

Infinitesimally Punctured Physics and Extended Nonstandard Analysis: A Unified Neutrosophic Geometric Framework

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This work develops a comprehensive connection between Infinitesimally Punctured Physics and Extended Nonstandard Analysis, introduced in recent publications. Through the introduction of MoBiNad manifolds, pierced binads, and neutrosophic triplets (T, I, F), we establish a mathematical bridge linking nonstandard infinitesimals to physically interpretable punctured structures in wave-particle duality, field theory, and geometry. The resulting framework extends the Gauss-Bonnet and Chern-Weil formalisms into neutrosophic topology, generating hyperreal-valued characteristic classes and infinitesimally varying curvature that correspond to Smarandache Geometries — spaces where axioms behave differently from region to region. Applications include nonstandard quantum field dynamics, MoBiNad Chern-Simons actions, and the unification of logical and geometric indeterminacies in physical models.

Notations

Binad — A directed infinitesimal neighborhood consisting of asymmetric left-right monads. In the MoBiNad framework, binads encode infinitesimal geometric asymmetry and generate curvature sign fluctuations.

Characteristic class (neutrosophic) — A hyperreal extension of classical Euler, Chern, or Pontryagin classes whose value decomposes into truth, indeterminacy, and falsity components under MoBiNad curvature forms.

Christoffel symbol (MoBiNad) — The connection coefficient derived from the punctured metric $g_M = g + \mu(0)$, containing infinitesimal corrections that vanish under standard-part projection.

Extended Nonstandard Analysis (ENSA) — A refinement of classical nonstandard analysis incorporating monads, binads, and pierced neighborhoods to model structured infinitesimal geometry.

Infinitesimally Punctured Physics (IPP) — The programme treating spacetime and fields as smooth continua containing structured infinitesimal voids whose collective behavior reproduces classical laws under projection.

MoBiNad derivative — A symmetric infinitesimal difference operator D_M defined on monadic neighborhoods whose standard part equals the classical derivative.

MoBiNad manifold — A smooth manifold M_M endowed with a punctured metric and monadic neighborhood structure allowing infinitesimal geometric heterogeneity.

MoBiNad curvature tensor — The Riemann tensor computed from the MoBiNad connection, decomposed as $R_M = R + \mu_R(0)$.

Monad — The infinitesimal neighborhood $\mu(x)$ of a hyperreal point x , containing all points differing by an infinitesimal quantity.

Neutrosophic Euler characteristic — The triplet (χ_T, χ_I, χ_F) obtained from integrating MoBiNad Gaussian curvature, extending the classical Euler invariant.

Puncture density — The local hyperreal measure ρ_P of infinitesimal void concentration within a MoBiNad manifold, governing curvature oscillations.

Pierced monad — A monadic neighborhood excluding its central point, modeling an infinitesimal void embedded in a continuum.

Smarandache geometry — A geometric structure in which classical axioms hold in some regions and fail or become indeterminate in others; in this framework, it arises as the standard part projection of

MoBiNad curvature sign alternation.

Standard-part projection (st) — The map from finite hyperreal quantities to their real limit, ensuring classical geometry is recovered from MoBiNad structures.

1 MoBiNad manifold: a differential geometry framework

The term MoBiNad is a portmanteau first introduced in 2019 by the author [1, 2], combining the words *Monad* and *Binad*. It designates the unified nonstandard set that collects all standard reals, all nonstandard reals (infinitesimals and infinities), and all monads and binads of every standard real number. See Notations above and also [1, 2] for detail.

1.1 MoBiNad manifold definition

Let the MoBiNad manifold be

$$M_m = (M, g_m),$$

where g_m is the punctured metric tensor

$$g_M(x) = g^\circ(x) \cdot \{g^\circ(+, -), g^\circ(0)\}.$$

To capture infinitesimal asymmetries (monadic left-right deviations) [1–4], we get

$$\delta g^\circ = g^{\circ+} - g^{\circ-}, \quad g^{\circ \text{st}} = g^\circ(x),$$

where the standard part projection is

$$\text{st}(g_m) = g^\circ,$$

ensuring classical differential geometry re-emerges macroscopically.

