*LETTERS TO PROGRESS IN PHYSICS*

# On the Lambda Term in Einstein's Equations and Its Influence on the Cosmological Redshift

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Here we analyze our theory of the physical vacuum ($\lambda$-field filling de Sitter spaces), in which we calculated its physically observable properties (2001) and the parabolic (non-linear) cosmological redshift specific to de Sitter spaces (2013). To explain the recently discovered non-linear cosmological redshift, we consider the following options: 1) our Universe is a de Sitter world with $\lambda > 0$ and, therefore, with a parabolic cosmological redshift (because with $\lambda > 0$ the non-Newtonian gravitational forces acting in a de Sitter world are forces of repulsion, which decelerate photons), but in this case the physical vacuum has a negative density $\breve{\rho} < 0$, and the observable curvature radius of space is imaginary (the space geometry is hyperbolic); 2) our Universe is a de Sitter world with $\lambda < 0$, where the physical vacuum has a positive density $\breve{\rho} > 0$, and the observable curvature radius is real (the space geometry is spherical), but with a parabolic cosmological blueshift (and not a redshift) because with $\lambda < 0$ the non-Newtonian forces are forces of attraction (they accelerate photons); 3) our Universe is a de Sitter world with $\lambda > 0$ and, hence, a parabolic cosmological redshift, but the $\lambda g_{\alpha\beta}$ term in Einstein's field equations has the opposite (negative) sign. We vote for the 3rd option, because in this case a) the physical vacuum has a positive density $\breve{\rho} > 0$, which satisfies the physical requirement that any kind of observable matter must have a positive mass and density, b) the observable curvature radius of space is real (and not imaginary) and, hence, the space geometry is spherical (and not hyperbolic), c) the non-Newtonian gravitational forces are forces of repulsion (they decelerate photons, thereby producing a redshift), d) the event horizon in such a universe is outlined by the gravitational radius of the de Sitter sphere. All this confirms the supposition that our Universe is a huge de Sitter gravitational collapsar with $\lambda > 0$ and a non-linear (parabolic) cosmological redshift.

In the late 1980s and 1990s, we undertook a massive theoretical research on the motion of particles in the space-time of General Relativity, which we published in 2001 in our two monographs [1, 2]. In particular, we created a theory of the physical vacuum ($\lambda$-field filling de Sitter spaces), in which we calculated its physically observable properties such as density, pressure, etc. [2, Chapter 5]. In 2010, using the mentioned theory of the physical vacuum, Larissa created a theory of the de Sitter gravitational collapsar (de Sitter bubble), which she suggested as a model of the observable Universe [3]. In our third monograph [4], published in 2013, we predicted a parabolic (non-linear) cosmological redshift in a de Sitter space [4, §6.4–§6.5]. In 2018, we also published a short paper on the mentioned parabolic redshift [5].

Meanwhile there was a serious problem with determining the cosmological shift in the frequency of photons travelling in a de Sitter world. This problem arose due to the sign of the $\lambda$-term, because it is not specified in de Sitter's metric. So, our closest colleagues drew our attention to some confusion in our sequential publications on this subject.

First, in our theory of the physical vacuum, where we did not consider the cosmological redshift, we assumed $\lambda < 0$ and, therefore, a positive vacuum density $\breve{\rho} > 0$ in our Universe [2, §5.3]. In this case, the non-Newtonian gravitational forces acting in any de Sitter space are forces of attraction, and the physically observable three-dimensional curvature is positive $C > 0$. Since $C = \frac{1}{\Re^2}$, the latter means that such a universe has a real observable curvature radius $\Re$, and, therefore, the space geometry is spherical (this is what the geometry of a de Sitter space should be). But a few years later, when Larissa suggested a collapsed de Sitter sphere as a model of the observable Universe [3], and then, in our monograph on the internal constitution of stars, where we considered de Sitter collapsars [4, Chapter 6], and also in our subsequent paper on the cosmological redshift [5], it was assumed that $\lambda > 0$. This is because forces of attraction ($\lambda < 0$) accelerate photons, thereby producing a photon blueshift (gain of the photon energy), and forces of repulsion ($\lambda > 0$) decelerate photons, thereby producing a photon redshift (loss of the photon energy). But in the case of $\lambda > 0$ we should expect a negative vacuum density $\breve{\rho} < 0$, a negative physically observable curvature $C < 0$ and, hence, an imaginary numerical value of the observable curvature radius $\Re$ of such a universe (this means that the space geometry is hyperbolic).

To resolve this contradiction, a solution was proposed in §6.5 of our book [4] (§5.1 in the 1st edition, 2013). But this solution was not well recognized by the readers, because it was "lost" among many other new results presented in that book. As a result, the above confusion has created a problem in understanding the parabolic cosmological redshift that we had predicted for a de Sitter universe.

Note that the cosmological redshift problem was never our task or field of interest. The parabolic redshift in a de Sitter universe was just one of the spin-off theoretical results that we obtained in the course of our long-term work on other, much more important mathematical and applied problems in the General Theory of Relativity.* On the other hand, we must respond to our colleagues regarding the parabolic cosmological redshift in a de Sitter universe, which we had predicted "at the tip of the pen". It looks like it is time to dot all the i's in this problem.

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Let us begin. The $\lambda$-term was introduced in 1917 by Albert Einstein [6]. He added it multiplied by the fundamental metric tensor $g_{\alpha\beta}$ to the right-hand side of the field equations (known also as Einstein's equations), which thus acquired their final well-known form†

$$R_{\alpha\beta} - \frac{1}{2}\, g_{\alpha\beta} R = -\varkappa\, T_{\alpha\beta} + \lambda g_{\alpha\beta}\,.$$

He did this, because the metric of a static finite spherically symmetric space filled with a homogeneous and isotropic distribution of substance — Einstein's metric, which he initially considered as the basic cosmological model of our Universe, — does not satisfy the original field equations (which do not contain the $\lambda$-term). Mathematically, this means that substituting the components of the fundamental metric tensor $g_{\alpha\beta}$, taken from Einstein's metric, into the original field equations (without the $\lambda$-term), the left-hand side of the field equations does not equalize the right-hand side. But with the $\lambda$-term added to the right-hand side of the field equations, the fundamental metric tensor $g_{\alpha\beta}$ of a spherically symmetric static homogeneous and isotropic universe (*Einstein's cosmological model*) makes both sides of the equations equal to each other, so the equations vanish. For detail, see Einstein's 1917 paper [6] and the comprehensive review on this subject [7].

