

## A Quantitative Representation of Particle Entanglements via Bohm's Hidden Variable According to Hadronic Mechanics

Ruggero Maria Santilli

The Institute for Basic Research, 35246 U. S. 19N, Suite 215, Palm Harbor, FL 34684, USA.  
E-mail: research@i-b-r.org

In this note, we first recall the 1935 historical view by A. Einstein, B. Podolsky and N. Rosen according to which "*Quantum mechanics is not a complete theory*" (EPR argument), because of the inability by quantum mechanics to provide a quantitative representation of the *interactions* occurring in particle entanglements. We then show, apparently for the first time, that the completion of quantum entanglements into the covering *EPR entanglements* formulated according to hadronic mechanics provides a *quantitative representation of the interactions occurring in particle entanglements* by assuming that their continuous and instantaneous communications at a distance are due to the overlapping of the wave packets of particles, and therefore avoiding superluminal communications. According to this view, entanglement interactions result to be non-linear, non-local and not derivable from a potential, and are represented via Bohm's variable  $\lambda$  hidden in the quantum mechanical *associative* product of Hermitean operators  $AB = A \times B$  via explicit and concrete, axiom-preserving realizations  $A \hat{\times} B = A \lambda B$ , with ensuing *non-unitary* structure, multiplicative unit  $U1U^\dagger = \hat{I} = 1/\lambda$ ,  $\hat{I} \hat{\times} A = A \hat{\times} \hat{I} = A$ , *inapplicability* of Bell's inequalities and consequential validity of Bohm's hidden variables. We finally introduce, also apparently for the first time, the completion of quantum computers into the broader *EPR computers* characterizing a collection of extended electronic components under continuous entanglements, and show their apparent faster computation, better cybersecurity and improved energy efficiency.

According to clear experimental evidence dating back to the early part of the past century, particles that were initially bounded together and then separated, can continuously and instantaneously influence each other at a distance, not only at the particle level (see e.g. [1, 2] and papers quoted therein), but also at the classical level [3].

The above experimental evidence is generally *assumed* to be represented by quantum mechanics and, therefore, particle entanglements are widely called *quantum entanglement* (Figure 1). However, Albert Einstein strongly criticized such an assumption because it would imply superluminal communications that violate special relativity. This occurrence motivated the 1935 historical view by A. Einstein, B. Podolsky and N. Rosen according to which "*Quantum mechanics is not a complete theory*" (EPR argument) [4].

In fact, quantum mechanics can only represent interactions derivable from a potential while *no* quantum mechanical potential is conceivably possible to represent continuous and instantaneous interactions at a distance. More explicitly, the quantum mechanical equation for two interacting particles with coordinates  $r_k$ ,  $k = 1, 2$  on a Hilbert space  $\mathcal{H}$  over the field  $\mathcal{C}$  of complex numbers is given by the familiar Schrödinger equation (for  $\hbar = 1$ )

$$\left[ \sum_{k=1,2} \frac{1}{2m_k} p_k p_k + V(r) \right] \psi(r) = E \psi(r). \quad (1)$$

When the two particles are entangled, in view of the absence

of any possible potential  $V(r)$ , the above equation becomes

$$\begin{aligned} \sum_{k=1,2} \frac{1}{2m_k} p_k p_k \psi(r_1) \psi(r_2) &= \\ &= \left[ \sum_{k=1,2} \frac{1}{2m_k} \left( -i \frac{\partial}{\partial r_k} \right) \left( -i \frac{\partial}{\partial r_k} \right) \right] \psi(r_1) \psi(r_2) = \\ &= E \psi(r_1) \psi(r_2) \end{aligned} \quad (2)$$

and *can only represent two free particles* characterized by the individual wave functions  $\psi(r_k)$  without any possible or otherwise known interaction.

At the 2020 *International Teleconference on the EPR argument* [5–7], R. M. Santilli proposed the new notion of *Einstein-Podolsky-Rosen entanglement* (Sect. 7.2.3, p. 61 of [6]) which is based on the sole conceivable interaction responsible for particle entanglements, that due to the overlapping of the wave packets of particles (Figure 2), thus being non-linear as first suggested by W. Heisenberg [8], non-local as first suggested by L. de Broglie and D. Bohm [9] and non derivable from a potential as first suggested by R. M. Santilli at Harvard University under DOE support [13, 14], because of contact, thus continuous and instantaneous character, by therefore voiding the need for superluminal communications.

The non-linear, non-local and non-potential character of the assumed interactions render them ideally suited for their representation via the *isotopic* (i.e. *axiom-preserving branch of hadronic mechanics*, [15–17]), comprising *iso-mathematics* and *iso-mechanics* (see [18] for an outline, [19–21] for a

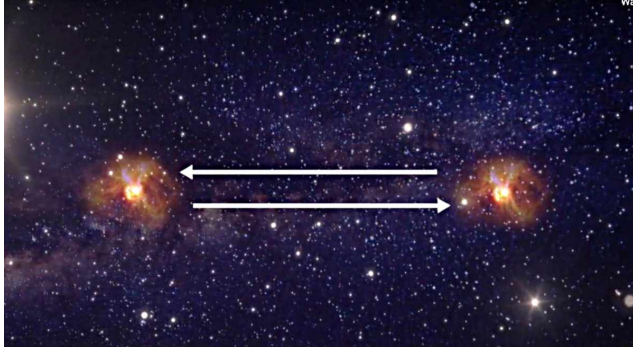


Fig. 1: In this figure, we illustrate the entanglement of particles with continuous and instantaneous interactions at a distance, and recall the argument by A. Einstein, B. Podolsky and N. Rosen on the lack of completeness of quantum mechanics due to its inability to represent said entanglement in a way compatible with special relativity [4].

