Defect Interactions in the Spacetime Continuum and Quantum Overlap Interactions

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In this paper, we consider the nature of defect interactions in the Spacetime Continuum and how this process provides an explanation of quantum overlap interactions within the Elastodynamics of the Spacetime Continuum (*STCED*). Strain energy is the fundamental defining energy characteristic of defects and their interactions in the spacetime continuum. In *STCED*, the interaction of dislocations and disclinations is mediated through the overlap interaction of their strain energy densities. One source of this overlap interaction comes from displacement defect interactions of different types of defects, characterized by their Burgers and Frank vectors. We derive the interaction terms of dislocations and disclinations arising from the defect displacements derived by deWit for the dislocation and disclination displacements and provide a sample calculation of dislocation strain energy densities from the dislocation displacements to calculate strain energy densities for defect interactions. Another approach considered is to calculate the force between the defects from the stress tensor, and obtain the strain energy of the overlap based on the work performed by the application of the force over a given distance.

1 Introduction

In this paper, we consider the nature of defect interactions in the Spacetime Continuum and how this process provides an explanation of quantum overlap interactions within the Elastodynamics of the Spacetime Continuum (*STCED*) [1–11].

1.1 Elastodynamics of the Spacetime Continuum

The Elastodynamics of the Spacetime Continuum is a natural extension of Einstein's General Theory of Relativity which combines continuum mechanics and general relativistic descriptions of the Spacetime Continuum. The introduction of strains in the Spacetime Continuum as a result of the energy-momentum stress tensor allows us to use, by analogy, results from continuum mechanics, in particular the stress-strain relation, to provide a better understanding of the general relativistic spacetime.

The stress-strain relation for an isotropic and homogeneous Spacetime Continuum is given by [1, 11]

$$2\,\bar{\mu}_0\,\varepsilon^{\mu\nu} + \bar{\lambda}_0\,g^{\mu\nu}\varepsilon = T^{\mu\nu} \tag{1}$$

where $\bar{\lambda}_0$ and $\bar{\mu}_0$ are the Lamé elastic constants of the Spacetime Continuum: $\bar{\mu}_0$ is the shear modulus (the resistance of the Spacetime Continuum to *distortions*) and $\bar{\lambda}_0$ is expressed in terms of $\bar{\kappa}_0$, the bulk modulus (the resistance of the Spacetime Continuum to *dilatations*):

$$\bar{\lambda}_0 = \bar{\kappa}_0 - \frac{1}{2}\,\bar{\mu}_0 \tag{2}$$

in a four-dimensional continuum. $T^{\mu\nu}$ is the general relativistic energy-momentum stress tensor, $\varepsilon^{\mu\nu}$ the Spacetime Continuum strain tensor resulting from the stresses, and

$$\varepsilon = \varepsilon^{\alpha}{}_{\alpha}\,, \tag{3}$$

the trace of the strain tensor obtained by contraction, is the volume dilatation ε defined as the change in volume per original volume $\Delta V/V$ [12, see pp. 149–152] and is an invariant of the strain tensor. It should be noted that the structure of (1) is similar to that of the field equations of General Relativity,

$$R^{\mu\nu} - \frac{1}{2}\,g^{\mu\nu}R = -\varkappa\,T^{\mu\nu} \tag{4}$$

where $R^{\mu\nu}$ is the Ricci curvature tensor, R is its trace, $\varkappa = 8\pi G/c^4$ and G is the gravitational constant (see [11, Ch. 2] for more details).

In *STCED*, as shown in [1, 11], energy propagates in the spacetime continuum (*STC*) as wave-like deformations which can be decomposed into *dilatations* and *distortions*. *Dilatations* involve an invariant change in volume of the spacetime continuum which is the source of the associated rest-mass energy density of the deformation. On the other hand, *distortions* correspond to a change of shape (shearing) of the spacetime continuum without a change in volume and are thus massless. Thus the deformations propagate in the continuum by longitudinal (*dilatation*) and transverse (*distortion*) wave displacements.

This provides a natural explanation for wave-particle duality, with the massless transverse mode corresponding to the wave aspects of the deformations and the massive longitudinal mode corresponding to the particle aspects of the deformations. The rest-mass energy density of the longitudinal mode is given by [1, see Eq. (32)]

$$\rho c^2 = 4\,\bar{\kappa}_0\,\varepsilon \tag{5}$$

where ρ is the rest-mass density, c is the speed of light, $\bar{\kappa}_0$ is the bulk modulus of the *STC* (the resistance of the spacetime continuum to *dilatations*), and ε is the volume dilatation as seen previously given by (3), which is an invariant of the strain tensor, as is the rest-mass energy density. Hence

$$mc^2 = 4\bar{\kappa}_0\,\Delta V \tag{6}$$

where m is the mass of the deformation and ΔV is the dilatation change in the spacetime continuum's volume corresponding to mass m. This demonstrates that mass is not independent of the spacetime continuum, but rather mass is part of the spacetime continuum fabric itself.

