On Action in the Spacetime Continuum

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In this paper, we investigate the role of action $\mathcal{S}$ in the Spacetime Continuum (*STC*) as provided by the Elastodynamics of the Spacetime Continuum (*STCED*). We find that energy applies to three-dimensional space, while action applies to four-dimensional spacetime. Planck's reduced constant $\hbar$ corresponds to an elementary quantum of action S_0, with action units being the same as those of angular momentum. We thus find that action is the fundamental four-dimensional spacetime scalar quantity corresponding to energy for three-dimensional space. This helps explain why equations of motion in the Spacetime Continuum are determined by minimizing action, not energy, using the principle of least (or stationary) action. The contribution of a path, in the path integral formulation of quantum mechanics and quantum field theory, depends on the number of elementary quanta of action S_0 in the path.

1 Introduction

In this paper, we investigate the role of action $\mathcal{S}$ in the Spacetime Continuum (*STC*) as provided by the Elastodynamics of the Spacetime Continuum (*STCED*) [1–3]. *STCED* is a natural extension of Einstein's General Theory of Relativity which blends continuum mechanical and general relativistic descriptions of the Spacetime Continuum. The introduction of strains in the Spacetime Continuum as a result of the energy-momentum stress tensor allows us to use, by analogy, results from continuum mechanics, in particular the stress-strain relation, to provide a better understanding of the general relativistic spacetime.

2 Elastodynamics of the Spacetime Continuum

The stress-strain relation for an isotropic and homogeneous Spacetime Continuum is given by [1, 3]

$$2\bar{\mu}_0\, \varepsilon^{\mu\nu} + \bar{\lambda}_0\, g^{\mu\nu}\varepsilon = T^{\mu\nu} \tag{1}$$

where $\bar{\lambda}_0$ and $\bar{\mu}_0$ are the Lamé elastic constants of the Spacetime Continuum: $\bar{\mu}_0$ is the shear modulus (the resistance of the Spacetime Continuum to *distortions*) and $\bar{\lambda}_0$ is expressed in terms of $\bar{\kappa}_0$, the bulk modulus (the resistance of the Spacetime Continuum to *dilatations*), in a four-dimensional continuum as:

$$\bar{\lambda}_0 = \bar{\kappa}_0 - \tfrac{1}{2}\,\bar{\mu}_0\,. \tag{2}$$

$T^{\mu\nu}$ is the general relativistic energy-momentum stress tensor, $\varepsilon^{\mu\nu}$ the Spacetime Continuum strain tensor resulting from the stresses, and

$$\varepsilon = \varepsilon^{\alpha}{}_{\alpha}\,, \tag{3}$$

the trace of the strain tensor obtained by contraction, is the volume dilatation ε defined as the change in volume per original volume [4, see pp. 149–152] and is an invariant of the strain tensor. It should be noted that the structure of (1) is similar to that of the field equations of general relativity,

$$R^{\mu\nu} - \tfrac{1}{2}\, g^{\mu\nu}R = -\varkappa\, T^{\mu\nu} \tag{4}$$

where $R^{\mu\nu}$ is the Ricci curvature tensor, R is its trace, $\varkappa = 8\pi G/c^4$ and G is the gravitational constant (see [2, Ch. 2] for more details).

In *STCED*, as shown in [1, 3], energy propagates in the Spacetime Continuum as wave-like *deformations* which can be decomposed into *dilatations* and *distortions*. *Dilatations* involve an invariant change in volume of the Spacetime Continuum which is the source of the associated rest-mass energy density of the deformation. On the other hand, *distortions* correspond to a change of shape (shearing) of the Spacetime Continuum without a change in volume and are thus massless.

Thus deformations propagate in the Spacetime Continuum by longitudinal (*dilatation*) and transverse (*distortion*) wave displacements. This provides a natural explanation for wave-particle duality, with the massless transverse mode corresponding to the wave aspects of the deformations and the massive longitudinal mode corresponding to the particle aspects of the deformations.

The rest-mass energy density of the longitudinal mode is given by [1, see Eq. (32)]

$$\rho c^2 = 4\bar{\kappa}_0\varepsilon \tag{5}$$

where ρ is the rest-mass density, c is the speed of light, $\bar{\kappa}_0$ is the bulk modulus of the *STC* as seen previously, and ε is the volume dilatation given by (3).

3 Action in the Spacetime Continuum

In a previous paper [5], we considered dislocations in the Spacetime Continuum as a framework for quantum physics. In a subsequent paper [6], we expressed Planck's constant in terms of the Burgers spacetime dislocation constant b_0, given by

$$\hbar = \frac{\bar{\kappa}_0\, b_0^4}{c}\,, \tag{6}$$

where $\bar{\kappa}_0$ is the Spacetime Continuum bulk modulus, b_0 is the Burgers spacetime dislocation constant, c is the speed of light *in vacuo* and $\hbar$ is Planck's reduced constant. This equation can be considered to be a definition of Planck's reduced constant $\hbar$. We consider this equation in greater detail.

On the right-hand side of the equation, we have the Spacetime Continuum bulk modulus constant $\bar{\kappa}_0$ in units of energy density [J m^{-3}], that is energy per 3-D volume. We can multiply $\bar{\kappa}_0$ by a 3-D volume to convert it to energy. However, $\bar{\kappa}_0$ is a Spacetime Continuum constant. We need a conversion in terms of the 4-D spacetime volume.

