Avoiding Negative Energies in Quantum Mechanics

G. G. Nyambuya

National University of Science and Technology, Faculty of Applied Sciences – Department of Applied Physics, Fundamental Theoretical and Astrophysics Group, P. O. Box 939, Ascot, Bulawayo, Republic of Zimbabwe.
E-mail: physicist.ggn@gmail.com

Quantum mechanical observables are naturally assumed to be real. Herein, we depart from this traditional and seemingly natural assumption whereby we consider a Quantum Mechanics (QM) whose operators have corresponding complex eigenvalues. The motivation for this is that complex eigenvalues lead us directly to positive definite energy solutions, hence mass. The resulting QM is able to qualitatively explain in a coherent manner some physical phenomenon that are currently inexplicable from a QM whose operators have corresponding real eigenvalues – e.g. one is now able to explain the instability of particles, their localization, the observed matter-antimatter asymmetry and the supposed variation of fundamental natural constants amongst others. In addition to this, there is the difficulty in Dirac's interpretation of negative energy states appearing in his theory. While Dirac's negative energy problem is not considered a problem anymore, we provide an alternative way out of this problem. We propose that eigenvalues corresponding to quantum mechanical operators associated with physical observables aught to be complex. From this seemingly simple hypothesis, we demonstrate that negative energy states leading to negative mass can be avoided altogether.

I cannot imagine a reasonable Unified Theory containing an explicit number which the whim of the Creator could just as easily have chosen differently … Numbers arbitrarily chosen by God do not exist. Their alleged existence relies on our incomplete understanding [of the Laws of Nature and how God designed and fashioned the Universe].

Albert Einstein (1879-1955)

1 Introduction

Looking back – thus far, one can most confidently and safely say that the time period of the first thirty years of the twentieth century was perhaps a special time in the intellectual discource of humanity with this period being a period of the greatest intellectual leaps in all the history of human thought and intellectual endeavour. For to date, these great intellectual leaps have found no equal. Perhaps, apart from CERN's famous 4^{th} of July 2012 announcement that a strong signal mimicking a *Higgs-like* particle has been detected in the LHC data, it appears as though real progress in Physics has hit a serious brick wall. In all probity, it aught to be said that there has not been any real noteworthy and new exciting discoveries this century as those witnessed at the beginning of the twentieth century, especially on the *frontiers of fundamental theoretical Physics*.

Take for example: in 1905, Germany's youthful 26 years' old third class Swiss patent clerk Albert Einstein (1879-1955) [1] discovered the Special Theory of Relativity (STR), and shortly thereafter, in the period 1923-4, France's aristocrat and physicist Louis Victor Pierre Raymond de Broglie (1892-1987) [2–5] opened *Pandora's Box* with his wave-particle duality hypothesis, Germany's great physicist Weiner Karl Heisenberg (1901-1976) [6] theoretically argued his uncertainty principle into existence and Austria's own theoretical physicist Erwin Rudolf Josef Alexander Schrödinger (1887-1961) [7, 8] discovered the key wave equation of Quantum Mechanics (QM) which now bears his name, *etc*.

Once QM was incepted in the mid-1920s, no sooner was it realised that there was a need to unite these two theories which stand to this day as a major part of the twin pillars of modern physics – i.e. the STR and QM. At the time of these great discoveries and revolutionary paradigm shifts, nobody yet knew how to make the two theories consistent with each other. In 1928 while QM was still in its nascence, the then little-known British preeminent Paul Adrien Maurice Dirac (1902-1984), who ranks as one of the greatest fundamental theoretical physicists of his time, then only 26 years' old, succeeded where others found it difficult. Dirac [9, 10] successfully unified Einstein [1]'s STR and de Broglie [2–5], Heisenberg [6] and Schrödinger [7, 8]'s QM.

Dirac [9, 10]'s unified theory was an unprecedented success, except for one detail: a quantum system could have either positive or negative energy. How can something have negative energy? For example, according to Einstein [1]'s mass-energy equivalence, the mass (m) of a particle is related to its energy (E) by the relation $m = E/c_0^2$ (where: $c_0 = 2.99792458 \times 10^8\,\mathrm{m\,s^{-1}}$ is the speed of light in *vacuo*), such that negative energy entails negative mass. For all we know, the measure of the resistance to any change of the state of motion of a given substance is a measure of its mass. Further, mass was and is understood as a measure of the quantity of matter in a substance. From this understanding, what does negative mass mean?

According to Newton's first law of motion, since a positive mass quantum system has the property that it has the tendency to preserve its current state of motion in such a manner that it resists all efforts to change this state of motion, does it then mean that a negative mass quantum system will have the exact opposite properties, that is, have the property that it has the tendency not to preserve the particle's current state of motion in such a manner that it does not resist any efforts to change the particle's current state of motion but only engenders it? Such are some of the plausible questions that have puzzled those that have attempted to comprehend what negative mass might actually be or mean. What will happen when positive energy-mass matter comes in contact with negative energy-mass matter? Will they nullify upon contact? These are just some of the pertinent questions out of many plausible ones that come to mind.

