The Dirac Equation and Its Relationship to the Fine Structure Constant According to the Planck Vacuum Theory

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The Dirac equation and the fine structure constant are complementary and cannot be understood separately. The manifestly covariant Dirac equation in the Planck vacuum (PV) theory (8) is a coupling-charge equation, where e_*^2 is the squared coupling charge that couples the equation to the PV state. The laboratory-measured electron or proton mass is denoted by m. The corresponding fine structure constant is $\alpha \equiv e^2/e_*^2$ where e^2 is the squared charge of the electron or proton as measured in the laboratory. Both the Dirac particle spin and the fine structure constant have their origin in the electron or proton coupling to the PV state. The electron g-factor, with radiative corrections, is calculated from the fine structure constant; and the proton g-factor is roughly estimated from the electron g-factor and the proton structure constant. The radiative corrections in the QED theory are the result of photon interactions taking place within the *pervaded* PV state. The apparent ability of the electron to emit and absorb photons is due to the ability of the PV state to emit and absorb photons to and from free space.

1 Introduction

The theoretical foundation [1] [2] [3] of the PV theory rests upon the unification of the Einstein, Newton, and Coulomb superforces:

$$\frac{c^4}{G}\left(=\frac{m_*c^2}{r_*}\right)=\frac{m_*^2G}{r_*^2}=\frac{e_*^2}{r_*^2} \tag{1}$$

where the ratio c^4/G is the curvature superforce that appears in the Einstein field equations. G is Newton's gravitational constant, c is the speed of light, m_* and r_* are the Planck mass and length respectively [4, p.1234], and e_* is the coupling charge.

The two particle/PV coupling forces

$$F_e(r)=\frac{e_*^2}{r^2}-\frac{m_ec^2}{r} \quad \text{and} \quad F_p(r)=\frac{e_*^2}{r^2}-\frac{m_pc^2}{r} \tag{2}$$

the electron core $(-e_*, m_e)$ and proton core $(+e_*, m_p)$ exert on the invisible PV state; along with their coupling constants

$$F_e(r_e)=0 \quad \text{and} \quad F_p(r_p)=0 \tag{3}$$

and the resulting Compton radii

$$r_e=\frac{e_*^2}{m_ec^2} \quad \text{and} \quad r_p=\frac{e_*^2}{m_pc^2} \tag{4}$$

lead to the important string of Compton relations

$$r_em_ec^2=r_pm_pc^2=e_*^2=r_*m_*c^2 \quad (=c\hbar) \tag{5}$$

for the electron and proton cores, where $\hbar$ is the reduced Planck constant. The Planck particle Compton radius is $r_*=e_*^2/m_*c^2$, which is derived by equating the Einstein and Coulomb superforces from (1). To reiterate, the equations in (2) represent the forces the free electron or proton cores exert on the invisible PV space, a space that is itself pervaded by a degenerate collection of Planck-particle cores $(\pm e_*, m_*)$ [5]. The positron and antiproton cores are $(+e_*, m_e)$ and $(-e_*, m_p)$ respectively.

Finally, the Lorentz invariance of the coupling constants in (3) lead to the energy

$$i\hbar\frac{\partial}{\partial t}=ie_*^2\frac{\partial}{\partial ct} \tag{6}$$

and momentum

$$-i\hbar\nabla=-i\frac{e_*^2}{c}\nabla \tag{7}$$

operators of the quantum theory [5]. It should be noted that the two operators are proportional to the squared coupling charge e_*^2.

Section 2 expresses the Dirac equation in terms of PV parameters. Section 3 discusses the fine structure constant. Section 4 discusses the gyromagnetic g-factor. Section 5 discusses the electron g-factor and Section 6, the proton g-factor. Sections 5 and 6 are a work in progress that seek to relate the QED radiative corrections to the PV coupling model. Section 7 presents some comments and conclusions.

2 Dirac equation

Using (5), the manifestly covariant form [6, p.90] [Appendix A] of the Dirac equation for the Dirac particle cores (electron, positron, proton, antiproton) can be expressed as:

$$\left(ic\hbar\gamma^\mu\frac{\partial}{\partial x^\mu}-mc^2\right)\psi=\left(ie_*^2\gamma^\mu\frac{\partial}{\partial x^\mu}-mc^2\right)\psi= \tag{8}$$

$$\left[ie_*^2\gamma^0\frac{\partial}{\partial x^0}+i\begin{pmatrix}0 & cS_j\\ -cS_j & 0\end{pmatrix}\frac{\partial}{\partial x^j}-mc^2\right]\psi=0 \tag{9}$$