review and [22–30] for independent studies) which are characterized by the isotopy  $\hat{\xi} : \{\hat{A}, \hat{B}, \dots, A \hat{\times} B\}$  of the universal enveloping associative algebra  $\xi : \{A, B, \dots, AB = A \times B\}$  of quantum mechanical Hermitian operators  $A, B, \dots$  with *iso-product* (first introduced in Eq. (5), p. 71 of [14])

$$\hat{A} \hat{\times} \hat{B} = \hat{A} \hat{T} \hat{B}, \quad \hat{T} > 0, \quad (3)$$

where  $\hat{T}$ , called the *isotopic element*, is positive-definite but possesses otherwise an unrestricted functional dependence on coordinates, momenta, wave function and any other needed local variable, with related *iso-unit*

$$\begin{aligned} \hat{I} &= 1/\hat{T} > 0, \\ \hat{I} \hat{\times} A &= \hat{A} \hat{\times} \hat{I} = \hat{A}, \quad \forall \hat{A} \in \hat{\xi}, \end{aligned} \quad (4)$$

completion of Lie's theory into the *Lie-Santilli iso-theory* [14] (see [23, 28] for independent studies) with iso-brackets for an  $N$ -dimensional iso-algebra

$$[X_i \hat{\times} X_j] = X_i \hat{\times} X_j - X_j \hat{\times} X_i = X_i \hat{T} X_j - X_j \hat{T} X_i = C_{ij}^k X_k, \quad (5)$$

*iso-Heisenberg's equation* (first proposed in Eq. (18), p. 163 of [14])

$$i \frac{dA}{dt} = [A \hat{\times} H] = A \hat{\times} H - H \hat{\times} A, \quad (6)$$

and related *iso-Schrödinger's equation*

$$H \hat{\times} |\hat{\psi}\rangle = H \hat{T} |\hat{\psi}\rangle = E |\hat{\psi}\rangle. \quad (7)$$

Since the isotopic element  $\hat{T}$  is *hidden* in the abstract axiom of associativity and becomes visible only in the isotopic realization (3)

$$A \hat{\times} (B \hat{\times} C) = (A \hat{\times} B) \hat{\times} C, \quad (8)$$

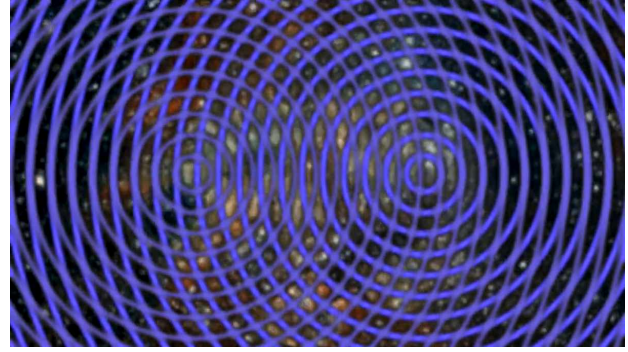


Fig. 2: In this figure, we illustrate the new Einstein-Podolsky-Rosen entanglement of particles introduced by R. M. Santilli in the 2020 overview [6] which is characterized by contact, therefore continuous and instantaneous interactions due to the overlapping of the wave packets of particles represented via Bohm's hidden variable (9), by therefore avoiding the need for superluminal interactions.

R. M. Santilli proposed in Sect. 6.8, p. 150 on, Eq. (5.8.19) in particular, of [16], as part of the isotopy of the SU(2) spin algebra and Pauli's matrices in particular, the *identification of the isotopic element  $\hat{T}$  with Bohm's hidden variable  $\lambda$*  [31]

$$\lambda = \hat{T}(r, p, \psi, \dots), \quad (9)$$

and consequential realization of the iso-unit  $\hat{I} = 1/\lambda$ , with a variety of explicit and concrete realizations that, for the entanglement of two particles, are of the type [21]

$$\begin{aligned} \lambda = \hat{T} = 1/\hat{I} &= \Pi_{\alpha=1,2} \text{Diag.} \left( \frac{1}{n_{1,\alpha}^2}, \frac{1}{n_{2,\alpha}^2}, \frac{1}{n_{3,\alpha}^2}, \frac{1}{n_{4,\alpha}^2} \right) e^{-\Gamma}, \\ n_{\mu,\alpha} > 0, \quad \Gamma > 0, \quad \mu &= 1, 2, 3, 4, \quad \alpha = 1, 2, \end{aligned} \quad (10)$$

providing:

1) A representation of the dimension and shape of particles via semi-axes  $n_{k,\alpha}^2$ ,  $k = 1, 2, 3$  normalized to  $n_{k,\alpha} = 1$ ,  $k = 1, 2, 3$ ,  $\alpha = 1, 2$  for the vacuum.

2) A representation of the density of particles via  $n_{4,\alpha}^2$  normalized to the value  $n_{\mu,\alpha}^2 = 1$  for the vacuum.

