

Space-Time Quantification

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The quantification of Length and Time in Kepler's laws implies an angular momentum quantum, identified with the reduced Planck's constant, showing a mass-symmetry with the Newtonian constant G . This leads to the Diophantine Coherence Theorem which generalizes the synthetic resolution of the Hydrogen spectrum by Arthur Haas, three years before Bohr. The Length quantum breaks the Planck wall by a factor 10^{61} , and the associated Holographic Cosmos is identified as the source of the Background Radiation in the Steady-State Cosmology. An Electricity-Gravitation symmetry, connected with the Combinatorial Hierarchy, defines the steady-state Universe with an invariant Hubble radius 13.812 milliard light-year, corresponding to 70.796 (km/s)/Mpc, a value deposited (1998) in a Closed Draft at the Paris Academy, confirmed by the WMAP value and the recent Carnegie-Chicago Hubble Program, and associated with the Eddington number and the Kotov-Lyuty non-local oscillation. This confirms definitely the Anthropic Principle and the Diophantine Holographic Topological Axis rehabilitating the tachyonic bosonic string theory. This specifies G , compatible with the BIPM measurements, but at 6σ from the official value, defined by merging discordant measurements.

1 The Diophantine Coherence Theorem (DCT)

For connecting different physical measurements, Physics uses multiplication while addition is forbidden. But multiplication is a generalization of addition [1]. This paradox may be suppressed by considering only numerical ratios of the same physical quantity, as in the third Kepler law, *introducing Space and Time quanta* L_1 and T_1 [15]. Considered as a Diophantine Equation, which uses only natural numbers n , it resolves directly:

$$\begin{aligned} (T_n/T_1)^2 &= (L_n/L_1)^3 \equiv n^6 \\ \Rightarrow T_n &= n^3 T_1 ; L_n = n^2 L_1 . \end{aligned} \quad (1)$$

This proceeds from the Holic Principle [27], a Diophantine form of the Holographic Principle, which states that Physics is described through the simplest degenerate Diophantine Equations, where the exponents identify with the dimensions 3 for Space, 2 for a 2D Time [39], 5 for Mass, and 7 for Field. The n -invariant L_n^3/T_n^2 is homogeneous to Gm_G , where G is Newton's gravitational constant, and m_G is a mass (here the usual central mass is divided by the factor $4\pi^2$). The other Kepler's law states that the orbital angular momentum per unit mass is an orbital invariant. Since the corresponding term L_n^2/T_n is proportional to n , this implies an orbital momentum quantum, identified to the reduced Planck constant, or *action quantum* \hbar , privileged by the particle physics in the spin concept. While the ratio of the kinematic parts of G and \hbar are homogeneous to a speed, these two universal constants presents a symmetry by respect to the mass concept, implying the as-

sociation of \hbar with a mass m_{\hbar} :

$$L_n^3/T_n^2 = Gm_G \quad ; \quad L_n^2/T_n = n\hbar/m_{\hbar} . \quad (2)$$

Any mass pair (m_G, m_{\hbar}) is associated to a series of Keplerian orbits (L_n, T_n) :

$$L_n = \frac{(n\hbar)^2}{Gm_G m_{\hbar}^2} \quad ; \quad T_n = \frac{(n\hbar)^3}{G^2 m_G^2 m_{\hbar}^3} . \quad (3)$$

For $n = 1$ and $m_G = m_{\hbar} = m$, the Special Non-Local Length and Time are:

$$L_{NL}(m) = \frac{\hbar^2}{Gm^3} \quad ; \quad T_{NL}(m) = \frac{\hbar^3}{G^2 m^5} . \quad (4)$$

Introducing the *formal velocity* $V_n = L_n/T_n$, this connects the reduced Planck energy $n\hbar/T_n$ with the gravitational potential energy pertaining to masses m_G and m_{\hbar} and the energy $m_{\hbar}V_n^2$:

$$\begin{aligned} V_n &= L_n/T_n = Gm_G m_{\hbar}/n\hbar \\ \Rightarrow n\hbar/T_n &= Gm_G m_{\hbar}/L_n = m_{\hbar}V_n^2 . \end{aligned} \quad (5)$$

With the Planck mass $m_P = \sqrt{\hbar c/G}$, where the light speed c is the third universal constant, this reads

$$\frac{n\hbar}{T_n} = \frac{Gm_G m_{\hbar}}{L_n} = m_{\hbar}V_n^2 \equiv m_{\hbar} \left(\frac{c}{nA} \right)^2 \quad ; \quad A = \frac{m_P^2}{m_G m_{\hbar}} . \quad (6)$$

This is called the *Diophantine Coherence Theorem* (DCT).