A year later, in 1918, Willem de Sitter [8] mathematically deduced that, in addition to Einstein's metric (which determines a space filled with a static finite spherically homogeneous and isotropic distribution of substance), there is also another space metric satisfying the field equations containing the $\lambda$-term. De Sitter's space metric has the form

$$ds^2 = \left(1 - \frac{\lambda r^2}{3}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{\lambda r^2}{3}} - r^2 \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

and describes a static finite spherically symmetric space filled with a homogeneous and isotropic distribution of the $\lambda$-field (determined by the $\lambda$-term) without any islands of mass or distributed substance. The above metric then became known as *de Sitter's cosmological model*.

Since the zero component $T_{00}$ of the energy-momentum tensor of a distributed matter is associated with the density of the matter [6], then, according to the right-hand side of the field equations, the $\lambda$-term divided by Einstein's constant $\varkappa$ should be associated with the density of the $\lambda$-field. Over the last century many astronomers tried to measure the numerical value of the $\lambda$-term, using various measurement methods; see the 2001 review on this subject [9]. However, all that they achieved does not differ from the result known in already the 1950s, according to which even the sign of the $\lambda$-term is under question, and the upper limit of its numerical value is $|\lambda| \leqslant 10^{-56}$ [cm$^{-2}$]. Even now the astronomers can only say that the $\lambda$-field is an extremely rarefied medium, the density of which cannot be surely detected within the current accuracy of astronomical measurements.

There was no theory of the $\lambda$-field until the mid-1990s, when we began our own research on this subject. As a result of our research, in Chapter 5 of our monograph [2], first published in 2001, we presented a mathematical theory of the $\lambda$-field. In the framework of this theory, we theoretically determined the physically observable properties of the $\lambda$-field and much more. As always in our studies, we used the mathematical apparatus of physically observable quantities in General Relativity, known also as the Zelmanov chronometric invariants [10–12]; see the most comprehensive review of this mathematical technique in our survey [13].

In [2] we called the $\lambda$-field the *physical vacuum*, because it has vacuum-like properties. We relied on the works of Erast Gliner [14,15], announced in 1966 by Andrew Sakharov [16]. Gliner selected and then studied a special state of distributed matter for which the energy-momentum tensor is $T_{\alpha\beta} = \mu\, g_{\alpha\beta}$, where $\mu$ is a constant number‡. He called this state of matter

*"As you know, our success depends on the fact that nearly all major scientific advances have been made while looking for something else, or following up curious observations." — David Jones, Editor of *New Scientist*, May 1981. Cited from: Jones D. The Inventions of Daedalus. W. H. Freeman & Co., Oxford, 1982, page 3.

†Here $R_{\alpha\sigma} = R_{\alpha\beta\sigma\cdot}^{\cdot\cdot\cdot\beta}$ [cm$^{-2}$] is Ricci's tensor (obtained as the contraction of the Riemann-Christoffel curvature tensor $R_{\alpha\beta\gamma\delta}$ by one index in each pair of its four indices), $R = g^{\alpha\beta} R_{\alpha\beta}$ is the scalar curvature, $\varkappa = 8\pi G/c^2 = 1.862 \times 10^{-27}$ [cm/gram] is Einstein's constant, $G = 6.672 \times 10^{-8}$ [cm$^3$/gram sec$^2$] is Gauss' gravitational constant, and $T_{\alpha\beta}$ [gram/cm$^3$] is the energy-momentum tensor of a distributed matter that fills the space.

Einstein's field equations show how in a Riemannian space the field of its four-dimensional curvature depends on the field of a distributed matter (substance or, say, electromagnetic field) that fills this space. Note that Einstein's field equations are not some kind of physical hypothesis. They follow mathematically from the geometric structure characteristic of any Riemannian space (as well as the Riemannian quadratic metric and its invariance throughout the entire space). For example, let us have a space determined by a metric. In order for this space to be Riemannian, it is necessary that its metric has the Riemannian quadratic form $ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$, be invariant throughout the entire space, and satisfy the field equations.

‡Gliner used the space signature $(-+++)$, resulting in $T_{\alpha\beta} = -\mu\, g_{\alpha\beta}$.

the *μ-vacuum*, because it is related to vacuum-like states of substance ($T_{\alpha\beta} \sim g_{\alpha\beta}$, which means $R_{\alpha\beta} = k\, g_{\alpha\beta}$, $k = const$), but is not the vacuum (because $T_{\alpha\beta} = 0$ in the vacuum). Following this way, we introduced a new geometric classification of the states of distributed matter (and of space-time itself) according to the form of the energy-momentum tensor. We called this new classification the *T-classification of matter*:

I. The emptiness is the state of space-time, for which the Ricci tensor is zero $R_{\alpha\beta} = 0$, which means the absence of both distributed substance ($T_{\alpha\beta} = 0$) and the physical vacuum ($\lambda = 0$).* So, the emptiness is the state of a space-time without any kind of distributed matter;

II. The physical vacuum or, simply, the vacuum is the state of distributed matter (space-time), which is determined by only the $\lambda$-field ($\lambda = const \neq 0$);

III. The $\mu$-vacuum is the state of distributed matter (space-time), which is determined by the energy-momentum tensor of the form $T_{\alpha\beta} = \mu\, g_{\alpha\beta}$ (where $\mu = const$). This is a vacuum-like state of matter, because $T_{\alpha\beta} \sim g_{\alpha\beta}$;

IV. Substance is the state of distributed matter (space-time) for which $T_{\alpha\beta} \neq 0$, but $T_{\alpha\beta} \nsim g_{\alpha\beta}$. This state comprises both ordinary substance and electromagnetic fields.