3) A representation of the non-potential character of the interactions due to the mutual penetration of particles via the exponential term  $e^\Gamma$ , where  $\Gamma(\hat{r}, \hat{p}, \hat{\psi}, \dots)$  is a positive-definite quantity with an unrestricted functional dependence on iso-coordinates  $\hat{r} = r\hat{I} = r/\lambda$ , iso-momenta  $\hat{p}$ , iso-wave-functions  $\hat{\psi}(\hat{r})$ , and other local variables.

By recalling the basic expression of the *iso-linear iso-momentum* characterized by the completion of the local Newton-Leibnitz differential calculus into the non-local *iso-differential calculus* [32] (see [29, 30] for independent studies)

$$\begin{aligned} \hat{p} \hat{\times} \hat{\psi}(\hat{r}) &= \hat{p} \hat{T}(\hat{r}, \dots) \psi(\hat{r}) = -i \frac{\partial}{\partial \hat{r}} \hat{\psi}(\hat{r}) = \\ &= -i \hat{I}(\hat{r}, \dots) \frac{\partial}{\partial \hat{r}} \hat{\psi}(\hat{r}) = -i \frac{1}{\lambda} \frac{\partial}{\partial r/\lambda} \hat{\psi}(r/\lambda), \end{aligned} \quad (11)$$

the non-relativistic version of the EPR entanglement is characterized by the iso-Schrödinger equation (see [16] for the relativistic extension)

$$\begin{aligned}
\Sigma_{k=1,2} \frac{1}{2m_k} \hat{p}_k \hat{\times} \hat{p}_k \hat{\times} \hat{\psi}(\hat{r}) &= \\
= \left[ \Sigma_{k=1,2} \frac{1}{2m_k} \left( -i\hat{I} \frac{\partial}{\partial \hat{r}_k} \right) \left( -i\hat{I} \frac{\partial}{\partial \hat{r}_k} \right) \right] \hat{\psi}(\hat{r}) &= \\
= \left\{ \Sigma_{k=1,2} \left[ -\frac{\hat{I}^2}{2m_k} \left( \frac{\partial}{\partial \hat{r}_k} \right) \left( \frac{\partial}{\partial \hat{r}_k} \right) - \frac{\hat{I}}{2m_k} \left( \frac{\partial \hat{I}}{\partial \hat{r}_k} \right) \left( \frac{\partial}{\partial \hat{r}_k} \right) \right] \right\} \hat{\psi}(\hat{r}) &= (12) \\
= \left\{ \Sigma_{k=1,2} \left[ -\frac{\hat{I}^2}{2m_k} \left( \frac{\partial}{\partial \hat{r}_k} \right) \left( \frac{\partial}{\partial \hat{r}_k} \right) - \frac{\Gamma}{2m_k} \left( \frac{\partial \Gamma}{\partial \hat{r}_k} \right) \left( \frac{\partial}{\partial \hat{r}_k} \right) \right] \right\} \hat{\psi}(\hat{r}) &= \\
= \hat{E} \hat{\times} \hat{\psi}(\hat{r}) = E \hat{\psi}(\hat{r}) = E \hat{\psi}(\hat{r}_1) \hat{\times} \hat{\psi}(\hat{r}_2), &
\end{aligned}$$

with the following primary characteristics:

1. Iso-equation (9) characterizes a *new entanglement interaction* represented by Bohm's hidden variable  $\lambda = \hat{T}$  which is absent in quantum mechanical equation (2);

2. The new entanglement interaction is manifestly *non-linear* (in the wave-function), yet the theory is *iso-linear* [15, 16], namely, it is linear on the Hilbert-Myung-Santilli iso-space  $\hat{\mathcal{H}}$  [33] with iso-states  $|\psi\rangle$  and iso-normalization [12]

$$\langle \psi | \hat{\times} | \hat{\psi} \rangle = \langle \psi | \lambda | \hat{\psi} \rangle = \lambda, \quad (12)$$

over Santilli iso-field  $\hat{C}$  of *iso-real*, *iso-complex* or *iso-quaternionic iso-numbers* [34]

$$\hat{n} = n \hat{I} = \frac{n}{\lambda} \quad (13)$$

with iso-superposition principle  $\psi(\hat{r}) = \hat{\psi}(\hat{r}_1) \hat{\times} \hat{\psi}(\hat{r}_2)$ ;

3. The new entanglement interaction is manifestly *non-local* in the sense of occurring in *volumes* represented by the iso-unit  $\hat{I} = 1/\lambda$  and characterized by the overlapping of two volumes  $V_k = (1/n_{1,k}^2, 1/n_{2,k}^2, 1/n_{3,k}^2)$ ,  $k = 1, 2$ , each being on ontological grounds as big as experimental measurements can allow;

4. The new entanglement interaction is manifestly of contact, zero-range character, thus not being derivable from a potential, and therefore avoiding the need for superluminal interactions required by quantum entanglements [4];

5. The new entanglement interaction verifies, by conception and construction, the abstract axioms of relativistic quantum mechanics although realized via the indicated universal iso-associative envelope [35–37].