1.2 Energy in the spacetime continuum

In *STCED*, energy is stored in the spacetime continuum as strain energy [5]. As seen in [1, see Section 8.1], the strain energy density of the spacetime continuum is separated into two terms: the first one $\mathcal{E}_\parallel$ expresses the dilatation energy density (the mass longitudinal term) while the second one $\mathcal{E}_\perp$ expresses the distortion energy density (the massless transverse term):

$$\mathcal{E} = \mathcal{E}_\parallel + \mathcal{E}_\perp \tag{7}$$

where

$$\mathcal{E}_\parallel = \frac{1}{2}\,\bar{\kappa}_0\,\varepsilon^2 \equiv \frac{1}{32\,\bar{\kappa}_0}\,\rho^2 c^4\,, \tag{8}$$

ρ is the rest-mass density of the deformation, and

$$\mathcal{E}_\perp = \bar{\mu}_0\,e^{\alpha\beta}e_{\alpha\beta} = \frac{1}{4\bar{\mu}_0}\,t^{\alpha\beta}t_{\alpha\beta}\,, \tag{9}$$

with the strain distortion

$$e^{\alpha\beta} = \varepsilon^{\alpha\beta} - e_s\,g^{\alpha\beta} \tag{10}$$

and the strain dilatation $e_s = \frac{1}{4}\,\varepsilon^\alpha{}_\alpha$. Similarly for the stress distortion $t^{\alpha\beta}$ and the stress dilatation t_s. Then the dilatation (massive) strain energy density of the deformation is given by the longitudinal strain energy density (8) and the distortion (massless) strain energy density of the deformation is given by the transverse strain energy density (9).

The strain energy W of the deformation is obtained by integrating (7) over the volume V of the deformation to give

$$W = W_\parallel + W_\perp\,, \tag{11}$$

where $W_\parallel$ is the (massive) longitudinal strain energy of the deformation given by

$$W_\parallel = \int_V \mathcal{E}_\parallel\,\mathrm{d}V \tag{12}$$

and $W_\perp$ is the (massless) transverse distortion strain energy of the deformation given by

$$W_\perp = \int_V \mathcal{E}_\perp\,\mathrm{d}V\,, \tag{13}$$

where the volume element $\mathrm{d}V$ in cylindrical polar coordinates is given by $r\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}z$ for a stationary deformation.

2 Quantum particles from STC defects

In [8, 10, 11], we show that quantum particles can be represented as defects in the spacetime continuum, specifically dislocations and disclinations. *Dislocations* are translational deformations, consisting of screw and edge dislocations, while *disclinations* are rotational deformations consisting of wedge and twist disclinations. As shown in [10], Table 1 below provides a summary of the identification of quantum particles and their associated spacetime dislocations and disclinations.

The basic Feynman diagrams can be seen to represent screw dislocations as photons, edge dislocations as bosons, twist and wedge disclinations as fermions [10], and their interactions. The interaction of defects results from the overlap of their strain energy densities. In QED, the exchange of virtual particles in interactions can be seen to be a perturbation expansion representation of the forces resulting from the overlap of the strain energy densities of the dislocations and disclinations. In *STCED*, the perturbative expansions are replaced by finite analytical expressions for the strain energy density interactions of individual screw dislocations as photons, edge dislocations as bosons, twist and wedge disclinations as fermions [10].

3 Strain energy density interactions of STC defects

Strain energy is the fundamental defining energy characteristic of defects and their interactions in the spacetime continuum. In this paper, we consider the interactions of defects in *STCED*, and how they relate to quantum overlap interactions. In *STCED*, the interaction of dislocations and disclinations is mediated through the overlap interaction of their strain energy densities.

We find that this interaction of the strain energy density results from the overlap of the strain energy densities of defects, a process akin to the wavefunction overlap of quantum mechanics, which is physically explained by this process in *STCED*. One source of this overlap interaction comes from displacement defect interactions of different types of defects, characterized by their Burgers and Frank vectors as derived in [8, 11]. Another source results from the overlap of the strain energy densities of multiple defects, including of the same type, as will be shown in §4.

3.1 Strain energy densities from defect displacements

We derive the interaction terms of dislocations and disclinations arising from the defect displacements derived by deWit for the dislocation and disclination displacements [13–16].

With the constants $\bar{\alpha}_0$ and $\bar{\beta}_0$ given by

$$\bar{\alpha}_0 = \frac{\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0}\,, \tag{14}$$

$$\bar{\beta}_0 = \frac{\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0}\,, \tag{15}$$

STC defect	Type of particle	Particles
Screw dislocation	Massless boson	Photon
Edge dislocation	Massive boson	Spin-0 particle
		Spin-1 Proca eqn
		Spin-2 graviton
Wedge disclination	Massive fermion	Quarks
ℓ^3 Twist disclination	Massive fermion	Leptons
ℓ Twist disclination	Massless fermion	Neutrinos

Table 1: Identification of quantum particles and their associated STC defects; quoted from [10].