2 The atom H and the Holographic Cosmos

Three years before Bohr, Arthur Haas [3] considered *the electron orbital period* in the Rutherford model, and the corresponding Planck energy $nhv = nh/T_n = n\hbar v_n/L_n$ where $v_n = 2\pi V_n$ is the orbital velocity. The correct Hydrogen spectrum is obtained by equalizing it with the electric potential energy $\hbar c/aL_n$, where $a \approx 137.0359991$ is the electric constant, and the double (virial) kinetic electron energy $m_e v_n^2$ (the useful physical constants are listed in Table 1):

$$n\hbar \frac{v_n}{L_n} = \frac{\hbar c}{aL_n} = m_e v_n^2 \equiv m_e \left(\frac{c}{na}\right)^2. \quad (7)$$

Note that the so-called “properties of vacuum” ϵ_0 and μ_0 are unnecessary: they are only introduced for historical reasons, leading to the cumbersome, but official, choice of electrical units, hiding the true “electrical constant” a , whose inverse α , called “the fine structure constant” is of minor importance. For $n = 1$, this gives the bare Haas-Bohr radius: $r_{HB} = a\lambda_e$, where $\lambda_e \equiv \hbar/(m_e c)$ is the Reduced Electron wavelength (the effective electron mass effect defines the Bohr radius $r_B = r_{HB}/(1 + 1/p)$). This double equation shows up the same form that the above DCT (6), where additional 2π factors are integrated in the definitions of m_G and m_{\hbar} . The identification of potential energy terms implies $m_G m_{\hbar} = m_p^2/a$, thus in this case $A = a$. The simplest choice $m_{\hbar} = m_e$ implies the following m_G , where $m_N = am_e$ is the Nambu mass, a quasi-quantum in Particle Physics [17]:

$$m_{\hbar} = m_e \quad ; \quad m_G = \frac{m_p^2}{m_N} \quad ; \quad A = a. \quad (8)$$

This last mass is $m_G \approx 3.7939 \times 10^{12}$ kg, whose corresponding Special Length (4) is:

$$d_0 = L_{NL}(m_p^2/m_N) \approx 3.051 \times 10^{-96} \text{ meter}. \quad (9)$$

This is the Cosmic Space Quantum d_0 breaking the “Planck Wall” by a factor 10^{61} which has been associated to the Cosmos holographic radius R_{hol} [14]:

$$\pi \left(\frac{R_{hol}}{l_p}\right)^2 = 2\pi \frac{R_{hol}}{d_0}. \quad (10)$$

This is the Bekenstein-Hawking Entropy formula of the Holographic Principle [6] where the *Planck Length*

$$l_p \equiv (G\hbar/c^3)^{1/2} \equiv L_{NL}(m_p)$$

is a basic holographic length. The Cosmos radius R_C has been defined by the natural *mono-chromatic* holographic extension:

$$\pi \left(\frac{R_{hol}}{l_p}\right)^2 = 2\pi \frac{R_{hol}}{d_0} = 2\pi \frac{R_C}{l_p}, \quad (11)$$

leading to:

$$\begin{aligned} R_{hol} &= 2L_{NL}(m_N) \approx 18.105 \text{ Giga light-year (Glyr)} \\ R_C &= 2L_{NL}(m_N^2/m_p) \approx 9.075 \times 10^{86} \text{ meter}. \end{aligned} \quad (12)$$

Table 2 shows this symmetry between the Nambu mass m_N and the Planck mass m_p , whose large value is the source of the “Hierarchical Problem” [41]. From $P/\sqrt{a} \approx a_w n_i^3$, where $P = m_p/m_e$, these formula leads to a confirmation of the optimal G value in the ppb domain (Table 1), where $\beta = (H - p)^{-1}$

$$\left(\frac{P}{a_w}\right)^3 \approx \left(\frac{4\pi}{\sqrt{a}}\right)^8 \frac{(pH\beta^2)^5}{2} \approx \frac{aW}{137Z} (pH)^5 (16 \text{ ppm}), \quad (13)$$

showing the role of the geometrical factor 4π .

Now $L_{NL}(\sqrt{m_p m_N}) \approx \lambda_{CMB}/2a_s^2 (2a_s^2 \sim a)$, tying to 0.3% the strong coupling a_s , and the nominal wavelength $\hbar c/kT_{CMB}$ of the Cosmic Microwave Background (CMB), whose source is lacking in the steady-state cosmology [7]. The simplest hypothesis is that the above Cosmos is this source. Indeed, the Wien CMB wavelength λ_{Wn} enters (0.1%):

$$4\pi \left(\frac{R_{hol}}{\lambda_{Wn}}\right)^2 \approx e^a. \quad (14)$$

This perfect holographic formula suggests that *the background would be coherent, meaning it brings information*. This could be the real significance of the CMB Anisotropy Statistics [29].