It should be indicated that (12) can be equally derived via a *non-unitary transformation* of quantum mechanical equation (2)

$$\begin{aligned}
U \times 1 \times U^\dagger &= \frac{1}{\lambda} = \hat{I}, \\
U \times (A \times B) \times U^\dagger &= A' \lambda B', \quad \lambda = (U \times U^\dagger)^{-1}, \quad (14) \\
A' &= U \times A \times U^\dagger, \quad B' = U \times B \times U^\dagger.
\end{aligned}$$

The *invariance* of the numeric value of Bohm's hidden variable is then assured by the *Lorentz-Poincaré-Santilli iso-symmetry* [20] with structure [38]

$$\begin{aligned}
U &= \hat{U} \times \hat{T}^{1/2} \\
\hat{U} \hat{\times} \hat{U}^\dagger &= \hat{U}^\dagger \hat{\times} \hat{U} = \frac{1}{\lambda}. \quad (15)
\end{aligned}$$

It may be of some interest to indicate the expected EPR completion of other branches of physics, such as the completion of quantum computers into new computers, here suggested under the name of *EPR computers*, for the description of extended electronic constituents in global, continuous and instantaneous communications, by therefore approaching the new notion of living organisms attempted in [39], with the following expected advances:

**1) Faster computations**, since all values of Bohm hidden variable  $\lambda$  are very small according to all available fits of experimental data [6], with ensuing rapid convergence of isoperturbative series (see also Corollary 3.7.1, p. 128 of [20]). As a confirmation of this expectation, we recall the achievement via iso-mathematics and iso-chemistry of the first known *attractive* force between the *identical* electrons of valence coupling (see Chapter 4 of [40]), resulting in a *strong valence bond* that allowed the first known numerically exact representation of the experimental data for the hydrogen [41] and water [42] molecules with iso-perturbative calculations at least one thousand times faster than their conventional chemical counterparts.

**2) Better cybersecurity**, due to the formulation via iso-mathematics, with the consequential availability of iso-cryptograms equipped with an algorithm changing the numeric value of the iso-unit with such a frequency to prevent a solution within a finite period of time (Appendix 2C, p. 84 of [15] and [43]).

**3) Increased energy efficiency**, due to the fact that EPR entanglements are caused by *interactions without potential energy*, thus being more energy efficient than quantum computers.

The following comments are now in order:

**I. Verifications of the EPR argument.** In 1964, J. S. Bell [44] released a theorem according to which, under the assumption of quantum mechanics, the representation of the spin 1/2 of particles via Pauli's matrices, statistical independence and other assumptions, *a system of point-like particles with spin 1/2 does not admit a classical counterpart*, by therefore preventing any possibility of recovering Einstein determinism [4] under the indicated assumptions. The theorem was proved by showing that a certain expression  $D^{Bell}$  (whose explicit value depends on the relative conditions of the particles) is always *smaller* than the corresponding classical value  $D^{Clas}$ ,

$$D^{Bell} < D^{Clas}, \quad (16)$$

for all possible values of  $D^{Bell}$ .

By assuming that nuclear forces have a non-linear, non-local and non-potential component represented via the isotopic element  $\hat{T}$ , R. M. Santilli initiated in 1981 [10] the studies on the inevitable *completion of Heisenberg's uncertainties and Bell's inequalities for strong interactions*.

Following the achievement of maturity for iso-mathematics and iso-mechanics, by representing the *extended* character of nuclear constituents via the *isotopic completion of Pauli's matrices with Bell's hidden variables* [16], today called the *Pauli-Santilli iso-matrices* formulated on a Hilbert-Myung-Santilli iso-space [33] over Santilli iso-fields  $C$  [34] (see Eqs. (6.8.20), p. 254 of [16])

$$\begin{aligned}\hat{\sigma}_k &= U\sigma_k U^\dagger, \quad UU^\dagger = \hat{I} = 1/\hat{T} = \text{Diag.}(\lambda^{-1}, \lambda), \\ \hat{\sigma}_1 &= \begin{pmatrix} 0 & \lambda \\ \lambda^{-1} & 0 \end{pmatrix}, \quad \hat{\sigma}_2 = \begin{pmatrix} 0 & -i\lambda \\ i\lambda^{-1} & 0 \end{pmatrix}, \\ \hat{\sigma}_3 &= \begin{pmatrix} \lambda^{-1} & 0 \\ 0 & -\lambda \end{pmatrix},\end{aligned}\quad (17)$$

with *iso-commutation rules*

$$[\hat{\sigma}_i, \hat{\sigma}_j] = \hat{\sigma}_i \hat{\sigma}_j - \hat{\sigma}_j \hat{\sigma}_i = i2\epsilon_{ijk} \hat{\sigma}_k, \quad (18)$$

and *conventional spin 1/2 iso-eigenvalues*

$$\begin{aligned}\hat{\sigma}_3 \hat{\sigma}_3 |\hat{b}\rangle &= \hat{\sigma}_3 \hat{T} |\hat{b}\rangle = \pm |\hat{b}\rangle, \\ \hat{\sigma}_3^2 \hat{\sigma}_3 |\hat{b}\rangle &= (\hat{\sigma}_1 \hat{T} \hat{\sigma}_1 + \hat{\sigma}_2 \hat{T} \hat{\sigma}_2 + \hat{\sigma}_3 \hat{T} \hat{\sigma}_3) \hat{T} |\hat{b}\rangle = 3 |\hat{b}\rangle.\end{aligned}\quad (19)$$

R. M. Santilli proved in 1998 the following *completion of Bell's inequalities for strong interactions* (Eq. (5.8), p. 189 of [11])

$$D^{HM} = \frac{1}{2}(\lambda_1 \lambda_2^{-1} + \lambda_1^{-1} \lambda_2) D^{Bell}, \quad (20)$$

where  $\lambda_1$  and  $\lambda_2$  are the hidden variables of the two particles. Additionally, Santilli proved that  $D^{HM}$  can indeed be equal to the corresponding  $D^{Class}$  with specific examples, by therefore confirming Einstein's view on the possible recovering of classical determinism.

