Ultrafaint Dwarf Galaxies and the Baryonic Tully-Fisher Relation (BTFR) Derived by Quantum Celestial Mechanics (QCM)

Franklin Potter

Huntington Beach, CA USA. E-mail: frank11hb@yahoo.com

All rotationally supported galaxies such as Andromeda and the Milky Way obey the baryonic Tully-Fisher relation (BTFR) of $V_f^4 \propto M_b$ for rotational velocity V_f and detected amount of baryonic mass M_b, as do all the pressure supported dwarf galaxies. However, the ultrafaint dwarf galaxies in the Local Group do not obey BTFR. Including dark matter does not resolve the issue. We investigate whether Quantum Celestial Mechanics (QCM) with its detailed derivation of the BTFR offers a reasonable resolution.

1 Introduction

At least three general types of galaxies are known: rotationally supported galaxies such as Andromeda and the Milky Way, pressure supported dwarf galaxies, and ultrafaint dwarf galaxies. The rotationally supported galaxies obey the original 1977 Tully-Fisher relation connecting their luminosity L to their outer circular velocity V [1]

$$L = V^{\alpha}\,, \tag{1}$$

where α is a constant that is dependent upon the specific physical properties of each galaxy and the physics model.

When more data became available in the 1990s, the baryonic Tully-Fisher relation (BTFR) determined by MOdified Newtonian Dynamics (MOND) revealed that the flat rotation velocity V_f of a rotationally supported galaxy such as Andromeda or the Milky Way depends upon its baryonic mass M_b exactly to the 4th power, i.e. $V_f^4 \propto M_b$. This relationship even holds true for dwarf galaxies that are pressure supported [2]. Or, as it is more commonly expressed [3],

$$V_f \propto M_b^{\frac{1}{4}}\,. \tag{2}$$

However, for the numerous ultrafaint dwarf galaxies of low mass with no gas component that surround the Milky Way and are gravitationally bound collections of stars only, the BTFR appears to fail [4]. The rotation velocities are too great for the amount of detected baryonic mass. Hence the possibilty exists for a significant amount of dark matter in addition to baryonic mass in their location.

In Fig. 1 are shown some of the many rotationally supported galaxies (circles), some of the numerous pressure supported galaxies including some dwarf galaxies (diamonds), and a representative sample of the ultrafaint dwarf galaxies (squares) with their very large uncertainties because of difficulties in their measurement [5]. These ultrafaints are usually too faint to be detected beyond the nearby Universe, so the sample is largely limited to the Local Group.

The straight line in Fig. 1 is the BTFR fit of the rotationally supported galaxies and the pressure supported galaxies, both of which contain a significant amount of baryonic gas among the stars, as determined [2,6] by MOdified Newtonian Dynamics (MOND) to be

$$V_f = \left(0.379\,\mathrm{km\,s^{-1}}\,M_{Sun}^{-\frac{1}{4}}\right) M_b^{\frac{1}{4}}\,. \tag{3}$$

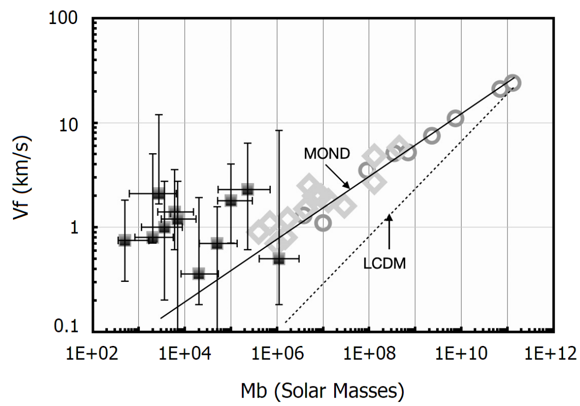

Fig. 1: $\log V_f$ vs. $\log M_b$ for rotationally supported galaxies (circles), pressure supported dwarf galaxies (diamonds), and ultrafaint dwarf galaxies (squares) in relation to the MOND predicted BTFR (straight line) at exactly V_f^4 vs. M_b, plus an example of a ΛCDM prediction (dashed line). The top two circles are Andromeda and the Milky Way. Uncertainties not shown are approximately symbol size.