3 The gravitational hydrogen molecule

The Haas method was already applied to the *special three-body* dihydrogen molecule [13, p.391]:

$$n\hbar \frac{v_n}{L_n} = \frac{Gm_p m_H}{L_n} = m_e v_n^2, \quad (15)$$

The comparison with the above Haas equation implies the substitution: $a \rightarrow a_G = m_p^2/m_p m_H$, corresponding to the following m_G value:

$$m_{\hbar} = m_e \quad ; \quad m_G = m_{bc} \quad ; \quad A = a_G \quad (16)$$

where $m_{bc} = m_p m_H/m_e$ is close to the DNA bi-codon mass, which shows a central position in the Topological Axis [13], corresponding to the dimension 16. Indeed the topological term $f(16) = e^{16}$ is close to pH , and, more precisely, to $2n_i^4/a^3$ (0.04%).

For $n = 1$, this Haas-Sanchez radius R_{H_2} shows a direct Electricity-Gravitation symmetry, by respect to the Reduced Electron wavelength $\lambda_e = \hbar/m_e c$:

$$\begin{aligned} r_{HB} &= a\lambda_e = a \frac{\hbar}{m_e c} \\ R_{H_2} &= a_G \lambda_e = \frac{\hbar^2}{Gm_e m_p m_H} \equiv L_{NL}(m_0), \end{aligned} \quad (17)$$

where $m_0 = (m_e m_p m_H)^{1/3}$. Note that a and a_G are very close to the last two terms of the Combinatorial Hierarchy 137 and $N_L + 137$, with $N_L = 2^{127} - 1$, the Lucas Number [12].

Table 1: Physical constants

Quantity	Value	Unit	10^{-9}
Electrical Constant a	137.035999084(21)	-	0.15
Electron Excess Magnetic moment d_e	1.00115965218096	-	0.26
Official Strong Coupling constant	8.45(5)	-	
Optimal Strong Coupling Constant a_s [15]	8.434502914	-	
Proton/Electron mass ratio p	1836.152 673 43	-	0.06
Proton/Electron Wyler mass ratio p_W [33]	$6\pi^5$	-	exact
Neutron/Electron mass ratio n_t	1838.683 661 7	-	0.5
Hydrogen/Electron mass ratio H	1837.152 660 14	-	0.06
Hydrogen Relativist correction factor $\beta = 1/(H - p)$	1.0000266	-	
Optimal Muon/Electron mass ratio μ [14]	206.768 286 9	-	
Optimal Higgs Boson mass m_{H_g} [15]	$495^2 m_e$	-	
Action quantum \hbar	$1.054 571 81 10^{-34}$	J s	exact
Official Gravitation Constant G_{off}	$6.674 30 \times 10^{-11}$	$\text{kg}^{-1} \text{m}^3 \text{s}^{-2}$	
Optimal Gravitation Constant G	$6.67545272 \times 10^{-11}$ [14]	$\text{kg}^{-1} \text{m}^3 \text{s}^{-2}$	
Speed of light in vacuum c	299 792 458	m s^{-1}	exact
Optimal Fermi Constant $G_F = \hbar^3/cm_F^2$	$1.435 851 10^{-62}$	J m^3	
Optimal Fermi mass ratio $m_F/m_e = F = a_w^{1/2}$	573007.3652	-	
W boson mass ratio $W = m_W/m_e$	157298 ± 23	-	1.5×10^5
Z boson mass ratio $Z = m_Z/m_e$	178450 ± 4	-	2.3×10^4
Electron mass m_e	$9.109 383 701 5 10^{-31}$	kg	0.3
Boltzmann Constant k	$1.380649 10^{-23}$	J K^{-1}	exact
Reduced Electron Wavelength λ_e	$3.861 592 675 10^{-13}$	m	0.3
Measured CMB temperature T_{CMB}	2.725 5(6)	Kelvin	
Optimal CMB Temperature T_{CMB}	2.725 820 138 [14]	K	
Optimal CMB Wien wavelength λ_{Wn}	$1.063 082 472 10^{-3}$ [14]	m	
Optimal CMB reduced wavelength $\hbar\lambda_{CMB} = \hbar c/kT_{CMB}$	$8.400 716 617 10^{-4}$ [14]	m	
Optimal CNB Temperature $T_{CNB} \equiv T_{CMB}(11/4)^{-1/3}$	1.945 597 [14]	Kelvin	
Optimal CNB reduced wavelength $\lambda_{CNB} = \hbar c/kT_{CNB}$	$1 176 956 918 10^{-3}$ [14]	m	
Optimal critical density $\rho_{cr} = 3c^2/8\pi GR^2$	$9.411 979 89 10^{-27}$	$\text{kg m}^{-1/3}$	
Kotov P_0 period t_K	9600.606(12) [19]	s	1200