1.2 MoBiNad Christoffel symbols

Generalized connection coefficients are

$$\Gamma_{\mu\nu\rho}^M = \frac{1}{2} g^{M\mu\rho} \left(D^{M\nu} g^{M\sigma\rho} + D^{M\rho} g^{M\sigma\nu} - D^{M\sigma} g^{M\nu\rho} \right),$$

where the MoBiNad derivative $D^M f(x) = \text{st}(\Delta^\circ(f))$ incorporates monadic asymmetry. Expanding to the first order, we have

$$\Gamma_{\mu\nu\rho}^M = \Gamma_{\mu\nu\rho}^\circ + \delta\Gamma_{\mu\nu\rho} \cdot \varepsilon(0),$$

$$\Gamma_{\mu\nu\rho}^\circ = \frac{1}{2} g^{\mu\sigma} (\partial_\nu g_{\sigma\rho} + \partial_\rho g_{\sigma\nu} - \partial_\sigma g_{\nu\rho}).$$

Here $\Gamma_{\mu\nu\rho}^M$ are infinitesimal corrections describing geometric punctures.

1.3 MoBiNad Riemann and Ricci tensors

The extended curvature operator is

$$R_{\nu\rho\sigma}^{M\mu} = D^{M\rho}\Gamma_{\nu\sigma}^{M\mu} - D^{M\sigma}\Gamma_{\nu\rho}^{M\mu} + \Gamma_{\rho\lambda}^{M\mu}\Gamma_{\nu\sigma}^{M\lambda} - \Gamma_{\sigma\lambda}^{M\mu}\Gamma_{\nu\rho}^{M\lambda}.$$

Expansion and contraction give

$$R_{\mu\nu}^M = R_{\mu\nu} + \delta R_{\mu\nu} \cdot \varepsilon(0),$$

$$R^M = g^{M\mu\nu} R_{\mu\nu}^M = R + \delta_R \cdot \varepsilon(0).$$

Here the infinitesimal parts correspond to microscopic curvature jitter — the geometric foundation of Smarandache curvature variations [5].

1.4 MoBiNad connection forms and structure equations

In moving-frame (vielbein) language

$$\omega_b^a = \Gamma_{\mu b}^a dx^\mu + \Gamma_{\varepsilon b}^a \cdot \varepsilon(0).$$

Structure equations (Cartan I-II type) have the form

$$T_M^a = de^a + \omega_b^a \wedge e^b = \delta_T^a \cdot \varepsilon(0),$$

$$R_{bM}^a = d\omega_b^a + \omega_c^a \wedge \omega_b^c = R_b^a + \delta_R^a \cdot \varepsilon(0).$$

Regions, where $\delta_R^a(0)$ flips sign between neighboring monads correspond to Smarandache zones [5].

1.5 MoBiNad Gauss-Codazzi equations for surfaces

For an embedded MoBiNad surface $S^M \subset M^M$ with coordinates (u, v) , we have

$$K^M = \frac{e^M g^M - f^{2M}}{E^M G^M - F^{2M}},$$

$$H^M = \frac{e^M G^M - 2f^M F^M + g^M E^M}{2(E^M G^M - F^{2M})}.$$

Curvatures K^M and H^M contain infinitesimal oscillations $\delta_{K(0)}$, $\delta_{H(0)}$. Sign variations of K^M define Smarandache surface regions — elliptic ($K^M > 0$), hyperbolic ($K^M < 0$), or flat/indeterminate [5].

1.6 Neutrosophic interpretation of operators

At each differential operator level, we assign a neutrosophic logic triple (T, I, F) [1–4, 6]. See Table 1 for detail.

Operator	T (truth)	I (indeterminacy)	F (false)
D_s^M	$1 - \delta + \varepsilon^-$	ε	ε^+
$\Gamma_{\nu\rho\phi}^M$	$1 - \delta\Gamma$	$\delta\Gamma$	0
$R_{\nu\rho\phi}^{MT}$	$1 - \delta_R \cdot \varepsilon(0)$	$\delta_R \cdot \varepsilon$	0
K^M, H^M	$\text{fn}(\delta_{K(0)})$	δ_K	sign flip

Table 1: Neutrosophic operator table. Note that ε^- , ε , ε^+ denote infra-, standard, and supra-infinitesimals; δ denotes an infinitesimal oscillation; $\text{fn}(\delta_{K(0)})$ denotes a function of the curvature oscillation at 0 [1–4].

