

# A Model of the Universe Expanding at a Constant Speed

Rostislav Szeruda

Roznov p.R. 75661, Czech Republic. E-mail: rostislav.szeruda@seznam.cz

This article deals with the possibility of finding an alternative model to the expanding universe model which can be in accordance with our astronomical observations. This is considered an easy but not usual model of closed universe with  $k = 1$ ,  $\Lambda = 0$  and  $q = 0$  which provides that mass of this universe is not constant but stepwise increasing.

## 1 Basic ideas and existing work

This article is based on four basic ideas:

1. Model of the universe expanding at a constant speed [3]. Such a model of the universe is not by itself consistent with observation. We observe that the rate of expansion of our universe is accelerating.
2. The idea that the universe may be a black hole is dealt with in [2].
3. The universe was born from a single quantum of energy. The mass of the universe, its size, and the instant speed of particles with a non-zero rest mass are inter-related. The idea was inspired by the book [1].
4. The relative particle size shrinking. This effect makes it seem to us that the universe is not expanding at a constant speed, but that the speed of its expansion is increasing. This idea is new.

## 2 Constant speed expanding universe

We assume that there is no difference between what our universe is and how it appears to us. But is that true? Let us imagine that we are in a room the walls of which are expanding, and we are shrinking just as quickly at the same time. How would this room seem to us?

Let us consider a universe expanding at a constant rate. The elementary particles try to move at the maximum possible speed. The speed of expansion of the universe is a limitation of the instantaneous velocity of the elementary particles within. Thus, particles with zero rest mass (photons) can move as fast as the universe expands:

$$\dot{a} \equiv c \tag{1}$$

where:

- $\dot{a}$  – speed of the universe expansion
- $c$  – speed of light in vacuum.

Further, let's suppose that particles with non zero rest mass have a tendency to move at the speed of light in vacuum too but due to their non zero rest mass they are not able to achieve that speed. The more their speed gets closer to the speed of light in vacuum, the higher their mass becomes and prevents them from moving faster.

Let's have a model of the universe described by Friedmann equations:

$$3\left(\frac{\dot{a}}{a}\right)^2 + \frac{3kc^2}{a^2} - \Lambda c^2 = 8\pi G\rho \tag{2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\rho c^2 + 3p) + \frac{\Lambda c^2}{3} \tag{3}$$

where:

- $G$  – gravitational constant
- $\rho = \rho(t)$  – matter density in universe
- $p = p(t)$  – pressure in the universe
- $a$  – expansion factor of the universe
- $\ddot{a}$  – acceleration of the universe expansion ( $\ddot{a} = 0$  for the model)
- $k$  – parameter of the universe curvature
- $\Lambda$  – Einstein cosmological constant.

Let's consider a Riemann space with a positive curvature, where:

1.  $k = 1$ .
2.  $\Lambda = 0$ .

The Friedmann equations are simplified to:

$$\rho = \frac{3c^2}{4\pi G a^2} \tag{4}$$

$$p = -\frac{1}{3}c^2\rho. \tag{5}$$

The density of the universe is then inversely proportional to the square of the expansion factor  $a$ . It means that the linearly expanding universe is possible only on the condition that its mass is not constant but it rises proportionally to  $a$ . The more matter the universe contains, the larger it becomes and vice versa.

For a closed universe ( $k = 1$ ), we can call the expansion factor  $a$  as the radius of the universe. Its volume is an elementary inter-sphere with surfaces  $4\pi a^2 \sin^2 \psi$  and its thickness  $a d\psi$  ( $0 \leq \psi \leq \pi$ ). We get it by integration:

$$V = a^3 4\pi \int_0^\pi \sin^2 \psi d\psi = 2\pi^2 a^3. \tag{6}$$

The universe mass is then given by the equation:

$$M_v = \frac{3\pi c^2 a}{2G}. \tag{7}$$

### 3 Initial parameters of the universe

Consider that the universe didn't begin its existence with all of the matter contained therein today, but was born from a single energy quantum  $M_0$  in a space of the size of the minimal quantum packet:

$$a_0 = \frac{\hbar}{2M_0 c} = \frac{G\hbar}{3\pi a_0 c^3}. \tag{8}$$

Thence

$$a_0 = \sqrt{\frac{G\hbar}{3\pi c^3}} \cong 5.26 \times 10^{-36} \text{ m}. \tag{9}$$