Finally, by combining the results of [10] and [11], in 2019 R. M. Santilli [12] (see the review in [20]) proved the following *completion of Heisenberg's uncertainties for strong interactions* (Eq. (35), p. 14 of [12])

$$\Delta r \Delta p \approx \frac{1}{2} |\langle \hat{\psi}(\hat{r}) | \hat{r} | \hat{\psi}(\hat{r}) \rangle| \ll \frac{1}{2} \hat{T} = \frac{\lambda}{2} \quad (21)$$

establishing that *the standard deviations  $\Delta r$  and  $\Delta p$ , individually as well as their product, progressively approach Einstein's classical determinism with the increase of the density in the interior of hadrons, nuclei, and stars, and achieve a full classical determinism at the limit of Schwarzschild's horizon for which  $\lambda = \hat{T} = 0$ .*

In essence, verifications [10–12] of the EPR argument establish that Bell's inequalities are *valid* for the electromagnetic interactions of point-like particles, including electrons and photons, with ensuing lack of hidden variables.

By contrast, following half a century of mathematical, theoretical and experimental studies in the field, this author believes that Bell's inequalities are *inapplicable*, in favor of their completions via hadronic mechanics [10–12], for composite systems of particles at short mutual distances, including hadrons [20] and leptons [47], because the exact representation of their experimental data requires non-unitary transforms of quantum models, under which none of Bell's assumptions can be formulated, with ensuing validity of Bohm's hidden variables.

It then follows that any experiment proving the *violation* of Bell's inequalities, as defined by the above equations, is a direct experimental verification of hadronic mechanics.

**II. Conditions of validity of hadronic mechanics.** Recall that the wave packet of one electron is identically null only at infinity and, consequently, the universe is a single integrated structure much similar to the total EPR entanglement of living organisms [39]. Recall also that the universe will never admit one single final theory for the representation of all its complexities. Under these recollections, this author believes that the isotopic branch of hadronic mechanics can indeed provide a first axiomatically consistent representation of *stable*, thus time reversible systems, while the genotopic branch of hadronic mechanics can provide a first axiomatically consistent representation of energy-releasing processes, including nuclear fusions and fossil fuel combustion [20], and the hyperstructural branch of hadronic mechanics can at least initiate the search for an axiomatically consistent representation of life [39], here referred to a quantitative representation of the difference between organic and inorganic molecules.

**III. Conditions of validity of quantum mechanics.** By recalling that “point-like wave packets” do not exist in nature, and that quantum mechanics is identically and uniquely recovered by iso-mathematics and iso-mechanics for  $\lambda = \hat{T} = 1$ , this author believes that the Copenhagen interpretation of quantum mechanics provides an excellent representation of *stable systems of particles at mutual distances allowing their effective approximation as being point-like*, by therefore solely admitting action-at-a-distance potential interactions, with ensuing validity for atomic structures, particles in accelerators, crystals and numerous other systems. Despite these justly historical achievements, following half a century of studies in the field, this author believes that *quantum mechanics cannot be exactly valid for particle entanglements, as shown in this note, as well as for all composite systems of particles at short mutual distances, thus including leptons, hadrons, nuclei and stars* [17].

It is appropriate in the latter respect to note that the widespread assumption that a single theory, quantum mechanics, can represent all possible complexities of the universe, has

been kept for about one century in oblivion of:

A) Clear experimental evidence in various fields of *deviations* of physical reality from quantum predictions in favor of *exact* representations via hadronic mechanics, including deviations in: nuclear physics [45]; electrodynamics [46–48]; nuclear physics [45]; condensed matter physics [49]; heavy ion physics [50]; time dilation for composite particles [51]; Bose-Einstein correlation [52, 53]; cosmology [54, 55]; and other fields.

B) The insufficiencies of quantum mechanics in nuclear physics due to its inability over one century under large public funds to achieve [19–21]: a quantitative representation of the synthesis of the neutron from the hydrogen in the core of stars; an exact representation of nuclear magnetic moments; an exact representation of the spin of nuclei in their true ground state (that without the usual orbital excitations); a representation of the stability of nuclei despite the huge Coulomb repulsion between nuclear protons; a representation of the stability of neutrons when members of a nuclear structure; and other insufficiencies.

C) The inability by quantum mechanics to allow a consistent treatment of energy-releasing processes, including nuclear fusions, due to their time irreversibility compared to the known time-reversibility of quantum mechanics (e.g. because of the invariance of Heisenberg's equation under anti-Hermiticity and for other reasons). Under these conditions, the same Schrödinger equation has to be applied for both, the forward and backward time evolutions, with ensuing violation of causality due to unavoidable solutions in which the effect precedes the cause. This violation of causality may explain the lack of achievement to date of controlled nuclear fusion [56].

Consequently, the continuation of the century-old use and support of quantum mechanics for all possible conditions existing in the universe in oblivion of the teaching by Einstein, Podolsky and Rosen, in oblivion of vast opposing experimental evidence, and in oblivion of fundamental unresolved nuclear problems, may continue to have widely negative implications for our rapidly deteriorating environment.

## Acknowledgements

The author would like to thank for critical comments the participants of the *2020 International Teleconference on the EPR argument*. Additional thanks are due to Mrs. Sherri Stone for linguistic control of the manuscript.