1.7 Example — a punctured Gaussian surface

Let $r^M(u, v) = (u, v, \sqrt{(u^2 + v^2) \cdot (1 + \delta(u, v))})$ with $\delta(u, v) \in \varepsilon(0)$. Then

$$K^M = 1 + \delta_K \cdot \varepsilon(0),$$

$$H^M = 1 + (\delta + \delta_H \cdot \varepsilon(0)),$$

where $\varepsilon(0)$ denotes an infinitesimal neighborhood of 0, and δ , K^M , H^M are infinitesimal oscillation parameters.

The Smarandache surface r^M is a punctured cone with Gaussian perturbation; its curvatures deviate from unity by infinitesimal amounts. Where δ oscillates through zero, there the curvature sign flips — a Smarandache surface is derived from MoBiNad operators.

1.8 Standard part projection conditions

Macroscopic consistency conditions of the MoBiNad framework are:

- $\text{st}(g^M) = g^\circ$ (metric recovers classically);
- $\text{st}(D^M) = D^\circ$ (derivative operator recovers classically);
- $\text{st}(K^M) = K^\circ$ (curvature recovers classically).

Smarandache zone behavior exists only at the infinitesimal/monadic level. Classical differential geometry is fully recovered under the standard part projection [5, 7].

2 MoBiNad-Gauss-Bonnet theorem: a neutrosophic extension

2.1 Classical Gauss-Bonnet theorem

For a compact, oriented, smooth surface S without a boundary embedded in \mathbb{R}^3 , we have

$$\iint_S K \, dA = 2\pi \chi(S),$$

where K is the Gaussian curvature and $\chi(S) = V - E + F$ is the Euler characteristic, a topological invariant [7].

2.2 Extension to MoBiNad surface

On a MoBiNad surface $S^M \subset M^M$, the area element is $dA^M = \sqrt{E^M G^M - F^{2M}} \, du \, dv$, and the Gaussian curvature is $K^M =$

$K^\circ + \delta_K \cdot \varepsilon(0)$. Define the infinitesimal puncture density

$$\rho_p(u, v) = \frac{N_p}{N_0},$$

where $\rho_p \in \mathbb{R}$, and $\rho_p \in \varepsilon(0)$ for microscopic scales.

2.3 MoBiNad Gaussian curvature form

In differential-form language

$$\Omega^M = K^M dA^M = (K^\circ + \delta_K \cdot \varepsilon(0)) \sqrt{EG - F^2} du dv,$$

and curvature decomposition is $\Omega^M = \Omega^\circ + \Omega_\varepsilon + \Omega_I$, where:

- $\Omega^\circ = K^\circ dA$ (standard curvature 2-form);
- $\Omega_\varepsilon = \delta_K \cdot \varepsilon(0) dA$ (even infinitesimal distortion);
- Ω_I encodes logical indeterminacy from inconsistent curvature signs.

2.4 Integrating over a neutrosophic region

Let a region $S^M = S_T \cup S_I \cup S_F$, partitioned as:

Region	Curvature behavior	Symbol
S_T	Determinate (standard, smooth)	Contributes Ω_T
S_I	Fluctuating/indeterminate	Contributes Ω_I
S_F	Discontinuous punctures/voids	Contributes Ω_F

Integral decomposition

$$\iint_{S^M} K^M dA^M = 2\pi(\chi_T + \chi_F) + 2\pi \cdot \delta_K \cdot \varepsilon(0) + 2\pi \cdot I_\chi,$$

where $\delta_K(0)$ is the infinitesimal deviation and I_χ is the indeterminacy component (statistical or logical weight of S_I).

