Santilli's Recovering of Einstein's Determinism

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We study the recent series of papers by the Italian-American physicist, Ruggero Maria Santilli based on the Lie-isotopic branch of hadronic mechanics, which imply that a system of extended protons and neutrons in conditions of partial mutual penetration in a nuclear structure verifies the following properties: 1) Admits, for the first time, explicit and concrete realizations of Bohm's hidden variables. 2) Violates Bell's inequalities by therefore admitting classical counterparts. 3) Verifies the broadening of Heisenberg's indeterminacy principle for electromagnetic interactions of point-like particles in vacuum into the *isouncertainty principle* of hadronic mechanics, also called *Einstein's isodeterminism*, for extended hadrons in conditions of partial mutual penetration, which new principle allows a progressive recovering of Einstein's determinism in the transition from hadrons to nuclei and stars and its full recovering at the limit of Schwartzschild's horizon. We then indicate some of the far reaching advances that are possible for hadronic mechanics and Einstein's isodeterminism but impossible for quantum mechanics and Heisenberg's indeterminacy principle.

1 Hadronic mechanism

Experimental foundations. This paper is based on the experimental evidence that protons [1] and neutrons [2] (collectively called nucleons) have an *extended* charge distribution with the radius $R_N = 0.87$ fm in conditions of partial mutual penetration when they are members of a nuclear structure [3–5] (e.g., the charge radius of the Helium [4] $R_{He} = 1.67$ fm is 0.07 fm smaller then the nucleon diameter $D_N = 1.74$ fm), resulting in the expectation that strong nuclear interactions have a conventional potential, thus Hamiltonian component and a new, contact, thus zero-range and non-Hamiltonian component.

Origination of hadronic mechanics. The Italian-American physicist, R. M. Santilli initiated his studies on extended particles under potential/Hamiltonian and contact/non-Hamiltonian interactions during his graduate studies at the University of Torino, Italy. By recalling that quantum mechanics is reversible over time while nuclear fusions are known to be irreversible and inspired by the 1935 Einstein-Podolsky-Rosen (EPR) argument that *Quantum mechanics is not a complete theory*, [6] (see the recent studies [7–9]), Santilli dedicated his 1965 Ph.D. thesis to the EPR irreversible "completion" of quantum mechanics via the *Lie-admissible generalization of Lie's theory and Heisenberg's equation* [10–12]).

After joining Harvard University under DOE support in September 1977 for the study of irreversible processes, Santilli resumed his research on the Lie-admissible formulation of irreversibility as one can see from his 1978 papers [13, 14], his Springer-Verlag monographs [15, 16] and his axiomatic formulation of irreversibility in the 1981 paper [17] released under his affiliation at Harvard's Department of Mathematics and proposed the continuation of the studies under the name of *hadronic mechanics* which is intended to denote a mechanics for strong interactions (see p. 112 of [16] for the first appearance of the name "hadronic mechanics").

Hamiltonian interactions, which are collectively referred to interactions that are linear, local and derivable from a potential, thus being fully representable by the conventional Hamiltonian of quantum mechanics.

Non-Hamiltonian interactions, which are collectively referred to interactions that are: *Nonlinear* (in the wave function) as pioneering by Werner Heisenberg [18]; *Nonlocal* (distributed in a volume not reducible to points) as pioneered by Louis de Broglie and David Bohm [19]; *Nonpotential* (of contact zero-range type) as pioneered by R. M. Santilli in the 1978 monograph [15] via the *conditions of variational selfadjointness* according to which Hamiltonian interactions are variationally selfadjoint (SA), while non-Hamiltonian interactions are variationally nonselfadjoint (NSA).

Lie-isotopic branch of hadronic mechanics. In this paper, we use the axiom-preserving, time reversible *Lie-isotopic branch of hadronic mechanics* introduced in Charts 5.2, 53 and 5.4, p. 165 on of [16] for the representation of *stable* (thus, time-reversal invariant) systems of *extended* collections of particles at short mutual distances under Hamiltonian/SA and non-Hamiltonian/NSA interactions.

Santilli's Lie-isotopic methods are based on the generalization of the conventional universal enveloping associative algebra ξ with generic product $AB = A \times B$ and related unit 1, $1 \times A = A \times 1 \equiv A$ into the associativity-preserving isoenveloping algebra $\hat{\xi}$ with isoproduct and related isounit (first presented in Eq. (5), p. 71 of [16] and Chart. 5.2 p. 154 for treatment)

$$A \hat{\times} B = A\hat{S}\,B, \quad \hat{S} > 0\,,$$
$$\hat{1} = 1/\hat{S}, \quad \hat{1} \times A = A \times \hat{1} \equiv A\,, \tag{1}$$

where S, called the isotopic element (or the Santillian) is positive-definite but possesses otherwise an unrestricted functional dependence on all needed local variables.

The lifting $\xi \rightarrow \hat{\xi}$ was proposed for the consequential generalization of all branches of Lie's theory into the axiom-preserving *Lie-Santilli isotheory* (presented in Charts 5.3, 5.4 from p.114 on of [16]) (see also [20, 21]) with particular reference to the lifting of n-dimensional Lie algebras with (Hermitean) generators $X_k, k = 1, 2, \ldots, n$ and conventional brackets into the form

$$[X_i \hat{,} X_j] = X_i \hat{\times} X_j - X_j \hat{\times} X_i = \\ = X_i \hat{S} X_j - X_j \hat{S} X_i = C^k_{ij} X_k \,. \tag{2}$$

The fundamental dynamical equation of the isotopic methods are given by the *Lie-isotopic generalization of Heisenberg equation* (Eq. (18a), p. 153 of [16])

$$idA/dt = A \hat{\times} H - H \hat{\times} A = A \hat{S} H - H \hat{S} A \,, \tag{3}$$

where the Hamiltonian H represents all SA interactions while the Santillian $\hat{S}$ represents the extended character of particles and their *new* class of NSA interactions.