Be that as it may, Dirac was an extraordinary brilliant man who sought beauty in his work. He did not think of the negative energy quantum systems implied by his equation in ordinary terms, but thought of them mathematically and quantum mechanically. The negative energy solutions first appeared in the Klein [11] and Gordon [12] theory (KG-theory) on whose shoulders the Dirac's theory stands. In order to get rid of these negative energies, some notable figures of the time suggested that these negative energy solutions must be discarded with the simple remark that "these solutions have no correspondence with physical and natural reality". To that, Dirac [9] replied:

> One gets over the difficulty on the classical theory by arbitrarily excluding those solutions that have a negative E. One cannot do this in the QM, since in general a perturbation will cause transitions from states with *E-positive* to states with *E-negative*.

So, it would strongly appear that negative energy states were here to stay – at least in theory. They needed a satisfying physical explanation.

While Dirac's theory was met with both enthusiasm and scepticism (e.g. by physicists Werner Heisenberg, Wolfgang Ernst Pauli (1900-1958), Ernst Pascual Jordan (1902-1980), George Gamow (1904-1968), amongst others), the enthusiasm was on the latent power wielded by the equation, e.g. the equation solved the difficult contemporary problem of spin; and scepticism was with respect to the negative energy solutions. Against this scepticism, Dirac [13] further proposed that the *vacuo* was an unobservable infinite sea of negative energy states, such that all negative energy states were filled! This invisible sea of negative energy states became known as the Dirac Sea.

According to *Pauli [14]'s Exclusion Principle* (PEP) that forbids more than two fermions to be in the same quantum state, a Universe in which there exists a Dirac Sea would forbid the transitions of positive energy quantum states to transit into negative energy states thereby resulting in a Universe that has stable positive energy states. Transitions from states with *E-positive* to states with *E-negative* are forbidden because the *E-positive* state once in the *E-negative* state is going to be in the same quantum state as the *E-negative* state thus violating the PEP, hence, forbidden by Nature. In this way, the Dirac [9, 10] theory was safe.

To further clarify Dirac's theory, the preeminent American physicist Richard Phillips Feynman (1924-1987) proposed that the negative energy states be interpreted as antiparticles: they move backwards in time such that, in a Universe where time moves in a forward direction, these quantum states would appear as positive energy states. This is the current *de facto* interpretation of antiparticles. Other than the negative energy problem, Dirac [9, 10]'s equation exhibits a perfect symmetry and this property of the equation has no correspondence nor bearing with physical and natural reality as we know it. Often, the theory has had to be patched [15, for example] in order to measure up to physical and natural reality. These patches often propose that the combined Charge (C) and Space or Parity (P) reversal symmetry (CP violation) must explain the apparent matter-antimatter asymmetry [16]. While CP violation has been observed [17–22, for example], it is yet to be verified by experiment as the mechanism responsible for the observed matter-antimatter asymmetry.

We must hasten to say that, while this work will touch on other subjects that we had not intended to cover, the original and sole aim of this work is twofold:

1. To demonstrate that Dirac's negative energy solutions can be eliminated altogether by resorting to particles endowed with Complex Energy and Momentum (CEM) wherein under this new proposal, the energy and momentum of the quantum system of concern is measured as the magnitude of these complex quantities.
2. To show that the resultant energy from the resulting complex energy and momentum does solve without any need for exogenous ideas, the matter-antimatter asymmetry problem that the Dirac theory has so far failed to solve and possibly the recent issue to do with the plausible variation of *Fundamental Natural Constants* (FNCs).

To achieve our desired objective, we adopt the working hypothesis, that in general, all quantum mechanical observables such as the energy and momentum of quantum mechanical systems can take complex values ($z = x + i\,y$) such that the resultant observable that we measure in the laboratory is the magnitude of this complex quantity in question (i.e. $|z| = \sqrt{x^2 + y^2} \geq 0$). This assumption is all that we shall require in our exploration. As a result, we shall formulate a new basis for the further development of a QM that allows for observables to take complex values and from thereon, proceed to show that the theory resulting from the CEM hypothesis not only provides a plausible and perdurable solution to Dirac's problem of negative energies, but that, it also provides a plausible solution to the matter-antimatter asymmetry problem which the bare Dirac theory is unable to solve by its own.

In closing this introductory section, we give the synopsis of the reminder of this paper. That is: in §2, we discuss the idea of complex quantum mechanical observables: we discuss how this idea may provide a perdurable solution to Dirac's negative energy problem. In §3, we write down the usual Dirac equation and thereafter proceed to incorporate into its structure the CEM hypothesis. In §4, we apply the idea of complex quantum mechanical observables to the notion of the variation of FNCs. In §5, we work out the symmetries of the new CEM-Dirac equation, and lastly, in §6 and §7, we give a general discussion and the conclusion drawn thereof, respectively.