This note has been written from 9-20 to 9-30, 2022, at the Pietra Blue Resort, Polignano a Mare, Bari, Italy.

Received on October 9, 2022

## References

1. Aspect A. *et al.* Experimental Realization of Einstein-Podolsky-Rosen-Bohm Gedankenexperiment: A New Violation of Bell's Inequalities. *Phys. Rev. Lett.*, 1982, v. 49, 91–94. <http://ui.adsabs.harvard.edu/abs/1982PhRvL..49...91A>

2. Eigen G. Measurements of the Polarization Correlation of the Two-Photon System Produced in Positron-Electron Annihilation. *Proceedings of the 2020 Teleconference on the EPR argument*, Curran Associates, New York, 823–843, 2021.
3. Berkowitz R. Macroscopic systems can be controllably entangled and limitlessly measured. *Physics Today*, July 2021, 16–18.
4. Einstein A., Podolsky B., and Rosen N. Can quantum-mechanical description of physical reality be considered complete? *Phys. Rev.*, 1935, v. 47, 777–791. <http://www.eprdebates.org/docs/epr-argument.pdf>
5. Beghella-Bartoli S. and Santilli R. M., eds, Proceedings of the 2020 Teleconference on the Einstein-Podolsky-Rosen argument that “Quantum mechanics is not a complete theory”. Curran Associates, New York, NY, 2021. <http://www.proceedings.com/59404.html> (printed), 60007.html (electronic). <http://www.world-lecture-series.org/level-xii-epr-teleconference-2020> (recorded lectures).
6. Santilli R. M. Overview of historical and recent verifications of the Einstein-Podolsky-Rosen argument and their applications to physics, chemistry and biology. APAAV - Accademia Piceno Aprutina dei Velati, Pescara, Italy, 2021. <http://www.santilli-foundation.org/epr-overview-2021.pdf>
7. Dunning-Davies J. A Present Day Perspective on Einstein-Podolsky-Rosen and its Consequences. *Journal of Modern Physics*, 2021, v. 12, 887–936. <http://www.scirp.org/journal/paperinformation.aspx?paperid=109219>
8. Heisenberg W. *Nachr. Akad. Wiss. Gottingen*, 1953, v. IIa, 111. [http://link.springer.com/chapter/10.1007/978-3-642-70079-8\\_23](http://link.springer.com/chapter/10.1007/978-3-642-70079-8_23)
9. Goldstein S. Bohmian (de Broglie-Bohm) Mechanics. Stanford Encyclopedia of Philosophy, 2021. <http://plato.stanford.edu/entries/qm-bohm/>
10. Santilli R. M. Generalization of Heisenberg's uncertainty principle for strong interactions. *Hadronic Journal*, 1981, v. 4, 642–657. <http://www.santilli-foundation.org/docs/generalized-uncertainties-1981.pdf>
11. Santilli R. M. Isorepresentation of the Lie-isotopic SU(2) Algebra with Application to Nuclear Physics and Local Realism. *Acta Applicandae Mathematicae*, 1998, v. 50, 177–190. <http://www.santilli-foundation.org/docs/Santilli-27.pdf>
12. Santilli R. M. Studies on the classical determinism predicted by A. Einstein, B. Podolsky and N. Rosen. *Ratio Mathematica*, 2019, v. 37, 5–23. <http://www.eprdebates.org/docs/epr-paper-ii.pdf>
13. Santilli R. M. Foundation of Theoretical Mechanics. Vol. I, The Inverse Problem in Newtonian Mechanics. Springer-Verlag, Heidelberg, Germany, 1978. <http://www.santilli-foundation.org/docs/Santilli-209.pdf>
14. Santilli R. M. Foundation of Theoretical Mechanics. Vol. II, Birkhoffian Generalization of Hamiltonian Mechanics. Springer-Verlag, Heidelberg, Germany, 1983. <http://www.santilli-foundation.org/docs/santilli-69.pdf>
15. Santilli R. M. Elements of Hadronic Mechanics. Vol. I, Mathematical Foundations. Ukraine Academy of Sciences, Kiev, 1995. <http://www.santilli-foundation.org/docs/Santilli-300.pdf>
16. Santilli R. M. Elements of Hadronic Mechanics. Vol. II, Theoretical Foundations. Ukraine Academy of Sciences, Kiev, 1995. <http://www.santilli-foundation.org/docs/Santilli-301.pdf>
17. Santilli R. M. Elements of Hadronic Mechanics. Vol. III, Experimental Verifications. Ukraine Academy of Sciences, Kiev, 2016. <http://www.santilli-foundation.org/docs/elements-hadronic-mechanics-iii.compressed.pdf>
18. Anderson R. Outline of Hadronic Mathematics, Mechanics and Chemistry as Conceived by R. M. Santilli. *American Journal of Modern Physics*, 2016, v. 6, 1–16. <http://www.santilli-foundation.org/docs/HMMC-2017.pdf>