the deWit dislocation displacements are given by [16] and [11, §9.5]

$$
\begin{aligned}
u_x &= \frac{b_x^{(e)}}{2\pi}\left(\theta + \bar{\beta}_0\,\frac{xy}{r^2}\right) + \frac{b_y^{(g)}}{2\pi}\left(\bar{\alpha}_0\,\ln r + \bar{\beta}_0\,\frac{y^2}{r^2}\right),\\
u_y &= -\frac{b_x^{(e)}}{2\pi}\left(\bar{\alpha}_0\,\ln r + \bar{\beta}_0\,\frac{x^2}{r^2}\right) + \frac{b_y^{(g)}}{2\pi}\left(\theta - \bar{\beta}_0\,\frac{xy}{r^2}\right), \qquad (16)\\
u_z &= \frac{b_z^{(s)}}{2\pi}\,\theta\,,
\end{aligned}
$$

where we have specifically appended superscripts to the Burgers vectors for clarity: $b_x^{(e)}$ for the edge dislocation proper, $b_y^{(g)}$ for the gap dislocation, and $b_z^{(s)}$ for the screw dislocation. In general, we will not append these superscripts except where required for clarity. We can obtain specific expressions for screw dislocations by putting $b_x = 0$ and $b_y = 0$, for edge dislocations by putting $b_y = 0$ and $b_z = 0$, and for gap dislocations by putting $b_x = 0$ and $b_z = 0$, and similarly for the other expressions below.

The deWit disclination displacements are given by [16] and [11, §10.5]

$$
\begin{aligned}
u_x &= -\frac{\Omega_x^{(s)}}{2\pi}\,z\left[\bar{\alpha}_0 \ln r + \bar{\beta}_0\,\frac{y^2}{r^2}\right] + \frac{\Omega_y^{(t)}}{2\pi}\,z\left[\theta + \bar{\beta}_0\,\frac{xy}{r^2}\right] -\\
&\quad - \frac{\Omega_z^{(w)}}{2\pi}\left[y\,\theta - \bar{\alpha}_0\,x\,(\ln r - 1)\right],\\
u_y &= -\frac{\Omega_x^{(s)}}{2\pi}\,z\left[\theta - \bar{\beta}_0\,\frac{xy}{r^2}\right] - \frac{\Omega_y^{(t)}}{2\pi}\,z\left[\bar{\alpha}_0 \ln r + \bar{\beta}_0\,\frac{x^2}{r^2}\right] +\\
&\quad + \frac{\Omega_z^{(w)}}{2\pi}\left[x\,\theta + \bar{\alpha}_0\,y\,(\ln r - 1)\right], \qquad (17)\\
u_z &= \frac{\Omega_x^{(s)}}{2\pi}\left[y\,\theta - \bar{\alpha}_0\,x\,(\ln r - 1)\right] -\\
&\quad - \frac{\Omega_y^{(t)}}{2\pi}\left[x\,\theta + \bar{\alpha}_0\,y\,(\ln r - 1)\right],
\end{aligned}
$$

where we have specifically appended superscripts to the Frank vectors for clarity: $\Omega_x^{(s)}$ for the splay disclination, $\Omega_y^{(t)}$ for the twist disclination proper, and $\Omega_z^{(w)}$ for the wedge disclination. As for dislocations, in general, we will not append these superscripts except where required for clarity. We can obtain specific expressions for wedge disclinations by putting $\Omega_x = 0$ and $\Omega_y = 0$, for splay disclinations by putting $\Omega_y = 0$ and $\Omega_z = 0$, and for twist disclinations proper by putting $\Omega_x = 0$ and $\Omega_z = 0$, and similarly for the other expressions below.

Sample calculation of dislocation strain energy densities

We first provide a sample calculation of dislocation strain energy densities from the dislocation displacements to calculate strain energy densities for defect interactions.

From the deWit dislocation displacements (16), the components of the strain tensor are derived from $\varepsilon^{\mu\nu} = \frac{1}{2}(u^{\mu;\nu} + u^{\nu;\mu})$ and the volume dilatation ε from (3) to obtain:

$$
\varepsilon = -\frac{1}{\pi}\,\frac{\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0}\,\frac{b_x y - b_y x}{r^2}\,. \qquad (18)
$$

The negative sign arises in this formula from the convention used to define the Burgers vector. It can be eliminated by using the **FS**/*RH* convention instead of the **SF**/*RH* convention for the Burgers vector [17, see p. 22]) and can be disregarded by taking the absolute value of (18). Substituting (18) in (8), the dislocation longitudinal dilatation strain energy density is then given by

$$
\mathcal{E}_{\parallel} = \frac{1}{2\pi^2}\,\frac{\bar{\kappa}_0\,\bar{\mu}_0^2}{(2\bar{\mu}_0 + \bar{\lambda}_0)^2}\,\frac{(b_x y - b_y x)^2}{r^4}\,. \qquad (19)
$$