2 Complex energy and momentum

The negative probabilities manifesting in the KG-theory are a result of the fact that the emergent quantum probability (P_{Q}) expression in this theory is directly proportional to the energy (E) of the quantum system in question – i.e. $P_{\mathrm{Q}} \propto E$, the consequential meaning of which is that, for negative energy quantum systems, the corresponding quantum probability will be negative. From this very fact $P_{\mathrm{Q}} \propto E$, Dirac hatched the idea that these negative energies appearing in the quantum probability of the KG-theory could be removed if a theory linear in the temporal and spatial derivatives were possible because a linear system of equations will always have one solution, a quadratic two, a cubic three, a quad four, *etc*.

Further on his effort to eliminate these meaningless negative probabilities, Dirac hoped that with his linear solution, he might also eliminate the negative energy solutions as well. Because of the pivotal constraint that he imposed on his theory, namely that when his equation is *squared* it must yield the quantum mechanical wave equation of the KG-theory, this directly translates to the fact that the energy solutions of Dirac's quantum systems would exactly be as those obtained in the KG-theory, thus leading back to the same problem of negative energies faced by the KG-theory. The only way to eliminate these supposedly *nagging* negative energy solutions would be to build a theory from an energy-momentum equation that only admits positive definite energy solutions from the outset. This is the approach that we take here. We make use of a property of complex numbers – namely that the magnitude of a complex number is always a positive definite quantity.

To that end, we postulate that every physical observable ($\mathcal{O} \in \mathbb{C}$) shall be considered to have two parts to it, namely: the real part ($\mathcal{O}_R \in \mathbb{R}$), and the imaginary part ($\mathcal{O}_I \in \mathbb{R}$), that is to say:

$$\mathcal{O} = \mathcal{O}_R + i\,\mathcal{O}_I\,. \tag{1}$$

The subscripts R and I in (1) are used to label the real and imaginary parts of the complex physical quantity in question. For example, if the energy of a quantum system were complex, then $E = E_R + i\,E_I$, where E_R and E_I are the real and imaginary parts of the energy respectively. The imaginary part of the energy may lead to the possibility of naturally explaining the phenomenon of particle decay. In the case of momentum, $\vec{p} = \vec{p}_R + i\,\vec{p}_I$, where $\vec{p}_R$ and $\vec{p}_I$ are the real and imaginary parts of the momentum respectively. Likewise, the imaginary part of the momentum may very well lead one to be able to naturally explain why particles are localised. These are interesting issues that can be tackled in a separate paper in the future.

Once the energy and momentum are complex physical variables, the rest-mass m_0 cannot be spared – i.e. $m_0 = m_R + i\,m_I$, where $(m_R, m_I) \in \mathbb{R}$. In summary:

$$E = E_R + i\,E_I\,, \tag{2a}$$

$$\vec{p} = \vec{p}_R + i\,\vec{p}_I\,, \tag{2b}$$

$$m_0 = m_R + i\,m_I\,. \tag{2c}$$

What (2) implies is that the four momentum p_μ, will have two parts to it – with one part that is associated with the real part and the other with the imaginary part, i.e.

$$\begin{aligned} p_\mu &= \left(\vec{p}, \frac{i\,E}{c_0^2}\right) \\ &= \left(\vec{p}_R, \frac{i\,E_R}{c_0^2}\right) + i\left(\vec{p}_I, \frac{i\,E_I}{c_0^2}\right) \\ &= p_\mu^R + i\,p_\mu^I\,, \end{aligned} \tag{3}$$

where:

$$\vec{p}_R = p_1^R\vec{\hat{i}} + p_2^R\vec{\hat{j}} + p_3^R\vec{\hat{k}}\,, \tag{4a}$$

$$\vec{p}_I = p_1^I\vec{\hat{i}} + p_2^I\vec{\hat{j}} + p_3^I\vec{\hat{k}}\,. \tag{4b}$$

For p^μ, we will have $p^\mu = (\vec{p}, i\,E/c_0^2)^* = (\vec{p}^*, -i\,E^*/c_0^2)$, so that the relativistic invariant quantity $p^\mu p_\mu$ is now such that $p^\mu p_\mu = m_0^* m_0 c_0^2$, i.e.

$$|E|^2 - |\vec{p}|^2 c_0^2 = |m_0|^2 c_0^4\,, \tag{5}$$

where:

$$|E| = \sqrt{E^* E} = \sqrt{E_R^2 + E_I^2} \geq 0\,, \tag{6a}$$

$$|\vec{p}| = \sqrt{\vec{p}^* \vec{p}} = \sqrt{\left|\vec{p}_R\right|^2 + \left|\vec{p}_I\right|^2} \geq 0\,, \tag{6b}$$

$$|m_0| = \sqrt{m_0^* m_0} = \sqrt{m_R^2 + m_I^2} \geq 0\,, \tag{6c}$$

hence, when written in full, (5) is given by:

$$\left(E_R^2 + E_I^2\right) - \left(\left|\vec{p}_R\right|^2 + \left|\vec{p}_I\right|^2\right) c_0^2 = \left(m_R^2 + m_I^2\right) c_0^4\,. \tag{7}$$