Subsequent studies. For advances on hadronic mechanics that occurred in the decades following the 1978 proposal [13–16], the interested reader can inspect: the overview [8] with applications in various fields; the classification of hadronic mechanics [22] (including, in addition to the *Lie-isotopic branch*, the *Lie-admissible branch* for the representation of irreversible processes; *hyperstructural branch* for biological structures and the *isodual branch* for antiparticles); the introductory reviews [23–25]; the AO collection of recent papers [26]; the list of early workshops and conferences [27]; independent monographs [28–36]; and the general presentation [37–39].

Realization of the isotopic element. To render this paper minimally self-sufficient, let us recall the generally used realization of the Santillian [8]

$$\hat{S} = \hat{S}_{4\times 4} = \Pi_{\alpha=1,2,3,4}\, \text{Diag}\left(\frac{1}{n^2_{1,\alpha}}, \frac{1}{n^2_{2,\alpha}}, \frac{1}{n^2_{3,\alpha}}, \frac{1}{n^2_{4,\alpha}}\right) \times \\ \times e^{-\Gamma(r,p,a,E,d,\tau,\pi,\psi,\ldots)} > 0 \,, \\ n_{\mu,\alpha} > 0, \;\; \Gamma > 0, \tag{4}$$

where:

1) The representation of the dimension and shape of the individual nucleons is done via semi-axes $n^2_{k,\alpha}$, $k = 1, 2, 3$ (with n_3 parallel to the spin) and normalization for the vacuum $n^2_{k,\alpha} = 1$.
2) The representation of the density is done via the characteristic quantity $n^2_{4,\alpha}$ per individual nucleons with normalization for the vacuum $n^2_{4,\alpha} = 1$.
3) The representation of the nonlinear, nonlocal and non-potential interactions between extended nucleons is done via the positive-definite exponential term Γ with an arbitrary dependence on relative coordinates r, momenta p, accelerations a, energy E, density d, temperature τ, pressure, π, wave functions ψ or any needed local variable.

When representing nucleons and their NSA interactions, the space dimension of the issotopic element is restricted not to surpass the range of strong interactions $R = 1$ fm $= 10^{-13}$ cm. However, the space dimension of the isotopic element can be, in general, infinite.

Elementary construction of hadronic mechanics. Despite their apparent mathematical complexity, all isotopic formulations can be constructed via the following simple *quantum mechanical nonunitary transformation* unit $1 = \hbar$, and therefore, of all related formulations according to the simple rules [40]

$$\begin{aligned} &1 \;\rightarrow\; U1U^\dagger = \hat{1} \neq 1, \\ &AB \;\rightarrow\; U(AB)U^\dagger = \\ &= (UAU^\dagger)(UU^\dagger)^{-1}(UBU^\dagger) = \hat{A}\hat{S}\,\hat{B}, \\ &[X_i, X_j] \rightarrow U[X_i, X_j]U^\dagger = [\hat{X}_i \hat{,} \hat{X}_j] \,, \end{aligned} \tag{5}$$

which transformations essentially complete a quantum mechanical model for point-like particles into a hadronic model for extended particles under new interactions.

Invariance of isotopic formulations. All quantum mechanical nonunitary models, thus including models (5), are affected by serious inconsistency problems, such as the general lack of conservation of Hermiticity/observability, causality, etc. These problems were resolved by Santilli in the 1998 Ref. [40] via the completion of unitary law (4) into the *isounitary law*

$$\hat{W} \hat{\times} \hat{W}^\dagger = \hat{W}^\dagger \hat{\times} \hat{W} = \hat{1}, \tag{6}$$

completed by the identical reformulation of transformations (5) into the isounitary form

$$\begin{aligned} &U = \hat{W}\hat{S}^{\,1/2}, \\ &UU^\dagger = \hat{1} \rightarrow \hat{W} \hat{\times} \hat{W}^\dagger = \hat{W}^\dagger \hat{\times} \hat{W} = \hat{1}, \\ &\hat{1} \rightarrow \hat{W} \hat{\times} \hat{1} \hat{\times} \hat{W}^\dagger = \hat{1}' \equiv \hat{1}, \\ &\hat{A} \hat{\times} \hat{B} \rightarrow \hat{W} \hat{\times} (\hat{A} \hat{\times} \hat{B}) \hat{\times} \hat{W}^\dagger = \hat{A}' \hat{\times}' \hat{B}' = \hat{A}' \hat{S}' \hat{B}', \\ &\hat{S}' \equiv \hat{S} = (W^\dagger \hat{\times} \hat{W})^{-1}, \end{aligned} \tag{7}$$

with consequential resolution of the problematic aspects of quantum nonunitary models (5), as well as the prediction by isotopic formulations, in view of properties (7), of the same numerical values under the same conditions at different times.

Experimental verifications. Santilli hadronic mechanics has been verified in virtually all physics fields by the exact and invariant representation of experimental data generally not representable via quantum mechanics, such as: direct experimental verifications of the EPR argument [41–43]; electrodynamics [44–47]; large ion physics [48]; particle physics

[49, 50]; Bose-Einstein correlation [51, 52]; propagation of light within physical media [53]; cosmology [54, 55]; neutron synthesis from the Hydrogen [56]; Deuteron magnetic moment [57]; Deuteron spin and rest energy [58]; and other fields.