Expanding the quadratic term, we obtain

$$
\mathcal{E}_{\parallel} = \mathcal{E}_{\parallel}^{E} + \mathcal{E}_{\parallel}^{G} + \mathcal{E}_{\parallel\,int}^{E-G}\,, \qquad (20)
$$

where the edge dislocation longitudinal dilatation strain energy density $\mathcal{E}_{\parallel}^{E}$, the gap dislocation longitudinal dilatation strain energy density $\mathcal{E}_{\parallel}^{G}$, and also the edge-gap interaction longitudinal dilatation strain energy density $\mathcal{E}_{\parallel\,int}^{E-G}$ are given by the formulae

$$\mathcal{E}_{\parallel}^{E} = \frac{1}{2\pi^2}\,\bar{\kappa}_0\bar{\alpha}_0^2\,\frac{b_x^{(e)\,2}y^2}{r^4}\,, \tag{21}$$

$$\mathcal{E}_{\parallel}^{G} = \frac{1}{2\pi^2}\,\bar{\kappa}_0\bar{\alpha}_0^2\,\frac{b_y^{(g)\,2}x^2}{r^4}\,, \tag{22}$$

$$\mathcal{E}_{\parallel\,int}^{E-G} = -\frac{1}{\pi^2}\,\bar{\kappa}_0\bar{\alpha}_0^2\,\frac{b_x^{(e)}b_y^{(g)}xy}{r^4}\,. \tag{23}$$

The screw dislocation longitudinal dilatation strain energy density $\mathcal{E}_{\parallel}^{S} = 0$ and is massless.

The distortion strain energy density is calculated from (9) and (10),

$$\mathcal{E}_{\perp} = \bar{\mu}_0\,e^{\alpha\beta}e_{\alpha\beta}\,,$$
$$e^{\alpha\beta} = \varepsilon^{\alpha\beta} - e_s\,g^{\alpha\beta}\,,$$

where $e_s = \frac{1}{4}$. Then

$$e^{\alpha\beta}e_{\alpha\beta} = \left(\varepsilon^{\alpha\beta} - \frac{1}{4}\,\varepsilon g^{\alpha\beta}\right)\left(\varepsilon_{\alpha\beta} - \frac{1}{4}\,\varepsilon g_{\alpha\beta}\right) \tag{24}$$

and the distortion strain energy density equation becomes

$$\mathcal{E}_{\perp} = \bar{\mu}_0\left(\varepsilon^{\alpha\beta}\varepsilon_{\alpha\beta} - \frac{1}{4}\,\varepsilon^2\right) \tag{25}$$

for $g^{\alpha\beta} = \eta^{\alpha\beta}$.

This expression is expanded using the non-zero elements of the strain tensor to give

$$\begin{aligned}\mathcal{E}_{\perp} = {}& \frac{\bar{\mu}_0}{8\pi^2}\,\frac{b_z^2}{r^2} + \frac{\bar{\mu}_0}{4\pi^2}\,\bar{\alpha}_0^2\,\frac{(b_x y - b_y x)^2}{r^4} + \\ &+ \frac{\bar{\mu}_0}{2\pi^2}\,\bar{\beta}_0^2\,\frac{(b_x x + b_y y)^2}{r^4} - \frac{3}{2\pi^2}\,\frac{\bar{\mu}_0\bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0}\,\frac{b_x b_y\,xy}{r^4}\,.\end{aligned} \tag{26}$$

Expanding the quadratic terms, we obtain

$$\mathcal{E}_{\perp} = \mathcal{E}_{\perp}^{S} + \mathcal{E}_{\perp}^{E} + \mathcal{E}_{\perp}^{G} + \mathcal{E}_{\perp\,int}^{E-G} \tag{27}$$

where the screw dislocation distortion strain energy density $\mathcal{E}_{\perp}^{S}$, the edge dislocation distortion strain energy density $\mathcal{E}_{\perp}^{E}$, the gap dislocation distortion strain energy density $\mathcal{E}_{\perp}^{G}$, and the edge-gap interaction distortion strain energy density $\mathcal{E}_{\perp\,int}^{E-G}$ are given by

$$\mathcal{E}_{\perp}^{S} = \frac{\bar{\mu}_0}{8\pi^2}\,\frac{b_z^{(s)\,2}}{r^2}\,, \tag{28}$$

$$\mathcal{E}_{\perp}^{E} = \frac{\bar{\mu}_0}{4\pi^2}\,\frac{b_x^{(e)\,2}\left(\bar{\alpha}_0^2 y^2 + 2\bar{\beta}_0^2 x^2\right)}{r^4}\,, \tag{29}$$

$$\mathcal{E}_{\perp}^{G} = \frac{\bar{\mu}_0}{4\pi^2}\,\frac{b_y^{(g)\,2}\left(\bar{\alpha}_0^2 x^2 + 2\bar{\beta}_0^2 y^2\right)}{r^4}\,, \tag{30}$$

$$\mathcal{E}_{\perp\,int}^{E-G} = -\frac{\bar{\mu}_0}{2\pi^2}\left(\bar{\alpha}_0^2 - 2\bar{\beta}_0^2 + 3\,\bar{\gamma}_0\right)\frac{b_x^{(e)}b_y^{(g)}xy}{r^4}\,, \tag{31}$$

where

$$\bar{\gamma}_0 = \frac{\bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0}\,. \tag{32}$$

Note that there are no interaction terms (cross-terms) between screw and edge dislocations for a dislocation line although there are interaction terms between edge dislocations (edge and gap dislocations).