While the energy and momentum of the quantum system are complex, what we measure as the energy, momentum and the rest-mass of the quantum system are the magnitudes of these complex quantities. These magnitudes can only take positive values. So from (5), we will have:

$$|E| = mc_0^2 = \sqrt{\left|\vec{p}\right|^2 c_0^2 + |m_0|^2 c_0^4} \geq 0\,. \tag{8}$$

In this way, we find a clever and clear mathematical fix to Dirac [13]'s long-standing issue of negative mass and energies as these are now positive definite (i.e. $|E| \geq 0$; $m = |E|/c_0^2 \geq 0$) as we would naturally expect. As a disclaimer, we must say that we are not saying that this is the scheme which *Nature* has chosen in order to solve this problem, but that this is a plausible solution which can be taken seriously. In the next section, we will show how this idea of complex observables can be applied to the supposed problem of temporal and spatial variation of *Fundamental Constants of Nature* (FNCs).

3 CEM-Dirac equation

What kind of a Dirac equation does one get from the CEM hypothesis? Before we can answer this important question, we write down, for completeness purposes, the usual Dirac equation that assumes real-valued physical observables. Written in Dirac [23]'s *Bra-Ket notation*, the Dirac equation is given by:

$$\left[i\hbar\gamma^{\mu}\partial_{\mu} - m_0 c_0\right]|\psi\rangle = 0\,, \tag{9}$$

where:

$$|\psi\rangle = \left|\begin{array}{c}\psi_0\\ \psi_1\\ \psi_2\\ \psi_3\end{array}\right\rangle, \tag{10}$$

is a four-component wavefunction which can further be written as a composition of two spinors, the left-hand $|\psi_L\rangle$ and the right-hand $|\psi_R\rangle$ spinors respectively, i.e.:

$$|\psi\rangle = \left|\begin{array}{c}\psi_L\\ \psi_R\end{array}\right\rangle, \tag{11a}$$

$$|\psi_L\rangle = \left|\begin{array}{c}\psi_0\\ \psi_1\end{array}\right\rangle, \tag{11b}$$

$$|\psi_R\rangle = \left|\begin{array}{c}\psi_2\\ \psi_3\end{array}\right\rangle, \tag{11c}$$

and

$$\gamma^0 = \begin{pmatrix} I_2 & 0\\ 0 & -I_2\end{pmatrix}, \text{ and } \gamma^i = \begin{pmatrix} 0 & \sigma^i\\ -\sigma^i & 0\end{pmatrix}, \tag{12}$$

are the 4×4 Dirac gamma matrices with I_2 and 0, being the 2×2 identity and null matrices respectively. Throughout this paper, the Greek indices will be understood to mean $\mu, \nu, \cdots = 0, 1, 2, 3$; and lower case English alphabet indices: $i, j, k, \cdots = 1, 2, 3$. The matrices σ^j are the three 2×2 Pauli [24] matrices and are given by:

$$\sigma^1 = \begin{pmatrix} 0 & 1\\ 1 & 0\end{pmatrix}, \tag{13a}$$

$$\sigma^2 = i\begin{pmatrix} 1 & 0\\ 0 & -1\end{pmatrix}, \tag{13b}$$

$$\sigma^3 = \begin{pmatrix} 0 & -1\\ 1 & 0\end{pmatrix}. \tag{13c}$$

The Dirac equation admits free particle solutions of the form $\psi = ue^{-iS/\hbar}$, where $u = u(E, \vec{p})$, is a 4×1 component object and $S = p_{\mu}x^{\mu} = \vec{p}\cdot\vec{r} - Et \in \mathbb{R}$ is the phase of the quantum system in question. The Quantum Probability Amplitude (QPA) ρ of such a quantum system is such that $\rho = u^{\dagger}u$, and this QPA has no temporal nor spatial dependence. As we shall soon find out, for the CEM version of the Dirac equation, things are very different.

The phase of the CEM-Dirac quantum system is such that $S = S_R + iS_I$, where $S_R = p^R_{\mu}x^{\mu} \in \mathbb{R}$ and $S_I = p^I_{\mu}x^{\mu} \in \mathbb{R}$ are the real and imaginary parts of the phase of the quantum system in question. Another major difference is that the rest-mass will be a complex quantity as opposed to it being real as is the case with the original Dirac equation. As will be demonstrated in §5, this complex rest-mass leads to a Lorentz [25–27] invariant C, CP, CT, and CPT-violating equation. The QPA of a CEM-Dirac quantum system is such that $\rho = u^{\dagger}ue^{S_I/\hbar}$, and unlike the QPA of the normal Dirac particle, the QPA of a CEM-Dirac quantum system does have an explicit temporal and spatial dependence. It is this explicit temporal and spatial dependence that we strongly believe will lead to an explanation of why particles decay and why they appear to be localized. Like we said (in the text above), we are not in the present going to investigate this issue, but shall leave it for a future paper. This we have done so that we keep our focus on the paramount issue at hand.