2 Einstein's isodeterminism

EPR entanglement. Experimental evidence well known since Einstein's times establishes that particles, which are initially bounded together and then separated, can influence each other continuously and instantaneously at arbitrary distances [59]. Albert Einstein strongly objected against the very terms "quantum entanglement" on grounds that the sole possible representation of particle entanglements via the Copenhagen interpretation of quantum mechanics would require superluminal communications that violate special relativity.

For the intent of honoring the generally forgotten Einsteins view, Santilli [62] proved that the sole possible representation of particle entanglement by the Copenhagen interpretation of quantum mechanics is that for which *the particles are free*, evidently because the sole possible interactions admitted by said interpretation are those derivable from a potential which is identically null for particles at large mutual distances.

By recalling that the wave packet of particles is identically null solely at infinite distance, Ref. [62] then pointed out that the sole interactions that are continuous, instantaneous and at arbitrary distances are given by the mutual penetration of wave packets of particles which, being nonlinear, nonlocal and nonpotential, thus NSA [15], are beyond any hope of treatment via quantum mechanics.

Thanks to the prior development of isomathematics for the representation of NSA interactions [33, 36, 37], Santilli [62] proposed the axiom-preserving completion of quantum into hadronic entanglement under the suggested name of *EPR entanglements* which does indeed represent particle entanglements with non-zero, yet non-Hamiltonian-NSA interactions.

Note that the EPR entanglement of particles requires a conceptual and and technical revision of the notion of interactions, e.g., because nuclear constituents admit nontrivial NSA interactions even when they are at a mutual distance bigger than that of strong interactions.

More recently, the EPR entanglement has been experimentally proved to hold at arbitrary *classical* distances [60]. This important feature appears to support Santilli's suggestion [15] that contact forces dating back to Newton, when turned into an operator form, are plausible candidates for the *fifth interactions* intended as *nonlinear, nonlocal, continuous and instantaneous interactions at arbitrary distances* due to the overlapping of the weave packets of particles (see Sect. 1.5.C of [80]). Their lack of identification to date is easily explained by their lack of existence in quantum mechanics. Therefore, in the event such a view is accepted, Santilli's 1978 monograph [15] can be considered the birth of the fifth interactions.

Note also that paper [62] confirms Einstein's additional view that *"The wave function of quantum mechanics does not provide a complete description of the entire physical reality"* [6].

Bohm's hidden variables. As it is well known, in an attempt of reconciling Einstein's determinism with quantum mechanics, D. Bohm [63, 64] submitted in 1952 the hypothesis that quantum mechanics admits *hidden variables* λ, that is, variables which are hidden in its formalism. Following half a century of failure to achieve explicit realizations, a rather general consensus (confirmed by Bell's inequalities outlined next) is that *Bohm's hidden variables do not exist within the formalism of quantum mechanics.*

In 1995, R. M. Santilli [38] proved that *hidden variables do exist within the context of hadronic mechanics, they are hidden in the axiom of associativity of quantum mechanics and are quantitatively represented by the isotopic element* (Sect. 4.C.3, p. 170 on and Sect. 6.8, p. 254 on of [38], e.g.,

$$\lambda = \hat{S}\,,$$
$$A\hat{\times}B = A\lambda B, \quad A\hat{\times}(B\hat{\times}C) = (A\hat{\times}B)\hat{\times}C. \tag{8}$$

It should be noted that, despite its apparent elementary character of the isotopic product (1), the quantitative study of the indicated realization of Bohm's hidden variables required collegial efforts in the nonlocal lifting of the entire 20th century applied mathematics, including the Newton-Leibnitz differential calculus [65] (see also studies [36]). Nowadays, there exists a number of explicit and concrete realization of hidden variables, among which we mention the realization used for the first numerically exact and time invariant representation of the Deuteron magnetic moment [66, 67] which achievement resulted to be impossible for quantum mechanics in one century.

Bell's inequalities. In the 1964, J. S. Bell [68] proved a number of quantum mechanical inequalities, the first one of which essentially states that systems of point like particles with spin $1/2$ represented via quantum mechanics do not admit classical counterparts. This view was assumed by mainstream physicists for over half a century to be the final disproof of the EPR argument and of Bohm's hidden variables.

Again thanks to the prior development of isomathematics as well as of explicit and concrete realizations of hidden variables, Santilli [71] proved in 1998 a number of hadronic inequalities essentially stating that *systems of extended particles with spin* $1/2$ *represented via the Lie-isotopic branch of hadronic mechanics do indeed admit classical counterparts*, while providing explicit examples.

Santilli's hadronic inequalities are confirmed by the direct experimental verifications of the EPR-argument [41–43] establishing the existence in nature of particle conditions which violate Bell's inequalities.

Note that the above theoretical and experimental works imply the expectation that Heisenberg's uncertainties principle is correspondingly violated by strong interactions between extended nucleons in conditions of mutual penetration.

Einstein's isodeterminism. Soon after joining Harvard University in late 1977, R. M. Santilli expressed doubts on the exact validity for strong interactions of Heisenberg's uncertainty principle (also called indeterminacy principle) and other quantum mechanical laws, as one can see from the *titles* of the 1978 memoir [14] (see also the subsequent papers [69, 70]). Santilli's argument underlying such a conviction is that Heisenberg's standard deviations for coordinates Δr, momenta Δp and their product are certainly valid for the conditions of their original conception, i.e., for point-like charged particles under electromagnetic interactions, because a point-like particle can move within a star by solely sensing action-at-a-distance interactions due to its dimensionless character.