The right-hand side of (6) also includes the term b_0^4 which can be taken to be the 4-D volume of a four-dimensional elementary hypercube of side $b_0 = 1.616 \times 10^{-35}$ m. This 4-D hypervolume has units of [m^4] while the four-dimensional Spacetime Continuum hypervolume consists of three space dimensions and one time dimension with units [m^3 s]. This requires that one of the four-dimensional hypercube dimensions b_0 be divided by c to convert it to a time elementary dimension $t_0 = b_0/c = 5.39 \times 10^{-44}$ s as is observed in (6). Eq. (6) can thus be written as

$$\hbar = \bar{\kappa}_0 \, b_0^3 \, \frac{b_0}{c} = \bar{\kappa}_0 \, b_0^3 \, t_0 = \bar{\kappa}_0 \, V_0^{STC} \tag{7}$$

where V_0^{STC} is the four-dimensional elementary Spacetime Continuum hypervolume and $\hbar$ has units of [J s] which are units of action $\mathcal{S}$.

Hence multiplying $\bar{\kappa}_0$ by a 3-D space volume converts it to energy, while multiplying it by a 4-D spacetime volume converts it to action. Energy applies to three-dimensional space, while action applies to four-dimensional spacetime. From (7), we see that Planck's reduced constant corresponds to an elementary quantum of action S_0:

$$\hbar = \bar{\kappa}_0 \, V_0^{STC} = S_0 \tag{8}$$

which has units of [J s]. Action units are the same as those of angular momentum, but this equivalence is accidental. The basic nature of $\hbar$ is an action, not an angular momentum. Calling $\hbar$ a "spin" quantity is an unfortunate misnomer from the early days of quantum mechanics. It needs to be called more appropriately an action quantity, i.e. the fundamental quantum of action of the Spacetime Continuum.

We thus find that action is the fundamental four-dimensional spacetime scalar quantity corresponding to energy for three-dimensional space. This helps explain why equations of motion in the Spacetime Continuum are determined by minimizing action, not energy, using the principle of least (or stationary) action given by

$$\delta\mathcal{S} = 0 \tag{9}$$

where the action $\mathcal{S}$ is expressed in terms of the Lagrangian L of the system as

$$\mathcal{S} = \int_{t_1}^{t_2} L\,(q(t), \dot{q}(t), t)\; dt \tag{10}$$

where $q = (q_1, q_2, ..., q_N)$ are the N generalized coordinates defining the configuration of the system and $\dot{q}$ denotes the time derivative of q.

In Lagrangian field theory, the action is written in terms of the Lagrangian density $\mathcal{L}$ specified in terms of one or more fields $\phi(x)$ and their derivatives $\partial_\mu \phi$ as [7, see p. 15ff]

$$\mathcal{S} = \int_{x_1}^{x_2} \mathcal{L}\left(\phi(x), \partial_\mu \phi\right)\, d^4x\,. \tag{11}$$

The path integral formulation of quantum mechanics and quantum field theory is a generalization of the action principle of classical mechanics [8]. Interestingly enough, Feynman who developed this formulation [9]

> ... belie[ved] that the path integral captures the fundamental physics, and that hamiltonians and Hilbert space are merely mathematical methods for evaluating path integrals. [10, see p. 143]

In *STCED*, the path integral between two points x_1 and x_2 can be understood to be equivalent to the different possible wave paths between the two points.

The propagation amplitude $G(x_2; x_1)$ between the points x_1 and x_2 is determined from the path integral using the appropriate action for the system under consideration. One can see that since the contribution of a path is proportional to $e^{i\mathcal{S}/\hbar}$ [10, see p. 146], then, from (8), it is equivalent to $e^{i\mathcal{S}/S_0}$. In other words, the contribution of a path depends on the number of elementary quanta of action S_0 in the path.

The quantization of action implied by the above, points to the approach required to achieve quantization of path integrals in quantum physics. Coupled with the understanding that equations of motion in the Spacetime Continuum are determined by minimizing action as per (9) provides an indication for its potential application to the development of a quantized theory of path integrals.

4 Discussion and conclusion

In this paper, we have investigated the role of action $\mathcal{S}$ in the Spacetime Continuum as provided by the Elastodynamics of the Spacetime Continuum (*STCED*). We have found that multiplying the Spacetime Continuum bulk modulus constant $\bar{\kappa}_0$ by a 3-D space volume converts it to energy, while multiplying it by a 4-D spacetime volume converts it to action. Hence energy applies to three-dimensional space, while action applies to four-dimensional spacetime. Planck's reduced constant $\hbar$ corresponds to an elementary quantum of action S_0, with action units being the same as those of angular momentum. We thus find that action is the fundamental four-dimensional spacetime scalar quantity corresponding to energy for three-dimensional space. This helps explain why equations of motion in the Spacetime Continuum are determined by minimizing action, not energy, using the principle of least (or stationary) action. In particular, the contribution of a path, in the path integral formulation of quantum mechanics and quantum field theory, depends on the number of elementary quanta of action S_0 in the path.

Received on August 24, 2022

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