The minimum time interval then:

$$t_0 = \frac{a_0}{c} = \sqrt{\frac{\hbar G}{3\pi c^5}} \cong 1.76 \times 10^{-44} \text{ s}. \tag{10}$$

The first quantum mass  $M_0$  of the universe is given by the relation:

$$M_0 = \sqrt{\frac{3\pi\hbar c}{4G}} \cong 3.34 \times 10^{-8} \text{ kg}. \tag{11}$$

The universe we describe here resembles a black hole. Its size is directly proportional to the amount of matter it contains:

$$a_{\bullet} = \frac{2GM_{\bullet}}{c^2} \tag{12}$$

$a_{\bullet}$  – radius of a black hole (Schwarzschild radius, horizon of events)

$M_{\bullet}$  – mass of a black hole.

The mass of the first quantum of the universe  $M_0$  is big enough to create a black hole, because the minimum mass of a black hole is given by Planck's relationship:

$$M_{\bullet_0} = \sqrt{\frac{\hbar c}{4G}} \cong 1.09 \times 10^{-8} \text{ kg}. \tag{13}$$

Thus, the initial quantum was below the event horizon, which began at a distance given by the minimum size of the black hole:

$$a_{\bullet_0} = \sqrt{\frac{G\hbar}{c^3}} \cong 1.62 \times 10^{-35} \text{ m}. \tag{14}$$

A black hole of this mass is characterized by temperature:

$$T_{\bullet_0} = \frac{\hbar c^3}{8\pi k G M_{\bullet_0}} = 1.13 \times 10^{31} \text{ K}. \tag{15}$$

### 4 The evolution of the universe

Let the mass of the universe be varied by quanta corresponding to the mass of the first quantum  $M_0$ . Then the size of the universe will change in discrete values, and the passage of time won't be continuous, but it will flow in elementary jumps:

$$M_v = nM_0 \tag{16}$$

$$a = na_0 \tag{17}$$

$$t = nt_0 = n \frac{a_0}{c} \tag{18}$$

where:

$n$  – natural number higher then zero.

The space where the initial quantum can occur is limited by the expansion function of the universe  $a$ . As the mass of the universe starts to grow,  $a$  will increase and matter will have more space to be located and to move. The total energy of the universe is permanently zero.

The universe can have zero total energy if the total gravitational potential energy that holds all its components together is negative and in absolute value is exactly equal to the sum of all positive energy in the universe contained in the masses and movements of the particles.

The matter growth within the universe does not occur by locally emerging new matter, but by increasing the velocity of motion of the initial quantum of energy to a speed close to the speed of light in vacuum:

$$M_v = \alpha_{v_m} M_0 = \frac{M_0}{\sqrt{1 - \frac{v_m^2}{c^2}}} = nM_0 \tag{19}$$

where:

$v_m$  – instant speed of all elementary particles with non zero rest mass. Consider that this speed is the same for all the quantum of energy in the universe. However, the resulting velocity of the particles made up of these quanta appear to be slower as the quantum of energy can move back and forth through space.

The instant speed of particles with rest mass is given by:

$$v_m = \dot{a} \sqrt{1 - \frac{1}{n^2}} = c \sqrt{1 - \frac{1}{n^2}}. \tag{20}$$

The older the universe is, the closer the instant speed of particles with a non zero rest mass is to the universe expansion speed.

$$\lim_{n \rightarrow \infty} v_m = c.$$

At the present time, the two values are not practically distinguishable.