In closing this section, we must say that what we have presented herein is what we have coined the *CEM-Dirac equation*. While the CEM-Dirac equation and the usual Dirac equation are identical in their symbols – i.e. the way we write these two equations down, the main difference between them is that the energy, momentum and rest-mass of the CEM-Dirac equation are complex physical variables while in the usual Dirac equation these are real physical variables. The real part of the energy and momentum $(E_R, \vec{p}_R)$ can perhaps be understood as the four-momentum of the quantum system that we measure in the laboratory while the imaginary part $(E_I, \vec{p}_I)$, can be understood as the energy responsible for the decay and localization of the particle in question. Once more, we shall reiterate that these are issues for a separate paper. In the next section, we shall apply the *CEM-hypothesis* to the contemporary issue of the supposed variation of FNCs.

4 Variation of fundamental physical constants

In this section, we show that the supposed variation of fundamental physical constants such as the dimensionless Fine Structure Constant (FSC) (or the Sommerfeld [28] constant) α_0 can be explained from the idea just laid down above – i.e. the idea of CEM eigenvalues. The FSC is given by:

$$\alpha_0 = \frac{e^2}{4\pi\varepsilon_0\hbar c_0}\,. \tag{14}$$

Present measurements give $1/\alpha_0 = 137.035999084(21)$ (CODATA, 2018). If the FSC is varying, it could be any one, or a combination, of the constituents making up this dimensionless quantity, namely e, ε_0, $\hbar$, c_0, or any one of the combination of these supposed constants.

The idea that fundamental constants may vary during the course of the Universe's evolution was first considered by the preeminent British physicists Edward Arthur Milne (1896-1950) and Dirac [29]. Independently, Milne [30] and Dirac [29] considered cosmological models which incorporated a time-variable gravitational constant G, thus setting the ball rolling for the serious theoretical consideration of FNCs. In the intervening years 1938 to about 1999, the idea that FNCs may vary over cosmological times had no backing from experimental philosophy, and because of this, the idea was considered as purely nothing more than an academic pursuit, speculation and curiosity with no bearing whatsoever to do with physical and natural reality. With Web *et al.* [31]'s ground breaking work, this position has since changed as further and strong evidence from observational experience suggesting a plausible variation of the supposedly sacrosanct constants of Nature has been put forward for serious consideration [32–36, for example]. The question is: *Is there a fundamental basis for this variation?* We think that the QM of complex eigenvalues might have something to say about this.

Without any doubt whatsoever, FNCs (e.g. e, ε_0, $\hbar$, c_0, *etc*) are observables since they cannot only be measured in the laboratory, but are intimately, intrinsically and inherently associated with quantum systems. With that having been said, it is clear that if a physical observable such as an FNC is a true constant of Nature, i.e. having no spatial nor temporal variation, then its total (and not partial) time derivative must vanish identically – i.e. $d\langle O\rangle/dt \equiv 0$. The total (and not partial) time derivative operator is given by:

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla}\,. \tag{15}$$

Applying this to the expectation value $\langle O\rangle = \langle\Psi|\widehat{\mathcal{T}}|\Psi\rangle$ of an arbitrary observable O, one gets:

$$i\hbar\frac{d\langle O\rangle}{dt} = \left\langle\Psi\left|\left[\widehat{\mathcal{T}}^{\dagger},\widehat{\mathcal{H}}\right]\right|\Psi\right\rangle + \vec{v}\cdot\left\langle\Psi\left|\left[\widehat{\mathcal{T}}^{\dagger},\vec{\widehat{P}}\right]\right|\Psi\right\rangle, \tag{16}$$

where:

$$\left[\widehat{\mathcal{T}}^{\dagger},\widehat{\mathcal{H}}\right] = \widehat{\mathcal{T}}^{\dagger}\widehat{\mathcal{H}} - \widehat{\mathcal{H}}^{\dagger}\widehat{\mathcal{T}}\,, \tag{17a}$$

$$\left[\widehat{\mathcal{T}}^{\dagger},\vec{\widehat{P}}\right] = \widehat{\mathcal{T}}^{\dagger}\vec{\widehat{P}} - \vec{\widehat{P}}^{\dagger}\widehat{\mathcal{T}}\,, \tag{17b}$$

and $\vec{\widehat{P}} = -i\hbar\vec{\nabla}$ is the quantum mechanical momentum operator and $\vec{v}$ is the velocity of the quantum system under consideration. We must say that it is more appropriate to think of this velocity:

$$\vec{v} = \frac{\hbar}{m}\,\mathrm{Im}\left(\frac{\Psi^{\dagger}\vec{\nabla}\Psi}{\Psi^{\dagger}\Psi}\right), \tag{18}$$

as the Bohmian [37–39] velocity* field of the quantum system in question and the possible justification for this has been provided in [40].