The situation is conceptually, mathematically, theoretically and experimentally different when considering extended nucleons in conditions of mutual penetration because, in view of their "strength", strong interactions imply the creation of a *pressure* on a given nucleon by its surrounding nucleons, according to a view pioneered by L. de Broglie and D. Bohm with their nonlocal theory [19]. It is then evident that the standard deviations for the indicated nucleon Δr and Δp *cannot* be the same as the corresponding deviations for an electron in vacuum, thus implying the need for a suitable completion of Heisenberg's uncertainty principle for strong interactions.

Thanks to the original works [14, 69, 70] and the recent works [62, 71], Santilli [72] finally achieved in 2019 the axiom-preserving EPR completion of Heisenberg's uncertainty principle into the *isouncertainty principle of hadronic mechanics*, also called *Einstein's isouncertainies*, for extended nucleons under electromagnetic, weak and strong interactions whose derivation can be outlined as follows.

Let $\mathcal{H}$ be the Hilbert-Myung-Santilli isospace [73] of isomechanics with *isostates* $|\hat{\psi}\rangle$ and isoinner product $\langle\psi|\hat{\times}|\hat{\psi}\rangle$ (for a review, see Sect. 4 of [23]). Assume the isonormalization which is necessary for a constant Santillian

$$\langle\hat{\psi}|\hat{\times}|\hat{\psi}\rangle = \langle\hat{\psi}|\hat{S}\,|\hat{\psi}\rangle = \hat{S}\,, \tag{9}$$

the Schrödinger-Santilli isoequation [16, 38]

$$\begin{aligned}&\hat{H}\hat{\times}|\hat{\psi}\rangle = \\ &= [\Sigma_{k=1,2,...n}\,\frac{1}{2m_k}\,\hat{p}_k\hat{\times}\hat{p}_k + \hat{V}(r)]\,\hat{S}\,(r,p,\psi,\dots)\,|\hat{\psi}\rangle = \\ &E\times|\hat{\psi}\rangle,\end{aligned} \tag{10}$$

the isolinear momentum [65]

$$\hat{p}\hat{\times}|\hat{\psi}\rangle = -i\,\hat{1}\,\partial_{\hat{r}}\hat{\psi}, \tag{11}$$

and the isocommutation rules

$$\begin{aligned}&[\hat{r}_i\hat{,}\hat{p}_j]\hat{\times}|\hat{\psi}\rangle = -i\,\hat{1}\,\delta_{i.j}|\hat{\psi}\rangle,\\ &[\hat{r}_i\hat{,}\hat{r}_j]|\hat{\psi}\rangle = [\hat{p}_i\hat{,}\hat{p}_j]|\hat{\psi}\rangle = 0.\end{aligned} \tag{12}$$

Then the isounitary transformation (7) of Heisenberg's uncertainty principle

$$\Delta r\Delta p = \frac{1}{2}\,|\langle\psi|\,[r\hat{,}p]\,|\psi\rangle| \geqslant \frac{1}{2}\,\hbar, \tag{13}$$

uniquely and unambiguously yields the *isouncertainty principle of hadronic mechanics*, also called *Einstein's isodeterminism*, whose projection on our spacetime (as needed for experiments) is given by [72] (see [23] for an extended derivation)

$$\begin{aligned}&\hat{\Delta}r\hat{\Delta}p = \frac{1}{2}\,|\langle\hat{\psi}|\hat{\times}\,[\hat{r}\hat{,}\hat{p}]\,\hat{\times}|\hat{\psi}\rangle| = \\ &= \frac{1}{2}\,|\langle\hat{\psi}|\hat{S}\,[\hat{r}\hat{,}\hat{p}]\,\hat{S}\,|\hat{\psi}\rangle| \approx \frac{1}{2}\,\hbar\hat{S}\, = \frac{1}{2}\,\hbar e^{-\Gamma(r,p,a,E,d,\tau,\pi,\psi,\dots)} \approx \\ &\approx \frac{1}{2}\,\hbar\,[1-\Gamma(r,p,a,E,d,\tau,\pi,\psi,\dots)+\dots] \ll \frac{1}{2}\,\hbar\,,\end{aligned} \tag{14}$$

where the Santillian $\hat{S}$ is given by Eq. (4) and we assumed, in first approximation, that all nucleons are perfectly spherical.

It should ne mentioned that completion (14) of Heisenberg's uncertainty principle includes as particular cases most of the existing generalized uncertainty relations known to this author (see, e.g., [74–76] and papers quoted therein).

In particular, the standard isodeviations $\hat{\Delta}r$ and $\hat{\Delta}p$ progressively and individually tend to zero with the increase of the density of the hadronic medium, thus in the transition from hadrons to nuclei and stars.

Note that the completion of the value $\geqslant \frac{1}{2}\,\hbar$ into the form $\approx \frac{1}{2}\,\hbar\hat{S}$ is due to the nonlocality of hadronic mechanics which requires a redefinition of the very notion of standard deviations due to the very big pressure exercised on a nucleon by the surrounding nucleons under "strong" interactions [24, 72].

To achieve the full validity of Einstein's determinism, Santilli [77, 78] decomposes the Riemannian metric $g(x)$ in four dimensions into then product of the Minkowskian metric $\eta = -\mathrm{Diag}, (1, 1, 1, -1)$ and the gravitational isotopic element $\hat{S}$

$$g(x) = \hat{S}_{4\times 4}\;\eta_{4\times 4}, \tag{15}$$

with particular values for the Schwartzschild metric

$$\hat{S}_{kk} = \frac{1}{1-2M/r}\,, \quad \hat{S}_{44} = 1-2M/r\,. \tag{16}$$

It is then easy to see that Einstein's determinism [6] is fully recovered at the limit of the Schwartzschild horizon.