Considering quanta of energy as moving one-dimensional objects, their size should appear smaller due to relativistic contraction:

$$l = \frac{l_0}{\alpha_{v_m}} = l_0 \sqrt{1 - \frac{v_m^2}{c^2}}. \tag{21}$$

The inner observer doesn't know that he is shrinking together with his entire planet, his solar system or his galaxy, at the same time that the universe itself expands. Because he measures the expansion of the universe in comparison with himself, it will seem to him that the universe is expanding faster than it actually is. Due to the contraction of distance, the gravitational force will appear to him stronger. He will attribute it to the greater mass of interacting objects.

Therefore, from the perspective of the internal observer, the size and mass of the universe will appear:

$$a_i = \alpha_{v_m}^2 a_0 = n^2 a_0 \quad (22)$$

$$M_i = \alpha_{v_m}^2 M_0 = n^2 M_0. \quad (23)$$

The fact, that the universe expands with speed  $\dot{a} = c$  perpendicular to our three-dimensional space and to all speed vectors in it, can be expressed by adding an imaginary mark before the value of the expansion speed. Generally, we can express the speed of a mass object  $w$  this way:

$$w = v + i\dot{a} \quad (24)$$

where:

$v$  – an object speed in our three-dimensional space.

The square of  $w$  can be expressed in the form:

$$w^2 = v^2 - \dot{a}^2 \left(1 - \frac{v^2}{c^2}\right). \quad (25)$$

Now the Einstein relativistic coefficient  $\alpha$  gets the more general form:

$$\begin{aligned} \alpha_w &= \frac{1}{\sqrt{1 - \frac{w^2}{c^2}}} = \alpha_v \alpha_{\dot{a}} \\ &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \frac{1}{\sqrt{1 + \frac{\dot{a}^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \frac{1}{\sqrt{2}}. \end{aligned} \quad (26)$$

The first coefficient  $\alpha_v$  in the relation (26) is the standard form of Einstein coefficient  $\alpha$ . The second coefficient  $\alpha_{\dot{a}}$  is related with the speed of the universe expansion and it is constant. So the universe will appear to us  $\sqrt{2}$  times bigger but not more massive.

$$a_i = \sqrt{2} n^2 a_0 \quad (27)$$

$$M_i = \frac{\sqrt{2} n^2 3\pi c^2 a_0}{2\sqrt{2}G} = n^2 M_0. \quad (28)$$

The universe density will seem to be equal with the critical density:

$$\rho_i = \frac{3c^2}{4\pi G \rho \left(\frac{a}{a_0}\right)^2} = \frac{3\dot{a}^2}{8\pi G a^2} = \frac{3H^2}{8\pi G} = \rho_k \quad (29)$$

where:

$H$  – Hubble constant:

$$H \equiv \frac{\dot{a}}{a} = \frac{c}{a}. \quad (30)$$

The density of the universe, in case of inner observer, thus seems to be equal to the critical density. It corresponds to our observation. In contrast to the inflation model it happens not only effectively. Therefore, the entire universe appears to be non-curved - flat, even though it is closed.

## 5 The universe pressure

The change of the internal energy of the universe corresponds with the change of its energy. The universe can't exchange heat with its surroundings. Then the first theorem of thermodynamics has an easy form by which we can express a change of the universe energy as:

$$dU = -pdV = c^2 dM. \quad (31)$$

Mass movement in the direction of the expansion of the universe and its rise with time induce a force, which has size:

$$F = v^2 \frac{dM}{dt} c = -\frac{3\pi c^4}{2G}. \quad (32)$$

This force acts on the surface:

$$S = 6\pi^2 a^2. \quad (33)$$

This creates a pressure that is already known from the relation (5):

$$p = \frac{-c^4}{4G\pi a^2} = -\frac{1}{3} c^2 \rho. \quad (34)$$

The pressure in the universe is negative at a positive energy density. However, matter and radiation create positive pressure. It thus appears rather a local phenomenon operating in three-dimensional space, which has no effect on the four-dimensional universe as a whole.