where the second term in (9) is summed over $j = 1, 2, 3$ and

$$\begin{pmatrix} 0 & cS_j \\ -cS_j & 0 \end{pmatrix} = \begin{pmatrix} 0 & e_*^2\sigma_j \\ -e_*^2\sigma_j & 0 \end{pmatrix} \tag{10}$$

where one of the charges in e_*^2 belongs to the free particle and the other to any one of the Planck-particle cores within the degenerate PV state. The $e_*^2\sigma_j/c$ from the 4x4 matrix in (10) are the 2x2 spin components of the S-vector

$$\overrightarrow{S} = \frac{e_*^2}{c}\overrightarrow{\sigma} \;\left(= \hbar\overrightarrow{\sigma}\right) \tag{11}$$

that applies to all the Dirac particles. $\overrightarrow{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ is the Pauli spin vector, where the σ_js are 2x2 matrices.

3 Fine structure constant

Using the expressions in (5), the fine structure constant can be expressed as

$$\alpha = \frac{e^2}{e_*^2} = \frac{e^2}{r_* m_* c^2} = \frac{e^2}{r_p m_p c^2} = \frac{e^2}{r_e m_e c^2} \tag{12}$$

where e is the magnitude of the laboratory-observed electron/proton charge. If $e = e_*$, then the Compton relations in (5) yield $\alpha = 1$ for the Dirac equation. Thus it is clear that the fine structure constant provides the "bridge" over which the Dirac equation connects to the charge e.

4 Gyromagnetic ratio *g*

For (8) and (9), the g-factor is exactly $g = 2$ [7, p.667]. This gyromagnetic ratio represents the magnetic to mechanical moment-ratio (13) for the Dirac equation without radiative corrections.

In general (radiative corrections or not), the intrinsic magnetic moment $\overrightarrow{\mu}$ is related to the spin vector $\overrightarrow{s} = \overrightarrow{S}/2$ through the equations [6, p.81]

$$\overrightarrow{\mu} = g\mu_B\overrightarrow{s} \quad \longrightarrow \quad g\mu_B = \frac{\mu}{s} \tag{13}$$

where g is the g-factor and μ_B is the Bohr magneton

$$\mu_B = \frac{e\hbar}{2m_e c} = \frac{ec\hbar}{2m_e c^2} = \frac{ee_*^2}{2m_e c^2} = \frac{er_e}{2} \tag{14}$$

where r_e is the electron Compton radius. Although the g-factor in (13) is exactly 2 for the Dirac equation, there is an anomalous-moment increase to this value due to radiative corrections [6, p.298].

Note that for the Dirac particles where $g = 2$, (13) yields

$$\overrightarrow{\mu} = er_e\overrightarrow{s} \quad \longrightarrow \quad \frac{\mu}{s} = er_e\,. \tag{15}$$

However, this is an unacceptable result for the Dirac proton; so (13) is replaced here by

$$\overrightarrow{\mu} = g\mu_c\overrightarrow{s} \quad \longrightarrow \quad \frac{\mu}{s} = g\mu_c \tag{16}$$

where $\mu_c = er_e/2$ for the electron and $\mu_c = er_p/2$ for the proton. Thus the correct baseline moments, normalized by their common spin, for the Dirac particles are given by (16) with $g = 2$, where

$$\frac{\mu_e}{s} = er_e \quad \text{and} \quad \frac{\mu_p}{s} = er_p \tag{17}$$

are the electron and proton magnetic dipole moments.

5 Electron *g*-factor

When radiative corrections are included with (8) and (9), photon exchanges taking place within the vacuum state lead to a small increase in the electron g-factor and a large increase in the proton g-factor. Using $\alpha^{-1} = 137.0$ [7, p.722] for the inverse fine structure constant in the Schwinger calculation [8] [6, p.298], the relative change in the electron magnetic moment is

$$\frac{\delta\mu}{\mu} = \frac{g}{2} - 1 = \frac{e^2}{2\pi c\hbar} = \frac{1}{2\pi}\frac{e^2}{e_*^2}$$

$$= \frac{\alpha}{2\pi} = 0.001162 \tag{18}$$

where one of the e_*s in the squared coupling charge e_*^2 belongs to the electron and the other to any one of the Planck-particle cores within the degenerate PV state.

In the QED theory, the result in (18) is considered to be a first order (in $\alpha/2\pi$) [6, p.82] radiative correction. Like this first order correction, the higher-order corrections are difficult to calculate, but produce increasingly accurate results based on the QED methodology.