3 Concluding remarks

Despite one century of studies under large public funds, nuclear physics has been unable to achieve the controlled nuclear fusion; the recycling of radioactive nuclear waste; the exact representations of nuclear data; the synthesis of the neutron from the Hydrogen atom in the core of stars; the nuclear stability despite the natural instability of the neutron and extremely repulsive protonic Coulomb forces; and other open problems.

A main point which is attempted to convey in this paper is that the indicated open nuclear problems appear to be due to the *theoretical assumption* that Heisenberg's uncertainty principle for point-like particles under electromagnetic interactions is also valid for extended nucleons under strong interactions.

As an illustration, Heisenberg's uncertainty principle prohibits a structural representation of the synthesis of the neutron from the electron and the proton in the core of stars, because the standard deviation Δr_e for the coordinate of the electron is much bigger than the size of the neutron and the standard deviation Δp_e of the momentum implies a kinetic energy of the electron bigger than the rest energy of the neutron,

$$\begin{gathered}\Delta r_e > R_n = 0.87 \times 10^{-13}\ \text{cm},\\ \Delta v_e > \frac{\hbar}{\Delta r_e \times m_e} > 10^{10}\ \text{m/s}, \\ \Delta K_e = \frac{1}{2m_e} \times (\Delta p_e)^2 > m_n = 939.56\ \text{MeV/c}^2.\end{gathered} \tag{17}$$

By comparison, the study of the neutron synthesis via hadronic mechanics under isouncertainty principle (14), implies standard isodeviations for which Eqs. (17) become

$$\begin{gathered}\hat{\Delta} r_e = \hat{S}\,\Delta r_e \leqslant R_n = 0.87 \times 10^{-13}\ \text{cm},\\ \hat{\Delta} v = \hat{S}\,\Delta v_e \ll 10^{10}\ \text{m/s}, \\ \hat{\Delta} K_e = \hat{S}\,\Delta K_e \ll m_n = 939.56\ \text{MeV/c}^2,\end{gathered} \tag{18}$$

thus allowing a quantitative representation of the neutron synthesis from the Hydrogen [79] with far reaching advances that cannot be formulated in quantum mechanics, let alone treated, such as [80–82]: 1) The prediction of means for recycling radioactive nuclear waste by nuclear power plants via *new* stimulated decays; 2) The possible return to the continuous creation of matter in the universe to explain the 0.782 MeV missing in the neutron synthesis; 3) The apparent reduction of all matter in the universe to protons and electrons.

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References

1. Pohl R., Antognini A. and Kottmann F. The size of the proton. *Nature*, 2010, v. 466, 213–216. www.nature.com/articles/nature09250
2. Wietfeldt F. E. Measurements of the Neutron Lifetime. *Atroms*, 2018, v. 6, 1–19. www.mdpi.com/2218-2004/6/4/70
3. IAEA, Nuclear data. www.iaea.org/about/organizational-structure/department-of-nuclear-sciences-and-applications/division-of-physical-and-chemical-sciences/nuclear-data-section
4. ScienceDirect, Helium nucleus. www.sciencedirect.com/topics/mathematics/helium-nucleus
5. Rau S. et al. Penning trap measurements of the deuteron and the HD^+ molecular ion. *Nature*, 2020, v. 585, 43–47. doi.org/10.1038/s41586-020-2628-7
6. Einstein A., Podolsky B. and Rosen N. Can quantum-mechanical description of physical reality be considered complete? *Phys. Rev.*, 1935, v. 47, 777–780. www.eprdebates.org/docs/epr-argument.pdf
7. Beghella-Bartoli S. and Santilli R. M, Ed. Proceedings of the 2020 Teleconference on the Einstein-Podolsky-Rosen argument that "Quantum mechanics is not a compete theory". Curran Associates, New York, NY, 2021. Printed version: www.proceedings.com/59404.html
8. Santilli R. M. Overview of historical and recent verifications of the Einstein-Podolsky-Rosen argument and their applications to physics, chemistry and biology. APAV - Accademia Piceno Aprutina dei Velati, Pescara, Italy, 2021. www.santilli-foundation.org/epr-overview-2021.pdf
9. Dunning-Davies J. A. Present Day Perspective on Einstein-Podolsky-Rosen and its Consequences. *Journal of Modern Physics*, 2021, v. 12, 887–936. www.scirp.org/journal/paperinformation.aspx?paperid=109219
10. Santilli R. M. Embedding of Lie-algebras into Lie-admissible algebras. *Nuovo Cimento*, 1967, v. 51, 570–585. //www.santilli-foundation.org/docs/Santilli-54.pdf
11. Santilli R. M. Dissipativity and Lie-admissible algebras. *Meccanica*, 1969, v. 1, 3–12.
12. Santilli R. M. An introduction to Lie-admissible algebras. *Suppl. Nuovo Cimento*, 1968, v. 6, 1225.
13. Santilli R. M. On a possible Lie-admissible covering of Galilei's relativity in Newtonian mechanics for nonconservative and Galilei form-non-invariant systems. *Hadronic Journal*, 1978, v. 1, 223–423. www.santilli-foundation.org/docs/Santilli-58.pdf.
14. Santilli R. M. Need of subjecting to an experimental verification the validity within a hadron of Einstein special relativity and Pauli exclusion principle. *Hadronic Journal* 1978, v. 1, 574–901. www.santilli-foundation.org/docs/santilli-73.pdf
15. Santilli R. M. Foundation of Theoretical Mechanics, Vol. I, The Inverse Problem in Newtonian Mechanics. *Springer-Verlag*, Heidelberg, Germany, 1978. www.santilli-foundation.org/docs/Santilli-209.pdf
16. Santilli R. M. Foundation of Theoretical Mechanics. Vol. II, Birkhoffian Generalization of Hamiltonian Mechanics. *Springer-Verlag*, Heidelberg, Germany, 1983. www.santilli-foundation.org/docs/santilli-69.pdf
17. Santilli R.M. Initiation of the representation theory of Lie-admissible algebras of operators on bimodular Hilbert spaces. *Hadronic J.*, 1979, v. 3, 440–467. www.santilli-foundation.org/docs/santilli-1978-paper.pdf
18. Heisenberg W. *Nachr. Akad. Wiss. Gottingen*, 1953, v. IIa, 111–133. www.link.springer. com/chapter/10.1007/978-3-642-70079-8_23