## 6 The universe age and mass

Three-dimensional black holes radiate energy from their horizon into the surrounding space. The horizon of a black hole bound up to the universe produces radiation which is moving on the surface of a four-dimensional sphere and remains part of it. As the universe expands, it cools down in such a way that its temperature corresponds to the current temperature of the black hole horizon.

The temperature of the radiation emitted at the beginning of the universe is now the same as the temperature of the radiation from the event horizon. The universe thus appears as the interior of the black body, where the radiation density is given by:

$$U = \frac{\pi^2 (kT)^4}{15 (\hbar c)^3}. \quad (35)$$

For the temperature of the relic radiation  $T_r = 2.726$  K results the energy density  $U \cong 4.18 \times 10^{-14} \text{ J m}^{-3} \cong 0.26 \text{ eV cm}^{-3}$

out of the relation (34). This corresponds to the measured value of the density of the relict radiation  $0.25 \text{ eV cm}^3$ .

If the temperature of the universe at its beginning corresponded to the temperature of the black hole horizon according to the relation (15), today it should correspond to the temperature:

$$T_{\bullet_n} = \frac{\hbar c^3}{8\pi k G M_{\bullet_n}} = \frac{T_{\bullet_0}}{n}. \quad (36)$$

If the temperature of the relict radiation  $T_r = 2.726 \text{ K}$  corresponds to the present temperature of the universe and at the same time to the temperature of the radiation from the early universe:

$$n \cong 4.14 \times 10^{30}. \quad (37)$$

The size of the universe (from the perspective of an inner observer) is then:

$$a_i = \sqrt{2} n^2 a_0 \cong 1.27 \times 10^{26} \text{ m}. \quad (38)$$

The Hubble constant  $H$  is then according to (29):

$$H = \frac{c}{a} \cong 2.35 \times 10^{-18} \text{ s}^{-1} \cong 72.63 \text{ km s}^{-1} \text{ Mpc}^{-1}. \quad (39)$$

This value is consistent with the value of the Hubble constant determined in 2018:  $H = 73.52 \pm 1.62 \text{ km s}^{-1} \text{ Mpc}^{-1}$ . The actual age of the universe is therefore:

$$t = \frac{1}{H} \cong 13.5 \times 10^9 \text{ yrs}. \quad (40)$$

The mass of the universe (from the point of view of an internal observer) is:

$$M_i = n^2 M_0 \cong 5.72 \times 10^{53} \text{ kg}. \quad (41)$$

## 7 Visible and invisible matter

We already know how the mass and size of the universe as a whole changes. How can the mass and size of its parts change? The mass of all objects has to change, for an internal observer, according to:

$$m_2 = m_1 \frac{t_2}{t_1} = m_1 \frac{n_2^2}{n_1^2} \quad (42)$$

where:

$m_1$  – object mass at time  $t_1$  ( $\sim n_1^2$ )

$m_2$  – object mass at time  $t_2$  ( $\sim n_2^2$ ).

The fact that this relationship is true can be seen in the motion of matter around the centers of galaxies. Outside the galaxy, the mass should move with velocity according to the standard model (see curve A in Fig. 1)

$$v^2 = \frac{GM_g}{r} \quad (43)$$

where:

$M_g$  – galaxy mass

$r$  – distance from the galaxy center.

If the mass at the edge of the galaxies is pulled away from the center of the galaxy due to the universe expansion and grows with distance ( $M_g(r) \sim r$ ) according to the relation (42), although most of this mass cannot be observed, their velocity around the galaxy's gravitational center remains the same in Fig. 1 – the rotation curve becomes flat from a certain distance from the center.

