place a highly charged object, whose strong electromagnetic fields are shielded by a conductor shell that is grounded, nearby one path of the LIGO laser beams. The space surrounding the charged object and the vacuum travelled through by the laser beam back and forth are polarized by the scalar field of the charged object. This polarization extends the optical path length of the laser beam to be significant enough for the accurate LIGO to detect the scalar field dark energy.

When $\alpha \gg 1$, δn is about linearly increasing with α. Then, the change of the optical path length of the polarized space or vacuum can be obtained by the following path line integration

$$\delta l = \int_C \delta n\, ds. \tag{23}$$

To quantitatively estimate the polarization, we consider a metal (e.g. copper) sphere with radius $R = 0.5$ m. From the mass density of copper $\rho = 9 \times 10^3$ kg/m^3, we can find the mass of the sphere to be $M = 4\pi\rho R^3/3 \sim 4.7 \times 10^3$ kg. Now, if the sphere is electrically charged up to $V = 10^5$ V, we can also calculate the charge Q and charge-mass ratio α of the sphere as $Q = 4\pi\epsilon_0 RV \sim 5.6 \times 10^{-6}$C $= 1.7 \times 10^4$ esu and $\alpha \sim 7$, respectively. Then, from Eq. (22), we can find the change of the refractive index in the space surrounding the charged sphere to be $\delta n = 1.2 \times 10^{-23}$. Here, we have chosen as an example the radial distance to be 4 radii of the object, i.e. $r = 2$ m. This result indicates that the scalar field of the charged object can extend the optical path length relatively by $\sim 1.2 \times 10^{-23}$ m for each meter, which is significant enough for the accurate LIGO detectors to detect.

Now, we suggest to place this charged object into the LIGO system nearby the middle of the path of one of the two perpendicular arms or laser beams (Figure 4). Then, the variation of the optical path length due to the space polarization by the scalar field dark energy can be estimated by,

$$\Delta L = (N+1)\int_{-L/2}^{L/2} \delta n\, ds$$

$$\Delta L = \frac{(N+1)\sqrt{1+\alpha^2}\,GM}{c^2}\ln\frac{L+\sqrt{L^2+4d^2}}{-L+\sqrt{L^2+4d^2}}, \quad (24)$$

where N is the number of reflections of the laser beam, L is the geometric length of the arm, d is the minimum distance from the center of the charged object to the laser beam, and s is the coordinate of position to be integrated along the path from $-L/2$ to $L/2$. For the LIGO working parameters, we can choose $N = 280$ and $L = 4$ km. The distance can be chosen again as 4 radii of the charged object, i.e. $d = 2$ m. Then, we can obtain that the optical length of the 4 km path of the LIGO laser beam with 280 times reflections is increased due to the space polarization by $\Delta L \sim 10^{-18}$ m, about the amount of one order higher than that being detectable by LIGO. Similarly to the gravitational-wave strain defined in [15], we can define a strain for the space polarization by scalar field, h, as the change of the optical length dividing by the length of the LIGO arm L,

$$h \equiv \frac{\Delta L}{L} \simeq \frac{2(N+1)\sqrt{1+\alpha^2}\,GM}{c^2 L}\ln\frac{L}{d}. \quad (25)$$

Here, we have approximate the expression or Eq. (24) by considering $d \ll L$. For $\alpha \gg 1$, we have that the strain is proportional to the charge Q but independent of the mass M.

$$h \simeq \frac{(N+1)\sqrt{G}\,Q}{c^2 L}\ln\frac{L}{d} \propto Q. \quad (26)$$

Here, the cgs units are adapted since we have used Eq. (14).

To see the charge dependence, we plot in Figure 5 the increase of the optical path length as a function of the voltage of the charged object. The result indicates that the extension of the optical path length remains a constant as the mass is fixed when the object is weakly charged ($V < 500$ V) and linearly increases with the voltage when the object is highly charged. For instance, when $V = 40$ kV, the charged object can cause the optical path length of one laser beam in a LIGO arm with 280 times reflections to extend up to about $\Delta L \sim 4 \times 10^{-19}$ m (or the strain $h \sim 10^{-22}$), which is the amount of 4 times greater than that to be detectable by the LIGO detectors [15]. For LIGO to detect the scalar field dark energy or to test the 5D gravity, we can switch on and off the power to the object and check whether the converging laser beams become out of phase and thus the interference pattern varies or not. In addition, to have a timely varying optical length difference in a specific range of 20-2000 Hz that LIGO can measure, we consider a harmonically varying voltage or power to charge the sphere, $V(t) = V_0 \sin(2\pi f t)$, with $V_0 = 10^5$ V and $f = 100$ Hz. Figure 6 plots the the varying optical length change between two laser beams as a function of time. Therefore, the accurate LIGO detectors that have recently detected first ever the gravitational waves from a binary black hole merger are capable to be detectors and testers for the scalar field dark energy of 5D gravity. This study provides a creative approach for LIGO to detect the vacuum polarization by the scalar field of 5D gravity, a candidate of dark energy that drives the universe in its accelerating expansion. It should be noted that this paper only focuses on the variation in optical length due to the vacuum polarization by the scalar field. To include the variation in optical length due to other fields, we need compute it based on the full solution of all fields. This leaves for future study.