19. Stanford Encyclopedia of Philosophy, Bohmian (de Broglie-Bohm) Mechanics. 2021. plato.stanford.edu/entries/qm-bohm
20. Sourlas D. S and Tsagas Gr. T. Mathematical Foundation of the Lie-Santilli Theory. Ukraine Academy of Sciences, 1993. www.santilli-foundation.org/docs/santilli-70.pdf
21. Kadeisvili J. V. An introduction to the Lie-Santilli isotopic theory. *Mathematical Methods in Applied Sciences*,1996, v. 19, 134–172. www.santilli-foundation.org/docs/Santilli-30.pdf
22. Anderson R. Outline of Hadronic Mathematics, Mechanics and Chemistry as Conceived by Santilli R. M. *American Journal of Modern Physics*, 2016, v. 6, 1–16. www.santilli-foundation.org/docs/HMMC-2017.pdf
23. Santilli R. M. Studies on A. Einstein, B. Podolsky and N. Rosen prediction that quantum mechanics is not a complete theory, I: Basic methods. *Ratio Mathematica*, 2020, v. 38, 5–69. eprdebates.org/docs/epr-review-i.pdf
24. Santilli R. M. Studies on A. Einstein, B. Podolsky and N. Rosen prediction that quantum mechanics is not a complete theory, II: Apparent proof of the EPR argument. *Ratio Mathematica*, 2020, v. 38, 71–138. eprdebates.org/docs/epr-review-ii.pdf
25. Santilli R. M. Studies on A. Einstein, B. Podolsky and N. Rosen prediction that quantum mechanics is not a complete theory, III: Illustrative examples and applications. *Ratio Mathematica*,2020, v. 38, 139–222. eprdebates.org/docs/epr-review-iii.pdf
26. Anderson A. Collected OA Papers on Hadronic Mathematics, Mechanics and Chemistry. www.santilli-foundation.org/docs/HMMC.pdf
27. Anderson R. List of early OA international workshops and conferences in hadronic mechanics. www.santilli-foundation.org/docs/workshops-conf-hm.pdf
28. Aringazin A. K, Jannussis A., Lopez F., Nishioka M. and Veljanosky B. Santilli's Lie-Isotopic Generalization of Galilei and Einstein Relativities. Kostakaris Publishers, Athens, Greece, 1991. www.santilli-foundation.org/docs/Santilli-108.pdf
29. Sourlas D. S. and Tsagas Gr. T. Mathematical Foundation of the Lie-Santilli Theory. Ukraine Academy of Sciences, 1993. www.santilli-foundation.org/docs/santilli-70.pdf
30. Lohmus J., Paal E. and Sorgsepp L. Nonassociative Algebras in Physics. Hadronic Press, Palm Harbor, 1994. www.santilli- foundation.org/docs/Lohmus.pdf
31. Kadeisvili J. V. Santilli's Isotopies of Contemporary Algebras, Geometries and Relativities. Ukraine Academy of Sciences, Second edition, 1997. www.santilli-foundation.org/docs/Santilli-60.pdf
32. Jiang C-X. Foundations of Santilli Isonumber Theory. International Academic Press, 2001. www.i-b-r.org/docs/jiang.pdf
33. Ganfornina R. M. F. and Valdes J. N. Fundamentos de la Isotopia de Santilli. International Academic Press, 2001. www.i-b-r.org/docs/spanish.pdf
English translation: *Algebras, Groups and Geometries*, 2015, v. 32, 135–308. www.i-b-r.org/docs/Aversa-translation.pdf
34. Davvaz B. and Vougiouklis Th. A Walk Through Weak Hyperstructures and H_v-Structures. World Scientific, 2018.
35. Gandzha I. and Kadeisvili J. V. New Sciences for a New Era: Mathematical, Physical and Chemical Discoveries by Ruggero Maria Santilli. Sankata Printing Press, Nepal, 2011. www.santilli-foundation.org/docs/RMS.pdf
36. Georgiev S. Foundations of IsoDifferential Calculus. Nova Publishers, New York:
Vol. 1: Iso-Differential and Iso-Integral Calculus for Iso-Functions in One Variable. 2014;
Vol. 2: Iso-Differential and Iso-Integral Calculus for Iso-Functions in Several Variables. 2014;
Vol. 3: Iso-Ordinary Iso-Differential Equations. 2014;
Vol. 4: Iso-Difference Equations. 2015;
Vol. 5: Iso-Stochastic Iso-Differential Equations. 2015;
Vol. 6: Theory of Iso-Measurable Iso-Functions. 2016.