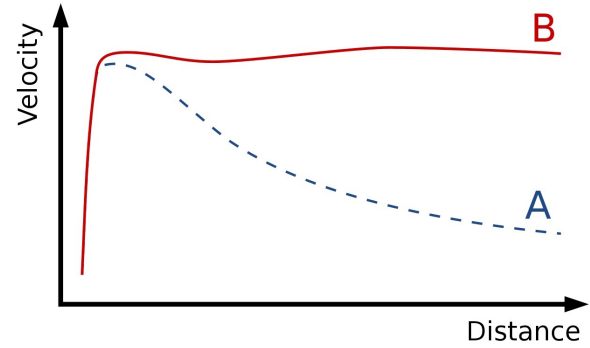


Fig. 1: Dependence of orbital velocity on distance from center of galaxy

The relation (42) describes the total amount of matter (perceivable or non-perceivable) that increases depending on space-time expansion. So what about the perceivable matter? If the relation (42) also applies to photons, and we still observe a redshift, this means that the first quantum of energy must be fragmented into a larger number of smaller quanta. For photons:

$$m_{f2} = m_{f1} \frac{n_{f1}}{n_{f2}} \frac{t_2}{t_1} \quad (44)$$

where:

$n_{f1}$  – number of photons at time  $t_1$

$n_{f2}$  – number of photons at time  $t_2$

$m_{f1}$  – photon mass at time  $t_1$

$m_{f2}$  – photon mass at time  $t_2$ .

If the universe with temperature  $T_1$  at time  $t_1$  contained  $n_{f1}$  particles then it will have at time  $t_2$  temperature  $T_2$  and will contain  $n_{f2}$  particles:

$$\frac{n_{f2}}{n_{f1}} = \frac{p_2 V_2}{p_1 V_1} \frac{T_1}{T_2} = \frac{a_2}{a_1} \frac{T_1}{T_2} = \frac{t_2}{t_1} \frac{T_1}{T_2} = \frac{n_2^3}{n_1^3}. \quad (45)$$

After insertion into (44):

$$m_{f2} = m_{f1} \frac{T_2}{T_1} = m_{f1} \frac{n_1}{n_2} \quad (46)$$

$$\lambda_2 = \lambda_1 \frac{T_1}{T_2} = \lambda_1 \frac{n_2}{n_1}. \quad (47)$$

As the temperature of the universe is decreasing, the mass of the photons has to decrease too. The radiation on the way through the universe gets colder, but the number of observable photons increases – as if the universe in the past contained the same amount of matter as today.

The particulate mass with non-zero rest mass will grow by (42) but simultaneously their wavelength will lengthen according to (47). Their mass is then given:

$$m_2 = m_1 \frac{T_2}{T_1} \frac{t_2}{t_1} = m_1 \frac{n_2}{n_1} \quad (48)$$

where:

$m_1$  – mass of a “cold” particle at time  $t_1$

$m_2$  – mass of a “cold” particle at time  $t_2$ .

As the mass of the universe increases, the number of quanta energies increases faster and thus their energy decreases. The smallest quanta of energy now have mass:

$$M_{0n} = \frac{n^2 M_0}{n^3} = \frac{M_0}{n} \cong 8.08 \times 10^{-39} \text{ kg}. \quad (49)$$

Relationships (46) and (48) describe observable mass. This is obviously lesser than the mass objects should have by the equation (42). Mass corresponding to the difference we can't directly observe, but we can observe its gravitational effect. The matter we name: “dark matter”. This is the “missing” matter around the galaxies.

## 8 Observable quantity of energy

A standardized wave packet is related with the whole universe and it moves in direction of the universe expansion [4]:

$$|\psi(a; t)|^2 = \frac{1}{\sqrt{2\pi} \Delta a_t} \exp \left[ -\frac{(a - ct)^2}{2(\Delta a_t)^2} \right]. \quad (50)$$

The wave packet related to the universe shows a dispersion which causes it to seem higher. For as much that the mass of the universe increases linearly with time, the dispersion is independent of time:

$$\begin{aligned} \Delta a_t &= \sqrt{(\Delta a_0)^2 + \left( \frac{\Delta(m_0 \dot{a})}{m} t \right)^2} \\ &= \sqrt{a_0^2 + \left( \frac{m_0 c t_0}{m_0} \right)^2} = a_0 \sqrt{2}. \end{aligned} \quad (51)$$

This result is in agreement with  $\alpha_a = 1/\sqrt{2}$  from the relation (26). The amplitude of this wave package relative to  $a_0$  is then:

$$|\psi(a = ct; t)|^2 a_0 = \frac{1}{2\sqrt{\pi}} \cong 0.282. \quad (52)$$

It means that if the universe size is  $a$ , then on quantum level corresponding to this size it is about 28.2 % of the whole universe energy. The rest of the universe energy 71.8 % occurs on near quantum levels.

