Fermi Scale and Neutral Pion Decay

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A modified Fermi coupling of the weak interactions is proposed and in analogy with the Planck units, a Fermi scale is defined. We define a second Fermi length, a Fermi mass (related to the threshold energy for unitarity), and a Fermi time. The holographic principle (HP) is then applied to some two-dimensional objects, where the unit cell size is given by the second Fermi length. With the aid of non-linear Dirac equation, a formula is obtained relating the Fermi, the nucleon, and the electron masses. Another relationship is found, linking the second Fermi length to cosmological constant and Planck scales. Finally HP in 2-d is employed in a stationary condition for the free energy, as a means to evaluate the neutral pion decay time.

1 Introduction

Once fixed the separation of them, the gravitational interaction between two particles of equal masses goes with the product of the Newton's gravitational constant G times the mass squared. Analogously, the electrostatic interaction of two equal charges is given by the product of the K_e-constant, let us call it the Coulomb constant, times the electric charge squared.

In quantum mechanics (QM) or in quantum field theory (QFT), by considering for instance only the absolute value of the proton-electron attraction in the hydrogen atom, it is convenient to write

$$K_e e^2 = \alpha \hbar c. \tag{1}$$

In (1) we have: e the quantum of elementary electric charge, $\hbar$ the reduced Planck constant, c the speed of light in vacuum, and α is the electromagnetic coupling strength. Relation (1) can be translated to the gravitational interaction case and takes the form

$$GM^2 = \alpha_g \hbar c. \tag{2}$$

According to the QFT the couplings are running with the energy [1], and we may define an energy (mass) scale such that we have $\alpha_g = 1$ [2]. We call this mass the Planck mass, and using the value $\alpha_g = 1$ in (2), we obtain

$$M_{Pl} = \sqrt{\frac{\hbar c}{G}}\,. \tag{3}$$

The Compton length of a particle with the Planck mass gives the Planck length, and the Planck time can be also defined by using c. We have

$$L_{Pl} = \frac{\hbar}{M_{Pl}\, c} = \sqrt{\frac{\hbar G}{c^3}}\,, \tag{4}$$

$$t_{Pl} = \frac{L_{Pl}}{c} = \sqrt{\frac{\hbar G}{c^5}}\,. \tag{5}$$

An alternative way to obtain the Planck scale (units) is to compare the Compton length of a particle with its Schwarzschild radius [3]. As is posted in Wikipedia [4]:

"Originally proposed by the German physicist Max Planck, these units are also known as natural units because the origin of their definition comes only from properties of fundamental physics theories and not from interchangeable experimental parameters."

The idea of the Planck length as being the minimal length (related to a discreteness of the space-time?), was first proposed by C.A. Mead [5,6]. The difficulty to publish his results is commented by Mead [8] and also highlighted by Sabine Hossenfeld [9], in a more recent essay.

In reference [10], the Fermi coupling constant G_F was used as a means to estimate the muon decay time. The way of using G_F in those calculations resembles the employment of Newton gravitational constant G in the Newtonian mechanics. This has inspired the present author to look at the possibility of defining a Fermi scale (units) in an analogous way as the Planck's case (relations (3) to (5) of this work). Indeed in a recent paper [11], Roberto Onofrio conjectured that weak interactions could be a manifestation of gravity when investigated through high energy probes (short distances).

In section 2, we use estimates of G_F quoted in the literature, in order to evaluate numerically the principal Fermi units, namely the second Fermi length, the Fermi mass (the second Fermi energy), and the Fermi time. The second Fermi length is named this way, to avoid confusion with the usual Fermi length related to the electrical conductivity of metals, for instance.

In section 3, we use the Holographic Principle (HP) in two dimensions (2-d) plus a simple Dirac- like equation, besides a relation connecting the wave function to the entropy, as a means to obtain a closed relation encompassing the Fermi, the electron and the nucleon masses.

In section 4, the HP in 2-d is used again, relating the second Fermi length to a length related to the cosmological constant [12], the Planck length and the electromagnetic coupling α.

Section 5 provides an estimate of the neutral pion radius.

In section 6, the HP in 2-d is used to evaluate the neutral pion decay time.