2.5 Neutrosophic Euler characteristic

Let up introduce

$$\chi_N(S^M) = (T, I, F),$$

with

$$T = \frac{1}{2} \iint_{S_T} K^\circ dA^M,$$

$$I = \text{amplitude} \cdot \iint_{S_I} K^\circ \cdot \varepsilon(0) dA^M,$$

$$F = \frac{1}{2} \iint_{S_F} K^M dA^M.$$

As a result, each surface now possesses a triplet topological signature:

- T-part \approx classical topology;
- I-part \approx punctured indeterminacy;
- F-part \approx missing curvature due to punctures or voids.

2.6 MoBiNad-Gauss-Bonnet theorem. Formal statement

For a compact MoBiNad surface S^M without a boundary, we have the following

$$\iint_{S^M} K^M dA^M = 2\pi\chi_T + 2\pi\delta_K \cdot \varepsilon(0) - 2\pi I_\chi.$$

Equivalently, in neutrosophic form it has the form

$$\iint_{S^M} K^M dA^M = 2\pi\chi_N(S^M) = 2\pi(T, I, F).$$

From the above, we see the following:

- the standard component χ_T corresponds to the usual Euler characteristic;
- $\delta_K(0)$ records microscopic curvature inventory from punctures;
- I_χ measures spatial variability where curvature cannot be consistently classified.

2.7 Example — a neutrosophic sphere with infinitesimal punctures

Let us now begin from a classical sphere of a radius r . Puncture its surface with infinitesimal holes of total fractional area $\alpha \in \varepsilon(0)$. Then

$$\iint_{S^M} K^M dA^M = 4\pi(1 - \alpha) + 4\pi \cdot \delta_K \cdot \alpha \cdot \varepsilon(0),$$

$$T = 2(1 - \alpha), \quad I = \delta_K \cdot \alpha, \quad F = \alpha,$$

as a result, we have:

- a perfect sphere $\rightarrow (2, 0, 0)$;
- an infinitesimally punctured sphere $\rightarrow (2 - 2\alpha, \delta_K \cdot \alpha, \alpha)$.

Thus, topology is neutrosophically diluted by the puncture distribution.

2.8 Generalization to Smarandache geometries

On a 2-manifold partitioned into mixed curvature zones, we have

$$\iint_{MSG} K^M dA^M = 2\pi(\chi_T + \delta_K \cdot \varepsilon(0) + I_\chi) = 2\pi\chi_N,$$

so we arrive at the conclusions:

- regions with $K^M > 0$ are elliptic (axioms valid);
- $K^M < 0$ are hyperbolic (axioms negated);
- $K^M \in \varepsilon^+$ are indeterminate (axioms uncertain).

Note that mixed axiomatic behavior characteristic of Smarandache geometry corresponds exactly to the neutrosophic partition of curvature forms via ENSA differential operators [5].

2.9 Physical interpretation

Terms in the theorem	Physical meaning
χ_T	Classical topological invariant (macroscopic shape)
$\delta_\chi \cdot \varepsilon(0)$	Infinitesimal energy from curvature fluctuations (quantum foam)
I_χ	Probabilistic/indeterminate vacuum effects (field fluctuations in punctured zones)

2.10 Synthesis of §2

Concept	Classical	MoBiNad/Neutrosophic extension
Curvature 2-form	$K dA$	$K^M dA^M = K dA + \delta_K \cdot \varepsilon(0) dA + \Omega_I$
Gauge interpretation	Levi-Civita connection	MoBiNad connection with infinitesimal torsion
Euler invariant	Integer	Hyperreal triplet (T, I, F)
Smarandache linkage	Uniform geometry	Mixed geometry via puncture zones
Physical reading	Static topology	Fluctuating quantum topological charge

3 Neutrosophic Chern-Weil theory on MoBiNad manifolds

3.1 Classical Chern-Weil theory

Given a principal G-bundle $P \rightarrow M$ with connection 1-form ω and curvature 2-form

$$\Omega = d\omega + \omega \wedge \omega.$$

An invariant polynomial $P(\Omega)$ on the Lie algebra \mathfrak{g} produces a closed $2k$ -form whose cohomology class $[P(\Omega)]$ is independent of the chosen connection. Characteristic examples of the above are

$$e(\Omega) = Pf(\Omega), \quad p_k(\Omega) = Tr(\Omega^{2k}), \quad c_k(\Omega) = Tr(\Omega^k).$$

Integrating these forms over M gives the corresponding characteristic numbers.