If we are situated on quantum level at the size  $a$  from imaginary centre of our universe, we are able to observe only the mass situated on the same quantum level. It means that the rest of our universe mass is not observable for us even though it gravitationally influences our universe as a whole.

## 9 Cosmological shift of spectrum

Perception (measurement) of time flow was obviously different than it is today. Physical process lasting 1 s at present time lasted  $n_2/n_1$  times longer in the past. Dimensions of mass objects were  $n_2/n_1$  times bigger and photons radiated from them had  $n_2/n_1$  times longer wavelength than they have in the same process today.

The shift of the spectrum of the radiation of the cosmological objects is defined:

$$z \equiv \frac{\lambda_r - \lambda_e}{\lambda_e}. \quad (53)$$

This relation presumes that the spectrum of cosmological source was the same in the past and today and the cosmological shift has happened during the travel from the source to an observer in consequence of the universe expansion. If the particles that create mass had smaller mass in the past than today then the energy radiated from them was equivalently smaller than today. We should rather write:

$$z = \frac{\lambda_r - \lambda_{e\text{-today}}}{\lambda_{e\text{-today}}}. \quad (54)$$

In case that the mass of elementary particles were smaller in the past, then:

$$\lambda_e = \lambda_{e\text{-today}} \frac{n_r}{n_e}. \quad (55)$$

According to (42), (54) and (55) results (as in classical theory):

$$z + 1 = \frac{\lambda_r}{\lambda_{e\text{-today}}} = \frac{\lambda_r}{\lambda_e} \frac{n_r}{n_e} = \frac{a_r}{a_e}. \quad (56)$$

## 10 Luminosity of cosmological sources

If the red shift does not exist, the apparent luminosity  $l$  of a cosmological source would be given by relation:

$$l = \frac{L}{S} \quad (57)$$

where:

$L$  – absolute luminosity of a cosmological source

$S$  – area on which photons from the cosmological source fall to.

The radiation energy from a cosmological source decreases in three ways:

1. The energy of the detected photons is lower then their original energy due to red shift according to (57).

- Photons radiated during time interval  $t_{e\text{-today}}$  (time the process would last today) will reach target during time interval  $\Delta t_r$ :

$$\frac{\Delta t_r}{\Delta t_{e\text{-today}}} = \frac{\lambda_r}{\lambda_{e\text{-today}}} = 1 + z. \quad (58)$$

- We can't forget influence of lesser particle mass in the past:

$$\frac{\lambda_e}{\lambda_{e\text{-today}}} = \sqrt{\frac{a_r}{a_e}} = \sqrt{1 + z}. \quad (59)$$

The relative luminosity  $l$  of a typical cosmological source (cosmological candle) can be then written in the form:

$$l = \frac{L}{4\pi d_L^2} = \frac{L}{4\pi r_e^2 a^2 (1 + z)^{2.5}} \quad (60)$$

where:

$d_L$  – distance of a cosmological source given by:

$$d_L = r_e a (1 + z)^{1.25}. \quad (61)$$

The variable  $r_e$  is given by [5] for  $k = 1$  and  $\ddot{a} = 0$  by the relation:

$$r_e = c \sin \left( \int_{t_e}^{t_r} \frac{dt}{a} \right) = \sin \left( \ln \frac{t_r}{t_e} \right) = \sin [\ln (1 + z)]. \quad (62)$$

The relative magnitude of stars  $m$  is defined by the Pogson equation [6]:

$$m = -2.5 \log \left( \frac{I}{I_0} \right) \quad (63)$$

where  $I_0$  is the bolometric reference value  $2.553 \times 10^{-8} \text{ W m}^{-2}$ .