The dislocation strain energy is calculated by integrating the dislocation strain energy density over the volume of the dislocation, as per (12) for the longitudinal massive component and (13) for the transverse massless component.

3.2 Strain energy densities of mixed defects

The sample calculation procedure can be used for dislocations or disclinations using (16) or (17) respectively. For mixed dislocations and disclinations, we need to use defect displacements that combine both to calculate the resulting strain energy density interactions.

We consider the interaction terms of dislocations and disclinations arising from the general displacements derived from the general combined deWit dislocation and disclination displacements, see [16] and [11, §17.2], to obtain:

$$\begin{aligned}u_x = {}& \frac{b_x}{2\pi}\left(\theta + \bar{\beta}_0\,\frac{xy}{r^2}\right) + \frac{b_y}{2\pi}\left(\bar{\alpha}_0\,\ln r + \bar{\beta}_0\,\frac{y^2}{r^2}\right) - \\ & - \frac{\Omega_x}{2\pi}\,z\left(\bar{\alpha}_0\ln r + \bar{\beta}_0\,\frac{y^2}{r^2}\right) + \frac{\Omega_y}{2\pi}\,z\left(\theta + \bar{\beta}_0\,\frac{xy}{r^2}\right) - \\ & - \frac{\Omega_z}{2\pi}\left(y\,\theta - \bar{\alpha}_0\,x\,(\ln r - 1)\right)\,, \\ u_y = {}& -\frac{b_x}{2\pi}\left(\bar{\alpha}_0\,\ln r + \bar{\beta}_0\,\frac{x^2}{r^2}\right) + \frac{b_y}{2\pi}\left(\theta - \bar{\beta}_0\,\frac{xy}{r^2}\right) - \\ & - \frac{\Omega_x}{2\pi}\,z\left(\theta - \bar{\beta}_0\,\frac{xy}{r^2}\right) - \frac{\Omega_y}{2\pi}\,z\left(\bar{\alpha}_0\ln r + \bar{\beta}_0\,\frac{x^2}{r^2}\right) + \\ & + \frac{\Omega_z}{2\pi}\left(x\,\theta + \bar{\alpha}_0\,y\,(\ln r - 1)\right)\,, \\ u_z = {}& \frac{b_z}{2\pi}\,\theta + \frac{\Omega_x}{2\pi}\left(y\,\theta - \bar{\alpha}_0\,x\,(\ln r - 1)\right) - \\ & - \frac{\Omega_y}{2\pi}\left(x\,\theta + \bar{\alpha}_0\,y\,(\ln r - 1)\right)\,,\end{aligned} \tag{33}$$

where again

$$\bar{\alpha}_0 = \frac{\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0}\,, \tag{34}$$

$$\bar{\beta}_0 = \frac{\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0}\,. \tag{35}$$

The components of the strain tensor are derived from

$$\varepsilon^{\mu\nu} = \frac{1}{2}(u^{\mu;\nu} + u^{\nu;\mu})\,.$$

As this is a linear operation, the combined components of the strain tensor are obtained from (33). The volume dilatation ε is then obtained from (3):

$$\begin{aligned}\varepsilon = &-\frac{1}{\pi}\frac{\bar{\mu}_0}{2\bar{\mu}_0+\bar{\lambda}_0}\frac{b_x y - b_y x}{r^2} - \\ &-\frac{1}{\pi}\frac{\bar{\mu}_0}{2\bar{\mu}_0+\bar{\lambda}_0}\left(\Omega_x x + \Omega_y y\right)\frac{z}{r^2} + \\ &+\frac{\Omega_z}{\pi}\left(\frac{\bar{\mu}_0}{2\bar{\mu}_0+\bar{\lambda}_0}\ln r + \frac{1}{2}\frac{\bar{\mu}_0+\bar{\lambda}_0}{2\bar{\mu}_0+\bar{\lambda}_0}\right).\end{aligned} \tag{36}$$

The mass energy density is calculated from (37), specifically

$$\rho c^2 = 4\bar{\kappa}_0\,\varepsilon = 2(2\bar{\lambda}_0+\bar{\mu}_0)\,\varepsilon\,. \tag{37}$$

Substituting for ε from (37), using the absolute value of ε as seen previously, the mass energy density of the combined deWit dislocation and disclination displacements is given by

$$\begin{aligned}\rho c^2 = \Bigg| &\frac{4}{\pi}\frac{\bar{\kappa}_0\bar{\mu}_0}{2\bar{\mu}_0+\bar{\lambda}_0}\frac{b_x y - b_y x}{r^2} - \\ &+\frac{4}{\pi}\frac{\bar{\kappa}_0\bar{\mu}_0}{2\bar{\mu}_0+\bar{\lambda}_0}\left(\Omega_x x + \Omega_y y\right)\frac{z}{r^2} + \\ &-\frac{4\bar{\kappa}_0\Omega_z}{\pi}\left(\frac{\bar{\mu}_0}{2\bar{\mu}_0+\bar{\lambda}_0}\ln r + \frac{1}{2}\frac{\bar{\mu}_0+\bar{\lambda}_0}{2\bar{\mu}_0+\bar{\lambda}_0}\right)\Bigg|\end{aligned} \tag{38}$$

and using $\bar{\alpha}_0$ and $\bar{\beta}_0$,

$$\begin{aligned}\rho c^2 = \Bigg| &\frac{4}{\pi}\bar{\kappa}_0\bar{\alpha}_0\frac{b_x y - b_y x}{r^2} - \\ &+\frac{4}{\pi}\bar{\kappa}_0\bar{\alpha}_0\left(\Omega_x x + \Omega_y y\right)\frac{z}{r^2} + \\ &-\frac{4\bar{\kappa}_0\Omega_z}{\pi}\left(\bar{\alpha}_0\ln r + \frac{1}{2}\bar{\beta}_0\right)\Bigg|\,.\end{aligned} \tag{39}$$