What (16) is telling us, is that if an observable is a true constant, that is, it does not vary neither with time nor space, then the operator corresponding to this observable must commute with both the Hamiltonian and the momentum operator – i.e. $\left[\widehat{\mathcal{T}}^{\dagger},\widehat{\mathcal{H}}\right] = 0$, and $\left[\widehat{\mathcal{T}}^{\dagger},\vec{\widehat{P}}\right] = 0$. Against the seemingly sacrosanct dictates of our current understanding of QM, the condition $\left[\widehat{\mathcal{T}}^{\dagger},\widehat{\mathcal{H}}\right] = 0$ is here found not to be sufficient to guarantee that the observable O will be a truly conserved quantity and constant quantity throughout all of space and time. If for some reason we have that $\left[\widehat{\mathcal{T}}^{\dagger},\widehat{\mathcal{H}}\right] \neq 0$, and $\left[\widehat{\mathcal{T}}^{\dagger},\vec{\widehat{P}}\right] \neq 0$, then for an observable to be a true constant, the spatial variation aught to be compensated by the temporal variation and *vice-versa*, and this will be in accordance with (16) under the setting $d\langle O\rangle/dt = 0$.

At this point, in order for us to proceed, we need to evaluate (16) in terms of *observable* quantities, i.e. we need to compute $\left\langle\Psi\left|\left[\widehat{\mathcal{T}}^{\dagger},\widehat{\mathcal{H}}\right]\right|\Psi\right\rangle$ and $\vec{v}\cdot\left\langle\Psi\left|\left[\widehat{\mathcal{T}}^{\dagger},\vec{\widehat{P}}\right]\right|\Psi\right\rangle$. To that end, we know that:

$$\begin{aligned}\left|\widehat{\mathcal{T}}\frac{\partial\Psi}{\partial t}\right\rangle &= \frac{1}{i\hbar}\left|\widehat{\mathcal{T}}i\hbar\frac{\partial\Psi}{\partial t}\right\rangle,\\ &= \frac{1}{i\hbar}\left|\widehat{\mathcal{T}}\widehat{E}\Psi\right\rangle,\\ &= -\frac{1}{i\hbar}E\left|\widehat{\mathcal{T}}\Psi\right\rangle,\end{aligned} \tag{19}$$

and that:

$$\begin{aligned}\left\langle\widehat{\mathcal{T}}\frac{\partial\Psi}{\partial t}\right| &= -\frac{1}{i\hbar}\left\langle\widehat{\mathcal{T}}i\hbar\frac{\partial\Psi}{\partial t}\right|,\\ &= -\frac{1}{i\hbar}\left\langle\widehat{\mathcal{T}}\widehat{E}\Psi\right|,\\ &= -\frac{1}{i\hbar}E^{*}\left\langle\widehat{\mathcal{T}}\Psi\right|.\end{aligned} \tag{20}$$

Multiplying (19) from the left by $\langle\Psi|$ and (20) from the right by $|\Psi\rangle$ respectively, and thereafter adding the resulting equations, we will have:

$$\begin{aligned}\left\langle\Psi\left|\left[\widehat{\mathcal{T}}^{\dagger},\widehat{\mathcal{H}}\right]\right|\Psi\right\rangle &= (E-E^{*})\langle O\rangle\,,\\ &= 2i\,E_I\langle O\rangle\,,\\ &= i\hbar\frac{\partial\langle O\rangle}{\partial t}\,,\end{aligned} \tag{21}$$

hence:

$$\left\langle\Psi\left|\left[\widehat{\mathcal{T}}^{\dagger},\widehat{\mathcal{H}}\right]\right|\Psi\right\rangle = -2i\,E_I\langle O\rangle\,. \tag{22}$$

*Im() is an operator which extracts the imaginary part of a complex quantity – i.e. if $z = x + iy$, then $\mathrm{Im}(z) = y$.

Further, we know that:

$$\begin{aligned} \left|\widehat{\mathcal{T}}\vec{\nabla}\Psi\right\rangle &= -\frac{1}{i\hbar}\left|\widehat{\mathcal{T}}(-i\hbar)\vec{\nabla}\Psi\right\rangle , \\ &= -\frac{1}{i\hbar}\left|\widehat{\mathcal{T}}\widehat{\vec{P}}\Psi\right\rangle , \\ &= -\frac{1}{i\hbar}\,\vec{p}\left|\widehat{\mathcal{T}}\Psi\right\rangle , \end{aligned} \tag{23}$$

and:

$$\begin{aligned} \left\langle\widehat{\mathcal{T}}\vec{\nabla}\Psi\right| &= \frac{1}{i\hbar}\left\langle\widehat{\mathcal{T}}(-i\hbar)\vec{\nabla}\Psi\right| , \\ &= \frac{1}{i\hbar}\left\langle\widehat{\mathcal{T}}\widehat{\vec{P}}\Psi\right| , \\ &= \frac{1}{i\hbar}\,\vec{p}^{*}\left\langle\widehat{\mathcal{T}}\Psi\right| . \end{aligned} \tag{24}$$