New Edition of Vol. 1: Iso-Differential and Iso-Integral Calculus for Iso-Functions in One Variable. 2022. Iso-Mathematics. Lambert Academic Publishing, 2022.
37. Santilli R. M. Elements of Hadronic Mechanics. Vol. I, Mathematical Foundations. Ukraine Academy of Sciences, Kiev, 1995. www.santilli-foundation.org/docs/Santilli-300.pdf
38. Santilli R. M. Elements of Hadronic Mechanics. Vol. II, Theoretical Foundations. Ukraine Academy of Sciences, Kiev, 1995. www.santilli-foundation.org/docs/Santilli-301.pdf
39. Santilli R. M. Elements of Hadronic Mechanics. Vol. III, Experimental verifications. Ukraine Academy of Sciences, Kiev, 2016. www.santilli-foundation.org/docs/elements-hadronic-mechanics-iii.compressed.pdf
40. Santilli R. M. Invariant Lie-isotopic and Lie-admissible formulation of quantum deformations. *Found. Phys.*, 1997, v. 27, 1159–1177. www.santilli-foundation.org/docs/Santilli-06.pdf
41. Fadel M. et al. Spatial entanglement patterns and Einstein-Podolsky-Rosen steering in Bose-Einstein condensates. *Science*, 2018, v. 360, 409–415. www.santilli-foundation.org/Basel-paper.pdf
42. Colciaghi P. et al. Einstein-Podolsky-Rosen Experiment with Two Bose-Einstein Condensates. *Phys. Rev. X*, 2023, v. 13, 021031, 10 pp. journals.aps.org/prx/pdf/10.1103/PhysRevX.13.021031
43. Aspect A. et al. Experimental Realization of Einstein-Podolsky-Rosen-Bohm Gedankenexperiment: A New Violation of Bell's Inequalities. *Phys. Rev. Lett.*, 1982, v. 49, 91–94. ui.adsabs.harvard.edu/abs/1982PhRvL..49...91A
44. Miller J. P et al. Muon (g2): experiment and theory. *Rep. Prog. Phys.*, 2007, v. 70, 795–881. news.fnal.gov/2021/04/first-results-from-fermilabs-muon-g-2-experiment-strengthen-evidence-of-new-physics
45. Santilli R. M. Apparent Unsettled Value of the Recently Measured Muon Magnetic Moment. *Progress in Physics*, 2022, v. 18, 15–18. www.santilli-foundation.org/docs/muon-meanlife-2022.pdf
46. Santilli R. M. Representation of the anomalous magnetic moment of the muons via the Einstein-Podolsky-Rosen completion of quantum into hadronic mechanics. *Progress in Physics*, 2021, v. 17, 210–215. www.progress-in-physics.com/2021/PP-62-13.PDF
47. Santilli R. M. Representation of the anomalous magnetic moment of the muons via the novel Einstein-Podolsky-Rosen entanglement in: Garcia H. M. C., Guzman J. J. C., Kauffman L. H. and Makaruk H., Eds. Scientific Legacy of Professor Zbigniew Oziewicz. World Scientific, 2024. www.santilli-foundation.org/docs/santilli-546-(1).pdf
48. Schukraft K. Heavy-ion physics with the ALICE experiment at the CERN Large Hadron Collider. *Trans. Royal Soc.*, 2012, v. A370, 917–932. royalsocietypublishing.org/doi/10.1098/rsta.2011.0469
49. Cardone F., Mignani R. and Santilli R. M. On a possible energy-dependence of the K^0 lifetime. Part I. *J. Phys. G: Part. Phys.*, 1992, v. 18, L141–L144. www.santilli-foundation.org/docs/Santilli-32.pdf
50. Cardone F., Mignani R. and Santilli R. M. On a possible energy-dependence of the K^0 lifetime. Part II. *J. Phys. G: Part. Phys.*, 1992, v. 18, L61–L65. www.santilli-foundation.org/docs/Santilli-32.pdf
51. Santilli R. M. Nonlocal formulation of the Bose-Einstein correlation within the context of hadronic mechanics. *Hadronic J.*, 1992, v. 5, 1–50 and v. 15, 81–133. www.santilli-foundation.org/docs/Santilli-116.pdf
52. Cardone F. and Mignani R. Nonlocal approach to the Bose-Einstein correlation. *JETP*, 1996, v. 83, 435. www.santilli-foundation.org/docs/Santilli-130.pdf
53. Ahmar H. et al. Additional experimental confirmations of Santilli's IsoRedShift and the consequential lack of expansion of the universe. *Journal of Computational Methods in Sciences and Engineering*, 2013, v. 13, 321–375. www.santilli-foundation.org/docs/IRS-confirmations-212.pdf