The rest-mass density is thus the sum of the rest-mass density of edge dislocations, wedge and twist disclinations. Screw dislocations are not present as they are massless. The negative terms indicate a decrease in total rest-mass energy due to bound states.

4 Overlap interaction of multiple defects

In this section, we consider the interactions of dislocations of multiple defects, including of the same type, which are seen to result from the force resulting from the overlap of their strain energy density in the spacetime continuum [17, see p. 112]. There is no longitudinal interaction strain energy density between edge dislocations and screw dislocations in a mixed dislocation, as the latter are massless. In addition, as shown in the sample calculation, there are no interaction terms (cross-terms) between screw and edge dislocations for a dislocation line which is a mixed dislocation. This is due to the perpendicularity of the two types of dislocations. However, multiple dislocations separated by a distance R can undergo defect interactions due to the overlap of their strain energy densities, both dislocation and disclination interactions.

These interactions can be calculated using various methods. Till now, we have concentrated on the calculation of the strain energy of the overlap based on the defect displacements. Another approach [18, see Chapter 3] that can be used is to calculate the force between the defects from the stress tensor, and obtain the strain energy of the overlap based on the work W performed by the application of the force F over a given distance d, from the well-known relation

$$W = \mathbf{F}\cdot\mathbf{d}\,. \tag{40}$$

Conversely, the force can be calculated from the negative of the derivative of the strain energy W with position. For example, for a dislocation line parallel to the z-axis positive along the z-axis, the force on the dislocation line can be written as

$$\mathbf{F} = -\left(\frac{\partial W}{\partial x}, \frac{\partial W}{\partial y}\right). \tag{41}$$

This allows us to calculate the overlap interaction strain energy.

We consider the total strain energy W for two neighbouring defects in a 3-D spatial continuum. This can be written in terms of the strain energy density $\mathcal{E}$ for the two neighbouring defects as an integral over the volume

$$W = \int_V \mathcal{E}\,\mathrm{d}V\,. \tag{42}$$

$\mathcal{E}$ can be expressed in terms of the stress density σ using the stress tensor portion of (9). Following Weertman [18, see p. 62], as the stress density equations are linear, we separate the stress density σ into a sum of two terms

$$W = \frac{1}{2\bar{\mu}_0}\int_V (\sigma_1+\sigma_2)^2\,\mathrm{d}V\,, \tag{43}$$

where σ_2 is the stress density of the defect (dislocation or disclination) of interest in an otherwise strain-free continuum, and σ_1, known as the *internal stresses*, is the stress density of the rest of the continuum including the neighbouring defect(s), but excluding the defect of σ_2. Then $\sigma_1+\sigma_2$ represents the stress density of a continuum including a defect of interest and internal stresses.

The total strain energy W can then be written as

$$W = \frac{1}{2\bar{\mu}_0}\int_V \sigma_1^2\,\mathrm{d}V + \frac{1}{2\bar{\mu}_0}\int_V \sigma_2^2\,\mathrm{d}V + \frac{1}{\bar{\mu}_0}\int_V \sigma_1\sigma_2\,\mathrm{d}V\,. \tag{44}$$

The first two terms are position independent. Only the third term is position dependent, and leads to a force on the defect as per (41) and contributes to the interaction strain energy W_{12}:

$$W_{12} = \frac{1}{\bar{\mu}_0}\int_V \sigma_1\sigma_2\,\mathrm{d}V\,. \tag{45}$$

4.1 Overlap interaction strain energy

As seen in §1.2, the energy-momentum stress tensor $T^{\alpha\beta}$ is decomposed into a stress deviation tensor $t^{\alpha\beta}$ and a scalar t_s, according to

$$T^{\alpha\beta} = t^{\alpha\beta} + t_s\, g^{\alpha\beta}\,, \tag{46}$$

where $t_s = \frac{1}{4}\, T^{\alpha}{}_{\alpha}$. Separating $T^{\alpha\beta}$ into the sum of two terms as per (43), we have

$$T^{\alpha\beta} = {}_1T^{\alpha\beta} + {}_2T^{\alpha\beta}\,, \tag{47}$$

and using (46), we obtain

$$T^{\alpha\beta} = {}_1t^{\alpha\beta} + {}_1t_s\, g^{\alpha\beta} + {}_2t^{\alpha\beta} + {}_2t_s\, g^{\alpha\beta}\,. \tag{48}$$