Likewise, multiplying (23) from the left by $\langle\Psi|$ and (24) from the right by $|\Psi\rangle$ respectively, and thereafter adding the resulting equations, we will have:

$$\begin{aligned} \left\langle\Psi\left|\left[\widehat{\mathcal{T}},\widehat{\vec{P}}^{\dagger}\right]\right|\Psi\right\rangle &= -\left(\vec{p}-\vec{p}^{*}\right)\langle O\rangle \\ &= -2i\,\vec{p}_I\,\langle O\rangle \\ &= -i\hbar\vec{\nabla}\,\langle O\rangle , \end{aligned} \tag{25}$$

hence:

$$\vec{v}\cdot\left\langle\Psi\left|\left[\widehat{\mathcal{T}},\widehat{\vec{P}}^{\dagger}\right]\right|\Psi\right\rangle = -2i\,\vec{v}\cdot\vec{p}_I\,\langle O\rangle . \tag{26}$$

Now, inserting (22) and (26) into (16), we obtain:

$$i\hbar\frac{d\langle O\rangle}{dt} = -2i\hbar\left(E_I-\vec{v}\cdot\vec{p}_I\right)\langle O\rangle . \tag{27}$$

From this, it follows that a system will have all of its observables being constants *if-and-only-if*:

$$E_I-\vec{v}\cdot\vec{p}_I = 0 . \tag{28}$$

In passing – out of curiosity, we need to point out an indelible fact of experience namely that (28) has a seductive and irresistible semblance with Bartoli [41, 42] and Maxwell [43]'s energy-momentum dispersion relation for Light – i.e. $E - c_0 p = 0$. If any, what connection can one make of this (28) with the nature of the photon? At present, we can only exhibit our curiosity: that is, we shall leave it here and slate it for exploration in future papers.

Now, applying the above ideas to the case of the variation of the FSC and assuming the present Standard Big Bang Cosmology Model [44–46] which assumes co-moving coordinates [47–50], it would appear that this FSC variation aught to be temporal in nature, as logic dictates that it cannot be spatial since co-moving coordinates imply $\vec{v} \equiv 0$. This directly implies that those patches of the sky exhibiting different FSC-values aught to be of different ages! If the temporal homogeneity and isotropy of the Universe is to be preserved, then the only way to explain the variation of the FSC across the night-sky is to drop the assumption of co-moving coordinates! We are not going to say anything further on this matter of the variation of the FSC and complex observables, as this is something that requires a dedicated piece of work of its own. All that we wanted, we have achieved, and this has been to demonstrate the latent power in the seemingly alien idea of complex quantum mechanical observables that we have here suggested. We shall now move to the next section, where we shall consider the symmetries of the CEM-Dirac equation.

5 Symmetries of the CEM-Dirac equation

Now, if the electromagnetic coupled CEM-Dirac equation $\left[i\hbar\gamma^{\mu}\mathcal{D}_{\mu} - m_0c_0\right]|\psi\rangle = 0$, with $m_0 \in \mathbb{C}$, is to be symmetric,

$$\text{i.e. } q \longmapsto -q \Rightarrow \left[i\hbar\gamma^{\mu}\mathcal{D}^{*}_{\mu} - m_0c_0\right]|\psi\rangle = 0$$

under charge conjugation, then we need to show that there exists a set of mathematically legal operations that take this new charge conjugated equation $\left[i\hbar\gamma^{\mu}\mathcal{D}^{*}_{\mu} - m_0c_0\right]|\psi\rangle = 0$, back to the original CEM-Dirac equation – i.e. an equation without the $*$-*operation* on the covariant derivative $\mathcal{D}_{\mu}$. If we can find these legal mathematical operations, it would mean that the CEM-Dirac equation applies equally to particles as to antiparticles – hence, it is symmetric with respect matter and antimatter. On the contrary, if we fail to find the said legal mathematical operations, it invariably means that the CEM-Dirac equation is not symmetric under charge conjugation.