3.2 A nonstandard extension. MoBiNad curvature forms

On a MoBiNad manifold M^M , we introduce the punctured connection 1-form

$$\omega^M = \omega + \delta\omega \cdot \varepsilon(0).$$

Its curvature 2-form is

$$\Omega^M = d^M \omega^M + \omega^M \wedge \omega^M = \Omega + \delta\Omega \cdot \varepsilon(0),$$

where $\delta\Omega(0) \in \varepsilon(0)$ is an infinitesimal correction due to MoBiNad asymmetry ($\varepsilon^+ - \varepsilon^-$). Infinitesimally, d^M differs from d by antisymmetric monadic corrections

$$d^M = d + d(\varepsilon(0)).$$

3.3 Neutrosophic invariant polynomials

Any G-invariant polynomial P is extended to

$$P^M(X) = P(X) + \delta_P \cdot \varepsilon(0),$$

with $\delta_P(0) \in \varepsilon(0)$ carrying the uncertainty/indeterminacy of the polynomial evaluation on infinitesimally variable curvature. The MoBiNad Chern-Weil form is then

$$P^M(\Omega^M) = P(\Omega) + P'(\Omega) \cdot \delta\Omega \cdot \varepsilon(0) + \delta_P \cdot \varepsilon(0).$$

Each extra piece is hyperreal, adding neutrosophic terms to the integrated topological invariants.

3.4 Neutrosophic characteristic forms

Consider characteristic forms

$$e^M(\Omega^M) = e(\Omega) + \delta_e \cdot \varepsilon(0),$$

$$p_{k,M}(\Omega^M) = p_k(\Omega) + \delta_{p_k} \cdot \varepsilon(0),$$

$$c_{k,M}(\Omega^M) = c_k(\Omega) + \delta_{c_k} \cdot \varepsilon(0),$$

each with $\delta_{(\cdot)}(0) \in \varepsilon(0)$. In neutrosophic-logic form, we obtain

$$(e^M, p_k^M, c_k^M) \sim (T, I, F) = (1 - |\delta_{(\cdot)}|, |\delta_{(\cdot)}|, 0^+).$$

therefore:

- T component \rightarrow standard form (truth);
- I component \rightarrow infinitesimal uncertainty;
- F component \rightarrow vanishing falsity except at puncture voids.

3.5 Neutrosophic topological invariants

Integrating over M^M , we obtain

$$E_N = \int_{M^M} e^M(\Omega^M) = E + \delta_E \cdot \varepsilon(0) + I_E,$$

$$p^{k,N} = \int_{M^M} p_{k,M}(\Omega^M) = p_k + \delta_{p_k} \cdot \varepsilon(0) + I_{p_k},$$

$$c^{k,N} = \int_{M^M} c_{k,M}(\Omega^M) = c_k + \delta_{c_k} \cdot \varepsilon(0) + I_{c_k}.$$

The standard part reproduces classical topology; $\varepsilon(0)$ parts record infinitesimal topological fluctuations; $I_{(\cdot)}$ encodes regions of indeterminate topology inside Smarandache zones. These are the *Neutrosophic Characteristic Numbers*.

3.6 An example — the neutrosophic Chern class on a punctured complex surface

Let M be a complex 2-manifold with Hermitian connection forming curvature Ω . Introduce punctured curvature

$$\Omega^M = \Omega + \delta\Omega \cdot \varepsilon(0).$$

Then the resulting curvature, taking MoBiNad terms (marked by the index M) into account, takes the form

$$c_{1M} = \text{Tr}(\Omega^M) = c_1 + \text{Tr}(\delta\Omega) \cdot \varepsilon(0),$$

$$\int_{M c_{1M}} = \chi_T + \delta_\chi \cdot \varepsilon(0) + O(\varepsilon(0))^2,$$

where the infinitesimal puncture contribution δ_χ represents microscopic gauge flux escaping through punctures. This explains how slightly broken gauge fields (quantum anomalies) manifest as neutrosophic corrections to integral invariants [8,9].