Hence we can write

$$\begin{aligned} t^{\alpha\beta} &= {}_1t^{\alpha\beta} + {}_2t^{\alpha\beta}\,, \\ t_s &= {}_1t_s + {}_2t_s\,. \end{aligned} \tag{49}$$

Going back to (42) and using (12) and (13), we have

$$W = \int_V \left(\mathcal{E}_\parallel + \mathcal{E}_\perp\right)\, \mathrm{d}V\,, \tag{50}$$

where

$$\mathcal{E}_\parallel = \frac{1}{32\bar{\kappa}_0}\left(\rho c^2\right)^2 \equiv \frac{1}{2\bar{\kappa}_0}\, t_s^2\,, \tag{51}$$

where ρ is the mass energy density of the edge dislocation, and

$$\mathcal{E}_\perp = \frac{1}{4\bar{\mu}_0}\, t^{\alpha\beta} t_{\alpha\beta}\,. \tag{52}$$

Substituting (51) and (52) into (50), we obtain

$$W = \int_V \left(\frac{1}{2\bar{\kappa}_0}\, t_s^2 + \frac{1}{4\bar{\mu}_0}\, t^{\alpha\beta} t_{\alpha\beta}\right) \mathrm{d}V \tag{53}$$

and using (49), (53) becomes

$$\begin{aligned} W = \int_V \Big[& \frac{1}{2\bar{\kappa}_0}\left({}_1t_s + {}_2t_s\right)^2 + \\ & + \frac{1}{4\bar{\mu}_0}\left({}_1t^{\alpha\beta} + {}_2t^{\alpha\beta}\right)\left({}_1t_{\alpha\beta} + {}_2t_{\alpha\beta}\right)\Big] \mathrm{d}V\,. \end{aligned} \tag{54}$$

Expanding and re-arranging, we obtain

$$\begin{aligned} W = \int_V \Big[& \frac{1}{2\bar{\kappa}_0}\, {}_1t_s^2 + \frac{1}{4\bar{\mu}_0}\, {}_1t^{\alpha\beta}\, {}_1t_{\alpha\beta} + \\ & + \frac{1}{2\bar{\kappa}_0}\, {}_2t_s^2 + \frac{1}{4\bar{\mu}_0}\, {}_2t^{\alpha\beta}\, {}_2t_{\alpha\beta} + \\ & + \frac{1}{\bar{\kappa}_0}\, {}_1t_s\, {}_2t_s + \frac{1}{4\bar{\mu}_0}\left({}_1t^{\alpha\beta}\, {}_2t_{\alpha\beta} + {}_2t^{\alpha\beta}\, {}_1t_{\alpha\beta}\right)\Big] \mathrm{d}V\,, \end{aligned} \tag{55}$$

which can be written as

$$\begin{aligned} W = & \int_V \left[\mathcal{E}_\parallel + \mathcal{E}_\perp\right]_1 \mathrm{d}V + \int_V \left[\mathcal{E}_\parallel + \mathcal{E}_\perp\right]_2 \mathrm{d}V + \\ & + \int_V \left[\mathcal{E}_\parallel + \mathcal{E}_\perp\right]_{12} \mathrm{d}V\,, \end{aligned} \tag{56}$$

which is equivalent to

$$W = W_1 + W_2 + W_{12}\,. \tag{57}$$

Hence the overlap interaction strain energy is given by the third term from (55):

$$W_{12} = \int_V \left[\frac{1}{\bar{\kappa}_0}\, {}_1t_s\, {}_2t_s + \frac{1}{4\bar{\mu}_0}\left({}_1t^{\alpha\beta}\, {}_2t_{\alpha\beta} + {}_2t^{\alpha\beta}\, {}_1t_{\alpha\beta}\right)\right] \mathrm{d}V\,. \tag{58}$$

Substituting from (46) into (58), we obtain

$$\begin{aligned} W_{12} = & \frac{1}{16\bar{\kappa}_0}\int_V {}_1T^{\alpha}{}_{\alpha}\, {}_2T^{\alpha}{}_{\alpha}\, \mathrm{d}V + \\ & + \frac{1}{4\bar{\mu}_0}\int_V \Big[\left({}_1T^{\alpha\beta} - \frac{1}{4}\, {}_1T^{\alpha}{}_{\alpha}\, g^{\alpha\beta}\right)\left({}_2T_{\alpha\beta} - \frac{1}{4}\, {}_2T^{\alpha}{}_{\alpha}\, g_{\alpha\beta}\right) + \\ & + \left({}_2T^{\alpha\beta} - \frac{1}{4}\, {}_2T^{\alpha}{}_{\alpha}\, g^{\alpha\beta}\right)\left({}_1T_{\alpha\beta} - \frac{1}{4}\, {}_1T^{\alpha}{}_{\alpha}\, g_{\alpha\beta}\right)\Big] \mathrm{d}V\,. \end{aligned} \tag{59}$$