For the above reasons, we called the mathematical theory of the $\lambda$-term, which we presented in Chapter 5 of our monograph [2], the *theory of the physical vacuum*.

The energy-momentum tensor and physical properties of the physical vacuum ($\lambda$-field) were derived as follows. We presented the field equations in the form

$$R_{\alpha\beta} - \frac{1}{2}\, g_{\alpha\beta} R = -\varkappa\, \widetilde{T}_{\alpha\beta}\,,$$

where $\widetilde{T}_{\alpha\beta} = T_{\alpha\beta} + \breve{T}_{\alpha\beta}$ is the joint energy-momentum tensor that describes both distributed substance and the physical vacuum, and

$$\breve{T}_{\alpha\beta} = -\frac{\lambda}{\varkappa}\, g_{\alpha\beta}$$

is the energy-momentum tensor of the physical vacuum. The physically observable properties of a medium are expressed with the projections of its energy-momentum tensor $T_{\alpha\beta}$ onto the time line and the three-dimensional spatial section of the observer [10–13]: the observable density $\rho$, the observable momentum density $J^i$ and the observable stress tensor $U^{ik}$

$$\rho = \frac{T_{00}}{g_{00}}\,, \qquad J^i = \frac{c\, T_0^i}{\sqrt{g_{00}}}\,, \qquad U^{ik} = c^2 T^{ik},$$

which, when calculated for the energy-momentum tensor of the physical vacuum $\breve{T}_{\alpha\beta} = -\frac{\lambda}{\varkappa}\, g_{\alpha\beta}$, have the form

$$\breve{\rho} = \frac{\breve{T}_{00}}{g_{00}} = -\frac{\lambda}{\varkappa}\,,$$

$$\breve{J}^i = \frac{c\, \breve{T}_0^i}{\sqrt{g_{00}}} = 0\,,$$

$$\breve{U}^{ik} = c^2\, \breve{T}^{ik} = \frac{\lambda}{\varkappa}\, c^2 h^{ik} = -\breve{\rho}\, c^2 h^{ik},$$

where $h^{ik}$ is the upper-index form of the physically observable three-dimensional metric tensor $h_{ik} = -g_{ik} + \frac{1}{c^2}\, v_i v_k$.†

From here we see that the physical vacuum ($\lambda$-field) is a *uniformly distributed matter* (since it has a constant density $\breve{\rho} = const$), and also is a *non-emitting medium* (the energy flux in the physical vacuum is zero $c^2 \breve{J}^i = 0$).

The equation of state of the physical vacuum‡ follows from the general formula of the stress tensor

$$U_{ik} = p_0\, h_{ik} - \alpha_{ik} = p\, h_{ik} - \beta_{ik}\,,$$

which is applicable to any medium [12]. Here $p_0$ is the equilibrium pressure of the medium, $p$ is the true pressure, $\alpha_{ik}$ is the viscosity of the 2nd kind, and $\beta_{ik} = \alpha_{ik} - \frac{1}{3}\, \alpha\, h_{ik}$ is the anisotropic part of $\alpha_{ik}$ ($\alpha = \alpha_i^i$ is its trace), called the viscosity of the 1st kind and manifested in anisotropic deformations of the medium. Since the physical vacuum ($\lambda$-field) is the only filler of any de Sitter space, and de Sitter's metric means a spherically symmetrical, homogeneous, isotropic and static (non-deforming) space, then in the physical vacuum $\breve{\alpha}_{ik} = 0$ and $\breve{\beta}_{ik} = 0$ (it is a *non-viscous medium*). Hence, the energy-momentum tensor of the physical vacuum has the form

$$\breve{U}_{ik} = \breve{p}\, h_{ik} = -\breve{\rho}\, c^2 h_{ik}\,,$$

and the equation of state of the physical vacuum is

$$\breve{p} = -\breve{\rho}\, c^2,$$

which means the *state of inflation*: if the density of a medium is positive, then the pressure inside it is negative (the medium expands).

So, we have obtained that the physical vacuum has the following physical properties:

Since the observable density of matter is positive, $\rho = \frac{T_{00}}{g_{00}} = -\mu > 0$, he obtained negative numerical values of $\mu$. We always use the space signature $(+---)$, because in this case the three-dimensional observable interval is positive [10–13]. Therefore, we have $\mu > 0$ and $T_{\alpha\beta} = \mu\, g_{\alpha\beta}$.

*In an empty space, we have the field equations $R_{\alpha\beta} - \frac{1}{2}\, g_{\alpha\beta} R = 0$ or, in the mixed form, $R_\alpha^\beta - \frac{1}{2}\, g_\alpha^\beta R = 0$. After their contraction $R_\alpha^\alpha - \frac{1}{2}\, g_\alpha^\alpha R = 0$, we obtain $R - \frac{1}{2}\, 4R = 0$. Hence, the scalar curvature of any empty space is $R = 0$, and the field equations in an empty space have the form $R_{\alpha\beta} = 0$.

†The physically observable three-dimensional spatial interval is determined as $d\sigma^2 = h_{ik}\, dx^i dx^k$, where $h_{ik} = -g_{ik} + \frac{1}{c^2}\, v_i v_k$ for which $h^{ik} = -g^{ik}$ and $h_k^i = -g_k^i = \delta_k^i$. Here $v_i = -c\, \frac{g_{0i}}{\sqrt{g_{00}}}$ is the linear velocity with which the reference space of the observer rotates (in the case, if it rotates), for which $v^i = -c\, g^{0i} \sqrt{g_{00}}$, $v_i = h_{ik} v^k$, $v^2 = v_k v^k = h_{ik} v^i v^k$. See the mathematical apparatus of chronometric invariants [10–13] for detail.