3.7 Connection with Smarandache geometry

Smarandache feature	ENSA equivalent	Chern-Weil interpretation
Mixed curvature signs	Alternating $\delta R_{(0)}$	Alternating local Euler densities
Variable local axioms	Fluctuating Ω^M structures	Partial deformation of curvature 2-forms
Indeterminate regions	$I_{(\cdot)}$ components	Logical indeterminacy in class numbers
Global invariants multivalued	Hyperreal integrals	Combined (T, I, F) class numbers

Each Smarandache geometry is the standard projection of a MoBiNad manifold, whose characteristic class triplets vary infinitesimally from region to region [5].

3.8 Neutrosophic Chern-Weil identity (theorem)

For any invariant polynomial P^M evaluated on a punctured curvature Ω^M

$$d^M P^M(\Omega^M) = 0, \quad \text{st}(P^M(\Omega^M)) = P(\Omega).$$

Hence the cohomology class decomposes as

$$[P^M(\Omega^M)] = [P(\Omega)] + [\delta_P \cdot \varepsilon(0)] + [I_P],$$

giving a triplet cohomology class $H^N = (T_H, I_H, F_H)$.

The neutrosophic Chern classes live in the extended cohomology ring

$$H_N^{2k}(M^M) = H^{2k}(M) \oplus \varepsilon(0) \oplus \mathbf{i},$$

bridging differential topology, infinitesimal analysis, and logical indeterminacy [1–7].

3.9 Physical interpretations

Neutrosophic term	Physical meaning
$\delta_P \cdot \varepsilon(0)$	Quantum-scale topological fluctuation (vacuum instanton density variation)
I_P	Uncertainty of topological charge due to mixed geometry (Smarandache zones)
T_P	Classical quantized charge (integer Chern number)

MoBiNad characteristic numbers yield fractional hyperreal generalizations of integer invariants, modeling quantized topological effects with infinitesimal deviations — the deterministic substratum of quantum anomalies [8,9].

4 Neutrosophic Chern number on a punctured torus — explicit computation

4.1 Classical reference — torus curvature and Chern number

For an ordinary complex torus $T^2 = \mathbb{C}/\Lambda$ (flat metric, $U(1)$ bundle): field strength $F = dA = iB dx \wedge dy$, first Chern form $c_1 = \frac{B}{2\pi} dx \wedge dy$, and ordinary Chern number is

$$C_1 = \int_{T^2} c_1 = \frac{B \cdot \text{Area}_{T^2}}{2\pi} = n \in \mathbb{Z},$$

when flux quantization holds.

4.2 MoBiNad torus T^{2M}

Let each cell of T^2 carry infinitesimal punctures of density $\rho_P(x, y) \in \varepsilon(0)$. Define a MoBiNad-deformed connection

$$A^M = A + \delta A, \quad \delta A = \varepsilon(x, y) f(x, y) dx, \quad \varepsilon(x, y) \in \varepsilon(0).$$

Its field strength has the form

$$F^M = d^M A^M = F + d(\delta A) + d(\varepsilon(0)) = iB dx \wedge dy + \delta_F \cdot \varepsilon(0).$$

4.3 Neutrosophic first Chern form

$$c_{1M} = \text{Tr}(F^M) = \frac{B}{2\pi} dx \wedge dy + \delta_{c_1} \cdot \varepsilon(0), \quad \delta_{c_1}(0) \in \varepsilon(0).$$

4.4 Integration — the neutrosophic Chern number

$$C_1^N = \int_{T^{2M}} c_{1M} = \int_{T^2} c_1 + \int T^2 \delta_{c_1} \cdot \varepsilon(0) = n + \delta C_1,$$

$$\delta C_1 \in \varepsilon(0).$$

The standard part n is the integer flux number; the residual δC_1 is an infinitesimal correction representing puncture deformation.