In R_{H_2} the speed c is eliminated: for this reason, a precise approximation was immediately guessed by the c -free “dimensional analysis”, the so-called *Three Minutes Formula*, from the ternary symmetry Electron-Proton-Neutron (Closed Letter to the Paris Science Academy, March 1998) [22] (see Table 2). The associated Special time $T_{NL}(m_0)$ is very close (0.9%) to the time associated to the triplet: \hbar , the Fermi constant G_F and the associated critical steady-state density $\rho_{cr} =$

$3c^2/8\pi GR^2$ where $R = 2R_{H_2}$ and it is

$$\hbar^4/G_F^{5/2}\rho_{cr}^{3/2} \approx 3m_p^2 R_{hol}/c m_e m_Z$$

(0.01%), comforting the following steady-state Universe.

4 The Steady-State Universe revisited

A salient feature of the Universe is its critical character, relating its horizon radius R with its mass by $R = 2GM/c^2$. How-

Table 2: Values of the DCT Fundamental ($n = 1$) Radius $\hbar^2/Gm_Gm_{\hbar}^2$ for specific values of m_G and m_{\hbar} . Planck mass: m_P . Nambu mass: $m_N = am_e$. Holographic ratio $u = R_{hol}/R$. Proton mass: m_p . Hydrogen mass: m_H . Mean Atomic mass: $m_0 = (m_e m_p m_H)^{1/3}$. Bicondon mass $m_{bc} = m_p m_H / m_e$. Photon mass $m_{ph} = \hbar/c^2 t_K \approx 1.2222 \times 10^{-55}$ kg. Graviton mass: $m_{gr} = m_{ph}/a_w \approx 3.7223 \times 10^{-67}$ kg [14]. Optimal Higgs boson mass: $m_{Hg} = 495^2 m_e$.

m_G	m_{\hbar}	Length	Symbol	Precision/offset
m_P^2/m_N	m_P^2/m_N	Space Quantum	d_0	exact
m_P^2/m_0	m_P^2/m_0	Topon	λ_M	exact
m_{bc}/a_w	$m_e \sqrt{a_w a_G}$	Reduced Electron Wavelength	λ_e	exact
m_P^2/m_N	m_e	Hass-Bohr radius $r_{HB} = a\lambda_e = r_B/(1 + 1/p)$	r_{HB}	exact
$a^3 m_P$	$\sqrt{m_p m_H}$	Background Wien Wavelength	λ_W	3.2×10^{-4}
m_{bc}	m_{bc}	Twice Kotov Length	$2l_K$	6.3×10^{-3}
m_{Hg}	m_{Hg}	$R\lambda_e/4\lambda_{CMB}$ $Ra_w^{1/2}/WZ^2$		- 0.23% + 0.25%
m_{bc}	m_e	Half Universe Radius	$R_{H_2} \equiv R/2$	exact
m_N	m_N	Half Holographic Cosmos radius	$R_{hol}/2$	exact
m_N^2/m_P	m_N^2/m_P	Half Cosmos Radius	$R_C/2$	exact
$u \times m_{bc}$	$\sqrt{m_{ph} m_{gr}}$	Cosmos radius	R_C	1.7×10^{-3}

ever, in the initial “flat universe” model [32], the total mass M is only matter, while in the present Λ CDM standard model, it is separated between a material part with relative density Ω_m and a so-called “dark energy” part with relative density $1 - \Omega_m$ [29]. We have noted that Ω_m is compatible with 3/10, which is both the density of the classical gravitational energy of a critical homogeneous ball and the density of the steady-state *non-relativist* recession kinetic energy [14]. While the standard cosmology uses an ad-hoc inflation to justify this observed critical condition, we consider rather the Universe as a particle (Topon) in the above Cosmos, with the Topon wavelength $\lambda_M \equiv \hbar/Mc = 2\hbar G/Rc^3 \equiv 2l_p^2/R$. Then, the critical condition results from the Bekeinstein-Hawking entropy holographic relation, as above (10), where the Topon appears as a secondary Length-Quantum, since the wavelength λ_m associated for any particle of mass m is a whole multiple n_m of the Topon, in conformity with the Field Quantum Theory. The geometrical interpretation is clear: it is a sphere area described by a whole number of sweeping circles, illustrating the fact that multiplication is a series of additions:

$$4\pi \left(\frac{R_{HB}}{l_p} \right)^2 = \pi \left(\frac{R}{l_p} \right)^2 = 2\pi \frac{R}{\lambda_M} \equiv 2\pi n_m \frac{R}{\lambda_m} \quad (18)$$

$$\Rightarrow M = \frac{Rc^2}{2G} \equiv \frac{R_{H_2} c^2}{G},$$

identifying twice the above Haas-Sanchez’s gravitational radius R_{H_2} with R , the steady-state Universe horizon radius, which is also the limit of a theoretical star radius when its

number of atoms shrinks to one [21], a central length in astrophysics, leading to the Machian formula:

$$R = 2 \frac{\hbar^2}{Gm_e m_p m_H} \Rightarrow M = \frac{m_P^4}{m_e m_p m_H}. \quad (19)$$