‡The equation of state of a distributed matter is the relationship between the pressure and density in the medium. For instance, $p = 0$ is the equation of state of a dust medium, $p = \rho c^2$ is the equation of state of a matter inside atomic nuclei, $p = \frac{1}{3}\, \rho c^2$ is the equation of state of an ultra-relativistic gas.

The physical vacuum, i.e., the $\lambda$-field, is a homogeneous ($\breve{\rho} = const$), non-viscous ($\breve{\alpha}_{ik} = 0$, $\breve{\beta}_{ik} = 0$) and non-emitting ($\breve{J}^i = 0$) medium, which is in the state of inflation ($\breve{p} = -\breve{\rho}c^2$).

We are able to calculate the numerical value of the physically observable density of the physical vacuum $\breve{\rho} = -\frac{\lambda}{\varkappa}$ and, therefore, the numerical value of the $\lambda$-term, if the physically observable three-dimensional curvature of the observable space will once be somehow measured in astronomical observations. How to do this is explained below.

In constant curvature spaces such as de Sitter spaces the Riemann-Christoffel curvature tensor is [17, Chapter VII]

$$R_{\alpha\beta\gamma\delta} = K\left(g_{\alpha\gamma}\,g_{\beta\delta} - g_{\alpha\delta}\,g_{\beta\gamma}\right), \quad K = const\,,$$

where $K$ is a constant number proportional to the constant scalar curvature $R$. Contracting it step-by-step we obtain the Ricci tensor $R_{\alpha\beta} = -3K\,g_{\alpha\beta}$, the scalar curvature $R = -12K$ (because $g_{\alpha\beta}\,g^{\alpha\beta} = 4$) and, as a result, the field equations in a constant curvature space

$$3K\,g_{\alpha\beta} = -\varkappa\,T_{\alpha\beta} + \lambda\,g_{\alpha\beta}\,,$$

which mean $(\lambda - 3K)\,g_{\alpha\beta} = \varkappa\,T_{\alpha\beta}$. Because $T_{\alpha\beta} = 0$ in any de Sitter space (its only filler is the $\lambda$-field), then the observable density of the physical vacuum in a de Sitter space is

$$\breve{\rho} = -\frac{\lambda}{\varkappa} = -\frac{3K}{\varkappa} = -\frac{3Kc^2}{8\pi G}\,,$$

and, since the physically observable three-dimensional curvature $C$ of a de Sitter space is $C = -6K$,* we obtain the physically observable density of the physical vacuum $\breve{\rho}$, the $\lambda$-term and the pressure $\breve{p}$ inside the vacuum (the latter follows from the equation of state of the physical vacuum, $\breve{p} = -\breve{\rho}c^2$), expressed with the physically observable space curvature $C$

$$\breve{\rho} = \frac{C}{2\varkappa}\,, \qquad \lambda = -\frac{C}{2}\,, \qquad \breve{p} = -\frac{c^2 C}{2\varkappa}\,,$$

or, since $C$ is known to be related to the physically observable curvature radius $\Re$ of a three-dimensional constant curvature space as follows $C = \frac{1}{\Re^2}$,

$$\breve{\rho} = \frac{1}{2\varkappa\Re^2}\,, \qquad \lambda = -\frac{1}{2\Re^2}\,, \qquad \breve{p} = -\frac{c^2}{2\varkappa\Re^2}\,.$$

Astronomical observations performed over the last century indicate that the physically observable event horizon in our Universe is approximately $10^{28}$ cm (within one order of magnitude). Therefore, if we assume that our Universe is a de Sitter sphere with the observable curvature radius $10^{28}$ cm, then we should expect (all within one order of magnitude):

$$\breve{\rho} \approx 3\times10^{-30}\ \text{gram/cm}^3,$$

$$\breve{p} \approx -2\times10^{-9}\ \text{gram/cm sec}^2,$$

$$|\lambda| \approx 5\times10^{-57}\ \text{cm}^{-2}.$$

Gravitational forces act in any de Sitter space, but they are non-Newtonian gravitational forces because they arise due to the non-Newtonian gravitational potential created by the physical vacuum ($\lambda$-field). Below explains why.

In general, gravitational forces are forces caused by the non-uniform distribution of the gravitational potential w determined by the zero component $g_{00}$ of the fundamental metric tensor $g_{\alpha\beta}$. As is known, in a general case [18],

$$g_{00} = \left(1 - \frac{\mathrm{w}}{c^2}\right)^2, \quad \text{hence} \quad \mathrm{w} = c^2\left(1 - \sqrt{g_{00}}\,\right),$$

and the physically observable vector of the gravitational inertial force, determined in the framework of the the mathematical apparatus of chronometric invariants [10–13], is

$$F_i = \frac{1}{\sqrt{g_{00}}}\left(\frac{\partial \mathrm{w}}{\partial x^i} - \frac{\partial v_i}{\partial t}\right).$$

In a space of de Sitter's metric (see the metric in the beginning of this article) we have

$$g_{00} = 1 - \frac{\lambda r^2}{3}\,, \quad \text{hence} \quad \mathrm{w} = c^2\left(1 - \sqrt{1 - \frac{\lambda r^2}{3}}\right)$$

is the *non-Newtonian gravitational potential* specific to de Sitter metric spaces. The mixed components $g_{0i}$ are zero in de Sitter's metric, hence $v_i = -c\,\frac{g_{0i}}{\sqrt{g_{00}}} = 0$ (which means that de Sitter spaces do not rotate). Using the above, we obtain that the physically observable gravitational inertial force in a de Sitter space has the following non-zero components

$$F_1 = \frac{\lambda c^2}{3}\,\frac{r}{1 - \frac{\lambda r^2}{3}}\,, \qquad F^1 = \frac{\lambda c^2}{3}\,r\,,$$

or, when expressed with the physically observable density of the physical vacuum $\breve{\rho}$ and the observable curvature radius of space $\Re$,

$$F_1 = -\frac{\varkappa\breve{\rho}c^2}{3}\,\frac{r}{1 + \frac{\varkappa\breve{\rho}r^2}{3}} = -\frac{c^2}{6\Re^2}\,\frac{r}{1 + \frac{r^2}{6\Re^2}}\,,$$

$$F^1 = -\frac{\varkappa\breve{\rho}c^2}{3}\,r = -\frac{c^2}{6\Re^2}\,r\,.$$

This is a *non-Newtonian gravitational force*, because it arises due to the non-Newtonian gravitational potential specific to de Sitter spaces, and also increases with the distance over which it acts (the force is proportional to $r$).