Finally section 7 is reserved for the concluding remarks.

2 The Fermi scale (units)

In reference [10], the muon decay time was estimated starting from the relation

$$m_\mu c^2 = \frac{1}{R_W} \frac{G_F\, c^2}{h^2} m_\mu^2. \tag{6}$$

In (6) m_μ is the muon mass, G_F is the Fermi constant of the weak interactions and R_W is the weak radius of the muon. We observe from an inspection of (6) that it is possible to define a modified Fermi constant G_F^*, namely

$$G_F^* = \frac{G_F\, c^2}{h^2}. \tag{7}$$

It is convenient to write the "inverse transform" of (7) as

$$G_F = G_F^* \frac{h^2}{c^2}. \tag{8}$$

We will call (8): all-classic to quantum relativistic transmutation. The reason to do so is: G_F^* could in principle to exist in the realm of the classical mechanics, while G_F only makes sense in a quantum relativistic treatment. Observe that given a finite G_F^*, G_F vanishes if $h \to 0$, or if $c \to \infty$, and naturally when both things happen. As can be verified in (6) and (7) G_F^* behaves for the weak interactions as G works in the case of the Newton's gravitational theory. As weak interactions are non- linear interactions, it is possible to write a set of equations similar to Einstein equations, putting in those equations G_F^* in the place of G.

The Schwarzschild-like metric for these equations gives the Weak-Schwarzschild radius R_{WS}. Here we apply this recipe to a particle with the muon mass. We have

$$R_{WS} = \frac{2\, G_F^*\, m_\mu}{c^2} = 2\, R_W. \tag{9}$$

Substituting (7) into (9), we get

$$R_{WS} = \frac{2\, G_F\, m_\mu}{h^2}. \tag{10}$$

The establishment of a modified Fermi coupling, namely G_F^* (please see (7)) permit us immediately to define the Fermi scale (units) in analogous way we have proceed in the Planck scale case. Therefore taking in account relations (3) to (5) we can write

$$M_F = \sqrt{\frac{\hbar\, c}{G_F^*}}\,, \tag{11}$$

$$L_{SF} = \frac{\hbar}{M_F\, c} = \sqrt{\frac{\hbar\, G_F^*}{c^3}}\,, \tag{12}$$

$$t_F = \frac{L_{SF}}{c} = \sqrt{\frac{\hbar\, G_F^*}{c^5}}\,. \tag{13}$$

With respect to (11) we notice that as is discussed on page 526 of the book by Rohlf [13], in a modern description of the weak interactions, the weak coupling constant is running with the energy of the probe used to measure it. According to Rohlf [13], "The weak interaction rate cannot increase forever with increasing energy. At some very large energy, this would violate conservation of probability or unitarity. Unitarity is violated at the energy where the weak coupling becomes unity." In the present treatment this happens just at the energy scale given by the Fermi mass (M_F).

In order to estimate the quantities (11) to (13), related to the Fermi scale of length, let us take the value of G_F as quoted in the book by Rohlf (formula 18.33, page 509).

$$G_F = 8.96 \times 10^{-8}\ \text{GeV fm}^3. \tag{14}$$

By using (7), we have

$$G_F^* = 2.94 \times 10^{21}\ \text{Nm}^2/\text{kg}^2. \tag{15}$$

Substituting G_F^* given by (15) into relations (11) to (13), we find

$$M_F \cong 1.84\ \text{TeV/c}^2. \tag{16}$$

$$L_{SF} \cong 1.07 \times 10^{-19}\ \text{m}, \tag{17}$$

$$t_F \cong 3.57 \times 10^{-28}\ \text{s}. \tag{18}$$

3 Deducing the Fermi mass

In this section it is proposed that the Fermi mass can be deduced by considering the holographic principle (HP) in 2-d, plus a non-linear Dirac-like equation (NLDE). A formula relating an entropy estimate via HP in 2-d and the wave function evaluated in the NLDE is considered. We are inspired in the neutron weak decay given a proton, an electron and a neutrino.

Inspired in McMahon [14], the holographic principle in 2-d can be stated as

- The total information content of a 2-d universe, in this case a spherical surface of radius R_x, can be registered in the perimeter of one of its maximum circles.
- The boundary of this spherical surface, here the perimeter of its maximum circle, contains at most a single degree of freedom per unit cell length.