Using (18) to second order in $\alpha/2\pi$ leads to

$$\frac{g}{2} - 1 = \frac{\alpha}{2\pi} - \left(\frac{\alpha}{2\pi}\right)^2 = 0.001160 \tag{19}$$

where the experimental g-factor is [6, p.298]

$$\left(\frac{g}{2} - 1\right)_{exp} = 0.0011596 \approx 0.001160\,. \tag{20}$$

The fortuitous agreement between (19) and (20) depends upon the choice of α in the first paragraph.

6 Proton *g*-factor

The electron is thought to be a true point particle [6, p.82] because it contains no internal structure, as does the proton [9]. In the present context, however, it is appropriate to associate the "size" of the electron and proton with their Compton radii, where the corresponding proton structure constant is defined here by

$$m_p = \frac{r_e}{r_p}m_e \quad \longrightarrow \quad \left(\frac{r_e}{r_p}\right) = \frac{m_p}{m_e} \approx 1836\,. \tag{21}$$

This suggests that the proton g-factor change be estimated from the electron change,

$$\frac{g}{2} - 1 = \left[\frac{\alpha}{2\pi} - \left(\frac{\alpha}{2\pi}\right)^2\right]\frac{r_e}{r_p} = 0.001160\,\frac{r_e}{r_p} = 2.13 \qquad (22)$$

where the experimental g-factor is [6, p.82]

$$\left(\frac{g}{2} - 1\right)_{exp} = 1.79\,. \qquad (23)$$

The agreement between (22) and (23) is remarkable, considering the large magnitude of r_e/r_p. It remains to be seen, however, whether or not (22) leads to something more substantial.

7 Summary and comments

It probably comes as a surprise that the charge associated with the Dirac equation and the Dirac particles is the coupling charge e_*, rather than the well known electron/proton charge e. That bewilderment is due to the collection of Planck particle cores that pervade the PV state. If there were no such pervasion, there would be no photon scattering taking place within the vacuum state and no resulting need for the coupling charge and the radiative corrections from the QED theory.

Sections 5 and 6 present calculations that suggest the PV theory may provide an aid to, or an alternative for, the difficult QED calculations that have been so spectacularly successful. That, of course, remains to be seen. But another hint that the PV theory may be a help is the Schwinger result in Section 5:

$$\frac{\alpha}{2\pi} = \frac{e^2/2\pi r_*}{m_* c^2} = \frac{e^2/2\pi r_p}{m_p c^2} = \frac{e^2/2\pi r_e}{m_e c^2} \qquad (24)$$

where, if r_* is the "radius" of the Planck-particle cores in the PV pervaded space, then $2\pi r_*$ is the "circumference" of the corresponding "spheres" surrounding those cores. Further work will be focused on developing a complete PV approach to the radiative correction phenomenon.

Feynman [10, p.129] notes that: "There is a most profound and beautiful question associated with the coupling constant, e—the amplitude for a real electron to emit or absorb a real photon. It is a simple number that has been experimentally determined to be close to -0.8542455. (My physicist friends won't recognize this number, because thy like to remember it as the inverse of its square: about 137.03597 with an uncertainty of about 2 in the last decimal place. It [the fine structure constant] has been a mystery ever since it was first discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it.)" The mystery of the fine structure constant α resides in the photon scattering that takes place within the pervaded PV state. It is also noted that the apparent electron emission/absorption of photons has its source in the pervaded nature of that state.

Appendix A: The γ and β matrices

The 4x4 γ, β, and α_i matrices used in the Dirac theory are defined here: where [6, p.91]

$$\gamma^0 \equiv \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \qquad (A1)$$

and ($j = 1, 2, 3$)

$$\gamma^j \equiv \beta\alpha_j = \begin{pmatrix} 0 & \sigma_j \\ -\sigma_j & 0 \end{pmatrix} \qquad (A2)$$

and where I is the 2x2 unit matrix and

$$\alpha_j = \begin{pmatrix} 0 & \sigma_j \\ \sigma_j & 0 \end{pmatrix} \qquad (A3)$$

where the σ_j are the 2x2 Pauli spin matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad (A4)$$

and $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$. The zeros in (A1)–(A3) and (A5) are 2x2 null matrices.

The mc in (8) and (9) represents the 4x4 matrix

$$mc\begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} \qquad (A5)$$

and ψ is the 4x1 spinor matrix.

The zero on the right side of (9) represents the 4x4 null matrix and the zeros in (10) represent 2x2 null matrices. The S_j and σ_j in (10) are 2x2 matrices; so their parentheses represent 4x4 matrices.

The coordinates x^μ are

$$x^\mu = (x^0, x^1, x^2, x^3), \quad x^0 \equiv ct. \qquad (A6)$$

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