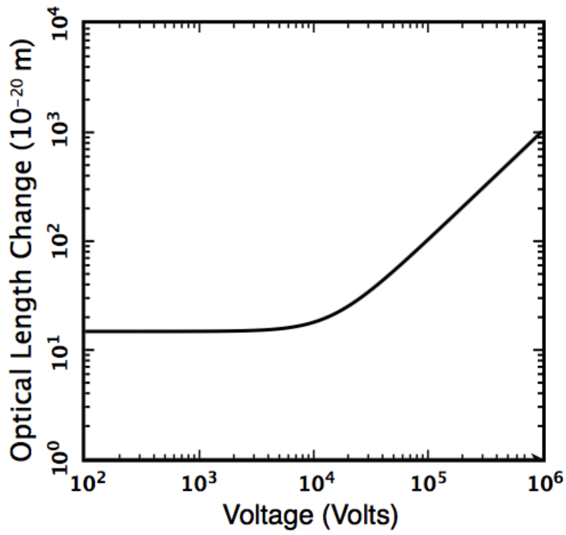

Fig. 5: Space polarization by the scalar field dark energy of 5D gravity. The increase of the optical path length of a laser beam in one LIGO arm that is polarized by a charged object is plotted as a function of the voltage applied to the object. In the case of the object to be only weakly charged ($V < 500$ V), the extension of the optical path length remains a constant as the mass is fixed. When the object is highly charged, however, the optical path length linearly increases with the voltage. At $V = 40$ kV, the charged object can extend the optical path length of one laser beam in a LIGO arm with 280 times reflections up to about $\Delta L \sim 4 \times 10^{-19}$ m (or the strain $h \sim 10^{-22}$), about one order higher than that to be detected by LIGO.

4 Discussions and conclusions

LIGO uses the interference pattern where the beams combine to determine if the optical length down the two laser arms is changing. Possible physical causes for the change of the optical length down the two laser beams can be various sources such as seismic disturbances, gravitational waves from binary black hole mergers, space polarizations by scalar field, and so on. When a gravitational wave passes through the interferometer, the spacetime in the local area is altered, disturbed, and curved. This results in an effective change in the optical length of one or both of the laser beams, which is estimated by $\Delta L(t) = h(t)L$, where $h(t)$ is the gravitational-wave strain amplitude projected onto the detector [15]. The advanced LIGO detectors can have sensitive responses to a strain of $h(t) \sim 10^{-21} - 10^{-23}$. This change of the optical length causes the light currently very slightly out of phase with the incom-

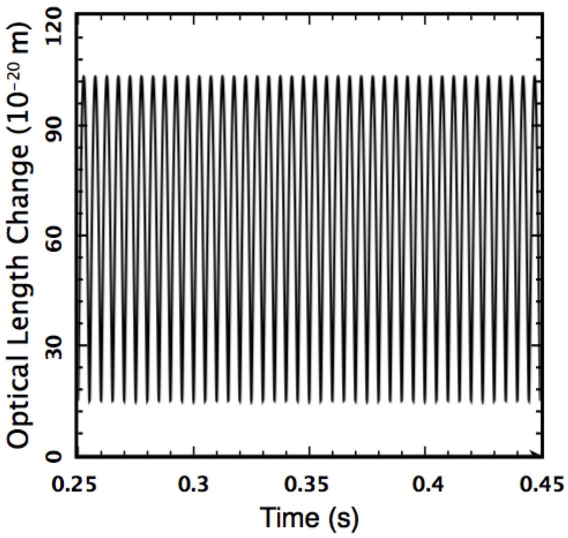

Fig. 6: The optical length difference between the two laser beams is plotted as a function of time when a 100-Hz harmonically varying voltage is applied to charge the sphere. LIGO detectors that have detected the gravitational waves from binary black hole mergers can measure the varying optical length change.

ing light and thus varies the interference pattern. The effective optical length change due to the spacetime disturbances and distortions by the passing of gravitational waves is calculated from the solution of the deviating geodesics equation with a gravitational wave from a binary black hole merger. For the space polarization by scalar field, as analyzed in this paper, we calculate the change of the optical length in accordance with the solution of the deviating index refraction. Seismic disturbances can also result in the converging laser beams being out of phase.

As a consequence, we have in terms of a 5D gravity found that a some-thousand-kilogram (e.g., 4700 kg) sphere electrically charged to some ten kilovolts (e.g. 40 kV) can polarize the vacuum by its scalar field dark energy and thus extend the optical path length of a laser beam that travels through one LIGO arm with some hundred (e.g. 280) reflections by approximately 4×10^{-19} m (or the strain of $h \sim 10^{-22}$), which is the amount of 4 times greater than that to be detected by the LIGO detectors. Switching on and off the power to the object allows to check whether the LIGO detectors can detect the scalar field dark energy and thus test the 5D gravity or not. For a harmonic voltage with frequency, e.g. 100 Hz, we have a varying optical length difference between the two laser beams in the frequency range of the LIGO detection. Therefore, being added a highly charged sphere into the experimental setup, LIGO, which has recently detected first ever the gravitational waves from the binary black hole merger, may directly discover first ever the scalar field dark energy of 5D gravity. This study also provides a design criterion for a new approach and experiment of discovering dark energy.

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