19. Santilli R. M. Studies on A. Einstein, B. Podolsky, and N. Rosen prediction that quantum mechanics is not a complete theory. I: Basic methods. *Ratio Mathematica*, 2020, v. 38, 5–69. [//eprdebates.org/docs/epr-review-i.pdf](http://eprdebates.org/docs/epr-review-i.pdf)
20. Santilli R. M. Studies on A. Einstein, B. Podolsky, and N. Rosen prediction that quantum mechanics is not a complete theory. II: Apparent proof of the EPR argument. *Ratio Mathematica*, 2020, v. 38, 71–138. [//eprdebates.org/docs/epr-review-ii.pdf](http://eprdebates.org/docs/epr-review-ii.pdf)
21. Santilli R. M. Studies on A. Einstein, B. Podolsky, and N. Rosen prediction that quantum mechanics is not a complete theory. III: Illustrative examples and applications. *Ratio Mathematica*, 2020, v. 38, 139–222. [//eprdebates.org/docs/epr-review-iii.pdf](http://eprdebates.org/docs/epr-review-iii.pdf)
22. Aringazin A. K., Jannussis A., Lopez F., Nishioka M. and Veljanosky B. Santilli's Lie-Isotopic Generalization of Galilei and Einstein Relativities. Kostakaris Publishers, Athens, Greece, 1991. <http://www.santilli-foundation.org/docs/Santilli-108.pdf>
23. Sourlas D. S. and Tsagas Gr. T. Mathematical Foundation of the Lie-Santilli Theory. Ukraine Academy of Sciences, 1993. <http://www.santilli-foundation.org/docs/santilli-70.pdf>
24. Lohmus J., Paal E. and Sorgsepp L. Non-associative Algebras in Physics. Hadronic Press, 1994. <http://www.santilli-foundation.org/docs/Lohmus.pdf>
25. Kadeisvili J. V. Santilli's Isotopies of Contemporary Algebras, Geometries and Relativities, Second edition. Ukraine Academy of Sciences, Kiev, 1997. <http://www.santilli-foundation.org/docs/Santilli-60.pdf>
26. Jiang C.-X. Foundations of Santilli Isonumber Theory. International Academic Press, 2001. <http://www.i-b-r.org/docs/jiang.pdf>
27. Ganfornina R. M. F. and Valdes J. N. Fundamentos de la Isotopia de Santilli. International Academic Press, Palm Harbor, FL, 2001. [www.i-b-r.org/docs/spanish.pdf](http://www.i-b-r.org/docs/spanish.pdf). English translation: Algebras, Groups and Geometries, 2015, v. 32, 135–308 (2015) <http://www.i-b-r.org/docs/Aversa-translation.pdf>
28. Gandzha I. and Kadeisvili J. V. New Sciences for a New Era: Mathematical, Physical and Chemical Discoveries of Ruggero Maria Santilli. Sankata Printing Press, Nepal, 2011. <http://www.santilli-foundation.org/docs/RMS.pdf>
29. Georgiev S. Foundations of IsoDifferential Calculus. Vol 1. – Iso-Differential and Iso-Integral Calculus for Iso-Functions in One Variable. Vol 2. – Iso-Differential and Iso-Integral Calculus for Iso-Functions in Several Variables. Vol. 3 – Iso-Ordinary Iso-Differential Equations. Vol. 4 – Iso-Difference Equations. Vol.5 – Iso-Stochastic Iso-Differential Equations. Vol. 6 – Theory of Iso-Measurable Iso-Functions. Nova Publishers, New York, NY, 2014 (I), 2014 (II), 2014 (III), 2015 (IV), 2015 (V), 2016 (VI), 2022 (I new ed.).
30. Georgiev S. Iso-Mathematics. Lambert Academic Publishing, 2022.
31. Bohm D. A Suggested Interpretation of the Quantum Theory in Terms of "Hidden Variables". *Physical Review*, 1952, v. 85, 166–182. <http://journals.aps.org/pr/abstract/10.1103/PhysRev.85.166>
32. Santilli R. M. Nonlocal-Integral Isotopies of Differential Calculus, Mechanics and Geometries. *Rendiconti Circolo Matematico Palermo, Suppl.*, 1996, v. 42, 7–82. <http://www.santilli-foundation.org/docs/Santilli-37.pdf>
33. Myung H. C. and Santilli R. M. Modular-isotopic Hilbert space formulation of the exterior strong problem. *Hadronic Journal*, 1982, v. 5, 1277–1366. <http://www.santilli-foundation.org/docs/myung-santilli-1982.pdf>
34. Santilli R. M. Isonumbers and Genonumbers of Dimensions 1, 2, 4, 8, their Isoduals and Pseudoduals, and "Hidden Numbers" of Dimension 3, 5, 6, 7. *Algebras, Groups and Geometries*, 1993, v. 10, 273–322. <http://www.santilli-foundation.org/docs/Santilli-34.pdf>
35. Santilli R. M. Lie-isotopic Lifting of Special Relativity for Extended Deformable Particles. *Lettere Nuovo Cimento*, 1983, v. 37, 545–555. <http://www.santilli-foundation.org/docs/Santilli-50.pdf>
36. Santilli R. M. Isotopic Generalizations of Galilei and Einstein Relativities. Intern. Academic Press, 1991. Vol. I: <http://www.santilli-foundation.org/docs/Santilli-01.pdf> Vol. II: <http://www.santilli-foundation.org/docs/Santilli-61.pdf>
37. Aringazin A. K., Jannussis A., Lopez F., Nishioka M. and Veljanosky B. Santilli's Lie-Isotopic Generalization of Galilei and Einstein Relativities. Notes from R. M. Santilli's 1990 Lectures at the ICTP, Trieste, Italy. Kostakaris Publishers, Athens, Greece, 1991. <http://www.santilli-foundation.org/docs/Santilli-108.pdf>
38. Santilli R. M. Invariant Lie-isotopic and Lie-admissible formulation of quantum deformations. *Found. Phys.*, 1997, v. 27, 1159–1177. <http://www.santilli-foundation.org/docs/Santilli-06.pdf>
39. Santilli R. M. and Vougiouklis T. New Conception of Living Organisms and its Representation via Lie-Admissible  $H_b$ -Hyperstructures. *Algebras, Groups and Geometries*, 2020, v. 37, 741–764. <http://www.santilli-foundation.org/docs/Santilli-Vougiouklis-2020-epr.pdf>
40. Santilli R. M. Foundations of Hadronic Chemistry, with Applications to New Clean Energies and Fuels. Kluwer Academic Publishers, 2001. <http://www.santilli-foundation.org/docs/Santilli-113.pdf>. Russian translation: Aringazin A. K. <http://i-b-r.org/docs/Santilli-Hadronic-Chemistry.pdf>
41. Santilli R. M. and Shillady D. D. A new isochemical model of the hydrogen molecule. *Intern. J. Hydrogen Energy*, 1999, v. 24, 943. <http://www.santilli-foundation.org/docs/Santilli-135.pdf>
42. Santilli R. M. and Shillady D. D. A new isochemical model of the water molecule. *Intern. J. Hydrogen Energy*, 2000, v. 25, 173. <http://www.santilli-foundation.org/docs/Santilli-39.pdf>
43. Wade M. I. and Gill T. L. Isonumbers and RGB Image Encryption. *Algebras, Groups and Geometries*, 2021, v. 37, 103–119. <http://www.santilli-foundation.org/docs/wade-gill-2021.pdf>
44. Bell J. S. On the Einstein Podolsky Rosen paradox. *Physics*, 1964, v. 1, 195–200. <http://cds.cern.ch/record/111654/files/vol1p195-200.001.pdf>
45. Santilli R. M. Iso-Representation of the Deuteron Spin and Magnetic Moment via Bohm's Hidden Variables. *Progress in Physics*, 2022, v. 18, 74–81. <http://www.santilli-foundation.org/docs/PiP-paper-3-22.pdf>
46. Miller J. P., de Rafael E. and Roberts B. Lee. Muon ( $g-2$ ): experiment and theory. *Rep. Prog. Phys.*, 2007, v. 70, 795–881. <http://news.fnal.gov/2021/04/first-results-from-fermilabs-muon-g-2-experiment-strengthen-evidence-of-new-physics/>
47. Santilli R. M. Representation of the anomalous magnetic moment of the muons via the Einstein-Podolsky-Rosen completion of quantum into hadronic mechanics. *Progress in Physics*, 2021, v. 17, 210–215. <http://www.santilli-foundation.org/muon-anomaly-pp.pdf>
48. Santilli R. M. Representation of the anomalous magnetic moment of the muons via the novel Einstein-Podolsky-Rosen entanglement. In: Guzman J. C., ed. Scientific Legacy of Professor Zbigniew Oziewicz: Selected Papers from the International Conference "Applied Category Theory Graph-Operad-Logic". Word Scientific, in press. <http://www.santilli-foundation.org/ws-rv961x669.pdf>
49. Fadel M., Zibold T., Decamps B. and Treutlein Ph. Spatial entanglement patterns and Einstein-Podolsky-Rosen steering in Bose-Einstein condensates. *Science*, 2018, v. 360, 409–415. <http://www.santilli-foundation.org/Basel-paper.pdf>
50. Schukraft J. Heavy-ion physics with the ALICE experiment at the CERN Large Hadron Collider. *Trans. R. Soc.*, 2012, v. A370, 917–932. <http://royalsocietypublishing.org/doi/10.1098/rsta.2011.0469>
51. Santilli R. M. Apparent Unsettled Value of the Recently Measured Muon Magnetic Moment. *Progress in Physics*, 2022, v. 18, 15–18. <http://www.santilli-foundation.org/docs/muon-meanlife-2022.pdf>