4.5 Neutrosophic triplet topology for T^{2M}

$$(T, I, F) = (1 - |\delta C_1|, |\delta C_1|, 0^+),$$

where we have

- T — fraction of torus retaining exact quantization;
- I — indeterminate layer from puncture density;
- F — vanishing except in deleted holes;

and also

$$\chi_{(C_1)}(T^{2M}) = (n, \delta_{c_1} \cdot \varepsilon(0), 0).$$

4.6 Connection with Smarandache geometry

Where puncture density $\rho_p(x, y)$ alternates its sign, there F^M flips locally between $+iB$ and $-iB$, generating mixed curvature zones:

Zone	Local K^M sign	Geometric type	Smarandache interpretation
Z_+	$+B$	Elliptic	Axiom valid
Z_-	$-B$	Hyperbolic	Axiom negated
Z_0	Fluctuating	Indeterminate	Axiom uncertain

so that

$$C_{1N} = \int_{\{Z_+\}} + \int_{\{Z_-\}} + \int_{\{Z_0\}} = n_+ - n_- + \delta_{top} \cdot \varepsilon(0).$$

This mixed-sign sum explicitly reproduces the Smarandache geometry condition: different regions of the same manifold satisfy different geometric axioms.

4.7 Generalization to sphere bundle S^{2M}

On a sphere with curvature

$$\Omega^M = i(1 + \delta(\theta, \varphi)) \sin(\theta) d\theta \wedge d\varphi,$$

we obtain

$$C_{1N}(S^{2M}) = \int_{S^{2M}} \Omega^M = 2\pi + \delta_\chi \cdot \varepsilon(0) = 2 + \delta C_1.$$

Even a sphere (classical $C_1 = 2$) becomes a neutrosophic surface with infinitesimal puncture overshoot or deficit in topology — the simplest Smarandache surface.

4.8 Equation summary

$$F^M = F + \delta_F \cdot \varepsilon(0),$$

$$c_{1M} = \text{Tr}(F^M) = c_1 + \delta_{c_1} \cdot \varepsilon(0),$$

$$C_{1N} = 3 \int_{T^{2M}} c_{1M} = n + \delta C_1,$$

$$\chi_{(C_1)}(T^{2M}) = (n, \delta C_1, 0^+).$$

The neutrosophic Chern number gives a continuous, hyperreal refinement of discrete topology, integrating MoBiNad infinitesimals, Smarandache mixed regions, and logical indeterminacy into a single invariant [1–6].

5 Neutrosophic Chern-Simons action

5.1 Classical Chern-Simons action

For a compact gauge group G with connection A (1-form valued in \mathfrak{g})

$$S_{CS}(A) = \frac{k}{4\pi} \int_M \text{Tr} \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right),$$

over a 3-manifold M . Gauge invariance restricts $k \in \mathbb{Z}$ (level quantization).

5.2 MoBiNad extension — punctured curvature and derivative

$$A^M = A + \delta A, \quad D^M = d + d(\varepsilon(0)),$$

$$F^M = D^M A^M + A^M \wedge^M A^M = F + \delta_F \cdot \varepsilon(0).$$

The wedge \wedge^M and derivative D^M contain monadic asymmetry; any differential form product now carries hyperreal remainder $(\cdot) \in \varepsilon(0)$.

5.3 Neutrosophic Chern-Simons action

$$S_N(A^M) = \frac{k^M}{4\pi} \int_{M^M} \text{Tr} \left(A^M \wedge^M D^M A^M + \frac{2}{3} (A^M)^3 \right),$$

where M^M is the MoBiNad 3-manifold, $k^M = k + \delta_k \cdot \varepsilon(0)$ is the neutrosophic Chern-Simons level, and fields carry infinitesimal and indeterminate components.

5.4 Expansion

Substituting $A^M = A + \delta A$ and $D^M = d + \delta d$, we obtain

$$S_M = S_{CS}(A) + \int_M \left(k \cdot \text{Tr}(A \wedge F) + \delta_k \cdot \varepsilon(0) \cdot \text{Tr}(A \wedge F) + S(\varepsilon(0)) \right).$$

Under standard-part projection, we have $\text{st}(S_N) = S_{CS}$.