Now we can calculate value  $l$  (for suitable  $L$ ) in the relation (60) and calculate the curve  $m = m(z)$  using the relation (63) (see Fig. 2). The best fit with real measured values of relative magnitude of supernovas type Ia [7] we get for  $L \cong 2.765 \times 10^{28} \text{ W}$ . It acknowledges that the model above can correspond with our reality.

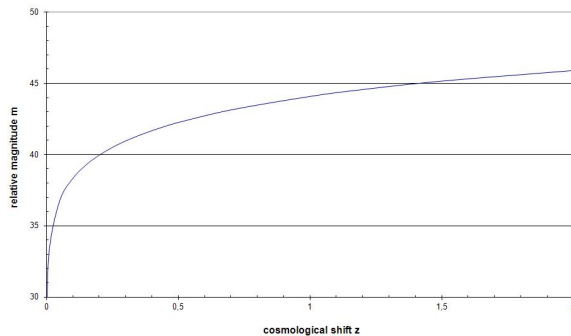


Fig. 2: Relative supernova magnitude – calculated for  $L = 2.765 \times 10^{28} \text{ W}$

We can construct the so-called residual Hubble diagram – relative luminosity of supernovas related to the case of an empty universe ( $\Omega = 0, k = -1, q = 0$ ) (see Fig. 3).

$$\Delta(m - M) = 5 \log \left( \frac{r_e}{r_{e0}} \right). \quad (64)$$

$$r_{e0} = \sinh[\ln(1 + z)]. \quad (65)$$

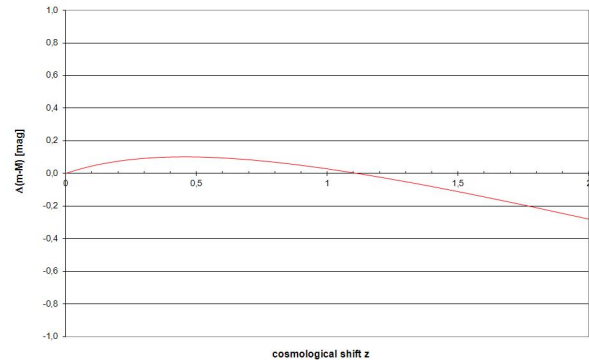


Fig. 3: Residual Hubble diagram – without consideration of dust influence

### 11 Conclusion

Our universe doesn't have to be necessarily open and accelerating its expansion in order to be in accordance with our present observation and knowledge. In this article, I tried to show that our universe can be closed and uniformly expanding supposing that its mass increases proportionally to its size and analogically its size increases proportionally to its mass, similarly as black holes do.

Received on July 22, 2020

### References

- Bohm D., Factor D., ed. *Unfolding Meaning: A weekend of dialogue with David Bohm*. Foundation House, Gloucestershire, 1985.
- Pathria R. K. The universe as a black hole. *Nature*, 1972, v. 240 (5379), 298–299.
- Skalský V. The model of a flat (Euclidean) expansive homogeneous and isotropic relativistic universe in the light of general relativity, quantum mechanics, and observations. *Astrophys. Space Sci.*, 2010, v. 330, 373–398.
- Cely J. *Zaklady kvantove mechaniky pro chemiky (Fundamentals of quantum mechanics for chemists)*. Univerzita Jana Evangelisty Purkyne, Bmo, 1986, p. 45.
- Misner C. W., Thorne K. S., Wheeler J. A. *Gravitation*. W. H. Freeman and Co., San Francisco, USA, 1973.
- <http://aldebaran.cz/studium/astrofyzika.pdf>, p. 7.
- [http://www.astro.ucla.edu/~wright/sne\\_cosmology.html](http://www.astro.ucla.edu/~wright/sne_cosmology.html) [18.4.2008].
- [http://aldebaran.cz/bulletin/2004\\_33\\_dma.html](http://aldebaran.cz/bulletin/2004_33_dma.html) [18.4.2008].