To that end, let us start our attempt by removing the $*$-*operator* on the covariant derivative $\mathcal{D}_{\mu}$ in the equation $\left[i\hbar\gamma^{\mu}\mathcal{D}^{*}_{\mu} - m_0c_0\right]|\psi\rangle = 0$. We will do this by taking the complex conjugate throughout this equation. So doing, we obtain $\left[i\hbar\gamma^{\mu*}\mathcal{D}_{\mu} + m^{*}_0c_0\right]|\psi^{*}\rangle = 0$, and because $\gamma^5\gamma^0\gamma^{\mu*} = -\gamma^{\mu}\gamma^5\gamma^0$, we can, in this resulting equation, remove the complex conjugate operator acting on $\gamma^{\mu*}$ and this we can achieve by multiplying throughout the resultant equation by $\gamma^5\gamma^0$ and then making use of the fact that $\gamma^5\gamma^0\gamma^{\mu*} = -\gamma^{\mu}\gamma^5\gamma^0$. So doing, we will have:

$$\left[i\hbar\gamma^{\mu}\mathcal{D}_{\mu} - m^{*}_0c_0\right]|\psi_{\rm c}\rangle = 0 , \tag{29}$$

where $|\psi_{\rm c}\rangle = \gamma^5\gamma^0\,|\psi^{*}\rangle$ is the wavefunction of the corresponding antiparticle. Clearly, if we have that $m_I \propto q$, or $m_I \propto q^n$, where $(n \in \mathbb{O}) = 3, 5, 7,$ *etc*, this would mean that the transformation $q \longmapsto -q$, would also lead to:

$$m_I \longmapsto -m_I \;\Rightarrow\; m^{*}_0 \longmapsto m_0 , \tag{30}$$

and in this way, (29) would simultaneously transform to:

$$\left[i\hbar\gamma^{\mu}\mathcal{D}_{\mu} - m_0c_0\right]|\psi_{\rm c}\rangle = 0 , \tag{31}$$

thus making this CEM-Dirac equation (whose rest-mass $(m_0 \in \mathbb{C})$ is a complex quantity) symmetric under charge conjugation. Less for the fact that the wavefunction $|\psi\rangle$ has been replaced by the new wavefunction $|\psi_{\rm c}\rangle$, (31) is the same

CEM-Dirac equation applicable to the particle counterpart. Since $|\psi_c\rangle$ represents the antiparticle, the original Dirac equation is said to be symmetric under charge conjugation. In Dirac [9, 10]'s original theory, m_0 is real, the meaning of which is that $m_I \equiv 0$, hence making this original Dirac [9, 10] equation symmetric under charge conjugation. In the new setting of the CEM-Dirac equation, if m_I is not related to the electrical charge of the particle as suggested in (29), then the CEM-Dirac equation (with $m_0 \in \mathbb{C}$) is going to be asymmetric with respect to charge conjugation. As the reader can verify for themselves, not only is the CEM-Dirac equation going to violate C symmetry, but also CP, CT, and CPT symmetries as well. The only preserved symmetries are the P, T and PT symmetries.

6 Discussion

We have shown herein that the issue to do with negative energies can be solved by way of making a proper choice of the energy and momentum eigenvalues of the energy and momentum operators, respectively. These eigenvalues need to be complex as opposed to them being real as is the case in the present formulation of QM. Once the energy and momentum eigenvalues are complex, the measurable values become the magnitude of the corresponding eigenvalues and these magnitudes are positive definite! In this way, the issues surrounding these negative energies vanish forthwith. What remains is whether or not this is the scheme which *Nature* has chosen in order to go round this problem. We are of the strong opinion that this may very well be the scheme *Nature* has chosen.

This issue of negative energies has similarities with negative probabilities. As already said in the main text, prior to the discovery of his equation, Dirac had hoped that the negative probabilities occurring in the KG-theory, if solved, would also solve, in his new anticipated theory, the issue of the negative energies as well. We now know that Dirac was wrong as his new anticipated theory, which has positive definite probabilities, also has these negative energies. We did show in [40] that the emergence of these negative probabilities in the KG-theory is a result of an improper choice of the quantum mechanical probability current density in the KG-theory. In the same vein, the emergence of negative energies in both the Dirac and the KG-theory is a result of an improper choice of the energy and momentum eigenvalues – they need to be complex as suggested therein.

While we have not explored the richness of the hypothesis of complex energy and momentum eigenvalues, we need to mention the latent power in this new way of thinking, namely that one may very well able to explain the variation of FNCs using this idea. Apart from this, it should be possible, using the complex part of the energy and momentum, to explain why particles decay, as well as the localization of particles into a finite region of space. What we had wanted here is to show that Dirac's negative energies can be done away with, once and for all!

7 Conclusion

The following conclusion is drawn on the *proviso* that the hypothesis of complex energy-momentum is acceptable:

1. The complex energy-momentum hypothesis when applied to both the Klein-Gordon and the Dirac theory, does solve the issue of negative energies. This problem ceases to exist as the energy of all particles now is positive definite.
2. Quantum mechanics as currently understood and constituted where *all* quantum mechanical operators are required to be hermitian so that the corresponding eigenvalues are real-valued, may have to be modified or reconsidered.
3. The long-standing issue of the asymmetry in the matter-antimatter constitution of the Universe can be explained by the C, CP, CT and CPT violation that arises from the complex energy-momentum hypothesis when applied to the Dirac equation.

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