5.5 Neutrosophic level quantization

A gauge transformation changes the action by

$$\Delta S_N = 2\pi(k + \delta_k \cdot \varepsilon(0)) N_{\text{wind}}, \quad N_{\text{wind}} \in \mathbb{Z}.$$

Gauge invariance holds for $\text{st}(k^M) = k$; the infinitesimal residue $\delta_k \cdot \varepsilon(0)$ corresponds to a soft symmetry breaking. This leads to locally varying topological phase in punctured regions, observable as minute phase differences or fractional quantum Hall-like plateaus [8, 9].

5.6 Neutrosophic gauge field triplet

Each local patch $U_i \subset M^M$ has $S_N^i = (T_i, I_i, F_i)$, with T_i the standard CS action weight, I_i the infinitesimal deviation from punctures, and F_i null except at deleted regions. Globally,

$$S_N = (S_{CS}, \delta S \cdot \varepsilon(0), 0^+).$$

5.7 A working example — MoBiNad 3-torus T^3

Use gauge group $U(1)$ on $T^{3M} = (x, y, z)$ with infinitesimal puncture function $\rho_p(x, y, z)$. Let $A = B_x \cdot x \cdot dy + B_y \cdot y \cdot dz + B_z \cdot z \cdot dx$. Then

$$A \wedge dA = (B_x B_y + B_y B_z + B_z B_x) dx \wedge dy \wedge dz,$$

$$A^M \wedge^M dA^M = (1 + \rho_p(x, y, z)) \cdot A \wedge dA,$$

$$S_N = \iiint (1 + \rho_p) A \wedge dA = S_{CS} + \iiint \rho_p A \wedge dA.$$

Since $\rho_p \in \varepsilon(0)$, the second term is infinitesimal. Across Smarandache zones, where ρ_p changes its sign, local topological orientation flips — elliptic \longleftrightarrow hyperbolic gauge sectors.

5.8 Four-dimensional extension — Chern-Simons current and instanton density

On a 4-manifold M_M^4 , the Pontryagin density has the form

$$p_{1M} = \frac{1}{8\pi^2} \text{Tr}(F^M \wedge^M F^M) = p_1 + \delta_{p_1} \cdot \varepsilon(0),$$

$$d(A^M \wedge^M dA^M + (A^M)^3) = \text{Tr}(F^M \wedge^M F^M).$$

Infinitesimal boundary corrections give the neutrosophic instanton number $N_I = n + \delta n \cdot \varepsilon(0)$.

5.9 Physical interpretations

Component	Interpretation in MoBiNad physics
k (integer)	Classical topological level
$\delta_k \cdot \varepsilon(0)$	Infinitesimal level deformation \longrightarrow local Chern-Simons phase variation
S_N (hyperreal)	Almost-invariant action \longrightarrow fractional topological quantization
Sign changes of ρ_p	Smarandache zones (Euclidean \longleftrightarrow Hyperbolic behavior)
Observables	Tiny shifts in quantized conductance/flux \longrightarrow near-integer fractions

5.10 Mathematical summary

$$A^M = A + \delta A, \quad F^M = F + \delta_F \cdot \varepsilon(0),$$

$$S_N = \frac{k^M}{4\pi} \int_{M^M} \text{Tr} \left(A^M \wedge^M D^M A^M + \frac{2}{3} (A^M)^3 \right),$$

$$\text{st}(S_N) = S_{CS}, \quad \Delta S_N = 2\pi(k + \delta_k \cdot \varepsilon(0))N_{\text{wind}},$$

$$S_N = (T, I, F) = (S_{CS}, \delta_S \cdot \varepsilon(0), 0^+).$$

Level	Structure	Meaning	Physical effect
Differential	ENSA-MoBiNad derivatives	Encode infinitesimal punctures	Micro-curvature
Geometric	Smarandache mixed zones	Regions of differing curvature sign	Domain walls
Logical	Neutrosophic triplet	Degree of gauge invariance	Phase uncertainty
Physical	Chern-Simons phase	Near-quantized topological field effect	Fractional Hall

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