Expanding the second integral, we obtain

$$\begin{aligned} W_{12} = & \frac{1}{16\bar{\kappa}_0}\int_V {}_1T^{\alpha}{}_{\alpha}\, {}_2T^{\alpha}{}_{\alpha}\, \mathrm{d}V + \\ & + \frac{1}{4\bar{\mu}_0}\int_V \Big[\, {}_1T^{\alpha\beta}\, {}_2T_{\alpha\beta} - \frac{1}{4}\, {}_1T^{\alpha\beta}\, {}_2T^{\alpha}{}_{\alpha}\, g_{\alpha\beta} - \\ & - \frac{1}{4}\, {}_1T^{\alpha}{}_{\alpha}\, g^{\alpha\beta}\, {}_2T_{\alpha\beta} + \frac{1}{16}\, {}_1T^{\alpha}{}_{\alpha}\, {}_2T^{\alpha}{}_{\alpha}\, g^{\alpha\beta} g_{\alpha\beta} + \\ & + {}_2T^{\alpha\beta}\, {}_1T_{\alpha\beta} - \frac{1}{4}\, {}_2T^{\alpha\beta}\, {}_1T^{\alpha}{}_{\alpha}\, g_{\alpha\beta} - \\ & - \frac{1}{4}\, {}_2T^{\alpha}{}_{\alpha}\, g^{\alpha\beta}\, {}_1T_{\alpha\beta} + \frac{1}{16}\, {}_2T^{\alpha}{}_{\alpha}\, {}_1T^{\alpha}{}_{\alpha}\, g^{\alpha\beta} g_{\alpha\beta}\Big] \mathrm{d}V\,. \end{aligned} \tag{60}$$

Using $g^{\alpha\beta} g_{\alpha\beta} = \eta^{\alpha\beta}\eta_{\alpha\beta} = 4$, expanding and simplifying the second integral, we obtain

$$\begin{aligned} W_{12} = & \frac{1}{16\bar{\kappa}_0}\int_V {}_1T^{\alpha}{}_{\alpha}\, {}_2T^{\alpha}{}_{\alpha}\, \mathrm{d}V + \\ & + \frac{1}{4\bar{\mu}_0}\int_V \Big[\, {}_1T^{\alpha\beta}\, {}_2T_{\alpha\beta} + {}_2T^{\alpha\beta}\, {}_1T_{\alpha\beta} - \\ & - \frac{1}{2}\, {}_1T^{\alpha}{}_{\alpha}\, {}_2T^{\alpha}{}_{\alpha}\Big] \mathrm{d}V\,, \end{aligned} \tag{61}$$

where ${}_1T^{\alpha}{}_{\alpha} = \rho_1 c^2$ and ${}_2T^{\alpha}{}_{\alpha} = \rho_2 c^2$.

Hence the overlap interaction strain energy can be written as

$$W_{12} = W_{12}^{mass} + W_{12}^{field}\,, \tag{62}$$

where W_{12}^{mass} is a pure mass longitudinal term given by

$$W_{12}^{mass} = \frac{1}{16\bar{\kappa}_0}\int_V {}_1T^{\alpha}{}_{\alpha}\, {}_2T^{\alpha}{}_{\alpha}\, \mathrm{d}V \tag{63}$$

and W_{12}^{field} is a pure massless transverse field term given by

$$W_{12}^{field} = \frac{1}{4\bar{\mu}_0} \int_V \Big[\, {}_1T^{\alpha\beta} \, {}_2T_{\alpha\beta} + \, {}_2T^{\alpha\beta} \, {}_1T_{\alpha\beta} - \\ - \frac{1}{2} \, {}_1T^{\alpha}{}_{\alpha} \, {}_2T^{\alpha}{}_{\alpha} \Big] \mathrm{d}V . \tag{64}$$

Alternatively, (61) can be written as

$$W_{12} = \frac{1}{4\bar{\mu}_0} \int_V \Big[\, {}_1T^{\alpha\beta} \, {}_2T_{\alpha\beta} + \, {}_2T^{\alpha\beta} \, {}_1T_{\alpha\beta} - \\ - \frac{\bar{\lambda}_0}{\bar{\mu}_0 + 2\bar{\lambda}_0} \, {}_1T^{\alpha}{}_{\alpha} \, {}_2T^{\alpha}{}_{\alpha} \Big] \mathrm{d}V . \tag{65}$$

It is important to note that the overlap interaction strain energy is usually expressed as energy per unit length of the defect W_{12}/ℓ.

5 Discussion and conclusion

In this paper, we have considered the nature of defect interactions in the Spacetime Continuum and how this process provides an explanation of quantum overlap interactions within the Elastodynamics of the Spacetime Continuum (*STCED*). The interaction of dislocations and disclinations is mediated through the overlap interaction of their strain energy densities.

We derived the interaction terms of dislocations and disclinations arising from the defect displacements derived by deWit for the dislocation and disclination displacements. We provided a sample calculation of dislocation strain energy densities from the dislocation displacements to calculate strain energy densities for defect interactions. We also considered another approach based on calculating the force between the defects from the stress tensor, and obtaining the strain energy of the overlap based on the work performed by the application of the force over a given distance. The results obtained are found to provide a physical explanation of quantum mechanical phenomena in terms of the interaction resulting from the overlap of defect strain energies in the spacetime continuum in *STCED*.

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