The effective electron mass $m'_e = m_e m_p / (m_p + m_e) \equiv M/n_e$, appears in the relation with Eddington number (Table 3) and introduces n_e , the *Universe Electron Quantum Number, canonical in Quantum Field Theory*. The Eddington Electron-Proton symmetry shows up in the following expression of the Large Number Correlation, where λ_{pH} is the geometrical mean of the reduced wavelengths of the proton and Hydrogen:

$$\frac{m_P^2}{m_p m_e} = n_e^{1/2} = \frac{R}{2\lambda_{pH}}, \quad (20)$$

which is extended by very precise dramatic expressions involving the symmetry between the weak bosons of masses $m_W = Wm_e$ and $m_Z = Zm_e$:

$$n_e^{1/2} \approx \frac{(WZ)^4}{2} \approx \left(\frac{m_F^2}{m_p m_H} \right)^7 \left(\frac{aZ}{W} \right)^3, \quad (21)$$

where appears as well a Planck-Fermi symmetry. It relates $a_G = m_P^2/m_p m_H$ to W and Z , specifying the known relation $a_G \approx W^8$ [5].

In the Topological Axis, the above Topon corresponds to the orbital number $k = 7$, while the gauge bosons correspond to $k = 3$ (weak bosons W, Z) and $k = 5$ (strong GUT boson

X), letting a single place $k = 1$ for a *non-standard massive Gluon* [14].

The particular values of the topological function $f(k) = \exp(2^{k+1/2})$ for $k = 7$ and 6 show up in (0.06%):

$$\begin{aligned} n_e &\approx f(7) \times 153^2 \\ R/\lambda_e &\approx f(6)/6, \end{aligned} \quad (22)$$

where $(f(6))^2 \equiv f(7)$ implies that $m_p/m_e \approx 1836 \equiv 6 \times 2 \times 153$, the Diophantine approximation of the Wyler formula $p_W = 6\pi^5$ [33]. The spectroscopic number associated to k is $2(2k+1)$, where 2 is the spin degeneracy and $2k+1$ the number of magnetic states [15]. For $k = 6$, this is 26 , the canonical dimension in the bosonic string theory [41].

This *invariable* Universe radius $R \approx 13.812$ Giga light-year (Glyr) of (19) is close to c times the variable standard Universe age. So the standard theoretical approach is correct, but not its Big Bang interpretation: it seems that a confusion is made somewhere between Time and Length, which readily occurs by putting $c = 1$. Moreover, the corresponding Hubble constant c/R is 70.793 (km/s)/Mpc, which is compatible with both the WMAP and the Carnegie-Chicago Hubble Program recent direct measurements (Table 3).

The above Universe gravitational potential energy $(3/10)Mc^2$ shows a Neutron Quantum Number (the number of neutron masses) very close (0.05%) to the large Eddington Number [14]. So it has nearly anticipated the correct Hubble Constant value (Table 3).

The Cosmos radius connects with the above radius R_{hol} and R by (27 ppm and 0.04%):

$$R_C(m_e/m_p)^2 \approx R_{hol} \left(\frac{WH}{3} \right)^2 \approx R(2FZ^2/3), \quad (23)$$

confirming very precisely, since $1/(H-p) \approx 27$ ppm, the optimal weak W boson mass [14] (Table 1).

5 The Cosmic Microwave Background (CMB)

This Universe radius $R = 2R_{H_2}$ enters a 1D-2D holographic relation: $2\pi R/\lambda_e = 4\pi\lambda_p\lambda_H/l_p^2$. The extension to the 3D holographic relation using λ_{H_2} , the reduced wavelength of the dihydrogen molecule H_2 , involves the reduced wavelength of the Cosmic Microwave Background (CMB) $\lambda_{CMB} = \hbar c/kT_{CMB}$:

$$2\pi \frac{R}{\lambda_e} = 4\pi \frac{\lambda_p\lambda_H}{l_p^2} \approx \frac{4\pi}{3} \left(\frac{\lambda_{CMB}}{\lambda_{H_2}} \right)^3, \quad (24)$$