Based on the results presented above, we arrive at the following conclusions:

*It was obtained by projecting the Riemann-Christoffel curvature tensor $R_{\alpha\beta\gamma\delta}$ onto the time line and the three-dimensional spatial section of the observer; see [2, §5.3] for detail.

1. If the $\lambda$-term is negative $\lambda < 0$, then the density of the physical vacuum is positive $\breve{\rho} > 0$, the pressure inside it is negative $\breve{p} < 0$ (which means that the vacuum expands), the non-Newtonian gravitational forces acting in the space are negative $F^i < 0$ (this means that they are forces of attraction), and the three-dimensional observable curvature is positive $C > 0$;
2. On the contrary, if the $\lambda$-term is positive $\lambda > 0$, then the density of the physical vacuum is negative $\breve{\rho} < 0$, the pressure inside it is positive $\breve{p} > 0$ (the vacuum contracts), and the non-Newtonian gravitational forces acting in the space are positive $F^i > 0$ (therefore, they are forces of repulsion). But in this case, the physically observable three-dimensional curvature is negative $C < 0$ and, therefore, the observable curvature radius of space $\Re$ has an imaginary numerical value $\Re = \frac{1}{\sqrt{C}} = \frac{1}{\sqrt{-2\lambda}}$, which means that the three-dimensional space geometry is hyperbolic (but not spherical, how the geometry of a de Sitter space should be).

Let us now study how the sign of the $\lambda$-term affects the frequency shift gained by a photon travelling in a de Sitter universe. It is called the *cosmological frequency shift*, because it is calculated for a photon fled a "cosmological" distance comparable to the radius of the Universe.

As we have shown and applied in our previous studies since 2009 [3–5], the physically observable cosmological frequency shift in photons is deduced by integrating the scalar equation of isotropic geodesic lines (trajectories of free light-like particles, e.g., free photons), which is the equation of the physically observable photon energy. This equation follows from the theory of chronometric invariants [10–13] (physical observables in the space-time of General Relativity), and is the chronometrically invariant (physically observable) projection of the four-dimensional equations of isotropic geodesics (the four-dimensional equations of motion of free photons) onto the time line of the observer, while the chronometrically invariant projection of the four-dimensional equations of isotropic geodesics onto the three-dimensional spatial section associated with the observer gives the three-dimensional physically observable equations of motion of free photons.

In particular, in 2011, following this calculation method applied to the equations of motion of mass-bearing particles, the cosmological mass defect of mass-bearing particles had been predicted [19]. The *cosmological mass-defect* is a new effect predicted according to General Relativity.

In short, the aforementioned calculation method applied to photons is as follows.

The four-dimensional (general covariant) equations of an isotropic geodesic line, which is the four-dimensional trajectory of a free photon*, when projected onto the time line and the three-dimensional spatial section associated with an observer, have two physically observable (chronometrically invariant) projections, which are†

$$\frac{d\omega}{d\tau} - \frac{\omega}{c^2}\,F_i c^i + \frac{\omega}{c^2}\,D_{ik}\,c^i c^k = 0\,,$$

$$\frac{d(\omega c^i)}{d\tau} - \omega F^i + 2\omega\left(D^i_k + A^{\cdot i}_{k\cdot}\right)c^k + \omega\Delta^i_{nk}\,c^n c^k = 0\,,$$

where $\omega$ is the photon's frequency, $c^i$ is the physically observable chr.inv.-vector of the light velocity ($h_{ik}\,c^i c^k = c^2$), and

$$d\tau = \sqrt{g_{00}}\,dt - \frac{1}{c^2}\,v_i\,dx^i$$

is the physically observable time interval. The factors under which photons move freely, aside of the chr.inv.-gravitational inertial force $F_i$ (explained above), are the chr.inv.-angular velocity tensor of the space rotation $A_{ik}$, the chr.inv.-tensor of the space deformation $D_{ik}$ and the chr.inv.-Christoffel symbols $\Delta^i_{jk}$ (they mean the non-uniformity of space)‡

$$A_{ik} = \frac{1}{2}\left(\frac{\partial v_k}{\partial x^i} - \frac{\partial v_i}{\partial x^k}\right) + \frac{1}{2c^2}\,(F_i v_k - F_k v_i)\,,$$

$$D_{ik} = \frac{1}{2}\frac{{}^*\partial h_{ik}}{\partial t}\,, \qquad D^{ik} = -\frac{1}{2}\frac{{}^*\partial h^{ik}}{\partial t}\,,$$

$$\Delta^i_{jk} = h^{im}\Delta_{jk,m} = \frac{1}{2}\,h^{im}\left(\frac{{}^*\partial h_{jm}}{\partial x^k} + \frac{{}^*\partial h_{km}}{\partial x^j} - \frac{{}^*\partial h_{jk}}{\partial x^m}\right).$$

We call the time projection (first equation) the *chr.inv.-energy equation*, because it gives the physically observable energy $E = \hbar\omega$ of the photon as it travels. The spatial (second) projection represents the chr.inv.-equations of the photon's motion in the three-dimensional space.

We calculate the cosmological frequency shift of a photon by integrating the chr.inv.-energy equation. De Sitter metric spaces do not rotate or deform§, therefore the only acting

*This is a photon, the motion of which is non-deviated by another force than the forces caused by the space itself (gravitation, rotation and deformation), so it travels along a shortest (geodesic) trajectory. If a photon is also under the action of another additional force, that force deviates it from the geodesic (shortest) path thereby making the photon's motion non-geodesic. Such a deviating non-geodesic force or forces appear in the right-hand side of the equations of motion thereby making the right-hand side of the equations nonzero and, thus, transforming them into the non-geodesic equations of motion. See our monograph [2] for detail.