52. Santilli R. M. Nonlocal formulation of the Bose-Einstein correlation within the context of hadronic mechanics. *Hadronic J.*, 1992, v. 15, 1–50 and v. 15, 81–133. <http://www.santilli-foundation.org/docs/Santilli-116.pdf>
  53. Cardone F. and Mignani R. Nonlocal approach to the Bose-Einstein correlation. *Europ. Phys. J.*, 1998, v. C4, 705–728.
  54. Ahmar H., Amato G., Kadeisvili J.V., Manuel J., West G. and Zogorodnia O. Additional experimental confirmations of Santilli's IsoRedShift and the consequential lack of expansion of the universe. *Journal of Computational Methods in Sciences and Engineering*, 2013 v. 13, 321–375. <http://www.santilli-foundation.org/docs/IRS-confirmations-212.pdf>
  55. Santilli R. M. Experimental Verifications of IsoRedShift with Possible Absence of Universe Expansion, Big Bang, Dark Matter, and Dark Energy. *The Open Astronomy Journal* 2010, v. 3, 124. <http://www.santilli-foundation.org/docs/Isoredshift-Letter.pdf>
  56. Santilli R. M. Apparent resolution of the Coulomb barrier for nuclear fusions via the irreversible Lie-admissible branch of hadronic mechanics. *Progress in Physcs*, 2022, in press. <http://www.santilli-foundation.org/docs/hyperfusion-2022.pdf>
-