leading to $T_{CMB} \approx (8G\hbar^4/3\lambda_p^5)^{1/3}/k \approx 2.729$ Kelvin, which is once more, apart the holographic factor $8/3$, a c -free three-fold (Mass, Length, Time) dimensional analysis, giving the energy kT_{CMB} from the constants G , \hbar , λ_p . Moreover, by substituting $a_G = R/2\lambda_e$ with the above Lucas Number N_L , this leads to a new holographic expression (analog to the area

of a 4D sphere), which gives T_{CMB} , compatible with the measured value $2.7255(6)$ Kelvin [14]:

$$\begin{aligned} N_L &\approx 2\pi^2 \frac{\lambda_{CMB}^3}{\lambda_e\lambda_H^2} \\ \Rightarrow T_{CMB} &= \frac{\hbar c}{k\lambda_{CMB}} \approx 2.7258205 \text{ Kelvin}. \end{aligned} \quad (25)$$

The standard cosmology predicts a Neutrino background with temperature $T_{CNB} = T_{CMB} \times (4/11)^{1/3} \approx 1.946$ Kelvin. The total CMB photon number is $n_{ph} = (3\xi(3)/8\pi)(R/\lambda_{CMB})^3$, exceeding the total Hydrogen number $n_H = M/m_H = R\lambda_H/2l_p^2$. But in terms of energy, the matter dominates. So one must consider also the ratio between the critical energy density $u_{cr} = 3c^4/8\pi GR^2$ and the total background energy density $u_{cmb+cnb} = yu_{cmb}$, with $y = 1 + (21/8)(4/11)^{4/3} \approx 1.681322$ [24] and $u_{cmb} = (\pi^2/15)\hbar c/\lambda_{CMB}^4$. We observed that these ratios are tied by an Eddington type relation:

$$\left(2 \frac{n_{ph}}{n_H} \right)^{1/2} \approx \frac{u_{cr}}{u_{cmb+cnb}} \Rightarrow T_{CMB} \approx 2.724 \text{ Kelvin}. \quad (26)$$

This confirms the existence of the Neutrino background. Now assuming that the total background Photon + Neutrino is the result of an ongoing Hydrogen-Helium transformation, producing $e_{He} = 6.40 \times 10^{14}$ Joule by kilogram of Helium, i.e. an efficiency $\epsilon_{He} = e_{He}/c^2 \approx 1/140$. The Helium mass density is $Y \times \rho_{bar}$; with the standard evaluation of baryonic density $\epsilon_{bar} = \rho_{bar}/\rho_{cr} \approx 0.045$ and $Y \approx 0.25$ [29], this leads to:

$$\begin{aligned} \left(\frac{\lambda_{CMB}^2}{l_p R} \right)^2 &\approx \frac{8\pi^3 y}{45Y\epsilon_{bar}\epsilon_{He}} \approx 1.15 \times 10^5 \\ \Rightarrow T_{CMB} &\approx 2.70 \text{ Kelvin}. \end{aligned} \quad (27)$$

In the standard model, the Universe age is far too small to explain a large Helium large density resulting from stellar activities [23]. Thus, it is not a real problem in the steady-state model.

6 The electron and the Kotov non-local period

This study confirms the central role of λ_e , the unit length in the Topological Axis [13]. So we look for a Diophantine series giving it for $n = 1$. This means:

$$\lambda_e \equiv \frac{\hbar}{m_e c} = \frac{\hbar^2}{Gm_G m_{\hbar}^2} \Rightarrow A \equiv \frac{m_p^2}{m_G m_{\hbar}} = \frac{m_{\hbar}}{m_e} \quad (28)$$

so that the fundamental ($n = 1$) energy is: $E \equiv m_{\hbar} c^2/A^2 = m_e c^2/A$. There is an *elimination of c* by considering the term A^2 as the product of the above gravitational constant $a_G = \hbar c/Gm_p m_H$ and the electro-weak one $a_w = \hbar^3/cG_F m_e^2$ [5], where G_F is the Fermi constant:

$$A^2 = a_G a_w \Rightarrow E = \frac{m_e c^2}{\sqrt{a_G a_w}} \quad (29)$$

Table 3: Prediction of Eddington Number ($N_E = 136 \times 2^{256}$) and Holo-physics formula for the *invariant* Hubble radius $R \approx 13.812$ Giga light-year (Gly) and the corresponding Hubble constant $H_0 = c/R$, which uses the length unit Megaparsec, compared to the main measurements. Lucas Number $N_L = 2^{127} - 1$. Topological Function $f(k) \equiv e^{2k+1/2}$. Holographic ratio $u = R_{hol}/R$. For comparison, the so-called standard “Universe Age” is also presented, with unit in the c ratio.