†For details on how these projections are deduced, see our first monograph [1], in which we considered geodesic particle motion in terms of chronomeric invariants (physically observable quantities in General Relativity), and also our second monograph [2] focused on non-geodesic motion.

‡The chr.inv.-derivation operators with respect to time and the spatial coordinates have the form: $\frac{{}^*\partial}{\partial t} = \frac{1}{\sqrt{g_{00}}}\frac{\partial}{\partial t}$ and $\frac{{}^*\partial}{\partial x^i} = \frac{\partial}{\partial x^i} + \frac{1}{c^2}\,v_i\,\frac{{}^*\partial}{\partial t}$.

§In de Sitter's metric (see it in the beginning of this article), we have $g_{0i} = 0$. This means that the linear rotational velocity of such a space is zero $v_i = -c\,\frac{g_{0i}}{\sqrt{g_{00}}} = 0$ and also the angular velocity tensor $A_{ik} = 0$. Hence, de Sitter metric spaces do not rotate. In addition, the chr.inv.-metric tensor in a de Sitter space has the form $h_{ik} = -g_{ik} + \frac{1}{c^2}\,v_i v_k = -g_{ik}$. It does not depend on time because the non-zero $g_{ik}$ components of de Sitter's metric, which are $g_{11} = -(1 - \frac{\lambda r^2}{3})^{-1}$, $g_{22} = -r^2$, $g_{33} = -r^2\sin^2\theta$, do not depend on time. Hence, the space deformation tensor is zero $D_{ik} = 0$. This means that de Sitter metric spaces do not deform, i.e., they are a kind of static spaces.

factor in the chr.inv.-energy equation is the non-Newtonian gravitational force $F_i$. Because the force and the physically observable time interval in a de Sitter space are

$$F_1 = \frac{\lambda c^2}{3} \frac{r}{1 - \frac{\lambda r^2}{3}}\,, \qquad d\tau = \sqrt{g_{00}}\, dt = \sqrt{1 - \frac{\lambda r^2}{3}}\, dt\,,$$

we obtain the chr.inv.-energy equation of a photon travelling along the radial direction $x^1 = r$ in a de Sitter space

$$\frac{d\omega}{d\tau} - \frac{\omega}{c^2} F_1 c^1 = 0$$

in the form

$$d \ln \omega = \frac{\lambda r}{3} \frac{dr}{1 - \frac{\lambda r^2}{3}}$$

or, since $d \ln\left(1 - \frac{\lambda r^2}{3}\right) = -\frac{2\lambda r}{3} \frac{dr}{1 - \frac{\lambda r^2}{3}}$,

$$d \ln \omega = -\frac{1}{2}\, d \ln\left(1 - \frac{\lambda r^2}{3}\right),$$

integrating which we obtain the photon's frequency $\omega$ and its cosmological shift $z$ (without specifying the sign of $\lambda$)*

$$\omega = \frac{\omega_0}{\sqrt{1 - \frac{\lambda r^2}{3}}} \simeq \omega_0 \left(1 + \frac{\lambda r^2}{6}\right),$$

$$z = \frac{\omega - \omega_0}{\omega_0} = \frac{1}{\sqrt{1 - \frac{\lambda r^2}{3}}} - 1 \simeq \frac{\lambda r^2}{6}\,.$$

As you can see from the above formula, we have obtained that in any de Sitter universe there is a parabolic (non-linear) cosmological shift in the frequency of photons. Two options of the cosmological frequency shift are conceivable, depending on the sign of $\lambda$:

1. In a de Sitter universe with $\lambda > 0$, we have $z > 0$ that means a *parabolic cosmological redshift* — the frequency of a photon decreases as it travels, because with $\lambda > 0$ the non-Newtonian gravitational forces acting in a de Sitter world are forces of repulsion, which decelerate photons travelling towards the observer. In this case, the physical vacuum has a negative density $\breve{\rho} < 0$, the pressure inside it is positive $\breve{p} = -\breve{\rho} c^2 > 0$ (which means that the physical vacuum contracts), the three-dimensional physically observable curvature $C = \frac{1}{\Re^2}$ is negative $C < 0$ and, therefore, the observable curvature radius of space $\Re$ has an imaginary numerical value. The latter means that the space geometry is hyperbolic (what the geometry of a de Sitter space should not be). In addition, $\breve{\rho} < 0$ contradicts the physical requirement that any kind of observable matter must have a positive mass and density;
2. In a de Sitter universe with $\lambda < 0$, we have $z < 0$ that means a *parabolic cosmological blueshift* — the frequency of a photon increases as it travels, since with $\lambda < 0$ the non-Newtonian gravitational forces acting in a de Sitter world are forces of attraction (they accelerate photons travelling towards the observer). In this case, the physical vacuum has a positive density $\breve{\rho} > 0$, its pressure is negative $\breve{p} = -\breve{\rho} c^2 < 0$ (which means that the physical vacuum is an expanding medium), the three-dimensional physically observable curvature is positive $C = \frac{1}{\Re^2} > 0$ and, therefore, the observable curvature radius of space $\Re$ has a real numerical value (this means that the space geometry is spherical as it should be in a de Sitter space).

In other words, in a de Sitter world with $\lambda > 0$ there is a non-linear (parabolic) cosmological redshift, and this our theoretical finding corresponds to the non-linearity of the cosmological redshift in the spectra of distant galaxies, which was recently discovered by astronomers†. But the $\lambda > 0$ case considered above does not satisfy such obvious physical requirements as a positive density of distributed matter and the real radius of the Universe. On the other hand, despite the fact that in a de Sitter world with $\lambda < 0$ there are no violations of the above physical requirements, in such a universe we have a parabolic cosmological blueshift.