But the ultrafaint dwarf galaxies (black squares) do not obey this BTFR relationship. Both MOND researchers and traditional lambda cold dark matter (ΛCDM) research groups have not been able to resolve this issue except by proposing that perhaps the Milky Way itself is possibly interfering with the stability of the ultrafaint systems., i.e. tidal disruptions from external gravitational field effects could be influencing a dispersion of the stars.

Moreover, the ΛCDM approach also has the fundamental problem that one cannot predict the 4th power relationship between the V_f and the baryonic mass M_b because the included dark matter in their galaxy models suggests the power relationship to be smaller than 4 as shown by the dashed line in Fig. 1.

However, the BTFR derived in quantum celestial mechanics (QCM) [7] offers a possible solution that explicitly includes more fundamental physical properties of the ultrafaint dwarf galaxies, including their baryon densities, radial sizes, and total angular momentum.

The successful application of QCM to galaxy clusters [8] explained how the dynamic baryon mass of a galaxy cluster, without requiring any dark matter, depends upon both the total angular momentum and the observed baryonic mass values. Therefore, we investigate the QCM expression for the BTFR to better understand the ultrafaint dwarf galaxies.

2 QCM derivation of the BTFR

QCM is derived from the general relativistic Hamilton-Jacobi equation [7]

$$g^{\alpha\beta}\,\frac{\partial S}{\partial x^\alpha}\,\frac{\partial S}{\partial x^\beta}\,-\mu^2 c^2 = 0 \tag{4}$$

via the transformation that defines the wave function

$$\Psi = e^{S'/H} \tag{5}$$

to obtain a scalar gravitational wave equation (GWE)

$$g^{\alpha\beta}\,\frac{\partial^2\Psi}{\partial x^\alpha \partial x^\beta}\,+\,\frac{\Psi}{H^2} = 0\,. \tag{6}$$

In these equations, S' is the classical action S divided by μc for a test particle of mass μ, with c being light speed in vacuum.

The characteristic distance H for a gravitationally bound system is defined to be

$$H = \frac{L_T}{M_T c} \tag{7}$$

where L_T is the total angular momentum for the system of total mass M_T.

In a gravitationally bound system in coordinates (t, r, θ, ϕ) obeying the Schwarzschild metric, as expected for planetary systems and galaxies, from the angular coordinates one derives the angular momentum *per unit mass* quantization condition

$$\frac{L}{\mu} = m\,\frac{L_T}{M_T} \tag{8}$$

for a mass μ in orbit with angular momentum L and for m an integer. All confirmed multi-planetary systems obey this relationship [9].

From the radial equation one obtains the energy *per unit mass* quantization

$$E = -\mu c^2\,\frac{r_s^2}{8n^2H^2}\,, \tag{9}$$

with r_s the Schwarzschild radius and integer n.

Application of the virial theorem for gravitation leads directly to the tangential rotation velocity for the test particle

$$v = \frac{r_s c}{2nH} \tag{10}$$

from which the QCM predicted BTFR can be derived.

If we assume that the galaxy is approximately a disc with a total baryonic mass $M_b = \pi h R^2 \rho_0$ for average density ρ_0, thickness h, and radius R, then its total angular momentum $L_T = \alpha M_b R^2 \omega$ with moment of inertia factor α and rotational velocity ω. With gravitational constant G, this BTFR becomes

$$V_f = M_b^{\frac{1}{4}}\,\sqrt{\frac{G}{n\alpha}\,\sqrt{\pi h \rho_0}}\,. \tag{11}$$

As one example, suppose h is 8 times greater than estimated, and the α and ρ_0 parameters can each vary by about a factor of 2. By combining them optimally, QCM predicts a maximum gain of about $\sqrt{8}$ times the previous value of V_f for the same baryonic mass value.

3 Conclusion

QCM predicts a BTFR relation depending upon the average density, thickness, and total angular momentum of a galaxy. Whether these three parameters in the BTFR expression derived by QCM will resolve the issue of the ultrafaint dwarf galaxy positions on the $V_f^4 \propto M_b$ graph remains to be determined when better data becomes available.

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