Making the requirement that the radius R_x coincides with the Compton wavelength of the nucleon λ_n and choosing the unit cell size as L_{SF}, we can write

$$S_1 = \frac{\pi\, \lambda_n}{L_{SF}}. \tag{19}$$

Meanwhile, let us consider the non-linear Dirac-like equation

$$\frac{\delta\phi}{\delta x} - \frac{1}{c}\frac{\delta\phi}{\delta t} = \frac{1}{\lambda_e}\phi - \frac{1}{\lambda_n}\phi^3. \tag{20}$$

In (20) λ_e stands for the Compton wavelength of the electron and the equation (20) is conceived within the structure of an abelian field theory. However in a paper dealing with the proton-electron mass ratio [15], a π-factor has appeared in an equation in order to take in account the curvature of the space due to the non-abelian character of the QCD. Therefore let us define

$$\phi = \pi\, \Psi. \tag{21}$$

Inserting (21) into (20), we look for the zero of the equation and we find

$$\Psi^2 = \frac{\lambda_n}{\pi^2\, \lambda_e}. \tag{22}$$

Now we combine the results of (19) and (22), but considering the possibility of an implicit spin-1 boson being at work. We write

$$3\, S_1\, \Psi^2 = 1. \tag{23}$$

The insertion of (19) and (22) into (23) gives

$$\lambda_n^2 = \frac{\pi}{3} L_{SF}\, \lambda_e. \tag{24}$$

Remembering that ($\hbar$ = c = 1)

$$\lambda_n = \frac{1}{m_n}, \quad \lambda_e = \frac{1}{m_e}, \quad L_{SF} = \frac{1}{M_F},$$

we finally obtain

$$3\, M_F\, m_e = \pi m_n^2. \tag{25}$$

Putting numbers in (25), we get

$$M_F \cong 1.8\ \mathrm{TeV/c^2}. \tag{26}$$

As we can see, the value here deduced for the Fermi mass, is very close to that obtained through of the use of the measured value of G_F displayed in (16).

4 Deducing the second Fermi length – II

In a previous section a modified Fermi coupling, G_F^*, was defined and we found that a Fermi scale could be constructed in analogy with the well-established Planck scale. Here we pursue another path towards the deducing o f the second Fermi length. The role played by relic neutrinos in cosmology and its possible connection with the cosmological constant problem [16,17], stimuli us to seek for a relationship between L_{SF} and R_Λ. Indeed according Cohen, Kaplan and Nelson [18], R_Λ may be thought as a geometric average between the ultraviolet (L_{Pl}) and the infrared (R_U) cut-offs of the gravitational interaction.

Meanwhile, although matter is globally electrically neutral, may be some connection to exist between charges fluctuations and the weak coupling. In this section we also intend to tie the Fermi scale L_{SF} to a new scale R_α, related to the electromagnetic coupling. Next we define R_α. We write

$$G\, M_\alpha^2 = \alpha^2\, \hbar\, c. \tag{27}$$

By taking $\hbar = c = 1$, we get from (27)

$$M_\alpha = \frac{\alpha}{\sqrt{G}}. \tag{28}$$

Based on (28) we take R_α as

$$R_\alpha = \frac{1}{M_\alpha} = \frac{L_{Pl}}{\alpha}. \tag{29}$$

Now let us consider a spherical surface universe of radius L_{SF}. We apply The HP in 2-d to it, which unit cell size of its maximum circle's perimeter is given by R_α, and we get the entropy S_2

$$S_2 = \frac{2\,\pi\, L_{SF}}{R_\alpha} = \frac{2\,\pi\,\alpha\, L_{SF}}{L_{Pl}}. \tag{30}$$

Turning to the relationship connecting the L_{SF} and the R_α scales, we may write the non-linear Dirac equation

$$\frac{\delta\Psi}{\delta x} - \frac{1}{c}\frac{\delta\Psi}{\delta t} = \frac{1}{R_\Lambda}\Psi - \frac{1}{L_{SF}}\Psi^3. \tag{31}$$