We were looking for a solution that would resolve this dilemma. As a result, we have arrived at a conclusion that the above contradiction is resolved if we take Einstein's field equations in the following form

$$R_{\alpha\beta} - \frac{1}{2}\, g_{\alpha\beta} R = -\varkappa\, T_{\alpha\beta} - \lambda g_{\alpha\beta}\,,$$

where the last term $\lambda g_{\alpha\beta}$ is taken with the opposite (negative) sign unlike Einstein's original equations, in which this term is positive. In this case,

$$R_{\alpha\beta} - \frac{1}{2}\, g_{\alpha\beta} R = -\varkappa \left(T_{\alpha\beta} + \frac{\lambda}{\varkappa}\, g_{\alpha\beta}\right),$$

where the right-hand side contains the sum $T_{\alpha\beta} + \frac{\lambda}{\varkappa}\, g_{\alpha\beta}$ (as it should be according to the logic of things) and the energy-

*The above $z$ is not a kind of the Doppler frequency shift and is therefore calculated using a different formula. The Doppler redshift $z$ is a decrease in the frequency of the signal emitted by a source moving away from the observer, and the Doppler blueshift is an increase in the signal's frequency when its source moves towards the observer. In contrast, in the case of a de Sitter space under consideration, the source of photons neither moves away nor approaches the observer (the distance $r$ between them remains unchanged). In this case, the photon frequency shift is due only to the non-Newtonian gravitational field attributed to such a space (see the chr.inv.-energy equation that above). In the formula for $z$, which we have obtained, $\omega_0$ is the frequency of the photon in the case, where its source coincides with the observer ($r = 0$), and $\omega$ is the photon's frequency in the case, where the source of the photon is located at a distance $r$ from him.

†See, for example, the surveys [20–22] and the original research results referred therein.

momentum tensor of the physical vacuum is*

$$\breve{T}_{\alpha\beta} = \frac{\lambda}{\varkappa}\, g_{\alpha\beta}\,.$$

In this case, in a de Sitter world with $\lambda > 0$ and, therefore, with a parabolic cosmological redshift, we have

$$\breve{\rho} = \frac{\lambda}{\varkappa} > 0\,, \qquad \breve{p} = -\breve{\rho}c^2 < 0\,,$$

and, therefore (following the same deduction as on page 7),

$$\breve{\rho} = \frac{C}{2\varkappa} = \frac{1}{2\varkappa\Re^2}\,, \qquad \lambda = \frac{C}{2} = \frac{1}{2\Re^2}\,,$$

$$\breve{p} = -\frac{c^2 C}{2\varkappa} = -\frac{c^2}{2\varkappa\Re^2}\,,$$

from which we obtain positive numerical values of the three-dimensional physically observable curvature $C = \frac{1}{\Re^2}$ and the observable curvature radius of space $\Re$

$$C = 2\lambda > 0\,, \qquad \Re = \frac{1}{\sqrt{C}} > 0\,.$$

In addition, there is one more property of de Sitter worlds with $\lambda > 0$. As follows from de Sitter's metric (see it in the beginning of this article) that the state of gravitational collapse (it is characterized by the condition $g_{00} = 0$) arises in a de Sitter space with $\lambda > 0$ under the obvious condition $\frac{\lambda r^2}{3} = 1$. As Larissa obtained in 2010 [3], "...since Schwarzschild's metric of the space inside a sphere of incompressible liquid transforms into de Sitter's metric by the collapse condition and the condition $\lambda = \frac{3}{a^2}$, we arrive at the conclusion: space inside a sphere of incompressible liquid, which is in the state of gravitational collapse, is described by de Sitter's metric, where the $\lambda$-term is $\lambda = \frac{3}{a^2}$. All these can be applied to the Universe as a whole, because it has mass, density, and radius such as those of a collapsar. Therefore, the Universe is a collapsar, whose internal space, being assumed to be a sphere of incompressible liquid, is a de Sitter space with $\lambda = \frac{3}{a^2}$ (here $a$ is the radius of the Universe)." Larissa called this model the *de Sitter bubble*.

Let us calculate the physically observable curvature radius $\Re$ of such a de Sitter space (it does not coincide with the metric radius $a$ of the de Sitter sphere). Since the collapse condition in a de Sitter world arises under $\frac{\lambda r^2}{3} = 1$, where the radial coordinate $r$ meets the metric radius $a$ of the de Sitter sphere ($r = a$), we have $\lambda = \frac{3}{a^2}$. On the other hand, the $\lambda$-term expressed with the three-dimensional physically observable curvature radius $C = \frac{1}{\Re^2}$ has the form $\lambda = \frac{C}{2} = \frac{1}{2\Re^2}$ (see the deduction above). As a result, we obtain that the observable curvature radius of a de Sitter space with $\lambda > 0$, expressed with the metric radius $a$ of the de Sitter sphere in the state of gravitational collapse, is

$$\Re = \frac{a}{\sqrt{6}} \simeq 0.41\, a\,,$$

which means that from the point of view of an observer located inside such a de Sitter bubble, the curvature radius of the bubble $\Re$ is less than its metric radius $a$ (which is the greatest metric distance in the space).† As is seen from the above deduction, this is an observable effect of General Relativity due to the physically observable distortion of space caused by the gravitational field ($\lambda$-field, in this case).