54. Mignani R. Quasars redshift in isominkowski space. *Physics Essay*, 1992, v. 5, 531. www.santilli-foundation.org/docs/Santilli-31.pdf
55. Santilli R. M. Experimental Verifications of IsoRedShift with Possible Absence of Universe Expansion, Big Bang, Dark Matter and Dark Energy. *The Open Astronomy Journal*, 2010, v. 3, 124–132. www.santilli-foundation.org/docs/Isoredshift-Letter.pdf
56. Norman R., Beghella Bartoli S., Buckley B., Dunning-Davies J., Rak J. and Santilli R. M. Experimental Confirmation of the Synthesis of Neutrons and Neutroids from a Hydrogen Gas. *American Journal of Modern Physics*, 2017, v. 6, 85–104. www.santilli-foundation.org/docs/confirmation-neutron-synthesis-2017.pdf
57. Santilli R. M. A quantitative isotopic representation of the Deuteron magnetic moment. In: Proceedings of the International Symposium "Dubna Deuteron-3e". Joint Institute for Nuclear Research, Dubna, Russia, 1994. www.santilli-foundation.org/docs/Santilli-134.pdf
58. Santilli R. M. The Physics of New Clean Energies and Fuels According to Hadronic Mechanics. Special issue of the Journal of New Energy, 1998. www.santilli-foundation.org/docs/Santilli-114.pdf
59. Bengtsson I. and Życzkowski K. An Introduction to Quantum Entanglement, second, revised edition. Cambridge University Press, 2006. chaos.if.uj.edu.pl/ karol/geometry.htm
60. Berkowitz R. Macroscopic systems can be controllably entangled and limitlessly measured. *Physics Today*, July 2021, 16–18. www.santilli-foundation.org/docs/particle-entanglement.pdf
61. Santilli R. M. Elements of nuclear physics according to hadronic mechanics. I: Exact Lie-isotopic representation of nuclear data. submitted for publication.
62. Santilli R. M. A quantitative representation of particle entanglements via Bohm's hidden variables according to hadronic mechanics. Progress in Physics, 2022, v. 18, 131–137. www.santilli-foundation.org/docs/pip-entanglement-2022.pdf
63. Bohm D. A Suggested Interpretation of the Quantum Theory in Terms of "Hidden Variables". *Phys. Rev.*, 1952, v. 85, 166–182. journals.aps.org/pr/abstract/10.1103/PhysRev.85.166
64. Bohm D. Quantum Theory. Prentice-Hall, New York, 1951.
65. Santilli R. M. Nonlocal-Integral Isotopies of Differential Calculus, Mechanics and Geometries. *Rendiconti Circolo Matematico Palermo, Suppl.*, 1996, v. 42, 7–82. www.santilli-foundation.org/docs/Santilli-37.pdf
66. Santilli R. M. Iso-Representation of the Deuteron Spin and Magnetic Moment via Bohm's Hidden Variables. *Progress in Physics*, 2022, v. 18, 74–81. www.progress-in-physics.com/2022/PP-63-12.PDF
67. Santilli R. M. and Sobczyk G. Representation of nuclear magnetic moments via a Clifford algebra formulation of Bohm's hidden variables. *Scientific Reports*, 2022, v. 12, 1–10. www.santilli-foundation.org/Santilli-Sobczyk.pdf
68. Bell J. S. On the Einstein Podolsky Rosen paradox. *Physics*, 1964, v. 1, 195. cds.cern.ch/record/111654/files/vol1p195-200_001.pdf
69. Santilli R. M. Generalization of Heisenberg's uncertainty principle for strong interactions. *Hadronic Journal*,1981, v. 4, 642. www.santilli-foundation.org/docs/generalized-uncertainties-1981.pdf
70. Santilli R. M. Isotopic lifting of Heisenberg's uncertainties for gravitational singularities. *Communications in Theoretical Physics*, 1994, v. 3, 47–66. santilli-foundation.org/docs/santilli-511.pdf
71. Santilli R. M. Isorepresentation of the Lie-isotopic $SU(2)$ Algebra with Application to Nuclear Physics and Local Realism. *Acta Applicandae Mathematicae*, 1998, v. 50, 177–190. www.santilli-foundation.org/docs/Santilli-27.pdf
72. Santilli R. M. Studies on the classical determinism predicted by A. Einstein, B. Podolsky and N. Rosen. *Ratio Mathematica*, 2019, v. 37, 5–23. www.eprdebates.org/docs/epr-paper-ii.pdf
73. Myung H. C. and Santilli R. M. Modular-isotopic Hilbert space formulation of the exterior strong problem. *Hadronic Journal*, 1982, v. 5, 1277. www.santilli-foundation.org/docs/myung-santilli-1982.pdf
74. Liang S.-D., Lake M. J. and Harko T., Eds. Generalized uncertainty relations: Existing paradigms and new approaches. Special Issue of *Frontiers in Astronomy and Space Sciences*, 2023. www.frontiersin.org/research-topics/25805/generalized-uncertainty-relations-existing-paradigms-and-new-approaches
75. Li J.-L., and Qiao C.-F. The Generalized Uncertainty Principle. *Annalen der Physik*, 2021, v. 533, 2000335. ui.adsabs.harvard.edu/abs/2021AnP...53300335L/abstract
76. Herdegen A. and Ziobro P. Generalized uncertainty relations. *Letters in Mathematical Physics*, 2017, v. 107, 659–671. link.springer.com/article/10.1007/s11005-016-0916-9
77. Santilli R. M. Isotopic quantization of gravity and its universal isopoincaré symmetry, in the Proceedings of The Seventh Marcel Grossmann Meeting on Gravitation. SLAC, 1992, Jantzen R. T., Keiser G. M. and Ruffini R., Editors, *World Scientific Publishers*, p. 500–505, 1994. www.santilli-foundation.org/docs/Santilli-120.pdf
78. Santilli R. M. Unification of gravitation and electroweak interactions, in the Proceedings of the Eight Marcel Grossmann Meeting on Gravitation. Israel, 1997, Piran T. and Ruffini R. Editors, *World Scientific*, p. 473–475, 1999. www.santilli-foundation.org/docs/Santilli-137.pdf
79. Santilli, R. M. Reduction of Matter in the Universe to Protons and Electrons via the Lie-isotopic Branch of Hadronic Mechanics. *Progress in Physics*, 2023, v. 19, 73–99. www.progress-in-physics.com/2023/PP-65-09.PDF
80. Santilli R. M. Elements of nuclear physics according to hadronic mechanics. I: Apparent insufficiencies of quantum mechanics in nuclear physics. *Acta Mathematica*, 2024, v. 52, in press. www.santilli-foundation.org/docs/RM-santilli-paper-1.pdf
81. Santilli R. M. Elements of nuclear physics according to hadronic mechanics. II: Exact Lie-isotopic representation of Deuteron data. *Acta Mathematica*, 2024, v. 52, in press. www.santilli-foundation.org/docs/RM-santilli-paper-2.pdf
82. Santilli R. M. Elements of nuclear physics according to hadronic mechanics. III: Exact Lie-isotopic representation of nuclear stability. *Acta Mathematica*, 2024, v. 52, in press. www.santilli-foundation.org/docs/RM-santilli-paper-3.pdf