Looking at the zero of (31), we get

$$\Psi^2 = \frac{L_{SF}}{R_\Lambda}. \tag{32}$$

Now we make the requirement that

$$S_2\, \Psi^2 = 1 \tag{33}$$

and we find

$$L_{SF}^2 = \frac{R_\Lambda\, L_{Pl}}{2\,\pi\,\alpha}. \tag{34}$$

To numerically evaluate (34), we consider $R_\Lambda = \sqrt{L_{Pl}\, R_U}$ with $L_{Pl} = 1.6162 \times 10^{-35}$ m and $R_U = 0.8 \times 10^{26}$ m [19], which yields

$$L_{SF} \cong 1.12 \times 10^{-19}\ \mathrm{m}. \tag{35}$$

As can be verified, this value is close to that experimentally determined (please see equation (17)).

5 The pion radius

In a paper dealing with the quark confinement related to the metric fluctuations [20], we have estimated a string constant K given by

$$K = \frac{m_q^2\, c^3}{\alpha_s\, \hbar} = \frac{m_q^2}{\alpha_s}, \quad (\hbar = c = 1). \tag{36}$$

In (36) the symbols m_q and α_s, stand for the quark constituent mass and the strong coupling, respectively. Now let us take

$$K\, 2\, R_\pi = m_\pi, \quad m_q = \frac{1}{2} m_\pi. \tag{37}$$

Combining the results of (36) and (37) and taking $\alpha_s \cong 1/3$ (please see ref. [21]), we obtain for the pion radius (being m_π the neutral pion mass)

$$R_\pi = \frac{2\,\alpha_s}{m_\pi} \cong \frac{2}{3\, m_\pi}. \tag{38}$$

Putting numbers in (38), we get

$$R_\pi \cong 0.98 \times 10^{-15} \text{ m} = 0.98 \text{ fm}. \quad (39)$$

6 Neutral pion lifetime from the holographic principle in 2-d

Let us consider the neutral pion decay, represented by the reaction

$$\pi^0 \to 2\,\gamma. \quad (40)$$

Taking in account the stationary condition for the free energy ($\Delta F = 0$), we get

$$\Delta U = T\,\Delta S. \quad (41)$$

Next we consider a 2-d universe, represented by a spherical surface of radius R_π and the entropy variation represented by the information contained on its maximum-circle perimeter, having a unit cell size equal to $2\,L_{SF}$. We can write

$$\Delta U = \frac{\alpha\,\hbar\,c}{R_\pi}, \quad \Delta S = \frac{\pi\,R_\pi}{L_{SF}}. \quad (42)$$

Besides this we consider

$$h\,\nu = \frac{h}{\tau} = T, \quad (k_B = 1). \quad (43)$$

Inserting the results of (42) and (43) into (41) and solving for τ, we obtain the neutral pion decay time given by

$$\tau = \frac{2\,\pi^2\,R_\pi^2}{\alpha\,c\,L_{SF}}. \quad (44)$$

Putting numbers in (44) we get

$$\tau_{estimated} = 0.81 \times 10^{-16} \text{ s}. \quad (45)$$

This value may be compared with [22]

$$\tau_{measured} = 0.84 \times 10^{-16} \text{ s}. \quad (46)$$

7 Concluding remarks

The estimate of the neutral pion decay time is usually obtained through the employment of the current algebra calculations. Partial conservation of the axial current (PCAC) prediction gives

$$\frac{h}{\tau} = \Gamma(\pi^0 \to 2\gamma) = \frac{\alpha^2\,m_\pi^3}{64\,\pi^3\,f_\pi^2}. \quad (47)$$

In the present paper, by using the concepts of the second Fermi length and the Hp in 2-d, we found a novel way to look at the neutral pion decay.

Meanwhile, Roberto Onofrio [11] conjectured that weak interactions should be considered as empirical evidences of quantum gravity at the Fermi scale. The "second" Fermi length estimated by Onofrio ($\sim 10^{-18}$ m) [11], is approximately one order of magnitude greater than that obtained in section 2 of this work. This comes from the fact that Onofrio used the expectation value of the Higgs field to fix the Fermi scale of energy, instead the unitary scale threshold we have used in the present work.

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