Date	Source $R = 2GM/c^2$	Hubble radius Gly	Hubble Cst. km s ⁻¹ /Mpc	Univ. “Age” Gyr
1945	Eddington Number [36] ; $N_E \approx (3/10) Mm_p/M_H m_n$	13.812	70.793	
1927	Lemaître [34]	1.6	620	
1929	Hubble [35]	1.8	540	
1956	Humason, Maydal and Sandage [37]	5.4	180	
1958	Sandage [38]	13	75	
1998	$2\hbar^2/Gm_e m_p m_n$ Twice (3 mn Form.= Clsd Draft)	13.800	70.852	
2006	$2\hbar^2/Gm_e m_p m_n$ [22]	13.800	70.852	
2006	$2 N_L \lambda_e$ [22]	13.889	70.397	
2017	$(WZ)^4 (\lambda_p \lambda_H)^{1/2}$ [5] [13]	13.796 ± 0.002	70.87 ± 0.01	
2017	$\lambda_e f(6)/6$ [13]	13.821	70.744	
2017	$\lambda_e (3^3)^3 / u$ [13]	13.812	70.793	
2017	$2\hbar^2/Gm_e m_p m_H$ [13] Machian Formula	13.812	70.793	
2017	$2(ct_K)^2 / a_w \lambda_e$ [13]	13.812	70.793	
2017	$(2/u)^{2 \times 3 \times 5 \times 7} \lambda_e$ [14] Complete Holic Principle	13.856	70.565	
2021	$(6/\pi)^{r_B/\lambda_e} \lambda_e$ [15]	13.776	70.975	
2022	$2N_L \lambda_e (1 - (137^2 + \pi^2 + e^2)/pH)$	13.812 (Machian prob)	70.793	
1998	PDG (Particle Data Group)	14 ± 2	70 ± 10	11.5 ± 1.5
2002	PDG	13.7 ± 0.3	71 ± 3	15 ± 3
2005	Hubble Space Telescope	13.6 ± 1.5	72 ± 8	13.7 ± 0.2
2012	WMAP [28]	14.1 ± 0.2	69.3 ± 0.8	13.77 ± 0.06
2019	Riess group [30]	13.2 ± 0.3	74.2 ± 1.4	
2020	Planck mission [29]	14.5 ± 0.1	67.4 ± 0.5	13.82 ± 0.04
2020	HOLICOW [31]	13.4 ± 0.3	73.3 ± 1.8	
2021	Carnegie-Chicago Hubble Program [18]	14.0 ± 0.3	69.8 ± 1.6	

with $t_e \equiv \hbar/m_e c^2$ the electron period, this corresponds to the time:

$$t_e \sqrt{a_G a_w} \approx 9600.60 \text{ s.} \tag{30}$$

The identification with the Kotov P_0 period $t_K \approx 9600.606$ (12)s [16, 19] corresponds to $G \approx 6.6754527$ SI, specified to 10^{-8} by the Single-Electron Radius $R_1 \approx (4\pi p/p_w)^2 a_w c t_K$ [14] and consistent with the BIPM measurements [25], but at 6σ from the official value, a mean between *discordant* measurements. With the Fermi mass $m_F = m_e \sqrt{a_w}$, close to the mean nucleotide mass [13], the Lepton Mu mass m_μ , $u = R_{hol}/R$, the critical density $\rho_{cr} = 3c^2/8\pi GR^2$, $m_{GF} =$

$(m_p m_F)^{1/2}$, this defines our optimal strong coupling a_s :

$$\begin{aligned}
 m_G &= \frac{m_e m_p m_H}{m_F^2} \\
 m_{\hbar}/m_P &= \frac{m_F}{(m_p m_H)^{1/2}} \equiv \frac{m_\mu^2}{m_e m_N} \equiv 2\pi \frac{a_s m_p m_H}{m_e m_F} \\
 (GG_F)^{1/2} &\equiv \left(\frac{\hbar}{m_{GF}} \right)^2 = \frac{\hbar}{(m_p m_H)^{1/2}} \frac{\lambda_e^2}{t_K} \\
 \frac{G_F}{G m_p^2 l_p^2} &\approx \frac{a^4 m_p m_\mu}{m_e^2} \text{ (0.2\%)} \\
 \frac{\hbar}{(G_F \rho_{cr})^{1/2}} &\approx \frac{\lambda_e^2}{u^{1/16} l_p} \text{ (0.01\%)}
 \end{aligned} \tag{31}$$

exhibiting a symmetry between canonical area speeds. Note that $2ct_K \approx L_{NL}(m_{bc})$, confirming once more the bi-codon

mass, which enters also a relation involving the Cosmos, the Photon and Graviton masses [14] (Table 3). Moreover, with $P = m_P/m_e$, $F = m_P/m_e$, $H = m_H/m_e$, $p = m_p/m_H$, and the precise variant (0.14 ppm) of the Golden Number: $\Phi_0 = P/(a_w H)^3 \approx ((4\pi/3)(H/p)^2)^{1/3}$, one observes:

$$\begin{aligned} \frac{L_{NL}(m_{GF})}{r_{HB}} &\equiv \left(\frac{P}{F^3}\right)^{1/2} \frac{1}{a} \approx \Phi_0^2 \quad (15 \text{ ppm}) \\ cT_{NL}(m_{GF}) &\equiv l_P \left(\frac{P}{F}\right)^{5/2} \\ &\approx \left(\frac{R_{hol} \lambda_e}{2}\right)^{1/2} \frac{1}{d_e^2} \quad (74 \text{ ppm}) \end{aligned} \quad (32)$$

where d_e is the canonical Excess Electron Magnetic Moment (Table 1). This specifies the holographic relations $a^2 \approx (4\pi/3)p^{3/2}$ and $F^5/Pa^3 \approx \eta$, with $\eta = 1 + 2/(3 \times 139)$ (ppb precision) [15], where 139 is the complete Atiyah form [26], adding the dimensions of the four algebra (octonion, quaternion, complex, real): $139 = 137 + 2 = 2^7 + 2^3 + 2^1 + 2^0 \approx i^{-i\pi}$, and $3 \times 139 + 2 = 419$, the positive crystallographic number [40] in the superstring dimensions 10D and 11D [41], see Table 7 in [15]. Moreover, $T_{NL}(m_{GF}) \approx 19.14$ ms, typical of the Human nervous system, and the third octave down the flat La tone (Lab) for $La_3(A_4) = 442.9$ Hz, an anthropic argument far more pertinent and precise than the rough standard ones, principally based on a *cosmic Big Bang scenario* [5].

7 Conclusions

The quantification of Length and Time implies, through the Diophantine treatment of the Kepler laws, an angular momentum quantum identified with the reduced Planck constant \hbar . This leads to the Diophantine Coherence Theorem (DCT) which has the same structure than the Hass formulation in the Hydrogen atom spectrum. The DCT shows that the real invariant quantity is the Frequency, so that the Energy conservation would mean a Frequency Accordance, or ‘‘Coherence Principle’’, mandatory in Practical Holography; the DCT conforms with the Harmony Principle of Pythagoras, the father of Natural Philosophy, the very root of Science. This confirms the pertinence of the Quantum Field Theory, where any Particle Field is defined by a whole number, entering the Holographic principle in the revisited critical steady-state Universe. In particular, both the Electron Quantum Number and the Neutron Quantum Number play a central role. The Universe Length Quantum (Topon) is associated to a Universe Time quantum (‘‘Chronon’’ $t_M = \lambda_M/c$), which may be looked as the period of the *Permanent Bang oscillation matter-antimatter* [42].

The DCT shows that the Haas-Bohr radius is a *pseudo* length quantum, while the Universe itself appears as a pseudo quantum in a Cosmos, defined by the Holographic Principle where the Planck length is an intermediate holographic length, instead of the standard quantum. The Cosmic Length

Quantum breaks the ‘‘Planck wall’’ by the factor 10^{-61} . The main pseudo length quantum is the reduced Electron Wavelength which shows, through the DCT and the Kotov non-local period, a symmetry between gravitation and electroweak interaction. The Kotov-Lyuty Non-Doppler oscillation was overlooked: it is however a sign of the non-local character of Quantum Cosmology. It is mandatory to check the Lyuty Non-Doppler Quasar measurements [16].

The Planck mass enters naturally in the DCT, but plays no role in Particle Physics. However, the standard spin formulation rejoins our conclusion that the reduced Planck constant \hbar plays a more fundamental role than h . This is confirmed by the spiraling trajectory interpretation of the Single-Electron cosmic model [14].

The standard speed limit c excludes any explanation of the wave packet reduction phenomena, which requires a non-local or tachyonic Physics. So, it is logical that the bosonic string theory, which introduces tachyon, is confirmed by the Diophantine Topological Axis. Indeed, the central bosonic dimension $d = 26$ corresponds to the non-local universe radius (Machian Formula). The Holographic Principle and the DNA bi-codon mass are both decisive. So the DNA could be an helix-hologram, opening the way towards bio-computing [20]. The c -free Elementary Non-Local Three Minutes Formula giving the Universe half-radius is now fully established: this means a tight harmony between the Universe and Human Consciousness, a special and decisive manifestation of the Anthropic Principle.

Acknowledgements

The authors are indebted to the philosophers Dominique Tasot and Nadia Guemidi, the computer scientists Laurent Gueroult, Denis Gayral and Joël Croissant, and the mathematicians Jacques Bailhache, Jacques de Wiele and Alain Molinier for useful discussions.

Received on April 20, 2022

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