Let us provide astronomers with a formula of physically observable cosmological distances inside a de Sitter universe with $\lambda > 0$. The theory of chronometric invariants (physical observables in the space-time of General Relativity) determines the square of the three-dimensional physically observable interval as $d\sigma^2 = h_{ik}\, dx^i dx^k$, where $h_{ik} = -g_{ik} + \frac{1}{c^2}\, v_i v_k$ is the chr.inv.-metric tensor, and $v_i = -c\,\frac{g_{0i}}{\sqrt{g_{00}}}$ is the linear rotational velocity of space. Since $g_{0i} = 0$ in de Sitter's metric (see the metric in the beginning of this article), de Sitter metric spaces do not rotate ($v_i = 0$). Therefore, $h_{ik} = -g_{ik}$ in any de Sitter space and, hence, $d\sigma^2 = h_{ik}\, dx^i dx^k = -g_{11}\, dr^2$ along the radial direction $x^1 = r$ in it. As a result, the three-dimensional physically observable interval along the radial direction in a de Sitter space has the form

$$d\sigma = \frac{dr}{\sqrt{1 - \frac{\lambda r^2}{3}}}\,,$$

the integration of which together with the collapse condition $\frac{\lambda a^2}{3} = 1$ gives the formula of physically observable cosmological distances inside the de Sitter bubble

$$\sigma = a \arcsin \frac{r}{a}\,.$$

At small metric distances $r \ll a$ between cosmic objects and an observer (compared to the metric radius of the de Sitter sphere $a$, which is the metric radius of the Universe), we have $\arcsin\frac{r}{a} \simeq \frac{r}{a}$. Therefore, at small metric distances $r \ll a$, the physically observable distances $\sigma$ to the cosmic objects are $\sigma \simeq r$. The farther a cosmic object is located from the observer, the greater the physically observed distance $\sigma$ to this object is than the metric distance $r$ to it. For the ultimately distant cosmic objects that are located at the distance equal to the metric radius of the Universe $r = a$ (the radius of the de Sitter bubble), the physically observable distance to them is

$$\sigma = a \arcsin 1 = \frac{\pi}{2}\, a \simeq 1.57\, a\,.$$

*And not $\breve{T}_{\alpha\beta} = -\frac{\lambda}{\varkappa}\, g_{\alpha\beta}$ as in the original version of Einstein's equations $R_{\alpha\beta} - \frac{1}{2}\, g_{\alpha\beta} R = -\varkappa\, T_{\alpha\beta} + \lambda\, g_{\alpha\beta}$ that means $R_{\alpha\beta} - \frac{1}{2}\, g_{\alpha\beta} R = -\varkappa\,(T_{\alpha\beta} - \frac{\lambda}{\varkappa}\, g_{\alpha\beta})$, where on the right-hand side is the energy-momentum tensor of distributed matter $T_{\alpha\beta}$, from which $\frac{\lambda}{\varkappa}\, g_{\alpha\beta}$ is subtracted.

†Note that the observable curvature radius is constant $\Re = const$ throughout a de Sitter space, because de Sitter metric spaces are a kind of constant curvature spaces by definition.

Finally, assuming the corrected version of Einstein's field equations (see above), we arrive at the third option to represent the observable Universe as a de Sitter world:

3. Our Universe is a de Sitter world with $\lambda > 0$, but the $\lambda g_{\alpha\beta}$ term in Einstein's field equations has the opposite (negative) sign unlike Einstein's original equations (in which this term is positive). In this case, the physical vacuum has a positive density $\breve{\rho} > 0$, the pressure inside it is negative $\breve{p} = -\breve{\rho}c^2 < 0$ (the physical vacuum expands), the three-dimensional physically observable curvature is positive $C = \frac{1}{\Re^2} > 0$ and, therefore, the observable curvature radius of space $\Re$ has a real numerical value (this means that the space geometry is spherical as it should be in a de Sitter space). Since $\lambda > 0$ the non-Newtonian gravitational forces acting in the space are forces of repulsion. They decelerate photons travelling towards the observer (the frequency of the photons decreases as they travel). As a result, the observer should register a *parabolic* (*non-linear*) *cosmological redshift* in the frequency of the photons arriving at him from the far cosmos.

We vote for the above 3rd option as a model of the observable Universe, because in this case:

a) the physical vacuum has a positive density $\breve{\rho} > 0$, which satisfies the obvious physical requirement that any kind of observable matter in the Universe must have a positive mass and density,

b) the observable curvature radius of space $\Re$ is real (and not imaginary) and, hence, the space geometry is spherical (and not hyperbolic),

c) the forces acting in such a space are the non-Newtonian gravitational forces of repulsion: they decelerate photons travelling towards the observer, thereby causing a non-linear (parabolic) redshift in the frequency of the photons,

d) the entire observable Universe is located inside a huge de Sitter gravitational collapsar (its gravitational radius outlines the observable event horizon).

Let us apply this model to calculate the metric distance $r$ to the galaxy JADES-GS-z13-0 that is the highest redshift galaxy known to date. It was discovered by astronomers in 2022, and its redshift is $z = 13.2$ [23]. Therefore, applying our parabolic redshift formula for this galaxy, we have

$$z = \frac{1}{\sqrt{1 - \frac{\lambda r^2}{3}}} - 1 = 13.2\,,$$

which with $\lambda = \frac{3}{a^2}$ taken into account (where $a$ is the metric radius of the collapsed de Sitter sphere, i.e., the metric radius of the Universe) gives

$$r = a\sqrt{1 - \frac{1}{(z+1)^2}} = 0.998\,a\,,$$

i.e., this galaxy is located on the very edge of the Universe. This fact is consistent with the observed non-linearity of the cosmological redshift discovered by astronomers [20–22] in the spectra of distant galaxies.

Even if galaxies with redshifts higher than $z = 13.2$ are discovered in the future, we will find that they are not much farther away from us than the aforementioned galaxy. This is thanks to our redshift formula, according to which the parabolic redshift curve $z$ rises very strongly upward at large distances $r$ even for very small increments of $r$. For example, a galaxy, the redshift of which is $z = 25$, according to our redshift formula is located at the distance $r = 0.999\,a$ from us, and the distance to a galaxy with $z > 100$ is $r = 0.99(9)\,a$.

For more or less nearby galaxies, the redshift of which is $z \simeq 0.1$, our formula that above gives $r \simeq 0.4\,a$.

All this confirms Larissa's suggestion, made in 2010 [3], according to which the observable Universe is a huge de Sitter gravitational collapsar (de Sitter bubble) with $\lambda > 0$, the gravitational radius of which outlines the observable event horizon.

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